LINEAR BLOCK CODES FOR BLOCK FADING CHANNELS BASED ON HADAMARD MATRICES

A Thesis

by

SPYROS SPYROU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2005

Major Subject: Electrical Engineering

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Approved by:

Chair of Committee,	Costas N. Georghiades
Committee Members,	Don R. Halverson
	Jim Ji
	Alexander Parlos
Head of Department,	Costas N. Georghiades

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ABSTRACT

Linear Block Codes for Block Fading Channels

Based on Hadamard Matrices. (December 2005)

Spyros Spyrou, B.Eng., National Technical University of Athens

Chair of Advisory Committee: Dr. Costas N. Georghiades

We investigate the creation of linear block codes using Hadamard matrices for block fading channels. The aforementioned codes are very easy to find and have bounded cross correlation spectrum. The optimality is with respect to the *metric-spectrum* which gives a performance for the codes very close to optimal codes. Also, we can transform these codes according to different characteristics of the channel and can use selective transmission methods.

To my parents

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I would like to thank Dr. Costas Georghiades for giving me the opportunity to work under him. I would like to thank him for his friendly and encouraging attitude and the support he provided. He has been a good teacher and a good advisor. I would also like to express my gratitude to my committee members, Dr. Don Halverson, Dr. Jim Ji and Dr. Alexander Parlos, for their time and support. Thanks are due to my friends and all of my current and former colleagues. Lastly, I would like to thank my parents. Without their support, I would have never come this far.

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CHAPTER I

INTRODUCTION

The fading multipath channel has been the object of research for many years since it serves as a model for signal transmission over many radio channels. Much research has been done in the direction of exploiting the special characteristics of fading channels. The main idea is to achieve diversity either by using multiple transmit and receive antennas or by finding codes that can achieve this. Some past research on the first idea has been done by [1, 2, 3] and it involves the capacity of such systems, the degrees of freedom and diversity and adaptive multiantenna transceiver for narrowband reception.

This thesis deals with the second idea which is channel coding. The work of other researchers in this area involves the efficient implementation of a maximum-likelihood detector for space-time block coded systems (Quasi-static channel) in [4], some codes for maximizing diversity by [5, 6] and some upper bounds on the probability of error when we have capacity achieving signaling in [7].

In this work, which is an extension of the work done in [8, 9], we develop some codes for the noncoherent block fading channel based on Hadamard matrices. The aforementioned codes are very easy to find and have bounded cross correlation spectrum. That gives a performance for the codes very close to optimal codes. The optimality is with respect to the *metric-spectrum* [8]. The metric spectrum is defined as the set of all the values of the metric characterizing the pairwise error probabilities.

In more details Chapter II gives all the background for this work, such as fading channels, block codes, union bound and optimal metric spectrum. In Chapter III

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we give the pairwise error probabilities for the block fading channel using different number of blocks and the theoretical performance of such codes. In Chapter IV we present codes for block fading channels created from Hadamard matrices with the special characteristic that the maximum cross correlation is 1/2 and follow the pairwise error probabilities of Chapter III. In Chapter V we introduce the selective transmission scheme where the codes are used in a more efficient way. In Chapter VI we present a wider area of codes with bounded maximum cross correlation and finally in the last chapter we summarize our results.

CHAPTER II

BACKGROUND

In this chapter we will briefly cover some definitions and background knowledge that is useful for the understanding of the following material.

A. Fading Channels

A block fading channel is a time-varying channel with a fade level that is assumed to be constant during a block of N bits and to vary between blocks according to a given probability distribution. The channel model for the block fading channel is

$$\begin{aligned} r_1 &= \alpha_1 x_1 + \nu_1 \\ r_2 &= \alpha_2 x_2 + \nu_2 \\ &\vdots \\ r_t &= \alpha_t x_t + \nu_t \end{aligned} \tag{2.1}$$

where t is the number of the blocks and $x = \sqrt{E_s}d$, $d = [d_1, d_2, \ldots, d_t]^T$. The fading variables α_i are modelled as zero-mean, circularly symmetric, complex Gaussian random variables of variance σ_α^2 and are independent of each other. The vectors v_i are i.i.d. zero mean, circularly symmetric complex Gaussian random variables with variance $\sigma^2 = N_0$. E_s is the energy per symbol. The modulation symbols d_j take values from the binary set {-1,1}. If $t = 1$ then we have the *quasi-static* channel where the fading variable is constant for all the codeword. The fading variable α is not known to the receiver (noncoherent detection).

B. Narrowband-Wideband Channels and Sum of Sinusoids Simulator by Jake

The distinction between narrowband and wideband channels is due to different characteristics of the receiving signal. In the case of the narrowband channel we are interested in simulating the random process with Rayleigh probability density function and the Doppler spectrum. In the case of the wideband channel we want to simulate the multipath effects on the signal.

1. Narrowband Channels

A way to simulate the Doppler spectrum is to use a sum of oscillators with different frequencies in a way to get the wanted results. This was done by Jake in [10] using the famous sum of sinusoids simulator (SOS). We will use this simulator to simulate the performance of codes in flat Rayleigh channels and in a frequency selective channel. A flat fading channel is when all frequency components of a received signal vary in the same proportion simultaneously and frequency selective is when that does't happen. Although Jake's model is for the flat fading channel it can be extended for the frequency selective channel. The Jake simulator models the received lowpass complex envelop of a flat fading channel under the assumption that there is no line of sight. A suitable model for this channel is a complex Gaussian random process with zero mean and uncorrelated real and imaginary parts. The Jake model gives a good approximation of the analytical model using a number of low frequency oscillators. The main characteristic of the procedure followed by Jake is to model a Gaussian random process by using a sum of low frequency oscillators. This principle is a result of the sum of sinusoids by Rice [11, 12, 13]. Before presenting the Jake simulator we have to note that much research has been done on the aforementioned simulator and some of the studies [14, 15, 16] presented weaknesses of the model concerning the assumptions that Jake used. What we present here is from the original work from Jake [10]. Jake started by writing the received signal as:

$$E(t) = Re[T(t)e^{i\omega_c t}] \tag{2.2}$$

where

$$T(t) = \frac{E_0}{\sqrt{N}}\left\{\sqrt{2}\sum_{n=1}^{N_0}[e^{j(\omega_m t\cos a_n+\phi_n)} + e^{-j(\omega_m t\cos a_n+\phi_{-n})}] + e^{j(\omega_m t+\phi_N)} + e^{-j(\omega_m t+\phi_{-N})}\right\} \tag{2.3}$$

and

$$N_0 = \frac{1}{2}\left(\frac{N}{2} - 1\right) \tag{2.4}$$

The term $\sqrt{2}$ is a normalizing factor. If N is large enough we can use the central limit theorem so that $T(t)$ is a good approximation of a Gaussian random process and the envelop $|T|$ is Rayleigh distributed. This approximation is good for $N \geq 6$. Then we get more information as far as N based on the autocorrelation of $E(t)$:

$$R(\tau) =< E(t), E(t+\tau) >= \frac{1}{2}Re\left[\left\langle T(t)T(t+\tau)e^{i\omega_c(2t+\tau)}\right\rangle + \left\langle T^*(t)T(t+\tau)e^{i\omega_c\tau}\right\rangle\right] \tag{2.5}$$

and finally we have:

$$R(\tau) = \frac{b_0}{N}\cos\omega_c\tau\left[4\sum_{n=1}^{N_0}\cos\left(\omega_m\tau\cos\frac{2\pi}{N}\right) + 2\cos(\omega_m\tau)\right] \tag{2.6}$$

Equation 2.6 is of the form $R(\tau) = g(\tau)\cos\omega_c\tau$ where $g(\tau)$ is a low frequency term multiplied by a bandpass term. For the model that we use $g(\tau) = b_0 J_0(\omega_m\tau)$ applies and we get:

$$4\sum_{n=1}^{N_0}\cos\left(\omega_m\tau\cos\frac{2\pi}{N}\right) + 2\cos(\omega_m\tau) = \frac{N}{2}J_0(\omega_m\tau) \tag{2.7}$$

So N_0 low frequency oscillators that have frequencies equal to the Doppler shift $\omega_m\cos(2\pi/N),\ \ n = 1, 2, \ldots, N_0$ plus one more oscillator with frequency ω_m are used to create signals with frequency deviation from the carrier frequency ω_c signal. The amplitudes of all these signals are equal to one except from the signal with frequency ω_m which has amplitude $1/\sqrt{2}$.

The phases β_n are chosen in a way that the probability density function is as close as possible to a uniform distribution. The quadrature components are:

$$x_c(t) = 2\sum_{n=1}^{N_0}\left(\cos\beta_n \;\cos\omega_n t + \sqrt{2}\cos\alpha \;\cos\omega_m t\right) \tag{2.8}$$

$$x_s(t) = 2\sum_{n=1}^{N_0}\left(\sin\beta_n \;\cos\omega_n t + \sqrt{2}\sin\alpha \;\cos\omega_m t\right) \tag{2.9}$$

The phase of the final signal $\widetilde{R}(t)$ must be random and uniformly distributed from 0 to 2π. This can be done in many different ways given that $< x_c^2 > \approx < x_s^2 >$ and $< x_c \; x_s > \approx 0$. Then

$$< x_c^2 >= N_0 + \cos^2\alpha + \sum_{n=1}^{N_0}\cos^2\beta_n \tag{2.10}$$

$$< x_s^2 >= \sin^2\alpha + 2\sum_{n=1}^{N_0}\sin^2\beta_n \tag{2.11}$$

$$< x_c x_s >= 2\sum_{n=1}^{N_0}\sin\beta_n \;\cos\beta_n + \sin\alpha\cos\alpha \tag{2.12}$$

By choosing $\alpha = 0$ and $\beta_n = \pi n/N_0$ we have that $< x_c x_s >= 0$ and $< x_c^2 >= N_0 + 1, < x_s^2 >= N_0$. So $\widetilde{R}(t)$ is a narrow band signal over the carrier frequency ω_c with Rayleigh fading characteristics and autocorrelation function approximately equal with the Bessel function.

2. Wideband Channels

For wideband channels the model of the statistical characteristics of the channel takes place in the time domain by simulating the impulse response of the multipath channel based on statistical characteristics. The method that we will describe is based on a mathematical model that describes the characteristics of the channel in the time domain. This model is the tapped delay line with discrete paths with the same or different delay. Every tap is a result of a number of multipath components (all the components arrive in a short period and cannot be distinguished) so we have fading. If $\widetilde{s}(t)$ is the complex envelop of the transmitted signal then the complex envelop of the received signal is:

$$\widetilde{r}(t) = \sum_{i=1}^{l} g_i(t)\widetilde{s}(t - \tau_i) \tag{2.13}$$

where l is the number of the paths and $g_i(t), \tau_i$ are the complex gains and delays of each path correspondingly. The impulse response of the channel is

$$g(t, \tau) = \sum_{i=1}^{l} g_i(t)\delta(t - \tau_i) \tag{2.14}$$

and it can be fully described by the tap gain vector

$$g(t) = (g_1(t), g_2(t), \ldots, g_l(t)) \tag{2.15}$$

and the tap delay vector

$$\tau = (\tau_1, \tau_2, \ldots, \tau_l) \tag{2.16}$$

Values for these vectors are formalized for different models like GSM or JTC. Different models and the values for the above vectors can be found in Chapter 6 of [17]. To summarize, the simulation of wideband channels is based on the extension of the SOS simulator for narrowband channels by adding more delayed fading signals.

C. Linear Block Codes

The binary (n, k) block code with *cardinality* $M = 2^k$ and block code length n is described by a matrix of the form

$$\mathcal{C}_{M,n} = \begin{pmatrix} c_{0,0} & c_{0,1} & \cdots & c_{0,n-1} \\ c_{1,0} & c_{1,1} & \cdots & c_{1,n-1} \\ \vdots & \ddots & \ddots & \vdots \\ c_{M-1,0} & c_{M-1,1} & \cdots & c_{M-1,n-1} \end{pmatrix}. \tag{2.17}$$

The block code is linear if it can be uniquely represented by a generator matrix G where

$$\mathcal{C}_{M,n} = mG \tag{2.18}$$

and m is the $M \times k$ matrix containing all possible M combinations of k bits. For our work we use codes of the form $\mathcal{C}_{M,M} = \mathcal{H}_M$ where $\mathcal{H}_M$ is the Hadamard matrix created by the Sylvester construction. The Sylvester construction works as follows: If there exist Hadamard matrices H_M and $H_k = [h_{ij}]$ of orders M and k, respectively, then the matrix obtained by replacing each $h_{ij} = \pm 1$ with $\pm H_M$ is a Hadamard matrix of order $M \times k$. By this construction a Hadamard matrix,H_M, of order M is a $M \times M$ matrix with elements 1's and -1's such that $H_M H_M^T = M I_M$. This implies that any two distinct rows of H_M are orthogonal and as a result these codes are optimal, in the sense of the optimal cross correlation spectrum, for the quasi-static channel. But their use in the block fading channel is not optimal and a reshuffling of the columns is to be made. Since $\mathcal{H}_M^T = \mathcal{H}_M$ a reshuffling of the columns means a reshuffling of the rows and as a consequence the code is still optimal for the quasi-static channel.

D. Union Bound

The probability of codeword error when we send one of the M equally likely transmitted codewords is bounded using the union bound [8] as seen in Equation 2.19.

$$P(e) \leq \frac{2}{M} \sum_{\mu \in \mathcal{M}} N_\mu P_2(\mu) \tag{2.19}$$

where $\mathcal{M}$ is the set of all pairwise values of μ and N_μ is the multiplicity of every μ.

E. Optimal Metric Spectrum

The metric that we use in our work is the absolute cross correlation metric. For a block code with codewords $d_1, d_2, \dots d_M$ the metric is $\mu = |\rho| = \frac{1}{n}|d_i^T d_j|$ where $i \neq j$. So the ρ-spectrum S_ρ of an (n,k) binary block code with an increasing pairwise probability of error with respect to ρ is the set of all pairs of absolute cross correlations and their multiplicities, i.e.,

$$S_\rho = \{(0, N_0), (1/n, N_{1/n}), \dots, (\rho_{max}, N_{\rho_{max}})\} \tag{2.20}$$

The optimal ρ-spectrum S_ρ^* over all the possible spectra S_ρ as defined in [8] is the one for which one of the following is true:

- $\rho^*_{max} < \rho_{max}$, or
- $\rho^*_{max} = \rho_{max}$, and there exists some λ:$\lambda/n = 0, 1/n, 2/n, \dots, \rho_{max}$, for which $N^*_{\lambda/n} < N_{\lambda/n}, N^*_{(\lambda+1)/n} = N_{(\lambda+1)/n}, \dots, N^*_{\rho_{max}} = N_{\rho_{max}}$.

CHAPTER III

PAIRWISE ERROR PROBABILITIES FOR THE NONCOHERENT BLOCK FADING CHANNEL

In this chapter we present the pairwise error probabilities for the two, three and four block fading channels. Since we use the Hadamard codes we cannot find optimal codes for the three block fading channel but we give the probability for completeness. First we present the channel model and then the pairwise error probabilities.

Here we give only the two block fading model since the others can easily be derived by it. The discrete time vector model for the two block fading channel is:

$$\begin{aligned} \boldsymbol{r_1} &= \alpha_1 \boldsymbol{x_1} + \boldsymbol{\nu_1} \\ \boldsymbol{r_2} &= \alpha_2 \boldsymbol{x_2} + \boldsymbol{\nu_2} \end{aligned} \tag{3.1}$$

where $\boldsymbol{x} = \sqrt{E_s}\boldsymbol{d}$, $\boldsymbol{d} = [\boldsymbol{d_1}, \boldsymbol{d_2}]^T = [d_{10}, d_{11}, \ldots, d_{1(n/2-1)}, d_{2n/2}, \ldots, d_{2(n-1)}]^T$. The fading variables α_i are modelled as zero-mean, circularly symmetric, complex Gaussian random variables of variance σ_α^2 and are independent of each other. The vectors $\boldsymbol{\nu_i}$ are i.i.d. zero mean, circularly symmetric complex Gaussian random variables with variance $\sigma^2 = N_0$. E_s is the energy per symbol. The modulation symbols d_j take values from the binary set {-1,1}. The noncoherent maximum likelihood detector can be shown to be (see Appendix E)

$$\hat{\boldsymbol{d}} = \arg \max_{\boldsymbol{d}} |\boldsymbol{r_1}^H \boldsymbol{d_1}|^2 + |\boldsymbol{r_2}^H \boldsymbol{d_2}|^2 \tag{3.2}$$

Just note that $\boldsymbol{d_1}$ and $\boldsymbol{d_2}$ belong to the same codeword. The pairwise error probability can be shown to be (see Appendix A)

$$P_2^w \equiv \frac{1}{2} - \frac{3}{4}\sqrt{\frac{\Lambda^2(1-\rho^2)}{\Lambda^2(1-\rho^2)+8\Lambda+16}} + \frac{1}{4}\left(\sqrt{\frac{\Lambda^2(1-\rho^2)}{\Lambda^2(1-\rho^2)+8\Lambda+16}}\right)^3 \tag{3.3}$$

where $\Lambda = \frac{\sigma_a^2}{\sigma^2} n E_s$ and ρ is the normalized cross-correlation of the first or the second part of the codewords $\boldsymbol{d_a}$ and $\boldsymbol{d_b}$, defined as

$$\rho = \frac{2}{n} \boldsymbol{d_{a1}}^T \boldsymbol{d_{b1}} = \frac{2}{n} \boldsymbol{d_{a2}}^T \boldsymbol{d_{b2}} \tag{3.4}$$

Although we made the assumption that $\frac{2}{n} \boldsymbol{d_{a1}}^T \boldsymbol{d_{b1}} = \frac{2}{n} \boldsymbol{d_{a2}}^T \boldsymbol{d_{b2}}$ for all the spectrum of the code in order to find the pairwise probability of error and therefore optimal codes, after simulations with optimal codes that don't follow that restriction the performance was the same as the aforementioned codes. The pairwise error probability for the three block channel is in (3.5) and for the four block fading channel in (3.6).

$$P_2^w \equiv \frac{1}{2} - \frac{15}{16} A + \frac{5}{8} (A)^3 - \frac{3}{16} (A)^5 \tag{3.5}$$

where $A = \sqrt{\frac{\Lambda^2(1-\rho^2)}{\Lambda^2(1-\rho^2)+12\Lambda+36}}$.

$$P_2^w \equiv \frac{1}{2} - \frac{35}{32} A + \frac{35}{32} (A)^3 - \frac{21}{32} (A)^5 + \frac{5}{32} (A)^7 \tag{3.6}$$

where $A = \sqrt{\frac{\Lambda^2(1-\rho^2)}{\Lambda^2(1-\rho^2)+16\Lambda+64}}$. Notice that $0 \leq \rho^2 \leq 1$ and from the above probabilities of error we see that they are minimized when $\rho^2 = 0$. Since ρ is the cross correlation for every block, the optimal block codes are the ones that have optimal spectrum for every block. In the next chapter we present codes with $\rho_{max} = 1/2$.

CHAPTER IV

NEW CODES FOR THE BLOCK FADING CHANNEL

This chapter deals with codes with spectrum of the form $S_\rho = \{0, \frac{1}{2}\}$. These codes are not optimal in any way but are very easy to find (no use of search methods) and perform the same with optimal codes.

A. Codes with Minimum $\rho_{max} = 1/2$ in a Two Block Fading Channel

In order to prove that we can always find a code with spectrum of the form $S_\rho = \{(0, N_1), (1/2, N_2)\}$ where N_1 and N_2 are the multiplicities for every ρ, we decompose the H_M Hadamard matrix as in Fig. 1.

The use of a Hadamard matrix H_M in the two block fading channel results to the creation of two $M \times (M/2)$ blocks by selecting half of the columns of the original Hadamard matrix for each new block. We start from the H_M matrix and the selection of the first block is going to be of the form

$$\left(\begin{array}{c|c} \multicolumn{2}{c}{\text{code 1}\{\ M/2 \times M/2\}} \\ \hline \text{code 2}\{\ M/2 \times n_1 \textit{ same columns} & M/2 \times n_2 \textit{ opposite columns}\} \end{array}\right)$$

where the terms *same* and *opposite* refer to the relationship between the columns

$$\left(\begin{array}{c|c} H_{M/2} & H_{M/2} \\ \hline H_{M/2} & -H_{M/2} \end{array}\right)$$

Fig. 1. H_M Hadamard matrix decomposed in the lower level of Hadamard matrices

$$\left(\begin{array}{c|c}
\multicolumn{2}{c}{H_{M/2}} \\
\hline
\scriptstyle First\ 3M/8\ columns\ of\ H_{M/2} & -H_{M/8} \\
 & +H_{M/8} \\
 & +H_{M/8} \\
 & -H_{M/8}
\end{array}\right)$$

Fig. 2. Code created in one block when $n_2 = M/8$

of *code* 1 and *code* 2. The optimum *code* 1 that we can find is the $H_{M/2}$ which has $|\rho_{max}| = 0$. As a result *code* 2 has $|\rho_{max}| = 0$ and we are interested in the absolute cross correlations between the codewords of *code* 1 and *code* 2. If *code* 2 is the same with *code* 1 ($H_{M/2}$) then the codeword d_i from *code* 1 $i = 1, 2, \ldots, M/2$ has cross-correlation equal to "1"($d_H = 0$) with the codeword d_i from *code* 2. The cross correlations of the codeword d_i from *code* 1 and d_j from *code* 2 $j = 1, 2, \ldots, M/2$ *and* $i \neq j$ are equal to *zero* ($d_H = M/4$). To generalize, if we have a Hadamard matrix H_M and we add q Hadamard matrices (same or opposite) vertical in order to create an $qM \times M$ code the new code has q absolute cross correlations equal to "1" and all the others equal to *zero*. Now if we choose $n_2 = M/8$ then we create a code like the one in Fig. 2. From the above theory we conclude that every codeword from *code* 1 has *four* absolute cross correlations equal to "1/2" with codewords from *code* 2 and all the others are *zero*. The resulting code for every block has cross correlation spectrum $S_\rho = \{(0, \frac{M\times(M-5)}{2}), (1/2, 2M)\}$.

Since

$$|\rho| = |1 - \frac{4d_H}{M}| \tag{4.1}$$

the Hamming distance between two codewords in a block is either $M/4$ or $M/8$. For *code* 1 (Hadamard matrix) each pair of codewords has a Hamming distance equal to $M/4$. As a result there is an easy way to find codes like this for the two block fading channel.

Lemma 1 *The selection of the first* $\frac{3M}{8}$ *columns and the last* $\frac{M}{8}$ *columns from the Hadamard matrix* H_M *to create the first block (the remaining columns go to the second block) results in a code with absolute cross correlation spectrum of the form* $S_\rho = \{(0, \frac{M\times(M-5)}{2}), (1/2, 2M)\}$ *for each block for the two block fading channel.*

Next we compare the performance of these codes with optimal codes for the Hadamard matrices with $M = 16, 32$. In Fig. 3 we have the performance of the codes H_{16} with cross correlation spectrums $S^*_\rho = \{(0, 40), (2/8, 64), (4/8, 16)\}$ and $S_\rho = \{(0, 88), (4/8, 32)\}$ for each block and the corresponding union bound. In Fig. 4 we have the performance of the H_{32} codes with spectrum $S^*_\rho = \{(0, 48), (2/16, 256), (4/16, 192)\}$ and $S_\rho = \{(0, 432), (8/16, 64)\}$.

It is obvious that the performance of the above codes is not very sensitive to changes of the maximum absolute cross correlation.

B. Codes with Minimum $\rho_{max} = 1/2$ in the Four Block Fading Channel

The use of a Hadamard matrix H_M in the four block fading channel results to the creation of four $M \times (M/4)$ blocks by selecting $M/4$ of the columns of the original Hadamard matrix for each new block. We can still create codes with spectrum of the form $S_\rho = \{(0, N_1), (1/2, N_2)\}$ for every block. The proof for this follows the same line as the previous section. We choose as *code* 1 a $M/2 \times M/4$ code that has spectrum $S_\rho = \{(0, N_1), (1/2, N_2)\}$ and is constructed according the previous section. *Code* 2 has the same spectrum. Then Fig. 2 becomes for the four block

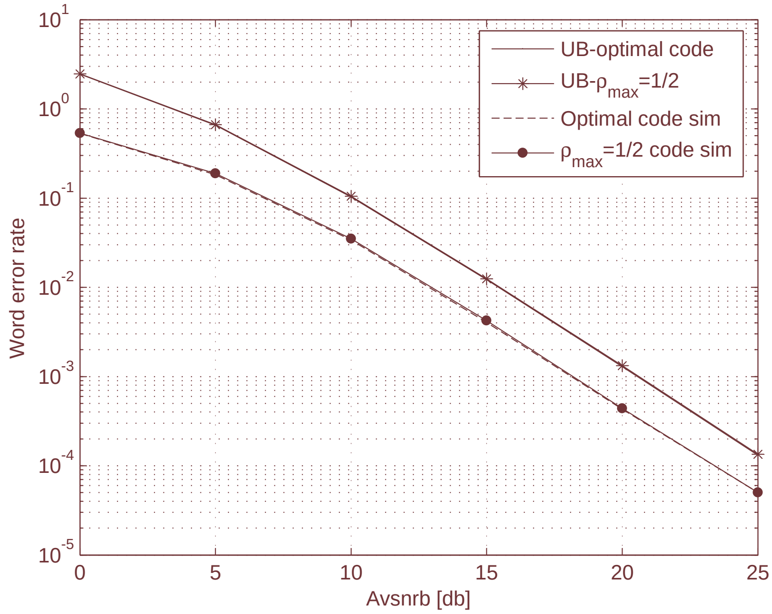

Fig. 3. Performance of an optimum H_{16} Hadamard code and a $\rho_{max} = 1/2$ code with the union bounds

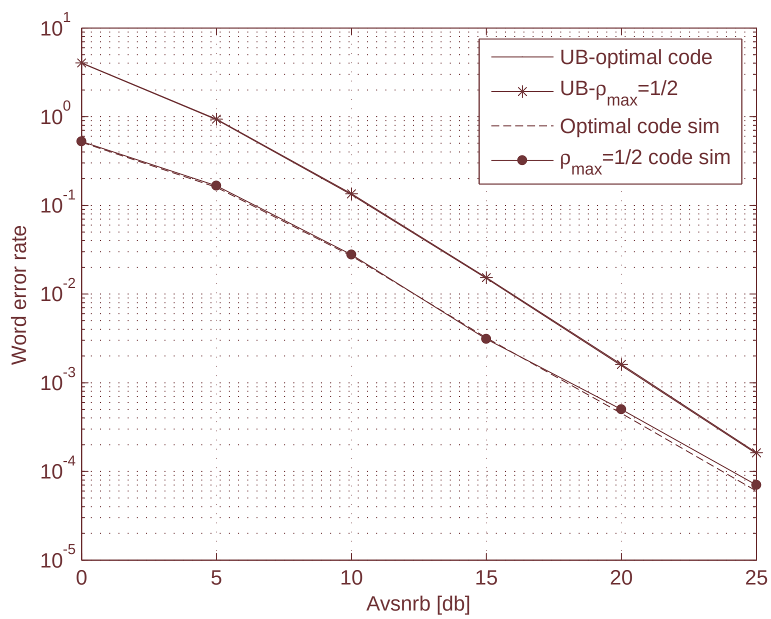

Fig. 4. Performance of an optimum H_{32} Hadamard code and a $\rho_{max} = 1/2$ code with the union bounds

$$\left(\begin{array}{c|c} \multicolumn{2}{c}{M/2\times\ M/4\ code\ (code\ 1)} \\ \hline 3M/16\ columns\ of\ code\ 1 & -M/16\ columns\ of\ code\ 1 \end{array}\right)$$

Fig. 5. Code created in one block when $n_2 = M/16$

$$\left(\begin{array}{c|c|c|c} \text{Block 1} & \text{Block 2} & \text{Block 3} & \text{Block 4} \\ 1,2,\ldots,M/4 & 1,2,\ldots,M/4 & 1,2,\ldots,M/4 & 1,2,\ldots,M/4 \end{array}\right)$$

Fig. 6. Grouping of columns for the four block fading channel

fading channel the one in Fig. 5. The code that is created has a spectrum for each block S_ρ $=\{(0, \frac{M\times(M-13)}{2}), (1/2, 6M)\}$.

The following algorithm is used to find the columns for each block using the Hadamard matrix H_M. First divide the Hadamard matrix H_M into four blocks and number the blocks $(1\ldots4)$ and the columns in every block $(1\ldots M/4)$. Then pair two consecutive blocks and select $M/4$ columns using the results of the previous section. This procedure is presented in Fig. 6.

Then for every pair of two blocks, let's say block i and block $i+1$ change the last $M/32$ columns of the i block with columns from the $i+2$ block that have the same number and the first $M/32$ columns of the $i+1$ block with columns from $i+3$ block that have the same number. This is repeated for $i = 1\ldots4$ in order to find the columns for the four blocks. For the four block fading channel we compare two codes created by H_{32}. The optimal code has spectrum $S^*_\rho = \{(0, 112), (2/8, 256), (4/8, 128)\}$ and the other code $S_\rho = \{(0, 304), (4/8, 192)\}$. The results are in Fig. 7 where we can see that the performance of these codes is the same.

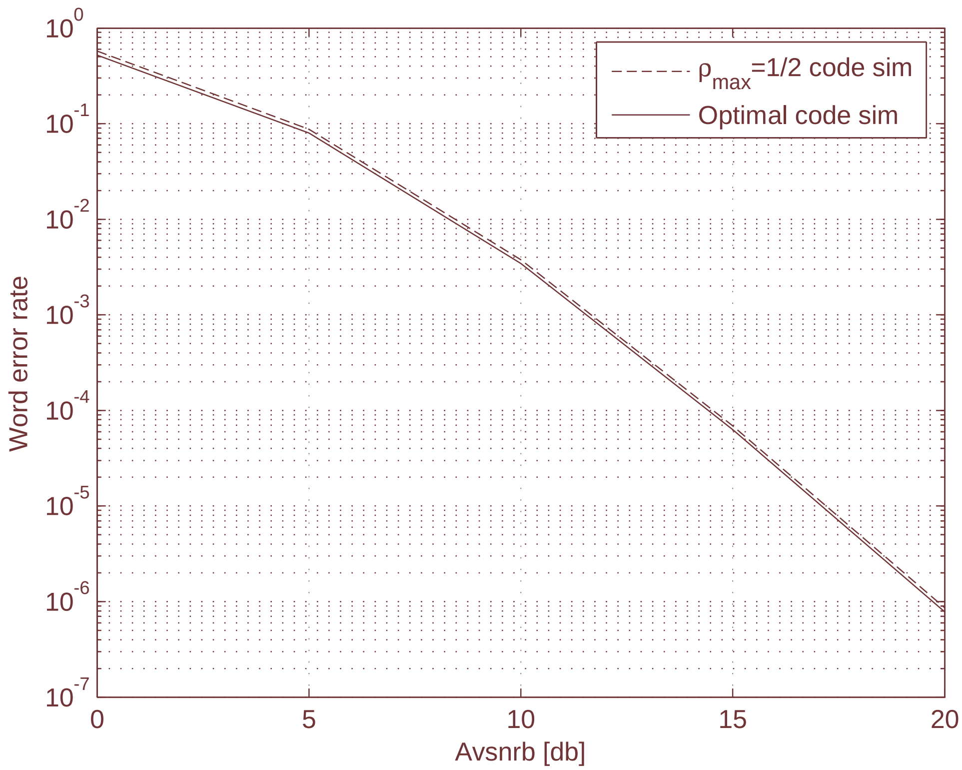

Fig. 7. Performance of an optimum H_{32} Hadamard code and a $\rho_{max} = 1/2$ code

C. Performance of Codes with Different Rate or Diversity Gain in the Block Fading Channel

This section deals with the effect of a rate change or a diversity gain change on the performance of a code. Diversity gain changes when the channel characteristics change. For the block fading channel that is the time that the fading variable is constant. The performance of optimum codes from [8] in the quasi-static channel with the change of the rate of the codes is presented in Fig. 8. We can see that although we have a significant reduction of the rate the change on the performance is very small. In Fig. 9 we see the performance of codes with $\rho_{max} = 1/2$ in the 2,4-block fading channel. Compared with the optimum code in the quasi-static channel the change in performance that we get by increasing the number of the blocks is very significant. So the next step is to find a way to transform these codes in order to get advantage of the diversity gain.

1. Codes Created by H_M (constant rate) When the Size of the Block Changes

For the N-block fading channel we assume that fading is constant for a block of $\frac{M}{N}$ bits. We want to see how the absolute cross correlation spectrum of these codes changes with N where $N = 2, 4$. The creation of a code for the 4-block fading channel according to section B results to a code with spectrum $S_\rho = \{(0, \frac{M\times(M-13)}{2}), (1/2, 6M)\}$ for every block. The possible Hamming distances are $d_H = 2, 4, 6$ (n=M/4). So if we create a 2-block code from a 4-block code (the code is the same but we use it in the 2-block channel) the possible Hamming distances are $d_H = 4, 6, 8, 10, 12$ (n=M/2) i.e. $\rho = 0, 1/4, 1/2$. The 2-block code has a spectrum $S_\rho = \{(0, \frac{M\times(M-11)}{2}), (1/4, 4M), (1/2, M)\}$. So if we want to keep the overall rate of the code constant k/M, a code created for the 4-block channel can be used for the 2-block channel (block size doubles) and

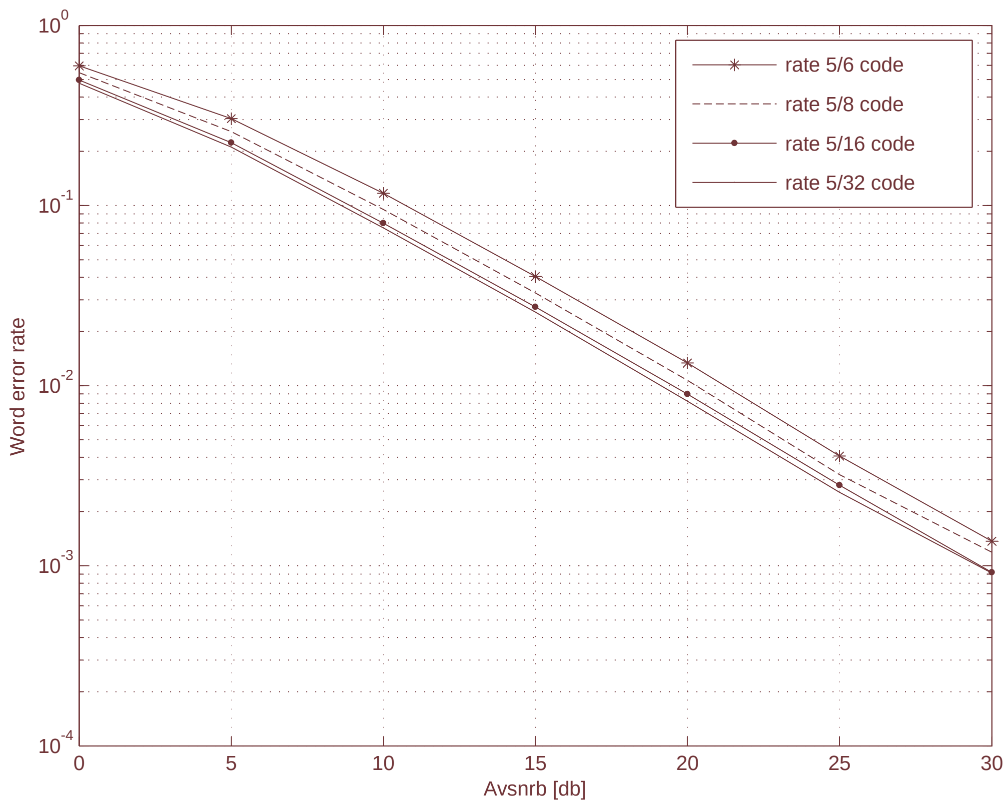

Fig. 8. Performance of 5/n codes in the quasi-static channel

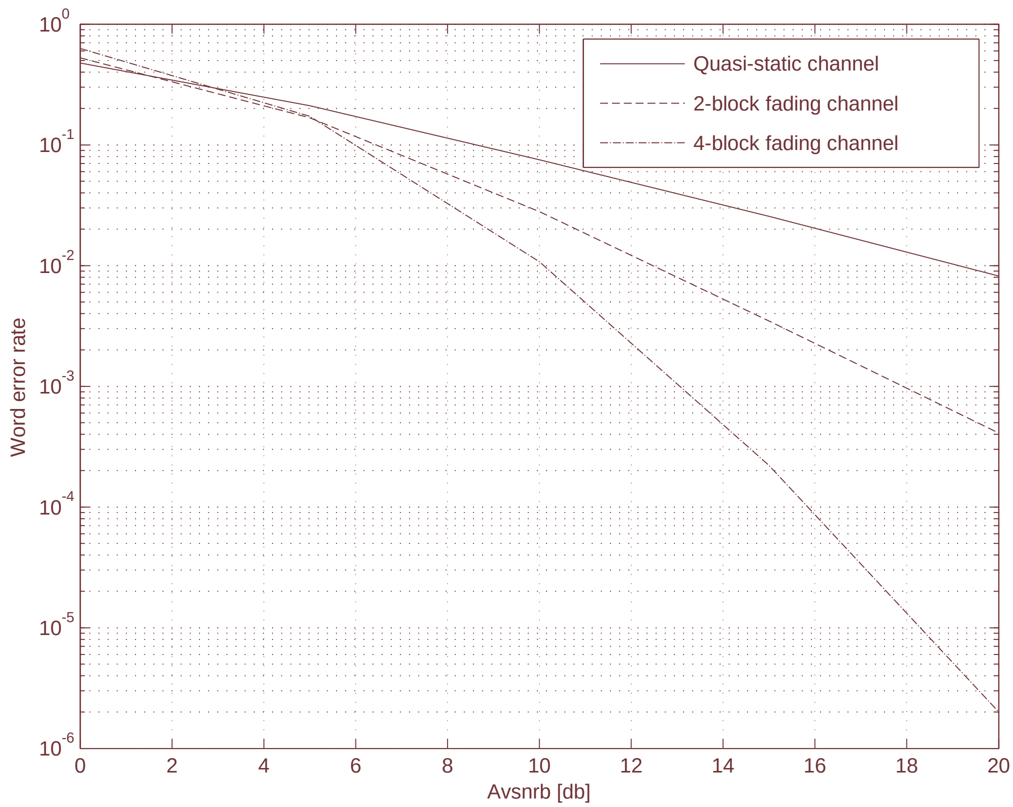

Fig. 9. Performance of a 5/32 code in quasi-static,2-block and 4-block channels

$\rho_{max} = 1/2$ for every block in any case.

2. Same Number of Blocks, Different Rate Codes

In order to use a code in the 2-block fading channel $M/2$ must be greater than k. As a result the higher rate code that we can use in the 2-block channel is the one created by H_8. For the 4-block fading channel $M/4$ must be greater than k. The code created by H_{32} is the higher rate code that we can use in the 4-block channel. In this section when the size of the block changes we change the rate of the code in order to keep the number of blocks the same. First we present a recursive way to create codes starting with the higher rate code. For the 2-block fading channel we have:

- Starting from the lower order Hadamard matrix that can be used (H_8) create a code for the 2-block fading channel with rate k/M using the method in section A .
- To create the rate $k + 1/2M$ code for the 2-block channel add to each block the corresponding columns from the second half of H_{2M} (for every column i select and add the $i + M$ column). The new code has spectrum $S_\rho = \{(0, \frac{M\times(M-5)}{2}), (1/2, 2M)\}$.

For the 4-block fading channel we have:

- Starting from the lower order Hadamard matrix that can be used (H_{32}) create a code for the 4-block fading channel with rate k/M using the method in section B .
- To create the rate $k + 1/2M$ code for the 4-block channel add to each block the corresponding columns from the second half of H_{2M} (for every column

rate 3/8 code

$$\left(\text{1,2,3,8} \;\middle|\; \text{4,5,6,7} \right)$$

$$\downarrow$$

rate 4/16 code

$$\left(\text{1,2,3,8,9,10,11,16} \;\middle|\; \text{4,5,6,7,12,13,14,15} \right)$$

$$\downarrow$$

rate 5/32 code

$$\left(\text{1,2,3,8,9,10,11,16,17,18,19,24,25,26,27,32} \;\middle|\; \text{4,5,6,7,12,13,14,15,20,21,22,23,28,29,30,31} \right)$$

$$\vdots$$

Fig. 10. Grouping of columns for the 2-block fading channel with rate change

i select and add the $i + M$ column). The new code has spectrum $S_\rho = \{(0, \frac{M\times(M-13)}{2}), (1/2, 6M)\}$.

This procedure is presented for the 2-block channel in Fig. 10 and for the 4-block channel in Fig. 11. The numbers represent the columns of the Hadamard matrix created by the Sylvester construction.

Using these recursive methods to create codes for either the 2-block fading channel or the 4-block fading channel gives us the following advantage. Let's say that we create a k/M code and the size of the block reduces to half of what it was, then by removing half (last half) of the columns from each block we have a rate $k - 1/(M/2)$ code with the same number of blocks. If the block size doubles then we do one more step of the above method and we get a rate $k + 1/2M$ code with the same number of blocks.

$$\text{rate } 5/32 \text{ code}$$

$$\left(\; 1,2,3,4,5,22,31,16 \; \right)$$

$$\downarrow$$

$$\text{rate } 6/64 \text{ code}$$

$$\left(\; 1,2,3,4,5,22,31,16,33,34,35,36,37,54,63,48 \; \right)$$

$$\downarrow$$

$$\text{rate } 7/128 \text{ code}$$

$$\left(\; 1,2,3,4,5,22,31,16,33,34,35,36,37,54,63,48,65,66,67,68,69,86,95,80,97,98,99,100,101,118,127,112 \; \right)$$

$$\vdots$$

Fig. 11. Grouping of columns for the 4-block fading channel with rate change (only one block)

rate 4/16 code

$$\left(\; 1,2,3,4,5,6,15,16 \;\middle|\; 7,8,9,10,11,12,13,14 \; \right)$$

$$\downarrow$$

rate 5/32 code

$$\left(\; 1,2,3,4,5,22,31,16 \;\middle|\; 7,9,10,11,12,13,24,30 \;\middle|\; 15,17,18,19,20,21,6,32 \;\middle|\; 23,25,26,27,28,29,8,14 \; \right)$$

Fig. 12. Transform of a code from the 2-block channel to the 4-block channel with rate change

3. Different Numbers of Blocks, Different Rate Codes

In the case that we want to improve the diversity gain and the block size is constant in order to go from a 2-block channel to a 4-block channel we have to change the rate. So if we use a rate k/M code in a 2-block fading channel and we change the rate to $k+1/2M$ the block size is the same but now the codeword size is double and the code is used in the 4-block fading channel. In order to do this we modify the algorithm in section 2. The method used to find the code for the 2-block fading channel is the one in section A.

We start from a rate k/M code in the 2-block channel. Use only the first step of the algorithm in section B to find the columns for the first block. Since the number of columns is the same per block we only have $M/8$ changed columns from the previous code per block. For the other blocks just add $M/2$ mod $2M$ to the previous block column numbers. The transition from a rate 4/16 code for the 2-block channel to a 5/32 for the 4-block channel is in Fig. 12.

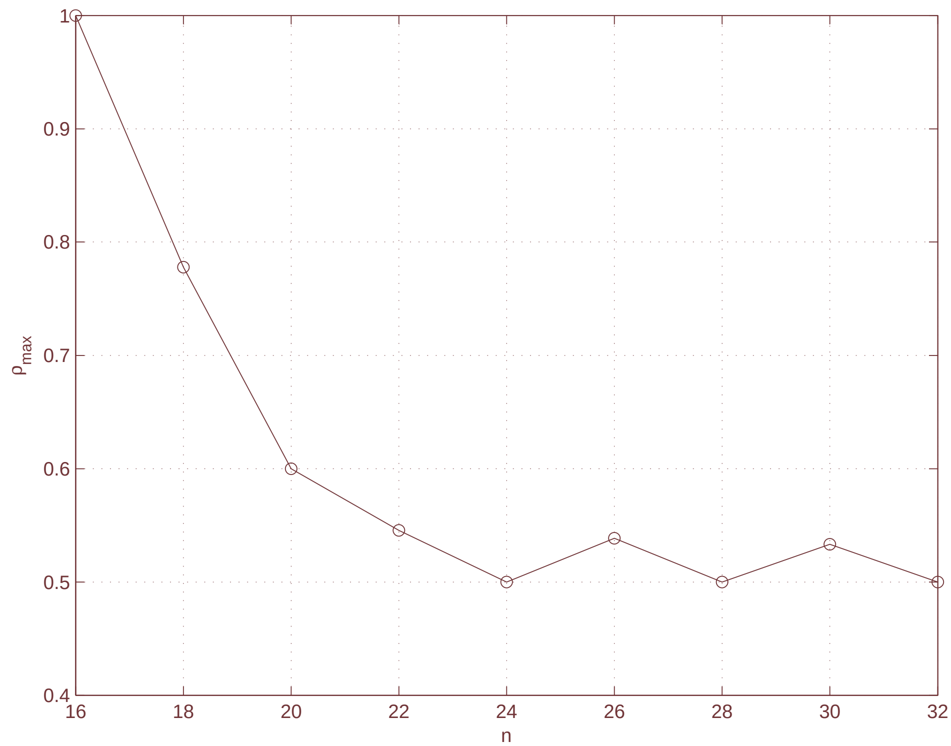

Fig. 13. M=64 (rate 6/n codes) ρ_{max} for our codes for one block (2-block channel)

4. Small Rate Changes

The codes described in section 2 perform very well when the rate of the code increases. More specific for the 2-block fading channel and for a k/M code for changes of $n = M/2, M/2-2, \ldots, 5M/16+2$ (for one block) the $min\ \rho_{max} = \frac{\lceil n/2 \rceil}{n}$ (see APPENDIX C). In Figs. 13,14 and 15 we can see how the $min\ \rho_{max}$ of our codes changes with the rate change for $k = 6, 7, 8$.

The performance of codes with different rates in the 2-block fading channel for $k = 6, 7$ is presented in Figs. 16 and 17. As we can see the performance of these codes does not change much with the change of the rate for the 2-block fading channel. For the 4-block fading channel the performance of codes with different rate is presented in

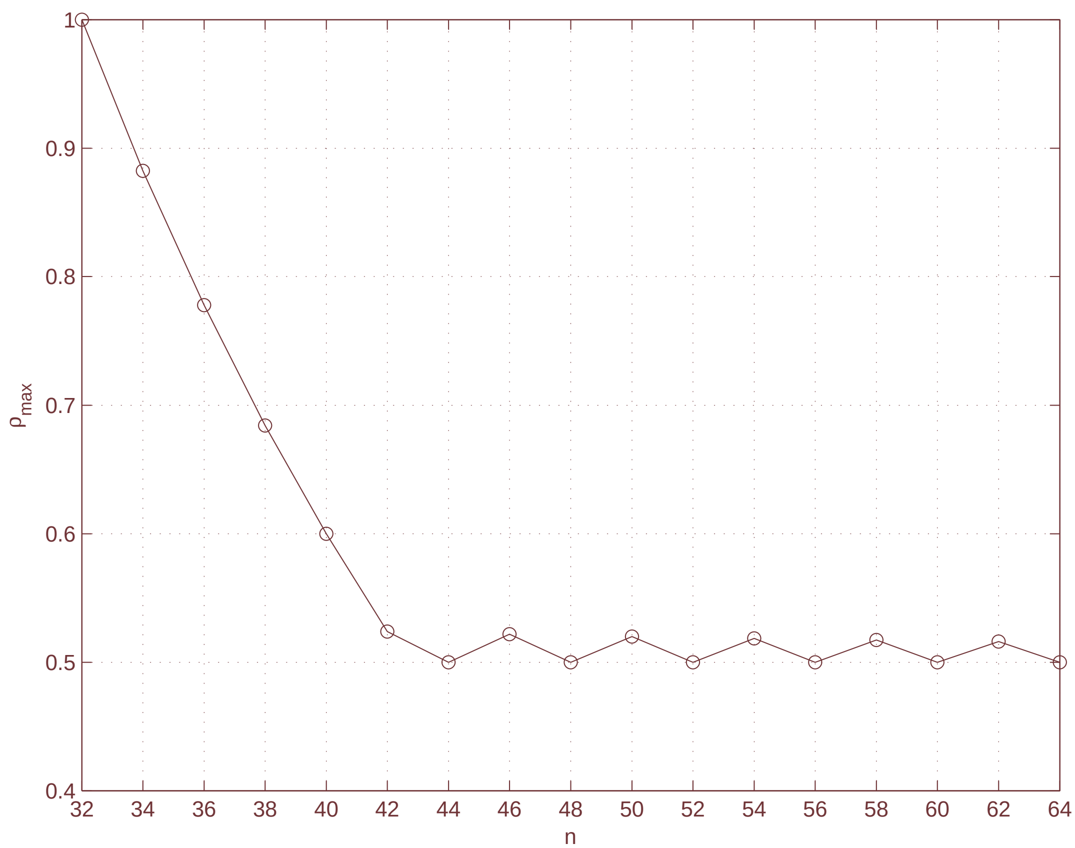

Fig. 14. M=128 (rate 7/n codes) ρ_{max} for our codes for one block (2-block channel)

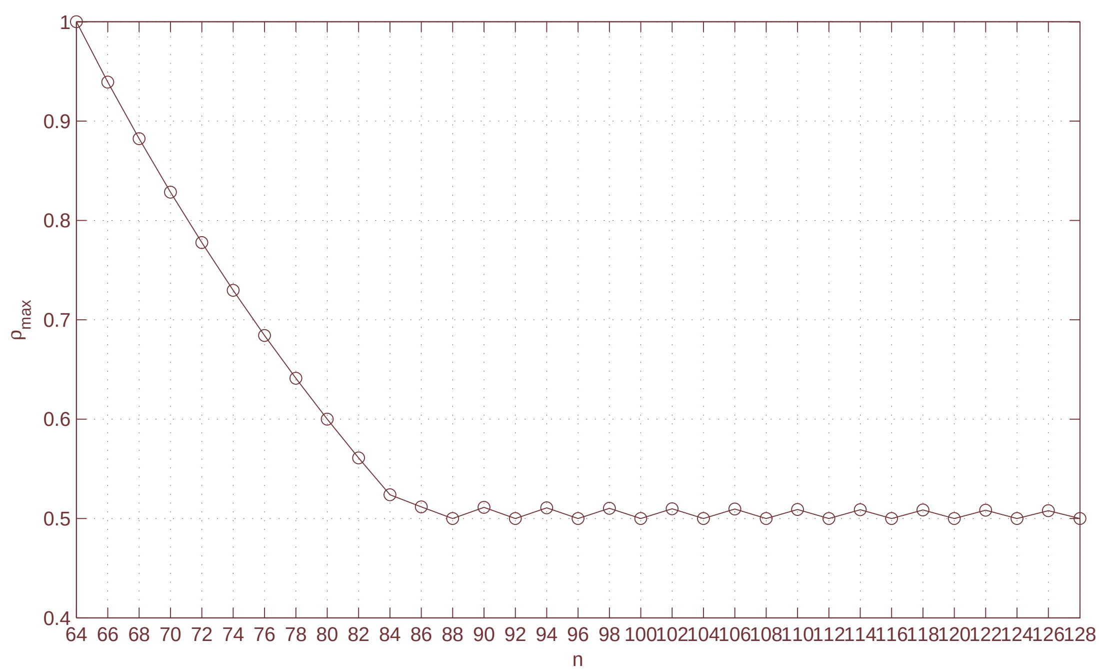

Fig. 15. M=256 (rate 8/n codes) ρ_{max} for our codes for one block (2-block channel)

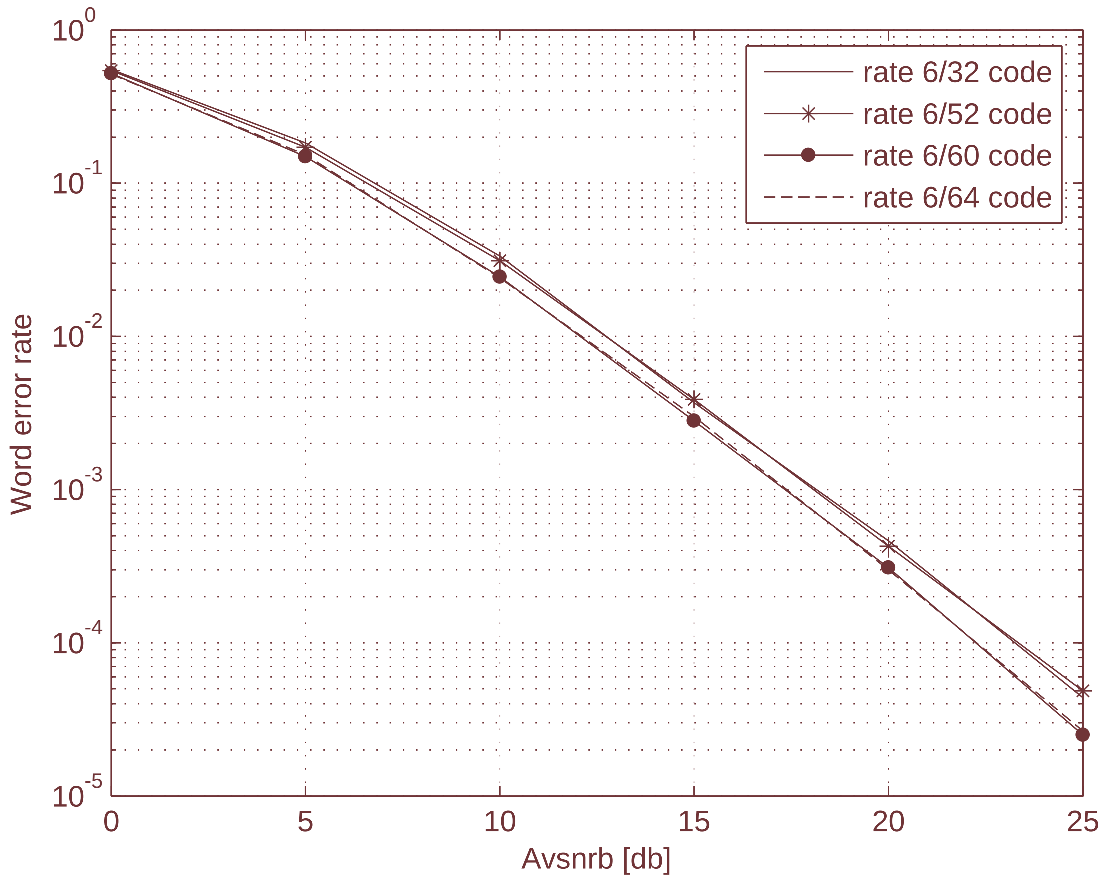

Fig. 16. Performance of $6/n$ codes in the 2-block fading channel

Figs. 18 and 19 for $k = 6$ and 7 respectively. This is the same result that we had for the quasi static channel. The advantage that we have though is that when the time slot that the fading is constant changes we can easily change the rate of the code and keep the performance at the same levels. For the 4-block fading channel the change of $min\ \rho_{max}$ per block is presented in Figs. 20,21 and 22 for $k = 8, 9, 10$.

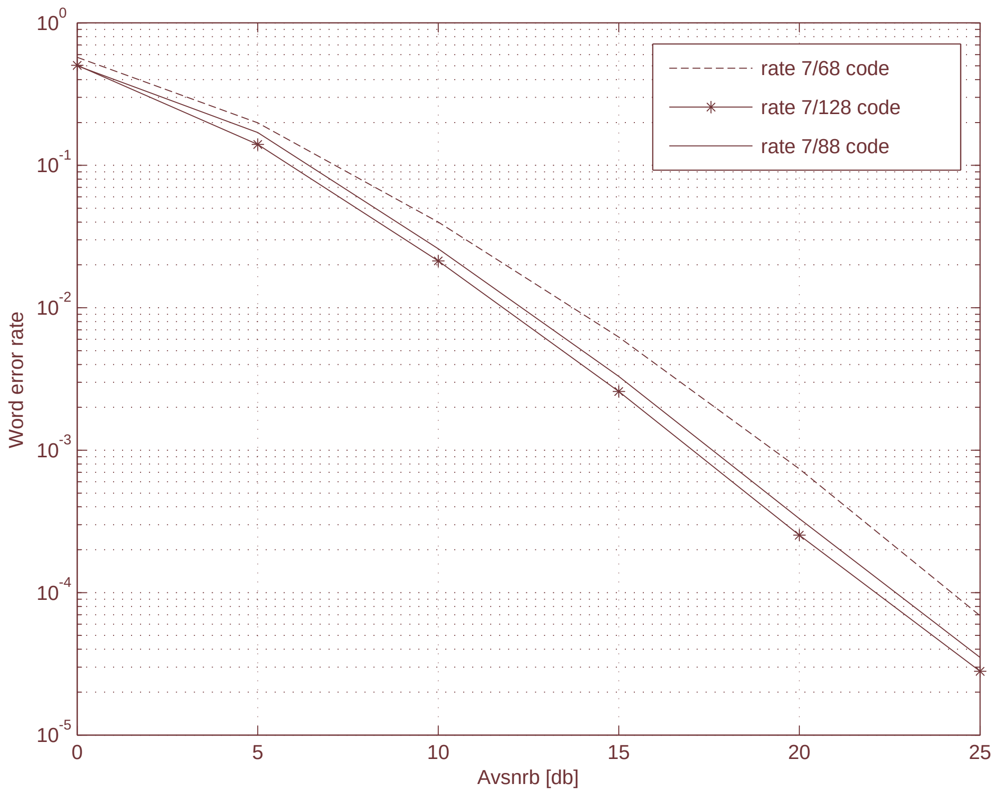

Fig. 17. Performance of $7/n$ codes in the 2-block fading channel

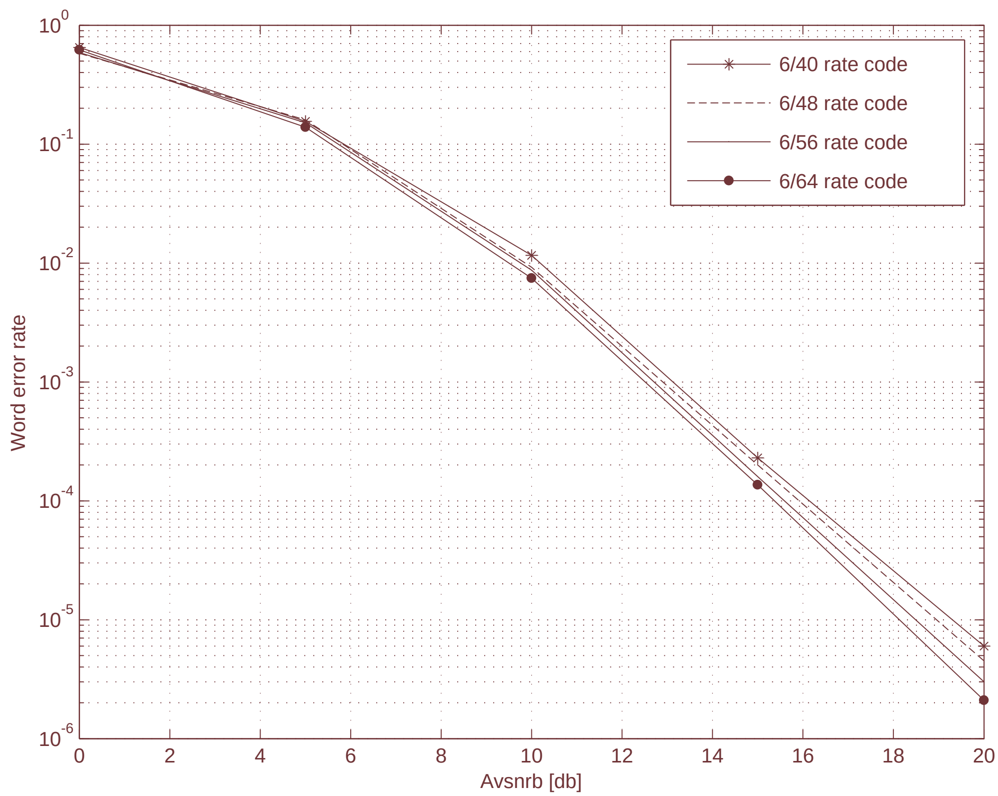

Fig. 18. Performance of $6/n$ codes in the 4-block fading channel

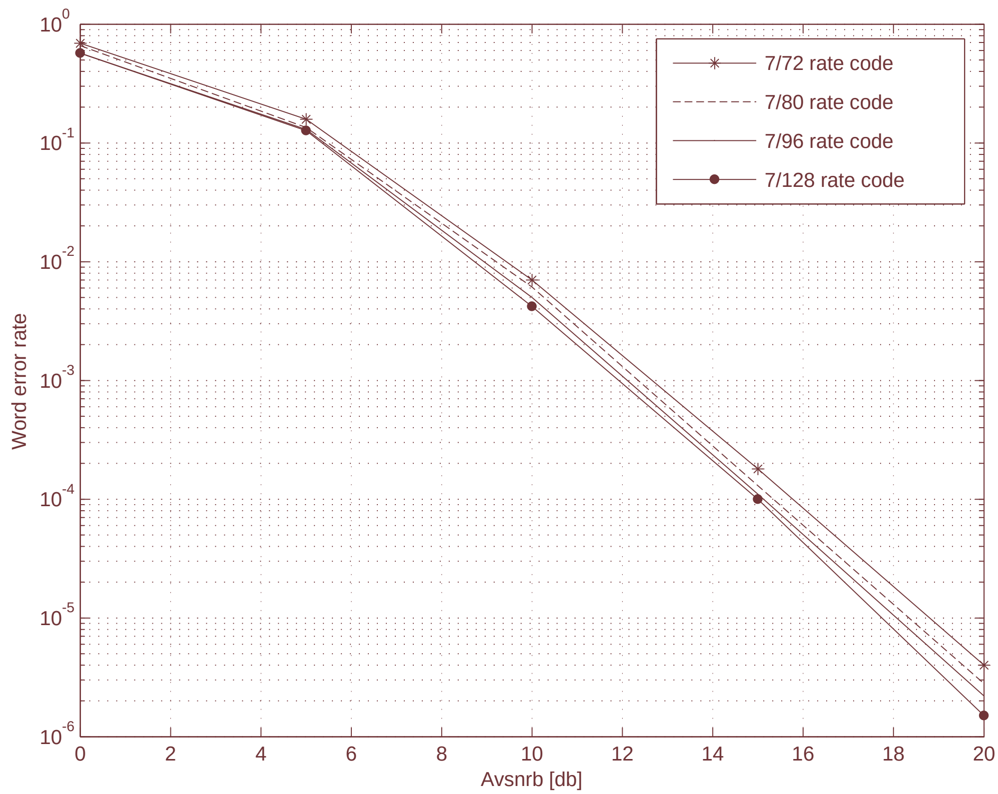

Fig. 19. Performance of $7/n$ codes in the 4-block fading channel

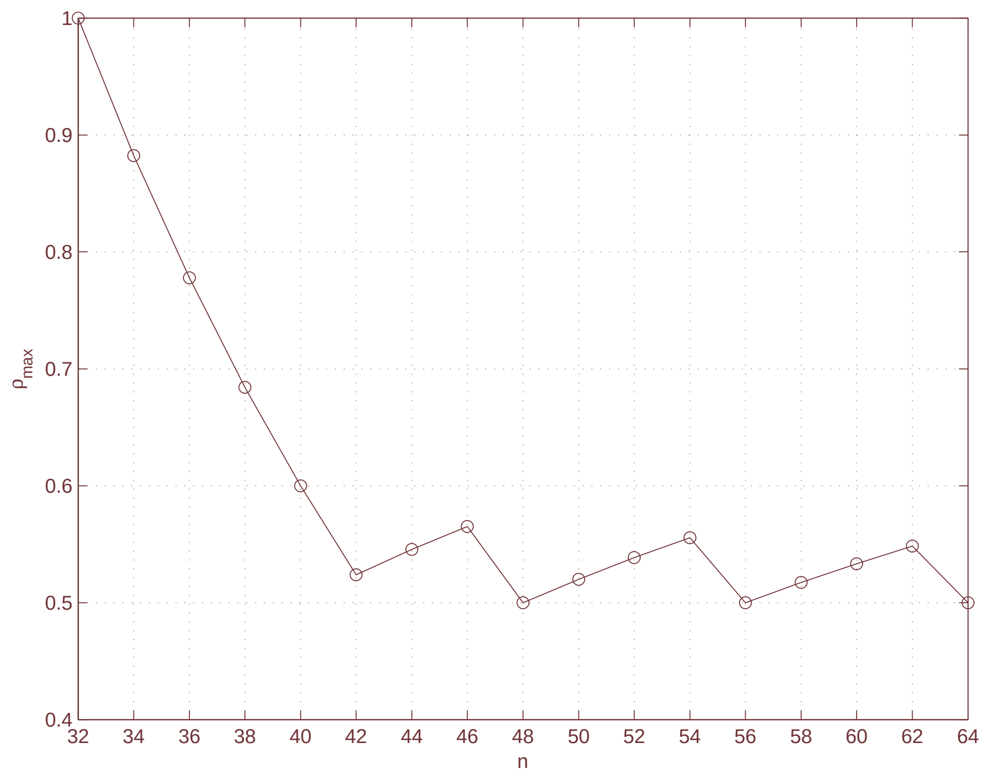

Fig. 20. M=256 (rate 8/n codes) ρ_{max} for our codes for one block (4-block channel)

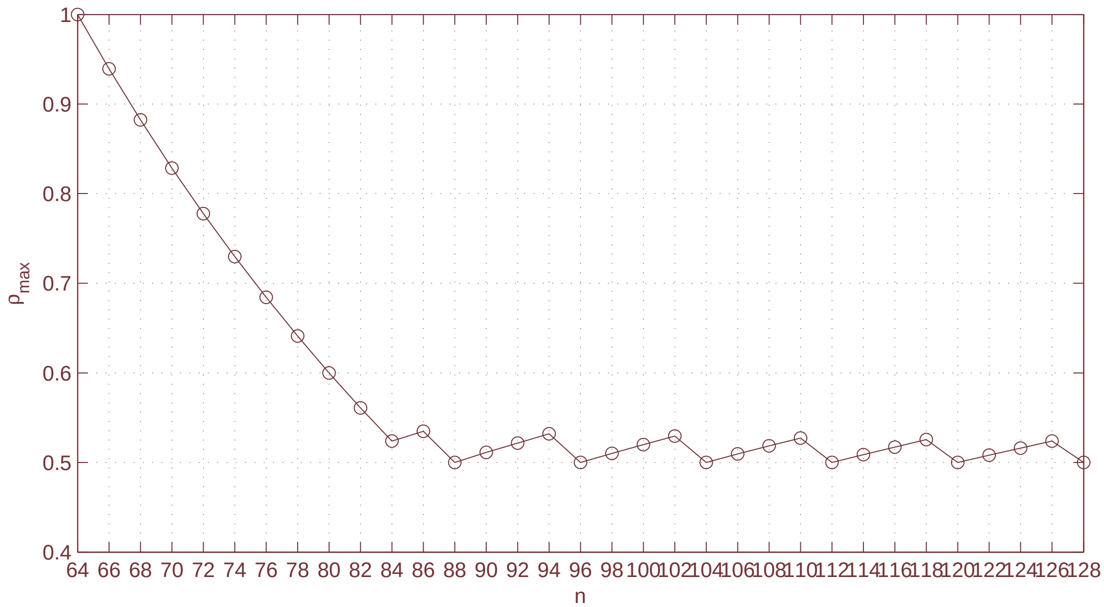

Fig. 21. M=512 (rate 9/n codes) ρ_{max} for our codes for one block (4-block channel)

D. Performance of $\rho_{max} = 1/2$ Codes in Fading Multipath Channels

In this section we present the performance of the rate 5/32 code in a flat or frequency selective channel with slow or fast fading. Small scale fading is characterized by two factors

- Time spread of the signal transmitted through the channel
- Frequency spread due to time variations in the structure of the medium.

The first factor determines the distortion of the signal due to inter-symbol interference (ISI) at the receiver and the second determines how fast the behavior of the channel changes. The two factors that effect small scale fading are independent so we have 4 kinds of small scale fading. The factors that determine the kind of fading that we have are the bandwidth of the signal, the specific environment of multipath propagation, the speed of the receiver and the speed of the objects around the receiver.

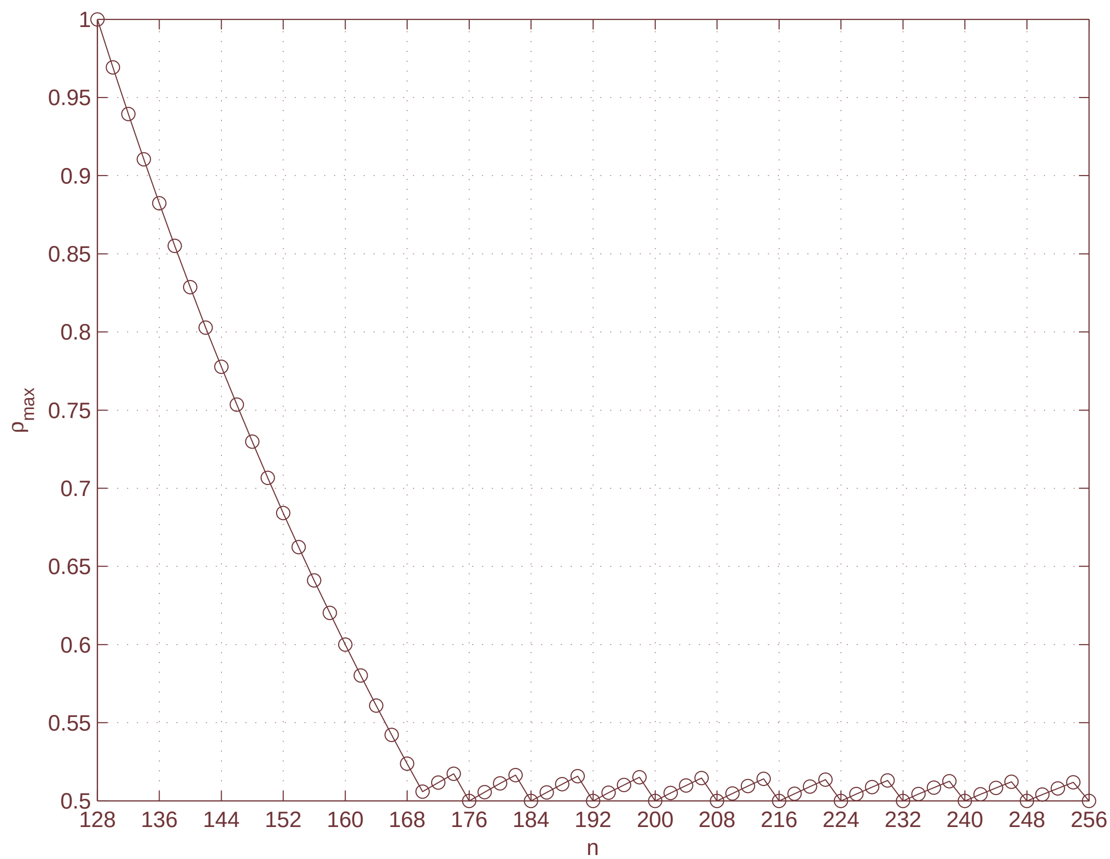

Fig. 22. M=1024 (rate 10/n codes) ρ_{max} for our codes for one block (4-block channel)

In order to quantify the multipath delay spread we use the *multipath spread of the channel* T_m which is the time period that the autocorrelation function of the channel is non-zero [18] and in the frequency domain the *coherence bandwidth* B_c which is the bandwidth that the channel is flat (constant gain and linear phase). The reciprocal of the multipath spread is the coherence bandwidth. That is,

$$B_c \approx \frac{1}{T_m} \tag{4.2}$$

So if the bandwidth of the transmitted signal is large compared to the coherence bandwidth then the channel is said to be frequency selective. On the other hand, if the signal bandwidth is small compared to the coherence bandwidth then the channel is said to be frequency nonselective or flat.

To quantify the time variations in the channel we use the *Doppler spread* B_d of the channel which is the bandwidth for which the Doppler power spectrum of the channel is non zero [18] and *coherence time* T_c. Again

$$T_c = \frac{1}{B_d} \tag{4.3}$$

So if the bandwidth of the transmitted signal is small compared to the Doppler spread then the channel is said to be fast fading. On the other hand, if the signal bandwidth is large compared to the Doppler spread then the channel is said to be slow fading.

To summarize, a signal with bandwidth B_s and symbol duration T_s is under flat fading if

$$B_s \ll B_c \tag{4.4}$$

and

$$T_s \gg T_m \tag{4.5}$$

A signal is under frequency selective fading if

$$B_s \succ B_c \tag{4.6}$$

and

$$T_s \prec T_m \tag{4.7}$$

A signal is under fast fading if

$$B_s \prec B_d \tag{4.8}$$

and

$$T_s \succ T_c \tag{4.9}$$

and under slow fading if

$$B_s \gg B_d \tag{4.10}$$

and

$$T_s \ll T_c \tag{4.11}$$

In Figs. 23 and 24 we can see the four kinds of fading that can occur according to the time duration of every symbol and the bandwidth of the signal correspondingly.

For our simulations we used the Jake SOS simulator for Rayleigh flat fading due to multipath propagation. According to Jake's model the received signal is given by

$$\begin{aligned} &r(t) = x(t) + jy(t) = \\ &= \left[\sqrt{\tfrac{2}{N_1+1}} \textstyle\sum_{n=1}^{N_1} \cos\left(\tfrac{\pi n}{N_1}\right) \cos\left\{2\pi f_d \cos\left(\tfrac{2\pi n}{N}\right) t\right\} + \tfrac{1}{\sqrt{N_1+1}} \cos(2\pi f_d)\right] + \\ &+ j\sqrt{\tfrac{2}{N_1}} \textstyle\sum_{n=1}^{N_1} \sin\left(\tfrac{\pi n}{N_1}\right) \cos\left\{2\pi f_d \cos\left(\tfrac{2\pi n}{N}\right) t\right\} \end{aligned} \tag{4.12}$$

where $N/2$ is odd and

$$N_1 = \frac{1}{2}\left(\frac{N}{2} - 1\right) \tag{4.13}$$

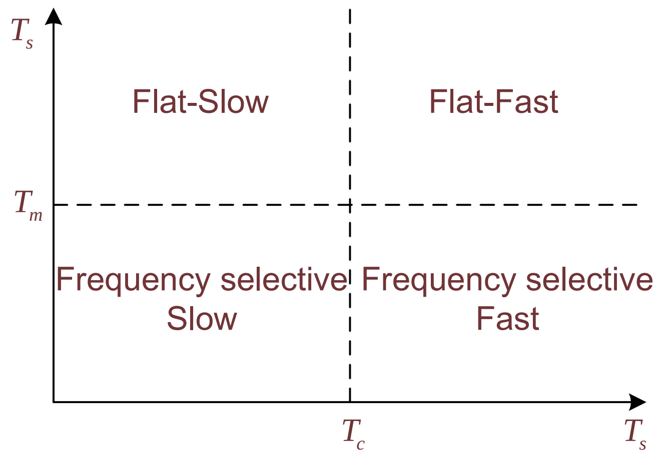

Fig. 23. Categories of fading according to the symbol duration

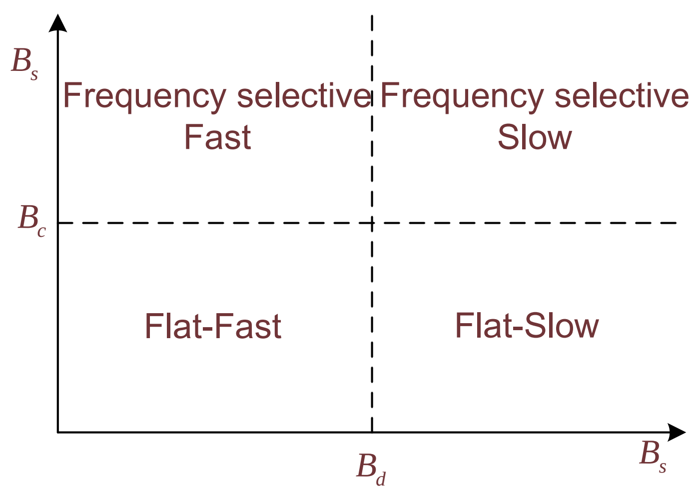

Fig. 24. Categories of fading according to bandwidth

According to equation 4.12 the following apply

$$\begin{aligned} &E[x^2(t)] = E[y^2(t)] = \tfrac{1}{2} \\ &E[x(t)y(t)] = 0 \end{aligned} \tag{4.14}$$

When N_1 is large enough $r(t)$ is a good approximation of a complex Gaussian random process and as a result has Rayleigh envelop and uniform phase in $[0, 2\pi]$. We multiply the data with the Rayleigh envelop and we do not take into consideration the phase which implies that the phase is slow changing and can be detected by the receiver.

In Fig. 25 we present the performance of the rate 5/32 code in a flat Rayleigh channel (one path) when we change the Doppler spread f_d. This means that for small values of f_d the channel is slow fading (quasi-static) and for larger values the channel becomes fast fading (block channel) and as a result we have better performance due to the advantage of diversity in a fast fading channel. In Fig. 26 we present the performance of the same code in a frequency selective channel (two path fading) when we change the Doppler spread f_d. We still have a great improvement in the performance due to diversity gain. The transmission rate used for these simulations is 256 $kbits/s$. Fig. 27 presents the performance of the same code in a slow frequency selective channel with different number of paths. A root raised cosine pulse shaping filter is used at the transmitter and at the receiver to avoid the effects of inter symbol interference and we can see that for more than $four$ paths the diversity gain is the same.

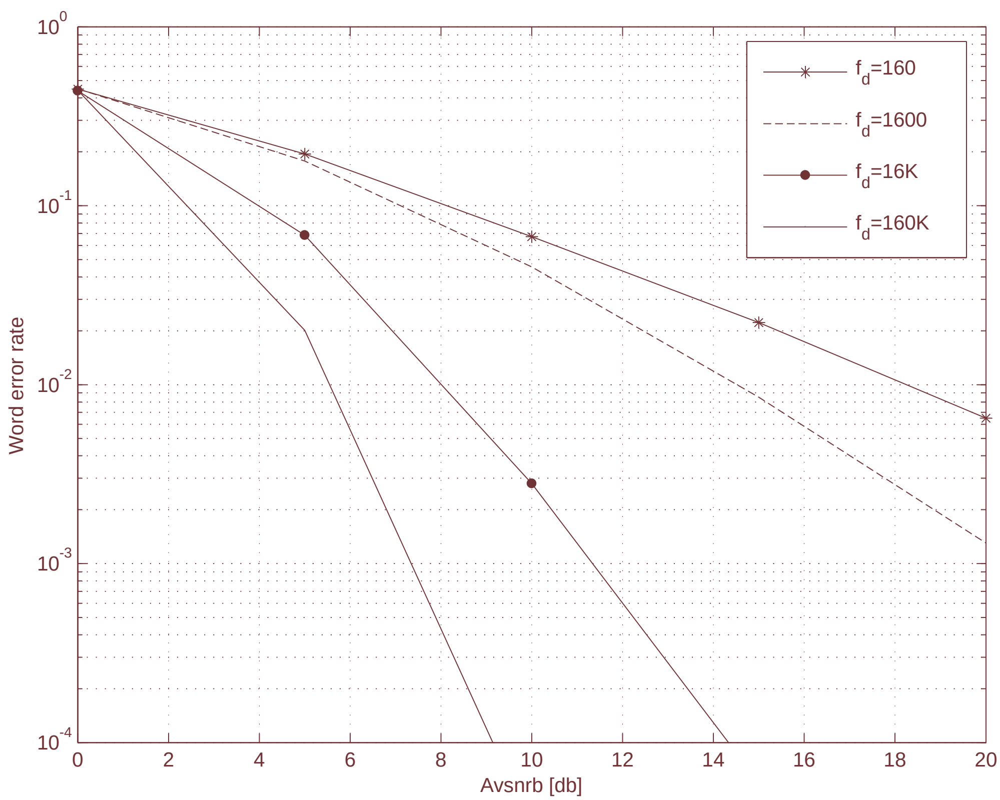

Fig. 25. Performance of a rate 5/32 code in flat fading channel (different Doppler spread)

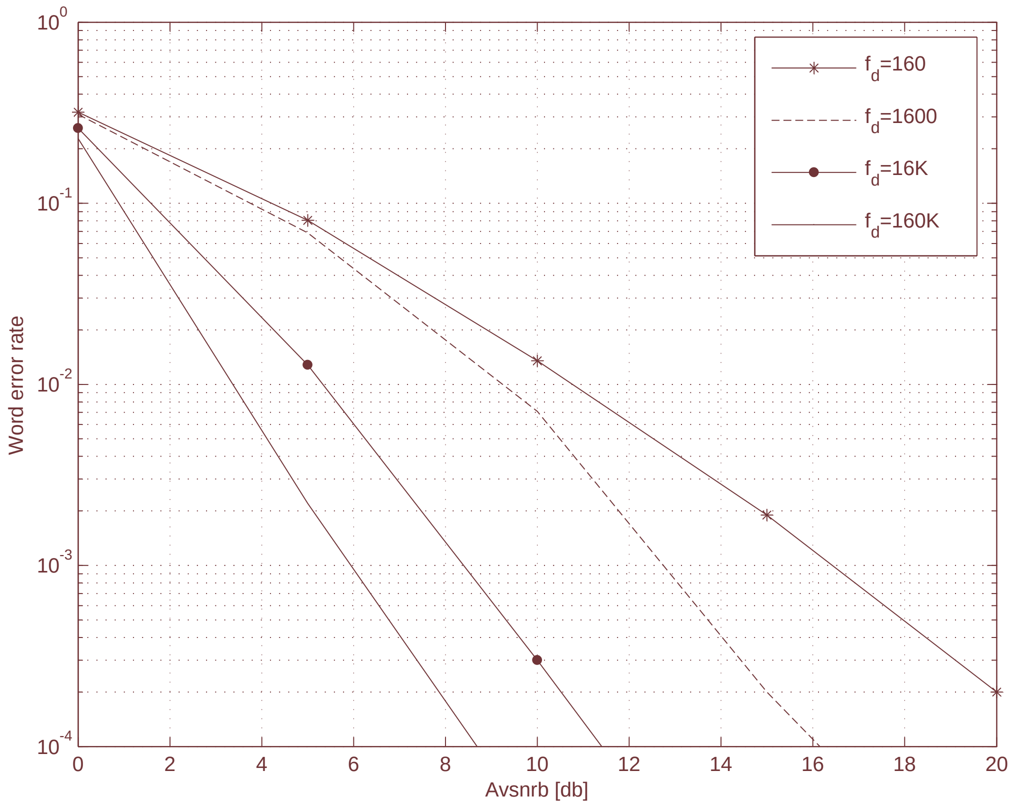

Fig. 26. Performance of a rate 5/32 code in frequency selective fading channel (different Doppler spread)

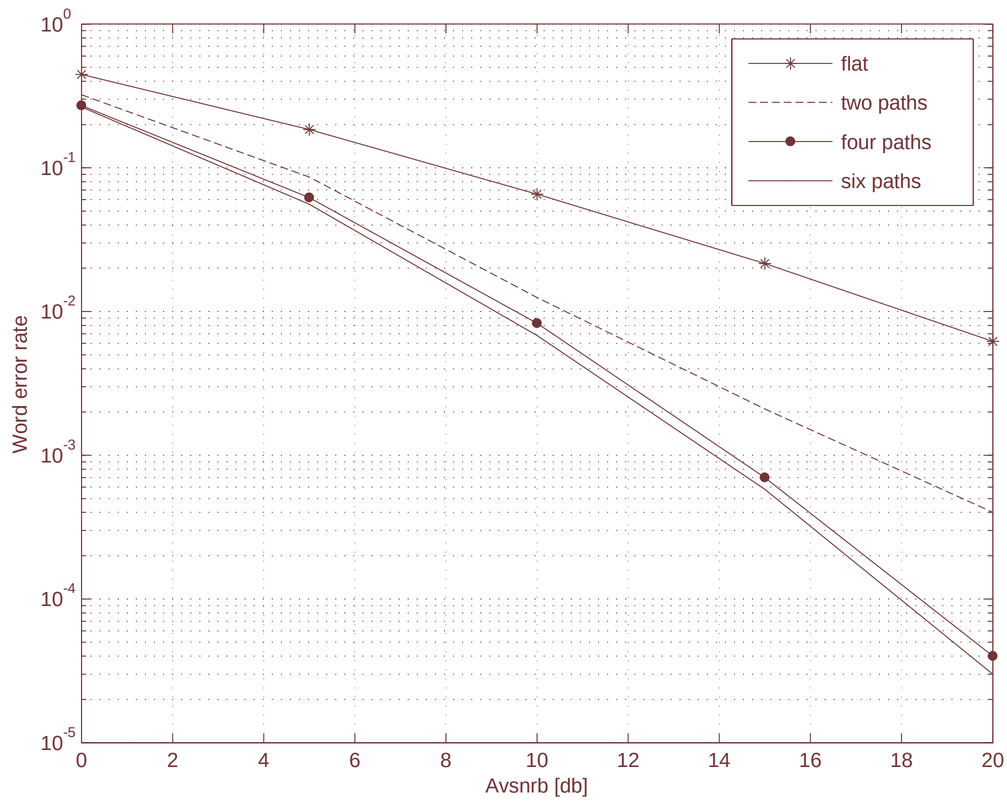

Fig. 27. Performance of a rate 5/32 code in a slow fading frequency selective channel with different number of paths

CHAPTER V

PERFORMANCE OF NEW CODES USING SELECTIVE TRANSMISSION

This chapter deals with the performance of the new codes using selective transmission. Selective transmission is the procedure where we transmit only the number of blocks necessary (we transmit part of the codeword) according to a reliability measure in order to get a performance as good as the code that transmits all the codeword.

The idea behind this is very simple. Since these codes are created for the block fading channel (every block has independent fading) every block of the code can be used independent as a new code. So if we transmit only the first block of a codeword and according to a reliability measure (we deal with the reliability measure later in this chapter) at the receiver there are no errors then we can uniquely decide which was the transmitted codeword and we use less energy to do that. If the receiver decides that the received codeword has errors then we send the next block. This procedure continuous until we either have no errors or we send all the codeword. So this system uses less energy, has higher throughput efficiency and less computational complexity compared to the same code without selective transmission. Furthermore, it still gets advantage of the diversity that we have from the independent blocks in a block fading channel. The drawback is that the receiver has to store the results of every decoding until the final decision and there is a delay between the transmission of the blocks. This idea of course is not new and past work using different codes and channels has been done by [19, 20, 21].

The maximum likelihood decoder for a k/n code used in the N-block fading channel $N = 2, 4$ is given by:

$$\widehat{d} = arg \max_{d} \left(\left| r_1^H d_{i1} \right|^2 + \left| r_2^H d_{i2} \right|^2 + \ldots + \left| r_N^H d_{iN} \right|^2 \right) \tag{5.1}$$

where $i = 1, 2, \ldots, M$. For the same code if selective transmission is used the decoding procedure is the following:

1. Set $j = 1$

2. Send the j^{th} block of the codeword

3. The receiver calculates the M dimensional vector $\left[\left|r_j^H d_{1j}\right|^2, \left|r_j^H d_{2j}\right|^2, \ldots, \left|r_j^H d_{Mj}\right|^2\right]$ and then creates the M dimensional vector

$$V = \left[\sum_{q=1}^{j} \left|r_j^H d_{1q}\right|^2, \sum_{q=1}^{j} \left|r_j^H d_{2q}\right|^2, \ldots, \sum_{q=1}^{j} \left|r_j^H d_{Mq}\right|^2\right] \tag{5.2}$$

   to make a decision where d_{ij} is the j^{th} block of the i^{th} codeword. The maximum value of V gives the decoded codeword. According to a reliability measure the receiver evaluates the decision made. Let's say that the maximum likelihood detector decides in favor of the m^{th} codeword. If the decision is good then the decoded codeword is the m^{th} codeword from the k/n code and $j = N$. If the decision is not good enough then the receiver requires a transmission of the next block and saves in a buffer the vector V.

4. If $j = N$ then transmit the next codeword or else set $j = j + 1$ and return to step 2.

The above procedure is presented in a flow chart form in Fig. 28.

As mentioned before this method of selective transmission uses less energy (always compared to a same rate code,same channel and without selective transmission scheme), has higher throughput efficiency and performs as well as a no-ST code. The factor that controls all these is the reliability measure that the receiver implements. It is obvious that there is a trade off between energy-performance and throughput efficiency. The goal is to achieve good performance with high throughput efficiency.

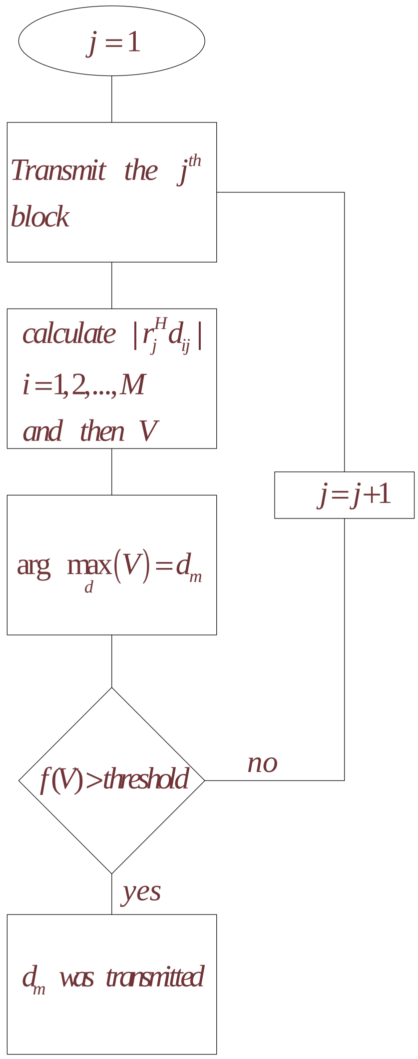

Fig. 28. Transmission procedure for the selective transmission scheme

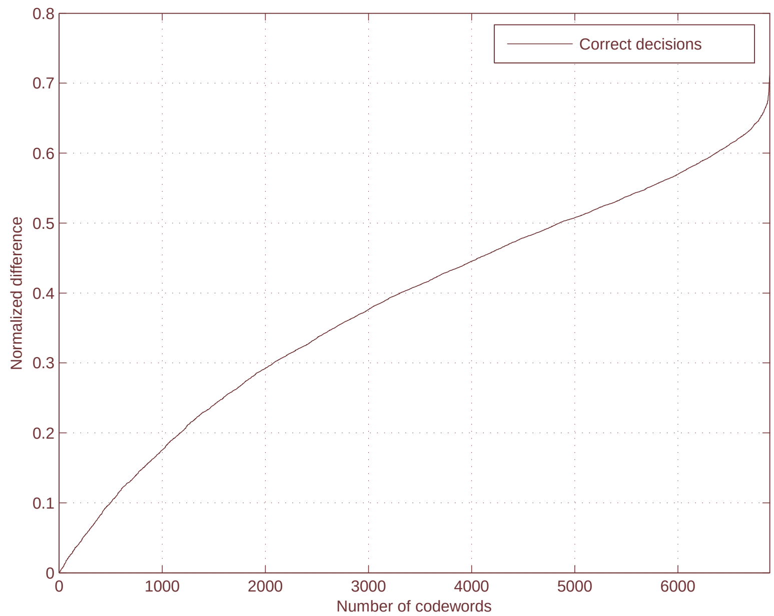

Fig. 29. Values of normalized difference for correct decisions

Whenever a block arrives at the receiver an M dimensional vector like in (5.2) is created and the decision is made in favor of the largest value. The reliability measure (RM) that we used is the normalized difference and is given by

$$RM = \frac{(maximum\ value\ of\ V) - (second\ largest\ value\ of\ V)}{(maximum\ value\ of\ V) - (minimum\ value\ of\ V)} \tag{5.3}$$

In Figs. 29 and 30 we present the values of the reliability measure for a 5/32 code in the 4-block fading channel for the correct decisions of the receiver and for the erroneous, respectively. We send 10000 codewords with average signal to noise radio per information bit 10 *db* and observe the values of the normalized difference.

It is obvious that if we want to choose a threshold that includes (all the values less than the threshold) all the error decisions in a transmission that would be 0.6. But that would also mean that we would include almost all of the correct decision codewords. A threshold that includes 90 % of the erroneous decisions let's say 0.3

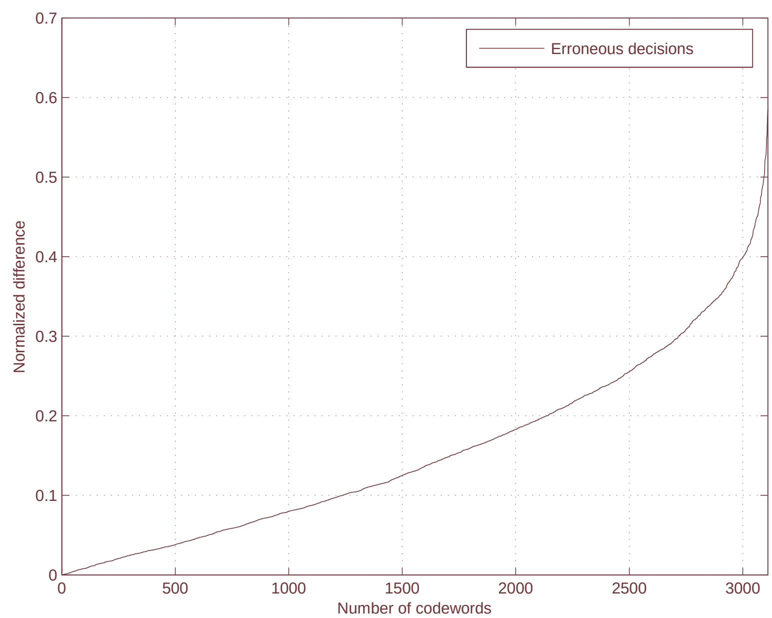

Fig. 30. Values of normalized difference for erroneous decisions

includes around 20 % of the right decisions. Using these graphs we found good values for the threshold of the normalized difference. So for every value of the normalized difference under the threshold the receiver requests for a transmission of the next block or else makes a decision based on the maximum value of the current V vector. Note here that these graphs are presented here just to give an inside of the trade off between throughput and performance. For constant signal to noise ratio the variations are very small of the order of ± 0.1. In higher signal to noise ratios where fewer errors occur the threshold should be such that extra block transmission is requested for all the erroneous blocks. In Fig. 31 we present the performance of the 5/32 code in the 4-block fading channel using different thresholds for the selective transmission (ST). For lower rate codes the performance is in Figs. 32,33 and 34 for the 6/64,7/80 and 7/128 rate codes respectively. As the SNR increases there is a point, for any value of the threshold, that the slope of the performance changes for the worst. That happens

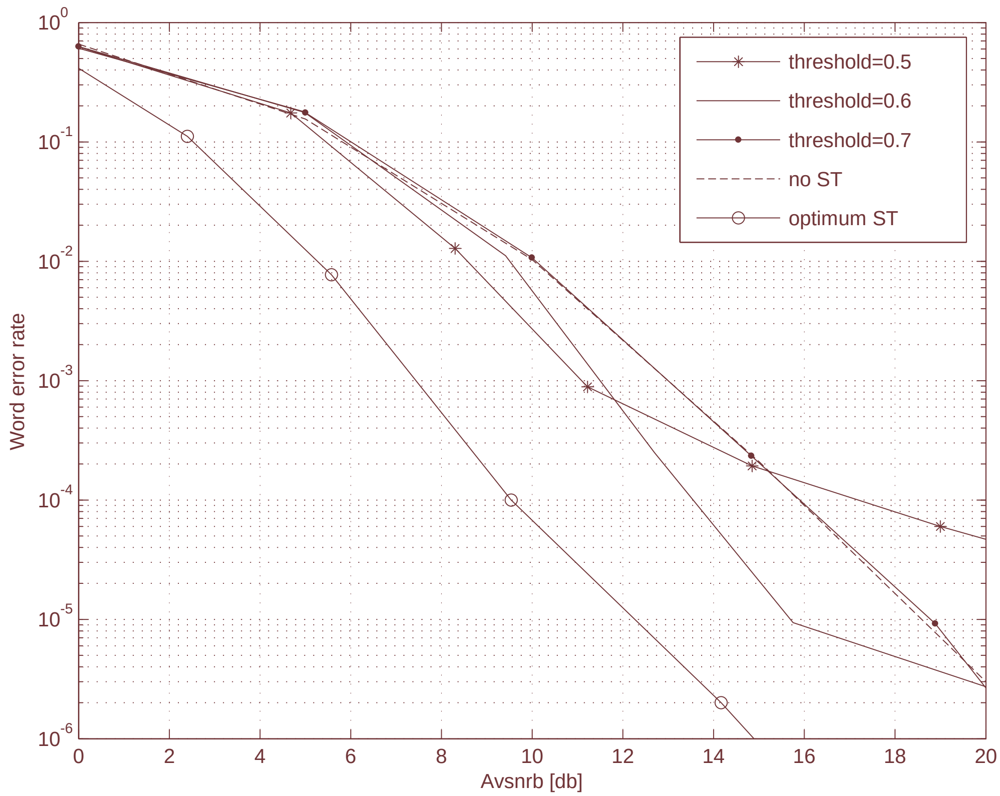

Fig. 31. Performance of a rate 5/32 code in the 4-block fading channel using selective transmission

because the number of the errors that take place when the receiver does not request a transmission of another block becomes comparable with the total number of errors.

In Fig. 31 the code with threshold 0.6 has better performance for higher SNR and the threshold 0.5 for lower SNR. Adaptive transmission (AT) is what we call when we use a different threshold for different values of SNR. For the threshold with value 0.7 we have the same performance to the code with no selective transmission since this thresholds results to the transmission of all the codeword on the average. The optimum selective transmission (the transmitted codeword is used at the receiver) which is used just as a lower bound on the performance for high SNR achieves an improvement on the performance close to the value of $\approx$ 6 *db*. That value is maximum

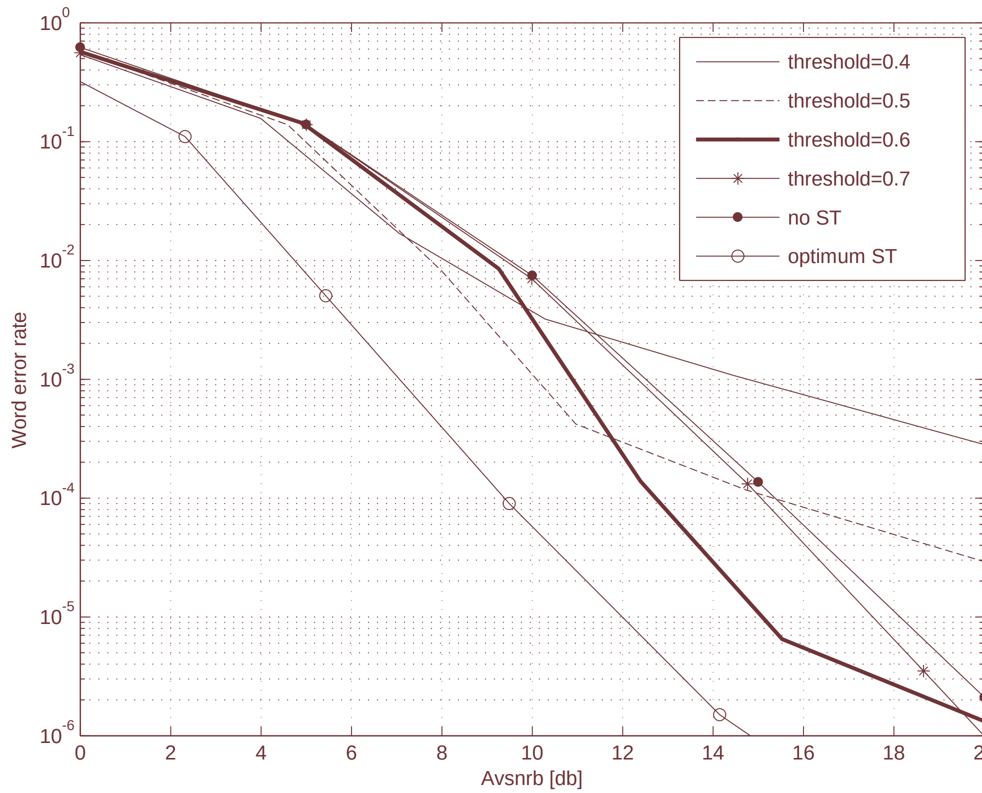

Fig. 32. Performance of a rate 6/64 code in the 4-block fading channel using selective transmission

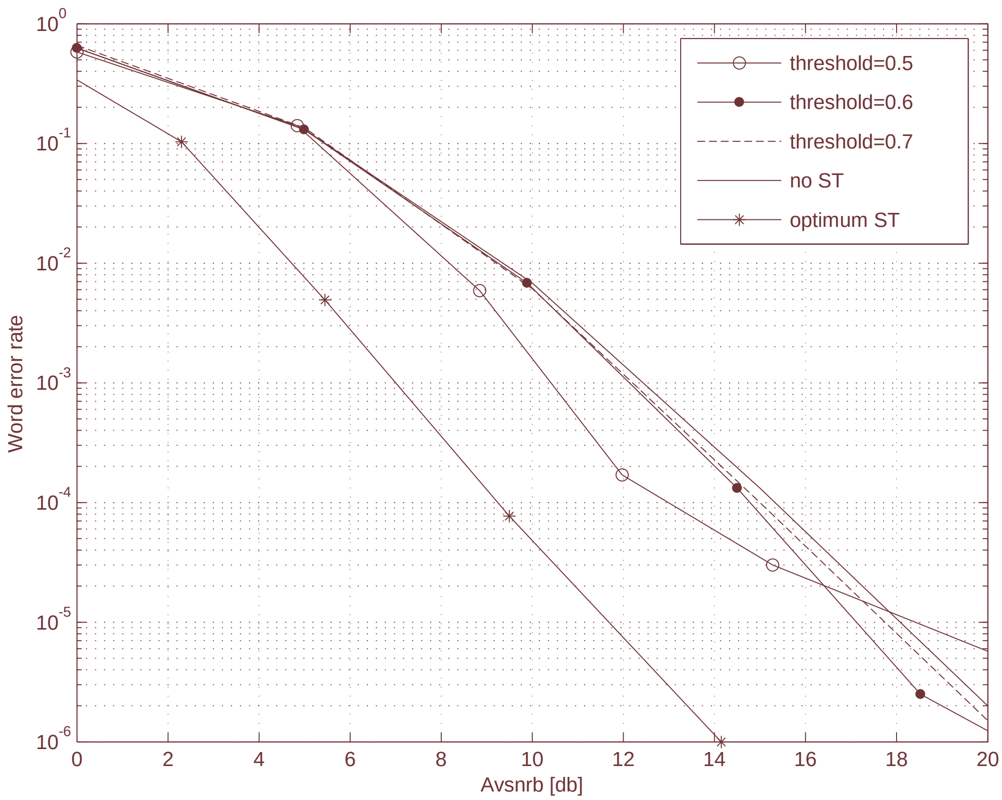

Fig. 33. Performance of a rate 7/80 code in the 4-block fading channel using selective transmission

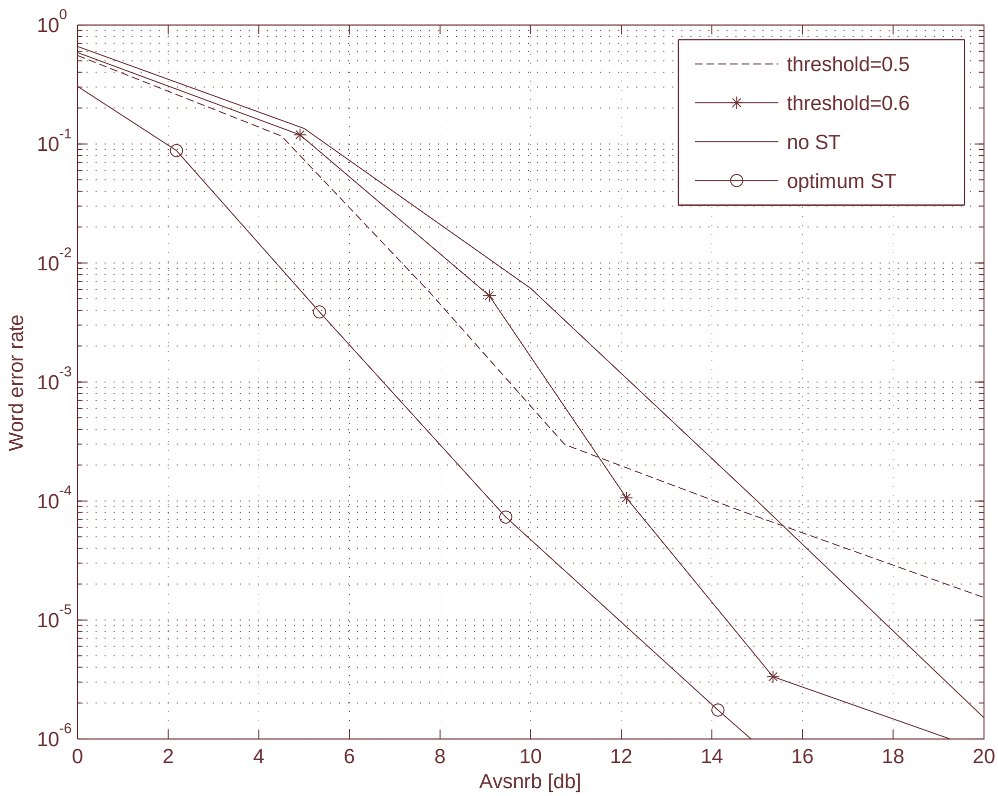

Fig. 34. Performance of a rate 7/128 code in the 4-block fading channel using selective transmission

since in the best case scenario we transmit only one of the four blocks. The scheme with one threshold achieves an improvement on the performance of the order of 3 *db* at high SNR. That is not satisfactory and that is why later in the chapter we use a second threshold.

As mentioned before we need codes with good performance and high throughput efficiency. The performance of the codes was just presented so the next thing that we deal with is throughput efficiency. The definition of throughput efficiency that we use is a bit different that the one used in [20].

Let N be the total number of blocks transmitted which compose a codeword and U the average number of blocks transmitted in order to get a codeword without errors (unless we transmit all the codeword). Then the throughput efficiency of our scheme is given by

$$\eta \triangleq \frac{N}{E[U]}\frac{k}{n} \tag{5.4}$$

where $E[U]$ is the expectation of U and k/n is the rate of the code. The code with rate k/n in the N-block fading channel with no selective transmission has constant throughput efficiency equal to the code rate k/n. Let A_1^c, A_1^e be the events that the first block of a codeword when it is transmitted contains no error and contains errors, respectively. Let B_i^c, B_i^e denote the events that the transmission of i blocks contains no error and contains errors, respectively. Then

$$\begin{aligned} Pr(A_1^c) + Pr(A_1^e) &= 1 \\ Pr(B_i^c) + Pr(B_i^e) &= 1 \end{aligned} \tag{5.5}$$

The average number of blocks transmitted is

$$E[U] = 1.Pr(A_1^c) + 2.Pr(A_1^e B_2^c) + \ldots + N.Pr(A_1^e B_N^c) \tag{5.6}$$

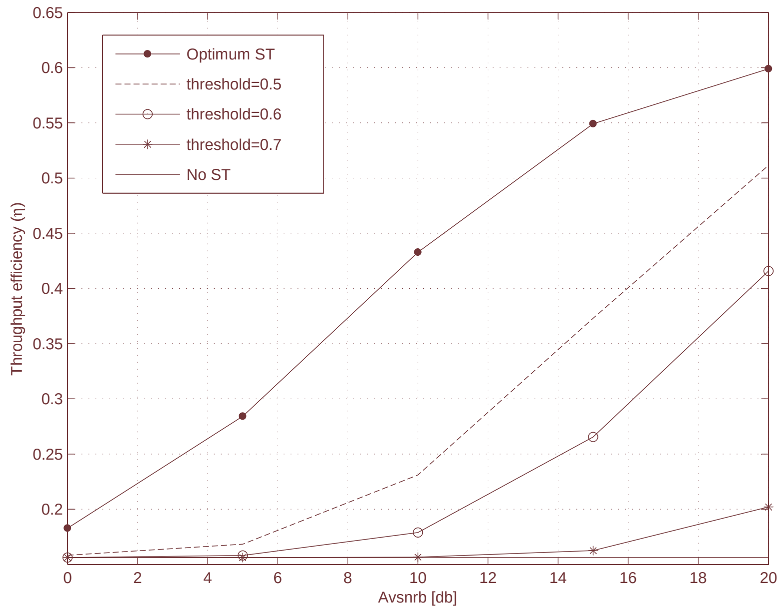

Fig. 35. Throughput efficiency of the rate 5/32 code

If any of the above events have errors or not is decided by the receiver using a reliability measure. The joint probabilities in (5.6) are hard to evaluate so we present the throughput efficiencies for the previous codes based on simulations. The lower bound for the throughput efficiency is given by the rate k/n code (=k/n) with no selective transmission and the upper bound by the same code using ideal selective transmission. For the previous codes with rates 5/32,6/64,7/80 and 7/128 we present the throughput efficiency plots in Figs. 35,36,37 and 38, respectively.

In Figs. 35,36,37 and 38 the codes that use $threshold = 0.6$ achieve high throughput efficiency only for high SNR. In Fig. 39 we presented the improvement on the throughput efficiency of a 6/64 rate code with selective transmission that we have for low SNR when we use different threshold.

The selective transmission scheme compared to the non selective transmission scheme uses less energy for better performance. The average number of blocks trans-

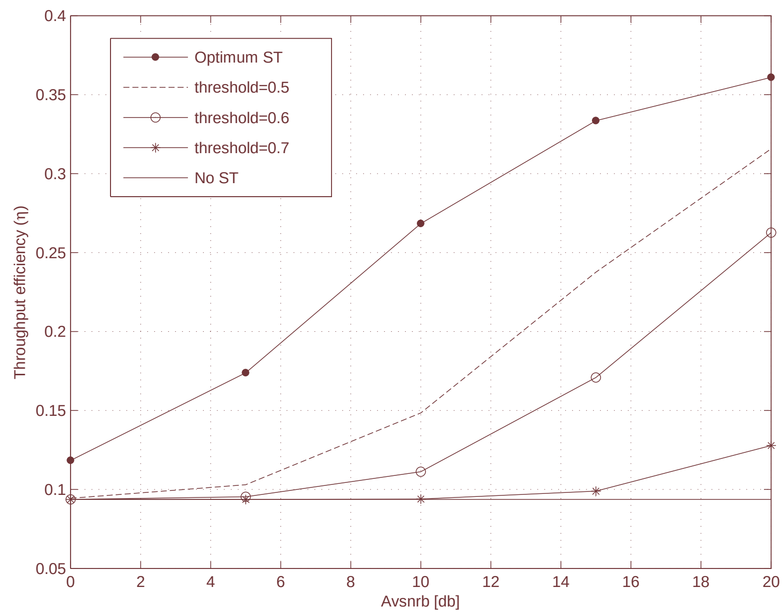

Fig. 36. Throughput efficiency of the rate 6/64 code

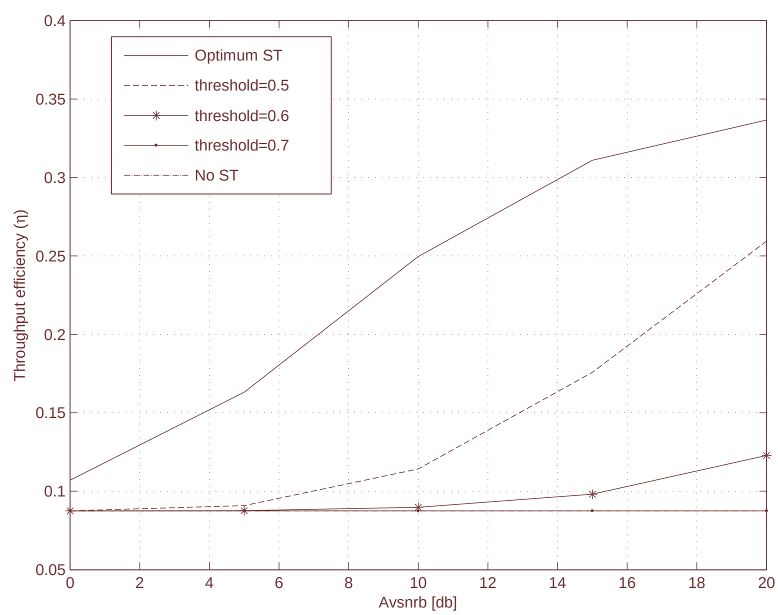

Fig. 37. Throughput efficiency of the rate 7/80 code

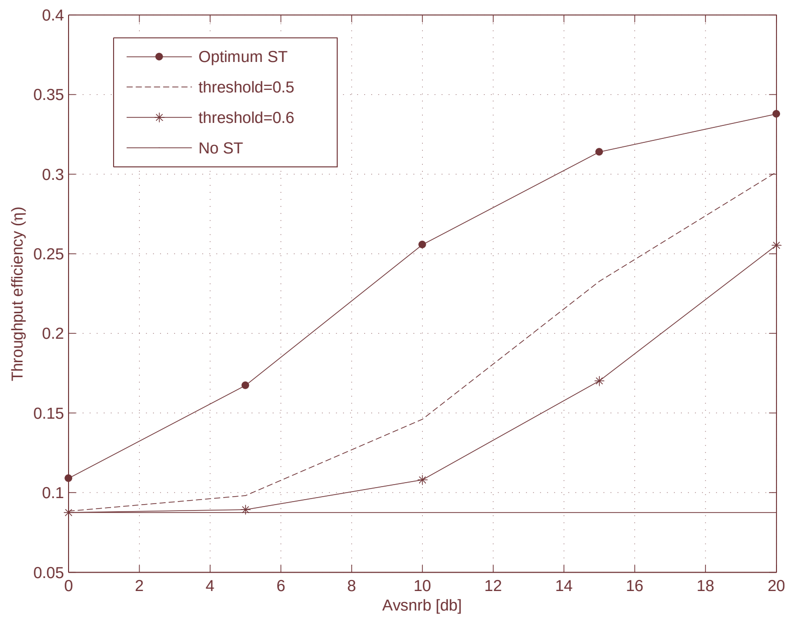

Fig. 38. Throughput efficiency of the rate 7/128 code

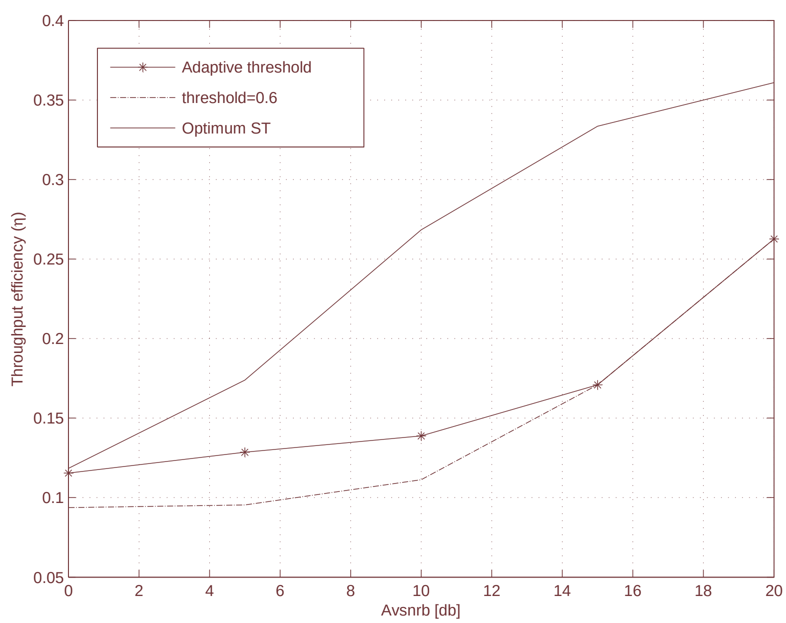

Fig. 39. Throughput efficiency of 6/64 code-constant threshold vs adaptive threshold

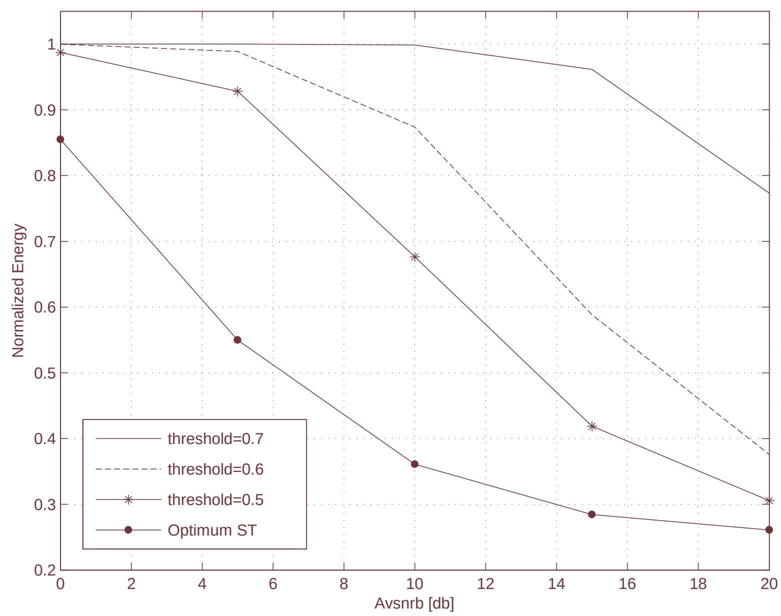

Fig. 40. Normalized energy of 5/32 code

mitted is given in (5.6). For the N-block fading channel $N = 2, 4$ and for the k/n rate code every codeword has energy $E_c = \frac{k}{n} E_b$ where E_b is the energy per information bit. Every block has energy E_c/N. If the average number of blocks transmitted is close to *one* then such a system can perform as well as the system that spends four times more energy. In Figs. 40,41 and 42 we present the average number of blocks transmitted over the total number (=4). The code with no selective transmission uses 4 blocks to transmit every codeword which means constant energy equal to one and the optimal selective transmission scheme uses the less energy.

An even greater increase of the throughput efficiency can result from increasing the complexity at the receiver by adding a second reliability measure. The receiver in this case must compute two reliability measures and compare each one with the corresponding threshold. The second reliability measure that we used is the normalized difference multiplied by the variance of the vector V ((5.2)). The values of the second

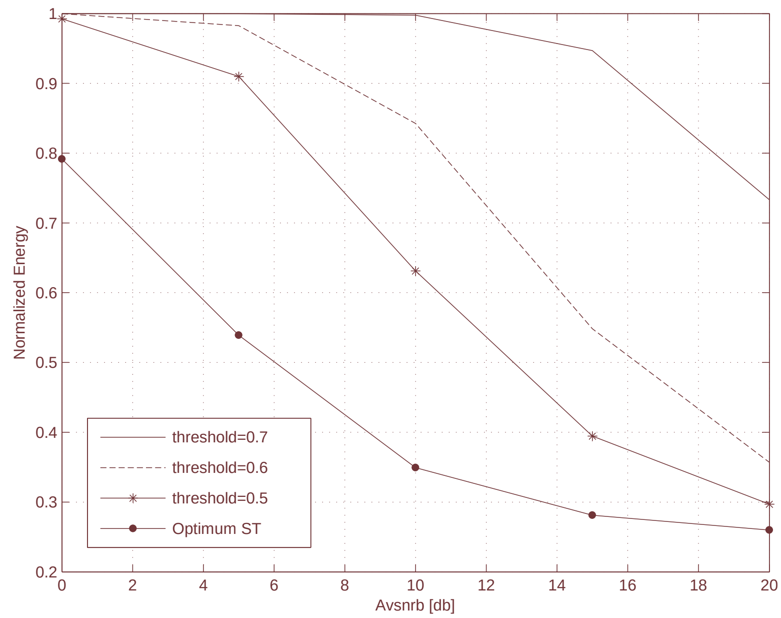

Fig. 41. Normalized energy of 6/64 code

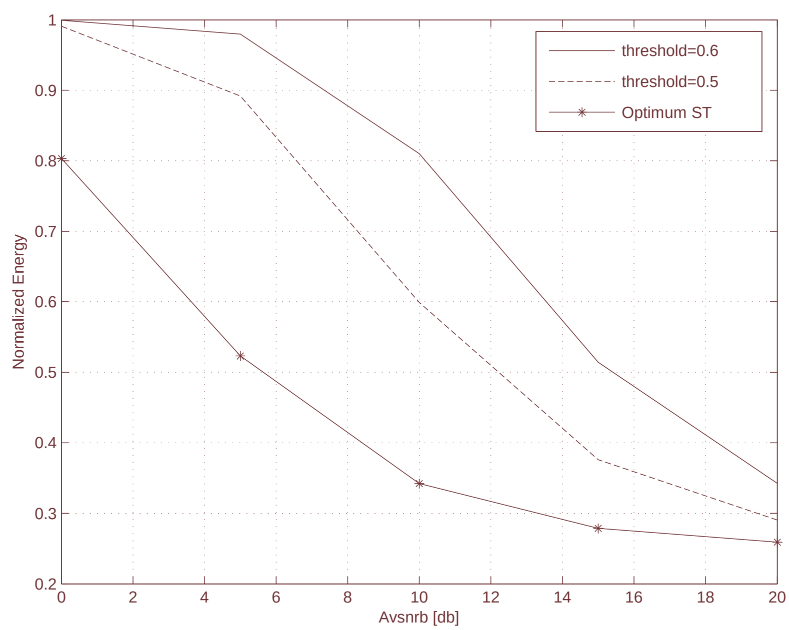

Fig. 42. Normalized energy of 7/128 code

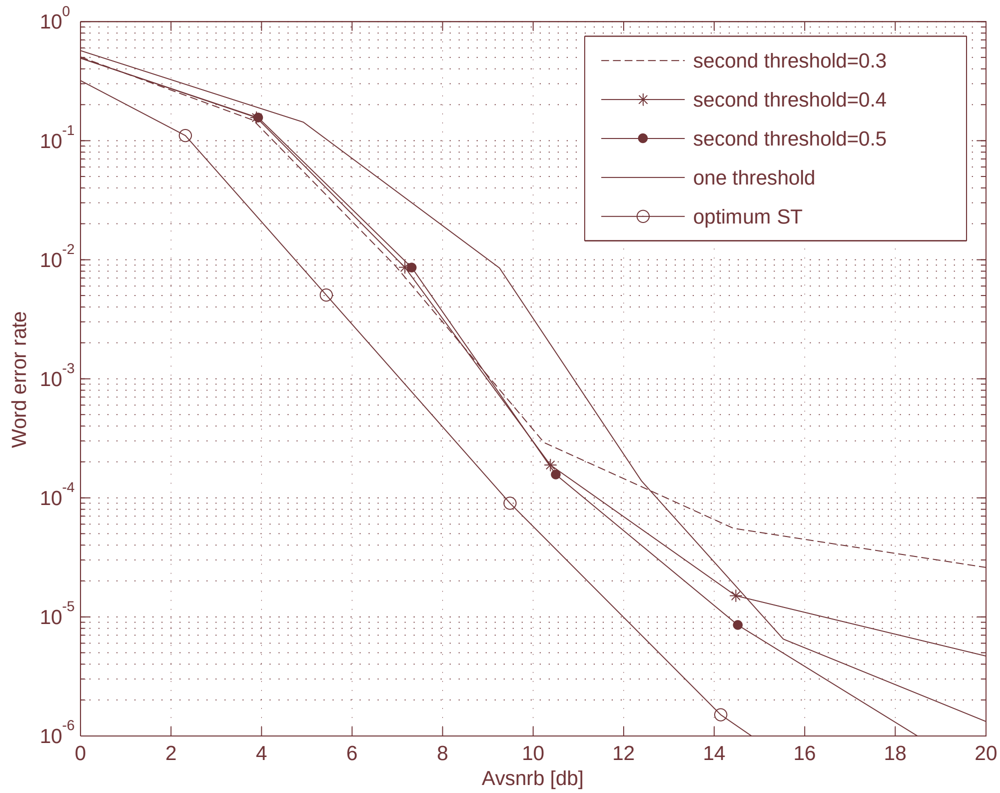

Fig. 43. Performance of the 6/64 rate code using two reliability measures

threshold that we used for our simulations are $0.3, 0.4$ and 0.5. We used an adaptive first threshold with constant the second threshold for all signal to noise ratios. The performance of the rate 6/64 code when we use two thresholds is in Fig. 43. When the second threshold is 0.5 we have a very good performance. The throughput efficiency and the normalized energy of the same code are in Figs. 44 and 45, respectively. It is obvious now that the second reliability measure increases significantly the throughput efficiency compared to one reliability measure.

To end this chapter the selective transmission scheme is used on 2-block fading codes. Although the expected maximum gain is 3 *db* any improvement of the performance without a trade off is welcome. In Figs.46, 47 and 48 we have the performance,

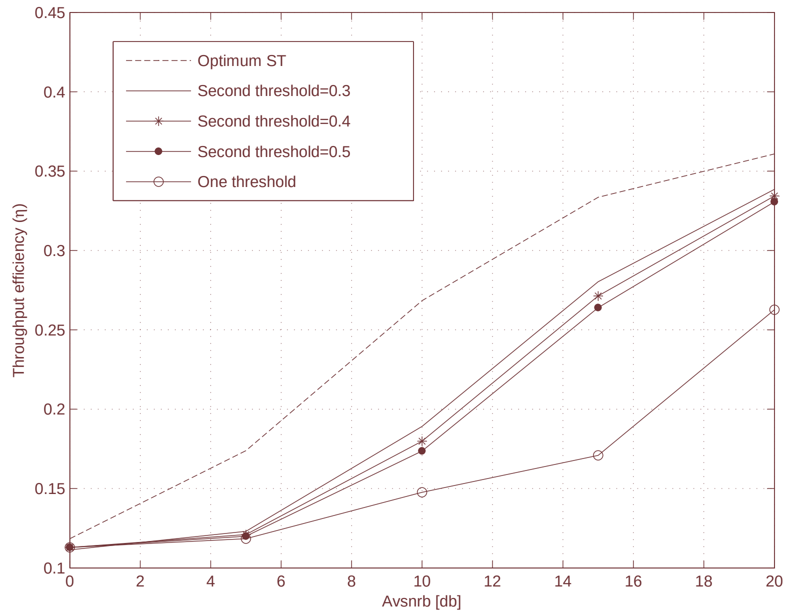

Fig. 44. Throughput efficiency of the 6/64 rate code using two reliability measures

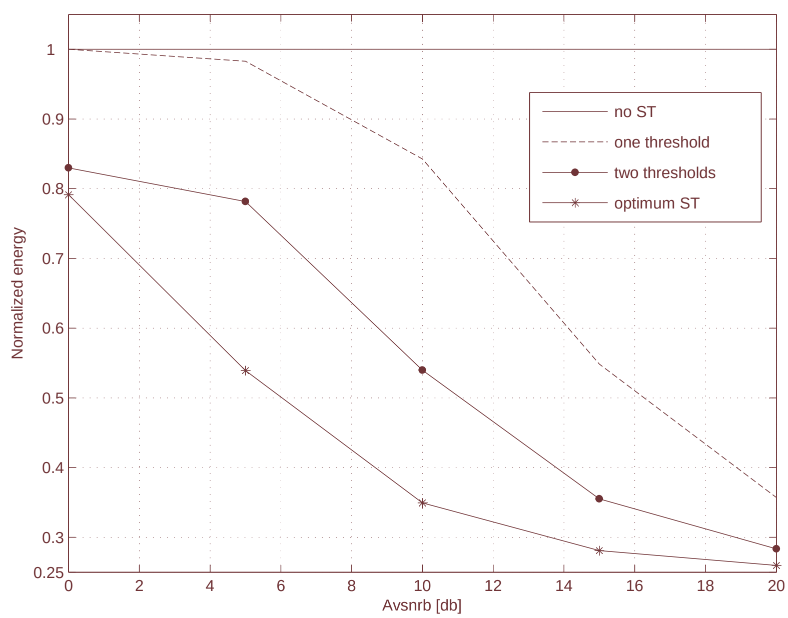

Fig. 45. Normalized energy of the 6/64 rate code using two reliability measures

the throughput efficiency and normalized energy, for the rate 5/32 code, respectively. For high SNR the use of two thresholds gives an even better performance than one threshold as in Fig. 49. The normalized energy for the same code with two thresholds is in Fig. 50. Finally in Figs. 51 and 52 we have the performance and normalized energy of a rate 6/64 code in the 2-block fading channel.

To sum up, in this chapter we presented the selective transmission scheme. We start with the transmission of one block and by using one or two reliability measures at the receiver we evaluate the received data. Accordingly the receiver decodes or requests for the next block. The selective transmission scheme with two reliability measures performs very close to a lower bound of an optimum retransmission scheme for high SNR.

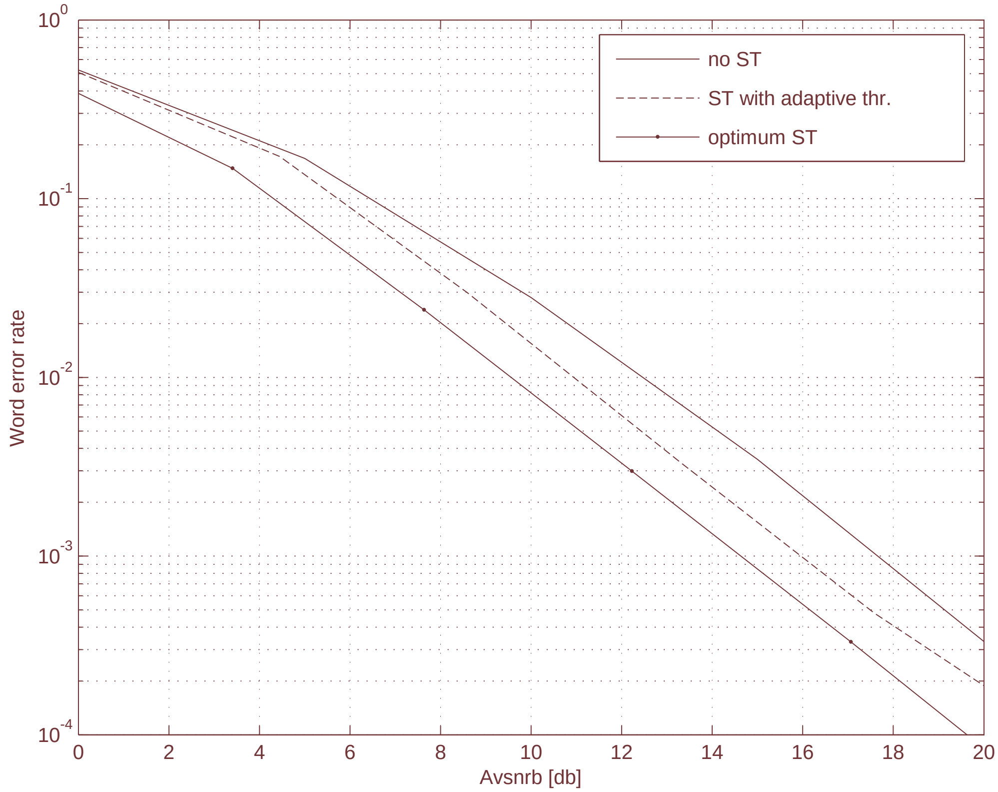

Fig. 46. Performance of the 5/32 rate code in a 2-block fading channel (one threshold)

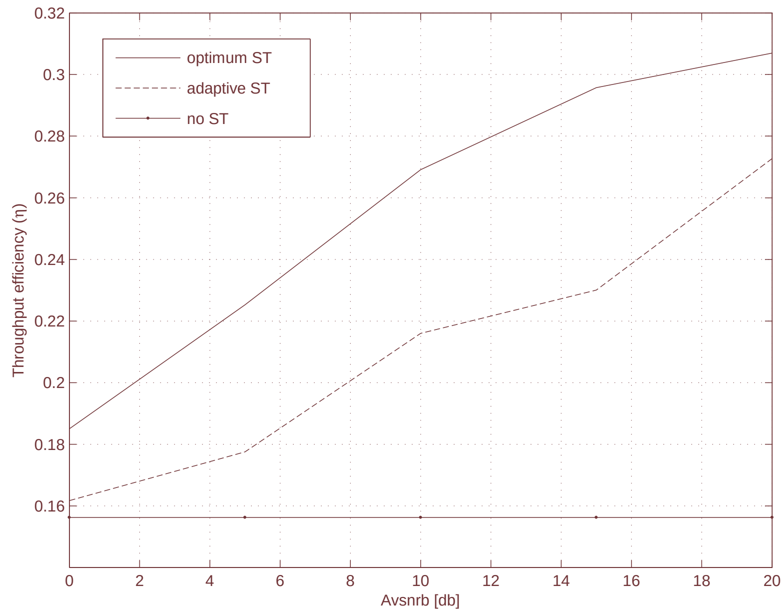

Fig. 47. Throughput efficiency of the 5/32 rate code in a 2-block fading channel (one threshold)

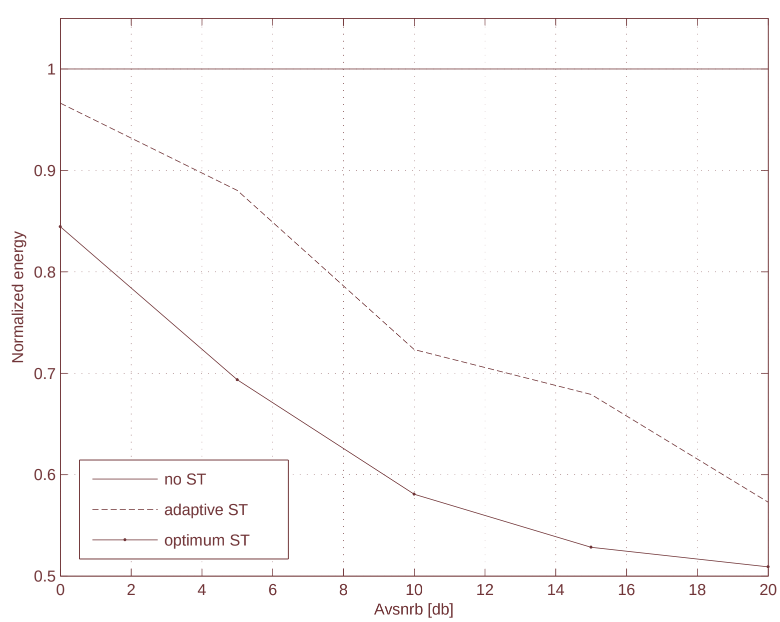

Fig. 48. Normalized energy of the 5/32 rate code in a 2-block fading channel (one threshold)

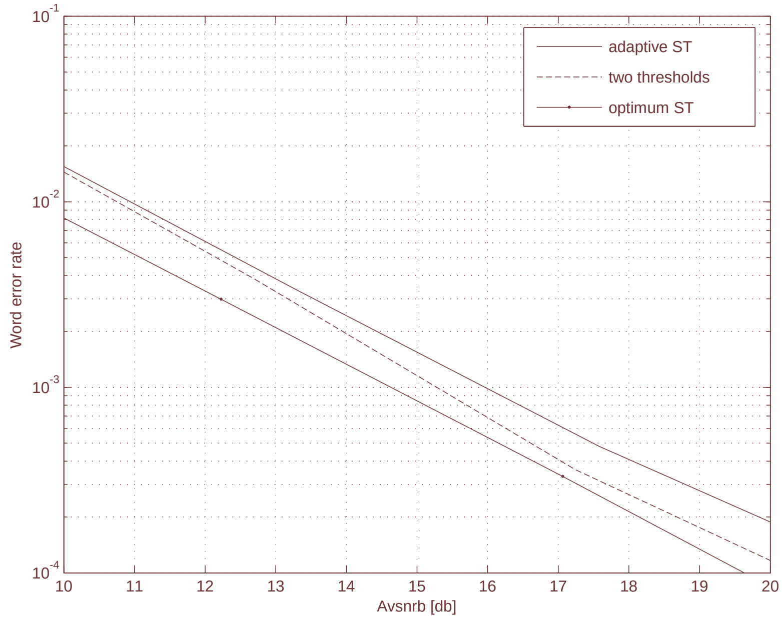

Fig. 49. Performance of the 5/32 rate code in a 2-block fading channel (two thresholds)

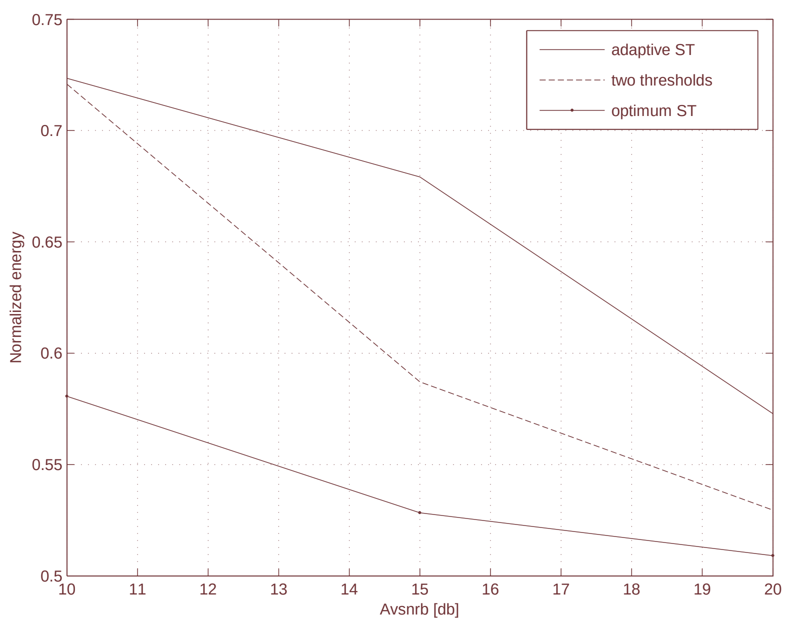

Fig. 50. Normalized energy of the 5/32 rate code in a 2-block fading channel (two thresholds)

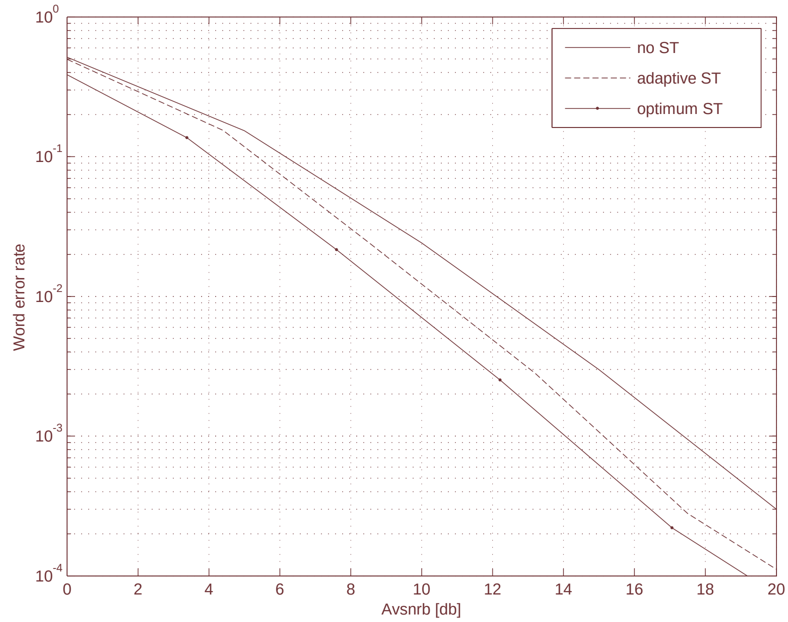

Fig. 51. Performance of the 6/64 rate code in a 2-block fading channel

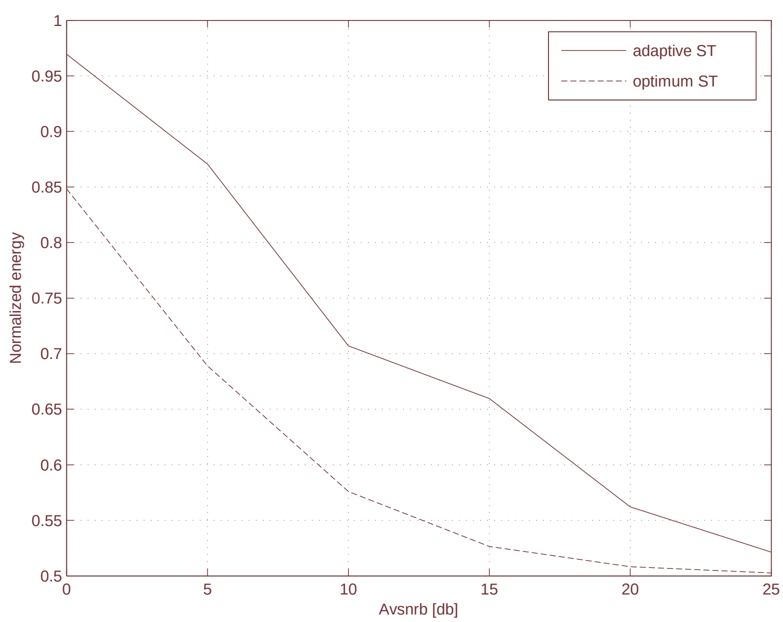

Fig. 52. Normalized energy of the 6/64 rate code in a 2-block fading channel

CHAPTER VI

CODES WITH $\rho_{max} \neq 1$ FOR THE 2-BLOCK FADING CHANNEL

Although in the previous chapters we found easy ways to create codes sometimes for fast or big changes on the channel we need to find a code with $\rho_{max} \neq 1$. By plotting the pairwise error probability we can see that the performance is not very sensitive to changes of ρ_{max}. It is very easy to show which codes are not optimal which means which combination of columns is to be avoided in order to get a good code. These codes have the worse performance according to the above probability of error because they have absolute cross correlations equal to one. So from now on, the term *non optimal code* refers to codes with some cross correlations between the codewords equal to one.

Since we are dealing with the two block fading channel we must divide the Hadamard matrix into two blocks. What we are trying to do is to select $\frac{M}{2}$ columns from the Hadamard matrix to create a new first block and of course the remaining columns create the second block. What applies for the first block applies for the second block too due to construction(as far as metric spectrum). Any reshuffling of the columns in a block results to a reshuffling of the rows, i.e. the code is the same. The following algorithm gives all the combinations of columns that result to a non optimal code.

1. We start with the Hadamard matrix $\mathcal{H}_M$ divided as follows:

$$\mathcal{H}_M = \left(\begin{array}{c|c|c|c|c|c} \frac{M}{2} & \cdots & 8 & 4 & 2 & 2 \end{array} \right) \tag{6.1}$$

   where the numbers note the number of columns involved in this grouping. Then select one of the two columns from the group at the right and move to the left by selecting the corresponding or complementary columns from the block to the

Table I. Algorithm for the H_{16} matrix

Step 1	16 15	14 13	12 11	10 9	8 7	6 5	4 3	2 1
Step 2	8	7	6	5	4	3	2	1
Step 3	4		3		2		1	
Step 4	2				1			

left. This involves k-1 steps to the left and a multiplication factor of two because at every step we can choose either the corresponding or the complementary columns. This step gives 2^{k-1} non optimal codes. By the terms corresponding and complementary columns, we mean that if we choose the right column from the most right block then from the next block to the left we can choose either the right column(corresponding) or the left(complementary).

2. Repeat the previous step but first group the previous matrix into blocks of two columns and consider each block as a column. Stop when there are only two columns in the final matrix.

In Table I we see how the algorithm works for the H_{16} matrix. The vertical lines show the grouping of the columns according to (6.1).

Using this algorithm we present in Table II as an example the non optimal codes using the $\mathcal{H}_8$ Hadamard matrix. Unfortunately some of the combinations include the systematic codes. The systematic columns of the Hadamard matrix H_M are M/2,M/4,. . .,2,1st (counting starts from zero).

As far as the rest of the codes in the case of the H_8 matrix are all optimal. In Fig. 53 we see how the pairwise error probability changes with ρ. In Fig. 54 we can see the performance of an optimal code in the quasi-static channel and in the two block fading channel. The achieving diversity is a result of the exploitation of the

Table II. Non-optimum codes using the H_8 Hadamard matrix

1^{st} block	2^{nd} block
1234	5678
1256	3478
1278	3456
1357	2468
1368	2457
1458	2367
1467	2358

independent fading between the first and the second part of the codeword.

In Figs. 55 and 56 we present the pairwise error probability of an optimum code and the pairwise error probability of the worst code (from the codes with $\rho_{max} \neq 1$). If we avoid codes that have absolute cross correlations equal to one then all the other codes achieve diversity but not all of them in an optimal way. The search for optimal codes is sometimes very difficult due to complexity.

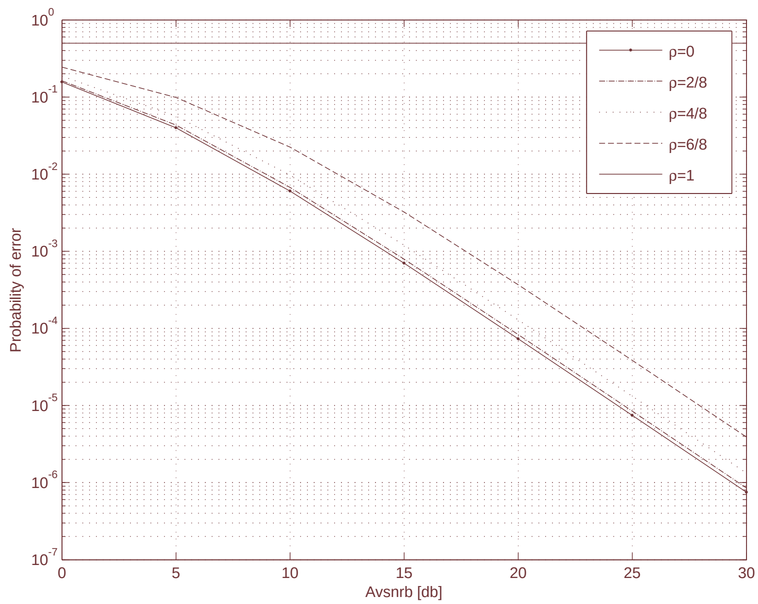

Fig. 53. Pairwise error probability for all the values of ρ when n=16

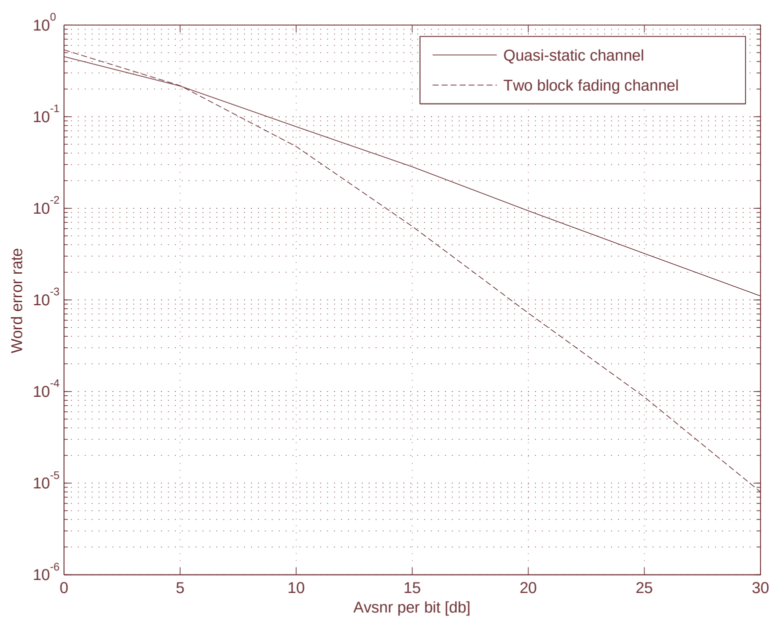

Fig. 54. Performance of an optimum H_8 Hadamard code in quasi-static and two block fading channel

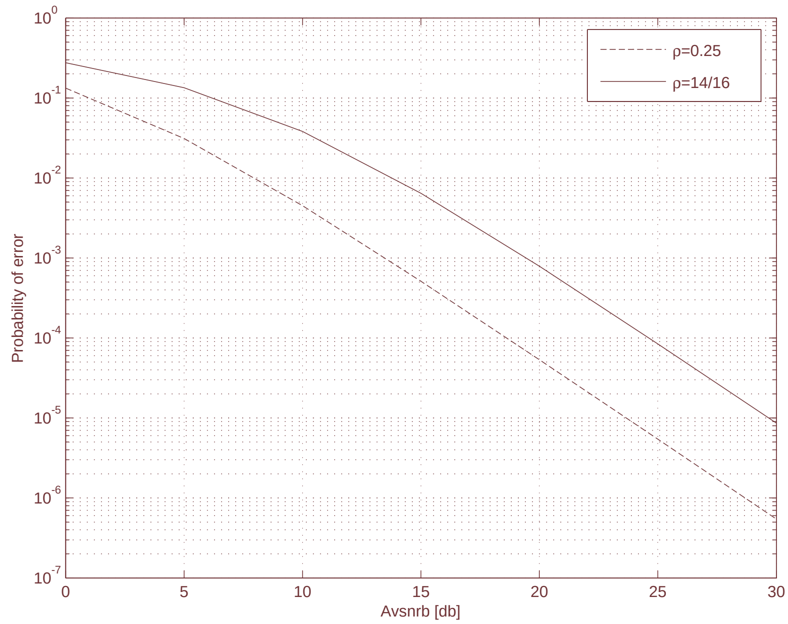

Fig. 55. Pairwise error probability of a code with $\rho_{max} = 0.25$ (optimum) and $\rho_{max} = 14/16$ where $k = 5$

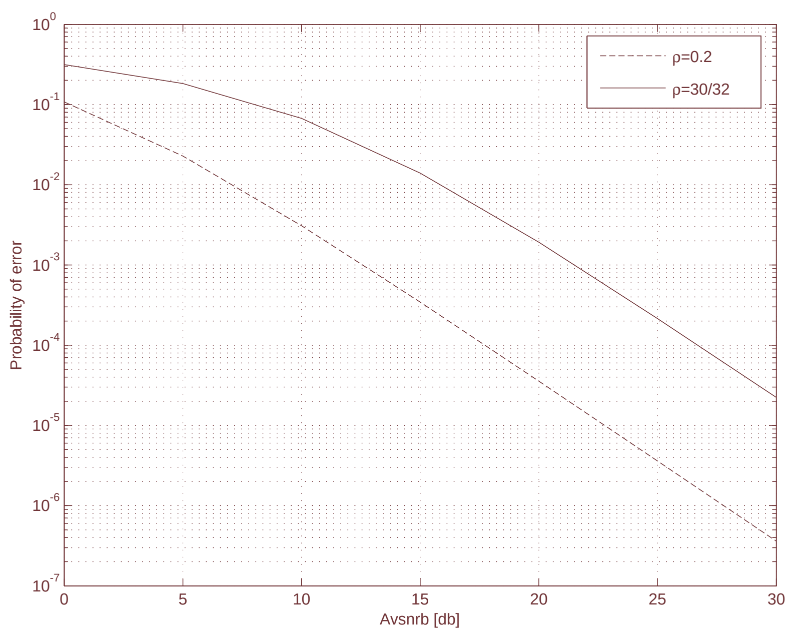

Fig. 56. Pairwise error probability of a code with $\rho_{max} = 0.2$ (optimum) and $\rho_{max} = 30/32$ where $k = 6$

CHAPTER VII

CONCLUSIONS

This work deals with the design and analysis of linear block codes based on Hadamard matrices for block fading channels. Codes that are easy to find without search methods are presented. Codes with $\rho_{max} = 1/2$ or with bounded ρ_{max} achieve diversity by exploiting the special characteristics of the block fading channel.

Analytically, codes that can be derived only by a simple selection of columns from Hadamard matrices are presented. These codes do not have optimum metric spectrum but their performance is very close. Furthermore easy transformations of these codes can take place in order to use the code in different channel (quasi-static,2-block or 4-block) with different rates.

Then we present a selective transmission scheme that exploits the diversity that the code achieves in a block fading environment and improves the performance and throughput efficiency of the code.

REFERENCES

[1] T. L. Marzetta and B. M. Hochwald, “Capacity of a mobile multiple-antenna communication link in rayleigh flat fading,” *IEEE Trans. Inform. Theory*, vol. 45, pp. 139–157, 1999.

[2] A. S. Y. Poon, D. N. C. Tse, and R. W. Brodersen, “An adaptive multiantenna transceiver for slowly flat fading channels,” *IEEE Trans. Commun.*, vol. 51, pp. 1820–1827, 2003.

[3] L. Zheng and D. N. Tse, “Diversity and freedom:a fundamental tradeoff in multiple antenna channels,” in *Proc., IEEE Int. Symp. on Inform. Theory*, Lausanne, Switzerland, p. 476, 2002.

[4] M. Uysal and C. N. Georghiades, “An efficient implementation of a maximum-likelihood detector for space-time block coded systems,” This paper was presented in part at 38th Allerton Conference on Communications, Control and Computing, University of Illinois, Urbana, October 2000, pp. 1–10.

[5] R. Knopp and P. Humblet, “Maximizing diversity on block fading channels,” in *Proc., IEEE Int. Conf. on Commun.*, Montreal, 1997, pp. 647–651.

[6] R. Knopp and P. A. Humblet, “On coding for block fading channels,” *IEEE Trans. Inform. Theory*, vol. 46, pp. 189–205, 2000.

[7] D. Lun, M. Medard, and I. Abou-Faycal, “An upper bound on the error probability of capacity-achieving signaling on broadband rayleigh fading channels,” *IEEE Trans. Veh. Tech.*, vol. 1, pp. 577–581, 2002.

[8] P. D. Papadimitriou, *Code Design Based on Metric Spectrum and Applications.* Ph.D. dissertation, Texas A&M University, College Station, 2004.

[9] P. D. Papadimitriou and C. N. Georghiades, "On binary code design for the non-coherent block fading channel," in *Proc., IEEE GLOBECOM Conference*, San Francisco, CA, 2003, pp. 1603–1607.

[10] W. C. Jakes, *Microwave Mobile Communications.* New York: Wiley, 1974.

[11] S. Rice, "Mathematical analysis of random noise," *Bell System Technical Journal*, vol. 23, pp. 282 –332, July 1944.

[12] S. Rice, "Mathematical analysis of random noise (continued)," *Bell System Technical Journal*, vol. 24, pp. 46–156, Jan 1945.

[13] S. Rice, "Statistical properties of a sine wave plus random noise," *Bell System Technical Journal*, vol. 27, pp. 109–157, Jan 1948.

[14] M. Ptzold and F. Laue, "Statistical properties of jakes fading channel simulator," *Vehicular Technology Conf. Rec. VTC98*, vol. II, pp. 712–718, May 1998.

[15] M. F. Pop and N. C. Beaulieu, "Limitations of sum of sinusoids fading channel simulators," *IEEE Trans*, vol. 49, no. 4, pp. 699–708, 2001.

[16] C. Xiao, Y. R. Zheng, and N. C. Beaulieu, "Second-order statistical properties of the wss jakes fading channel simulator," *IEEE Transactions on Communications*, vol. 50, pp. 888–891, June 2002.

[17] K. Pahlavan and A. Levesque, *Wireless Information Networks.* New York: Wiley, 1995.

[18] J. G. Proakis, *Digital Comminications.* 4th ed., Boston:McGraw-Hill, 2001.

[19] Y.-M. Wang and S. Lin, "A modified selective-repeat type-ii hybrid arq system and its performance analysis," *IEEE Trans. Commun.*, vol. Com-31, May 1983.

[20] S. Lin and P. S. Yu, "A hybrid arq scheme with parity retransmission for error control of satellite channels," *IEEE Trans. Commun.*, vol. Com-30, July 1982.

[21] J. J. Metzner, "Improvements in block retransmission schemes," *IEEE Trans. Commun.*, vol. Com-27, February 1979.

APPENDIX A

PROOF OF THE PAIRWISE PROBABILITY FOR THE 2-BLOCK FADING CHANNEL

The maximum likelihood detector of the problem can be shown to be

$$l\left(\boldsymbol{d}\right) = \arg\max_{\underline{\boldsymbol{d}}} \left(\left|{\boldsymbol{r_1}}^H \boldsymbol{d_1}\right|^2 + \left|{\boldsymbol{r_2}}^H \boldsymbol{d_2}\right|^2 \right) \tag{A.1}$$

where the indexes 1 and 2 are for the first and second block of the codeword respectively. Supposing the q^{th} codeword $\boldsymbol{d_q}$ was transmitted, the pairwise error probability is the probability that the detector will choose say the $\boldsymbol{d_m}$ codeword. So

$$\begin{aligned} &P_2^w = P\left(l\left(\boldsymbol{d_q}\right) - l\left(\boldsymbol{d_m}\right) < 0 | \boldsymbol{d_q} \quad transmitted \right) = \\ &P\left(\left|{\boldsymbol{r_1}}^H \boldsymbol{d_{q1}}\right|^2 + \left|{\boldsymbol{r_2}}^H \boldsymbol{d_{q2}}\right|^2 - \left|{\boldsymbol{r_1}}^H \boldsymbol{d_{m1}}\right|^2 - \left|{\boldsymbol{r_2}}^H \boldsymbol{d_{m2}}\right|^2 < 0 | \boldsymbol{d_q} \quad transmitted \right) \end{aligned} \tag{A.2}$$

This is a problem solved in [18] with

$$D = \left|\underbrace{{\boldsymbol{r_1}}^H \boldsymbol{d_{q1}}}_{X_1}\right|^2 + \left|\underbrace{{\boldsymbol{r_2}}^H \boldsymbol{d_{q2}}}_{X_2}\right|^2 - \left|\underbrace{{\boldsymbol{r_1}}^H \boldsymbol{d_{m1}}}_{Y_1}\right|^2 - \left|\underbrace{{\boldsymbol{r_2}}^H \boldsymbol{d_{m2}}}_{Y_2}\right|^2 \tag{A.3}$$

and

$$\begin{aligned} P_b = Q_1(\alpha, b) - I_0(\alpha b) exp[-\frac{1}{2}(\alpha^2 + b^2)] \\ + \frac{I_0(\alpha b) exp\left[-\frac{1}{2}(\alpha^2 + b^2)\right]}{(1 + \upsilon_2/\upsilon_1)^3} \sum_{k=0}^{1} \begin{pmatrix} 3 \\ \mathrm{k} \end{pmatrix} \left(\frac{\upsilon_2}{\upsilon_1}\right)^k \end{aligned} \tag{A.4}$$

where

$$Q_1(\alpha, b) = \int_b^\infty xexp[-\frac{1}{2}(\alpha^2 + x^2)]I_0(\alpha x)dx \tag{A.5}$$

$$I_0(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{2k}}{(k!)^2}, \ x \geqslant 0 \tag{A.6}$$

In our case

$$\begin{aligned} \upsilon_1 &= \sqrt{\tfrac{E\sigma_a^2}{4u^2} + \tfrac{1}{n^2 u(1-\rho^2)}} - \tfrac{E\sigma_a^2}{2u} \\ \upsilon_2 &= \sqrt{\tfrac{E\sigma_a^2}{4u^2} + \tfrac{1}{n^2 u(1-\rho^2)}} + \tfrac{E\sigma_a^2}{2u} \end{aligned} \tag{A.7}$$

and

$$\alpha = b = 0 \tag{A.8}$$

where $u = En\sigma^2\sigma_\alpha^2 + 2\sigma^4$. Now from A.4 we get

$$P_2^w = \frac{1}{2} - \frac{3}{4}\sqrt{\frac{\Lambda^2\left(1-\rho^2\right)}{16 + 8\Lambda + \Lambda^2\left(1-\rho^2\right)}} + \frac{1}{4}\left(\sqrt{\frac{\Lambda^2\left(1-\rho^2\right)}{16 + 8\Lambda + \Lambda^2\left(1-\rho^2\right)}}\right)^3 \tag{A.9}$$

where $\Lambda = \frac{\sigma_\alpha^2}{\sigma^2}nE$

APPENDIX B

$\rho_{max} = 1/2$ CODES

In this appendix we present some of the rate k/M block codes for the 2,4-block fading channel using *method* 1 (sections A,B) and *method* 2 (subsection 2). The codes are presented in the form of a row vector that contains the indices of H_M.

1. Method 1

- 2-block codes

- k=3

$$c_3 = [1, 2, 3, 8, 4, 5, 6, 7] \tag{B.1}$$

- k=4

$$c_4 = [1, 2, 3, 4, 5, 6, 15, 16, 7, 8, 9, 10, 11, 12, 13, 14] \tag{B.2}$$

- k=5

$$\begin{aligned} c_5 = [&1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 29, 30, 31, 32, 13, 14, 15, 16, \\ &17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] \end{aligned} \tag{B.3}$$

- k=6

$$\begin{aligned} &c_6{=}[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22, \\ &23,24,57,58,59,60,61,62,63,64,25,26,27,28,29,30,31,32,33,34, \\ &35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56] \end{aligned} \tag{B.4}$$

- k=7

$$c_7=[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112] \tag{B.5}$$

- k=8

$$c_8=[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,225,226,227,228,229,230,231,232,233,234,235,236,237,238,239,240,241,242,243,244,245,246,247,248,249,250,251,252,253,254,255,256,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,143,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180,181,182,183,184,185,186,187,188,189,190,191,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,208,209,210,211,212,213,214,215,216,217,218,219,220,221,222,223,224] \tag{B.6}$$

- 4-block codes

- k=5

$$c_5\text{=}[1,2,3,4,5,22,31,16,9,10,11,12,13,30,7,24,17,18,19,20,21,6,15,32,25,26,27,28,29,14,23,8] \tag{B.7}$$

- k=6

$$c_6\text{=}[1,2,3,4,5,6,7,8,9,10,43,44,61,62,31,32,17,18,19,20,21,22,23,24,25,26,59,60,13,14,47,48,33,34,35,36,37,38,39,40,41,42,11,12,29,30,63,64,49,50,51,52,53,54,55,56,57,58,27,28,45,46,15,16] \tag{B.8}$$

- k=7

$$c_7\text{=}[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,85,86,87,88,121,122,123,124,61,62,63,64,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,117,118,119,120,25,26,27,28,93,94,95,96,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,21,22,23,24,57,58,59,60,125,126,127,128,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,53,54,55,56,89,90,91,92,29,30,31,32] \tag{B.9}$$

- k=8

c_8=[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,
25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,169,170,171,172,173,
174,175,176,241,242,243,244,245,246,247,248,121,122,123,124,125,126,
127,128,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,
85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,233,
234,235,236,237,238,239,240,49,50,51,52,53,54,55,56,185,186,187,188,
189,190,191,192,129,130,131,132,133,134,135,136,137,138,139,140,141,
142,143,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,
159,160,161,162,163,164,165,166,167,168,41,42,43,44,45,46,47,48,113,
114,115,116,117,118,119,120,249,250,251,252,253,254,255,256,193,194,
195,196,197,198,199,200,201,202,203,204,205,206,207,208,209,210,211,
212,213,214,215,216,217,218,219,220,221,222,223,224,225,226,227,228,
229,230,231,232,105,106,107,108,109,110,111,112,177,178,179,180,181,
182,183,184,57,58,59,60,61,62,63,64]

(B.10)

2. Method 2

- 2-block codes

- k=3

$$c_3 = [1, 2, 3, 8, 4, 5, 6, 7] \tag{B.11}$$

- k=4

$$c_4 = [1, 2, 3, 8, 9, 10, 11, 16, 4, 5, 6, 7, 12, 13, 14, 15] \tag{B.12}$$

- k=5

$$c_5\text{=[1,2,3,8,9,10,11,16,17,18,19,24,25,26,27,32,4,5,6,7,12,13,14,}\\ \text{15,20,21,22,23,28,29,30,31]} \tag{B.13}$$

- k=6

$$c_6\text{=[1,2,3,8,9,10,11,16,17,18,19,24,25,26,27,32,33,34,35,40,41,42,}\\ \text{43,48,49,50,51,56,57,58,59,64,4,5,6,7,12,13,14,15,20,21,22,23,28,}\\ \text{29,30,31,36,37,38,39,44,45,46,47,52,53,54,55,60,61,62,63]} \tag{B.14}$$

- k=7

$$c_7\text{=[1,2,3,8,9,10,11,16,17,18,19,24,25,26,27,32,33,34,35,40,41,42,}\\ \text{43,48,49,50,51,56,57,58,59,64,65,66,67,72,73,74,75,80,81,82,83,88,}\\ \text{89,90,91,96,97,98,99,104,105,106,107,112,113,114,115,120,121,122,}\\ \text{123,128,4,5,6,7,12,13,14,15,20,21,22,23,28,29,30,31,36,37,38,39,44,}\\ \text{45,46,47,52,53,54,55,60,61,62,63,68,69,70,71,76,77,78,79,84,85,86,87,}\\ \text{92,93,94,95,100,101,102,103,108,109,110,111,116,117,118,119,124,125,}\\ \text{126,127]} \tag{B.15}$$

- k=8

$$c_8=[1,2,3,8,9,10,11,16,17,18,19,24,25,26,27,32,33,34,35,40,41,42,43,48,49,50,51,56,57,58,59,64,65,66,67,72,73,74,75,80,81,82,83,88,89,90,91,96,97,98,99,104,105,106,107,112,113,114,115,120,121,122,123,128,129,130,131,136,137,138,139,144,145,146,147,152,153,154,155,160,161,162,163,168,169,170,171,176,177,178,179,184,185,186,187,192,193,194,195,200,201,202,203,208,209,210,211,216,217,218,219,224,225,226,227,232,233,234,235,240,241,242,243,248,249,250,251,256,4,5,6,7,12,13,14,15,20,21,22,23,28,29,30,31,36,37,38,39,44,45,46,47,52,53,54,55,60,61,62,63,68,69,70,71,76,77,78,79,84,85,86,87,92,93,94,95,100,101,102,103,108,109,110,111,116,117,118,119,124,125,126,127,132,133,134,135,140,141,142,143,148,149,150,151,156,157,158,159,164,165,166,167,172,173,174,175,180,181,182,183,188,189,190,191,196,197,198,199,204,205,206,207,212,213,214,215,220,221,222,223,228,229,230,231,236,237,238,239,244,245,246,247,252,253,254,255] \tag{B.16}$$

- 4-block codes

- k=5

$$c_5=[1,2,3,4,5,22,31,16,9,10,11,12,13,30,7,24,17,18,19,20,21,6,15,32,25,26,27,28,29,14,23,8] \tag{B.17}$$

- k=6

$$c_6=[1,2,3,4,5,22,31,16,33,34,35,36,37,54,63,48,9,10,11,12,13,30,7,24,41,42,43,44,45,62,39,56,17,18,19,20,21,6,15,32,49,50,51,52,53,38,47,64,25,26,27,28,29,14,23,8,57,58,59,60,61,46,55,40] \tag{B.18}$$

- k=7

$$c_7=[1,2,3,4,5,22,31,16,33,34,35,36,37,54,63,48,65,66,67,68,69,86,\\
95,80,97,98,99,100,101,118,127,112,9,10,11,12,13,30,7,24,41,42,43,\\
44,45,62,39,56,73,74,75,76,77,94,71,88,105,106,107,108,109,126,103,\\
120,17,18,19,20,21,6,15,32,49,50,51,52,53,38,47,64,81,82,83,84,85,70,\\
79,96,113,114,115,116,117,102,111,128,25,26,27,28,29,14,23,8,57,58,\\
59,60,61,46,55,40,89,90,91,92,93,78,87,72,121,122,123,124,125,110,119,104] \tag{B.19}$$

- k=8

$$c_8=[1,2,3,4,5,22,31,16,33,34,35,36,37,54,63,48,65,66,67,68,69,86,\\
95,80,97,98,99,100,101,118,127,112,129,130,131,132,133,150,159,\\
144,161,162,163,164,165,182,191,176,193,194,195,196,197,214,223,\\
208,225,226,227,228,229,246,255,240,9,10,11,12,13,30,7,24,41,42,\\
43,44,45,62,39,56,73,74,75,76,77,94,71,88,105,106,107,108,109,126,\\
103,120,137,138,139,140,141,158,135,152,169,170,171,172,173,190,\\
167,184,201,202,203,204,205,222,199,216,233,234,235,236,237,254,\\
231,248,17,18,19,20,21,6,15,32,49,50,51,52,53,38,47,64,81,82,83,\\
84,85,70,79,96,113,114,115,116,117,102,111,128,145,146,147,148,149,\\
134,143,160,177,178,179,180,181,166,175,192,209,210,211,212,213,\\
198,207,224,241,242,243,244,245,230,239,256,25,26,27,28,29,14,23,\\
8,57,58,59,60,61,46,55,40,89,90,91,92,93,78,87,72,121,122,123,\\
124,125,110,119,104,153,154,155,156,157,142,151,136,185,186,187,\\
188,189,174,183,168,217,218,219,220,221,206,215,200,249,250,251,\\
252,253,238,247,232] \tag{B.20}$$

APPENDIX C

RATE CHANGE FOR $\rho_{MAX} = 1/2$ CODES

First we prove that the codes created in section 2 have cross correlations $\rho = \{0, 1/2\}$ and then prove that with rate change the ρ_{max} stays close to 1/2. For the 2-block fading channel in order to create a k/M rate code (we present the procedure only for the first block) we start from the $(k-1)/(M/2)$ rate code and the first block is

$$code_2 = \left(\begin{array}{c|c} code_1 & code_1 \\ \hline code_1 & -code_1 \end{array}\right) \tag{C.1}$$

where $code_1$ is the first block of the rate $(k-1)/(M/2)$ code with dimensions $M/2 \times M/4$ and $code_2$ is the first block of the rate k/M code. $Code_1$ has cross correlations $\rho = \{0, 1/2\}$ and corresponding Hamming distances $d_H = \{M/8, \{M/16, 3M/16\}\}$. Here we do not take into account the multiplicity of every Hamming distance (how many times a specific Hamming distance occurs between two codewords). We only want to prove that the cross correlations between codewords of $code_2$ remain 0 and 1/2.

The codewords of $code_2$ $d_i, i = 1, 2, \ldots M/2$ or the codewords $d_j, j = M/2 + 1, M/2+2, \ldots M$ between them have Hamming distances $d_H = \{M/4, \{M/8, 3M/8\}\}$ and from $\rho = |1 - 4d_H/M|$ the cross correlations are $\rho = \{0, 1/2\}$. It is obvious that the Hamming distance between any codeword with i numbering and any codeword with j numbering is $d_H = M/4$ and as a result $\rho = 0$. So $code_2$ has a spectrum $\rho = \{0, 1/2\}$.

To prove how ρ_{max} changes with the rate of the code we start from the code

$$\left(\begin{array}{c} code_1 \\ \hline code_1 \end{array}\right) \tag{C.2}$$

which has $\rho_{max} = 1$ and we add two columns at a time until we create $code_2$. This means we start from a rate $k/(M/4)$ code and go to $k/(M/2)$ rate code per block. Since the procedure that we use here is recursive and starts with $code_1 = first\ block\ of\ H_8$ if we divide the code into blocks of 4 columns the Hamming distances between codewords are $d_H = \{1, 2, 3\}$ and if two codewords have one of those values in one block then they have the same value in the other block too. By taking this into consideration let's divide the second half of $code_2$ into blocks of $four$ columns and t is the number of columns added to the code in C.2. So $4m \leq t \leq 4(m+1), m = 0, 1, \ldots, M/16 - 1$.

Again we split the analysis to the code with codewords from $code_2$ with $d_i = 1, 2, \ldots, M/2$ and $d_j = M/2+1, M/2+2, \ldots, M$ (the cross correlation spectrum of these codes is the same) and the cross correlation spectrum between codewords with i and j numbering. For the first case and for a random m ($t = 4m$) the Hamming distances are $d_H = \{M/16 + m, M/8 + 2m, 3M/16 + 3m\}$ and from $\rho = |1 - 2d_H/n|$ we get $\rho = \{1/2, 0, 1/2\}$ respectively. For $t = 4m + 2$ the minimum Hamming distances are $d_H = \{M/16 + m, M/8 + 2m, 3M/16 + 3m\}$ and

$$\rho = \left\{ \left|1 - \frac{M+16m}{2(M+16m+8)}\right|, \left|1 - \frac{M+16m}{(M+16m+8)}\right|, \left|1 - \frac{3(M+16m)}{2(M+16m+8)}\right| \right\} \tag{C.3}$$

For $t = 4(m+1)$ the Hamming distances are $d_H = \{M/16 + m + 1, M/8 + 2(m+1), 3M/16 + 3(m+1)\}$ and $\rho = \{1/2, 0, 1/2\}$. For the case of codewords with i and j numbering the Hamming distances are for every addition of two columns

$d_H = \{0, 2, 4, \ldots, M/4\}$ and the corresponding cross correlations are

$$\rho = \{1, |1 - 16/(M + 8)|, |1 - 32/(M + 16)|, \ldots, 0\} \quad \text{(C.4)}$$

Combining the previous results in order to find ρ_{max} we get the figures 13,14 and 15.

APPENDIX D

MATLAB CODES

1. This program creates codes for the 2-block fading channel using method-1 and method- 2 for k=7 and n=88

```
close all;
clear;
k=7;
M=2^k;
n=M/2;
cor=44; %number of columns per block
H=hadamard(M)*-1;
%**********************method-1**********************
u=1;
for i=1:3*M/8
    colnmd1b1(u)=i;
    colnmd1b2(u)=mod(i+M/2,M);
    u=u+1;
end
for i=(M-M/8+1):M
    colnmd1b1(u)=i;
    colnmd1b2(u)=mod(i+M/2,M);
    u=u+1;
end
```

```
colnmd1b2=sort(colnmd1b2);
for i=1:length(colnmd1b1)
    codemd1b1(:,i)=H(:,colnmd1b1(i));
    codemd1b2(:,i)=H(:,colnmd1b2(i));
end
code788md1=[codemd1b1,codemd1b2];
%**********************************************************
%Kf:all the 2^k binary combinations of length k
for i=2:M K(i)=i-1;end
K=dec2bin(K,k);
for i=1:M
    for j=1:k
        Kf(i,j)=str2double(K(i,j));
    end
end
%**************************method-2************************
M2=8;
u=1;
for i=1:3*M2/8
    colnmd2b1(u)=i;
    colnmd2b2(u)=mod(i+M2/2,M2);
    u=u+1;
end
for i=(M2-M2/8+1):M2
    colnmd2b1(u)=i;
    colnmd2b2(u)=mod(i+M2/2,M2);
```

```
    u=u+1;
end
colnmd2b2=sort(colnmd2b2);
for j=1:k-3
    help1=colnmd2b1+M2;
    colnmd2b1=[colnmd2b1,help1];
    help2=colnmd2b2+M2;
    colnmd2b2=[colnmd2b2,help2];
    M2=M2*2;
end
for i=1:length(colnmd2b1)
    codemd2b1(:,i)=H(:,colnmd2b1(i));
    codemd2b2(:,i)=H(:,colnmd2b2(i));
end
codemd2b1=codemd2b1(:,1:cor);
codemd2b2=codemd2b2(:,1:cor);
code788md2=[codemd2b1,codemd2b2];
%************************************************************
%creates the generator matrix
codeb=(code788md2+1)/2;
for i=1:M
    if sum(Kf(i,:))==1 [a,b]=max(Kf(i,:));g788md2(b,:)=codeb(i,:);end;
end
```

2. This program creates codes for the 4-block fading channel using method-2 for k=7 and n=96

```
close all;
clear;
k=7;
M=2^k;
n=M/4;
cor=24; %number of columns per block
H=hadamard(M)*-1;

%*******************************************************
%Kf:all the 2^k binary combinations of length k
for i=2:M K(i)=i-1;end
K=dec2bin(K,k);
for i=1:M
    for j=1:k
        Kf(i,j)=str2double(K(i,j));
    end
end
%***********************method-2*******************
%results from method 1 for k=4
block1=[1,2,3,4,5,22,31,16];
block2=[9,10,11,12,13,30,7,24];
block3=[17,18,19,20,21,6,15,32];
block4=[25,26,27,28,29,14,23,8];
M2=32;
for j=1:k-5
    help1=block1+M2;
```

```
    block1=[block1,help1];
    help2=block2+M2;
    block2=[block2,help2];
    help3=block3+M2;
    block3=[block3,help3];
    help4=block4+M2;
    block4=[block4,help4];
    M2=M2*2;
end
for i=1:length(block1)
    codeb1(:,i)=H(:,block1(i));
    codeb2(:,i)=H(:,block2(i));
    codeb3(:,i)=H(:,block3(i));
    codeb4(:,i)=H(:,block4(i));
end
codeb1=codeb1(:,1:cor);
codeb2=codeb2(:,1:cor);
codeb3=codeb3(:,1:cor);
codeb4=codeb4(:,1:cor);
code796md2=[codeb1,codeb2,codeb3,codeb4];
%*****************************************************
%creates the generator matrix
codeb=(code796md2+1)/2;
for i=1:M if sum(Kf(i,:))==1
[a,b]=max(Kf(i,:));g796md2(b,:)=codeb(i,:);end;
end
```

3. This program simulates the performance of the code with k=7 and n=88 in a 2-block fading channel

```
close all;
clear;
k=7;
n=88;
M=2^k;
%******************************************************************
load code788bf2m;
load g788bf2m;
code=code788bf2m;
codeb=(code+1)/2;
g=g788bf2m;
%******************************************************************
avsnrb = 0:5:25;
snrb = 10.^(avsnrb./10);
rate=k/n;
var1 = 1;
var2= snrb.*rate;
nloop=10000;

for u=1:length(snrb)
%******************** START CALCULATION 7/88 code*****************
nloop=nloop;  % Number of simulation loops
```

```
werror=0;

for iii=1:nloop
%******************* Data generation **********************
    data=rand(1,k)>0.5;  % rand: built in function
    data=mod(data*g,2);
%******************* BPSK Modulation **********************
    data1=data.*2-1;
%********************* Fading channel *********************
  fader = sqrt(var2(u))*randn(1);
  fadei = sqrt(var2(u))*randn(1);
  fade = fader+fadei*j;
  ifadea = fade.*data1(1:n/2);
  fader = sqrt(var2(u))*randn(1);
  fadei = sqrt(var2(u))*randn(1);
  fade = fader+fadei*j;
  ifadeb = fade.*data1(n/2+1:n);
%*********** Add White Gaussian Noise (AWGN) ***************
    awgnch1 = sqrt(var1)*randn(1,n);
    awgnch2 = sqrt(var1)*randn(1,n);
    awgnch = awgnch1+awgnch2*j;
    data4a=ifadea+awgnch(1:n/2);
    data4b=ifadeb+awgnch(n/2+1:n);
%**************** Maximum likelihood decoding **************
    for i=1:M
        h11(i)=abs(data4a*transpose(code(i,1:n/2)))^2;
```

```
        h12(i)=abs(data4b*transpose(code(i,n/2+1:n)))^2;
        h1(i)=h11(i)+h12(i);
    end
    [Q,W]=max(h1);
    demodata=codeb(W,:);
%****************** Word Error Rate (WER) ****************
    noe2=sum(abs(data-demodata));  % sum: built in function
    if noe2~=0 werror=werror+1;end
end % for iii=1:nloop

%******************** Output result *********************
wer = werror/nloop; wer788bf2m(u)=wer; end
%****************** end of file *************************
```

4. This program simulates the performance of the code with k=7 and n=96 in a 4-block fading channel

```
close all;
clear;
k=7;
n=96;
M=2^k;
%****************************************************************
load code796bf4m;
load g796bf4m;
code=code796bf4m;
```

```
codeb=(code+1)/2;
g=g796bf4m;
%*****************************************************************
avsnrb = 0:5:20;
snrb = 10.^(avsnrb./10);
rate=k/n;
var1 = 1;
var2= snrb.*rate;
nloop=10000;
for u=1:length(snrb)
%***************** START CALCULATION 7/96 code*****************
nloop=nloop;  % Number of simulation loops
werror=0; for iii=1:nloop
%******************* Data generation **********************
    data=rand(1,k)>0.5;  % rand: built in function
    data=mod(data*g,2);
%******************* BPSK Modulation **********************
    data1=data.*2-1;
%********************* Fading channel ********************
  fader = sqrt(var2(u))*randn(1);
  fadei = sqrt(var2(u))*randn(1);
  fade = fader+fadei*j;
  ifadea = fade.*data1(1:n/4);
  fader = sqrt(var2(u))*randn(1);
  fadei = sqrt(var2(u))*randn(1);
  fade = fader+fadei*j;
```

```
  ifadeb = fade.*data1(n/4+1:n/2);
  fader = sqrt(var2(u))*randn(1);
  fadei = sqrt(var2(u))*randn(1);
  fade = fader+fadei*j;
  ifadec = fade.*data1(n/2+1:3*n/4);
  fader = sqrt(var2(u))*randn(1);
  fadei = sqrt(var2(u))*randn(1);
  fade = fader+fadei*j;
  ifaded = fade.*data1(3*n/4+1:n);
 %************ Add White Gaussian Noise (AWGN) *************
    awgnch1 = sqrt(var1)*randn(1,n);
    awgnch2 = sqrt(var1)*randn(1,n);
    awgnch = awgnch1+awgnch2*j;
    data4a=ifadea+awgnch(1:n/4);
    data4b=ifadeb+awgnch(n/4+1:n/2);
    data4c=ifadec+awgnch(n/2+1:3*n/4);
    data4d=ifaded+awgnch(3*n/4+1:n);
%************* Maximum likelihood decoding *****************
    for i=1:M
        h11(i)=abs(data4a*transpose(code(i,1:n/4)))^2;
        h12(i)=abs(data4b*transpose(code(i,n/4+1:n/2)))^2;
        h13(i)=abs(data4c*transpose(code(i,n/2+1:3*n/4)))^2;
        h14(i)=abs(data4d*transpose(code(i,3*n/4+1:n)))^2;
        h1(i)=h11(i)+h12(i)+h13(i)+h14(i);
     end
    [Q,W]=max(h1);
```

```
    demodata=codeb(W,:);
   %***************** Word Error Rate (WER) ****************
    noe2=sum(abs(data-demodata));  % sum: built in function
    if noe2~=0 werror=werror+1;end
end % for iii=1:nloop

%********************** Output result **********************
wer = werror/nloop; wer796bf4m(u)=wer; end
%******************** end of file *************************
```

5. This program simulates the performance of the code with k=6 and n=64 in a 4-block fading channel using the selective transmission scheme with two thresholds

```
close all;
clear;
k=6;
n=64;
M=2^k;
%*****************************************************************
load code664bf4m;
load g664bf4m;
code=code664bf4m;
codeb=(code+1)/2;
g=g664bf4m;
%*****************************************************************
avsnrb = 0:5:20;
```

```
snrb = 10.^(avsnrb./10);
rate=k/n;
var1 = 1;
var2= snrb.*rate;
nloop=100000000;
thresh=[0.3 0.4 0.5 0.6 0.6];
thresh2=[3000 3000 3000 4000 5000];
%*****************************************************************
for u=1:length(snrb)
%****************** START CALCULATION 6/64 code******************
nloop=nloop;  % Number of simulation loops
werror=0; for iii=1:nloop
%******************** Data generation **********************
    data=rand(1,k)>0.5;  % rand: built in function
    data=mod(data*g,2);
%******************** BPSK Modulation **********************
    data1=data.*2-1;
%********************* Fading channel *********************
  fader = sqrt(var2(u))*randn(1);
  fadei = sqrt(var2(u))*randn(1);
  fade = fader+fadei*j;
  ifadea = fade.*data1(1:n/4);
  fader = sqrt(var2(u))*randn(1);
  fadei = sqrt(var2(u))*randn(1);
  fade = fader+fadei*j;
  ifadeb = fade.*data1(n/4+1:n/2);
```

```
 fader = sqrt(var2(u))*randn(1);
 fadei = sqrt(var2(u))*randn(1);
 fade = fader+fadei*j;
 ifadec = fade.*data1(n/2+1:3*n/4);
 fader = sqrt(var2(u))*randn(1);
 fadei = sqrt(var2(u))*randn(1);
 fade = fader+fadei*j;
 ifaded = fade.*data1(3*n/4+1:n);
%************ Add White Gaussian Noise (AWGN) **************
   awgnch1 = sqrt(var1)*randn(1,length(ifadea));
   awgnch2 = sqrt(var1)*randn(1,length(ifadea));
   awgnch = awgnch1+awgnch2*j;
   data4a=ifadea+awgnch;
   awgnch1 = sqrt(var1)*randn(1,length(ifadeb));
   awgnch2 = sqrt(var1)*randn(1,length(ifadeb));
   awgnch = awgnch1+awgnch2*j;
   data4b=ifadeb+awgnch;
   awgnch1 = sqrt(var1)*randn(1,length(ifadec));
   awgnch2 = sqrt(var1)*randn(1,length(ifadec));
   awgnch = awgnch1+awgnch2*j;
   data4c=ifadec+awgnch;
   awgnch1 = sqrt(var1)*randn(1,length(ifaded));
   awgnch2 = sqrt(var1)*randn(1,length(ifaded));
   awgnch = awgnch1+awgnch2*j;
   data4d=ifaded+awgnch;
%*************** Maximum likelihood decoding ****************
```

```
%******************** 1-block *****************************
  for i=1:M
        h11(i)=abs(data4a*transpose(code(i,1:n/4)))^2;
        h1(i)=h11(i);
  end
    help1=sort(h1);
    help2=(help1(M)-help1(M-1))/(help1(M)-help1(1));
    help3=(help1(M)-help1(M-1))/(help1(M)-help1(1))*cov(h1);
    if help2<thresh(u) & help3<thresh2(u) noe2=1;else noe2=0;end
    [Q,W]=max(h1);demodata=codeb(W,:);
    noe3=sum(abs(data(1:n/4)-demodata(1:n/4)));
%******************** 2-blocks *****************************
    if noe2~=0
        for i=1:M
            h12(i)=abs(data4b*transpose(code(i,n/4+1:n/2)))^2;
            h1(i)=h11(i)+h12(i);
        end
        help1=sort(h1);
        help2=(help1(M)-help1(M-1))/(help1(M)-help1(1));
        help3=(help1(M)-help1(M-1))/(help1(M)-help1(1))*cov(h1);
        [Q,W]=max(h1);demodata=codeb(W,:);
        noe3=sum(abs(data(1:n/2)-demodata(1:n/2)));
        if (help2<thresh(u) & help3<thresh2(u) ) noe2=1;
        else noe2=0;end
   end
%************************ 3-blocks ***********************
```

```
if noe2~=0
        for i=1:M
            h13(i)=abs(data4c*transpose(code(i,n/2+1:3*n/4)))^2;
            h1(i)=h11(i)+h12(i)+h13(i);
        end
    help1=sort(h1);
    help2=(help1(M)-help1(M-1))/(help1(M)-help1(1));
    help3=(help1(M)-help1(M-1))/(help1(M)-help1(1))*cov(h1);
    [Q,W]=max(h1);demodata=codeb(W,:);
    noe3=sum(abs(data(1:3*n/4)-demodata(1:3*n/4)));
    if (help2<thresh(u) & help3<thresh2(u) ) noe2=1;
    else noe2=0;end
 end
%*************************** 4-blocks **********************
if noe2~=0
        for i=1:M
           h14(i)=abs(data4d*transpose(code(i,3*n/4+1:n)))^2;
           h1(i)=h11(i)+h12(i)+h13(i)+h14(i);
        end
    [Q,W]=max(h1);demodata=codeb(W,:);
    noe3=sum(abs(data(1:n)-demodata(1:n)));
end
%******************* Word Error Rate (WER) *****************
    if noe3~=0 werror=werror+1;end
end % for iii=1:nloop
```

```
%********************** Output result ************************
wer = werror/nloop; wer664w2thr(u)=wer; end
%******************** end of file ****************************
```

APPENDIX E

NONCOHERENT MAXIMUM LIKELIHOOD DETECTOR FOR THE TWO BLOCK FADING CHANNEL

The discrete time vector model for the two block fading channel is according to 3.1:

$$\begin{aligned} \boldsymbol{r_1} &= \alpha_1 \boldsymbol{x_1} + \boldsymbol{\nu_1} \\ \boldsymbol{r_2} &= \alpha_2 \boldsymbol{x_2} + \boldsymbol{\nu_2} \end{aligned} \tag{E.1}$$

where $\boldsymbol{x} = \sqrt{E_s}\boldsymbol{d}$, $\boldsymbol{d} = [\boldsymbol{d_1}, \boldsymbol{d_2}]^T = [d_{10}, d_{11}, \ldots, d_{1(n/2-1)}, d_{2n/2}, \ldots, d_{2(n-1)}]^T$. The fading variables α_i are modelled as zero-mean, circularly symmetric, complex Gaussian random variables of variance σ_α^2 and are independent of each other. The vectors ν_i are i.i.d. zero mean, circularly symmetric complex Gaussian random variables with variance $\sigma^2 = N_0$. E_s is the energy per symbol. The modulation symbols d_j take values from the binary set {-1,1}. The maximum likelihood detector is given by

$$\hat{m}_{ML} = \arg\max_{\boldsymbol{x}} [\ln p(\boldsymbol{r}/\boldsymbol{x})] \tag{E.2}$$

where $p(\boldsymbol{r}/\boldsymbol{x}) = p(\boldsymbol{r_1}/\boldsymbol{x_1})p(\boldsymbol{r_2}/\boldsymbol{x_2})$ and

$$p(\boldsymbol{r_i}/\boldsymbol{x_i}) = c \exp\left[-\frac{1}{2}(\boldsymbol{r_i} - \mu_{\boldsymbol{r_i}/\boldsymbol{x_i}})^H Cov(\boldsymbol{r_i}/\boldsymbol{x_i})^{-1}(\boldsymbol{r_i} - \mu_{\boldsymbol{r_i}/\boldsymbol{x_i}})\right] \tag{E.3}$$

where c is a constant and $i = 1, 2$. Then

$$\begin{aligned} \ln p(\boldsymbol{r}/\boldsymbol{x}) = &-(\boldsymbol{r_1} - \mu_{\boldsymbol{r_1}/\boldsymbol{x_1}})^H Cov(\boldsymbol{r_1}/\boldsymbol{x_1})^{-1}(\boldsymbol{r_1} - \mu_{\boldsymbol{r_1}/\boldsymbol{x_1}}) \\ &-(\boldsymbol{r_2} - \mu_{\boldsymbol{r_2}/\boldsymbol{x_2}})^H Cov(\boldsymbol{r_2}/\boldsymbol{x_2})^{-1}(\boldsymbol{r_2} - \mu_{\boldsymbol{r_2}/\boldsymbol{x_2}}) \end{aligned} \tag{E.4}$$

with $\mu_{\boldsymbol{r_i}/\boldsymbol{x_i}} = 0$ and $Cov(\boldsymbol{r_i}/\boldsymbol{x_i}) = N_0 I_{n/2} + \boldsymbol{x_i}\sigma_\alpha^2\boldsymbol{x_i}^H$. Using the matrix inversion lemma we have for the inverse of the covariance matrix:

$$Cov(\boldsymbol{r_i}/\boldsymbol{x_i})^{-1} = \frac{I_{n/2}}{N_0} - \frac{\sigma_\alpha^2}{N_0}\boldsymbol{x_i}\left[I_{n/2}N_0 + \boldsymbol{x_i}^H\sigma_\alpha^2\boldsymbol{x_i}\right]\boldsymbol{x_i}^H \tag{E.5}$$

Then the detector is of the form

$$\ln p(\boldsymbol{r}/\boldsymbol{x}) = -\boldsymbol{r_1}^H\left(c_1 - c_2\boldsymbol{x_1}\boldsymbol{x_1}^H\right)\boldsymbol{r_1} - \boldsymbol{r_2}^H\left(c_3 - c_4\boldsymbol{x_2}\boldsymbol{x_2}^H\right)\boldsymbol{r_2} \tag{E.6}$$

After dropping all the terms that are not sequence dependent we get

$$\hat{\boldsymbol{d}} = \arg\ \max_{\boldsymbol{d}}|\boldsymbol{r_1}^H\boldsymbol{d_1}|^2 + |\boldsymbol{r_2}^H\boldsymbol{d_2}|^2 \tag{E.7}$$

VITA

Spyros Spyrou received the B.Eng. degree in Electrical and Computer Engineering from the National Technical University of Athens, Athens, Greece, in 2003. In August 2003, he joined the Electrical Engineering department at Texas *A&M* University, College Station, TX. His research interests include channel coding and baseband signal processing. Spyros Spyrou was a recipient of the Hellenic State Scholarships Foundation (I.K.Y) Scholarship for the years 1998-2003 and a Fulbright scholarship for 2003-2005.

The typist for this thesis was Spyros Spyrou.