GRINDING MEDIA OSCILLATION: EFFECT ON TORSIONAL VIBRATIONS IN TUMBLE MILLS

A Thesis

by

KIRAN KUMAR TORAM

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2005

Major Subject: Mechanical Engineering

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Committee Members, Luis San Andres
Doug Hensley
Head of Department, Dennis O'Neal

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ABSTRACT

Grinding Media Oscillation: Effect on Torsional Vibrations in Tumble Mills.

(August 2005)

Kiran Kumar Toram, B.Tech., JNT University

Chair of Advisory Committee: Dr. John M. Vance

Tumble mills are hollow cylindrical shells of large diameter carrying grinding media (a combination of rock/iron ore/chemical flakes and metal balls/rods), which, upon rotation of the mill, will be ground into fine powder. These mills rotate at low speeds using a gear reduction unit and often have vibration problems. These vibration problems result in increased gear wear and occasional catastrophic failures resulting in production loss. The objective of this research is to investigate the effect of oscillation of grinding media on torsional vibrations of the mill. A theoretical model was developed to determine the oscillating frequency of the grinding media. A 12" (0.3 m) diameter tumble mill test rig was built with a 0.5 hp DC motor. The rig is tested with sand and iron bb balls to simulate the industry process application. At low volume levels the grinding media oscillates like a rigid body as compared to higher volumes. It is shown that tumbling action of grinding media causes torsional excitation and hence its effect has to be considered in torsional vibration analysis. At starting, the load on the gears is much higher due to this oscillation.

DEDICATION

to Lord Siva

ACKNOWLEDGEMENTS

I wish to express my thanks to the following people. Thanks to Dr. John Vance for giving me an opportunity to work at the Turbomachinery Laboratory, for teaching me the basics of vibrations, and for his patience and support. Thanks to Dr. Luis San Andrés and Dr. Doug Hensley for agreeing to be on my committee.

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Thanks to my roommates Srinivas Chittla, Rajesh Pilla, Hari Prasad and all Telugu Aggies for their encouragement and valuable advice. Thanks to my parents and sisters for their love and support and making me what I am today.

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NOMENCLATURE

a	Overhung load distance from right bearing
D	Free fall distance
e	Distance between center of gravity of the grinding media and geometric center of the tumble mill
E	Modulus of elasticity
F	Overhung load applied on simply supported beam
g	Acceleration due to gravity
G	Center of gravity of the grinding media
I	Moment of Inertia of the grinding media about '*O*'
I_{area}	Area moment of inertia of rotor
$J_{assembly}$	Mass polar moment of inertia of rotor and mill assembly
J_{motor}	Mass polar moment of inertia of motor
l	Length of rotor
L	Length of the mill
L_c	Length of cable
m	Mass of grinding media in the mill
N	Normal reaction acting on the grinding media
O	Geometric center of tumble mill
R	Radius of tumble mill in ft
R^*	Effective radius of mill (internal radius of mill – radius of media)
R_S	Radius of coupling

R_c	Distance between cable and geometric center of rotor assembly
stime	Start-up time
t	Time in seconds
T	Time period for the free fall distance of D
V	Volume fill level
W	Weight of rotor assembly
$W1$	Weight of grinding media in lb
Ω	Angular speed of tumble mill
Ω^*	Critical speed of tumble mill
θ	Angle made by the center of gravity of grinding media and tumble mill geometric center with negative Y-axis
$\dot{\theta}$	Angular speed of grinding media in tumble mill
$\ddot{\theta}$	Angular acceleration grinding media in tumble mill
$\theta 1$	Maximum amplitude of oscillation of grinding media
$\theta 2$	Minimum amplitude of oscillation of grinding media
α	Angle made by normal reaction with Y axis
ρ	Fill density
μ_S	Static or Slip friction coefficient
μ_K	Kinetic friction coefficient
ω	Frequency of grinding media

CHAPTER I

INTRODUCTION

Tumble mills are used in cement, mining and chemical industries. These mills are hollow cylinders of large diameters used to grind the iron ore, minerals and rock. The grinding media to be ground is mixed with iron balls or rods and fed into the mill from one side. As the mill rotates at low speed the grinding media is ground into very fine particles due to the action of balls or rods on the grinding media. The ground products can be collected on other end. Figure 1.1 [1] shows the cut away section of a ball mill. Figure 1.2 [1] shows an industrial ball mill. The two types of tumbling mills are distinguished by the use of rods or balls. They are classified as rod-loaded and ball-loaded. The distinctive feature of the rod-loaded mill is a shell, usually cylindrical, revolving with axis horizontal and diameter to length ratio 1.5 to 2.5. Figure 1.3 [2] shows the picture of a rod - loaded mill. Ball-loaded mills may be classified as ball mills or tube mills. Ball mills have a length to diameter ratio not exceeding 1.5 and tube mills have length to diameter ratio more than 1.5. Figure 1.4 [2] shows the picture of a ball-loaded mill.

This thesis follows the style and format of the ASME Journal of Turbomachinery.

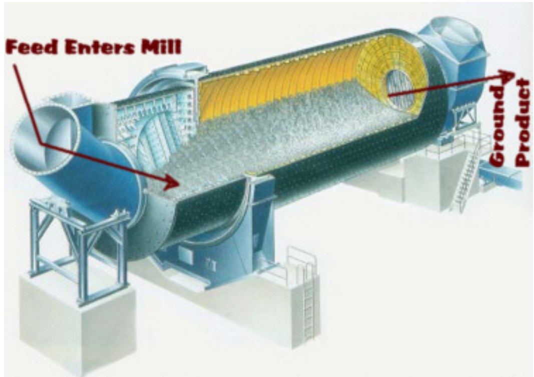

Figure 1.1 Cut away section of Ball Mill. Source: Ref. # 1

Figure 1.2 Industrial Ball Mill. Source: Ref. # 1

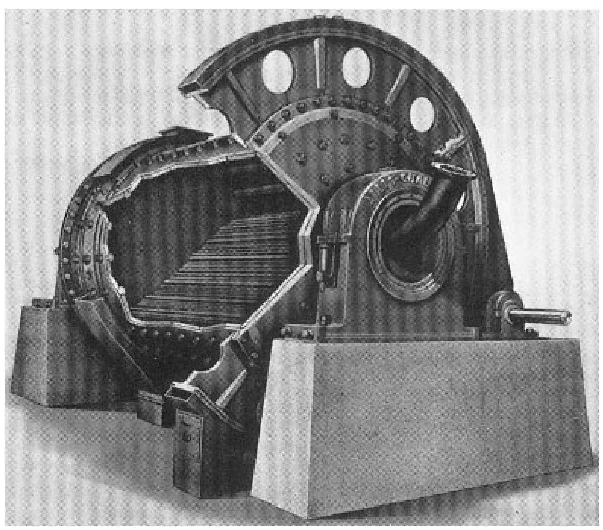

Figure 1.3 Rod Loaded Mill. Source: Ref. # 2

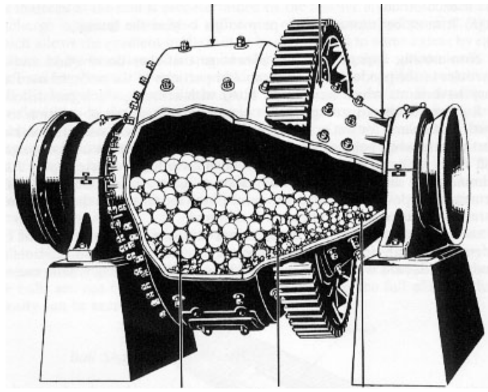

Figure 1.4 Ball Loaded Mill.   Source: Ref. # 2

A particular speed of rotation is necessary to achieve the most grinding for least power consumption. The speed has to generate enough “centrifugal force” to make the grinding media lift up on the shell lining to a height of about two-thirds of the diameter of the shell with least sliding. The optimum speed of the mill is about 0.6 to 0.75 times the “critical speed”. The critical speed of the mill is defined as the minimum speed at which the grinding media are held against the mill by centrifugal force, i.e. the speed at which the centrifugal force is just balanced by the weight of the grinding media. From figure 1.5, which shows the balance of forces, the critical speed is given by equation (1.2). So for a mill of diameter 25’ (7.62 m) the critical speed is 1.6 rad/sec or 15.31 rpm. The operating speed range of the mill in this case is 9.2 - 11.5 rpm.

$$m(\Omega^*)^2 R^* = mg \qquad (1.1)$$

$$\Omega = \sqrt{\frac{g}{R^*}} \qquad (1.2)$$

R^* = Effective Radius of Mill

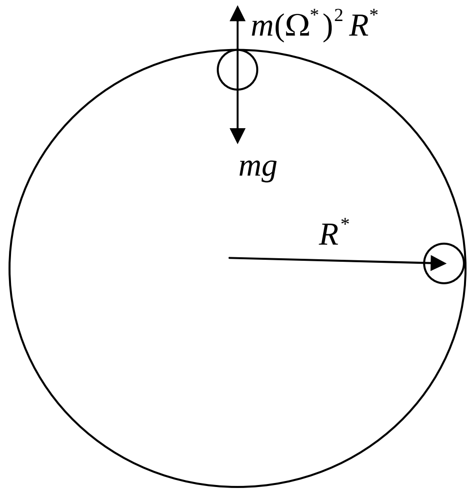

Figure 1.5 Condition for Centrifuging

The industrial mills shown in figures 1.2, 1.3 and 1.4 often require large power; some may even have thousands of horse power motors to run them. These mills have motors connected to them via a gear reduction unit. The largest built tube mill is a kiln (oven) in Clarksville, MO, which has a length of 760’ (231 m) and diameter 25’ (7.62 m) [3]. It has dual system drive consisting of two 1200 HP DC motors, flywheel and a speed reducer. The block diagram of the kiln is shown in Figure 1.6. After eight years of trouble free operation, the south side reducer of the dual kiln drive system had broken up inside. Torsional vibration measurements were taken at the drive, and it was found that there were three “critical speed*” ranges in the drive system; 0.311 rpm, 0.520 rpm and 0.805 rpm [3]. 0.805 rpm is also the operating speed of the kiln. An analytical model was developed as shown in figure 1.7 to determine the natural frequencies. The problem was fixed through tuning by changing inertia and stiffness of the system.

* Not the same critical speed as defined above.

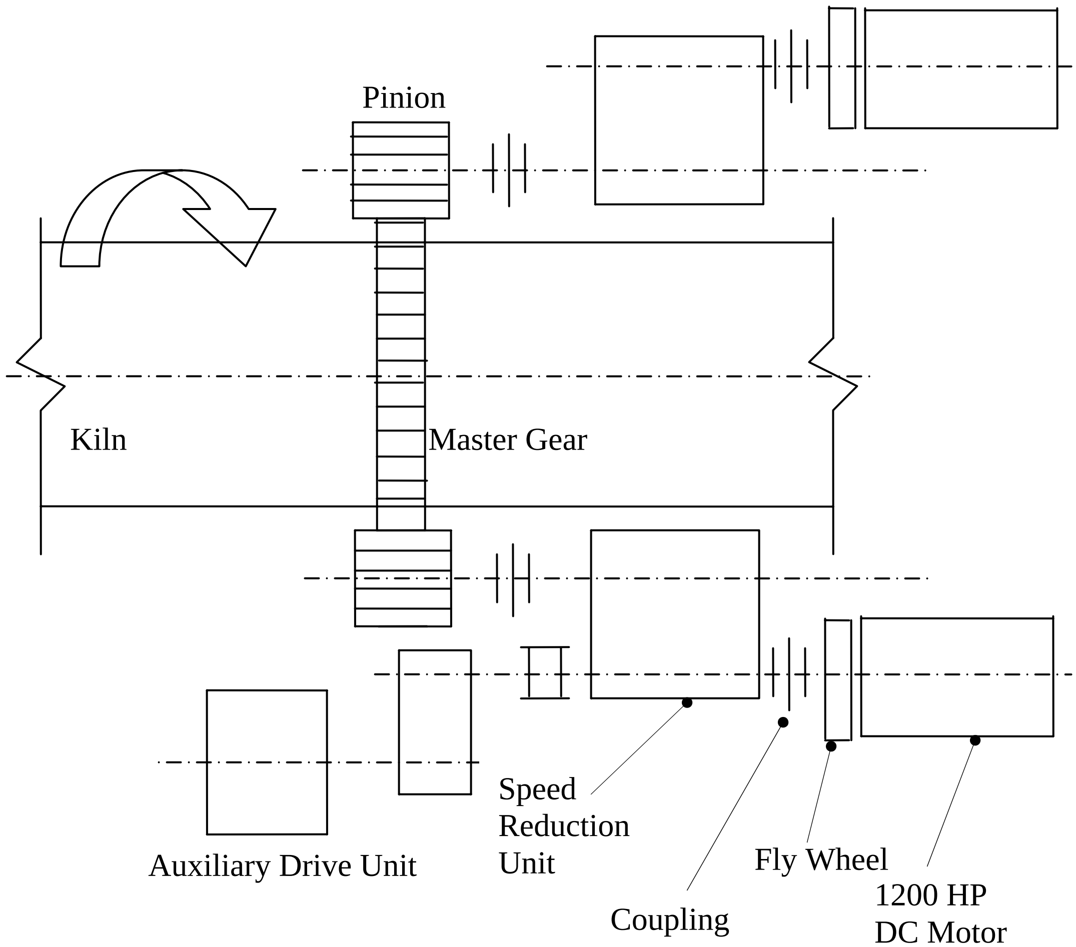

Figure 1.6 Block Diagram of Kiln at Clarksville

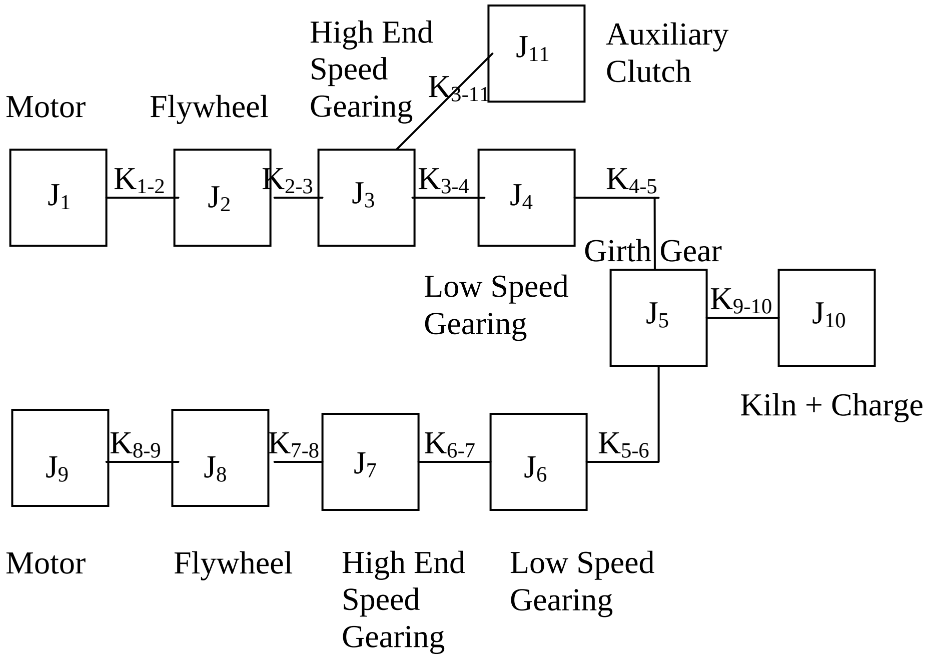

Figure 1.7 Spring Mass System of Kiln

In a case study by Dr. John M. Vance of Texas A&M University on a 10’ (3 m) length, 25’ (7.62 m) feet diameter tumble mill running at 12 rpm, it was observed that the gears were failing during start-up of the mill. The reason behind this failure was speculated to be excitations coming from the grinding media oscillation in the mill.

There has not been any work done by researchers to study the excitations coming from the mill. The motion of the grinding media inside the mill can be in two modes. In the first mode shown in figure 1.8, the grinding media, depending upon the rotation and friction of the mill inner surface, lifts up on the rising side of the mill to a certain height; the grinding media that has reached this height then rolls and drops down on the free surface of remaining part of the grinding media. As the mill rotates, jostling action in the lifting up and rolling and impact action during the fall down takes place in the grinding media. This is the intended motion required to get the grinding media to be ground into fine granule.

In the second mode the grinding media will behave almost as a rigid body and its motion is described in figure 1.8. As the mill rotates the grinding media lift up on the rising side of the mill to a certain height due to friction and rotation. Then the grinding media will slide down on the surface of the mill. This action repeats as the mill rotates. It can be noted that the second mode is periodic and can act as a torsional excitation. This unintended motion does not serve the purpose of a tumble mill, but one of the objectives of this project is to show that it can occur.

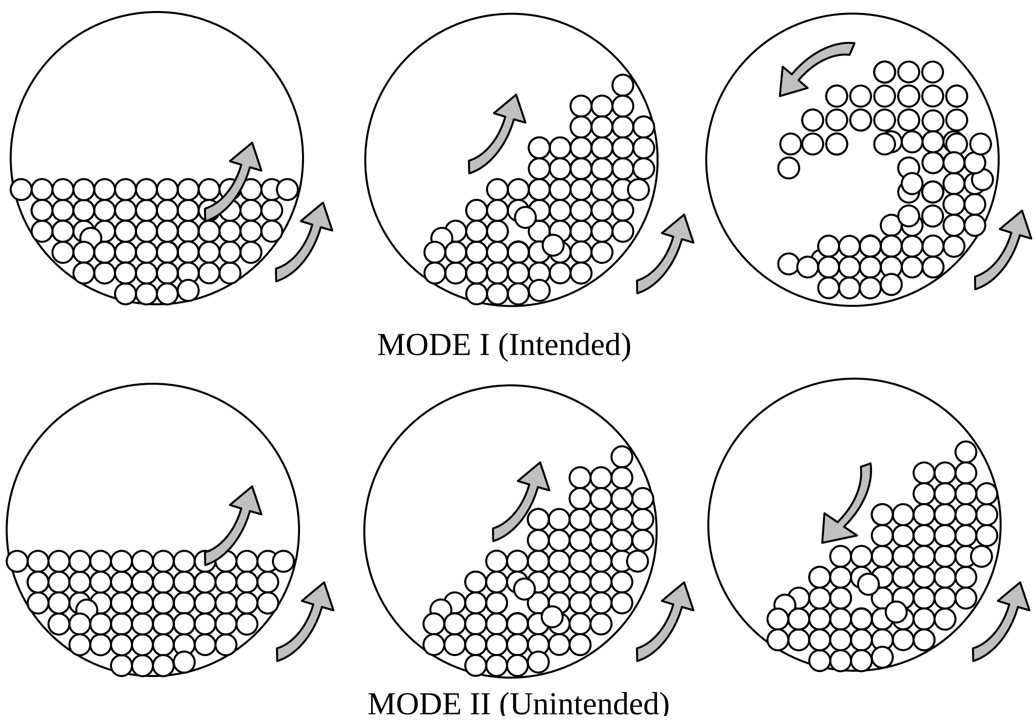

Figure 1.8 Grinding Media Motion in Tumble Mill

LITERATURE REVIEW

Yang Zhiqian [4] worked on a cement plant tube mill of 7.9’ (2.4 m) diameter and 42.65’ (13 m) length. He considered the mill operating system as a 3 degree of freedom consisting of motor, gear reduction unit and tube mill as shown in figure 1.9. He considered electromagnetic damping from the motor, oil damping in the gear reduction unit and grinding media damping in the tube mill. In his investigation it was assumed that the grinding media in the mill did not add inertia to the system. To determine the damping from the grinding media, a pulse excitation from the motor was collected at mill normal operation. This pulse excitation is a free torsional vibration shown in figure 1.10. From the plot the damping ratio ξ was determined. It was found that the second natural frequency of the system is 1.94 Hz (116.4cpm) from theory and 1.93Hz (116cpm) from test. He concluded that this natural frequency has to be considered carefully in the design of the mill.

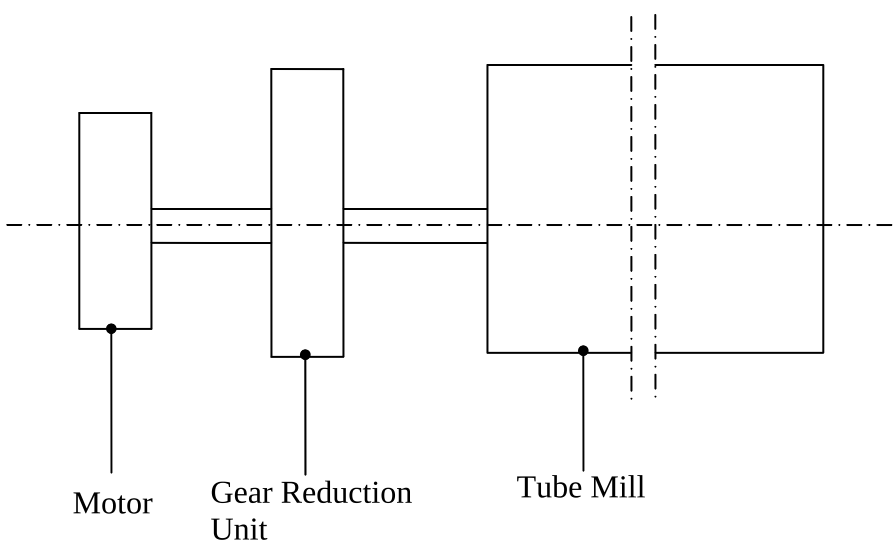

Figure 1.9 Tube Mill System

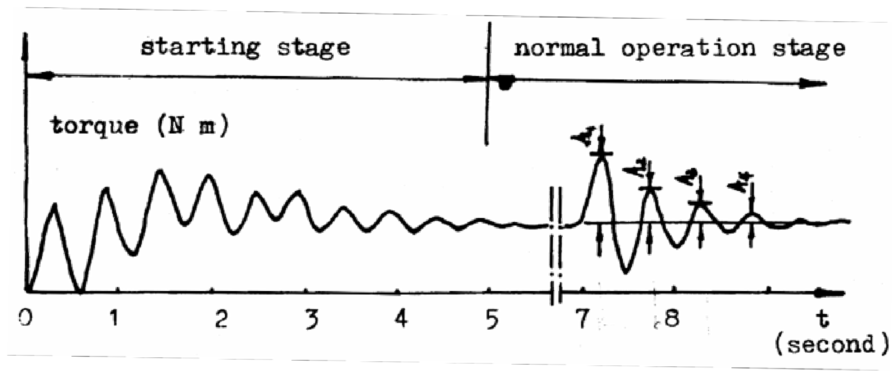

Figure 1.10 Response of Tube Mill Torsional Vibration. Source: Ref. # 2

Later Heidecker [5], in his research work considered mill plus grinding media as a single degree of freedom. He tested on a 12” (0.3 m) diameter and 12” (0.3 m) long ball mill test rig with wet & dry conditions of grinding media. He placed liners in the mill to protect the mill shell from wear and to reduce slip between the shell and grinding media. He used strain gauges and a telemetry system in measurements.

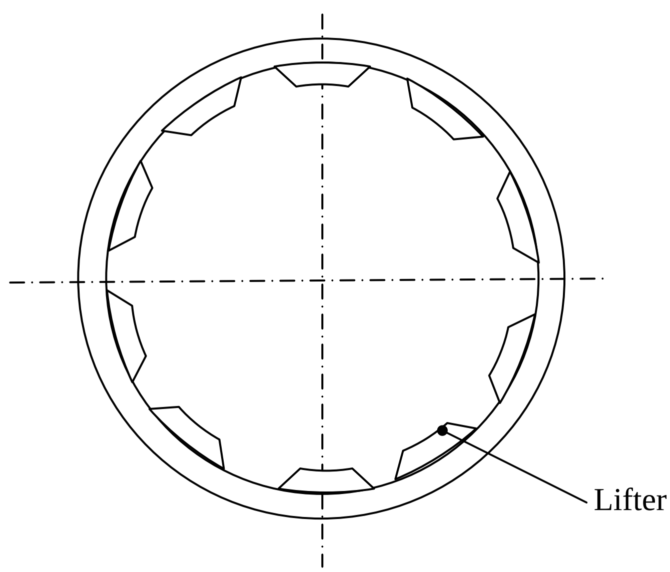

Figure 1.11 Tumble Mill with Lifters

He found that there is an increase in inertia when dense material is placed in the drum. When lifters (bars placed around the inner surface of the mill to prevent slippage between grinding media and mill shown in figure 1.11) are placed there is an increase in the inertia of the mill and also in mean torque. It was observed that the increase in mean torque was from the reduction in slippage between the media and the mill. An increase in damping occurred when media are placed in the mill. As the density of the grinding media increased, there is an increase in the damping. The placement of lifters also increased the damping.

The research analysis presented in this thesis, considers the grinding medium as a separate degree of freedom.

RESEARCH OBJECTIVE

The present research work aims to find whether the grinding media will oscillate in mode II as a rigid body. If it is oscillating in mode II, what is the torsional excitation frequency on the system? To accomplish the research objective the following tasks will be performed.

- Develop a theoretical model and computer program for the grinding media oscillation in a tumble mill, assuming the grinding media as a rigid body
- Design and build a test rig which can be conveniently tested in the Turbo machinery Laboratory
- Install a transparent Plexiglas plate on one side of the drum, to study the grinding media oscillations
- Develop an interface in Lab View to measure torsional vibrations

- Experimentally measure the oscillation frequencies of the grinding media on the test rig at different rotational speeds and at different fill volumes
- Compare theoretical and experimental results

CHAPTER II

THEORY

Assumptions:

1. The grinding media is assumed to be a rigid body in deriving the equation of motion
2. Dry friction is assumed to exist between the mill surface and the grinding media

The equation of motion for the grinding media is given in (equation 2.1). Sliding takes place between the grinding media and tumble mill surface, when the sum of the restoring and inertia moment exceeds the frictional moment. Figure 2.1 shows the free body diagram for the grinding media in the tumble mill. Figure 2.2 shows the free body diagram to find the normal reaction of the grinding media.

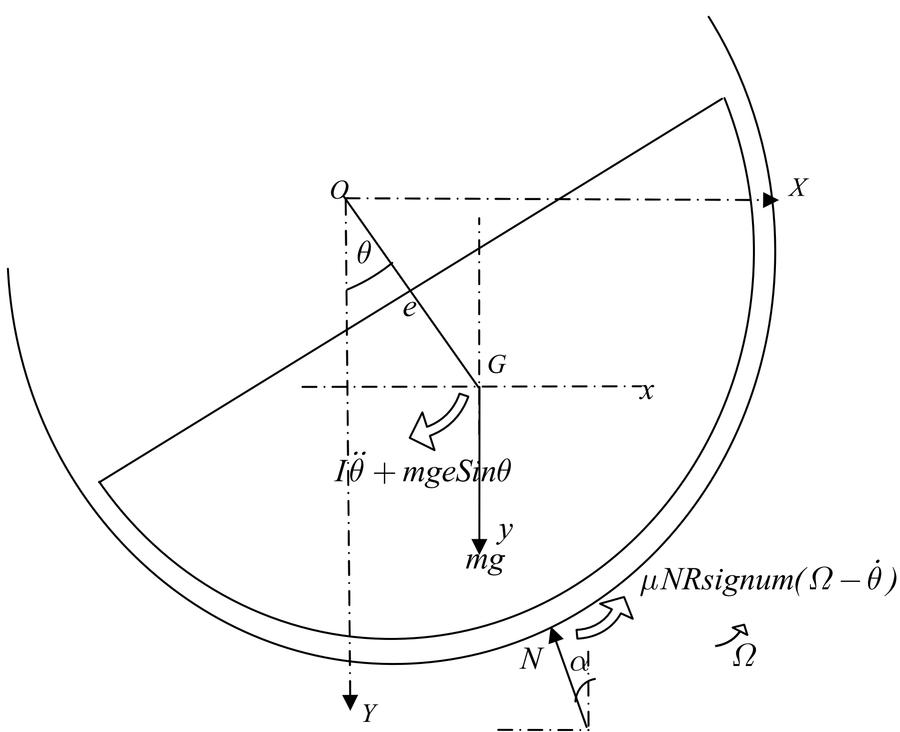

Figure 2.1 Free Body Diagram of Grinding Media in Tumble Mill

Inertia torque = $I\ddot{\theta}$

Restoring torque = $mgesin\theta$

Frictional torque = $\mu NRsignum(\Omega - \dot{\theta})$

signum(+ve) = +1, signum(0)= +1, signum(-ve)= -1

$\therefore$ Equation of motion is

$$I\ddot{\theta} + mgesin\theta = \mu NRsignum(\Omega - \dot{\theta}) \tag{2.1}$$

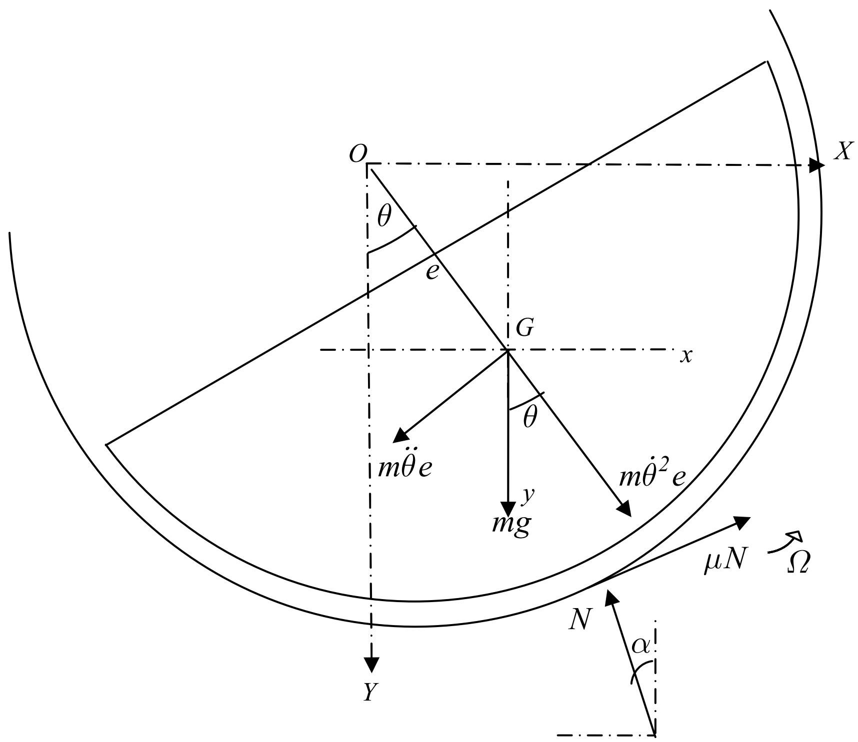

Figure 2.2 Free Body Diagram to find Normal Reaction

For an angle θ made by the grinding media with Y axis the resultant normal reaction N makes an angle α with the Y axis.

$$\sum F_x = 0$$

$$-NSin\alpha + m\dot{\theta}^2 eSin\theta - m\ddot{\theta}eCos\theta = 0 \tag{2.2}$$

$$\sum F_y = 0$$

$$-mg - m\dot{\theta}^2 eCos\theta - m\ddot{\theta}eSin\theta + NCos\alpha = 0 \qquad (2.3)$$

Solving these equations for N and α we get

$$N = \left[(mg)^2 + (m\dot{\theta}^2 e)^2 + (m\ddot{\theta}e)^2 + 2mg(m\dot{\theta}^2 eCos\theta + m\ddot{\theta}eSin\theta)\right]^{1/2} \qquad (2.4)$$

$$\alpha = tan^{-1}\left[\frac{m\dot{\theta}^2 eSin\theta - m\ddot{\theta}eCos\theta}{mg + m\dot{\theta}^2 eCos\theta + m\ddot{\theta}eSin\theta}\right] \qquad (2.5)$$

After substituting N from the above equations into the equation of motion, the final equation becomes

$$I\ddot{\theta} + mgeSin\theta = \mu\left[(mg)^2 + (m\dot{\theta}^2 e)^2 + (m\ddot{\theta}e)^2 + 2mg(m\dot{\theta}^2 eCos\theta + m\ddot{\theta}eSin\theta)\right]^{1/2} Rsignum(\Omega - \dot{\theta}) \qquad (2.6)$$

The resulting equation of motion, (2.6), is a second order, nonlinear differential equation. A Microsoft Excel spread sheet using Visual Basic macros was developed to simulate the motion of the grinding media. This XLTumbleMill computer program uses Runga - Kutta 4th order integration. It takes the radius of the mill (R), Length of the mill (L), volume fill level (V), fill density (ρ), static (μ_S) and kinetic (μ_k) friction coefficients, mill operating speed (Ω) and start up time (*stime*) to reach that speed as inputs. The user can vary the solver conditions like initial time, final time, step size, initial displacement, initial speed and band. Band is the difference between mill operating speed and grinding media speed.

$$(\Omega - \dot{\theta}) \leq band \Rightarrow \mu = \mu_S \qquad (2.7)$$

$$(\Omega - \dot{\theta}) > band \Rightarrow \mu = \mu_k \qquad (2.8)$$

If the difference between mill speed and grinding media speed, $(\Omega - \dot{\theta})$ is less than band the friction coefficient is static otherwise it will be kinetic. By running the code several times with different band conditions it is observed that 10% of mill operating speed as a band value gives better results. For example if the mill is operating at 10 rpm, for a band of 10%, the friction coefficient is static for the grinding media speed of 9 or more than 9 rpm. For less than 9 rpm the friction coefficient is kinetic. The program outputs the included angle i.e., angle between the two radii of the mill with the chord made by the fill shown in figure 2.3. This is used in calculating the center of gravity (*e*), cross sectional area (*m*) and area moment of inertia of grinding media (*I*). The program also outputs weight and volume of the fill. This program gives the angular distance, angular speed and toque of grinding media at each time step. The angular distance (θ) and angular speed ($\dot{\theta}$) of the grinding media with the mill speed (Ω) are directly plotted with time (*t*) as the program output.

The interface of the program is shown in figure 2.4. The results for a 12” (0.3m) mill diameter, 15% mill fill level of grinding media, 10 rpm mill running speed and 5 sec startup time are shown in figures 2.4, 2.5, 2.6 and 2.7.

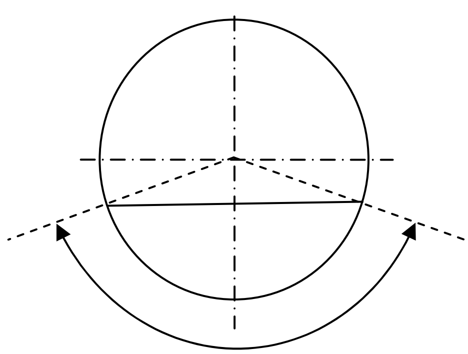

Figure 2.3 Included Angle of Grinding Media with Center of Mill Center

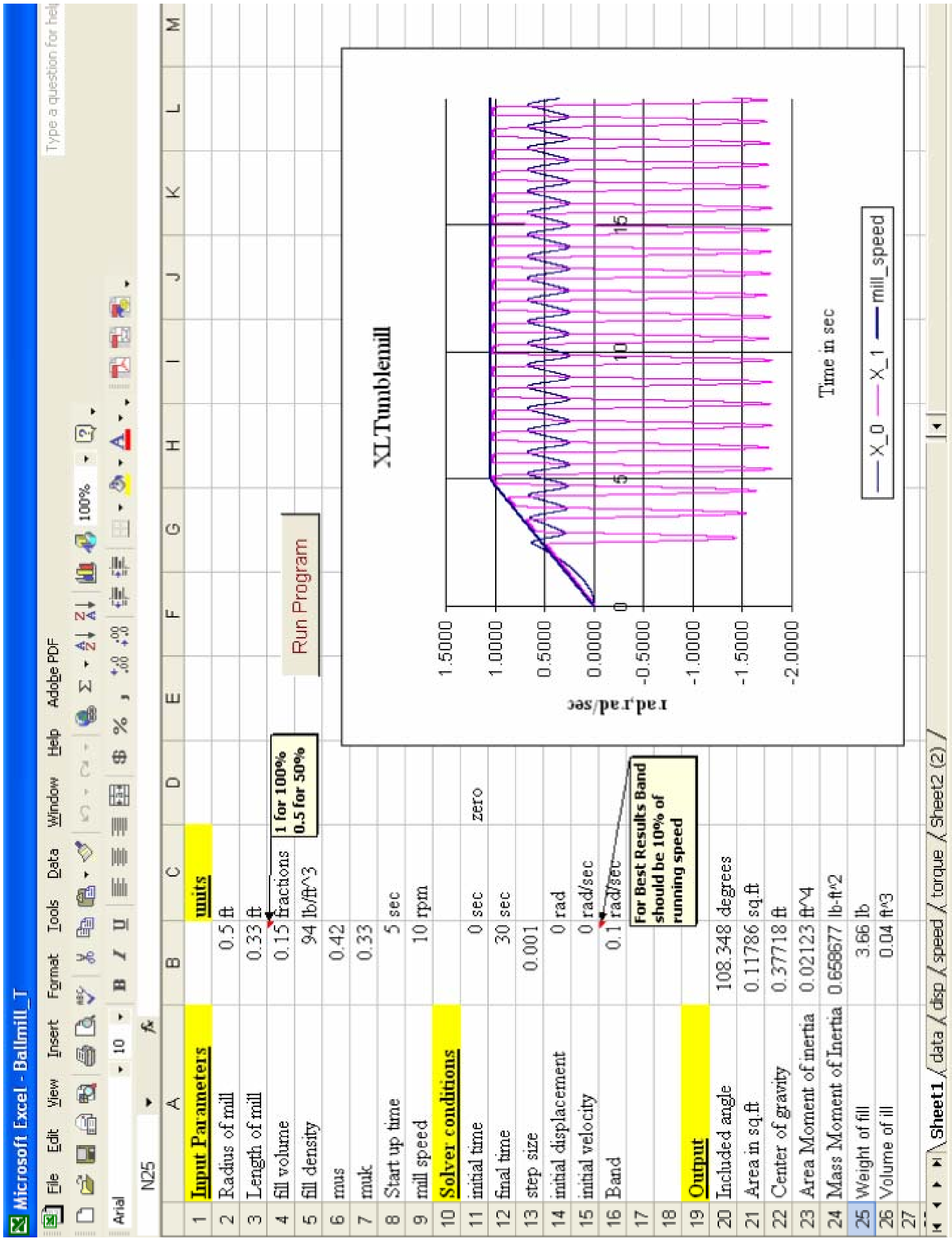

Figure 2.4 Input Data for XLTumbleMill

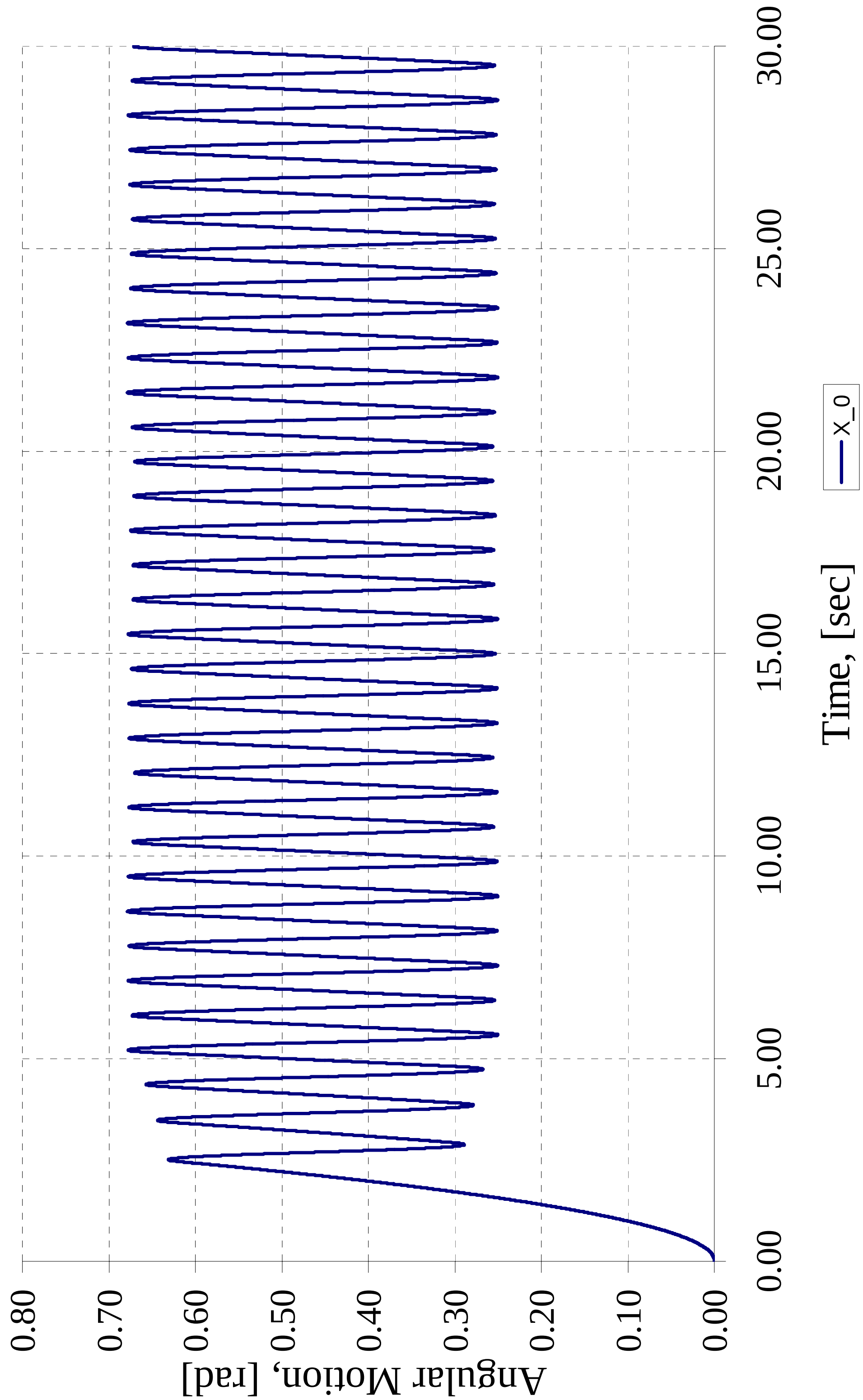

Figure 2.5 Displacement Response of XLTumbleMill

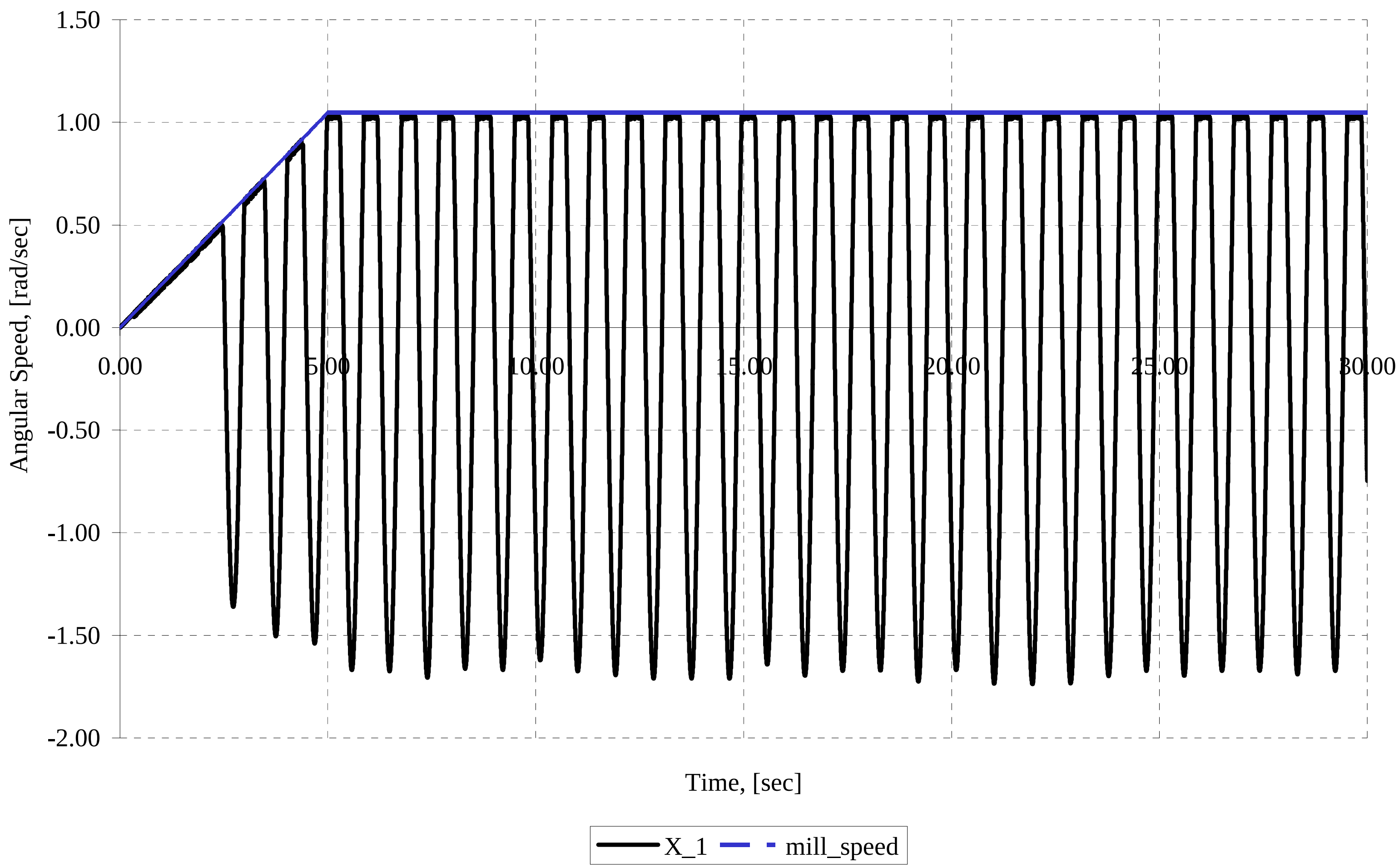

Figure 2.6 Speed Response of XLTumbleMill

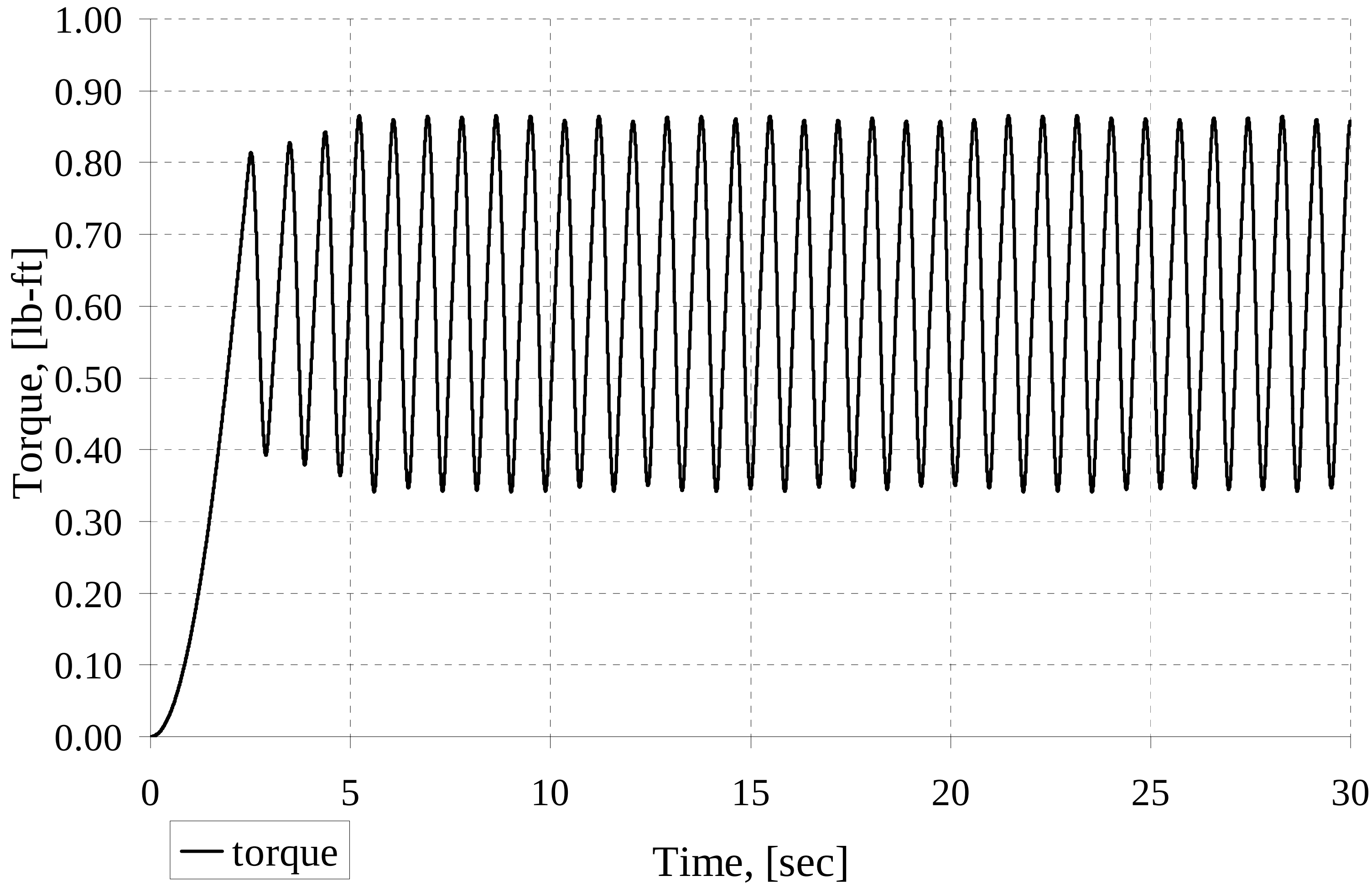

Figure 2.7 Torque Response of XLTumbleMill

CHAPTER III

DESIGN AND FABRICATION OF TEST RIG

A 12" (0.3 m) diameter tumble mill test rig was designed and fabricated for testing. The three dimensional assembled and exploded views of the designed test rig are shown in figures 3.1 and 3.2.

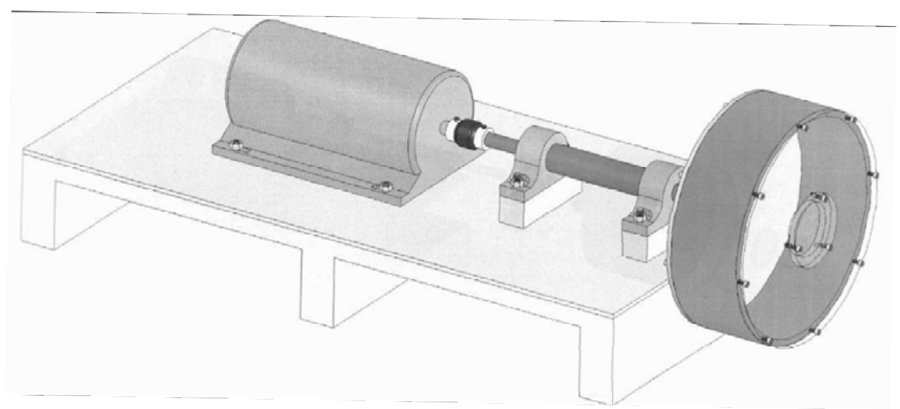

Figure 3.1 Assembled View of Test Rig

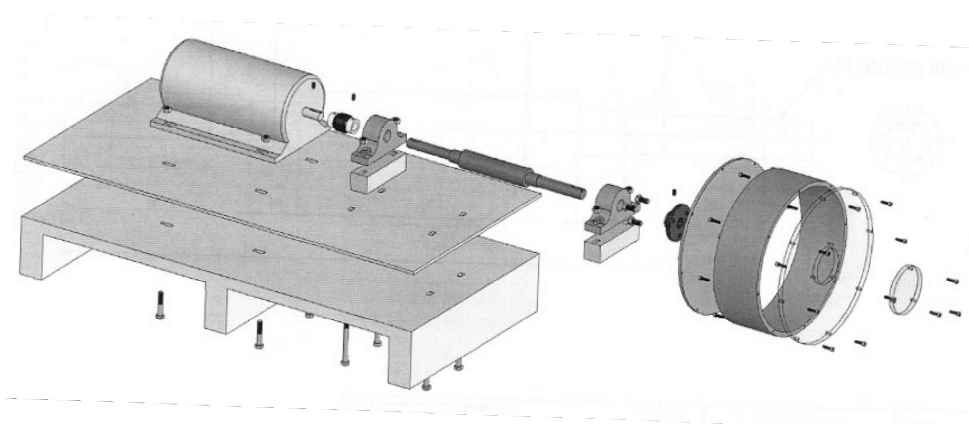

Figure 3.2 Exploded View of Test Rig

Figure 3.3 shows the schematic diagram of the tumble mill and its parts. The mill is made of a PVC pipe of 12” (0.3 m) inside diameter and 4” (0.1 m) length. The wall thickness of PVC tube is 3/8” (10 mm). One side of the mill is closed with a 12¾” (0.32 m) diameter, 3/8” (10 mm) thick Plexiglas so the operator can see the oscillation of the grinding media inside the mill. Another Plexiglas sheet of 3/8” (10 mm) thick and 3” (76 mm) diameter attached to this Plexiglas cover at its center to serve as a filling door. Through this filling door one can load or unload the mill with grinding media. The other side of the PVC mill is closed with a 12¾” (0.32 m) diameter, 3/8” (10 mm) thick Aluminum plate which is connected to the rotor via a steel flanged plate. The rotor is supported on two NTN Pillow Block, self aligning and narrow inner ring bearings with a bearing span of 6” (0.15 m). The inside diameter of each bearing is 1¼” (32 mm). Each bearing has the capability to take a 3000 lb (1360 kg) radial and 1000 lb (424 kg) axial load. The mill is connected to the outboard end of the rotor (an overhung load at a distance of 3” (76 mm) from the bearing). The other end of the rotor is connected to a 90 V DC, 0.5 HP motor via a 3 jaw coupling. The test rig uses a speed controller to carry out tests at different speeds.

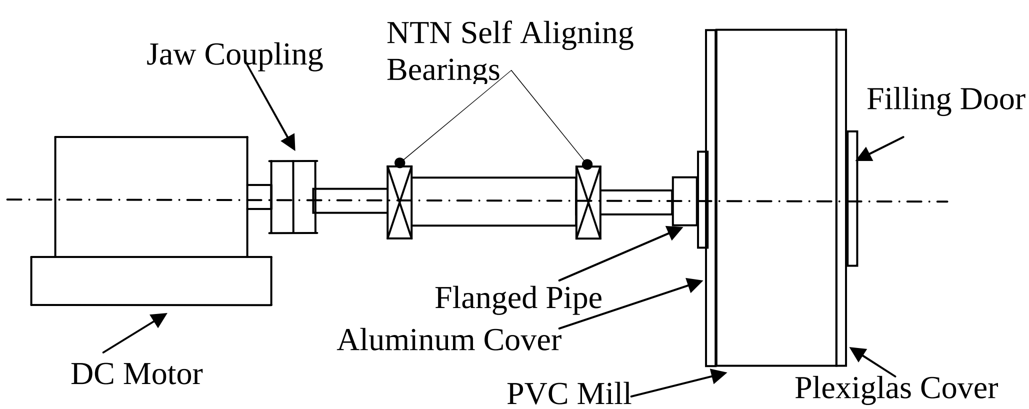

Figure 3.3 Line Diagram of Tumble Mill Test Rig

The entire setup is fixed on an 18” X 36” (0.45 m X 0.9 m) wooden base. This base has ¾” (19 mm) thick plywood and attached to it is a 3/8” (10 mm) aluminum plate. The pillow bearing blocks are elevated by 1¾” (45 mm) vertically to align with the motor outlet shaft. Figure 3.4 shows pictures of the tumble mill test rig. Appendix A presents the individual component drawings of the tumble mill. Table 3.1 list the items of the test rig with description and material of each part.

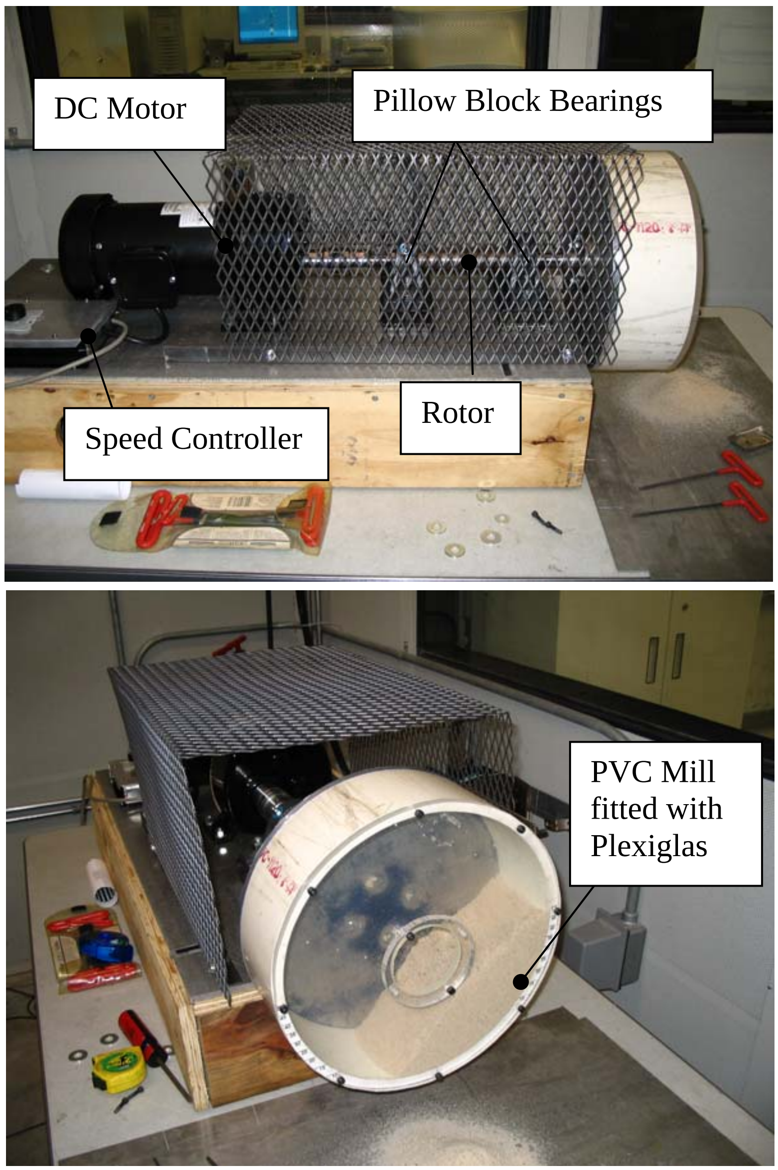

Figure 3.4 Tumble Mill Test Rig

Table 3.1 Items of Test Rig

ITEM NO.	QTY	PART NO. grainger	DESCRIPTION	MATERIAL
1	1	6Z418	Dayton ½ HP, 90V DC motor	-
2	1	6Z385	DC Speed Controller	-
3	1	2L036	Jaw type shaft coupling	Steel
4	1	2L039	Jaw type shaft coupling	Steel
5	1	2L070	Bronze Insert	Bronze
6	1	2L071	Synthetic Rubber Insert	Rubber
7	1	2L072	Polyurethane Insert	Polyurethane
8	1	-	Rotor	Steel
9	2	5NW87	Pillow Block Bearings	-
10	1	-	Flanged Pipe	Steel
11	1	-	Solid Plate	Aluminum
12	1	-	Mill	PVC
13	1	-	Plexiglas Plate	Plexiglas
14	1	-	Charging Door	Plexiglas
15	2	-	Bearing Supports	Steel
16	1	-	Base	Wood
17	1	-	Base Plate	Aluminum

CHAPTER IV

MEASUREMENTS AND INSTRUMENTATION

The instrumentation used in this study is a magnetic transducer along with National Instruments Lab VIEW™ and NI 4472 PCI 8-channel board for data acquisition. An Intel Pentium 1.4GHz, 512 RAM, Windows NT or XP personal computer is fitted with data acquisition boards. The Lab VIEW™ 7.0 Express with the Order Analysis and Sound and Vibration Toolset [6] is installed.

MAGNETIC TRANSDUCER

A characteristic unique to the magnetic pick up is that its output signal amplitude is proportional to peripheral speed when used with a gear. Further advantages of the magnetic transducer signal are that it requires no external power source, its output signal is very clean, approximating a sine wave if properly selected for use with a gear, and it generally produces a strong signal with minimum amount of noise. The pickup used in this study is a B&K MM0002 magnetic transducer. It has a linear response to velocity over a wide speed range, with output level a function of distance from the teeth passing by.

NI-4472 DATA ACQUISITION BOARD

The NI 4472 [7] is an 8-channel data acquisition device which, when used with the Sound and Vibration Toolset (Lab VIEW™), allows high precision vibration measurements. The test stand uses two such boards, thus possessing the capability of

16 channel synchronized data acquisition. Input channels incorporate Integrated Electronic Piezoelectric signal conditioning for accelerometers. The channels simultaneously digitize signals over a bandwidth of 0 to 45 kHz. Figures 4.1 and 4.2 show the data acquisition board and a personal computer installed with the board.

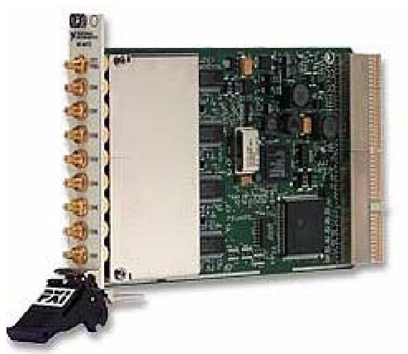

Figure 4.1 NI-4472 Data Acquisition Board

Figure 4.2 Data Acquisition Board Installed in PC

The LVTorsion (Lab VIEW Torsion) has the capability to measure torsional vibration measurements. To measure torsional vibrations the rotating object has to have a gear like structure. It uses either a magnetic or proximity probe or an optical pickup. In the case of an optical pickup a paper with black and white strips will work like a gear. In the gear case there is increase in voltage whenever the tooth comes near the probe. To implement this, the solid plate connected to the drum is wound with a galvanized iron chain of 70 links. The magnetic transducer is connected to the mount and installed to the base of the test rig. The Lab View installed measurement test setup and magnetic transducer with chain placed are shown in figures 4.3 and 4.4.

Figure 4.3 Lab View Installed PC

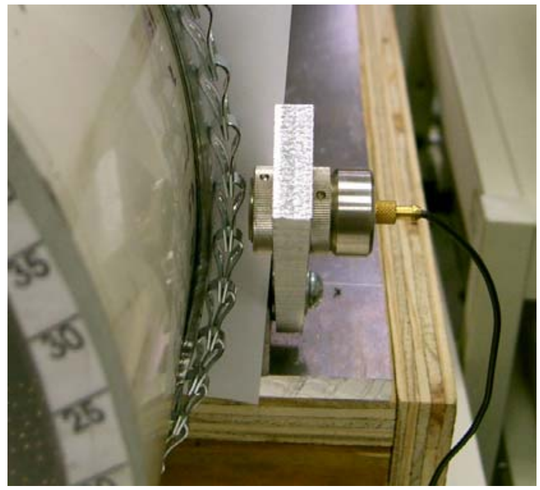

Figure 4.4 Magnetic Transducer with Chain

As the mill rotates the voltage pickup induced in the transducer is fed into the data acquisition board. The raw signal is shown in figure 4.5. The raw signal has peaks indicating the chain link passage. Whenever a link passes the transducer, there is an increase in voltage induced, which is shown in figure 4.5. The number of links passed over a fixed amount of time can be counted using peak/pulse counter. As the speed of the mill increases the voltage induced in the magnetic transducer also increases. So the threshold of the pulse can be changed accordingly to count the number of pulses. Figures 4.6 and 4.7 show the pulse detection.

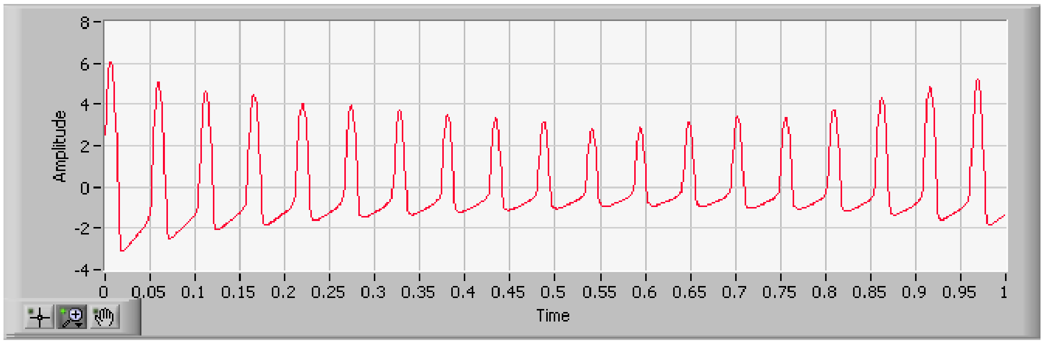

Figure 4.5 Raw Signal from the Magnetic Transducer

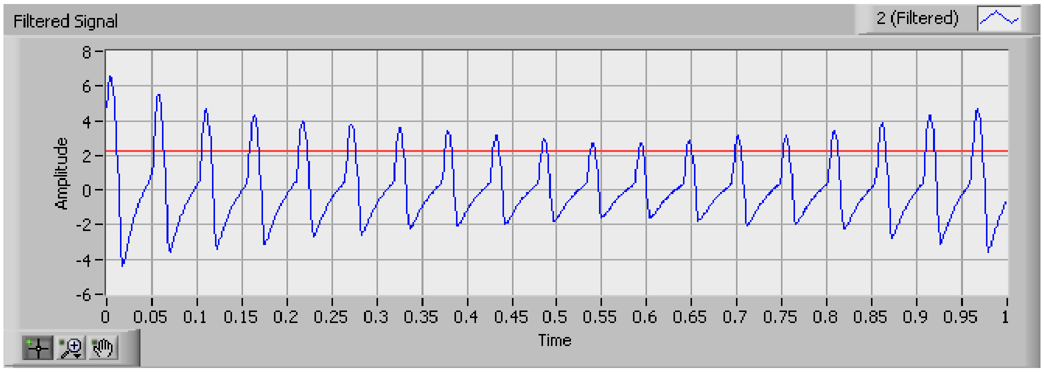

Figure 4.6 Setting Threshold Value to Count Pulses

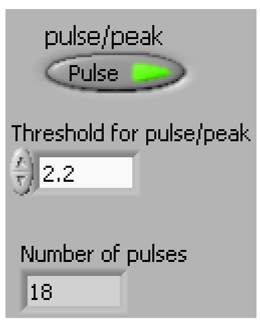

Figure 4.7 Threshold Value to Count Pulses and Number of Pulses

Filters can be used to refine the raw signal in this Lab View program. It has the capability to use a high pass filter, a band pass filter and no filter at all. Figure 4.8 shows the three filter configurations available. Whichever filter window is active that filter is activated.

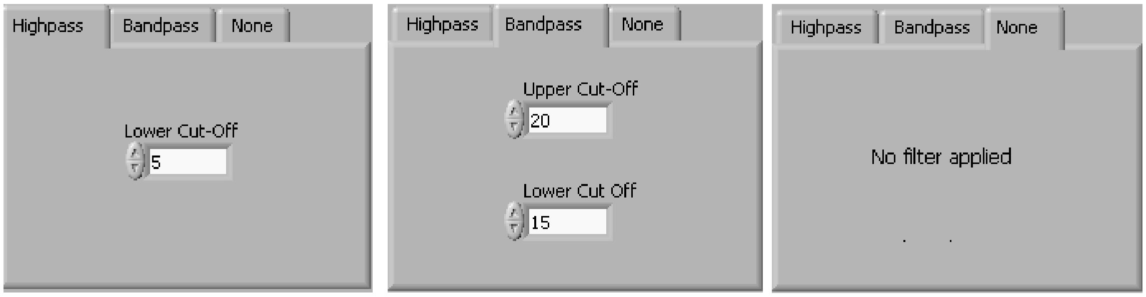

Figure 4.8 Filter Configuration

After pulse detection over a fixed time the instantaneous speed can be calculated through following procedure. The chain has 70 links along the periphery of the drum and it will take 70 pulses to complete one revolution. From figure 4.7 the number of pulses measured in 1 sec is 18. So the instantaneous speed at that particular moment is 18/70 rps or 18*60/70 rpm or 15.42 rpm. The instantaneous speed of the mill is shown in figure 4.9. This speed can be fed into the FFT to get spectrum. The Lab View contains an inbuilt spectrum virtual instrument and it is used to get the spectrum. This FFT has various windowing and averaging parameters available. The sample picture of the FFT from an instantaneous speed sample is in figure 4.10.

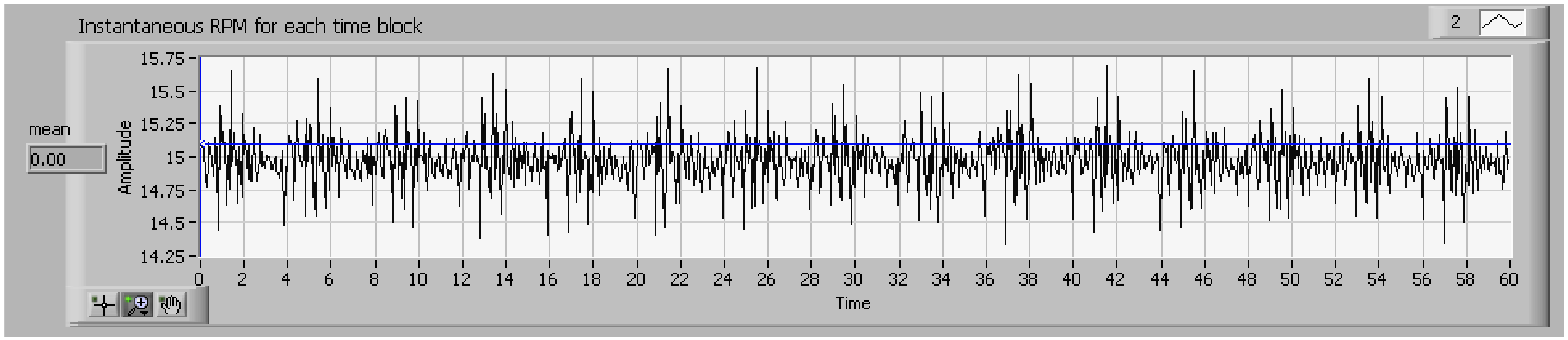

Figure 4.9 Instantaneous Speed of Mill

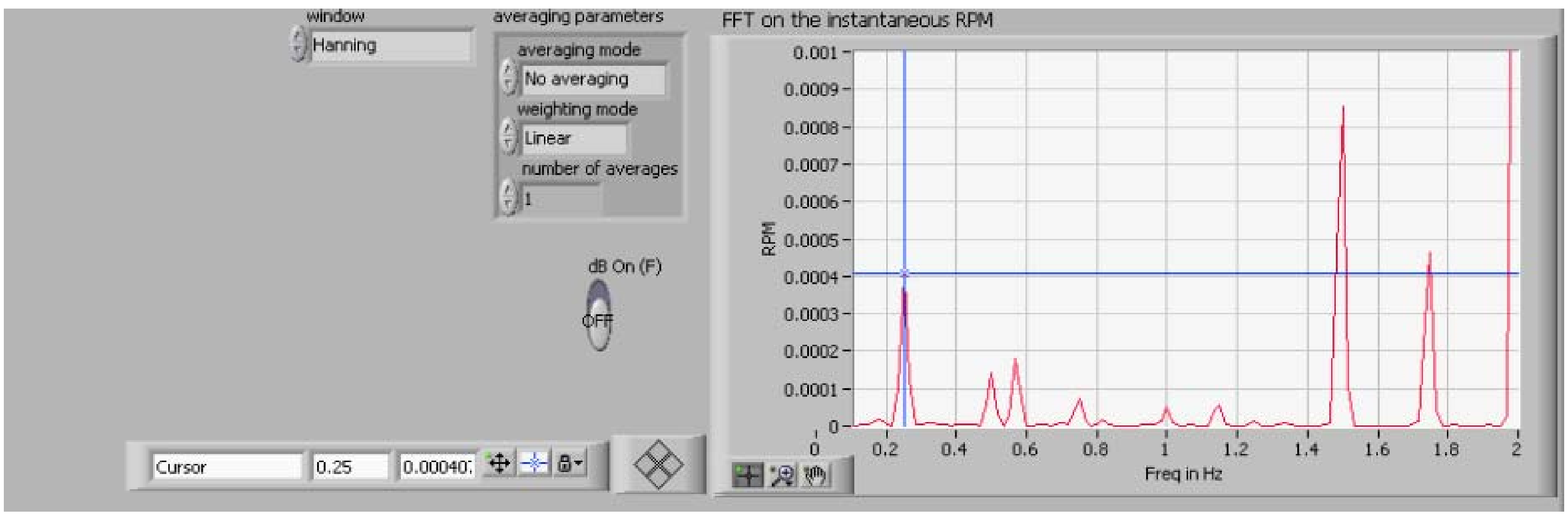

Figure 4.10 Spectrum of Instantaneous Speed of Mill

The Lab View program has 3 tabs; first one is channel configuration, second one Signal Analysis and third Spectrum. In channel configuration one can configure in accordance with the transducer using. The signal analysis page basically uses filter and pulse detection. The third one is instantaneous speed along with spectrum. The different pages of this program are shown in figures 4.11, 4.12 and 4.13. LVTorsion also has the capability to detect side bands. The raw signal from the transducer is fed into the FFT available in the Lab View to get tooth pass frequency and side bands are formed at torsional excitation frequency on both sides of the tooth pass frequency.

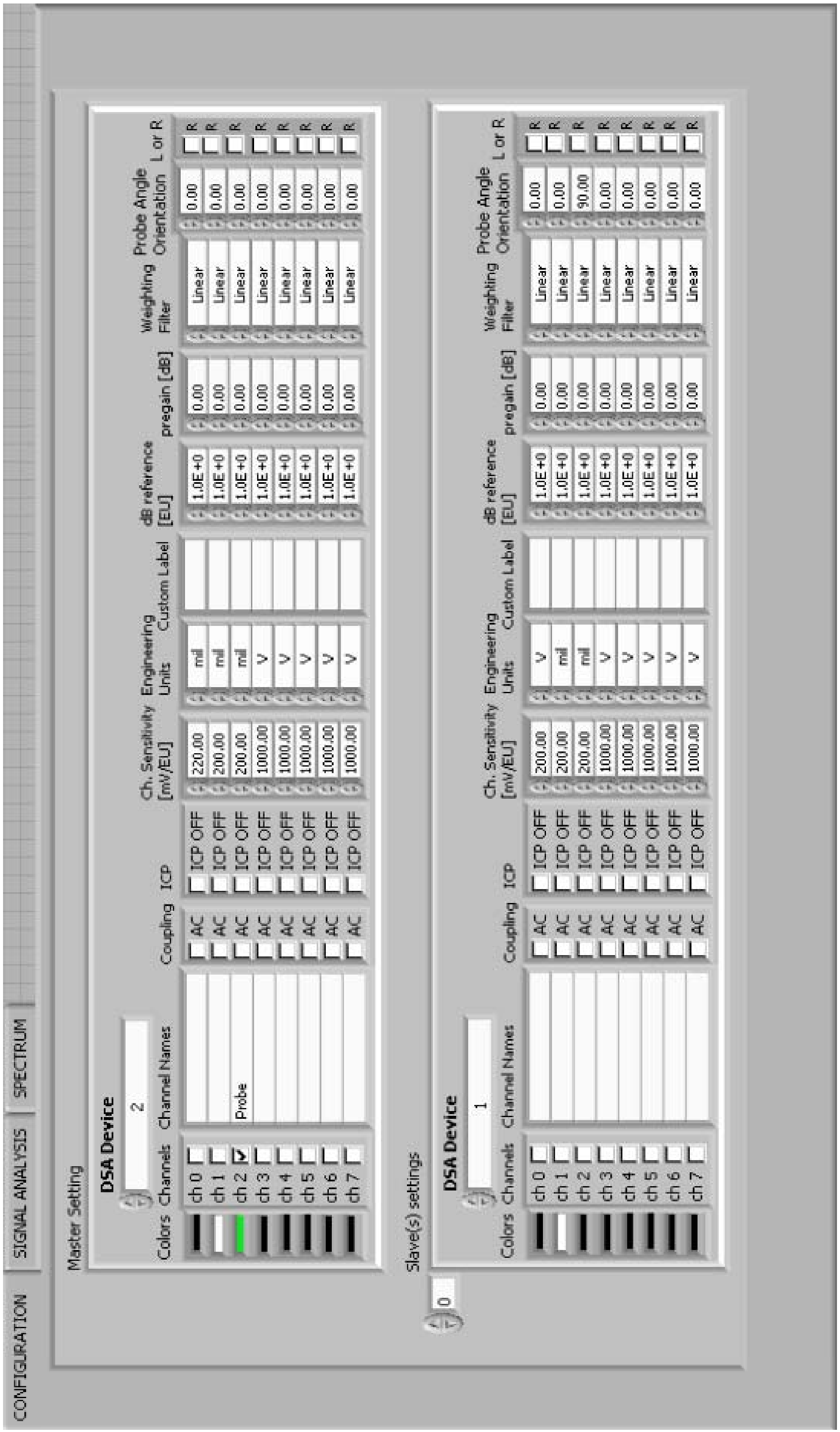

Figure 4.11 Channel Configuration

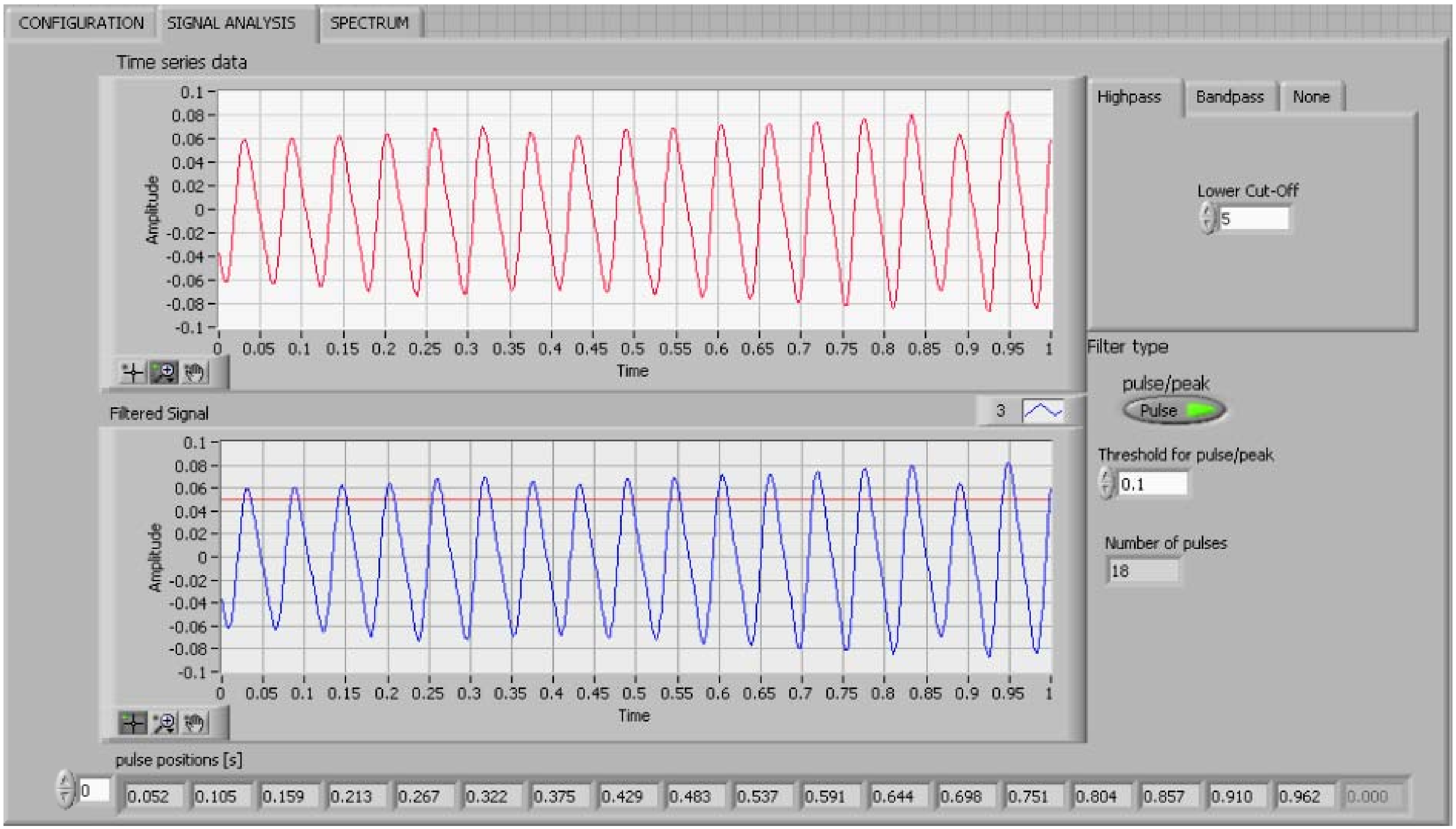

Figure 4.12 Signal Analysis

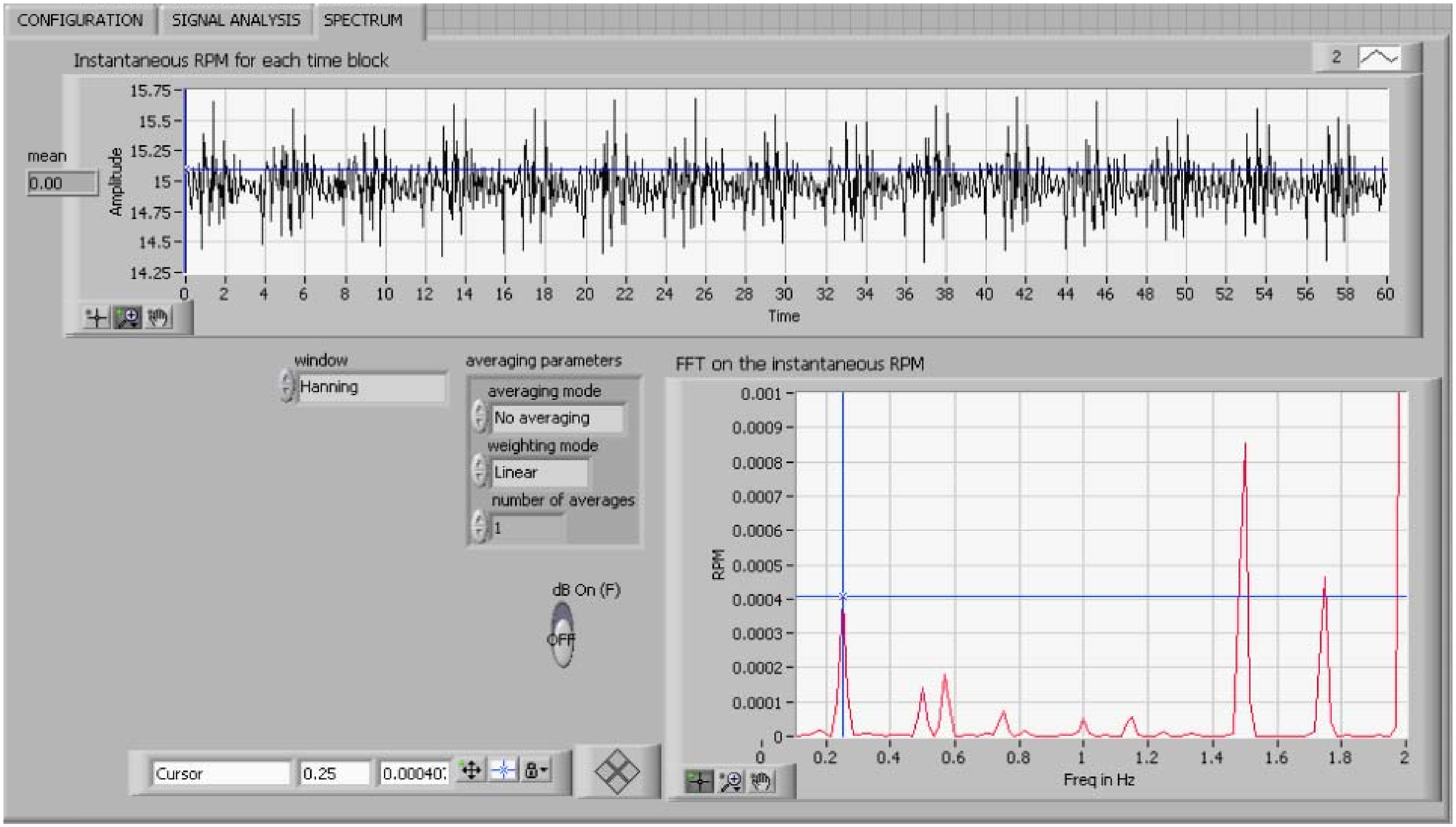

Figure 4.13 Instantaneous Speed and Spectrum

CHAPTER V

RESULTS AND DISCUSSION

The tumble mill test rig has a drum of diameter 12" (0.3 m). The "critical speed", $\sqrt{\frac{g}{R^*}}$, of the mill is 76 rpm and so the best operating speed for tumbling action is 45 rpm – 57 rpm. The tumble mill test rig is tested with four different types of material to observe the rigid body oscillation or second mode motion described in Chapter I. The four different materials tested are river rock, gravel, sand and BBs (small pellets fired from an air rifle). The mill is filled with each material (except BBs) with fill volume level of 15%, 30% and 45% and rotated to a maximum speed of up to 60 rpm. The second mode motion was observed with wet sand as the grinding media for a volume fill level of 15% and at 10 rpm. At this fill volume level and speed, the dry grinding media motion is in first mode. With the addition of a little water (less than 1% by volume), the sand behaves as a rigid body and the grinding media motion is in the second mode. The motion of the sand was observed through the Plexiglas cover.

GRINDING MEDIA: SAND

Captured video frames of the second mode motion of grinding media at 15% fill volume level and 10 rpm is depicted in figures 5.1 and 5.2. In figure 5.1, frames 1 to 6 describe the upward motion of the grinding media, which can be established through a vertical reference line. This vertical reference line passes through frames 1 to 6. It can be noted that the distance between the vertical reference line and the trailing edge of the grinding media is increasing from frames 1 to 6 indicating upward motion.

The downward motion of the grinding media is shown in frames 7 to 11 of figure 5.2. This motion can also be observed through the decrease in the distance between the vertical reference line and the trailing edge of the grinding media from frames 7 to 11.

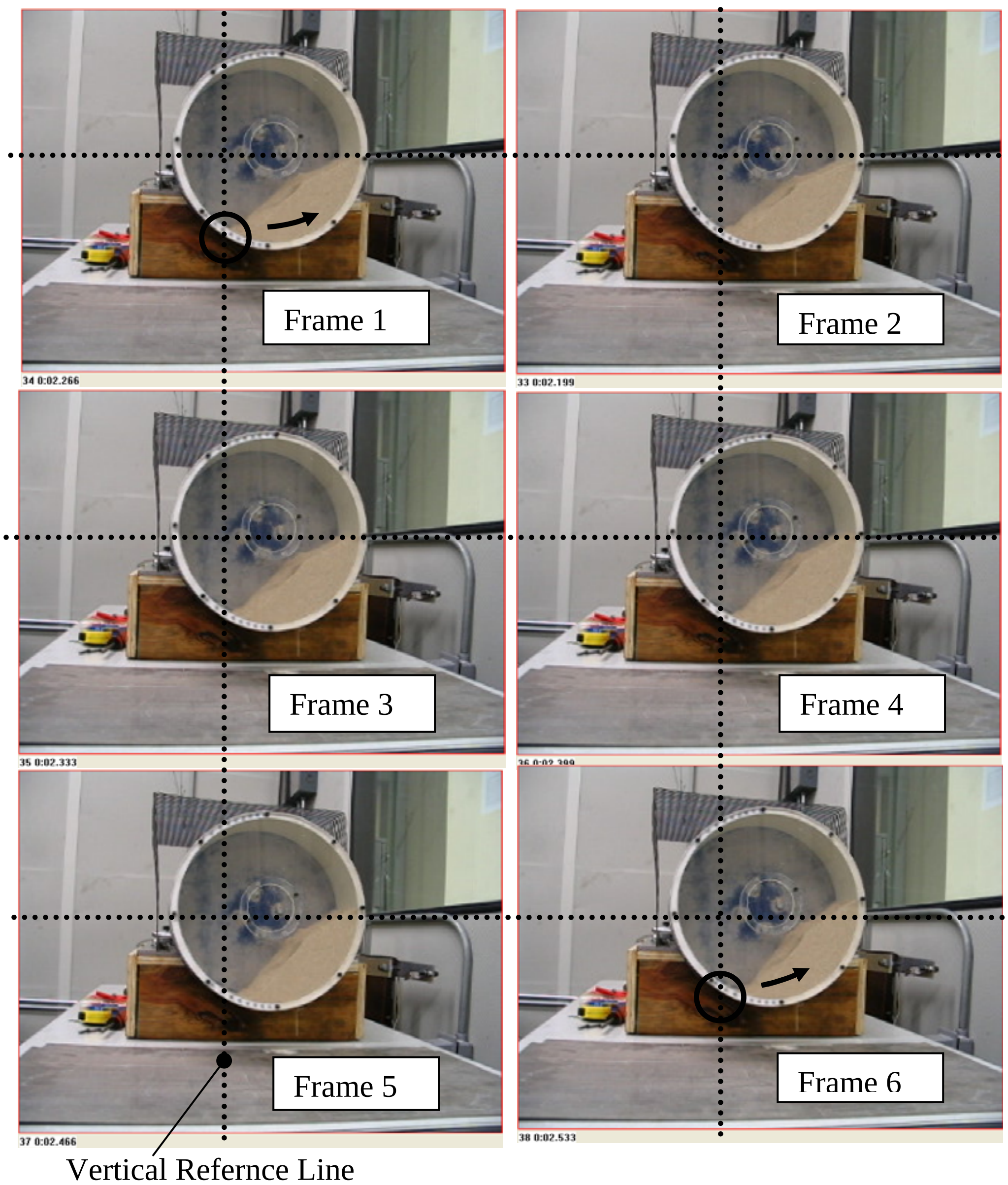

Figure 5.1 Grinding Media Upward Motion

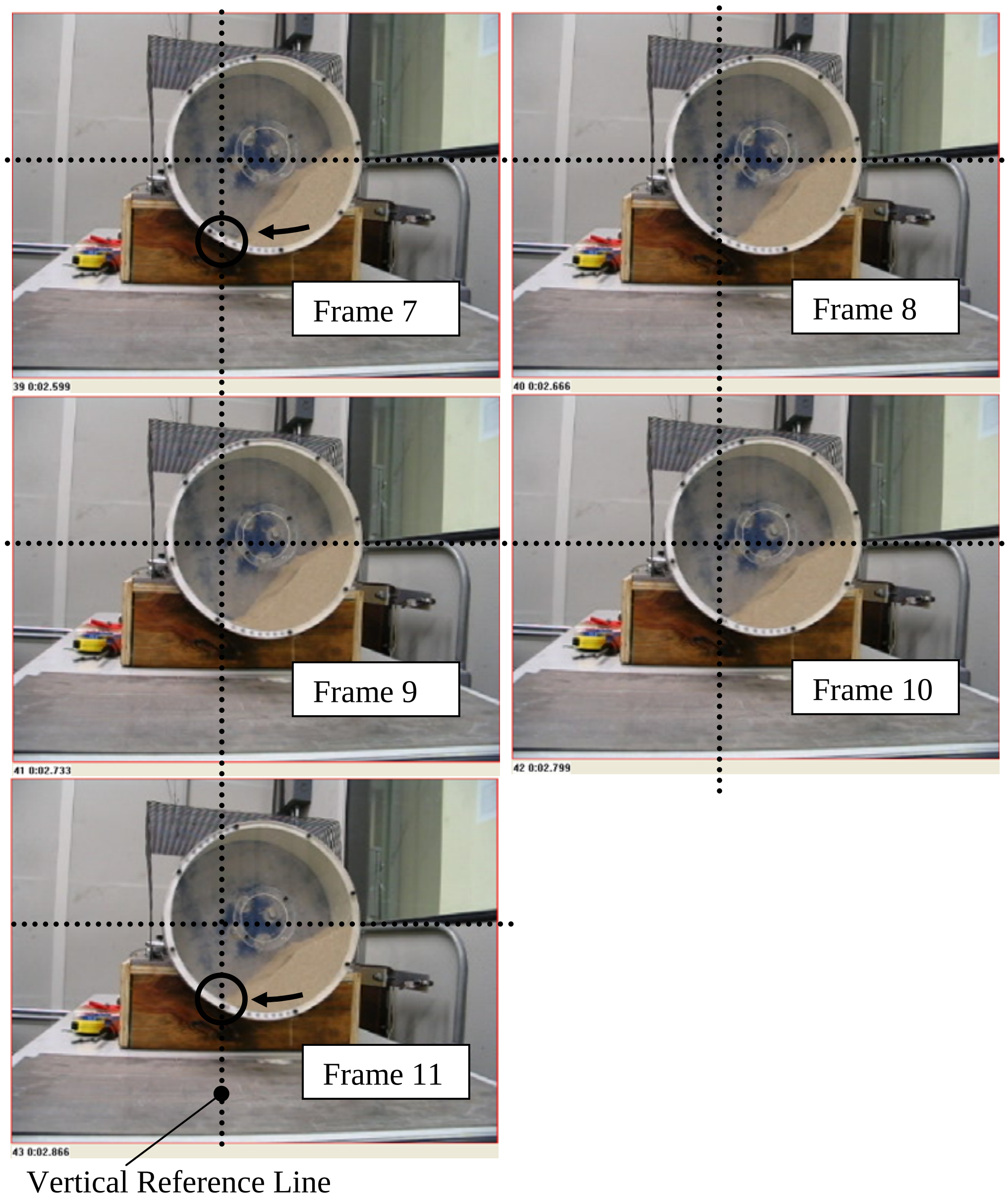

Figure 5.2 Grinding Media Downward Motion

At a fill volume level of 15% and a mill operating speed of 15 rpm, the grinding media oscillating frequency is found to be 80 cpm (1.33 Hz). This frequency is measured using a stop watch and observing the grinding media motion through Plexiglas. The spectrums from LVTorsion for the empty mill and the mill loaded at 15% volume fill level and 15 rpm running speed are shown in figures 5.3 and 5.4 respectively. The spectrum for the mill loaded with grinding media (sand in this case) clearly shows a peak at 80 cpm (1.33 Hz).

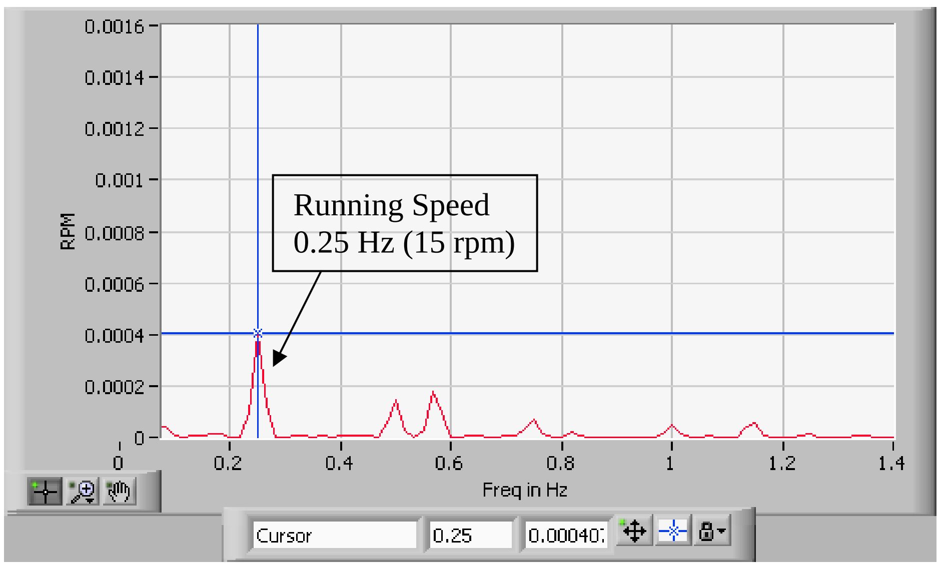

Figure 5.3 Spectrum of Mill at 15 rpm without Load

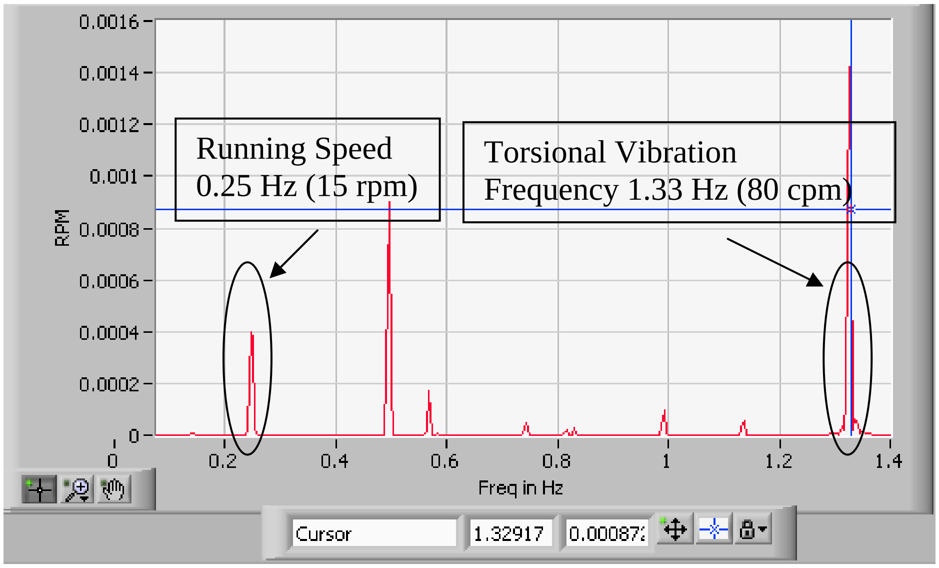

Figure 5.4 Spectrum of Mill at 15 rpm with Load

SIDE BANDS

The traditional method to measure torsional vibration frequencies is side bands. The torsional excitation frequency of the grinding media can also be established through the presence of side bands, but one has to know the frequencies to look for. The tooth pass frequency at the mean running speed of 14.87 rpm (approximately 15 rpm) is 14.87 multiplied by the number of teeth (70) or 1040.88 cpm (17.348 Hz). As the grinding media oscillates at 80 cpm (1.33 Hz), the side bands due to torsional vibrations should be at 18.68 Hz (17.348 Hz + 1.33 Hz) and 16.018 Hz (17.348 Hz - 1.33 Hz). Figure 5.5 shows the spectrum of the raw signal from magnetic transducer along with the side bands.

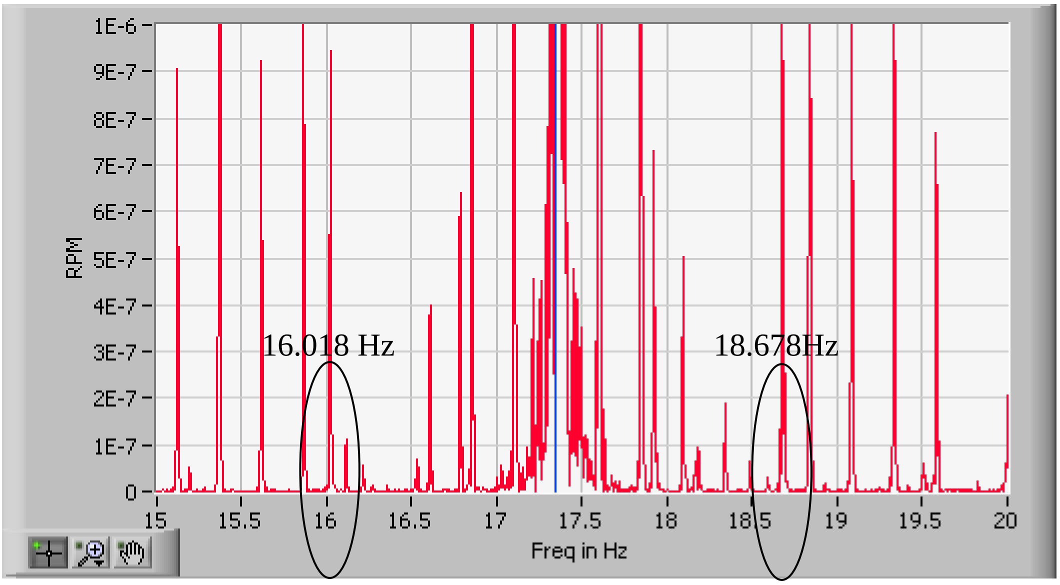

Figure 5.5 Spectrum of Raw Signal Showing Side Bands

EQUATION FOR FRICTION COEFFICIENT

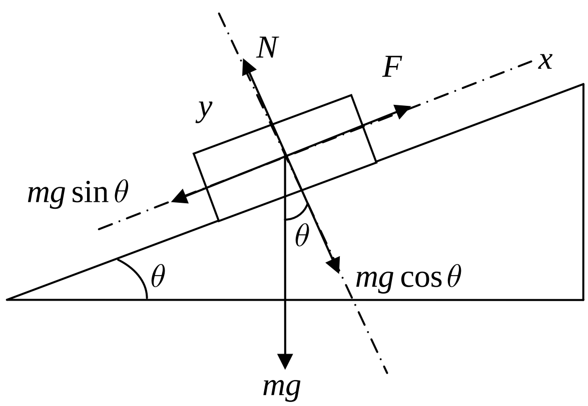

Figure 5.6 FBD to Determine Friction Coefficient

$$\sum F_x = 0$$

$$F = mg\sin\theta \tag{5.1}$$

$$\sum F_y = 0$$

$$N = mgcos\theta \qquad (5.2)$$

from $F = \mu N$

$$mg\sin\theta = \mu mg\cos\theta \qquad (5.3)$$

$$\mu = \tan\theta \qquad (5.4)$$

To determine the friction coefficient the sand was placed into a bottomless container and put onto an inclined surface (shown in figure 5.6) similar to the PVC mill surface. The inclination angle was increased until the sand began to slide. Plexiglas sheet was used in this testing as both materials are made of plastic. After testing with sand and BB's the static friction coefficients are found to be 0.52 and 0.3 respectively.

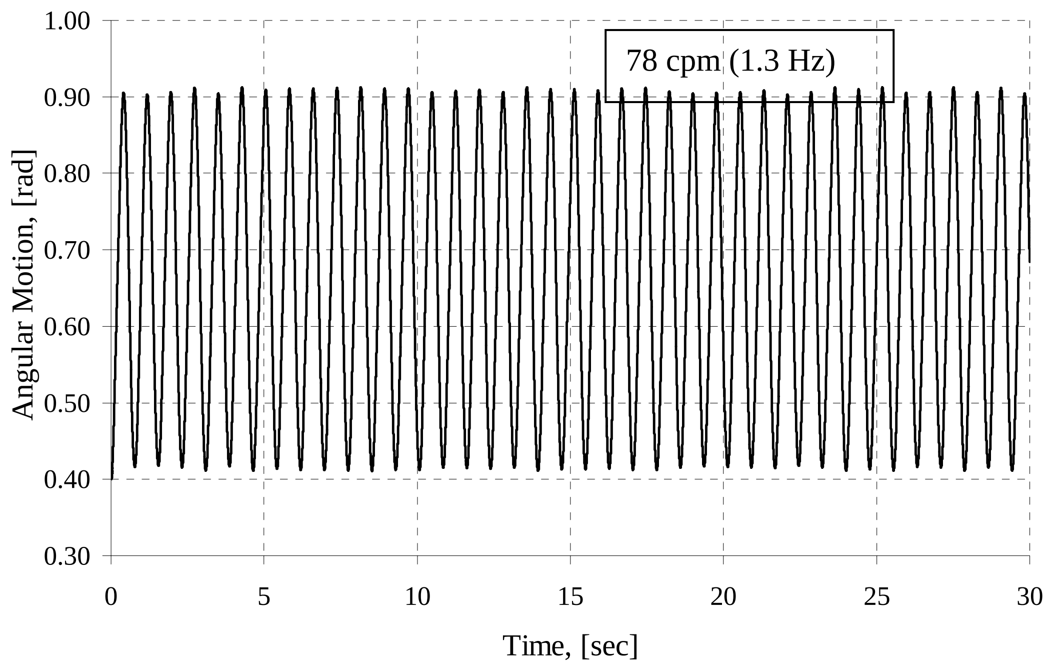

Figure 5.7 Angular Motion of Grinding Media in Mill from Simulation

Computer simulation of the grinding media for the given friction coefficients of 0.52 and 0.45, fill volume level of 15%, running speed of 15 rpm and 0 seconds start up time is shown in figure 5.7. This predicts a grinding media oscillation frequency of 78 cpm (1.3 Hz).

Video frames of the mill running at 15% fill volume level and 15 rpm running speed are used to measure the angle made by the center of gravity of the mill (*e*) with negative vertical axis. These measured angles are used in calculating excitation torque. Figure 5.8 shows a comparison of excitation torque for measured and predicted motion of the grinding media.

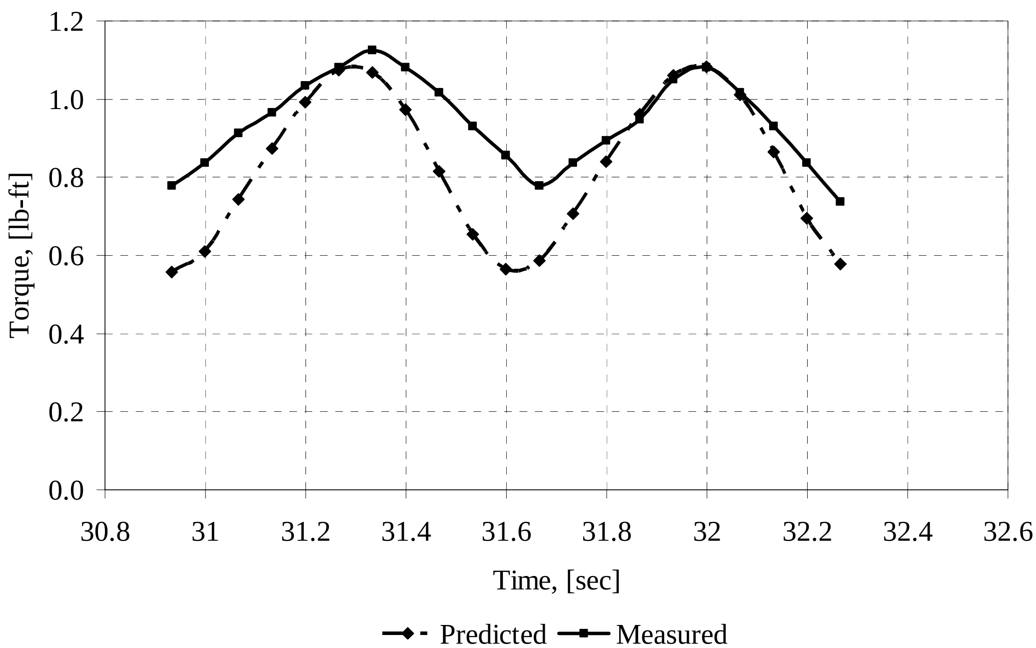

Figure 5.8 Torque Excitation of Grinding Media in Tumble Mill

The tumble mill is then tested at the same fill volume level of 15% and at running speeds of 20, 30 and 40 rpm. The comparisons between measurements and computer simulation are shown in table 5.1 and figure 5.9.

Table 5.1 Comparison between Measured and Predicted Frequencies

Mill Running Speed (RPM)	Grinding Media Oscillating Frequency from Measurements Mode II (CPM)	Grinding Media Oscillating Frequency from Predictions Mode II (CPM)
15	80	78
20	78	79
30	79.8	82
40	83.4	82

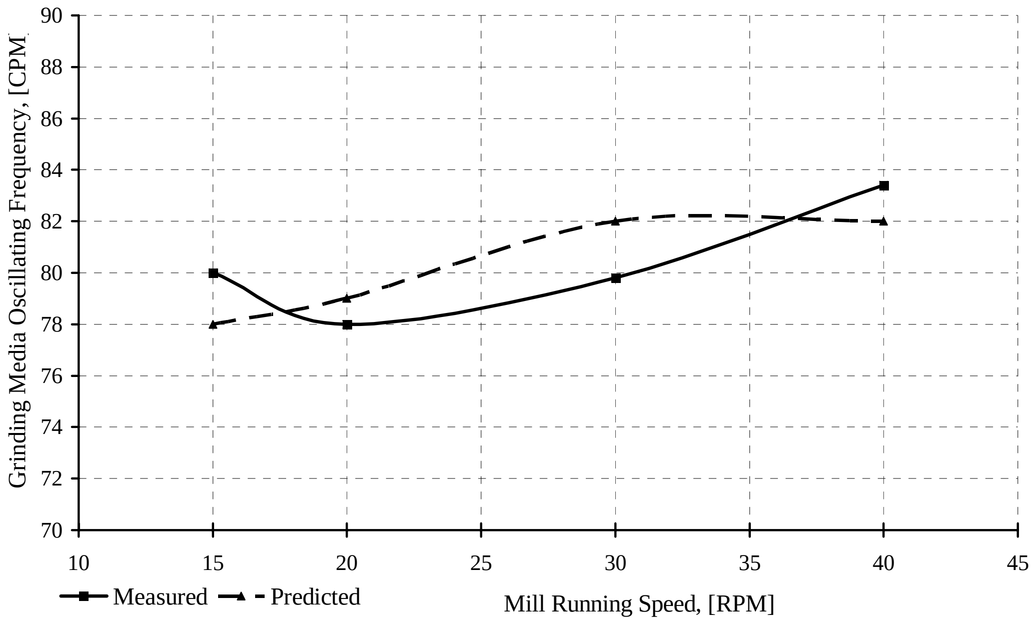

Figure 5.9 Comparison between Measurements and Predictions

It should be noted from figure 5.9 that as the mill running speed increases the oscillating frequency of the grinding media increases. The increase in this frequency is only 4% when the mill running speed is increased from 15 to 40 rpm. This is because of the increase in the normal reaction of the grinding media as the speed increases which increases the frictional torque pulling the grinding media up from sliding down. This can also be established through figure 5.10, which shows the predicted grinding media oscillation amplitude and frequencies as the speed increases.

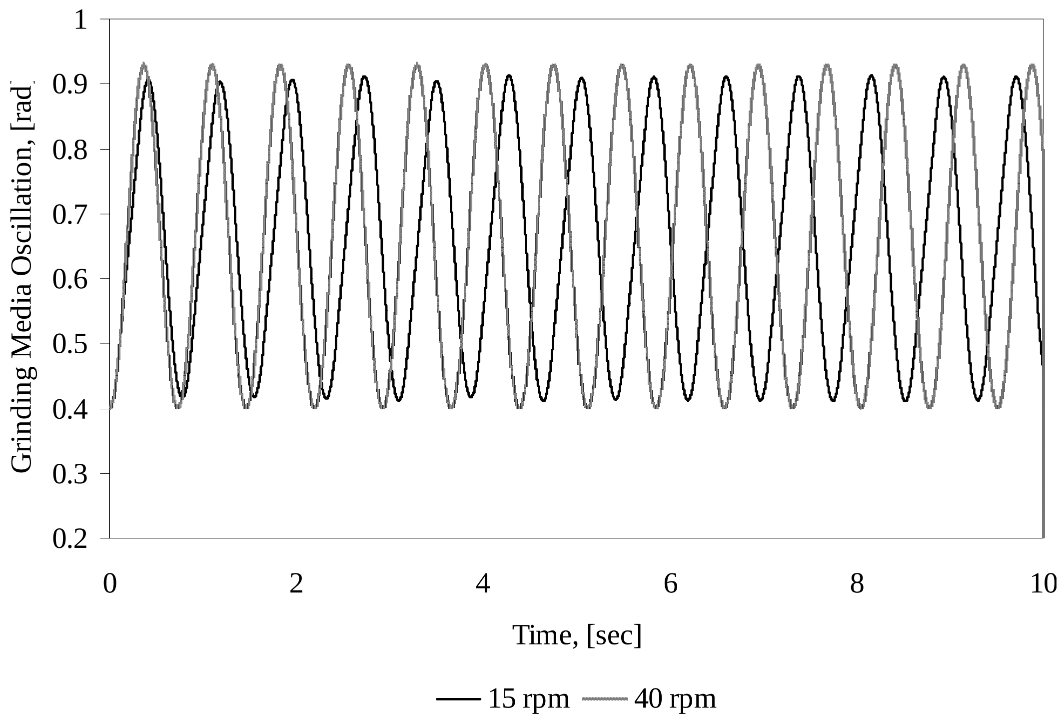

Figure 5.10 Grinding Media Oscillation Amplitude from Simulation

GRINDING MEDIA: BBs

The tumble mill test rig is tested with BBs at 15% fill volume level and at different running speeds. At 45 rpm a clear oscillating sound is heard but it was difficult to notice these oscillations visually. These audible oscillations are counted using a stopwatch and the BBs are found to oscillate at 100 cpm (1.67 Hz). The spectrum in figure 5.11 obtained using LVTorsion shows a peak at 102 cpm (1.7 Hz). During start-up of the mill with BBs it is clearly observed that there is a torsional excitation frequency (audibly noticed), these oscillations could not be measured as LVTorsion does not have the capability to measure torsional vibrations during run-up and coast down.

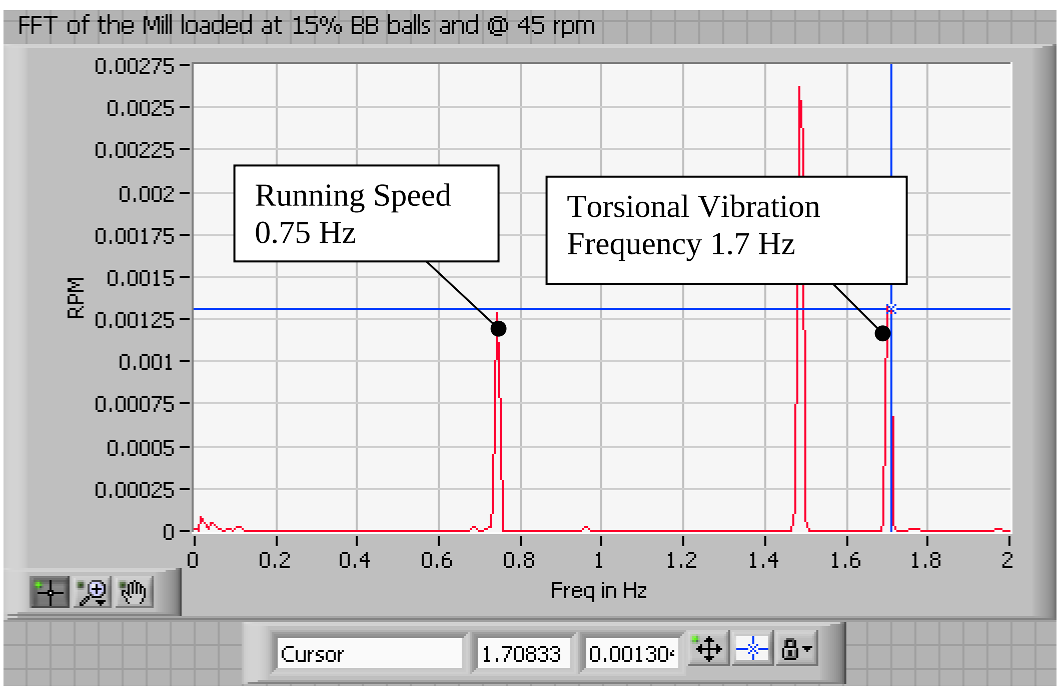

Figure 5.11 Spectrum of the Mill Loaded with BB Balls

GRINDING MEDIA: SAND

When the tumble mill is filled with more than 15% sand the grinding media no longer behaves as a rigid body and the motion of the grinding media is in the first mode as described in Chapter I. This means that the assumptions made in earlier chapters are invalid for cases with more than 15% fill. Regardless of this invalidity, it is of the interest to the industry to conduct tests on the mill with more than 15% fill volume, as 33% fill volume is the optimum level used in industry, [3]. Tests are performed at 30% and 45% fill volumes at different running speeds. The mill is loaded at 30% fill volume and run up to 60 rpm. There is no change on the spectrum between the cases of the mill loaded and unloaded up to 40 rpm. The spectrums of the mill at 50 rpm without load and with load at 30% of grinding media are shown in figures 5.12 and 5.13 respectively.

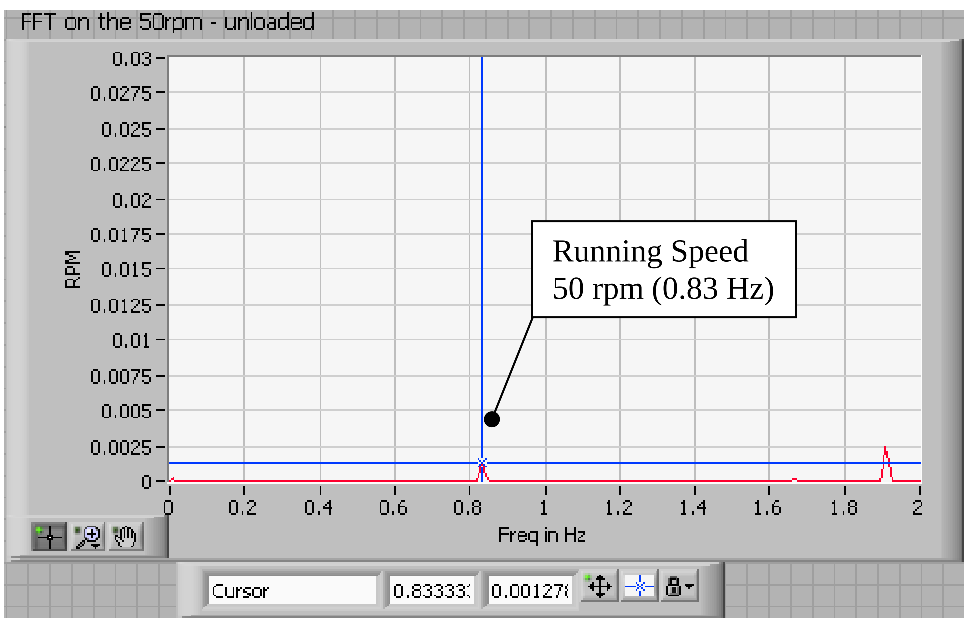

Figure 5.12 Spectrum of Tumble Mill without Load at 50 RPM

Figure 5.13 clearly shows a grinding media oscillating frequency of 90 cpm (1.51 Hz). The oscillating frequency is not mode II. It is an oscillating variant of mode I. The incorrectly predicted mode II motion from the computer program for 30% fill volume and 50 rpm is shown in figure 5.14 and the calculated frequency from the simulation is 72 cpm (1.2 Hz). Even though the mode of motion is not correctly predicted, the predicted excitation frequency is within 20%.

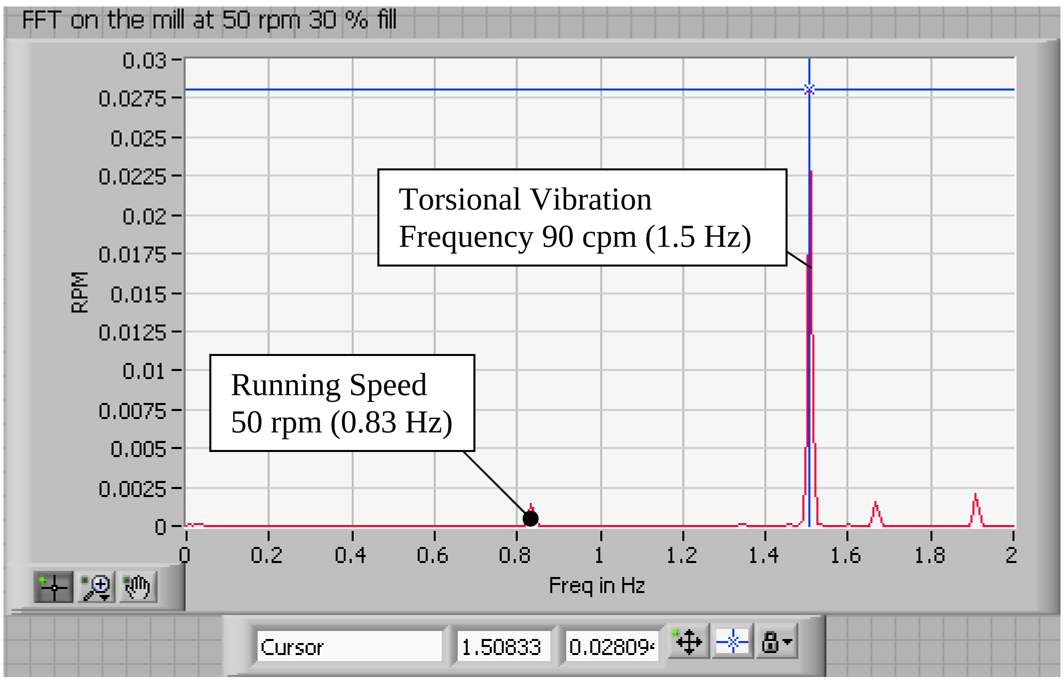

Figure 5.13 Spectrum of Tumble Mill with 30% Fill Volume at 50 RPM

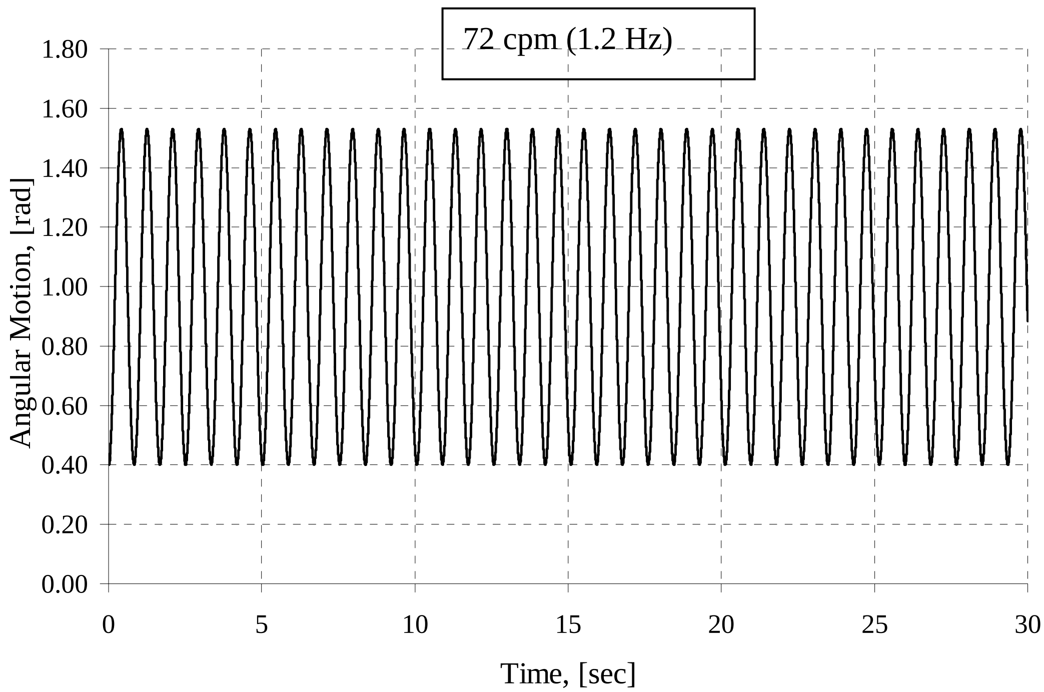

Figure 5.14 Angular Motion of Grinding Media in Mill from Simulation

The test results of the mill at fill volume levels of 30% and 45% and at running speeds of 50 and 60 rpm are shown in table 5.2 and figure 5.15 along with predicted frequencies from the computer program. It should be noted that the program assumes rigid body motion (mode II) for the grinding media inside the drum and so there is large difference between predicted and measured results at 30% and 45% fill volumes than at 15% fill volume level.

At 45% fill volume and 60 rpm the measured oscillating frequency is 89 cpm (1.483 Hz), which is less than that of 50 rpm running speed with oscillating frequency 114 cpm (1.9 Hz). This may be because at 60 rpm bigger fractions of grinding media ride to top of the mill compared to smaller fractions at 50 rpm. At 45% fill volume level and at 50 rpm and 60 rpm the prediction is that the grinding media does not oscillate as it is near the critical speed of 76 rpm.

Table 5.2 Measurements and Predictions for 30% and 45% Fill Volumes

	30% fill level		45% fill level	
Mill Running Speed (RPM)	Grinding Media Oscillating Frequency from Measurements Mode I (CPM)	Grinding Media Oscillating Frequency from Predictions Mode II (CPM)	Grinding Media Oscillating Frequency from Measurements Mode I (CPM)	Grinding Media Oscillating Frequency from Predictions Mode II (CPM)
50	90	72	114	-
60	94	72	89	-

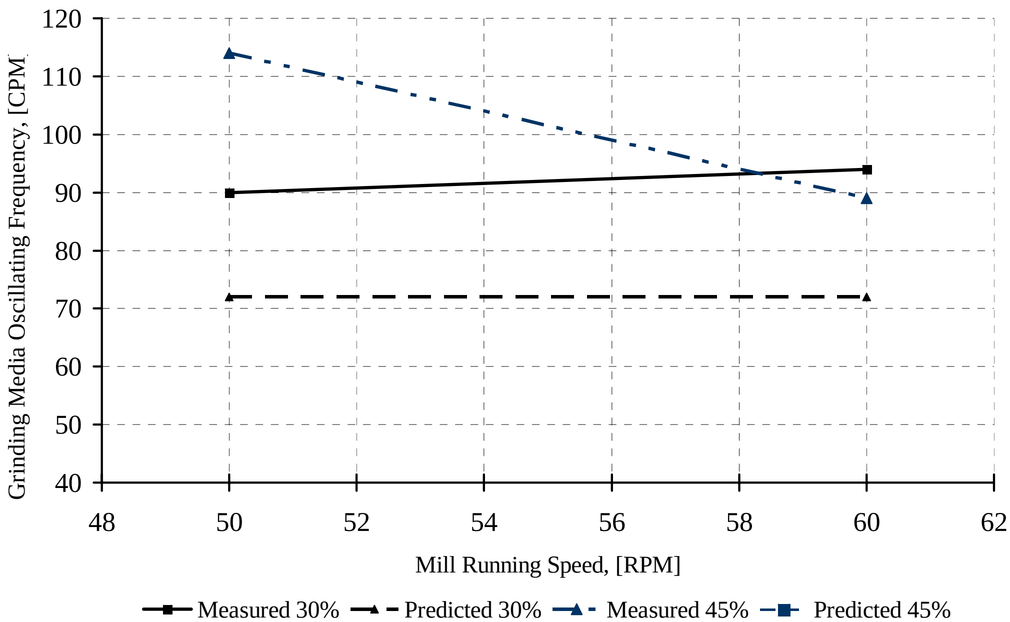

Figure 5.15 Comparison between Measurements and Predictions

The measurements from LVTorsion for the fill volume level of 30% at 60 rpm and 45% at 50 and 60 rpm are presented in Appendix F.

CHAPTER VI

CONCLUSIONS

The theoretical model for the grinding media motion inside the tumble drum is developed and is programmed in Microsoft Excel using Visual Basic macros. A 12” diameter tumble mill test rig is designed and fabricated. LVTorsion is used to measure torsional vibrations on the tumble mill due to grinding media oscillation.

The grinding media motion is in the second mode for a fill volume of 15% and induces torsional excitation. At higher fill volume levels it is seen that the grinding media motion is in the first mode, which also causes torsional excitation due to regular modulation of the tumbling action. These excitation frequencies should be considered in the design and failure analysis of tumble mills.

At 15% fill volume level the measurements and predictions agree well. As the fill volume level is increased, however the discrepancy between the measurements and predictions increases. The torque variation from the simulation is only valid for fill levels up to 15% for sand as the simulation is based on mode II motion.

When the tumble mill is loaded with BBs, experiments show that the grinding media oscillates in mode I during start-up of the mill. This may be an explanation of higher loads on the gears during start-up. However, this excitation amplitude and frequency was not measured because LVTorsion does not have the capability to measure torsional vibrations under start-up and coast down conditions.

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Supplemental Sources Consulted:

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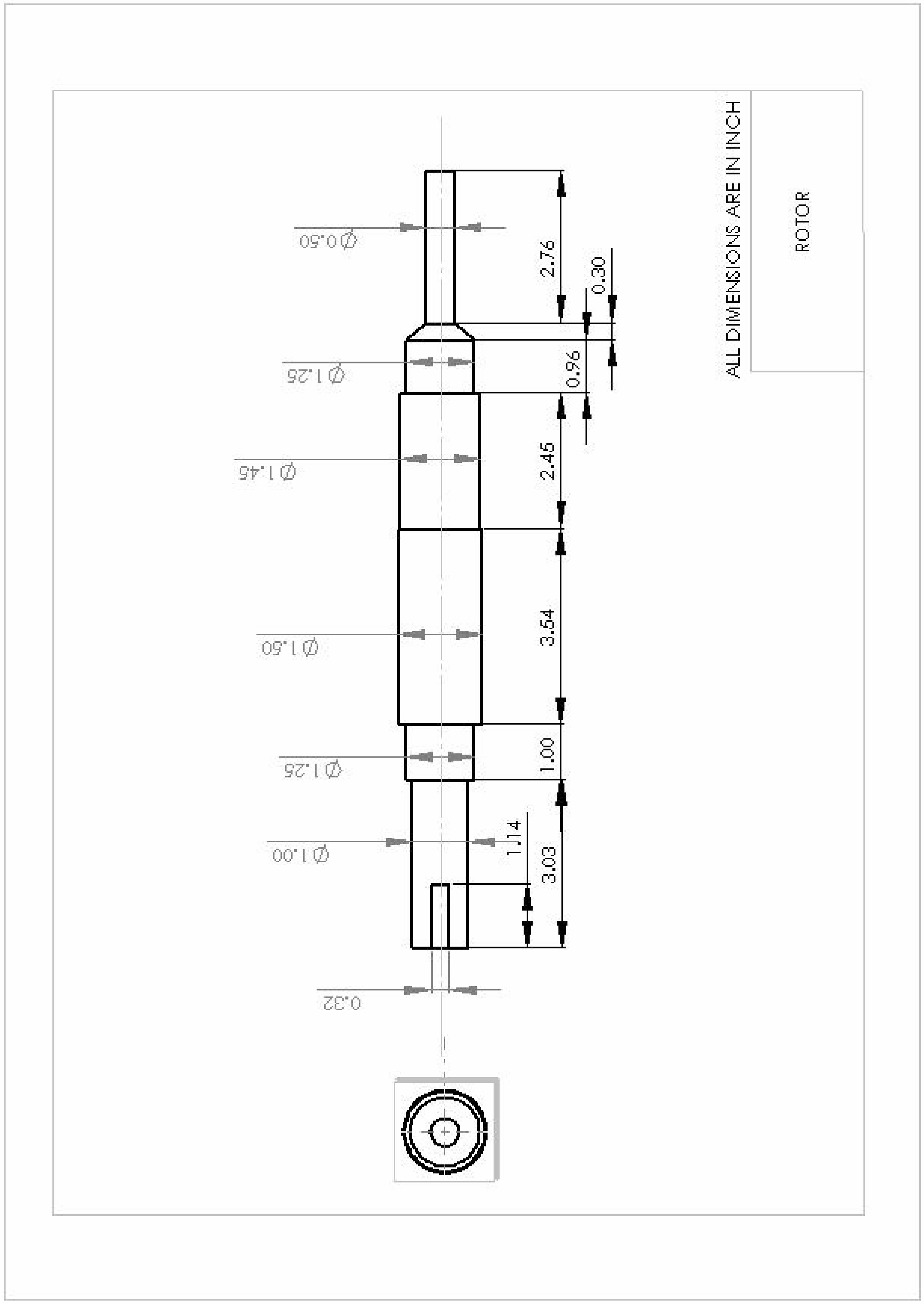
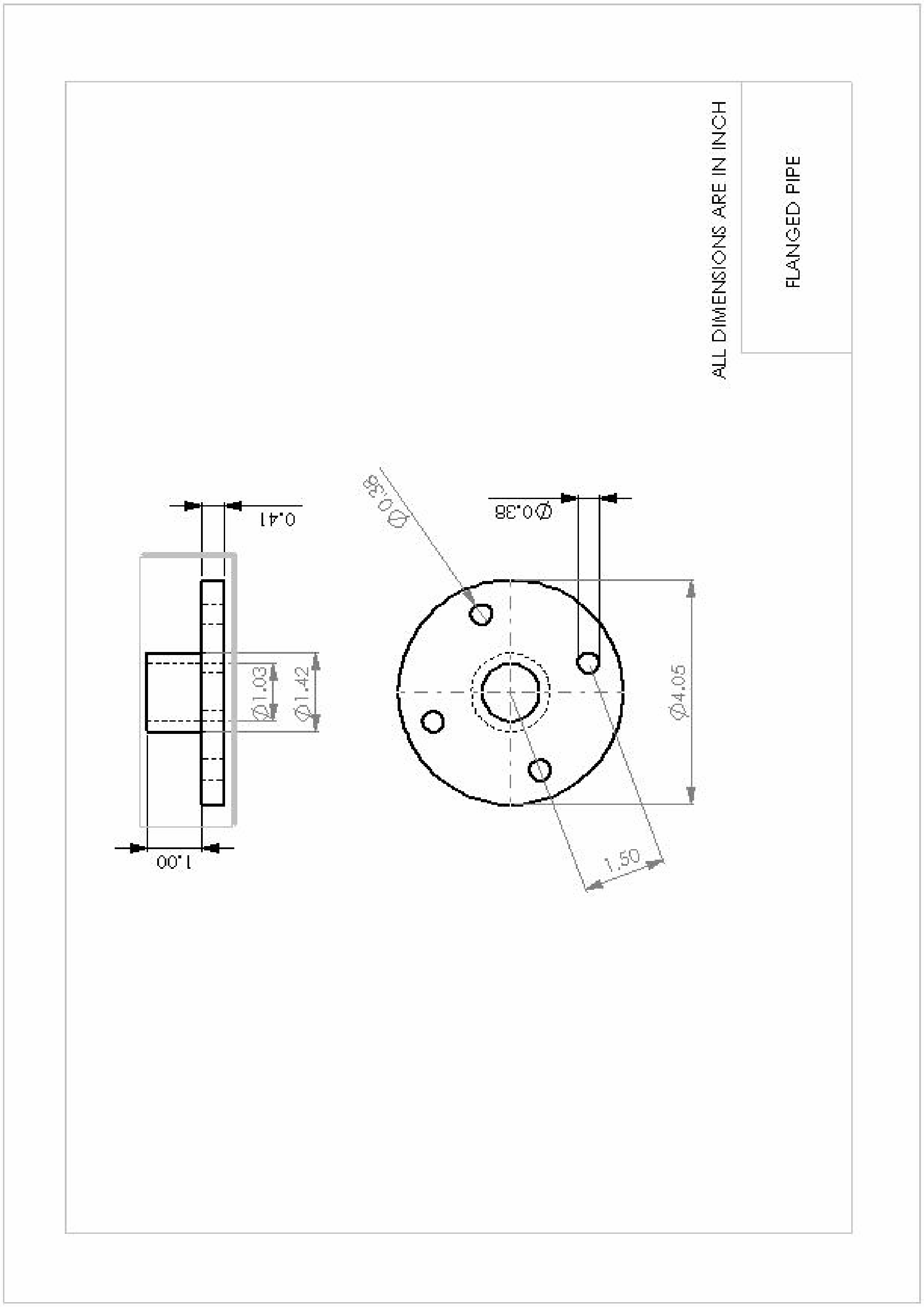
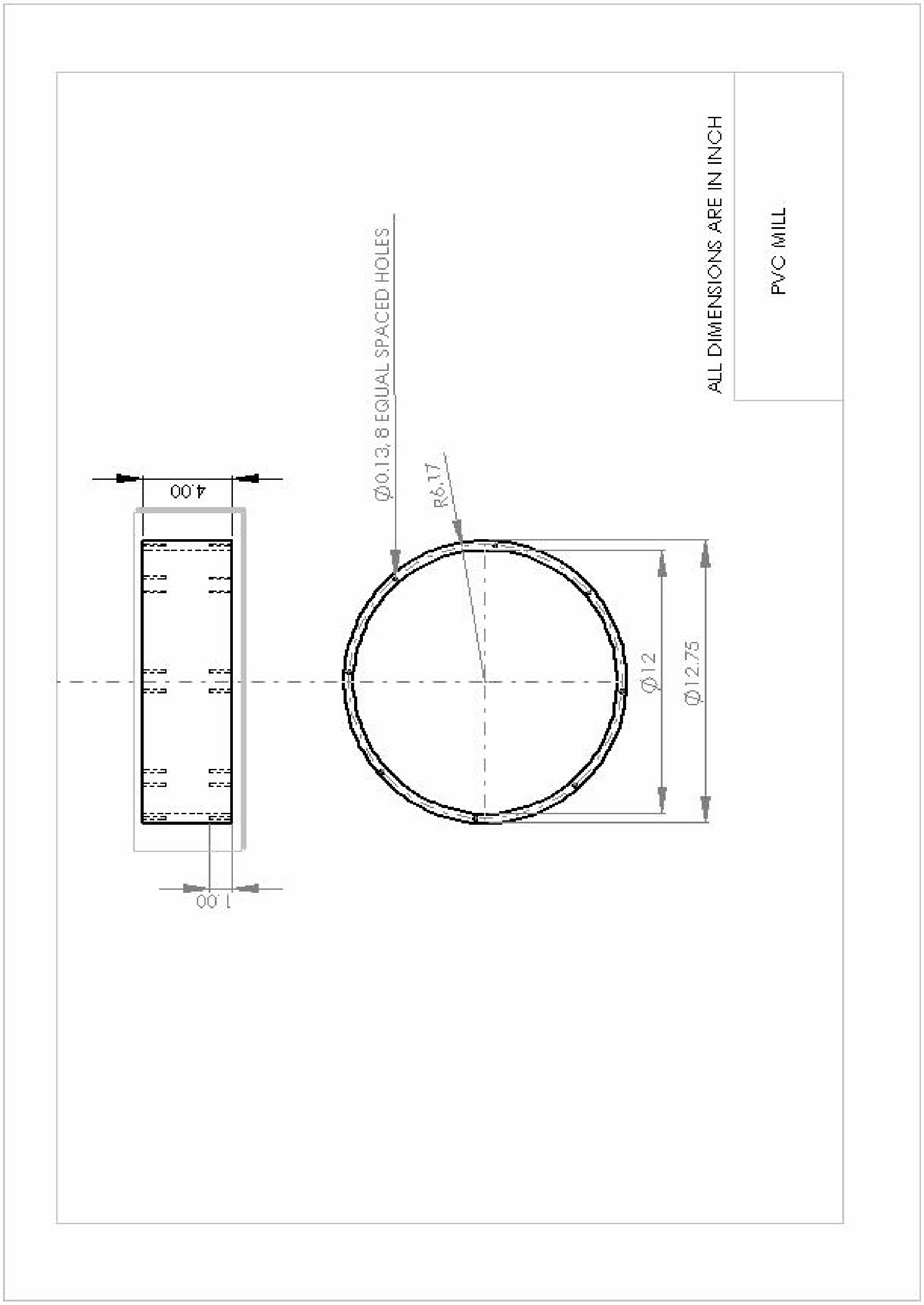
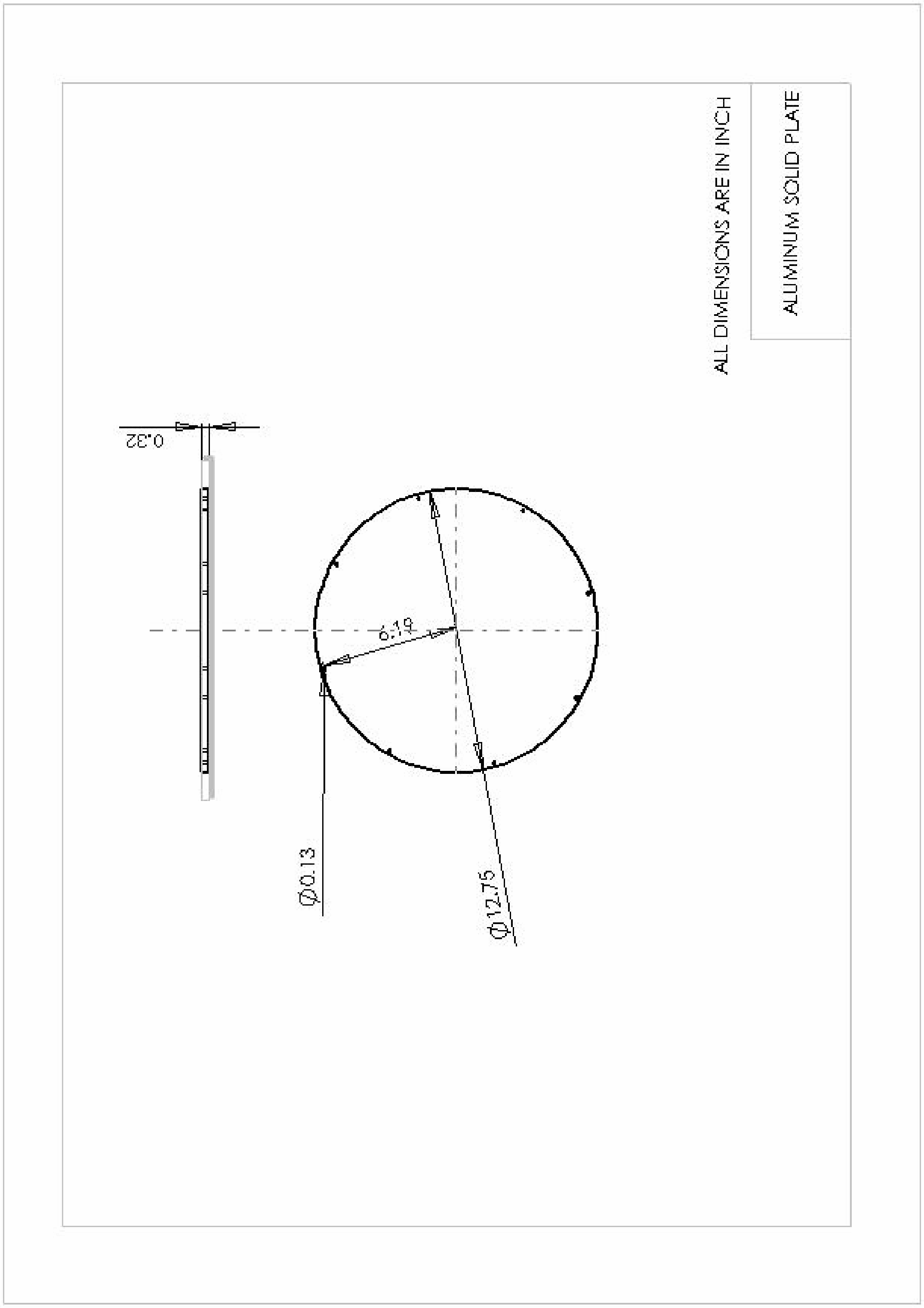
APPENDIX A

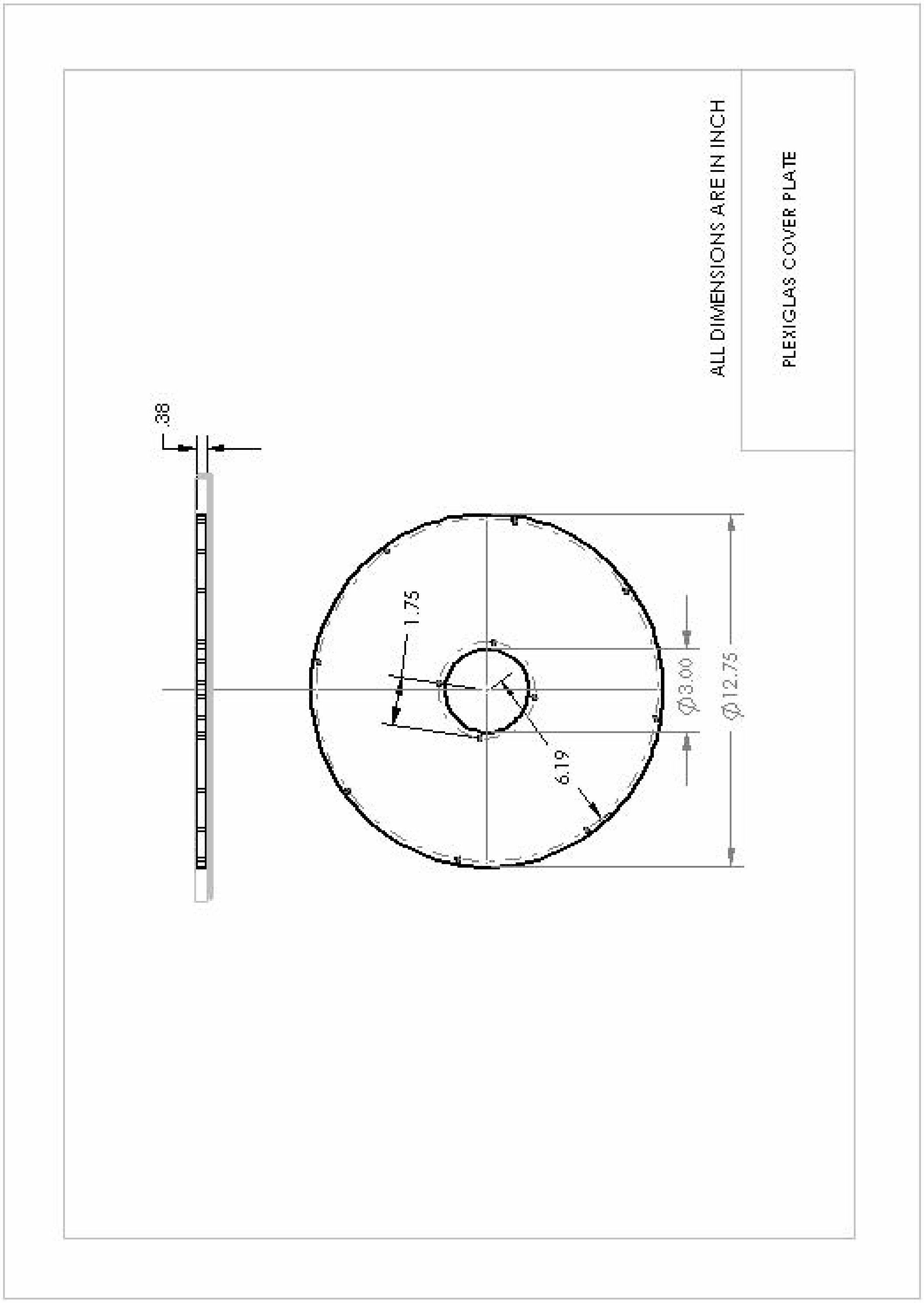
DRAWINGS

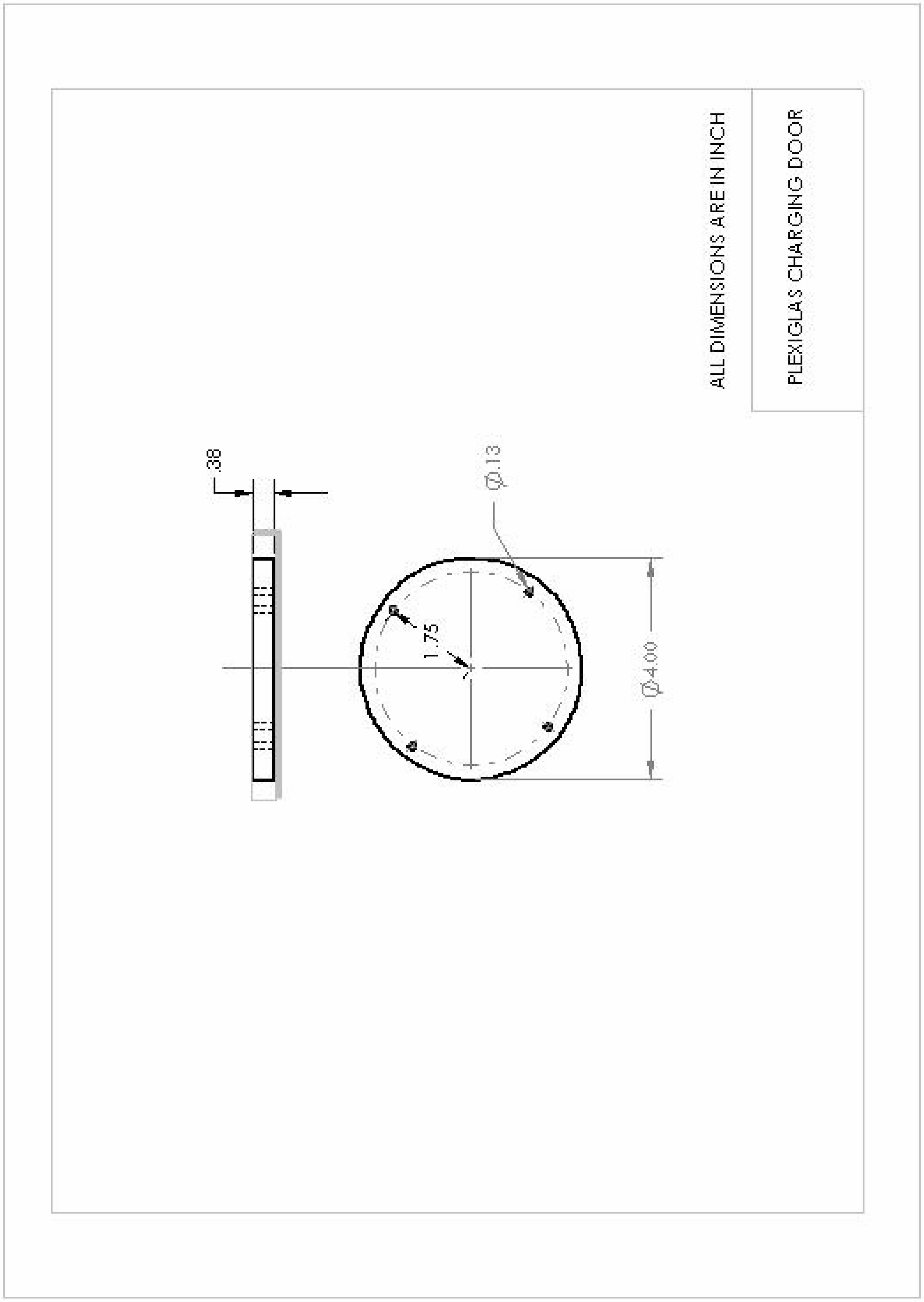

ALL DIMENSIONS ARE IN INCH
ROTOR
Ø0.50
2.76
0.30
Ø1.25
0.96
Ø1.45
2.45
Ø1.50
3.54
Ø1.25
1.00
Ø1.00
1.14
3.03
0.32


ALL DIMENSIONS ARE IN INCH
FLANGED PIPE
0.41
1.00
Ø1.03
Ø1.42
Ø0.38
Ø0.38
Ø4.05
1.50


ALL DIMENSIONS ARE IN INCH
PVC MILL
Ø0.13, 8 EQUAL SPACED HOLES
R6.17
Ø12
Ø12.75
4.00
1.00


ALL DIMENSIONS ARE IN INCH
ALUMINUM SOLID PLATE
0.32
6.19
Ø0.13
Ø12.75


ALL DIMENSIONS ARE IN INCH
PLEXIGLAS COVER PLATE
.38
1.75
6.19
Ø3.00
Ø12.75


ALL DIMENSIONS ARE IN INCH
PLEXIGLAS CHARGING DOOR
.38
Ø.13
1.75
Ø4.00

APPENDIX B

MASS POLAR MOMENT OF INERTIA OF TUMBLE MILL ASSEMBLY AND MOTOR

The mass polar moment of inertia of the assembly (made of rotor, flanged plate, solid aluminum plate, PVC mill, Plexiglas cover and Plexiglas filling door) is found through two procedures. The first procedure described in [8], involves assembling all the parts mentioned above together, suspending the assembly by two cables and measuring the period of oscillation as the assembly twists. The axis of oscillation is the axial geometric centerline of the assembly and the cable length is long enough to minimize translational motion; thereby to improving the accuracy of the measurement. The assembly suspended from a cable length of 94” (2.38 *m*) is shown in figure B1 (entire cable length not shown).

Figure B1 Tumble Mill Assembly Suspended from Cables

After suspending the assembly from the ceiling, the assembly was twisted manually and then released. The assembly oscillated torsionally back and forth and the period of oscillation was measured using a stop watch. The mass polar moment of inertia was computed using the following expression [8].

$$J_{assembly} = \frac{WR_c^{\,2}\,Period^{\,2}}{4\pi^2 L_c}$$

The average time taken over 10 iterations was 1.857 *seconds*. The distance between the axial center of the assembly to the cable (R_c) was 6.5” (0.16 *m*). The total weight of the assembly (W) was 16 *lb* (7.25 *kg*).

$$J_{assembly} = \frac{(16).(6.5)^2.(1.857)}{4.\pi^2.94} = 0.628015 lb.inch.\sec^2 (0.067 kg.m^2)$$

The second procedure for calculating the mass polar moment of inertia involves using component dimensions, material properties and CAD software. Using Solid Works® the mass polar moment of inertia for the given geometry and physical properties of each component were calculated. The individual components of the rotor, mill assembly were drafted and density of each component material like steel, aluminum and PVC are used to find the mass polar moment of inertia. Table B.1 gives each component of the assembly, mass polar moment of inertia and weight.

Table B.1 From Solid Works

Component	Polar Moment of Inertia	Weight
	lb-inch^2	lb
Coupling	0.570	1.200
Rotor	1.030	4.980
Flange Plate	3.100	1.590
Solid Plate	79.270	3.900
Drum	117.540	3.090
Plexiglas plate	41.700	1.940
Charging Door	0.410	0.200
TOTAL	243.620	16.900

Mass polar moment inertia of the total assembly $= \dfrac{243.620}{386}$

$$= 0.63114 lb.inch.\sec^2 (0.071 kg.m^2)$$

The percentage difference between procedure 1 and 2 is approximately 0.5%.

To find the mass polar moment inertia of gear motor the procedure described in [4] was followed. A weight of 11.35 *lb* (5.15 *kg*) was suspended through a cable wound around the coupling of the motor as shown in figure B2. The radius R_S of the coupling is 0.875” (22 *mm*). The time taken for the suspended weight to fall freely through a distance of 24” (0.61 *m*) was noted over 10 iterations. The average time *T* taken for the free fall distance *D* of 24” (0.61 *m*) was 31.5 *sec*. These values were substituted in the expression shown below to calculate the mass polar moment of inertia of the gear motor.

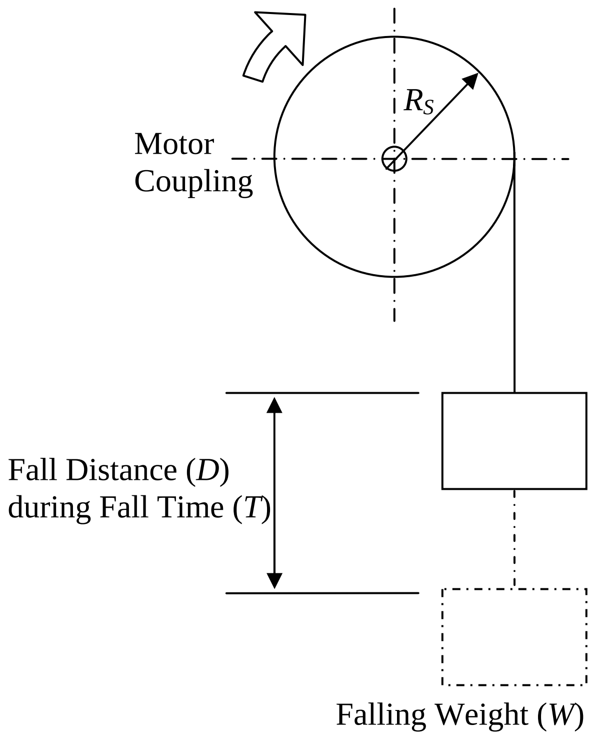


Figure B2 Mechanical Arrangement for

Mass Polar Moment Inertia of Gear Motor

$$J_{motor} = W.R_S^2.\left[\frac{T^2}{2.D} - \frac{1}{g}\right]$$

$$= (11.35).(0.875^2).\left[\frac{31.5^2}{(2).(24)} - \frac{1}{386.2}\right]$$

$$= 179.61 lb.inch.sec^2 (20.32 kg.m^2)$$

APPENDIX C

CATALOGS

24-Bit, 8-Channel Dynamic Signal Acquisition

Specifications

Typical for 25 °C unless otherwise noted.

Analog Input

Channel Characteristics

Number of channels	
NI 4472 Series	8, simultaneously sampled
NI 4474 Series	4, simultaneously sampled
Input configuration	Unbalanced differential
Resolution	24 bits, nominal
Type of ADC	Delta-sigma
Oversampling, for sample rate (f_s):	
1.0 kS/s ≤ f_s ≤ 51.2 kS/s	128 f_s
51.2 kS/s < f_s ≤ 102.4 kS/s	64 f_s
Sample rates (f_s)	1.0 to 102.4 kS/s in 190.7 µS/s increments for f_s > 51.2 kS/s or 95.36 µS/s increments for f_s ≤ 51.2 kS/s
Frequency accuracy	±25 ppm
Input signal range	±10 V peak
FIFO buffer size	1,024 samples
Data transfers	DMA

Transfer Characteristics

Offset (residual DC)	±3 mV, max
Gain (amplitude accuracy)	±0.1 dB, max, f_{in} = 1 kHz

Amplifier Characteristics

Input impedance (ground referenced)	
Positive input	1 MΩ in parallel with 60 pF
Negative input (shield)	50 Ω in parallel with 0.02 µF
Flatness (relative to 1 kHz)	±0.1 dB, DC to 0.4535 f_s, max, DC-coupled
-3 dB bandwidth	0.4863 f_s
Input coupling	AC or DC, software-selectable
AC -3 dB cutoff frequency	
NI 4472, NI 4474	3.4 Hz
NI 4472B	0.5 Hz
Overvoltage protection	
Positive input	±42.4 V
Positive inputs protected	CH<0..7>
Negative input (shield)	Not protected, rated at ±2.5 V
Common mode rejection ratio (CMRR)	
f_{in} < 1 kHz	>60 dB, minimum

Dynamic Characteristics

Alias-free bandwidth (passband)	DC (0 Hz) to 0.4535 f_s
Stop band	0.5465 f_s
Alias rejection	110 dB
Spurious-free dynamic range	130 dB, 1.0 kS/s ≤ f_s ≤ 51.2 kS/s,
118 dB, 51.2 kS/s < f_s ≤ 102.4 kS/s THD, f_{in} = 1kHz	
0 dBFS input	<-90 dB
20 dBFS input	<-100 dB
60 dBFS input	<-60 dB
IMD	<-100 dB (CCIF 14 kHz + 15 kHz)
Crosstalk[1] (channel separation), f_{in} = 0 to 51.2 kHz	
Between channels 0 and 1, 2 and 3, 4 and 5, or 6 and 7	
Shorted input	<-90 dB
1 kΩ load	<-80 dB
Other channel combinations	
Shorted input	<-100 dB
1 kΩ load	<-90 dB
Phase linearity	<±0.5 deg
Interchannel phase mismatch	<f_{in} (in kHz) x 0.018 deg + 0.082 deg
Interchannel gain mismatch	±0.1 dB
Filter delay through ADC	38.8 sample periods

Onboard Calibration Reference

DC level	5.000 V ±2.5 mV
Temperature coefficient	±5 ppm/°C maximum
Long-term stability	±20 ppm/$\sqrt{1{,}000 \text{ h}}$

Signal Conditioning

Constant current source (software-controlled)

Current	4 mA, ±5%
Compliance	24 V
Output impedance	>250 kΩ at 1 kHz
Current noise	<500 pA/$\sqrt{\text{Hz}}$

Triggers

Analog Trigger

Source	CH<0..7>
Level	-10 to +10 V, full scale, programmable
Slope	Positive or negative (software selectable)
Resolution	24 bits, nominal
Hysteresis	Programmable

Digital Trigger

Compatibility	5 V TTL/CMOS
Response	Rising or falling edge
Pulse width	10 ns, minimum

Bus Interface

Type	Master, slave

Power Requirements

+3.3 VDC	
PXI	400 mA, maximum
+5 VDC	
PCI	2.6 A, maximum
PXI	2.2 A, maximum
+12 VDC	120 mA, maximum
-12 VDC	120 mA, maximum

Physical

Dimensions (not including connectors)	
PCI	17.5 by 10.7 cm (6.9 by 4.2 in.)
PXI	16.0 by 9.9 cm (6.3 by 3.9 in.) (1 slot)
Analog I/O connectors	SMB male
Digital trigger connector	SMB male

Maximum Working Voltage

Maximum working voltage refers to the signal voltage plus the common-mode voltage.

Channel-to-earth	10 V, installation category I
Channel-to-channel	10 V, installation category I

Environmental

Operating temperature	0 to 50 °C
Storage temperature	-20 to 70 °C
Relative humidity	10 to 90%, noncondensing
Maximum altitude	2,000 m
Pollution degree (indoor use only)	2

Calibration

Internal – On software command; computes gain and offset corrections	
Interval	Whenever temperature is different from temperature at last internal calibration by more than ±5 °C
External – Internal voltage reference read and stored in nonvolatile memory	
Interval	1 year
Warm-up time	15 minutes

Certifications and Compliances

CE Mark Compliance CE

[1]Measured with full-scale (±10 V) input.

Dayton®

Repair Parts Available 1-800-323-0620

E47479

68769

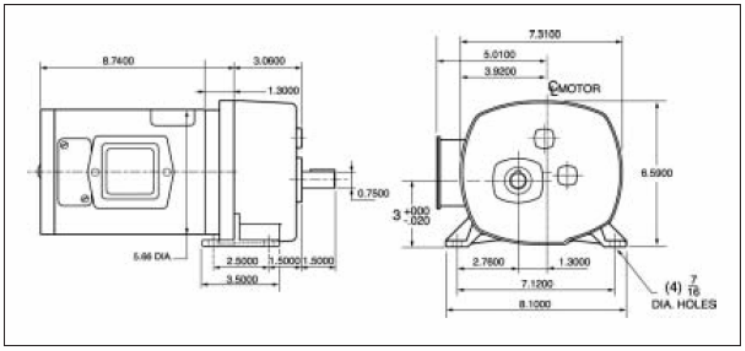
90 VDC Parallel Shaft Gearmotors

- Enclosure: TENV & TEFC
- Gearcase: Die-cast aluminum
- Lubrication: Permanent, heavy fluid gear oil
- Gears: Hardened steel, 1st stage helical, subsequent stages, spur
- Bearings: Heavy-duty ball and thrust balls on case; ball on motor
- Mounting: All position
- Rotation: Reversible
- Thermal Protection: None
- Brushes: Externally replaceable
- Ambient: 40°C
- Seals: Lip-type on input and output shafts

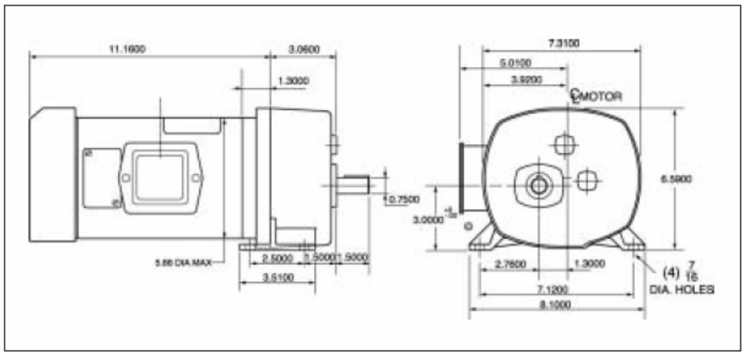
DAYTON 2-YEAR LIMITED WARRANTY

Text of warranty available upon request. See "Manufacturers' Warranties" on page opposite inside back cover.

No. 6Z406
TENV



No. 6Z413
TEFC



Nominal F/L RPM	Full-Load Torque (In.-Lbs.)	Over-hung Load (Lbs.)	Input HP	Full-Load Amps at 90 VDC	Gear Ratio	Stock No.	Each	Shpg. Wt.
90 VDC PERMANENT MAGNET TENV								
8.8	1087	325	1/4	2.0	196:1	6Z406	$520.00	29.0
13	960	325	1/4	2.8	131:1	6Z407	520.00	28.0
20	727	325	1/4	2.8	87:1	6Z408	520.00	28.0
40	353	325	1/4	2.8	41:1	6Z409	520.00	28.0
60	238	325	1/4	2.8	28:1	6Z410	520.00	32.0
90	152	325	1/4	2.8	19:1	6Z411	520.00	26.0
146	101	325	1/4	2.8	12:1	6Z412	520.00	26.0
90 VDC PERMANENT MAGNET TEFC								
20	1112	325	1/2	4.2	87:1	6Z413	563.50	35.0
34	822	325	1/2	5.8	50:1	6Z414	563.50	35.0
40	705	325	1/2	5.8	41:1	6Z415	563.50	34.0
60	476	325	1/2	5.8	28:1	6Z416	563.50	34.0
90	305	325	1/2	5.8	19:1	6Z417	563.50	33.0
146	202	325	1/2	5.8	12:1	6Z418	563.50	33.0

DART CONTROLS

Made in U.S.A.

DC Variable Speed Controls

Provide superior wide range adjustable speed control for permanent magnet or shunt wound DC motors and gearmotors. Solid-state design converts line voltage (120/240VAC) input to full wave DC power on constant or diminishing torque applications. Controls operate on 50/60 Hz at ±10% rated line voltage. Designed for reliable, repeatable operation and energy savings. Inhibit circuit allows Start/Stop without breaking AC lines. Speed controls are fully compatible with Dayton and GE PMDC motors.

Uses: Material handling, laboratory, food processing, pump, fan/blower, conveyor, machine tool, spooling.

- Adjustable IR comp., min./max. speed
- Operating temperature: -10° to 45°C
- Speed regulation ±1% with armature voltage feedback, ±0.5% with tach feedback
- Speed pot, knob, and dial plate provided with chassis models
- Built-in transient and surge suppression
- Fuse protected

No. 5JJ56

No. 5JJ57

No. 2M510

No. 5JJ58

No. 6Z385

No. 6Z386

No. 6Z387 & 5JJ59

No. 6Z388

No. 2M511

Stock No.	Current/ Torque Limit	Follower Circuit 0-10VDC	Linear Acceleration	Adjustable Linear Deceleration	Shunt Wound Field VDC	DC Tach Feedback 0-6VDC	Dynamic Braking	External Switches	Agency Approvals
5JJ56	None	No	Fixed 0.5 sec.	No	100/200	No	No	None	C-UL, UL, CE
5JJ57	None	No	Fixed 0.5 sec.	No	100/200	No	No	None	C-UL, UL, CE
5JJ58	None	No	Fixed 0.5 sec.	No	100/200	No	No	On/Off	C-UL, UL, CE
2M510	Adj.	Yes	Fixed 6 sec.	No	100/200	Yes	No	None	C-UL US
6Z385	Adj.	Yes	Adjustable	No	100/200	Yes	No	None	C-UL US
6Z386	Adj.	Yes	Adjustable	No	100/200	Yes	No	On/Off	C-UL US
6Z387	Adj.	Yes	Adjustable	Yes	100/200	Yes	No	None	C-UL US
6Z388	Adj.	Yes	Adjustable	Yes	100/200	Yes	No	On/Off	C-UL US
2M511	Adj.	Yes	Adjustable	Yes	100	Yes	Yes	On/Off, Fwd/Rev	—
5JJ59	Adj.	Yes	Adjustable	Yes	100/200	Yes	No	None	—

Enclosure Type	Output HP 90VDC	Output HP 180VDC	Input VAC	Max. Current	Output to Motor VDC	Speed Range	Dimensions (In.) W	L	D	Dart Model	Stock No.		Each	Shpg. Wt.
Chassis	1/50-1/6	1/25-1/3	120/240	2.0	90/180	25:1	2.80	1.30	3.30	15DV1A	5JJ56	✓	$49.30	0.4
Chassis	1/50-1/6	1/25-1/3	120/240	2.0	90/180	25:1	2.80	1.50	3.30	15DV2A	5JJ57	✓	49.30	0.4
NEMA 4/12	1/50-1/2	1/50-1/2	120/240	3.0	90/180	25:1	3.81	5.50	3.50	15DVE	5JJ58	✓	144.90	1.0
Chassis	1/8-1/2	1/4-1	120/240	5.0	90/180	50:1	3.63	4.25	1.30	125DV-C-K	2M510	✓	77.40	0.7
Chassis	1/8-1	1/4-2	120/240	10.0	90/180	50:1	5.53	7.00	1.63	253G-200C	6Z385	✓	173.50	1.4
NEMA 4/12	1/8-1	1/4-2	120/240	10.0	90/180	50:1	5.53	7.25	2.75	253G-200E	6Z386	✓	223.75	2.0
Chassis	1/8-1	1/4-2	120/240	10.0	90/180	50:1	6.70	9.00	2.00	530BC	6Z387	✓	380.25	3.0
NEMA 4/12	1/8-1	1/4-2	120/240	10.0	90/180	50:1	6.70	10.00	4.75	530-BRE	6Z388	✓	428.25	3.8
NEMA 4/12	1/8-1	—	120	10.0	90	50:1	6.70	10.00	4.75	530-BRE-36MA	2M511	✓	514.00	4.7
Chassis	1/8-1½	1/4-3	120/240	15.0	90/180	50:1	6.70	9.00	2.00	533BC	5JJ59	✓	625.00	2.9

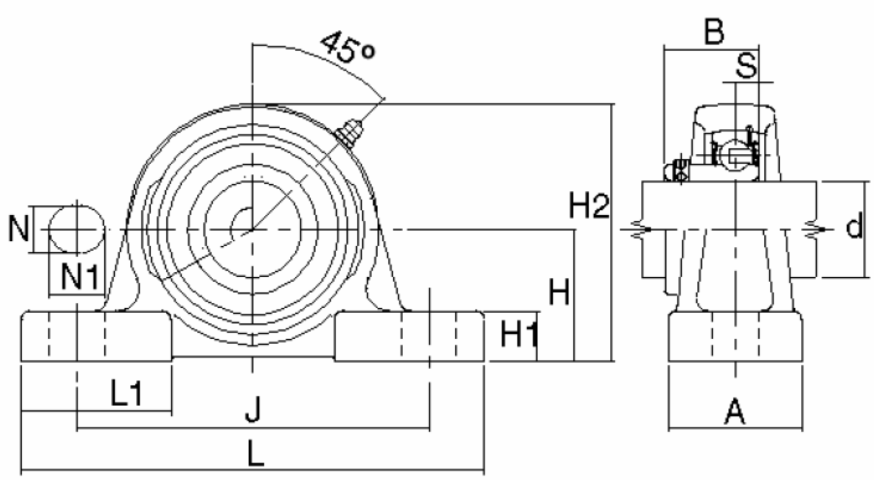
Mounted Unit Data Sheet

Ultra-Class Pillow Block High Base - Narrow Inner Ring - Set Screw Type

NTN Part Number ARP-1.1/8
Bearing Insert # A-AR206-102D1
Housing # P206D1V50
Weight N/A (lbs) / N/A (kg)

Dimensions			Features / Options		
Dimension	Imperial	Metric	Option	Code	Description
Bore d	1.1250 (in)	28.575 (mm)	Bearing Insert	AR	Narrow inner ring, set screw locking, nylon cage
H	1.6875 (in)	42.863 (mm)	Housing	P	High base, pillow block
L	6.5000 (in)	165.100 (mm)			
J	4.6250 (in)	117.475 (mm)			
A	1.8750 (in)	47.625 (mm)			
N	0.5625 (in)	14.288 (mm)			
N1	0.9375 (in)	23.813 (mm)			
H1	0.6563 (in)	16.669 (mm)			
H2	3.2813 (in)	83.344 (mm)			
L1	2.1250 (in)	53.975 (mm)			
B	1.1811 (in)	30.000 (mm)			
S	0.3540 (in)	8.992 (mm)			
Lube Hole of Housing	1/8-27NPT	1/8-27NPT			
Basic Load Ratings				**Limiting Speeds**	
Static	0 (lbf)	0 (N)		Oil	N/A
Dynamic	0 (lbf)	0 (N)		Seals	N/A
				Grease	N/A

Reference Drawing



APPENDIX D

TUMBLE MILL ASSEMBLY MODEL AS SIMPLY SUPPORTED BEAM WITH OVERHUNG LOAD

The tumble mill assembly can be modeled as a simply supported beam with an overhung load. Using this model the deflection of the rotor due to a load on the drum can be determined. The two bearing locations are the supports on which the rotor is supported. The load applied is a point load at the end of the outboard bearing. The model for such a standard problem is shown in figure D1. The deformation of the beam due to the overhung load can be determined by equation (D.1). For simplicity the beam is assumed to have a uniform cross section of 0.875" (22 *mm*). The maximum load applied on the beam is 80 *lb* (36 *kg*).

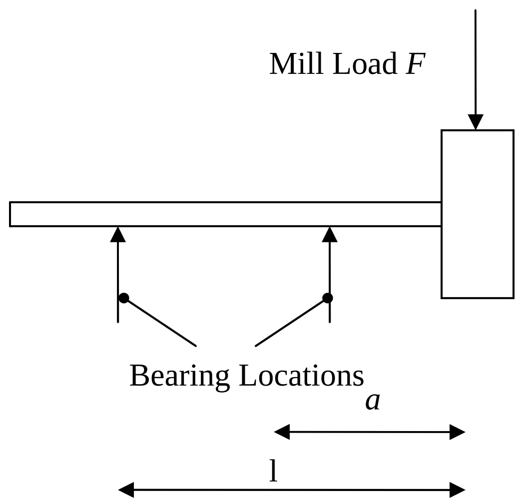


Figure D1 Tumble Mill Model as Simply

Supported Beam with Overhung Load

$$Y_C = F.a^2.\left[\frac{l+a}{3.E.I_{area}}\right] \quad \text{(D.1)}$$

$$= (80).(3)^2.\left[\frac{11+3}{(3).(30.10^6).(0.0575)}\right]$$

$$= 0.0019 inch (0.048 mm)$$

APPENDIX E

NATURAL FREQUENCIES FROM THE NONLINEAR EQUATION

VS THE LINEARIZED EQUATION EIGEN VALUE

The equation of motion for the grinding media in tumble mill, equation (E.1) is a nonlinear differential equation. This equation was linearized assuming small changes in θ about an operating point of 0°. Therefore the functions of $sin\theta$ and $cos\theta$ are θ and 1 respectively, and the homogeneous equation is

$$I\ddot{\theta} + mgeSin\theta = 0 \qquad \text{(E.1)}$$

$$sin\,\theta \rightarrow \theta$$

$$\Rightarrow I\ddot{\theta} + mge\theta = 0 \qquad \text{(E.3)}$$

$$\omega_n = \sqrt{\frac{mge}{I}} \qquad \text{(E.4)}$$

For 15% fill volume the weight (*mg*), the distance between center of gravity and geometric center of the mill (*e*) and the mass moment of inertia of grinding media (*I*) are 3.66 *lb*, 0.377 *ft* and 0.2048 $lb.ft.sec^2$ respectively. Substituting these in equation (E.4) the eigen value is given by 8.208 *rad/sec* or 78.4 *cpm*. Table E.1 gives the measured and predicted grinding media oscillating frequencies. The predicted values come from the nonlinear equation of motion and from the linearized equation of motion for a fill volume level of 15% at different speeds. Figure E1 shows the plot of these frequencies.

Table E.1

Mill Running Speed (RPM)	Grinding Media Oscillating Frequency from Measurements (CPM)	Grinding Media Oscillating Frequency from Simulation (CPM)	Grinding Media Oscillating Frequency from Linearization (CPM)
15	80	75	78.4
20	78	76	78.4
30	79.8	78	78.4
40	83.4	79	78.4

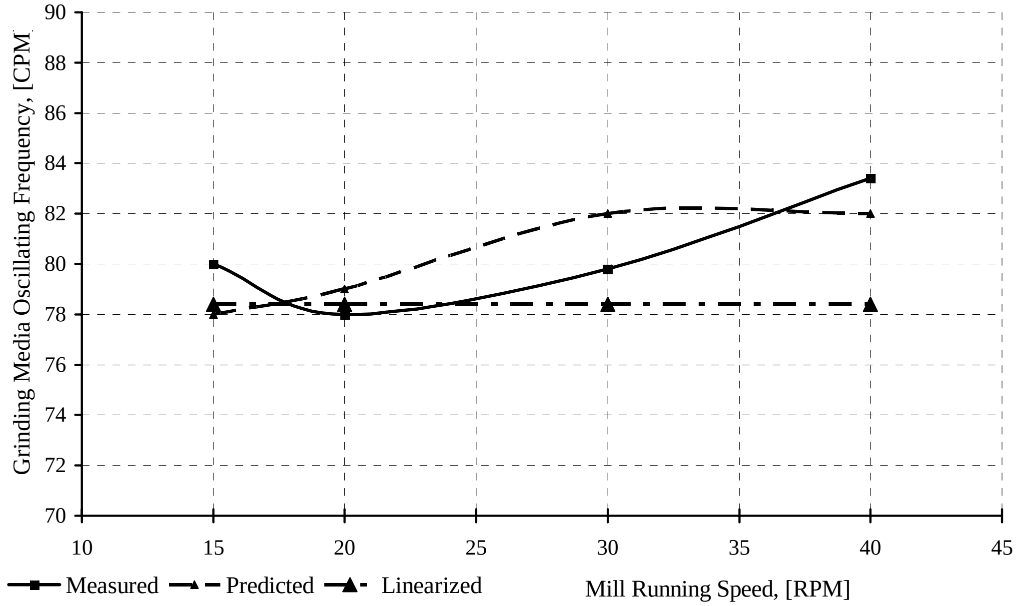


Figure E1 Comparison of Frequencies of Grinding Media

APPENDIX F

MEASUREMENTS FROM LVTORSION

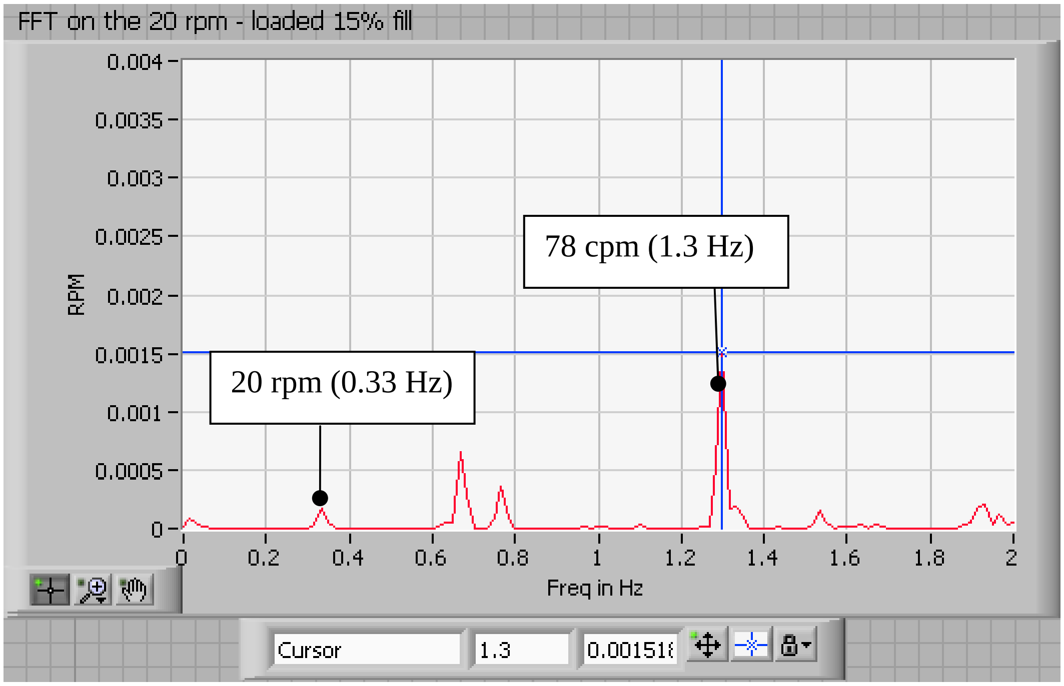


Figure F.1 Spectrum of Tumble Mill with 15% Load at 20 RPM

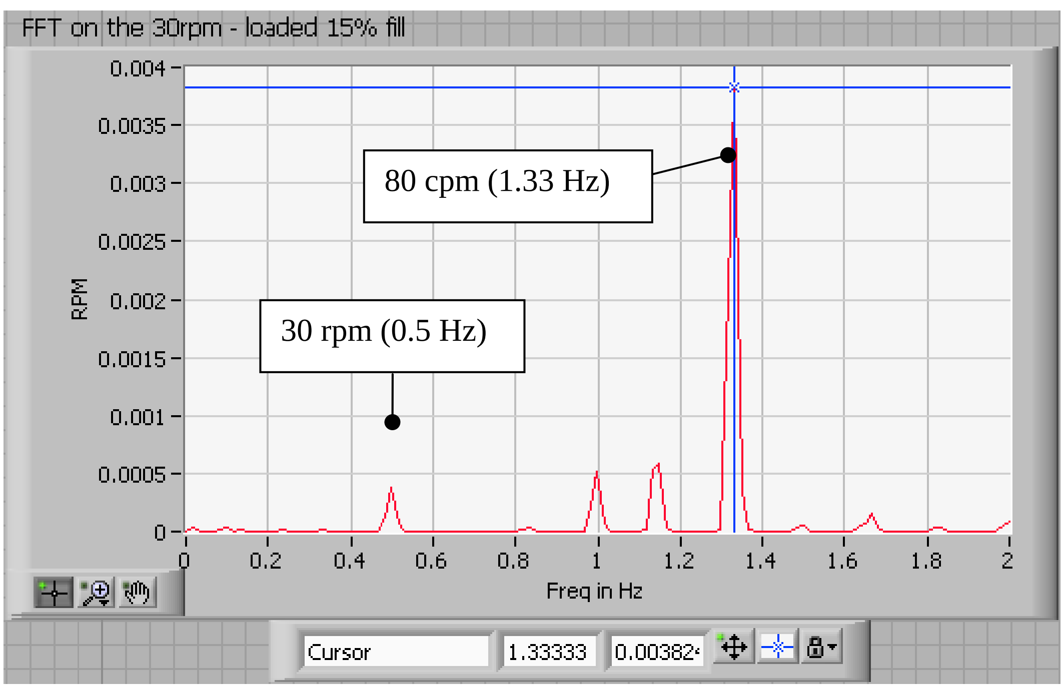


Figure F.2 Spectrum of Tumble Mill with 15% Load at 30 RPM

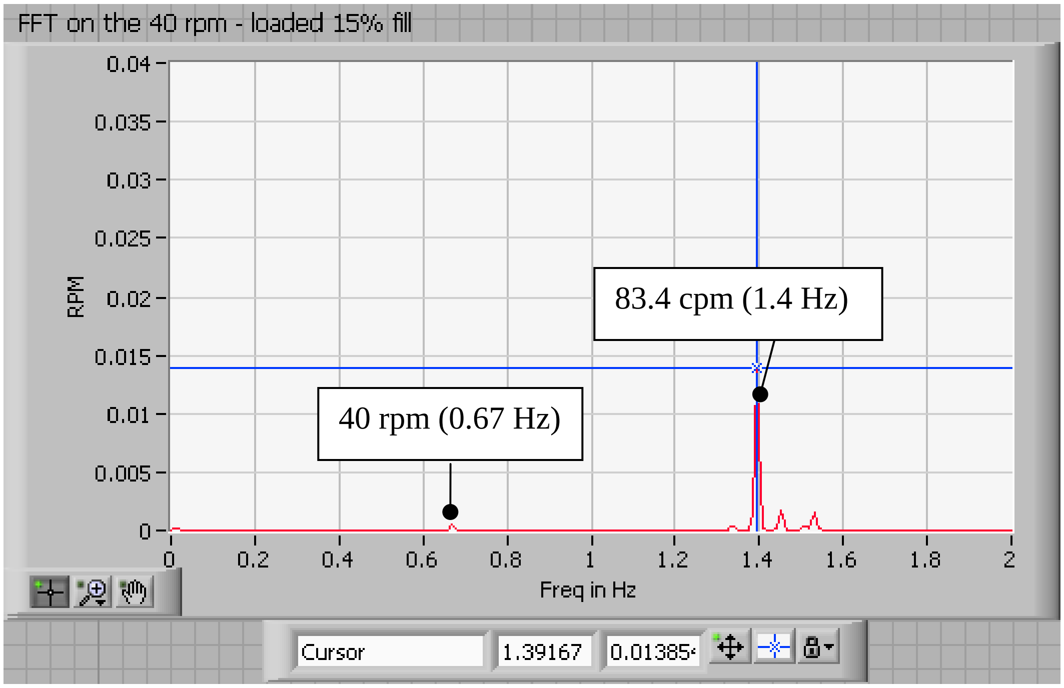


Figure F.3 Spectrum of Tumble Mill with 15% Load at 40 RPM

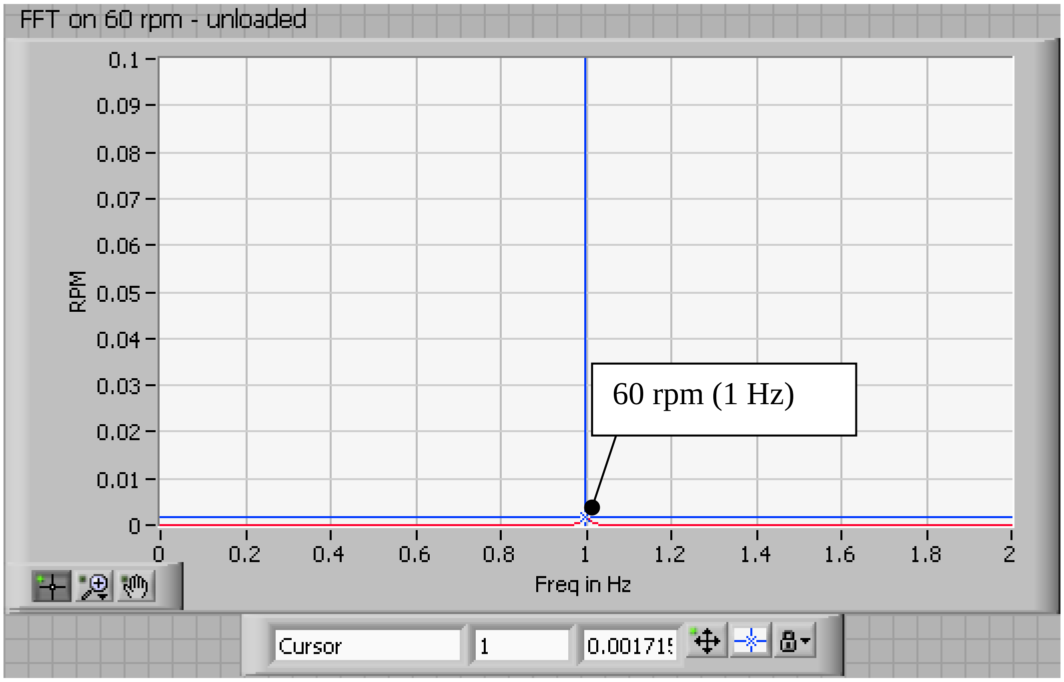


Figure F.4 Spectrum of Tumble Mill with out Load at 60 RPM

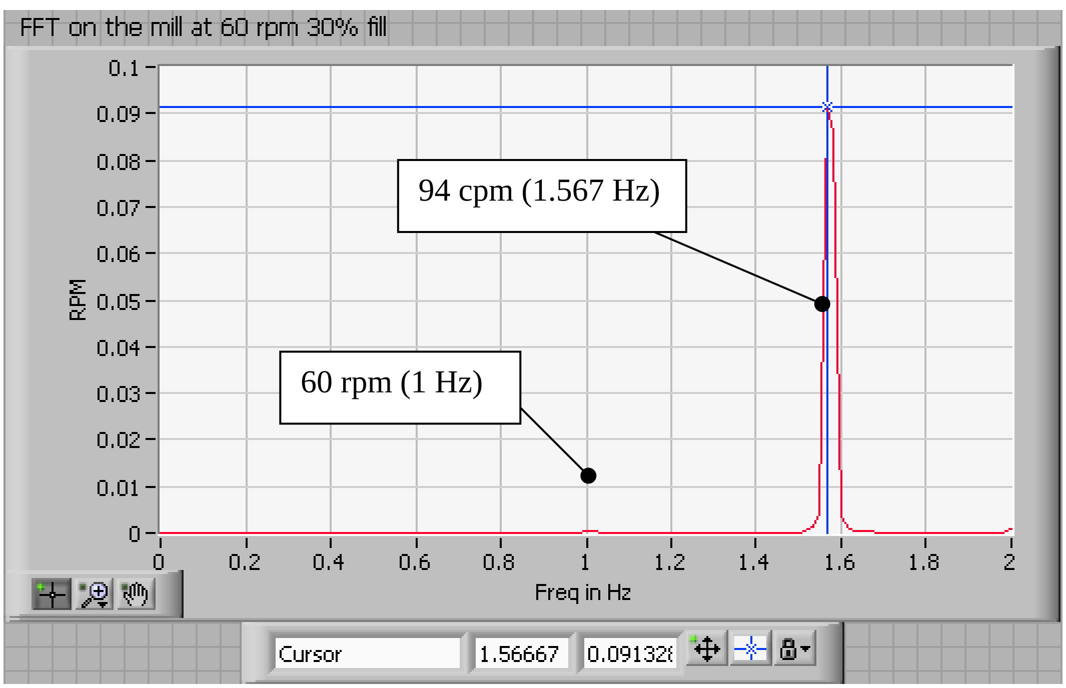


Figure F.5 Spectrum of Tumble Mill with 30% Load at 60 RPM

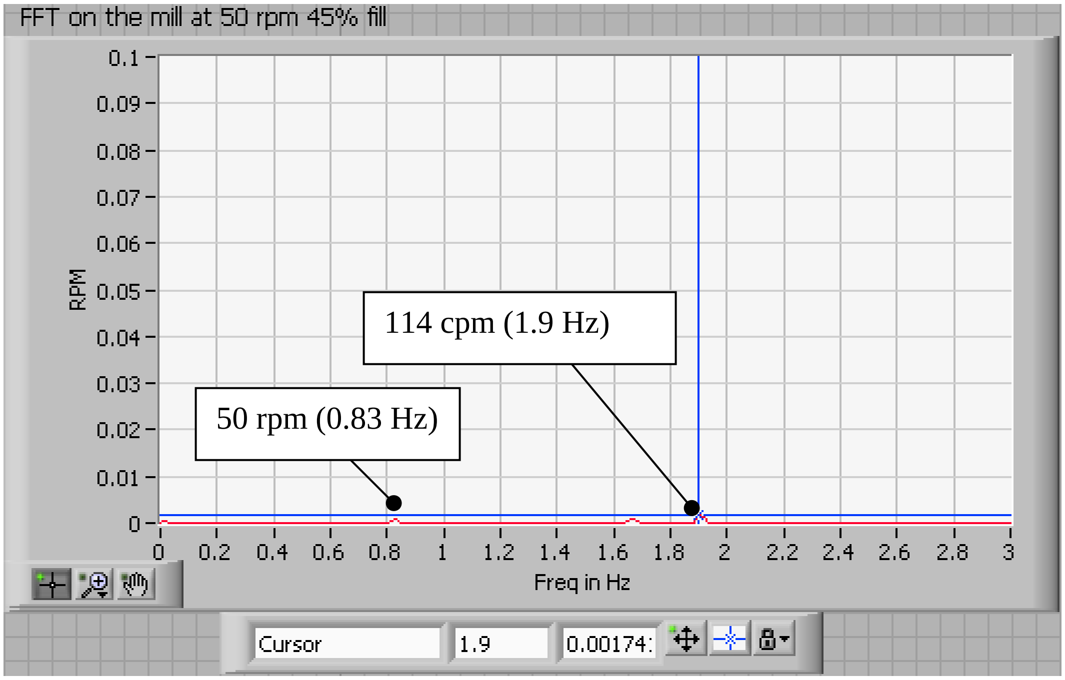


Figure F.6 Spectrum of Tumble Mill with 45% Load at 50 RPM

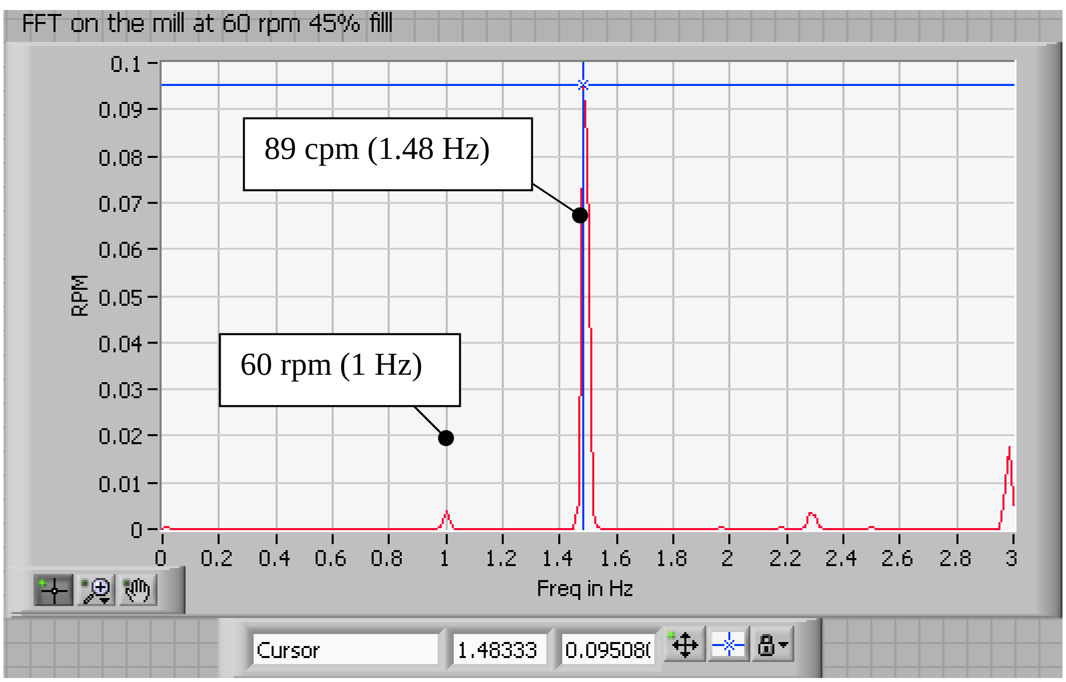


Figure F.7 Spectrum of Tumble Mill with 45% Load at 60 RPM

VITA

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