ADVERTISING AND CONSUMER SEARCH

IN DIFFERENTIATED MARKETS

A Dissertation

by

KEVIN KENTON HARRIOTT

Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

August 2005

Major Subject: Economics
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Approved by:
Chair of Committee,  Hae-Shin Hwang
Committee Members, William S. Neilson
Steven Puller
H. Alan Love
Head of Department,  Leonardo Auernheimer

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ABSTRACT

Advertising and Consumer Search in Differentiated Markets. (August 2005)
Kevin Kenton Harriott, B.Sc.; M.Sc., University of the West Indies, Jamaica
Chair of Advisory Committee: Dr. Hae-Shin Hwang

This dissertation, in its most general context, is an investigation into the modeling of markets with imperfectly informed agents. In such markets, there will invariably be incentives for informed agents to take advantage of information asymmetries by disseminating the relevant information to uninformed agents. Similarly, there will be incentives for uniformed agents to reduce the adverse effects of information asymmetries by acquiring the relevant information. The primary purpose of this dissertation is to demonstrate that the understanding of such markets can be greatly enhanced by explicit modeling both channels of information flow as omitting either channel could eliminate important interaction effects.

The arguments in this dissertation are narrowly framed within a familiar differentiated market in which firms advertise and each consumer is imperfectly informed about which product is most suited to his taste. However, the conclusions drawn in the dissertation are applicable to more general economic systems in which it is costly for agents to acquire information relevant to the decision-making process.

There is a long-standing debate in the literature about whether or not advertising is purely informative. Although there is extensive research on advertising models and consumer search models, little is known about differentiated markets in which firms advertise and consumers search. In modeling advertising and consumer search, this dissertation questions the relevance of two pieces of evidence that have been offered against the view that advertising is informative.

In the first instance, I demonstrate that firms may use purely informative advertising and still maintain market power in the long-run in monopolistically
competitive markets; this finding thus rejects the argument that firms rely on manipulating consumer preferences in order to maintain market power in these markets. In the second instance, I demonstrate that advertisements without any information about the product being advertised may still be informative to some consumers; this finding thus rejects the argument that the widespread use of uninformative television advertisements is inconsistent with the view that advertising is informative in nature.

This dissertation shows that our understanding of the nature of advertising (information dissemination mechanism) is greatly enhanced by modeling consumer search (information acquisition mechanism).
To Dimples, who gave everything, so that I would want for nothing.
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I would like to thank the members of my Committee: Dr. Hae-Shin Hwang (Chair), Dr. William Neilson, Dr. Steve Puller and Dr. Alan Love. I benefited tremendously from their respective expertise during conversations about my research.

I would especially like to thank my Chair for demonstrating immense patience and unwavering poise as he guided my maiden voyage in the sea of original, scholarly research. Thanks a lot, Dr. Hwang, for helping me to organize my thoughts.
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CHAPTER I

INTRODUCTION

This dissertation demonstrates the benefits derived from explicitly modeling both channels of information flow in markets characterized by imperfect information. Specifically, I model an oligopolistic market characterized by imperfectly informed consumers with heterogeneous preferences for the products. There are two channels through which information may flow in a market: informative advertising and consumer search. Within the context of differentiated markets analyzed in this research, I show the information dissemination mechanism (advertising technology) plays a greater role than the information acquisition mechanism (consumer search technology) in shaping the decentralized market equilibrium. The simultaneous modeling of advertising and consumer search enriches our understanding of the nature of advertising by capturing its interactive effects with consumer search.

There is a long-standing debate on the effect of advertising on the performance of the market. The traditional view of advertising maintains that advertising is used by firms to retard the otherwise competitive forces in a market; supporters of this view label advertising of this nature as combative, persuasive or goodwill advertising. The modern view maintains that advertising is pro-competitive in nature as it assuages the friction generated by information asymmetries among market participants; supporters of the modern view have labeled advertising of this nature as informative advertising. In Chapters III and IV, I re-examine two of the arguments that have been offered in support of the traditional view of advertising.

This dissertation follows the style and format of the RAND Journal of Economics.
In Chapter III, I explore whether the use of purely informative advertising is consistent with long-run equilibrium in a truly monopolistically competitive market; the economists who developed this market structure intimated that only persuasive advertising could prevail because firms maintained market power despite competing in a large market.

In chapter IV, I examine the issue of whether the amount of information in an advertisement can adequately determine the informativeness of the advertisement. This is an important analysis given the sizeable proportion of television commercials that (seemingly) provides no relevant product information. I show the use of commercials with no brand specific information is not necessarily inconsistent with the modern view of advertising as a pro-competitive marketing device.

Economists have been curious about the underlying factors that would generate a monopolistically competitive market since the theoretical market structure was independently advanced by Chamberlin and Joan Robinson. The main feature of the market structure is that an arbitrarily large number of firms competing with horizontally differentiated products maintain market power while earning normal profits in the long run. The main object of curiosity in this market structure, as postulated, was the source of the market power firms are able maintain even as the number of firms operating in the market arbitrarily expanded. The theory of monopolistic competition sparked a debate on the role of advertisement in the performance of the market. It is the contention of Joan Robinson (and Chamberlin, to a lesser extent) that each firm uses advertising to attach consumers to its product by unduly influencing consumer preferences.

Joan Robinson is one of many economists who subscribes to the traditional view that advertising is persuasive by design and socially undesirable since it is an anti-competitive tool used to reduce the perceived substitutability among products. This

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1 Market power is defined as the ability of firms to charge prices in excess of the marginal cost of production.
view seems unacceptable to economic theorists since it relies on the assumption that consumers are irrational and capable of being unduly influenced by firms.

With the formal introduction of imperfect information into mainstream economics in the 1960s, a modern view of the role of advertising emerged. The modern view argues that advertisements promote greater substitutability among products by subsidizing the cost of information acquisition and is consequently a competitive tool to be socially desired. The modern view is more acceptable since it relies on consumer ignorance as opposed to consumer irrationality [See Bagwell (2004) for a more complete treatment of the modern and traditional views of advertising].

Despite the relative success of the modern view in countering the arguments of the traditional view of advertising, the use of informative advertising seems to be inconsistent with the features of a truly monopolistically competitive market. If we accept the modern view of the informative role of advertising, an immediate question arises in relation to the theory of monopolistically competitive markets. Under what conditions, if any, could firms use informative advertising and maintain market power in differentiated product markets with unlimited entry? A related question is, if advertising isn’t the source of market power, from where else could it be derived? The questions arise because if in fact the use of advertising promotes greater substitutability among products, then firms should lose all market power in the limit as arbitrarily large number substitutable products enter the industry, in contradiction to the long run equilibrium conditions in monopolistically competitive markets.

Many of the advertisements observed in the various media do not convey complete product information [See references cited in Anderson and Renault (2004) for empirical studies on the issue]. Some have held up these “uninformative” advertisements as evidence that firms attempt to manipulate consumer preferences as opposed to reducing information asymmetries.
Resnik and Stern (1977) quote a former head of Consumer Protection at the Federal Trade Commission saying:

Those forms of advertising which are essentially non-informative in character may raise questions as to their fundamental conformity with [the modern view of advertising]…and the extent to which such advertising is designed to exploit such fears or anxieties as social acceptance or personal wellbeing without fulfilling the desires raised.

In Chapter IV, I address the sentiments addressed in the quote: is the use of “non-informative” advertisements consistent with the modern views of advertising? Alternatively, is there any merit to the claim that the use of uninformative advertisements provides substantial evidence in support of the view that advertising has an anti-competitive effect on the performance of the market? Another issue addressed in this dissertation is whether there is any theoretical underpinning for the observed predominant use of “non-informative” advertisements given that consumer preferences are not manipulated.

I argue the information content of an advertisement *in and of itself* can not be used to assess the effect of advertising on the performance of the market. In Chapter IV, I present a model that is most favorable to the anti-competitive view: that is advertising with *no brand-specific information*. I do not question whether there are instances when advertising may have anti-competitive effects. The point I argue is that advertising content is an inadequate means of evaluating the effect of an advertising regime on the performance of the market.

To reconcile the modern view of advertising with advertisements that lack brand-specific information, I make the distinction between the costs of becoming aware of product characteristics as opposed to becoming aware of the unique characteristics of the various brands. I assume the cost of learning about product characteristics (discovery) is
considerably greater than the cost of learning about brand characteristics (search). I argue television commercials without any brand-specific information may be informative in the sense that they alert the consumer to general characteristics (e.g., existence and intended use) of the product.

I make the distinction between two types of information acquisition costs faced by consumers. Product search cost (henceforth, *discovery cost*) refers to the cost of learning the general characteristics of a class of products (e.g. Personal Computers); I argue that every advertisement convey this information to consumers. Brand search cost (henceforth, *search cost*) refers to the cost of learning about the idiosyncratic features of each brand within a product class (e.g. Dell, Hewlett-Packard, Apple); this information is only conveyed in informative advertisements from a specific firm. I find that discovery cost is of greater import than search cost in the firm’s decision on how much information to disclose in advertisements. Most papers modeling consumer search consider only brand search cost.

In Chapter II, I present a survey of papers that have examined markets in which agents are imperfectly informed in differentiated markets and focus on the channels of information explicitly examined. I develop a general n firm model of imperfect markets in Chapter III. In this model, firms disseminate advertisements to consumers with heterogeneous preferences. Consumers in my model also search for the product most suited to their tastes. The explicit modeling of consumer search with heterogeneous information sets represents an innovation over extant consumer search models of differentiated products. This innovation allows me to demonstrate that while the maintenance of market power under unlimited entry is consistent with the use of informative advertising, it crucially depends on the nature of advertising technology. I find when initial marginal efforts to advertise is costless, firms lose all market power with unlimited entry but when initial advertising effort is costly, firms maintain market power even with unlimited entry.

In Chapter IV, I use a duopoly version of my general model to assess the adequacy of using advertising content to measure the informativeness of advertising.
The crucial feature in this chapter is the distinction between discovery and search costs. In Chapter V, I summarize the main findings of the dissertation.
CHAPTER II
LITERATURE SURVEY

In this dissertation, I take the more favorable view of advertising and explore its implications for the empirical relevance of monopolistically competitive markets; that is, advertisements in our model do not alter consumer preference for the product advertised.2 Proponents of both the traditional and modern views of advertising recognize that markets operate under imperfect information. They advance their opinion on the role of advertising using models that fail to account for both active channels of information flows. The main purpose of this dissertation is to highlight the benefits to be gained from modeling both active channels of information flow. Along the way, I develop a model of consumer search that incorporates consumers with heterogeneous prior information sets and make a significant contribution to the ongoing debate about the role of advertising in economics.

Attempts to model imperfect information within a theoretical framework have been a part of the economics literature since the seminal work of Stigler (1961). Within these studies, a consumer makes decisions based on incomplete information since gathering all the required information is too expensive (search models). Consumers may also passively obtain information from firms (advertising models). Most models of imperfect information focus on only one of these channels of information flow.

2 A few studies [Tremblay (2001), Tremblay and Martins-Filho (2001), Bloch and Manceau (1999), von der Fehr and Stevik (1998) and Dixit and Norman (1978, 1979, 1980)] posit that firms use advertising to influence the perceived desirability of the advertised good to consumers. We make no such assertion in this paper.
In what follows, I organize the literature on imperfect information by the channels on information explicitly modeled. I show the majority of these studies are partial analyses of imperfect information markets in the sense they fail to explicitly incorporate the incentives of the informed agents to disseminate information (supply side analysis) and the incentives of the uninformed agents to acquire information (demand side analysis).

2.1. Advertising Models

The supply-side of the market for product information is examined in advertising models. The focus of advertising models has been predominantly confined to exploring the role of advertising and examining whether or not a decentralized market would devote the efficient amount resources to inform consumers. Our research is more closely related to papers that view advertisements as being purely informative [Butters (1977), Grossman and Shapiro (1984) and Stegeman (1991)]. When goods are homogenous, Butters (1977) shows advertising is optimal but Stegeman shows that advertising is undersupplied. For differentiated products, Grossman and Shapiro (1984) show advertising levels are socially excessive with greater product diversity. The model in Chapter III is developed by appending consumer search technology to the model envisioned by Grossman and Shapiro where consumer search is precluded. Surprisingly, long-run equilibrium strategies are identical in both models suggesting firms are not influenced by the ability of consumers to search. In Chapter III, I offer an explanation for this invariance of equilibrium strategies to consumer search.

2.2. Search Models

The demand-side issues of the market for product information, studied in consumer search models, have focused largely on characterizing the effects of search costs on price dispersion and the performance of the market [Anderson and Renault (1999, 2000), Fischer and Harrington (1996), Stahl (1994, 1996) and Diamond (1971)]. My research examines symmetric, pure strategies in Bertrand competition and so price
dispersion (mixed strategies behavior) is ruled out as an equilibrium outcome. Wolinsky (1986) unwittingly bolstered the evidence in favor of the modern view of advertising by making an important contribution to the theory of monopolistic competition. He provides an alternative source of market power by developing a consumer search model for a differentiated product market in which firms do not advertise. He shows that search activities of imperfectly informed consumers could generate a truly monopolistically competitive market. Although Wolinsky’s study shows that advertising is not central to the theoretical underpinnings of monopolistically competitive markets, its major deficiency with regard to this dissertation is that it could not address the issue of whether the use of informative advertising is consistent with monopolistic competition.

Given that the use of advertising is a prominent feature of many industries, it suggests Wolinsky’s model of imperfect information lacks an important component that would allow consumers to have information prior to search and allow firms to maintain market power. Indeed, once the advertising mechanism is explicitly modeled alongside the search mechanism whereby the number of consumers with prior information is endogenously determined, Chapter III provides conditions under which the monopolistically competitive environment is restored. In deriving these conditions, I generalize the search model developed by Wolinsky by incorporating consumers with heterogeneous prior information sets.

2.3. Advertising and Search Models

I use a simple model for simultaneously examining demand and supply in the market for information. In this regard, models of advertising and consumer search such as Konish and Sandhort (2002) and Anderson and Renault (2004), Robert and Stahl (1993) and Butters (1977) are closer in spirit to my set up. In Konish and Sandhort (2002) with horizontally differentiated products, advertising only conveys information about the price of the product. This would be uninformative in our paper since consumers do not search for prices in our model; they rationally anticipate the
equilibrium price of the products and search instead for the product more suited to their
taste. In order for an advertisement to be informative in our model, it must convey some
information about match value. Butters (1977) finds that advertising level is inadequate
when consumers search but the conclusion was reached when consumers engage in a
sub-optimal search strategy where as I model consumers with an optimal search strategy.

The model in Robert and Stahl is very close to the one developed in Chapter III. It is the first to model optimal consumer search behavior in an advertising model. Their research shows advertising technology plays a greater role than consumer search technology in determining the equilibrium. I also discover asymmetric effects between advertising and consumer search technology. The main point of departure between their model and the one outline in Chapter III is that I consider differentiated products while they considered homogenous products.

My model is a hybrid of advertising models developed in Grossman and Shapiro (1984) and Robert and Stahl (1993). In my model, products are differentiated and firm advertising is modeled along with consumer search. In Grossman and Shapiro, consumer search is explicitly prohibited and in Robert and Stahl, goods are homogenous. A comparison of results from these studies with my results suggests neither consumer search behavior nor product heterogeneity are as important as advertising technology in the performance of markets with many firms. In large markets, the pricing strategy uncovered in Chapter III is identical to that outlined in Grossman and Shapiro suggesting that the inclusion of search in our model does not affect the pricing strategy of the firms. I also find that in large markets, the advertising technology determines whether equilibrium price is competitive or monopolistic. This is also a finding of Robert and Stahl and suggests that the homogenous product assumption is not crucial to this result.

Marshall, Chamberlin and Robinson were the early proponents of the traditional view of advertising [See references cited in Bagwell (2004) for the works of these and other proponents of the traditional view of advertising]. They argue advertisements that convey no direct economic information represent attempts by firms to manipulate the preferences of unsuspecting consumers.
This view is not widely accepted in mainstream economics since it relies on the irrationality of consumer behavior. Another explanation, grounded in rational consumer behavior, is that the use of uninformative advertisements conveys indirect information by signaling to the consumers that the product is of high quality [Nelson (1974), and Milgrom and Roberts (1986)]. Of course, this explanation is only appropriate in vertically differentiated markets where products differ in quality. An explanation for this lack of informative advertising in horizontally differentiated markets is noticeable absent from the previous literature and is the focus of Chapter IV.

The explanation offered in this dissertation is grounded in the cost of information acquisition. As alluded to in Comanor and Wilson (1974), any advertisement will at least inform the consumer of intended use of the product class. Even if no brand-specific information is included, information regarding the existence of the product class is useful to consumers who find it prohibitively expensive to acquire the information on their own. I formally model this feature in my research; as I show in Chapter IV, this is a crucial feature in generating an equilibrium in which firms advertise.

My research presented in Chapter IV, is related to the research of Anderson and Renault (2004). They consider a monopoly market and examine the incentives to the firm to supply information about the price and product characteristics to consumers. In their study, the firm advertises to all consumers, but decides on the price and content of the advertisements. I consider a Duopoly market where firms decide the amount of information, the price and the number of consumers to inform. Both studies conclude that more product specific information will be disclosed in advertisements when information acquisition cost is sufficiently high. However, Anderson and Renault (2004) focus on search cost while I focus on discovery cost.

2.4. Advertising Intensities in Real Markets

Empirically, promotional activities are important aspects of firms in a wide cross-section of industries. It has been argued that the promotion of a product is as important to the success of a firm as the attributes of the product itself; advertising is big
business. During 2003, firms in industries with the top 100 advertising budgets spent resources equivalent to, on average, to 9.6 percent of their profit margins. The advertising to margin ratio varies starkly by industry. Industries with the highest margins include health services (83.6 percent), Household Audio and Video equipment (44.9 percent), Distilled and Blended Liquor (43.7 percent) and Loan Brokers (43.2 percent). This is in contrast to the relatively low ratios reported by agricultural products (crops), Commercial Printing, Petroleum Refining and Refuse Systems, all of which reported a ratio below 1 percent during 2003.

The top six advertising agencies generated $1.82 billion in revenues in the ME during 2003. The next 100 highest ranked advertising entities generated $2,946 billion during 2003, a 6.32 percent increase over 2002. The means of advertising is just as telling. The top 100 media houses (a sub-group of advertising agencies) generated $179.65 billion in 2003; $34.26 billion (19.07 percent) through newspaper, $14.90 billion (8.03 percent) through magazine, $32.86 billion (18.29 percent) through TV, $10.08 billion (5.61 percent) through radio, $59.77 billion (33.27 percent) through cable and $27.79 billion (15.47 percent) was generated via the Internet.

The documented importance of advertising in actual markets therefore justifies our curiosity in developing richer theoretical models of advertising to aid in our understanding of how these considerable expenditures on advertising might be affecting the performance of these markets.

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*Profit margin is calculated as net sales minus cost of goods sold. All data on advertising expenditures are taken from Advertising Age (2004).*
CHAPTER III
ADVERTISING IN MONOPOLISTICALLY COMPETITIVE MARKETS WHEN CONSUMERS SEARCH

This model is a hybrid of advertising models developed in Grossman and Shapiro (1984) and Robert and Stahl (1993). In this model, brands are differentiated and firm advertising is modeled along with consumer search. In Grossman and Shapiro, consumer search is explicitly prohibited and in Robert and Stahl, goods are homogenous.

A description of the general model is provided in Section 3.1. In Section 3.2, firm’s expected demand is derived based on heterogeneity in consumers’ information set; strict attention is paid to how the consumer’s buying decision is influenced by their information available prior to search. In Section 3.4, I endogenize heterogeneity in consumer information sets by incorporating informative advertising in the consumer search model. A discussion of results is included in Section 3.5. The chapter concludes in Section 3.6.

3.1. General Model

To fix ideas, I discuss the market for personal computers (PCs). The computers may be purchased at one of \( n \) firms. The PCs are identically designed with respect to processing speed, storage capacity, graphics capabilities etc., but each firm specializes in different colors. Firms are identical in every other respect. Consumers have no idea of the cost of the computers. More importantly to the discussion, not all consumers know which color computer best would appeal to his taste.

Each consumer purchases at most one PC (unit demand). He may learn how well a particular color computer is suited to his taste in one of two ways. The first involves traveling to the firm to view the computer on the display shelf. The effort involved in traveling to the store exacts a negative utility, \( c \), which I henceforth refer to as his search cost. The other way involves receiving an advertisement brochure from the firm with a
colored picture of the computer. It is assumed the brochure is sufficiently informative to allow the individual to learn his taste for that particular color computer. This assessed value is referred to as his match value for that color (brand) PC. Each consumer strictly prefers buying a computer to not buying any at all (perfectly inelastic demand).

The results of this paper are applicable to a much broader market environment. Products sold by rival firms could be differentiated along an attribute other than color. The essential features of the PC market are that firms compete on prices, the products are purely horizontally differentiated, each consumer demands exactly one unit of the good and at least some of the consumers must have no prior idea of his match value or the price of the product; this information must be costly to send as well as to acquire.

3.1.1. Extensive Form Description of the Model

There are three stages \{0,1,2\} and three sets of players in this game: Nature, \(n\) firms \((k = 1,2,\ldots,n)\) and a continuum of consumers, \(i \in [0,1]\).\(^4\) In Stage 0, Nature assigns each consumer with a match value for each brand. The specific assignment of match values to consumer is neither observed by consumer nor firm. However, it is common knowledge that the distribution of match values for each product is given by the continuous and twice differentiable cumulative distribution \(F\) on \([a,b]\) which is identically and independently distributed (iid) across products.

In Stage 1, firms simultaneously choose price \(p_k \in [0,\infty)\) and advertising intensity \(\phi_k \in [0,1]\). The payoffs of actions taken by the firms is realized is Stage 2 and represented by the expected profit function \(\pi_k\).

In Stage 2, a consumer decides which store to visit and which brand to purchase. Consumers must buy one unit of the good from one of the firms and payoffs are given by his (indirect) utility function \(v_i\), minus search costs. It is common knowledge that there are only \(n\) brands available in the market and that consumers search sequentially with

\(^4\) The terms firm, brand and store are identical concepts in this dissertation and are used interchangeably throughout.
perfect recall. At the beginning of Stage 2, consumers know their match value and price of a brand if and only if an advertisement is received from the firm in Stage 1. A visit to a store is sufficient for the consumer to learn his specific match value for that brand in addition to the price.

3.1.2. Equilibrium Concept

The equilibrium concept used in this model is perfect Bayesian equilibrium. Within the context of this model, I look for symmetric pure strategies in $p$ and $\phi$. It requires that each firm choose price and advertising intensity to maximize his own payoffs (expected profits) based on firm’s beliefs about its rivals’ strategies as well as its knowledge of consumers’ search strategy in Stage 2. It further requires consumers to buy a brand to maximize net indirect utility based on the information (if any) obtained from firms in Stage 1, as well as information obtained during the search process while anticipating the price charged by firms from which they did not receive an advertising message. In equilibrium expectations are met. The Bayesian inference in this solution concept requires us to state how a consumer’s beliefs are updated when he comes across a price other than $p$ at a particular store. For this I assume that he will continue to anticipate a price $p$ for products yet to be searched.

Note that the firm’s pay-off (via expected demand) is affected by the extent of consumer search; also, the extent of consumer search is affected by the amount of advertising. It is the interaction between information dissemination and information acquisition that I contend is present in all markets of imperfect information.

3.1.3. Firms

I assume there are $n$ single product firms producing horizontally differentiated products. Firms are otherwise identical. I normalize production costs to zero and assume that each firm’s advertising technology is represented by $A(\phi; \lambda)$ with $A_{\phi} > 0, A_{\phi\phi} > 0, A_{\lambda} > 0, A_{\phi\lambda} > 0$ and $A(0; \lambda) = 0$. The function $A$ indicates the cost incurred by a firm to advertise its brand to a proportion $\phi \in [0,1]$ of the consumer
population. The parameter $\lambda$ shifts the advertising cost curve and is thus a measure of the efficiency of the advertising technology.

3.1.4. Consumers

I assume the number of consumers in this model is normalized to one. I also assume consumers have heterogeneous preferences with a perfectly inelastic unit individual demand for the product. The distribution of match values is independently and identically distributed (iid) and represented by twice differentiable and continuous distribution $F$. These match values are distributed over the interval $\varepsilon \in [a,b]$. The identical assumption means the distribution of match values is the same for each brand and the independence assumption means that the consumer can not use a match value realized for one brand to predict the match value he will realize for another brand.

Consumers are aware of the existence of the product but have no a priori information about their match value for the various $n$ brands. Conditional on consuming brand $k \in \{1,2,\ldots,n\}$, the indirect utility enjoyed is given by

\[
(3.1) \quad v_i(p_k) = y - p_k + \varepsilon_k,
\]

where $\varepsilon_k$ is the match value for brand $k$, $p_k$ is the price, and $y$ is income. A consumer incurs a search cost $c$ whenever he visits a store for the first time. The net utility received by participating in this market is therefore given as $v_i(p) - c.z$ where $z$ is the number of stores visited by the consumer. I now discuss the consumer’s optimal search strategy.

3.1.5. Consumer Search Behavior

Consumers know the number of firms in the market but are unaware of the extent to which they would like a particular brand. One way in which a consumer may become aware of their match for a brand is through an advertisement received from that firm; otherwise, he becomes aware of how much he likes the brand by searching (buying
information about the brand). I assume a *sequential search* behavior pattern with consumers having perfect recall. Perfect recall means a consumer never forgets his match value for a brand already searched. Further, I assume once the consumer incurs the cost of searching a particular store, he may return at no additional cost. However, if the consumer is informed about a product through advertisement, he must pay the search cost if he chooses to visit the store. In this search pattern, each consumer visits at least one firm. Consumers rationally anticipate that each unsampled firm charges the same price.

The gain from engaging in these costly visits is finding the brand with a higher match value. Based on the distribution of match values, consumers have a *reservation value* for the product. This reservation value represents the minimum match value a consumer could realize in order to make the cost of another search outweigh the expected gain of an additional search. This implies a consumer will stop searching (and buy from that firm) if he realizes a match value at least as great as his reservation value.

3.1.5.1. Search without Prior Information

The reservation value for a given consumer is intimately linked to the information available to him prior to search. I now consider uninformed consumers: those without information on any brand. I assume the distribution of match values, $F$ is such that the expected surplus from the very first visit exceeds the cost of search. This means that an uninformed consumer is willing to pay the search cost $c$ and visit a randomly selected store. Without loss of generality, I label the surplus gained at this store as $(\varepsilon - p_i)$.

How high would this surplus need to be to deter the consumer from paying an additional search cost in order to sample an additional store? If another store is sampled and the consumer gets a surplus of $(\varepsilon - p)$, the consumer only benefits if $(\varepsilon - p) > (\varepsilon_i - p_i)$. The marginal benefit of visiting a second store as opposed to buying from the first store is therefore $\{\varepsilon - (\varepsilon_i - p_i + p)\}$ if $\varepsilon > (\varepsilon_i - p_i + p)$ and 0 if $\varepsilon \leq (\varepsilon_i - p_i + p)$. Since $(\varepsilon_i - p_i)$ is known and $p$ is rationally anticipated, $\varepsilon$ is the only
random variable. The expected benefit from searching an additional store is therefore given as $E_x[\max\{\varepsilon - (\varepsilon_i - p_1 + p), 0]\}$. Define $\hat{x}$ as the match value from store 1 (net of price differential) that equates the marginal benefit of search with the marginal cost of search.

(3.2) $E_x[\max\{(\varepsilon - \hat{x}), 0]\] = c$

**Proposition 1.** The reservation value match $\hat{x} \in \Re$ is a decreasing, convex function of search costs.

**Proof of Proposition 1.** Rewrite equation (3.2) as $\int (\varepsilon - \hat{x})dF(\varepsilon) - c = 0$. Totally differentiating the Left Hand Side with respect to $\hat{x}$ and $c$ gives $-(1 - F(\hat{x}))d\hat{x} - dc = 0$.

This implies $\frac{d\hat{x}}{dc} = -\frac{1}{(1 - F(\hat{x}))} < 0$ and $\frac{d^2\hat{x}}{dc^2} = -\frac{d\hat{x}}{dc} \frac{f(\hat{x})}{(1 - F(\hat{x}))^2} > 0$. *Q.E.D.*

It is obvious that the left hand side of equation (3.2) decreases in $\hat{x}$. This implies that for $(\varepsilon_i - p_1 + p > \hat{x})$, the expected marginal benefit of searching the second store is less than the cost of search. For uninformed consumers, therefore, the reservation match value is given as $(\hat{x} + p_1 - p)$; for $\varepsilon_i > \hat{x} + p_1 - p$ searching an additional store is not expected be worth the cost whereas for $\varepsilon_i < \hat{x} + p_1 - p$ the search will be worthwhile. The optimal sequential search strategy for a consumer without prior information is therefore:

**Lemma 1.** Search Strategy I (Uninformed Consumers)

i. Randomly select the first store to search.

ii. Stop searching (buy from the store) if the realized match value exceeds $(\hat{x} + p_1 - p)$; otherwise randomly select another store and continue to search until the condition is met.
iii. If the stop rule in step ii is not met and all stores have been searched, then return to the store that offers the highest surplus.

3.1.5.2. Search with Prior Information

I now outline an optimal search strategy for partially informed consumers: those with prior information about at least one brand but not every brand. Without loss of generality, label as brand 1, the brand that offers the highest surplus among those the consumer is informed about. The surplus from buying brand 1 is therefore \((c_1 - p_1 - c)\) since the consumer must still incur the search cost even though he is informed about the brand’s characteristics prior to search; this highlights the primary difference between information acquired prior to search and information gathered during search. Since the consumer obtains information without visiting the store, the effective price must be considered as \((p + c)\) instead of \(p\) for consumers who gather the information during search.

A consumer with prior information (via advertising) may decide to visit an unadvertised store \(k\) where he realizes a surplus \((c_k - p)\). The partially informed consumer’s decision of whether to accept the advertised offer from firm 1 or search the other stores hinge critically on his expected benefits from searching. Recall the discussion in the previous section that if his best offer exceeds \((\hat{x} + p_1 - p)\), with \(\hat{x}\) defined in equation (3.2) above, then the consumer is not expected to benefit from search and will consequently go directly to buy an advertised brand. A consumer will buy immediately from the set of advertised brands if \([c_i > \hat{x} + (p_1 + c) - p]\). Once the consumer decides against buying an advertised product immediately, his reservation value for each unadvertised brand is identical to that of uninformed consumers, \((\hat{x} + p_k - p)\). The optimal sequential search strategy for a consumer with prior information is therefore:
Lemma 2. Search Strategy II (Partially Informed Consumers)

i. Consider the product that offers the highest surplus from among the advertised brands. Label this brand 1.

ii. Buy brand 1 if the surplus exceeds \((\hat{x} + p_{1} - p + c)\); otherwise randomly select a store to visit.

iii. Stop searching (buy from the store \(k\)) if the realized match value exceeds \((\hat{x} + p_{k} - p)\); otherwise randomly select another store and continue to search until the condition is met.

iv. If the stop rule in step (iii) is not met and all stores have been searched, then return to the brand (advertised or not) that offers the highest surplus.

A key distinction between the strategy of the uniformed and partially informed consumers is the determination of which store is given first consideration. For uninformed consumers, the store that is given first consideration is determined randomly while for partially informed consumers, first consideration is given to stores the consumer has prior information on. This suggests each firm has an incentive to ensure that consumers have prior information about its product and immediately suggest a role for informative advertising prior to consumer search.

3.2. Demand Estimation when Consumers Search

I now outline the expected demand faced by a representative firm, say firm 1, based on its beliefs about rivals’ price strategies and consumers’ search strategies. Here I consider only symmetric equilibrium strategies where firm 1 believes each rival charges price \(p\) and consumers anticipate this price at all stores yet to be search. Firms 1’s expected demand then shows how firm 1 anticipates how demand for his brand to change if he charges \(p_{1} \in [0, \infty)\), given that rivals charge \(p\) and consumers expect each firms to charge \(p\). The chance a given consumer purchases firm 1’s brand depends on the number of brands the consumer is informed about prior to search. I therefore derive the expected demand for consumers based on the number of products consumers are informed about. Consumers will be segmented into three groups based on the amount of
brands they are informed about prior to search; fully informed consumers, uninformed consumers and partially informed consumers.

3.2.1. Fully Informed Consumers

It is assumed in this section that consumers have prior information on all brands available in the market. These consumers go directly to the brand offering the highest surplus. Consider a consumer who receives a match value of \( \varepsilon_i \) for brand 1. The consumer will visit (and buy) from firm 1 if the surplus is lower at all other stores. The chance this occurs is given as \( \Pr[\varepsilon_2 - p < \varepsilon_1 - p_1, \ldots, \varepsilon_n - p < \varepsilon_1 - p_1] \). Since the distribution match values is iid across brands,

\[
\Pr[\varepsilon_2 - p < \varepsilon_1 - p_1, \ldots, \varepsilon_n - p < \varepsilon_1 - p_1] = \prod_{j=2}^{n} F_{\varepsilon_j} (\varepsilon_1 - p_1 + p) \text{ (by independence)}
\]

\[
= F^{n-1}(\varepsilon_1 - p_1 + p) \quad \text{ (by identical)}
\]

where we use the notation \( F^{n-1}(\cdot) \equiv [F(\cdot)]^{n-1} \).

The expected demand for firm 1’s product from fully informed consumers is then found by aggregating over match values. That is,

\[
(3.3) \quad D^f(p_1, p) = L_n \cdot \int_{a}^{b} F^{n-1}(\varepsilon - p_1 + p) dF(\varepsilon)
\]

where \( L_n \) is the number of fully informed consumers.

3.2.2. Uninformed Consumers

In this Section, I derive the expected demand for brand 1 from consumers who have no prior information on any brand; this is first developed in Wolinsky (1986) and based on the consumer behavior outlined in Lemma 1. The order in which the stores are searched is randomly determined; subsequently, each store has an equal chance of being the \( s^{th} \) \( (s = 1, \ldots, n) \) store visited by a consumer. Since a consumer continues to search
only if he does not realize his reservation value, the probability firm 1 is searched on the
$s^{th}$ visit by the consumer is \( \frac{1}{n} F^{s-1}(\hat{x}) \) because it must be the case that the consumer
realize a match value below his reservation level on visits to the previous \((s - 1)\) stores.
The probability that the consumer buys brand 1 on his \( s^{th} \) overall trip is therefore \( \frac{1}{n} F^{s-1}(\hat{x})(1 - F(\hat{x} + p_1 - p)) \).

The demand from consumers who stops and buys at firm 1 (step (ii) in Lemma 1) is given by
\( \frac{1}{n} \sum_{s=1}^{s} F^{s-1}(\hat{x})(1 - F(\hat{x} + p_1 - p)) \). Consumers who receive a match value at firm 1 lower than the reservation value might still return to purchase brand 1 if the match value at all other stores is even lower than that received at firm 1. These are consumers who reach step (iii) in Lemma 1. Since these consumers have searched all the stores, their expected demand for firm 1 is similar to the expected demand from fully informed consumers. That is, the probability that such a consumer who receives a match value of \((\varepsilon < \hat{x} + p_1 - p)\) at store 1 buys brand 1 is given as
\[ \int_a^d p + p dF(a) \] .

**Proposition 2.** The expected demand from uninformed consumers for firm 1’s product is given as
\[
D_1^u(p_1, p; c) = L_0 \left[ \frac{1}{n} (1 - F(\hat{x} + p_1 - p)) \frac{1}{1 - F(\hat{x})} + \int_a^{\hat{x} + p_1 - p} F^{n-1}(\varepsilon - p_1 + p) dF(\varepsilon) \right]
\]
where \( L_0 \) is the number of uninformed consumers.

3.2.3. Partially Informed Consumers

In this section, I derive the expected demand for firm 1 from consumers who have prior information on exactly \( r \) \((r = 1, \ldots, n - 1)\) brands. These consumers follow the search strategy outlined in Lemma 2. There is a measure \( L_r \) of consumers who have prior information about \( r \) products. The demand for brand 1 from partially informed
consumers depends on how many of these consumers have prior information about brand 1. Let us use a (+) superscript to denote the number of consumers who have prior information about brand 1 and a (−) superscript to denote the measure of consumers without prior information about brand 1. By definition, therefore, $L_r = L_r^+ + L_r^−$. Similarly, let $M_r$ be the set of brands that are successfully advertised to a given consumer with $M_r^+$ (respectively, $M_r^−$) indicating that brand 1 is included (respectively, excluded) from this set.

**Proposition 3.** The expected demand for brand 1 is given as

(3.5) \[ D_1^d(p_1, p; c) = \sum_{r=1}^{n-r} [L_r^+ D_r^+ + L_r D_r^−] \]

with

(3.6) \[ D_r^+ \equiv \int_{\hat{\tau} + p - p - c}^{\hat{\tau} + p - p - c} F^{r-1}(\varepsilon - p_i + p) dF(\varepsilon) + \int_{\hat{\tau} + p - p - c}^{\hat{\tau} + p - p - c} F^{a-r}(\varepsilon - p_i + p) \cdot F^{n-r}(\varepsilon - p_i + p - c) dF(\varepsilon) \]

(3.7) \[ D_r^− \equiv \frac{r}{n-r} \int_{\hat{\tau} + p - p - c}^{\hat{\tau} + p - p - c} F^{r-1}(\varepsilon) \cdot [1 - F^{n-r-1}(\varepsilon - c) F(\varepsilon + p_i - p - c)] dF(\varepsilon) \]

**Proof of Proposition 3.** The term $D_r^+$ represents the demand for brand 1 from a consumer with prior information about brand 1 and $(r-1)$ other brands in the market, $r = 1, \cdots, n-1$. These $r$ firms are in direct competition with each other and only the firm that offers the highest surplus to a given consumer has a chance of being selected by the consumer. Let us label the advertised brands from 1 to $r$, that is $M_r^+ = \{1, \ldots, r\}$. The probability firm 1 is the consumer’s best offer is given by

\[ \Pr[\varepsilon_1 - p_1 > \max_{j \in M_r^+} (\varepsilon_j - p)] = F^{r-1}(\varepsilon_1 - p_1 + p) \]
The first term in $D^+_r$ captures the idea consumers buy brand 1 directly whenever their match value is in excess of the reservation value ($\epsilon_1 > \hat{x} + p_1 - p + c$); this is in keeping with step (i) of Lemma 2. Consumers whose best offer is below the reservation value ($\epsilon_1 \leq \hat{x} + p_1 - p + c$) search the other $(n-r)$ stores before consider buying the best advertised brand. During search, they realize a match value $\epsilon_s \in (a, b), s \in M^+_r$ at an anticipated price $p$. These consumers return to buy brand 1 only if the surplus from brand 1 exceeds the highest surplus realized at the other stores. The probability a consumer finds a lower match value at the other $(n-r)$ stores is given as

$$\Pr[\epsilon_1 - p_1 - c > \max_{s \in M^+_r} (\epsilon_s - p)] = F^{n-r}(\epsilon_1 - p_1 - c)$$

The second term therefore represents the number of consumers who finds brand 1 better than the other advertised brands as well as better than the unadvertised brands. The expected measure of consumers that buys brand 1 is found by taking expectations over $\epsilon_1 < \hat{x} + p_1 - p + c$.

The term $D^-_r$ represents the demand from consumers without prior information about brand 1. Label the set of advertised brands as $M^-_r = \{n-r+1, \ldots, n\}$. Note that brand 1 is not advertised to these consumers ($1 \not\in M^-_r$). Recall that firm 1 believes each rival charges identical price $p$. Firm 1 then believes that with probability $F^{r-1}(\epsilon)$, a consumer with a match value $\epsilon$ will search at least one of the other $(n-r)$ stores. The total amount of consumers who do not receive an advertisement from firm 1 but decides to search is given by $r \cdot F^{r-1}(\epsilon)$. Further, the probability that each of these consumers find a greater surplus for an unadvertised brand (which includes firm 1) is given by $[1 - F^{n-r-1}(\epsilon - c)F(\epsilon + p_1 - p - c)]$. Since each of these $(n-r)$ has an equal chance of being sampled during search, the probability that firm 1 is selected is $\frac{r}{n-r} \cdot F^{r-1}(\epsilon)[1 - F^{n-r-1}(\epsilon - c)F(\epsilon + p_1 - p - c)]$ and the expected demand from these
partially informed consumers is then found by aggregating over possible values \( \varepsilon_k \in [a, \hat{a} + c] \). Q.E.D.

Theorem 1. The expected demand facing each firm when consumers with heterogeneous prior information engage in sequential search strategies is given as

\[
D_i(p_1, p; c) = L_nD_{i1}(p_1, p) + \sum_{r=1}^{s} [L_r^+D_r^+(p_1, p; c) + L_r^-D_r^-(p_1, p; c)] + L_0D_{in}(p_1, p; c)
\]

with \( \sum_{s=0}^{n} L_s = 1 \)

The demand in equation (3.8) above is a generalization of the demand generated by those commonly used models in the extant literature. For instance, when \( L_0 = 1 \) the model specifies to Wolinsky (1986) whereas for \( L_n = 1 \), the model specifies the demand generated under markets with complete information.

3.3. Existence and Uniqueness of Price Equilibrium

A sufficient condition for the existence and uniqueness of the equilibrium price strategy is for the Demand curve [equation (3.8)] to be concave in \( p_1 \). To establish uniqueness, therefore, I appeal to aggregation results in Caplin and Nalebuff (1991).

The demand function for the fully informed (Section 3.2.1) and uninformed consumers (Section 3.2) have already been shown to be concave in the Literature. I now proceed to establish concavity of the demand for partially informed consumers (Section 3.2). In Appendix B, I show that demand from partially informed consumer can be written

\[
D'_i(p_1, p; c) = L_r^+[\int_{\varepsilon \in E_1} g(\varepsilon)de + \int_{\varepsilon \in E_2} g(\varepsilon)de] + L_r^+ \left[ \frac{r}{n-r} \int_{\varepsilon \in E_3} g(\varepsilon)de \right]
\]

with \( \varepsilon = \{\varepsilon_1, \ldots, \varepsilon_n\} \), \( \varepsilon_j \in [a, b] \), \( j = 1, \ldots, n \), \( g(\varepsilon) = \prod_{j=1}^{n} f(\varepsilon_j) \).
\[ E_1 = \{ \varepsilon : \varepsilon_i - p_i > \max_{j \neq i, j \in M^*_i} (\varepsilon_j - p) \text{ and } \varepsilon_i - p_i > \hat{x} - p + c \}, \]

\[ E_2 = \{ \varepsilon : \varepsilon_i - p_i > \max_{j \neq i, j \in M^*_i} (\varepsilon_j - p) \text{ and } \varepsilon_i - p_i < \hat{x} - p + c \text{ and } \] 
\[ \varepsilon_i - p_i - c > \max_{s \in M^*_i} (\varepsilon_s - p) \}, \]

\[ E_3 = \{ \varepsilon : \varepsilon_k - p > \max_{j \neq k, j \in M^*_i} (\varepsilon_j - p) \text{ and } \varepsilon_k - p < \hat{x} - p + c \text{ and } \] 
\[ \varepsilon_k - p - c < \max_{s \neq 1, s \in M^*_i} (\varepsilon_i - p, \varepsilon_s - p) \} \]

Caplin and Nalebuff show that as long as preferences are linear in \( \varepsilon \), then the integration of \( g \) over any convex subset of its domain induces certain properties on the shape of the integral. Specifically, if \( g^\rho \) is concave, then \( \left[ \int_{x \in \mathbb{R}} x^\rho g(x) dx \right]^{\frac{1}{1+\rho}} \) is also concave.

They show that integrating over a log-concave distribution will result in a concave function. As long as \( f \) is log-concave, \( g \) is log-concave since it is the product of log-cave functions. The expected demand is a linear combination of integrals of joint cumulative distribution of match values. Using the result by Caplin and Nalebuff, these integrals will result in a concave function as long as the distribution of match values \( f \) is log concave. The family of log concave distributions includes the Uniform, Normal, Beta, exponential and many others commonly used as statistical distributions.

The aggregate demand for brand 1 is therefore concave since it is a linear combination of concave functions.

3.3.1. Symmetric Equilibrium

In this section, I determine the symmetric equilibrium pricing strategies of firms. I assume fixed costs of production are zero and firms produce at a constant marginal cost that we normalize to zero. Assuming zero marginal cost of production means that the equilibrium price strategies derived should be interpreted as the price-cost margin as opposed to nominal price levels. The expected profit of firm 1 given its beliefs that rival firms charge \( p \) and consumers follow a sequential search strategy is given by
The FOC for profit maximization for firm 1 is given as

\[(3.10) \quad D(p_1, p; c) + p_1 \frac{\partial D}{\partial p_1} = 0\]

Imposing symmetry in equilibrium strategies \((p_1 = p)\) gives

\[(3.11) \quad p = \frac{1}{-n \frac{\partial D}{\partial p_1}(p, p; c)}\]

The discussion in the previous section guarantees the uniqueness of the pricing strategy outlined in equation \((3.11)\) so long as the distribution of match values \(f\) is log-concave. Without loss of generality, therefore, I make the following assumption for computational ease.

**Assumption 1.** Consumer match values are uniformly distributed over the unit interval.

It is straightforward to show that firm 1’s expected demand is decreasing in own price and (under assumption 1) is given as

**Proposition 4.**

\[(3.12) \quad \frac{\partial D_1}{\partial p_1}(p_1, p) = - \left[ L_n + \sum_{r=1}^{n-1} J^r + \sum_{r=1}^{n-1} K^r \left\{ \frac{r}{n-r} \left[ F^r - F^{r-1} (\hat{\varepsilon}) \cdot F^{n-r-1} (\varepsilon - c) dF(\varepsilon) \right] \right\} \right] - L_0 \left\{ \frac{1}{n} \frac{1 - F^n (\hat{x})}{1 - F(x)} \right\} < 0\]

**Proof of Proposition 4.** (See Appendix A)
Assumption 2. (i) \( L_r = \frac{n-r}{r} L_r \) and (ii) \( L_r = L_r \frac{n}{r} \)

Under assumption 2, I can show

Proposition 5.

(3.13) \( D_i(p, p) = \frac{1}{n} \)

Proof of Proposition 5. \( D'(p, p; c) = \frac{b}{a} F^{n-1}(\varepsilon) dF(\varepsilon) = \frac{1}{n} \),

\[ D^+(p, p; c) = \int_{\hat{x}+c}^{b} F^{-1}(\varepsilon) dF(\varepsilon) + \int_{a}^{\hat{x}+c} F^{-1}(\varepsilon) F^{n-r}(\varepsilon - c) dF(\varepsilon) \]

\[ D^-(p, p; c) = \frac{r}{n-r} \int_{a}^{\hat{x}+c} F^{-1}(\varepsilon) [1 - F^{n-r}(\varepsilon - c)] dF(\varepsilon), \]

\[ D^a(p, p; c) = \frac{1}{n} [1 - F^n(\hat{x})] + \frac{\hat{x}}{a} F^{n-1}(\varepsilon) dF(\varepsilon) = \frac{1}{n} \quad \text{Under assumption 2 and equation (3.8)}, \]

\[ D_i(p, p; c) = L_n \frac{1}{n} + \sum_{r=1}^{n-1} L_r \left[ \int_{a}^{b} F^{-1}(\varepsilon) dF(\varepsilon) \right] + \frac{1}{n} L_0 \]

\[ = L_n \frac{1}{n} + \sum_{r=1}^{n-1} L_r \frac{1}{r} + \frac{1}{n} L_0 \]

\[ = \frac{1}{n} \left[ L_n + \sum_{r=1}^{n-1} L_r \frac{n}{r} + L_0 \right] \]

\[ = \frac{1}{n} \left[ L_n + \frac{n}{r} L_r + L_0 \right] \]

\[ = \frac{1}{n} \quad Q.E.D. \]
3.3.2. Limiting Equilibrium

Since primary interest is in long run equilibrium of monopolistically competitive markets, I now derive the equilibrium pricing strategy for firms as the number of firm in the industry becomes arbitrarily large \( n \to \infty \). By taking the limits of equation (3.12) and substituting in equation (3.11), the unique equilibrium price strategy in this large group market where consumers are heterogeneous along preference and prior information set is given as

\[
\lim_{n \to \infty} p = \left[ \frac{L_0}{1 - F(\hat{x}(c))} + \lim_{n \to \infty} n \left( L_n + \sum_{r=1}^{n-1} L_r^* \right) \right]^{-1}
\]

This means equilibrium profits in the market is given as

\[
\lim_{n \to \infty} \pi_1 = \left[ \frac{L_0}{1 - F(\hat{x}(c))} + \lim_{n \to \infty} n \left( L_n + \sum_{r=1}^{n-1} L_r^* \right) \right]^{-1} \cdot \frac{1}{n} = 0
\]

The search model proposed in Wolinsky (1986) is a special case of my model whereby every consumer is uninformed. This is evident by substituting \( L_0 = 1, L_r^* = L_n = 0, r = 1, \cdots, n - 1 \) (no consumer has prior information) into the equations above and comparing them to the relevant expressions in equations (19) and (20) in Wolinsky (1986). In order for this pricing strategy to represent a truly monopolistic competitive market, the limit equilibrium price must be above the marginal cost of production (that is, \( p > 0 \)). This condition will only be met if \( \lim_{n \to \infty} n (L_n + \sum_{r=1}^{n-1} L_r^*) \) is finite.

This condition is met in Wolinsky (1986) by assuming \( (L_n + \sum_{r=1}^{n-1} L_r^*) = 0 \). I need not make such a strong assumption here. All that is required is for the expression \( (L_n + \sum_{r=1}^{n-1} L_r^*) \) to approach zero at a rate at least as fast as \( \frac{1}{n} \); otherwise price equals marginal cost in the
limit. Since \( L_n + \sum_{r=1}^{n-1} L_r \) represents the number of consumers informed about a given product, the intuitive interpretation of this condition is: for firms to maintain market power in the limit equilibrium, is it is sufficient that very few consumers have prior information about a given product. The equilibrium has the desirable property of price uniformly approaching competitive level whenever cost of information acquisition approaches zero, that is \( \lim_{c \to 0} \lim_{h \to c} p = 0 \).

The main point of Wolinsky is to point out firms maintain market power \( (p > 0) \) in large markets if consumers are imperfectly informed and acquiring the information is costly. But the sensitivity of the properties of the equilibrium pricing strategy to the number of consumers informed about a given product consumers suggests Wolinsky omitted a least one essential component of true monopolistically competitive market structures. I argue that such a missing component is the market determined distribution of information prior to search.

In order to endogenize the distribution of information prior to search, I must first specify a means of information transmission. This is presented in the next section which describes a model of informative advertising.

3.4. Advertising and Consumer Search

The analysis above describes a search model in which consumers have heterogeneous prior information. A limitation of previous models in the Industrial Organization literature is the lack of attention paid to consumers with heterogeneous prior information sets. An exception is Anderson and Renault (2000) who consider a search model with fully informed and uninformed consumers (that is \( L_r = 0, r = 1, \ldots, n - 1 \)). As far as one the major objective of this paper is concerned, the obvious short-coming of their model is that it does not facilitate analysis of consumers with information about a subset of the brands available in the market (partially informed consumers). In this chapter, I capture a market with imperfect information in which there is active information acquisition and information dissemination on the part of
economic agents. It is very unlikely any information-disseminating technology used by non-colluding firms would lead to such discrete prior information sets for consumers as in Anderson and Renault (2000). The model outlined in Section 3.2 is sufficient to capture more continuous levels of heterogeneity in consumer’s prior information. In the next section, I complete the model by specifying how the heterogeneity in consumer information is endogenously generated through informative advertising.

3.4.1. Advertising Technology

In what follows, I describe a model informative advertising. Firm \( k \in \{1, \ldots, n\} \) randomly disseminates information about its brand (i.e., advertise) to a measure \( \phi_k \in [0,1] \) of consumers. The advertising cost is an increasing convex function of \( \phi \), the advertising reach. Denote this function as \( A(\phi, \lambda) \) with \( A(0; \lambda) = 0, A_\phi > 0, A_{\phi\phi} > 0 \), \( \lambda > 0 \).

Assume advertising technology is identical across firms and the decision about how many consumers to inform is set independently by each firm and no firm has the ability to target any specific segment of the consumer population. Since I am primarily concerned with the symmetric equilibrium, denote \( \phi_k = \phi, k = 2, \ldots, n \) and focus on firm 1’s optimal advertising strategy given its belief that rivals select \( \phi \). These assumptions allow me to capture heterogeneity of consumer prior information by quantifying the expected number of messages received by consumers.

**Proposition 6.** (i) \( L_r = \binom{n-1}{r-1} \phi \phi^{r-1} (1-\phi)^{n-r} \), (ii) \( L_r = \binom{n-1}{r} \phi^r (1-\phi)^{n-r-1} (1-\phi_1) \)

(iii) \( L_0 = (1-\phi)^{n-1} (1-\phi_1) \), (iv) \( L_n = \phi^{n-1} \phi_1 \)

**Proof of Proposition 6.** (i) Statistical principles tells us that if you have \( n \) firms trying to send information to consumers, the total number of different combination in which a consumer successfully receives an advertisement from \( r \in \{1, \ldots, n-1\} \) firms (and fails to
receive an ad from \((n-r)\) firms) is given by \(\binom{n}{r} = \frac{n!}{(n-r)!r!}\). Further, the number of

ways that an advertisement from firm 1 is one of those \(r\) advertisements successfully transmitted to the consumer is given as \(\binom{n-1}{r-1}\). This implies that the number of consumers who receive an advertisement from firm 1 and \((r-1)\) other firms is given as

\[ L^+ = \binom{n-1}{r-1} \cdot \phi \phi^{r-1} (1-\phi)^{n-r}. \]

(ii) Similarly, the number of ways the message firm 1 is not successfully transmitted is given as \(\binom{n-1}{r}\) which implies that number of consumers who receive an advertisement from \(r\) firms but none from firm 1 is given as

\[ L^- = \binom{n-1}{r} \cdot \phi^r (1-\phi)^{n-r-1}(1-\phi) \].

(iii) Similarly, there is only one way in which no consumer receives any advertisement from any firm, \(L_0 = (1-\phi)^{n-1}(1-\phi_1)\) and (iv) only one way in which each consumer receives an advertisement from every firm, \(L_n = \phi \phi^{n-1}\). \(Q.E.D.\)

It is easy to verify that assumption 2 is also satisfied for this advertising technology.

**Lemma 3.** For \(\phi_i = \phi\), \(L_n + \sum_{r=1}^{n-1} L^+_r = \phi\)

**Proof of Lemma 3:** By Proposition 6,

\[
L_n + \sum_{r=1}^{n-1} L^+_r = \phi^n + \sum_{r=1}^{n-1} \binom{n-1}{r-1} \phi^r (1-\phi)^{n-r} = \phi^n + \phi \sum_{r=1}^{n-1} \binom{n-1}{r-1} \phi^{r-1} (1-\phi)^{n-r}
\]

\[
= \phi^n + \phi \left[ \binom{n-1}{0} (1-\phi)^{n-1} + \binom{n-1}{1} \phi (1-\phi)^{n-2} + \cdots + \binom{n-1}{n-2} \phi^{n-2} (1-\phi) \right]
\]

\[
= \phi^n + \phi(1-\phi^{n-1}) = \phi. \quad Q.E.D.
\]
The demand for brand 1 is obtained by substituting the expressions for \( L_r, r = 0,1,\ldots, n \) in Theorem 1. That is

\[
\begin{align*}
(3.16) & \\
D_1(p_1, p; \phi_1, \phi) = \phi \phi^{n-1} \cdot D_1^f + \sum_{r=1}^{n-1} \left( \frac{n-1}{r-1} \right) \phi \cdot \phi^{r-1} (1-\phi)^{n-r} \cdot D_r^+ \\
& + \sum_{r=1}^{n-1} \left( \frac{n-1}{r} \right) \phi^{1-r} (1-\phi)(1-\phi)^{n-r-1} \cdot D_r^- + (1-\phi)(1-\phi)^{n-1} \cdot D_1^u
\end{align*}
\]

where \( D_1^f \) is given in equation (3.3), \( D_r^+ \), \( D_r^- \) and \( D_1^u \) are given in equations (3.6), (3.7) and (3.4) respectively.

3.4.2. Symmetric Equilibrium

In this section, I derive the symmetric equilibrium strategies in price and advertising strategies.

**Definition 1.** Market equilibrium is a pair \( \{p, \phi\} \) such that

\[
\{p, \phi\} \in \arg\max_{\{p, \phi\}} \pi_1(p_1, p, \phi_1, \phi; c, \lambda) = p_1 D(p_1, p, \phi_1, \phi; c) - A(\phi_1, \lambda)
\]

The FOCs for this problem is given as

\[
\begin{align*}
(3.17) & \\
& \frac{\partial D}{\partial p_1}(p, p; \phi, \phi) - D(p, p; \phi, \phi) = 0 \quad (\pi_p = 0) \\
(3.18) & \\
& \frac{\partial D}{\partial \phi_1}(p, p; \phi, \phi) - A_1(\phi, \lambda) = 0 \quad (\pi_{\phi} = 0)
\end{align*}
\]

3.4.3. Limit Equilibrium

In this section, I analyze equilibrium in this advertising model when the number of firms is large. Specifically we look at the limiting case as the number of firms gets infinitely large.
Proposition 7.

(i) \[
\frac{\partial D_1}{\partial \phi_i} = \frac{1}{n} \left\{ \phi^{n-1} - (1 - \phi)^{n-1} \right\} - \frac{1}{n(1 - \phi)} \left\{ 1 - (1 - \phi^n - \phi^n) \right\} + \frac{1}{n\phi(1 - \phi)} \left\{ [1 - \phi^n - \phiF(\hat{x} + c) + (1 - \phi)]^n + [\phiF(\hat{x} + c)]^n \right\}
\]

(ii) \[
\lim_{n \to \infty} n \frac{\partial D_1}{\partial p_1}(p, \phi) = -n\phi
\]

(iii) \[
\lim_{n \to \infty} \frac{\partial D_1}{\partial \phi_i}(p, \phi) = \frac{1}{n\phi}
\]

Proof of Proposition 7. (See Appendix A)

Proposition 7 implies that for the large group market with differentiated products, equilibrium is characterized by

\[(3.19) \quad p = \frac{1}{n\phi} \]

\[(3.20) \quad (n\phi)^2 \frac{\partial A}{\partial \phi}(\phi; \lambda) = 1 \]

The equilibrium price strategy in equation (3.19) is equivalent in form to the equilibrium price in equation (3.14) after substituting the endogenously determine distribution of prior information. To see this, note that in the advertising equilibrium,

\[
\lim_{n \to \infty} L_0(1 - \phi)^n = 0 \quad \text{and} \quad \lim_{n \to \infty} L_n(\phi^n = 0
\]

Equation (3.19) is then recovered by substituting these terms into equation (3.14). The pricing strategy in (3.19) is not surprising. Under perfect information with \(n'\) firms competing, the equilibrium price is given as \(\frac{1}{n'}\). (To see this substitute \(\phi = 1\) in
equation (3.19) to derive the full information result). As I discuss in detail in the following section, the effective number of firms competing for each consumer is 
\[ n' = n\phi \]
hence the equilibrium price should in fact be 
\[ \frac{1}{n\phi} \].

**Theorem 2.** As \( n \to \infty \)

(i.) The number of consumers informed about a given product is \( \phi \to 0 \)

(ii.) The equilibrium price strategy is 
\[ p \to \sqrt{A_{\phi}(0;\lambda)} \]

(iii.) The average number of products consumers are informed about is 
\[ n\phi \to \left[ A_{\phi}(0;\lambda) \right]^{\frac{1}{2}} \]

**Proof of Theorem 2:** (i) To demonstrate that \( n\phi \) increase as \( n \) increases, we use the deduction put forth in Grossman and Shapiro (1984) by examining the equilibrium condition in equation (3.20). As \( n \) increases, \( \phi \) must decline to keep the equation balance. But if \( \phi \) falls, so must \( \phi_{A} \) since \( 0 > \phi_{A} \). Further, if \( \phi_{A} \) falls, \( n\phi \) must increase to keep the equation balanced. (ii) From equation (3.20), 
\[ n\phi = \left[ A_{\phi}(0;\lambda) \right]^{\frac{1}{2}} \]. Substitute this into equation 3.19 to complete proof. (iii) This result is immediate from (ii) above. Q.E.D.

An interesting property of this equilibrium is 
\[ \lim_{n \to \infty} \frac{\partial (\text{lim}_{p} p)}{\partial c} = 0 \]; in the limit as the market expands, the pricing strategy of firms is not affected by the search cost. All that is required to sustain this equilibrium is for \( c > 0 \). This does not mean, however, that there is no actual search in equilibrium. Consumers actively search in equilibrium as long as the advertising technology is not too efficient. Firms are not influenced by the search cost but not because consumers do not search; rather, it is because no individual firm expects to generate sales from consumers who search.
3.5. Discussion

My model of advertising is a hybrid of advertising models of Grossman and Shapiro (1984) with differentiated products but no search technology and Robert and Stahl (1993) with consumer search but homogenous products. Accordingly, my model nests these models as special cases. In what follows, I compare these models and highlight the additional insights gleaned by modeling both channels of information transmission in differentiated products markets.

The market equilibrium strategies given in Theorem 2 are striking since they are identical to the equilibrium strategies in Grossman and Shapiro (1984). It must then be the case that either consumers do not search in equilibrium in my model or firms are not influenced by consumers who search; I show that the latter is indeed correct. The fact strategies are identical is a consequence of the large group assumption. In a large market, no individual firm expects to generate revenues from consumers who search since the probability that any individual firm is randomly selected approaches zero whenever the market is large. This means that firms expect to generate sales revenue exclusively from consumers who observe their advertisements and this is observationally equivalent to the model developed in Grossman and Shapiro. The key distinction is that in my model consumers may or may not search; the extent of consumer search varies in my model and is endogenously determined by the advertising and consumer search technologies. In Grossman and Shapiro a lack of consumer search is exogenously imposed.

Robert and Stahl (1993) highlight the asymmetric effects of the advertising and search information channels on market equilibrium. They show that the advertising technology is solely responsible for determining whether or not firms maintain market power in equilibrium. I also observe this asymmetric effect in my paper and show that advertising has more far reaching effects on the equilibrium. In what follows, I show the dominant role of the advertising technology extends to other features of equilibrium.

Another feature of equilibrium in Robert and Stahl (1993) is that each consumer is informed about almost all the brands available in the market. A consequence of this
result, therefore, is that there is no real search in equilibrium. In my model, a consumer need not be informed about that many products in equilibrium and search occurs when the advertising technology is sufficiently inefficient. The advertising and search technologies jointly determine the extent of consumer search in equilibrium; however advertising plays a greater role. For the advertising technology described in the previous section, the number of advertisements that are successfully transmitted to consumers in equilibrium follows a binomial random variable with $n$ independent trials, each with $\phi$ probability of success. Therefore an average of $n\phi$ advertisements is successfully transmitted to each consumer. A consumer searches only if no advertised brand meets his reservation value. The probability that a consumer searches in equilibrium is therefore $F^{n\phi}(\hat{x} + c)$. This term captures the idea that consumer search in equilibrium is jointly determined by advertising technology (via $n\phi$) and search technology (via $\hat{x} + c$). Whenever the advertising technology is relatively efficient ($A_y(0, \lambda)$ is small) so that $n\phi \to \infty$ (by Theorem 2), each consumer is informed about a large number of brands and so search is unlikely in equilibrium, regardless of the search technology. Whenever the advertising technology is relatively inefficient ($A_y(0, \lambda)$ is large), each consumer is informed about a few products and is therefore expected to search at least one product. In this case the extent of search (number of brands searched) is determined by the search technology.

Theorem 2 shows that extent of market power maintain in equilibrium depend on the value of $A_y(0, \lambda)$; price exceeds marginal cost whenever $A_y(0, \lambda) > 0$. What is clear from the result, however, is that market power in large markets does not necessarily imply advertising is not fully informative in nature.

3.6. Conclusion

Economists often encounter situations in which agents with asymmetric information interact in a non-cooperative manner. In such cases, there is an incentive for the agent in the disadvantaged position to take active steps to acquire the information as
well as incentives for agents to make this information available to the disadvantaged. I argue explicitly modeling both channels of information flow is crucial for a more complete understanding of these markets. Although the arguments are advanced within the confines of a differentiated product market, the implications are relevant to a much broader spectrum of economic analysis.

The main result of this chapter reconciles the modern view of advertising with the theory of monopolistic competition. I show advertising can be a pro-competitive tool that increases the substitutability among products while allowing firms maintain market power in the limit as the number of firms becomes arbitrarily large. This is an important contribution to the debate on the role of advertising. Monopolistic competition has long since been held up to delineate the persuasive or combative purpose of advertising which allow firms to manipulate consumer preferences in favor of the advertised product in order to maintain market power. I have shown under informative advertising, firms maintain market power in the limit if the initial effort to disseminate information is costly whereas firms will lose all market power if this initial effort is costless. Hopefully, this chapter has now moved the discussion of the source of market power in monopolistically competitive markets from the role of advertising (informative vs. persuasive) to the nature of the advertising technology.

Chapter III has also highlighted the dominant role played by advertising technology in achieving the (resource allocative) efficient market outcome. When the initial costs to send an advertisement to an additional consumer is zero, equilibrium price in large markets approaches marginal cost. This result is not new, as it was also present in Grossman and Shapiro (1984) and Robert and Stahl (1993). The value of this finding lies in the fact that it is now shown to be robust to diverse modeling assumptions as it is shown to be invariant to consumer search and heterogeneity in product characteristics.

An underlying assumption of search models is consumers have limited information about the product (its existence) that allows them to search brands that are not advertised to them; the source of this information was left unspecified. Knowledge of the existence of the product is invaluable information to consumers who would prefer
consuming any brand to not consuming any at all. This is especially true in circumstances where it is too expensive for consumers to acquire this information on their own.

This observation suggests a natural extension of the study presented in this chapter. It would be fruitful to devise a model that allows a firm to choose the amount of brand-specific information contained in its advertisements. The immediate issue would be to determine the circumstances in which firms prefer to disclose more information to less information in their advertisements.

To the best of my knowledge, the only study to explicitly model advertising content in the economic literature is Anderson and Renault (2004) who explores the issue in a monopoly market structure. Given the amount of resources spent by firms to advertise their products in oligopoly markets, the relationship between advertising content and the performance of these markets is of both theoretical and practical concern. These and other issues are developed further and explored in Chapter IV.
Is there any merit to the claim that the use of uninformative advertisements provides substantial evidence in support of the view that advertising has an anti-competitive effect on the performance of the market? I argue that the information content of the advertisement in and of itself can not be used to assess the effect of advertising on the performance of the market. In this chapter, a model of advertising that is most favorable to the anti-competitive view is presented: advertising with no brand-specific information. I do not dispute that there might be instances when advertising is uninformative. My point is that it is ill-advised to evaluate the informativeness of an advertisement regime based solely on its observable information content; cost of information acquisition must also be considered.

Information acquisition cost is at the heart of this analysis. I am not referring to search cost that was discussed in Chapter III. In this chapter, I discuss discovery cost. Discovery cost refers to the amount of resources expended by an individual to learn about the existence of a product. This is to be distinguished from the analysis of search costs in Chapter III which assumes each consumer is already aware of the existence of product but gathers information on particular brands of the product. When the discovery cost associated with a given product is prohibitively high, therefore, consumers will not participate in the market for that product unless information is disseminated to these consumers about the existence of the product. I argue in this chapter that advertisements without any brand specific information informs consumers that the product exists whenever discovery cost is high; it is in this sense I show that information content of an advertisement is an inadequate measure of its informativeness.

In Section 4.2, I derive alternative models of advertising based on information content and derive the equilibrium price, advertising intensities and expected economic rents under each regime within a broad class of consumer preference distribution. I specialize the models to the uniform distribution case and compare the profits under each
regime. I also carry out comparative static results and consider how various endogenous variables are affected by the parameters in the model. In Sections 4.4 and 4.5, I calibrate the parameters in my model with empirical data and discuss the implications of calibrated results. I conclude my study in Section 4.6 where I suggest areas of future investigations.

4.1. Duopoly Model

I consider a duopoly market with each firm producing a single brand. Brands are differentiated but consumers do not perceive any differences in quality of the brands. Consumers have a unit demand for the product. Firms compete in price, advertising level and advertising content. Each firm maximizes expected profits calculated as total expected revenue less advertising expenditures since production costs are normalized to zero. The number of consumers is normalized to 1. Consumers are unaware of how well either product is matched to their taste but they know match values for each product is identically and independently distributed by a log concave distribution $F$ on the interval $[a,b]$. Consumers must first decide whether to participate in the market and then decide which brand to select, conditional on deciding to participate. Consumers will only participate in the market if either discovery cost is low or an advertisement is received from at least one firm.

4.1.1. Description of the Game

There are four stages $\{0,1,2,3\}$ and three sets of players: two firms, a continuum of consumers and Nature. The timing of the game is as follows. At Stage 0, Nature assigns each consumer with a match value for each brand. The other players do not observe the individual assignments but each player knows the distribution $F$ from which the match values are drawn. At Stage 1, each firm decides on how much information $\{I,U\}$ to disclose in advertisements in order to maximize expected profits. An advertising regime is said to be Informative ($I$) if a consumer’s assigned match value and the price of the brand are revealed through the advertisement. A regime is
said to be *Uninformative* (*U*) if neither the price nor match value are revealed by the advertisement.

In Stage 2 the advertising regime of both firms is common knowledge. Further, each firm decides the price \( p \in [0, \infty) \) and advertising level \( \phi \in [0,1] \). The amount of information contained in each advertisement depends on the decision taken by firms at Stage 1.

In Stage 3, consumers select a brand to purchase. A consumer who receives an advertisement selects a brand based on the information contained in the advertisement as well as information acquired in subsequent searches. A consumer who does not receive an advertisement will not participate in the market if the discovery cost is high; otherwise, the consumer searches and select a brand based on information gathered during search.

4.1.2. Consumers

I assume there is a continuum of consumers with a size normalized to 1. Consumers are heterogeneous with respect to their preferences for the brands. The heterogeneity is captured in their *match value* for each brand. The match value \( \varepsilon \) is the amount of satisfaction the consumer receives by consuming a given brand. The distribution of consumer match values is assumed to be identically and independently distributed \( F \) for both brands over the interval \( \varepsilon \in [a, b] \). The assumption that the distribution of match values is identical for each product underpins the horizontal characteristics of the model. The independence assumption means a consumer is not able to use match value received for one brand to infer his match value for the other. Conditional on consuming brand \( k \), consumer \( i \)'s indirect utility is given as

\[
V^i = y + \varepsilon^i - p_k - c \cdot z
\]
where $y$ is disposable income, $\varepsilon^i_k$ is the match value received by consumer $i$ for product $k$, $p_k$ is the price, $c$ is the cost of a unit search and $z$ is the number of searches made (stores visited) by the consumer before a decision is taken. Each consumer maximizes the expected utility by following an optimal sequential search strategy.

4.1.3. Sequential Search

A consumer searches when either discovery cost is low or an advertisement is received from at least one firm. Conditional on one of these events occurring, I now describe an optimal sequential search strategy.

Consumers know there are two firms but are unaware of the extent to which they would like a particular brand. One way in which a consumer may become aware of their match for a brand is through an advertisement received from that firm. Otherwise, the consumer becomes aware of how much he likes the brand by searching. I assume a sequential search behavior pattern with the consumers having perfect recall. Perfect recall means a consumer never forgets his match value for a brand already searched. Further, I assume once the consumer incurs the cost of searching a particular brand, he may return to that store selling that brand at no additional cost. However, if the consumer is informed about a product through advertisement, he must pay the search cost if he chooses to purchase the brand (since he must visit the store in order to do so). In this search pattern, each consumer visits at least one firm once discovery cost is low or an advertisement is received from at least one firm.

Consumers rationally anticipate the price is the same for all unsearched brands; the gain from engaging in these costly visits is finding the brand with a higher match value. Based on the distribution of match values, consumers determine a reservation value for the product. This reservation value represents the minimum match value a consumer could realize to make the cost of another search outweigh the expected gain of an additional search. This means that a consumer will stop searching (and buy from that firm) if he realizes a match value at least as great as his reservation value.
4.1.3.1 Search without Prior Information

The reservation value for a given consumer is a function of the information available prior to search. I now consider consumers who do not receive any advertisement (uninformed consumers). An uninformed consumer randomly selects which store to visit first; without loss of generality, label the surplus gained at this store as \((\varepsilon_i - p_i)\). The consumer then decides whether to purchase brand 1 and enjoy a surplus of \((\varepsilon_i - p_i)\) with certainty or pay an additional unit of search cost in order to sample the other store. If the other store is sampled and the consumer gets a surplus of \((\varepsilon - p)\), the consumer only benefits from the additional search if \((\varepsilon - p) > (\varepsilon_i - p_i)\).

The marginal benefit of visiting the other store as opposed to buying from the first store is therefore \(\{\varepsilon - (\varepsilon_i - p_i + p)\}\) if \(\varepsilon > (\varepsilon_i - p_i + p)\) and 0 if \(\varepsilon \leq (\varepsilon_i - p_i + p)\). The expected benefit from searching is therefore given as \(E_\varepsilon[\max\{\varepsilon - (\varepsilon_i - p_i + p), 0\}]\).

Define \(\hat{x}\) as the match value from store 1 (net of price differential) that equates the marginal benefit of search with the marginal cost of search. That is,

\[
E_\varepsilon[\max\{(\varepsilon - \hat{x}), 0\}] = c
\]

It is obvious that the marginal benefit of search decreases in \(\hat{x}\). This implies that for \((\varepsilon_i - p_i + p > \hat{x})\), the expected marginal benefit of searching the second store is less than the cost of search. For uninformed consumers, therefore, the reservation match value is given as \((\hat{x} + p_i - p)\); for \(\varepsilon_i > \hat{x} + p_i - p\), searching the expected gain in surplus is lower than the cost whereas for \(\varepsilon_i \leq \hat{x} + p_i - p\) the expected gain is at least as great as the cost. The optimal sequential search strategy for a consumer without prior information is therefore

**Lemma 4.** Search Strategy I (uninformed consumers)

i. Randomly select the first store to search.
ii. Stop searching (buy from the store) if the realized match value exceeds \((\hat{x} + p_1 - p)\); otherwise visit the other store.

iii. Stop at the second store if the surplus exceeded the first store; otherwise return to the first store and buy the product.

4.1.3.2. Search with Prior Information

I now outline the reservation value for a consumer who receives an advertisement from only one firm (partially informed consumer). Without loss of generality, label as brand 1, the brand for which he receives an advertisement. The surplus from buying brand 1 is therefore \((c_1 - p_1 - c)\) since the consumer must still incur the search cost even if though he is informed about the product information prior to search. Alternatively, the consumer may decide to visit the other store of which he has no prior information where he realizes a surplus \((\epsilon - p)\). The consumer’s decision of whether to accept firm 1’s offer or search (visit the other store) hinge critically on his expected benefits from searching.

Recall from the previous discussion that if his best offer exceeds \(\hat{x}\), defined in equation (4.1) above, then the consumer will not expect to benefit from search and will consequently go directly to purchase the advertised product. This suggests, therefore, that the partially informed consumer will immediately purchase brand 1 if \((\epsilon_1 > \hat{x} + p_1 - p + c)\). Once the consumer decides not to buy the advertised product immediately, his search strategy is identical to that of uninformed consumers. The optimal sequential search strategy for a consumer with information about only one product is:

**Lemma 5. Search Strategy II (Partially Informed Consumers)**

i. Buy the advertised product if the surplus exceeds \((\hat{x} + p_1 - p + c)\); otherwise visit the other store.

ii. Buy from the other store if the realized match value exceeds the surplus from the advertised product; otherwise buy the advertised product.
A key distinction between the strategy of uninformed consumers and partially informed consumers is the determination of which brand is given first consideration. For uninformed consumers, each store has an equal chance of being given first consideration first while for partially informed consumers, first consideration is given to the brand for which he has prior information.

4.1.4. Firms

I assume there are two firms competing in price, advertising intensity and information content of advertisements. Firms produce horizontally differentiated products but are otherwise identical. By horizontal differentiation I mean that consumer perceive no difference in the quality characteristics of each brand. I normalize production costs to zero in order to focus attention to advertising costs. The advertising technology is given by the twice differentiable continuous function $A$.

The amount of resources needed to send advertisements to $\phi \in [0,1]$ consumers is given as $A(\phi; \lambda)$ where $\lambda$ is a parameter that shifts the cost curve thus controlling the efficiency of the technology. I further assume that $A$ is an increasing, convex function of $\phi$ with $A_x > 0$ and $A(0; \lambda) = 0$ such that higher values of $\lambda$ reflect less efficient advertising technologies and the firm does not incur any advertising expense unless an advertisement is disseminated.

4.1.5. Equilibrium Concept

The equilibrium concept is Perfect Bayesian Equilibrium. It requires that each firm choose strategies to maximize his own payoffs (expected profits) based on the firm’s beliefs about its rivals’ strategies as well as its knowledge of consumers’ search strategy in Stage 3. It further requires consumers to buy a brand to maximize net indirect utility based on the information (if any) obtained from firms in Stage 2, as well as information obtained during the search process while anticipating the price charged by firms from which they did not receive an advertising message. In equilibrium
expectations are met. The Bayesian inference in this solution concept requires us to state how a consumer’s beliefs are updated when he comes across a price other than \( p \) at a particular store. For this I assume that he will continue to anticipate a price \( p \) for brands yet to be searched.

4.2. Symmetric Equilibrium

To solve this game, I use backward induction that involves first solving Stage 3 of the game and work back to Stage 1. I focus on symmetric, pure strategy equilibria. The demand curve represents the solution to Stage 3. In what follows, we solve the rest of the stages.

4.2.1. Stage 2: Price and Advertising Level

In this section, I determine the price and advertising level selected by each firm. For a given advertising regime, each firm selects \( \{p, \phi\} \) to maximize expected profits. To derive an expression for expected profits, I need to derive the firm’s expected demand under the alternative advertising regimes.

4.2.1.1. Informative Advertising Regime

The use of informative advertising is now considered. The consumer desires to know his surplus from each brand to make an informed decision. Two pieces of information are needed to reveal a consumer’s surplus for a particular brand: the price and match value. An informative advertisement reveals both pieces of information.

Whenever discovery cost is low, the consumer will become aware of the general product class without receiving an advertisement. Whenever discovery cost is high, consumers become aware of the existence of the product if and only if they receive an advertisement from at least one of the firms. Conditional on receiving an advertisement, consumers may search between both stores for the brand that maximizes expected utility. The expected demand from consumers when discovery cost is high is therefore similar to the expected demand when discovery costs are low. The only difference is that a
measure \((1 - \phi_1)(1 - \phi)\) of consumers will not participate in the market when discovery cost is high since they do not receive an advertisement from either firm. To model consumer demand under this parameterization, I modify the Duopoly version of the advertising model developed in Chapter III in order to explicitly model discovery cost, \(d\).

I measure discovery cost as a discrete value taking on a value of 0 whenever discovery costs are insignificant and a value of 1 otherwise.\(^5\) Each firm independently distributes advertising messages to a fraction of the consumers. Each message reveals the consumer’s assigned match value for that product as well as the price. The expected demand for firm 1’s product, given that the rival firm charges \(p\) and \(\phi\) is given as

\[
D_1 = \phi \phi \int_a^b F(\varepsilon - p_1 + p)dF(\varepsilon)
\]

\[
+ \phi_1 (1 - \phi) \left( (1 - F(\hat{x} + p_1 - p + c)) + \int_a^{\hat{x} + p_1 - p + c} F(\varepsilon - p_1 + p - c)dF(\varepsilon) \right)
\]

\[
+ \phi (1 - \phi_1) \left( \int_a^{\hat{x} + p_1 - p} [1 - F(\varepsilon - p + p_1 - c)]dF(\varepsilon) \right)
\]

\[
+ (1 - d)(1 - \phi_1)(1 - \phi) \left( \frac{1}{2} (1 + F(\hat{x})) (1 - F(\hat{x} + p_1 - p)) + \int_a^{\hat{x} + p_1 - p} F(\varepsilon - p_1 + p)dF(\varepsilon) \right)
\]

with \(d = \{0,1\}\)

Note that \(D_1(p_1 = p, \phi_1 = \phi) = \frac{1}{2} \left[ \phi^2 + 2\phi(1 - \phi) + (1 - d)(1 - \phi)^2 \right].\) Further,

\[
\frac{\partial D_1}{\partial p_1}(p_1 = p, \phi_1 = \phi) = -\phi^2 \int_a^b f(\varepsilon) dF(\varepsilon) - \phi(1 - \phi) \left( \int_a^{\hat{x} + c} f(\hat{x} + c) + \int_a^{\hat{x} + p_1 - p} F(\varepsilon - c)f'(\varepsilon)dF(\varepsilon) \right)
\]

\[
- (1 - d)(1 - \phi)^2 \left( \frac{1}{2} (1 - F(\hat{x})) f(\hat{x}) + \int_a f(\varepsilon)dF(\varepsilon) \right)
\]

\(^5\) The results reported here are unaltered using a continuous measure of discovery cost.
\[
\frac{\partial D_1}{\partial \phi_1}(p_1 = p, \phi_1 = \phi) = \frac{1}{2}d(1 - \phi)
\]

The Firm’s expected profits is given as

\[
\pi_1 = p_1D_1(p_1, p, \phi_1, \phi) - A(\phi_1)
\]

Since I am seeking only symmetrical equilibrium strategies, I impose \( p_1 = p \) and \( \phi_1 = \phi \) in the FOCs and get

\begin{align*}
\pi_p : p \frac{\partial D_1}{\partial p_1}(p_1 = p, \phi_1 = \phi) + D_1(p_1 = p, \phi_1 = \phi) &= 0 \\
\pi_\phi : p \frac{\partial D_1}{\partial \phi_1}(p_1 = p, \phi_1 = \phi) - A'(\phi) &= 0
\end{align*}

Using the FOC given in \( \pi_p \), I can write the equilibrium price as

\[
p(\phi) = \frac{-1}{2 \cdot \frac{\partial D_1}{\partial \phi_1}}(\phi)
\]

Substituting the expression for equilibrium price in the FOC given in \( \pi_\phi \), the equilibrium advertising intensity is given implicitly given as

\[
- \frac{\partial D_1}{\partial \phi_1} - A'(\phi) = 0
\]

4.2.1.2. Uninformative Advertising Regime

In this section, I derive the equilibrium under uninformative advertising. By uninformative, I mean neither the price nor match value is disclosed in the advertisement
so the consumer has no idea of how much surplus the brand offers. The advertisement, however, does alert the consumer to the existence and intended use of the general class of product sold in the market and this information is useful when it is costly for consumers to acquire the information on their own.

With low product discovery costs, consumers may become aware of the product’s existence without receiving an advertisement. The expected demand from consumers who receive an advertisement is identical to the demand from consumers who do not receive any advertisement since these advertisements do not contain any information relevant to the brand selection decision. Whenever discovery cost is prohibitively high, consumers will not be aware of the existence of the product unless an advertisement from at least one of the firms. Consumers who receive uninformative advertisements behave in an identical manner to the set of uninformed consumer outlined Proposition 2 for \( n = 2 \). The firm’s expected demand under high discovery costs is similar to the demand under low discovery cost except for the fact that only consumers who receive at least one advertisement participates in the market. The expected demand is given by

\[
D_1 = \left[ 1 - (1 - \phi_1)(1 - \phi) \right] \left\{ \frac{1}{2} (1 + F(\hat{x}))(1 - F(\hat{x} + p_1 - p)) + \int_a^{\hat{x} + p_1 - p} \frac{F(\varepsilon - p_1 + p) dF(\varepsilon)}{1} \right\} + (1 - d)(1 - \phi_1)(1 - \phi) \left\{ \frac{1}{2} (1 + F(\hat{x}))(1 - F(\hat{x} + p_1 - p)) + \int_a^{\hat{x} + p_1 - p} \frac{F(\varepsilon - p_1 + p) dF(\varepsilon)}{1} \right\}
\]

It can be verified that \( D_1(p_1 = p, \phi_1 = \phi) = \frac{1}{2} [1 - d(1 - \phi)^2] \) and the slope of the demand, evaluated at the equilibrium, is given as

\[
\frac{\partial D_1}{\partial p_1} (p_1 = p, \phi_1 = \phi) = -[1 - d(1 - \phi)^2] \left\{ \frac{1}{2} [1 - F(\hat{x})] . f(\hat{x}) \right\} + \int_a^{\hat{x}} f(\varepsilon)dF(\varepsilon) \}
\]

\(^6\) Uninformative advertisements informs the consumer of the intended use of the product (and that the product exists) but not the search characteristics of the brand.

\(^7\) I thank Bill Neilson for subsidizing my discovery cost by directing me to the Nissan experiment.
Further, since \( \frac{\partial D_1}{\partial \phi_1}(p_1 = p, \phi_1 = \phi) = \frac{1}{3} d(1 - \phi) \), the FOCs for the equilibrium price and advertising level is given as

\[
(4.4) \quad p = \frac{\left[ 1 - d(1 - \phi)^2 \right]}{\left[ 1 - d(1 - \phi)^2 \right] \cdot \left[ 1 - F(\hat{x}) \right] f(\hat{x}) + 2 \int_a^b f(\varepsilon)dF(\varepsilon) } 
\]

\[
(4.5) \quad \frac{1}{3} d(1 - \phi)^2 \left[ 1 - d(1 - \phi)^2 \right] \cdot \left[ 1 - F(\hat{x}) \right] f(\hat{x}) + 2 \int_a^b f(\varepsilon)dF(\varepsilon) } = 0 
\]

4.2.2. Stage 1: Information Content

In this section, I compare the economic rents earned by firms under each advertising regime. Up until now, I have been somewhat general about the functional form of the preference distribution \( F \); any log-concave distribution function is sufficient to establish the existence and uniqueness of the equilibrium strategies in price and advertising level (see Section 3.3). Without loss of generality, therefore, I now assume a specific distribution for the match values and advertising technology.

Assumption 3. Consumer match values are uniformly distributed \( F \) over the interval \([0,1]\).

Assumption 4. The cost of reaching a proportion \( \phi \in [0,1] \) of the population is given as \( A(\phi; \lambda) = -\lambda \log(1 - \phi) \).

With these simplifying assumptions, I now determine the firm’s choice of advertising regime by comparing the equilibrium profits earned under each alternative.
4.2.2.1. Informative Advertising

Based on the previous section, the Propositions 8 and 9 can easily be verified:

\textit{Proposition 8.} For low discovery cost \((d = 0)\), the equilibrium price, advertising intensity and profits are given as:

\begin{align*}
(4.6) \quad p &= (1 + \hat{x})^{-1} \\
(4.7) \quad \phi &= 0 \\
(4.8) \quad \pi^H &= \frac{1}{2}(1 + \hat{x})^{-1}
\end{align*}

I now explain why there is no advertising in this equilibrium. The only reason a firm advertises is to generate additional sales for its brand. It is generally agreed that the increase store traffic comes from two sources. Firstly, advertising can increase the total demand for the product by drawing consumers away from other industries (products). Secondly, advertising might lead to a redistribution of consumers among the various brands within an industry. Informative advertising in my model potentially has both effects. But as I explain, these effects are neutralized by certain parameter restrictions in my model. In this section with low discovery costs, the size of aggregate demand is fixed at 1 (the size of the consumer population) and thus unaffected by advertising levels. Hence the only effect of informative advertising with low discovery cost is to lure consumers away from the rival firm (business stealing). It turns out that for uniform preferences in duopoly markets, the expected demand from consumers informed about a particular brand is identical to the expected demand from consumers not informed about the brand. This means there is no private benefit (to the firm) from advertising and there will be no advertising as long as it is costly to do.

This is however, an artifact of the joint assumptions on the preference distribution and number of firms in the market; another market structure with either more than two firms or a non-uniform preference distribution generates a positive amount of advertising.
**Proposition 9.** For high discovery cost \((d = 1)\), the equilibrium price, advertising intensity and profit are given as:

\[
p_{\text{HM}} = \frac{1}{\phi^2} \left[ 1 - (1 - \phi)^2 \right] \\
\phi_{\text{HM}} = \sqrt[3]{z_1} - \sqrt[3]{z_2} + \frac{4}{3} \\
\pi_{\text{HM}} = \frac{1}{8} \left[ 1 - (1 - \phi)^2 \right] (1 - \phi) + \lambda \log(1 - \phi)
\]

where expressions for \(z_1\) and \(z_2\) are given in the Appendix A.

\[
\frac{1}{\pi^*} \equiv \frac{1}{\lambda} (1 + \hat{x})^{-1}
\]

The equilibrium level of advertising intensity will in general be positive. With high discovery costs, potential consumers do not participate unless they receive an advertisement from at least one of the firms. In essence, therefore, advertising increases the total demand for the product class and is thus utilized in equilibrium.

4.2.2.2. Uninformative Advertising

I now use assumptions 3 and 4 to derive the expected equilibrium profit earned under the uninformative advertising regime. Based on the previous section, Propositions 10 and 11 can easily be verified:

**Proposition 10.** Under low discovery cost \((d = 0)\), the equilibrium price, advertising intensity and profits are given as:

\[
p = (1 + \hat{x})^{-1} \\
\phi = 0 \\
\pi_{\text{UL}} = \frac{1}{\lambda} (1 + \hat{x})^{-1}
\]

It is again apparent it is not optimal to advertise whenever discovery costs are low. Unlike the case for informative advertising, this result is not an artifact of the uniform preference distribution and duopoly assumptions. In my model, uninformative
advertisements are not expected to redistribute consumers among brands since they contain no brand specific information. The only benefit of uninformative advertising is to increase the total demand for the industry. With low discovery costs, total demand is fixed at 1 and so it is not optimal to advertise so long as doing so is costly.

**Proposition 11.** Under high discovery cost, the equilibrium price, advertising intensity and profits are given as:

\begin{align}
    p^{\text{UH}} & = (1 + \hat{x})^{-1} \\
    \phi^{\text{UH}} & = 1 - \sqrt{2\lambda(1 + \hat{x})} \\
    \pi^{\text{UH}} & \equiv \frac{1}{2}(1 + \hat{x})^{-1}(1 - 2\lambda(1 + \hat{x})) + \frac{1}{2}\lambda \log[2\lambda(1 + \hat{x})]
\end{align}

The nature of the alternative advertising regimes is quite transparent when one compares the pricing strategies $p^{\text{IH}}$ and $p^{\text{UH}}$ (equations 4.9 and 4.15 respectively). Equilibrium price under the informative regime is decreasing in level of advertising whereas the equilibrium price under uninformative regime is independent of the level of advertising. The intuition is that the information disclosed in informative advertisements places firms under more direct competition with each other and there is a tendency to under-cut prices. Firms still maintain some market power because of heterogeneity in preferences for the products. Under uninformative advertising, however, the competition between rivals is not as direct since consumers receive no brand specific information and will only know their surplus by visiting the stores. The following results are immediate:

**Result 1.**

i. $\phi^{\text{UH}} \geq \phi^{\text{IH}}$

ii. $p^{\text{UH}} \geq p^{\text{IH}}$

iii. $\pi^{\text{UH}} \geq \pi^{\text{IH}}$

**Proof of Result 1.** (i) The expected increase in demand generated by a marginal increase in advertising intensity is the identical under informative and uninformative advertising
regimes \( \left( \frac{1}{\delta}d(1-\phi) \right) \). Since (ii) shows that the price received under uninformative advertising is at least as great as the price under informative advertising, it implies the marginal benefit of advertising under uninformative advertising is at least as great as the marginal benefit under informative advertising hence \( \phi^{\text{UH}} \geq \phi^{\text{IH}} \). (ii) Suppose not. This means \( p^{\text{UH}} < p^{\text{IH}} \). This implies

\[
(1 + \hat{x})^{-1} < \left[ \phi^2 + \phi(1 - \phi)(1 + \hat{x}) \right]^{-1} \left[ 1 - (1 - \phi)^2 \right]
\]

\[
\Rightarrow 2[\phi^2 + \phi(1 - \phi)(1 + \hat{x})] < (1 + \hat{x})[1 - (1 - \phi)^2].
\]

Replacing \( 1 - (1 - \phi)^2 \equiv \phi^2 + 2\phi(1 - \phi) \), I obtain \( 1 < \hat{x} \), a contradiction since \( \hat{x} < b = 1 \). Hence \( p^{\text{UH}} \geq p^{\text{IH}} \).

(iii) \( \pi^{\text{UH}} - \pi^{\text{IH}} \equiv p^{\text{UH}}D^{\text{UH}} - p^{\text{IH}}D^{\text{IH}} + \lambda[\log(1 - \phi^{\text{UH}}) - \log(1 - \phi^{\text{IH}})] \). By Envelope Theorem, \( \frac{\partial(\pi^{\text{UH}} - \pi^{\text{IH}})}{\partial \lambda} = \log(1 - \phi^{\text{UH}}) - \log(1 - \phi^{\text{IH}}) \leq 0 \) since \( \phi^{\text{UH}} \geq \phi^{\text{IH}} \). But

\[
\lambda \in (0, \overline{\lambda}) \text{ with } \overline{\lambda} \equiv \left[ 2(1 + \hat{x})^{-1} \right].
\]

It is readily verified that \( \left( \pi^{\text{UH}} - \pi^{\text{IH}} \right)_{\lambda = \overline{\lambda}} = 0 \). This implies that \( (\pi^{\text{UH}} - \pi^{\text{IH}}) \geq 0 \) for all \( \lambda < \overline{\lambda} \). \( Q.E.D. \)

If I make the tie-breaking rule that a firm will choose to exclude information if it is otherwise indifferent between including and excluding information, then result (iii) tells me that as long as discovery cost is high, the unique pure strategy advertising equilibrium in my model is \( \{U, p^{\text{UH}}, \phi^{\text{UH}} \} \).

4.3. Equilibrium Properties

Result (iii.) above shows that uninformative advertising regime weakly dominates informative advertising regimes. This finding is consistent with the preponderance of empirical studies that find most advertisements to be uninformative. Since both informative and uninformative advertisements are observed empirically, I relax the tie-breaking rule to further explore the model when result (iii) holds with strict equality; this occurs when the advertising technology is least efficient. The results above show firms never advertise whenever discovery cost is low \( \phi^{\text{UL}} = \phi^{\text{UL}} = 0 \). I subsequently restrict my analysis to the case when discovery cost is high \( (d = 1) \). I now
present the following comparative static results that may be readily verified using expressions in Proposition 11:

**Result 2.**

iv. \( \frac{\partial \phi^{UH}}{\partial c} > 0, \frac{\partial \phi^{UH}}{\partial \lambda} < 0 \)

v. \( \frac{\partial p^{UH}}{\partial c} > 0, \frac{\partial p^{UH}}{\partial \lambda} = 0 \)

vi. \( \frac{\partial \pi^{UH}}{\partial c} > 0, \frac{\partial \pi^{UH}}{\partial \lambda} < 0 \)

4.3.1. Least Efficient Advertising Technology (LEAT)

In this section, I focus on key equilibrium patterns in my model as \( \lambda \to \overline{\lambda} \equiv \frac{1}{2}(1 + \hat{x})^{-1} \). I single out this extreme case because it is the only instance in which my model predicts the use of both regimes in equilibrium, i.e., there is a mixed strategy equilibrium in advertising regimes. I start with the equilibrium strategies under the uninformative advertising regime. Using Proposition 11, \( \phi^{UH} \to 0 \) as \( \lambda \to \overline{\lambda} \). Result 1 (i.) shows it must also be true that \( \phi^{III} \to 0 \). Using equation (4.9), by L’Hopital’s Rule

\[
p^{III} = \frac{\frac{1}{2} \left[ 1 - (1 - \phi)^2 \right]}{\phi^2 + \phi(1 - \phi)(1 + \hat{x})} \to \frac{1}{(1 + \hat{x})} = p^{UH} \text{ as } \phi \to 0
\]

The above expression shows that under LEAT, the price strategy is identical under each advertising regime. To conclude the characterization of equilibrium for this special case, I need only compare that FOC for advertising intensity under each regime and note that they are identical for \( p^{III} = p^{UH} \). This implies that \( \phi^{UH} \to \phi^{III} \) as \( \lambda \to \overline{\lambda} \).

This shows that only an extremely small number of consumers observe advertisements with this least efficient advertising technology in a highly concentrated
market. Since I am considering high discovery cost, this also implies only these consumers participate in the market. The following results may also be readily verified:

**Result 3.** As \( \lambda \to \bar{\lambda} \equiv \frac{1}{2}(1 + \hat{x})^{-1}, \)

vii. \( (\pi^{\text{UH}} - \pi^{\text{HI}}) \to 0 \)

viii. \( \pi^{\text{UH}} \to 0, \pi^{\text{HI}} \to 0 \)

ix. \( r_2^{\text{II}} = r_3^{\text{I}} \to 1 \)

(x) \( r_4^{\text{II}} = r_5^{\text{I}} \to \frac{1}{2} \)

Results (ix) and (x) track very important accounting ratios. The advertising: profit ratio (result ix) shows that for every dollar paid out in advertising expenditure, one dollar accrues to the firm as profit. The advertising to sales ratio (result x) shows that for every dollar paid out to advertise the product, two dollars are generated in sales revenue.

The analyses in the previous sections allow me to derive empirically testable hypothesis that I confront with observed events in the automobile market as well as data from an empirical study of information content in television advertising by Resnik and Stern (1977).

4.4. The Infiniti Experiment

From Section 4.1, it is clear uninformative advertisements are only used by firms whenever the discovery cost associated with the product is high (Propositions 8 through 11). A discussion of the factors that determine the magnitude of the discovery cost is outside the scope of this paper. However, since uninformative advertisements essentially inform consumers to the existence of the brand, it is reasonable to assume that the discovery costs associated with established (known) brands are relatively low where as the discovery costs associated with newly introduced brands are relatively high.

With this in mind, limited support of our theoretical description of the importance of discovery costs on information content of advertisements is provided by
the advertising campaign used by the Nissan Corporation to introduce the *Infiniti* model in the United States.

An account of this campaign strategy is provided in Sternthal (2005). The automobile industry provides a reasonable setting (but not ideal, as discussed later) to test the usefulness of the model since consumers typically purchase only one car (unit demand) and the automobile industry is somewhat concentrated.

In 1990, Nissan introduced the Infiniti automobile to the United States. The Infiniti was a luxury vehicle designed to compete with known (established) luxury model of *Mercedes-Benz*. At the same time, the Toyota Corporation also introduced the *Lexus* model as its luxury line. Toyota’s advertisements provided consumers with images, features and availability of the Lexus. Nissan’s strategy was unconventional in the sense that no information, other than the availability of the Infiniti, is conveyed in any of the advertisements; not even an image of the Infiniti appeared in the television advertisements. The advertising campaign was more successful for Nissan in the sense that consumers found the advertisements to be more memorable than advertisements for the Lexus. Despite the appeal of the uninformative advertisements, the Lexus sold more units. The appeal of the uninformative advertisements was lost very shortly after the cars were introduced and Nissan eventually began to produce informative advertisements.\(^{10}\) The difference between the advertising strategies used to launch the luxury models provides a natural experiment to assess the predictions of the model as it points out how the information content in advertisements is affected by changes in discovery costs.

### 4.4.1. Explained Results

I now relate these events to the theoretical model outlined in the previous sections. Toyota used informative advertisements while Nissan’s used uninformative advertisements. It is clear from Section 4.3 that uninformative advertisements are used

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\(^{10}\) Sternthal (2005) shows that Infiniti’s advertisements were ranked as third most memorable at the time the models were about to be introduced but slipped to ten most memorable a few months after introduction.
when discovery cost is high and only informative advertisements are observed whenever discovery cost is low. A discussion of the determinants of discovery cost is outside the scope of this paper. However, it seems reasonable to conjecture that a product’s life cycle is a determinant of the discovery cost associated with the product; all other things constant, the discovery cost for product should be higher earlier in its life cycle (newly introduced) than later in its life cycle (establish product).\(^{11}\) It is argued therefore, that the uninformative advertisements which launched the Infiniti were used when discovery cost was high; consistent with model predictions. After a few months, Nissan switched to an informative advertising regime. This is also consistent with the model since the Infiniti would no longer be considered a new good after a few months in the sense that word-of-mouth from person who viewed the initial advertisements would lower the discovery costs for consumers who did not view the initial advertising campaign.

4.4.2. Unexplained Results

There is, however, at least one inconsistency with the Infiniti experiment and the theoretical model. It is shown in Section 4.3 that the use of an uninformative advertising strategy is not dominated by the use of a fully informative strategy. The model therefore can not explain why Lexus sold more units that Infiniti and why Nissan switched to informative advertising. I can think of at least two reasons for the divergence between the predictions and the empirical results. The first reason might be advertisements are not fully informative in the automobile industry given that “test drives” are a prominent feature in marketing cars. The use of “test drives” by dealers indicates that advertisements do not fully capture the likely driving experience of a given model vehicle. This study does not allow for the transmission of partial information and it could very well be that advertisements with partial information dominate uninformative advertisements.

\(^{11}\) The discovery cost for existing goods may be low because consumers may learn of these goods through word-of-mouth from other consumers who may have purchased the goods. All other things constant, discovery costs for new products will be higher since the good is yet to be consumed by anyone, by definition, and so the word-of-mouth option is not available.
A second reason for the divergence between the theoretical model and empirical results is that Lexus and Infiniti might not be purely horizontally differentiated as assumed in the model, i.e., consumers may perceive Lexus to be a higher quality luxury vehicle. In theoretical models of vertical differentiation, the higher quality product always earns greater rents and this would explain why Lexus out-performs Infiniti.\footnote{For studies in pure vertical differentiation, see Shaked and Sutton (1982), Lehmann-Grube (1997) and Vandenbosch and Weinberg (1995). For studies with both horizontally and vertically differentiated products, see Economides (1989, 1993), Neven and Thisse (1989) and Degryse and Irmen (2001).}

4.5. The Usefulness of Content Analysis

In what follows, I confront my theoretical model with an empirical study of the informativeness of television commercials by Resnik and Stern (1977). Data were collected during April 1975. A total of 378 commercials from the three major television networks (ABC, CBS, NBC) were recorded. The information content of each commercial is evaluated using fourteen criteria reflecting product characteristics. The results of their study are reproduced in Table 1.

4.5.1. Data Issues

There are a couple of reasons it might be inappropriate to use the data in Resnik and Stern (1977) to evaluate the relevance of my theoretical model. The first concern is the class of products that are advertised. In my model, I assume consumers have discrete inelastic individual demand. This means consumers purchased one unit of the good and select only one brand to consume. This form of demand is not associated with any of the product categories listed in the table. The representative consumer and the CES models of individual demand that allow more that one brand to be purchased and consumed in variable units appears to more appropriately represent the product classes listed in the table.
TABLE 1
Informativeness Based on Advertisement Content

<table>
<thead>
<tr>
<th>Sample/ Condition</th>
<th>Number Evaluated</th>
<th>Informative Advertisements (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Sample of Advertisements</td>
<td>378</td>
<td>49.2%</td>
</tr>
<tr>
<td>Total Weekday Advertisements</td>
<td>189</td>
<td>50.3%</td>
</tr>
<tr>
<td>Weekday Morning Advertisements</td>
<td>63</td>
<td>57.1%</td>
</tr>
<tr>
<td>Weekday Afternoon Advertisements</td>
<td>63</td>
<td>33.3%</td>
</tr>
<tr>
<td>Weekday Evening Advertisements</td>
<td>63</td>
<td>60.3%</td>
</tr>
<tr>
<td>Total Weekend Advertisements</td>
<td>189</td>
<td>48%</td>
</tr>
<tr>
<td>Weekend Morning Advertisements</td>
<td>63</td>
<td>34.9%</td>
</tr>
<tr>
<td>Weekend Afternoon Advertisements</td>
<td>63</td>
<td>49.2%</td>
</tr>
<tr>
<td>Weekend Evening Advertisements</td>
<td>63</td>
<td>60.3%</td>
</tr>
<tr>
<td>Total Morning Advertisements</td>
<td>126</td>
<td>46%</td>
</tr>
<tr>
<td>Total Afternoon Advertisements</td>
<td>126</td>
<td>41.3%</td>
</tr>
<tr>
<td>Total Evening Advertisements</td>
<td>126</td>
<td>60.3%</td>
</tr>
<tr>
<td>Food Advertisements</td>
<td>144</td>
<td>45.8%</td>
</tr>
<tr>
<td>Institutional Advertisements</td>
<td>24</td>
<td>75%</td>
</tr>
<tr>
<td>Personal Care Product Advertisements</td>
<td>93</td>
<td>39.8%</td>
</tr>
<tr>
<td>Laundry and Household Product Advertisements</td>
<td>52</td>
<td>46.2%</td>
</tr>
<tr>
<td>Hobby, Toy and Transportation Advertisements</td>
<td>29</td>
<td>69%</td>
</tr>
<tr>
<td>“Other” Product Advertisement</td>
<td>36</td>
<td>58.3%</td>
</tr>
</tbody>
</table>

Source: Resnik and Stern (1977, Exhibit 2).
Note: Television commercials are classified by Resnik and Stern as informative if they contained at least one of the following fourteen (14) informational cues: price or value, quality, performance, components or contents, availability, special offers, taste, nutrition, packaging or shape, guarantees or warranties, safety, independent research, company research or new ideas.
However, an important result in Anderson, de Palma and Thisse (1992) suggests that my model predictions are robust to the specification of individual consumer demand. They show under certain conditions (which are satisfied in my specification), discrete individual demand (which I explicitly model) and CES individual demand yield identical aggregated demand functions. Since my results are based on the aggregate demand and not individual demand, I expect them to be robust to the modeling of individual consumer demand.

A second concern is my model is derived for industries with two single product firms. A look at Table 1 suggests that this might not be the case for the product classes reported in the study. The product list consists of industries with more than two rivals and multi-product firms with a given industry. This is especially true for Food and Personal Care Products. My hope is these industries are highly concentrated with a couple of dominant players despite having many firms. A good example of such an industry is the Cola market that has many firms but dominated primarily by Pepsi and Coke. With these caveats in mind, I now confront the data.

In the pursuant discussion, I assume my theoretical model accurately describes the actual markets from which the sample is drawn. I attempt to reconcile the observed pattern in the sample with my model in one of two ways. In the first instance, I highlight a classification scheme that could have induced bias in the sample. I then discuss the nature of the bias and show that the sample becomes more consistent with the predictions of my theoretical model as the bias is corrected. In the second instance, I assume the sample is indeed measured without bias and impute the parameter restrictions in my model that are required to generate such a sample.

A prediction of my model is that uninformative advertising weakly dominates informative advertising. That is, when firms decide to advertise, the expected rents earned from uninformative advertising exceed the rents from informative advertising when advertising technology is sufficiently efficient and expected rents are the same under both regimes when the advertising technology is sufficiently inefficient. My
model then predicts that uninformative advertisements should be most prevalent in the sample.

The information in Table 1 shows that 49.2 percent of the commercials in the sample were evaluated as informative. Such a big proportion of informative commercials arouses concerns since it lends little support to my prediction that firms weakly prefer uninformative commercials to uninformative ones. However, my concerns are lifted upon examining how the commercials in Table 1 were classified. In Table 1, a commercial is determined to be informative if it included at least one of the fourteen information cues used by consumers in making purchasing decision (the information cues are listed directly below Table 1). In my model, fully informative commercials would be those with more than one cue and so the sample in Table 1 might have an upward bias. Resnik and Stern (1997) point out that if the criterion for informativeness was changed to having at least two information cues, the proportion of informed commercials would be only 16 percent. Further, if the criterion were for the commercial to have at least three information cues, then only 1 percent of the commercials would be evaluated as informative. These lower percentages provide stronger support for my model.

My model does not allow partial information to be transmitted in commercials so I can not definitively argue whether partially informative commercials should be classified as either informative or uninformative. However, grouping them as uninformative seems to be more consistent with the prediction of my model. Indeed, I can see how partial product information might be of no value to consumers. In my model, sending only price information in commercials is of no informative value since consumers rationally anticipate this price in equilibrium. In that sense a commercial that reveals only the price would be classified as uninformative in my model.

Let us set aside the potential data measurement error in Table 1 and assume that the informativeness of commercials is accurately captured. I now show that the data presented in Table 1 is consistent features of my model under specific market parameters. My analysis in Section 4.2 shows that the expected profits are identical
under each advertising regime once the advertising technology is least efficient. Indifference between the two regimes means that I would expect one half of the sample to be informative commercials, conditional on using a least efficient advertising technology. Thus the sample value of 49.2 percent is remarkably close to the predictions of my model. If I trust that my model is correct, therefore, I must infer that television was a highly inefficient means of advertising in April 1975.

Despite the fact that 49.2 percent of all commercials in Table 1 are informative, there is a high variability across the six product class categories ranging from a low of 39.8 percent for Personal Care Products to a high of 75 percent for Institutional services. As pointed out, this variability might be induced by small sample biases within a couple of the categories. The general rule of thumb in statistics is a sample size of at least 30 is needed to avoid small sample bias. Once you ignore the results for Institutional category (with a sample of 24 commercials) and Hobby, Toy and Transportation category (with a sample size of 29), the proportion of informative commercials in the other four product classes are all within 10 percentage points of the hypothesized number of 50 percent.

While the variability in the proportion of informative commercials might just be reflecting sample biases in the data, variability in the number of commercials within each product class might have greater economic significance. The equilibrium advertising intensity in equation (4.16) is found to be dependent on both the advertising and search cost parameters. Since the advertising technology is constant in the sample (only television advertisements are considered), observed variability in the sample must be reflecting differences in the search costs across the product classes. I showed in result (iv.) that the equilibrium advertising intensity increases with search costs. I can therefore use this information to rank the relative cost of searching among different brands among the different product classes. My ranking of search cost, from highest to lowest would therefore be: (i.)Food, (ii.)Personal Care, (iii.)Laundry and Household Product, (iv.)Hobby, Toy and Transportation, (v.)“other” products (vi.) and Institutional.

Of course, it would be preferable to have an independent measure of the relative search costs within the product classes. Failing that, if I use “time spent shopping” as a
proxy for search costs then my results seems reasonable; individuals spend more time shopping for food and personal care items than say laundry and household products.

I have presented, two competing ways in which one could reconcile my model with the empirical study of Resnik and Stern (1977). The first implies that television is a relatively inefficient medium of information dissemination. While this seems plausible given that many homes might not have had access to television in the mid 1970s, I find this implication less tenable for the following reason. Stern and Resnik (1991) replicated their earlier study and classify the information content of television viewed in 1986. They conclude there is no statistical difference in the proportion of informative commercials reported in the 1975 and 1986 samples; this would imply that television was just as inefficient in 1986 as in 1975. There have been rapid technological advancements in consumer electronic devices since the 1970s. The commensurate free fall in the price of Personal Computers, Televisions, Radios and other electronic devices, means that many consumers would have owned television sets by the mid 1980s. It is unlikely that television would remain such an inefficient means of disseminating information to consumers in 1986.

I therefore find it more likely that the informativeness of television commercials was inaccurately measured in the empirical studies. Indeed it seems likely that consumer’s surplus from a given product would depend on more than one dimension in the product’s characteristics space.

I argue that discovery cost should also be considered when evaluating the informative value of advertisements. This should complement extant studies in the marketing literature that have focused on advertising content.

4.6. Conclusion

Previous theoretical studies in advertising have considered either advertising level or the content of advertising; this is the first paper to consider both dimensions. The results from this research shed some light on widespread use of seemingly uninformative advertisements in highly concentrated markets. I have demonstrated the
use of advertisements which do not provide any brand-specific information conveys other valuable information to consumers whenever discovery cost is sufficiently high. I have shown distributing advertisements without any product specific information weakly dominate those with complete brand specific information. This supports the empirical observations that informative advertisements are not as widely used as uninformative ones.

The model offers predictions about the information content of advertisements that I confront with empirical data from highly concentrated markets. Firstly, my study shows uninformative advertisements will never be used in duopolies with low discovery costs. Secondly, it shows that whenever informative advertisements are observed in markets with highly inefficient advertising technology, the advertising expenditure equals profits.

One obvious limitation of the study in this chapter is the restricted attention to duopoly market structures with respect to incentives to provide brand specific information; it would be interesting to see whether these results are relevant in less concentrated markets. Another shortcoming is the discrete nature in which the information is treated in this chapter. I assume that the information disclosed in advertisements provides the consumer with either the complete (brand specific) information he requires or no information at all. Another extension of this research would model situations in which firms disclose partial information in their advertisements, consistent with observed practices and the research of Anderson and Renault (2004). Notwithstanding, the chapter represents a useful first step at analyzing the incentives of oligopolistic firms to exclude product specific information from advertisements without influencing consumer preferences.
CHAPTER V
CONCLUSION

Decision-making is the central theme of any economic analysis. Although most analyses focus on explaining how agents behave in certain environments (deriving equilibrium strategies), the information used by these agents in the decision-making process is of no less importance. The obvious question then becomes how to model information as an intermediate good generated by the market. In this dissertation, I argue there are two means through which information flows in a market: information dissemination and information acquisition.

Information dissemination reflects the fact that there are incentives for those with the relevant information to provide it to uninformed agents. Similarly, information acquisition reflects the fact that there are incentives for uninformed agents to take steps to gather the relevant information relevant to the decision-making process. One needs only to think of information dissemination as the supply of information and information acquisition as the demand for information to realize that a market characterized by imperfect information can not be fully understood unless both channels of information are explicitly considered. To demonstrate this point, I analyze a differentiated product market with imperfectly informed consumers.

The main result of chapter III reconciles the modern view of advertising with the theory of monopolistic competition. I show advertising can be a pro-competitive tool that increases the substitutability among products while allowing firms maintain market power in the limit as the number of firms becomes arbitrarily large. This is an important contribution to the debate on the role of advertising. I have shown under informative advertising, firms maintain market power in the limit if the initial effort to disseminate information is costly whereas firms will lose all market power if this initial effort is costless. Hopefully, this chapter has now moved the discussion of the source of market power in monopolistically competitive markets from the role of advertising (informative vs. persuasive) to the nature of the advertising technology.
Chapter III has also highlighted the dominant role played by advertising technology in determining the equilibrium market outcome. I have shown this asymmetric effect is robust consumer search and heterogeneity in product characteristics.

In Chapter IV, I shed some light on widespread use of seemingly uninformative advertisements in highly concentrated markets. I demonstrate that the use of advertisements which do not provide any brand-specific information conveys other valuable information to consumers whenever discovery cost for the product is sufficiently high. The true role (information value) of these seemingly uninformative advertisements is only revealed by modeling information acquisition.

One obvious limitation of this study is the discrete nature of the information disseminated to consumers. I assume that the information disclosed in advertisements provides the consumer with either the complete information or no information at all. A useful extension of this research would model situations in which firms disclose partial information in their advertisements, consistent with observed practices and the research of Anderson and Renault (2004). Notwithstanding, the research should convince economists that both channels of information transmission should be explicitly modeled whenever attempting to explain any feature within markets characterized by imperfect information.
REFERENCES


APPENDIX A

Lemma A1. \[ \sum_{r=1}^{n-1} L_r^r \phi_r^{-1} (1-\phi)^{n-r-1} \cdot \frac{\phi}{r} = \frac{1}{n(1-\phi)} \left[ 1 - (1-\phi)^n - \phi^n \right] \]

Proof of Lemma A1.

\[ \sum_{r=1}^{n-1} L_r^r \phi_r^{-1} (1-\phi)^{n-r} \cdot \frac{\phi}{r} = \sum_{r=1}^{n-1} \left( \frac{n-1}{r} \right) \frac{1}{r n} \phi'(1-\phi)^{n-r} \]

\[ = \frac{1}{n} \sum_{r=1}^{n-1} \phi'(1-\phi)^{n-r} \]

\[ = \frac{1}{n} \left[ 1 - (1-\phi)^n - \phi^n \right] \]

Divide both sides by \((1-\phi)\) to complete the proof. \( Q.E.D. \)

Lemma A2.

\[ \sum_{r=1}^{n-1} \phi_r^{-1} (1-\phi)^{n-r-1} L_r^r D_r^r = \frac{1}{n \phi(1-\phi)} \left\{ (1-\phi)^n - \left[ \phi F(\hat{x} + c) + (1-\phi) \right]^p + \left[ \phi F(\hat{x} + c) \right]^p \right\} \]

Proof of Lemma A2.

\[ \sum_{r=1}^{n-1} \phi_r^{-1} (1-\phi)^{n-r} \]

\[ = \frac{1}{n} \sum_{r=1}^{n-1} \left( \frac{n-1}{r} \right) \left[ \int_{\hat{x} + c}^{b} F_r^{-1}(\varepsilon) dF(\varepsilon) + \int_{a}^{\hat{x} + c} F_r^{-1}(\varepsilon) \cdot F_r^{-1}(\varepsilon - c) dF(\varepsilon) \right] \]

Since \( F_r^{-1}(\varepsilon) \cdot F_r^{-1}(\varepsilon - c) \leq F_r^{-1}(\varepsilon) \cdot F_r^{-1}(\varepsilon) \rightarrow 0 \forall \varepsilon \in [a, \hat{x} + c] \) as \( n \rightarrow \infty \)

\[ \sum_{r=1}^{n-1} \phi_r^{-1} (1-\phi)^{n-r-1} L_r^r D_r^r = \sum_{r=1}^{n-1} \phi_r^{-1} (1-\phi)^{n-r-1} \left[ \int_{\hat{x} + c}^{b} F_r^{-1}(\varepsilon) dF(\varepsilon) \right] \]

\[ = \int_{\hat{x} + c}^{b} \frac{1}{n} \sum_{r=1}^{n-1} \phi_r^{-1} (1-\phi)^{n-r-1} \left[ \int_{\hat{x} + c}^{b} F_r^{-1}(\varepsilon) dF(\varepsilon) \right] \]

\[ = \int_{\hat{x} + c}^{b} \frac{1}{n} \sum_{r=1}^{n-1} \left[ F(\varepsilon) \phi \right]^{r-1} (1-\phi)^{n-r-1} dF(\varepsilon) \]
\[
\begin{align*}
&= \frac{1}{(1-\phi)} \int_{\hat{x}+c}^{b} \left( \phi F(\varepsilon) + (1-\phi) \right)^{n-1} - \left[ \phi F(\varepsilon) \right]^{n-1} dF(\varepsilon) \quad \text{(by Lemma A3)} \\
&= \frac{1}{n\phi(1-\phi)} \left[ \phi F(\varepsilon) + (1-\phi) \right]^n - \left[ \phi F(\varepsilon) \right]^n \bigg|_{\hat{x}+c}^{b} \\
&= \frac{1}{n\phi(1-\phi)} \left\{ 1 - \phi^n - \left[ \phi F(\hat{x}+c) + (1-\phi) \right]^n + \left[ \phi F(\hat{x}+c) \right]^n \right\}. \quad Q.E.D.
\end{align*}
\]

Lemma A3. \[ \sum_{r=1}^{n-1} \binom{n-1}{r-1} (h\phi)^{r-1} (1-\phi)^{n-r-1} = \frac{1}{(1-\phi)} \left[ h\phi + (1-\phi) \right]^{n-1} - (h\phi)^{n-1} \]

Proof of Lemma A3.
\[ \sum_{r=1}^{n-1} \binom{n-1}{r-1} (h\phi)^{r-1} (1-\phi)^{n-r} = \binom{n-1}{0} (h\phi)^0 (1-\phi)^{n-1} + \cdots + \binom{n-1}{n-1} (h\phi)^{n-1} (1-\phi)^0 \\
= [h\phi + (1-\phi)]^{n-1} \]

This implies \[ \sum_{r=1}^{n-1} \binom{n-1}{r-1} (h\phi)^{r-1} (1-\phi)^{n-r} = [h\phi + (1-\phi)]^{n-1} - (h\phi)^{n-1} \]

Divide both sides by \((1-\phi)\) to complete the proof. \quad Q.E.D.

Proof of Proposition 4.
\[ \frac{\partial D_a}{\partial p_i} (p_1, p) = -L_a \left\{ \binom{h}{a} (n-1) F^{n-2} (\varepsilon - p_1 + p) \cdot f(\varepsilon - p_1 + p) dF(\varepsilon) \right\} \]
\[ - \sum_{r=1}^{n-1} L_r \left\{ F^{r-1} (\hat{x}+c) \cdot f(\hat{x} + p_1 - p + c) \right\} \]
\[ - \sum_{r=1}^{n-1} L_r \left\{ (-1)^r r F^{r-2} (\varepsilon - p_1 + p) \cdot f(\varepsilon - p_1 + p) dF(\varepsilon) \right\} \]
\[ + \sum_{r=1}^{n-1} L_r \left\{ F^{r-1} (\hat{x}+c) \cdot F^{n-r} (\hat{x}) \cdot f(\hat{x} + p_1 - p + c) \right\} \]
Evaluating the slope of the demand curve at $p_1 = p$ gives

\[
\frac{\partial D_\varepsilon}{\partial p_1}(p, p) = -L_{n \varepsilon} \left\{ \int_a^n (n - 1) F^{n-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b F^{r-1}(\varepsilon) \cdot F^{n-r-1}(\varepsilon - c) f(\varepsilon - p_1 + p - c)dF(\varepsilon) \right\}
\]

\[
+ \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

\[
- \sum_{r=1}^{n-1} \int \frac{r}{n-r} \left\{ \int_a^b (r - 1) F^{r-2}(\varepsilon) \cdot f(\varepsilon)dF(\varepsilon) \right\}
\]

But
(A2) \[ \int_{\hat{c}}^{b} F^{r-1}(\varepsilon) \cdot f'(\varepsilon) d\varepsilon = F^{r-1}(\varepsilon) \cdot f(\varepsilon)|_{\hat{c}}^{b} - \int_{\hat{c}}^{(r-1)F^{r-2}(\varepsilon)} f(\varepsilon) d\varepsilon \]

(A3) \[ \int_{a}^{\hat{c}} F^{r-1}(\varepsilon) \cdot F^{n-r}(\varepsilon - c) \cdot f'(\varepsilon) d\varepsilon \]

= \[ F^{r-1}(\varepsilon) \cdot F^{n-r}(\varepsilon - c) \cdot f(\varepsilon)|_{\hat{c}}^{b} - \int_{\hat{c}}^{(r-1)F^{r-2}(\varepsilon)} F^{n-r}(\varepsilon - c) \cdot f(\varepsilon - c) d\varepsilon \]

(A4) \[ \int_{a}^{\hat{c}} F^{n-1}(\varepsilon) \cdot f'(\varepsilon) d\varepsilon = F^{n-1}(\varepsilon) \cdot f(\varepsilon)|_{a}^{\hat{c}} - \int_{a}^{(n-1)F^{n-2}(\varepsilon)} f(\varepsilon) d\varepsilon \]

Substituting the expressions for (A2), (A3) and (A4) in (A1) gives

(A5) \[ \frac{\partial D_{1}}{\partial p_{1}}(p, p) \]

= \[-L_{n} \left\{ \int_{a}^{b} (n-1)F^{n-2}(\varepsilon) \cdot f(\varepsilon) dF(\varepsilon) \right\} - \sum_{r=1}^{n-1} L_{r} \left\{ f(b) - \int_{\hat{c}}^{(r-1)F^{r-2}(\varepsilon)} F^{r-1}(\varepsilon) \cdot f'(\varepsilon) d\varepsilon \right\} \]

+ \sum_{r=1}^{n-1} L_{r} \left\{ \int_{a}^{\hat{c}} F^{r-1}(\varepsilon) \cdot F^{n-r}(\varepsilon - c) \cdot f'(\varepsilon) d\varepsilon \right\} \]

- \sum_{r=1}^{n-1} L_{r} \left\{ \frac{r}{n-r} \int_{a}^{\hat{c}} F^{r-1}(\varepsilon) \cdot F^{n-r-1}(\varepsilon - c) f(\varepsilon - c) dF(\varepsilon) \right\} \]

- \left\{ \frac{f(\varepsilon) 1 - F^{n}(\hat{x})}{n} - \frac{i}{1 - F(\hat{x})} + \int_{a}^{\hat{c}} F^{n-1}(\varepsilon) \cdot f'(\varepsilon) d\varepsilon \right\} \]

Now imposing uniform preferences,

(A6) \[ \frac{\partial D_{1}}{\partial p_{1}}(p, p) = -L_{n} \left\{ \int_{a}^{b} (n-1)F^{n-2}(\varepsilon) \cdot f(\varepsilon) d\varepsilon \right\} \]

- \sum_{r=1}^{n-1} L_{r} \left\{ |1| \right\} - \sum_{r=1}^{n-1} L_{r} \left\{ \frac{r}{n-r} \int_{a}^{\hat{c}} F^{r-1}(\varepsilon) \cdot F^{n-r-1}(\varepsilon - c) dF(\varepsilon) \right\} \]

- \left\{ \frac{1}{n} \left( 1 - F^{n}(\hat{x}) \right) \right\} \]

- \left\{ \frac{i}{1 - F(\hat{x})} + \int_{a}^{\hat{c}} F^{n-1}(\varepsilon) \cdot f'(\varepsilon) d\varepsilon \right\} \]
\[
= - \left[ L_n + \sum_{r=1}^{n-1} L_r^+ + \sum_{r=1}^{n-1} L_r^- \left\{ \frac{r}{n-r} \right\} \left[ \int F^{r-1}(\varepsilon) \cdot F^{n-r-1}(\varepsilon - c) dF(\varepsilon) \right] + L_0 \left\{ \frac{1}{n} \left( 1 - F^n(\chi) \right) \right\} \right]
\]

Q.E.D.

**Proof of Proposition 7.**

(i)

\[ F^{r-1}(\varepsilon) \cdot F^{n-r}(\varepsilon - c) \leq F^{r-1}(\varepsilon) \cdot F^{n-r}(\varepsilon) . \]

\[ = F^{n-1}(\varepsilon) \to 0 \text{ as } n \to \infty, \forall \varepsilon \in [a, \hat{x} + c] . \]

Using this result, by Proposition 4

\[ \frac{\partial D_1}{\partial p_1} = - \left[ L_n + \sum_{r=1}^{n-1} L_r^+ + L_0 \frac{1}{n} \frac{1}{(1 - F(\hat{x}))} \right] . \]

By Lemma 3 and Proposition 6,

\[ n \frac{\partial D_1}{\partial p_1} = - \left[ n\phi + \frac{(1 - \phi)^n}{(1 - F(\hat{x}))} \right] \to -n\phi \text{ as } n \to \infty . \]

(ii)

\[ \frac{\partial D}{\partial \phi_i}(p, p, \phi, \phi; c) = \phi^{n-1} \cdot D^f(p, p) - (1 - \phi)^{n-1} D^u(p, p) \]

\[ + \sum_{r=1}^{n-1} \frac{\phi^{r-1}}{\phi^r} (1 - \phi)^{n-r} L_r^+ \cdot D_r^+(p, p) - \phi^r (1 - \phi)^{n-r-1} L_r^- D_r^-(p, p) \]

Since \[ D^f(p, p) = D^u(p, p) = \frac{1}{n} , \]

\[ \frac{\partial D}{\partial \phi_i}(p, p, \phi, \phi; c) = \phi^{n-1} \cdot \frac{1}{n} - (1 - \phi)^{n-1} \cdot \frac{1}{n} \]

\[ + \sum_{r=1}^{n-1} \frac{\phi^{r-1}}{\phi^r} (1 - \phi)^{n-r-1} \left[ L_r^+ D_r^+ (p, p) - \phi \left[ L_r^- D_r^- (p, p) + L_r^+ D_r^+(p, p) \right] \right] \]

Noting that \[ L_r^- = \frac{n-r}{r} L_r^+ \text{ (by Assumption 2)} , \]

\[ \frac{\partial D}{\partial \phi_i}(p, p, \phi, \phi; c) = \phi^{n-1} \cdot \frac{1}{n} - (1 - \phi)^{n-1} \cdot \frac{1}{n} \]
\[ + \sum_{r=1}^{n-1} \phi^{-r+1} (1 - \phi)^{n-r-1} \cdot L_r \left[ D_r^+(p,p) - \phi \left\{ D_r^+(p,p) + \frac{n-r}{r} D_r^-(p,p) \right\} \right] \]

It is easily verified that \( D_r^+ + \frac{n-r}{r} D_r^- = \frac{1}{r} \), so

\[
\frac{\partial D}{\partial \phi_1} (p, p, \phi, \phi; c) = \phi^{-n} \cdot \frac{1}{n} - (1 - \phi)^{-n} \cdot \frac{1}{n} + \sum_{r=1}^{n-1} \phi^{-r+1} (1 - \phi)^{n-r-1} \cdot L_r D_r^+(p,p) \\
+ \sum_{r=1}^{n-1} \phi^{-r+1} (1 - \phi)^{n-r-1} \phi \]

By Lemmas A1 and A2,

\[
\frac{\partial D}{\partial \phi_1} = \frac{1}{n} \left[ \phi^{-n} - (1 - \phi)^{-n} \right] - \frac{1}{n(1 - \phi)} \left[ 1 - (1 - \phi)^{n} - \phi^{n} \right] \\
+ \frac{1}{n\phi(1 - \phi)} \left\{ 1 - \phi^{n} - \left[ \phi F(\hat{x} + c) + (1 - \phi) \right]^{n} + \left[ \phi F(\hat{x} + c) \right]^{n} \right\}
\]

This implies \( \frac{\partial D_1}{\partial \phi_1} \to \frac{1}{n\phi(1 - \phi)} - \frac{1}{n(1 - \phi)} = \frac{1}{n\phi} \cdot Q.E.D. \)

**Proof of Proposition 9.**

The FOC for \( \phi \) when \( d = 1 \) is given as

\[
\frac{1}{2} \frac{1}{\phi^2 + \phi(1 - \phi)(1 + \hat{x})} \cdot \frac{1}{2} (1 - \phi) - \frac{\lambda}{1 - \phi} = 0 . \quad \text{Rearranging gives} \]

\[
\phi^3 - 4\phi^2 + (5 - 4\lambda\hat{x}) + (4\lambda(1 - \hat{x}) - 2) = 0 . \quad \text{To solve this cubic equation, make the substitution} \]

\[
(A7) \quad \phi = y + \frac{4}{3} 
\]

This yields a new equation

\[
y^3 + \alpha y + \beta = 0 \quad \text{with} \quad \alpha \equiv -(\frac{1}{3} + 4\lambda\hat{x}) , \quad \beta \equiv 4\lambda(1 + \hat{x}) - \frac{2}{27} - \frac{16\lambda\hat{x}}{3} . 
\]

The only root I am interested in is given by

\[
y_1 = \sqrt[3]{\frac{1}{z_1}} - \frac{1}{3} \sqrt[3]{z_2} \quad \text{where}
\]
\[ z_1 = \frac{-3\beta \sqrt{3} + \sqrt[4]{4\alpha + 27\beta^2}}{6\sqrt{3}} \quad \text{and} \quad z_2 = \frac{-\beta \sqrt{3} + \sqrt[4]{4\alpha + 27\beta^2}}{6\sqrt{3}} \, . \]

Recover the solution for \( \phi \) by substituting the expression for \( y_i \) into equation (A7). \( \text{Q.E.D.} \)
APPENDIX B

I now show that firm 1’s expected demand from partially informed consumers, given that he anticipates all other firms to charge $p$, is given as

\[
(B1) \quad D'_1(p_1, p) = L_r \left[ \int_{\varepsilon \in E_1, g(\varepsilon)} d\varepsilon + \int_{\varepsilon \in E_2} g(\varepsilon) d\varepsilon \right] + L_r \left[ \frac{r}{n - r} \int_{\varepsilon \in E_3} g(\varepsilon) d\varepsilon \right]
\]

with $\varepsilon = \{\varepsilon_1, \ldots, \varepsilon_n\}$, $\varepsilon_j \in [a, b], j = 1, \ldots, n$ and $g(\varepsilon) = \prod_{j=1}^{n} f(\varepsilon_j)$ is the joint density function of match values.

where,

\[
E_1 = \{\varepsilon : \varepsilon_1 - p_1 > \max_{j \neq 1, j \in M_r^+} (\varepsilon_j - p) \text{ and } \varepsilon_1 - p_1 > \hat{x} - p + c\},
\]

\[
E_2 = \{\varepsilon : \varepsilon_1 - p_1 > \max_{j \neq 1, j \in M_r} (\varepsilon_j - p) \text{ and } \varepsilon_1 - p_1 < \hat{x} - p + c \text{ and } \varepsilon_1 - p_1 - c > \max_{s \in M_r} (\varepsilon_s - p)\},
\]

\[
E_3 = \{\varepsilon : \varepsilon_1 - p > \max_{j \neq k, j \in M_r} (\varepsilon_j - p) \text{ and } \varepsilon_k - p < \hat{x} - p + c \text{ and } \varepsilon_k - p - c < \max_{s \in M_r} \{\varepsilon_1 - p_1, \varepsilon_s - p\}\}
\]

Equation (B1) represents firm 1’s expected demand from consumers who are informed about $r$ products (uninformed about $n - r$). The expected demand from a consumer depends on whether or not the consumer is informed about firm 1’s product.

$M_r^+$ represents the set of products that are advertised to the consumers with the (+) superscript indicating that firm 1 is among the advertised products. $M_r^-$ represents the set of advertised products where the (−) superscript indicates that that firm 1 is not among the advertised products.

The expression in the first set of square brackets represents demand from consumers who are informed about firm 1’s product. The first term in this expression represents the demand from consumers who buy firm 1’s product directly without
searching. This group of consumers is captured in the set $E_1$. The definition of $E_1$ above shows that two criteria must be jointly met for consumers to buy product 1 without searching. These $r$ products are in direct competition with each other since consumers are fully informed about their surplus each. The first criterion ensures that firm 1 is the best offer for these consumers among the products they are informed about. The second criterion follows directly from the consumer’s sequential search strategy (outlined in Section 3.2) which dictates that the consumer buy the advertised product directly if $\varepsilon_i > \hat{x} + p_1 - p + c$.

The set $E_2$ gives the set of consumer types who are informed about product 1 but buys only after searching the other products. The definition of the set $E_2$ shows that three criteria must be jointly met for consumers who are informed about product 1 to purchase the product after searching the other stores. The first criterion states that firm 1 must represent the best offer among the products the consumers are informed about. The second criterion follows from the consumer’s search strategy and ensures that the best offer is sufficiently low that the consumer expects to gain additional surplus from searching the other $(n-r)$ products. Once the consumer decides to search, the consumer will only buy product 1 if the surplus from brand 1, $(\varepsilon_1 - p_1 - c)$ exceeds the surplus from a product searched, $(\varepsilon_s - p)$. The search cost associated with product 1 will only be incurred if the consumer decides to purchase (and hence visit) firm 1. The search cost associated with unadvertised products are sunk at the time of decision since it is incurred before the consumers acquires the information and so is not considered by the consumer. The third criterion stipulates that consumers will only buy product 1 when the surplus from firm 1 (net of search cost) exceeds the surplus of each unadvertised product.

The expression in the second pair of square brackets represents the demand from $L_c$ consumers who are not informed about product 1. Since firm 1 is not among the products advertised to these consumers, only these consumers that search could possibly buy product 1. The set $E_3$ represents the consumer types who reject the best offer from the set of advertised products (brand $k$) and search for a better offer among the
unadvertised brands. The first two conditions in the definition of $E_3$ provide the condition under which search takes place for each of the $r$ advertised brands and the third condition ensures that one of the $(n-r)$ unadvertised brands is purchased. Since there are $r$ advertised products and each of the $(n-r)$ unadvertised brands has an equal chance of being selected, I determine the demand for firm 1 by applying the factor $\frac{r}{n-r}$. 
Kevin Kenton Harriott

Email: kharriot@hotmail.com

Permanent Address: Dept. of Economics, TAMU, College Station, TX 77843-4228

Education

B.Sc. (Statistics), University of the West Indies, Jamaica, (July 1996)

M.Sc. (Economics), University of the West Indies, Jamaica, (July 1998)

Fields of Specialization

Primary: Industrial Organization and Econometrics

Secondary: Human Resource Economics

Professional Experience

Instructor:

-Texas A&M University

  Principles of Microeconomics (Five Semesters)
  Macroeconomic Theory (Summer 2003)
  Money and Banking (Summer 2004)

-University of the West Indies

  Statistical Methods (Summer 1998)
  Principles of Statistics (Summer 1999)
  Probability and Distribution Theory (Summer 2002)

Research Assistant:

  Central Bank of Barbados (Summer 1997)