

CAPITAL CONTROLS AND EXTERNAL DEBT TERM STRUCTURE

A Dissertation

by

EZA GHASSAN AL-ZEIN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2005

Major Subject: Economics

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ABSTRACT

Capital Controls and External Debt Term Structure. (August 2005)

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In my dissertation, I explore the relationship between capital controls and the choice of the maturity structure of external debt in a general equilibrium setup, incorporating explicitly the role of international lenders. I look at specific types of capital controls which take the form of date-specific and maturity-specific reserve requirements on external borrowing. I consider two questions: How is the maturity structure of external debt determined in a world general equilibrium? What are the effects of date- and maturity-specific reserve requirements on the maturity structure of external debt? Can they prevent a bank run?

I develop a simple Diamond-Dybvig-type model with three dates. In the low income countries, banks arise endogenously. There are two short-term bonds and one long-term bond offered by the domestic banks to international lenders. First I look at a simple model where international lending is modeled exogenously. I consider explicitly the maturity composition of capital inflows to a domestic economy. I show that the holdings of both short-term bonds are not differentiated according to date.

Second, I consider international lending behavior explicitly. The world consists of two large open economies: one with high income and one with low income. The high

income countries lend to low income countries. There exist multiple equilibria and some are characterized by relative price indeterminacy.

Third, I discuss date-specific and maturity-specific reserve requirements. In my setup reserve requirements play the role of a tax and the role of providing liquidity for each bond at different dates. I show that they reduce the scope of indeterminacy. In some equilibria, I identify a case in which the reserve requirement rate on the long-term debt must be higher than that on the short-term debt for a tilt towards a longer maturity structure.

Fourth, I introduce the possibility of an unexpected bank run. I show that some specific combination of date-and maturity-specific reserve requirements reduce the vulnerability to bank runs. With regard to the post-bank-run role of international lenders, I show that international lenders may still want to provide new short-term lending to the bank after the occurrence of a bank run.

To Gehad and Raghda, for a dedicated parenthood, and an intellectual friendship,
For teaching me perseverance and unfurling my ambitions
For their continuous love

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CHAPTER I

INTRODUCTION

[...] the right starting point for thinking about capital controls must be on very focused, temporary measures aimed at stemming massive temporary inflows or outflows of debt. (Rogoff, 2002, ¶ 6)

After the financial crises of the 1990s, preventing large speculative and volatile inflows of short-term debt became a major concern. Following the spread of liberalization policies in the 1990's, many developing countries have seen their private and financial sectors accumulate high short term borrowing. The increase in private short-term borrowing resulted in a situation where the private sector became unable to fully service the debt without new borrowing. This phenomenon is identified as short-term over-borrowing by the private sector. With high inherited short-term debt, any new borrowing becomes costlier. A debt overhang arises wherein the expected present value of output is lower than the accumulated debt. The private sector with a debt overhang signals its inability to pay back its debt and therefore may not be able to get any new borrowing. This in turn leads to default crises, bank runs or speculative attacks. Theoretical findings and the Mexican experience of 1994 suggested the importance of the maturity structure of external debt and the benefits of debt with longer maturity to prevent debt-related financial crises. Thus in the spirit of the quote by Rogoff (2002), the study of capital controls became popular again. The success of the Chilean experience with reserve requirements on external debt brought increasing attention to the use of

these types of capital controls. Despite the extensive literature analyzing capital controls and external debt the issue of reserve requirements on external borrowing, as a type of capital controls has received little attention. In my dissertation I look at how the maturity structure of private external debt is endogenously determined and the role of date- and maturity- specific reserve requirements in preventing bank runs.

How is the maturity structure of external debt determined? How are policies of capital controls on the external debt flows documented in the literature? What is the impact of capital controls on the composition of debt? Can they reduce the vulnerability to a bank run? To answer these questions, in this chapter I will first look at how the over-borrowing syndrome of short-term debt has been looked at in the existing literature on financial crises and how the maturity and interest rate term structure was considered in these models. Second I will review the controversial existing results regarding the effect of capital controls. There I will look specifically at how capital controls that take the form of date- and maturity- specific reserve requirements on external borrowing have been considered in the literature.

Note that theoretical work on debt crises has taken different turns over the last three decades. The first-generation models on crises have focused on the relation between fiscal deficit of governments and the resulting collapse of exchange rates (Krugman, 1979). The second-generation models started to explain currency crises as the result of investors' panic regarding the central bank's holding of international reserves to backup the exchange rate peg. Devaluation policies or the abandonment of the peg become self-fulfilling prophecies (Obstfeld, 1994). There the choice of government debt maturity was

exogenous, when considered. The Mexican 1994 crisis and subsequent widespread financial crises throughout the 1990s shifted the analysis from developing standard models of governments' management of debt and exchange rate regimes to analyzing the performance of the private sector. The third-generation models attempt to explain how the inflows and outflows of capital can drive both a currency crisis and/or a financial crisis. Within this strand of models, different approaches have been undertaken (See Chang and Velasco, 2000 and Aizenman and Turnovsky, 2002). In section I.1, I summarize the explanations regarding the choice of the maturity structure. In section I.2, I look at how the role of capital controls has been documented. Finally, section I.3 summarizes the findings of each chapter in the dissertation.

I.1. On the choice of the maturity structure of external debt

With the financial crises of the 1990s, and the Mexican crisis in 1994, the literature on crises shifted attention towards analyzing private flows of capital. In addition, looking at the behavior of investors and firms became necessary. One strand in the literature on the debt overhang in the late 1980s and early 1990s has developed models in which the country is treated like a firm. In Krugman (1988) and Bulow and Rogoff (1991), there is a high inherited debt of a country that is higher than the expected present value of its future stream of resources. In these types of models the debt is inherited and the maturity structure is not originally a choice variable. Lamont (1995) introduces the role of beliefs in a model with debt overhang. In his model he introduces two types of inherited debt: one with long-term maturity and one with short-term maturity, along with an exogenous maturity structure.

Another strand in the debt literature has looked at the relationship between asset prices and the debt accumulation of firms to explain why over-borrowing occurs. Kiyotaki and Moore (1997) and Auernheimer and Garcia-Saltos (2000) suggest that the presence of collateral magnifies the effect of transitory productivity shocks on asset prices and thus on consumption. The major common chain in the above cited literature is that over-borrowing becomes a vicious circle due to under-accumulation of capital and high accumulated debt that is maturing. The problem is exacerbated by the need for new financing that is available only at high interest rate. The interest rate on the debt usually reflects the expectations regarding the ability of borrowers to repay the debt. The classical definition of the interest rate given by Fisher (1930) as an “index of impatience” suggests that the interest rate on the short-term debt is lower than on the long-term debt because it carries less risk of default (pp. 52-53). However, such a definition is not in tune with theoretical findings that suggest the importance of longer maturity structure to avoid crises. Keynes (1935) perceived the interest rate as the “reward for parting with liquidity”(pp.167). Diamond (1991) suggests a model in which the choice of debt maturity reflects a trade-off between “liquidity risk” in the short-term and “a preference for short-term debt due to private information about credit rating” (pp.709). A borrower with high credit rating prefers short-term debt. A borrower with somewhat low to middle credit rating would prefer long-term debt. A borrower with very low credit rating would prefer to hide his type and thus borrow short-term debt. Despite the relation of the maturity structure to liquidity risk and credit rating, in this model, only the borrowers’

decisions are modeled explicitly. There, the behavior of lenders and interest rates on debt are assumed to be exogenous.

Several papers based on Diamond and Dybvig (1983) have analyzed the behavior of domestic banks in choosing their debts and the debt maturity structure. Originally, Diamond and Dybvig (1983) explain how banks subject to runs can attract deposits in a closed economy. They recommend government deposit insurance to produce superior contracts. In Aizenman and Turnovsky (2002), despite considering the indebtedness of the private sector, there is no role for a bank in their model because uncertainty arises from an aggregate shock and not the lack of information. Chang and Velasco (2000) develop a model with a small open economy in which they show that bank runs are associated with high short-term debt. They provide a set-up for a small open economy that captures the endogenous term structure along with an endogenous financial intermediary. Nevertheless, their analysis focuses only on the borrower's behavior. However in a general equilibrium context, the maturity structure of the debt should be the outcome of the behaviors of both lenders and borrowers. Allowing for only one type of bond-- a short-term bond-- Seo (2003) finds that there always exists an equilibrium with pessimistic foreign investors who make bank runs the equilibrium strategy for domestic agents.

For the purpose of comparison across modeling techniques, it is important to touch on how the term structure of debt has been explored in a parallel literature related to governments' default and sovereign debt rather than the debt of the private sector. In a partial equilibrium framework Calvo and Guiddotti (1990) show that long-term debt

becomes optimal only with some pre-commitment and indexation of the domestic debt. If there is no indexation, the optimal maturity for the debt is short term, due to the possibility of using inflation to wash out its nominal debt. Another approach is presented by Jeanne (2004). He uses a game theoretic framework in which efforts of having good or bad policy to increase or decrease the probability of high output conditional on a bad signal are within the utility function of a country. In his model international creditors' behavior is not explicitly formalized. "A risky [short-term] debt induces the country to implement a good policy by making bad policy relatively costly" (Jeanne, 2004, pp.10). This is parallel to the commitment problem that leads to short-term debt being optimal in Calvo and Guidotti (1990). In contrast, by exogenously imposing various maturity structures, Cole and Kehoe (2000) show that a longer maturity structure of the prevailing debt can reduce the possibility of a crisis by reducing the new borrowing needed by the government at each period from international lenders. In their model a general equilibrium framework with risk neutral international lenders and a domestic government is used. However the maturity structure is exogenously imposed.

To sum up, at least three features would be desirable in an environment to analyze an endogenous maturity structure: First, for the framework to have a structure in which it is not clear that short-term debt is preferred to long-term, liquidity risk must be present. Second, for the choice to be explicit between short-term and long-term debt, the possibility of issuing long-term and short-term debt simultaneously should be considered. Third to understand why new lending would be provided or not, the behavior of international lenders has to be considered in a world general equilibrium. The

literature has looked at the first two features described above, but none of the existing models have endogenized the interest rates and therefore they have not considered lenders' behavior explicitly.

It is true that for a debtor who pays interest, the interest is, to him, a real cost, and is debited on his books. But we need only to be reminded of the debit and credit bookkeeping of the first chapter to see that this item is counterbalanced on the books of the creditor, to whom this interest is by no means a cost, but on the contrary an item of income. (Fisher, 1930, pp.540)

In tune with the quote by Fisher (1930), a complete model for debt has to consider a general equilibrium environment in which the interest rate falls as a cost on borrowers and a return to lenders.

In the following section, I give a review on the mixed results in the literature regarding capital controls.

I.2. Capital controls

Volatile short-term borrowing has marked many default crisis episodes in the 1990's. The fundamental feature of the 1990's is the rapid growth of private short-term debt in developing countries. In addition the ratio of short-term debt to international reserves became quite high. In fact, capital inflows to developing countries faced strong reversals in 1997 and 1998. This indicates that the growth in short-term debt made developing economies more susceptible to liquidity runs (Dadush et al., 2000). In addition to the volatile aspect of short-term debt, Kaminsky and Reinhart (1999) show that banking crises are features of the crises in the 1980's and 1990's but not in the 1970s, when

financial markets were still highly regulated. They report the probability of a currency crisis conditioned on the beginning of a banking crisis as 46% when the reported unconditional probability is 29%. In their survey, most of the banking crises precede balance of payment crises. In fact, they find a link between financial liberalization and banking crisis: “[In] 18 out of the 26 banking crises studied (...), the financial sector had been liberalized during the preceding five years, usually less” (Kaminsky and Reinhart, 1999, pp.480). As pointed by Rogoff (2002) and Chang and Velasco (2000), debt crises related to large capital inflows in an open economy context are structurally related to the maturity structure. In tune with the quote by Rogoff (2002), the benefits of capital controls are in preventing short-term volatile debt flows.

Theoretically, the answer on the benefit of capital controls is not really straight forward because as much as it prevents excessive short-term debt ex-ante, it puts more constraint on the provision of liquidity ex-post. In other words, the reserve requirements could play the role of liquidity or liquidity guarantee for any issuance of debt ex-ante, and therefore may prevent a crisis ex-ante. But in the event of a liquidity shock, short-term borrowing becomes more difficult when reserve requirements are imposed and therefore might create a constraint to the role of short-term borrowing as a liquidity provider. These two effects working in opposite directions are consistent with controversial results related to the benefits or losses regarding capital controls. Diamond and Rajan (2000) argue against capital requirements. Moreover, Reinhart and Smith (2002) studied the effect of temporary controls on capital inflows. In the second part of their paper, they analyzed it in the presence of the over-borrowing distortion. They

derived the optimal tax rate on capital inflows. After calibrating the model, they find that temporary capital controls need to be very high to be effective. The welfare benefits of such taxes are estimated to be very low.

Analyzing the effects of reserve requirements without considering the maturity structure of the bond may be missing a fundamental chain: the tilt of the maturity structure of the debt. It is only in the prevention of destabilizing short-term capital inflows that capital controls could be effective in removing crises. Aizenman and Turnovsky (2002) analyze the effect of reserve requirements in a model with only short-term debt. The work by Aizenman and Turnovsky (2002) analyzing the effect of reserve requirements on borrowing, stands as an exception. They model a default crisis as endogenous to sovereign risk and moral hazard considering the behavior of both risk-neutral lenders and borrowers. They show the benefit of reserve requirements in reducing the probability of default and thus increasing welfare. However, their model does not focus on the dual aspect of this type of capital control: yet the expression “reserve requirement” suggests a dual role of this type of capital controls: the “reserve” function, i.e. the role of “liquidity provider”, and the requirement constraining function, i.e. the role of a tax. The first role would presumably be important in preventing default crises. Although the second role secures from over-borrowing, it may prevent the provision of liquidity in the event of a crisis. In addition, short-term debt constitutes an important source of liquidity for domestic banks; reserve requirements may have the adverse effect of reducing such role. Accordingly, it is impossible in the Aizenman and Turnovsky (2002) model to capture the dual feature of short-term borrowing: On one hand as a de-

stabilizer and on the other as a liquidity provider. In fact, De Gregorio et al. (2000) show that, in Chile, such policies have tilted the maturity composition of capital inflows towards longer maturity structure. Chile's financial liberalization dates since 1974. It had a banking crisis in September 1981 that peaked in March 1983. The closest balance of payment crisis was in August 1982 (Reinhart and Kaminsky, 1999). In the 1990's, Chile did not experience any financial crisis. Among many sound monetary policies, the literature explaining the success of the Chilean case refers to the use of reserve requirements as capital controls. In January 1992, 20 percent reserve requirements were imposed on deposits and loans in foreign currency held by commercial banks. The reserves had to be maintained for one year. In May 1992, the rate was increased to 30 percent and it was set such that the reserve requirement rate fell as the maturity increased. In September 1998, a year marked by financial crisis in other Latin American countries, the rate of reserve requirement was set to zero. The share of private debt in Chile in the 1990's has increased. In addition, short-term debt share has decreased significantly from 19.41% in 1990 to 5.08% in 1998 (Reinhart and Reinhart, 1999 and De Gregorio et al., 2000).

The above suggests that a rigorous analysis of reserve requirements should consider not only the maturity structure of external private debt, but also the behavior of banks and international lenders in the external debt market.

I.3. Overview of dissertation chapters

In my dissertation, I explore the relationship between capital controls and the choice of external debt maturity, in a general equilibrium setup, with endogenous international

lenders' behavior. More specifically I extend the basic Chang and Velasco (2000) framework to a general equilibrium model in which international lending is endogenous and analyze the effects of capital controls that take the form of reserve requirements on external borrowing. I develop a simple Diamond-Dybvig-type model with three dates and a two-period planning horizon. In the low-income countries, banks or coalitions of depositors arise endogenously. There are two short-term bonds and one long-term bond offered by the domestic banks. There is a single good each period, which is homogeneous across countries. This good cannot be produced: this is an endowment or pure exchange economy. The good is perishable if not invested or consumed. There is an investment technology that yields a real gross rate of return $R > 1$ at $t=2$, but produces only r units of the good if liquidated earlier at $t=1$, where $r^2 < R$. The environment is characterized by liquidation costs at $t=1$. In chapter II, exogenous lending behavior is assumed. In chapter III, international lenders' behavior is considered explicitly. In chapter IV, date- and maturity- specific reserve requirements are imposed on the domestic economy's borrowing. In all chapters II, III, and IV, there is no bank run. In chapter V, I introduce the possibility of an unexpected bank run at $t=1$. In chapter V, I first look at what combination of date- and maturity-specific reserve requirements can prevent a bank run. Second, when a bank run occurs, I explore the re-optimizing behavior of international lenders in deciding whether to bailout the borrowing bank.

1.3.1. Chapter II

In chapter II, for comparability, a la Chang and Velasco (2000), international lending behavior is not explicitly modeled, and I impose upper limits on the borrowing by

domestic borrowers. Unlike Aizenman and Turnovsky (2002), I consider explicitly the maturity composition of capital inflows.

First, when I allow for an environment with a rich maturity structure, the scope for locally-unique multiple equilibria increases even when credit is rationed. I also show that the scope of multiple equilibria and the ranking of utility across different types of equilibria depend on the opportunity cost of liquidation. Second with exogenous lending, it is impossible to generate equilibria with high demand for one of the short-term bonds without generating high demand for the other short-term bond. Third, credit rationing is exogenous when credit limits are imposed. The holdings of both short-term bonds are not differentiated according to date. However, to capture in a nontrivial manner the idea that capital controls are not only maturity-specific but also date-specific, I need to look at the behavior of international lenders as well.

1.3.2 Chapter III

This chapter is the foundation for chapters IV and V. Here I consider international lenders' behavior explicitly. The world consists of two blocks of large open economies: The high income and low-income countries' blocks. The low-income countries have the same technology and consumption schemes described in chapter II. The high-income countries benefit from lending to low-income countries. In the high-income block, it is assumed that the international creditors are homogeneous. There are two short-term bonds and one long-term bond offered by the domestic banks in the low-income countries to international lenders in the high-income countries.

After I solve both the international lenders and the domestic bank problems, I derive multiple sets of equilibria. In the world general equilibrium, the interest rates will be such that markets clear. Accordingly, the equilibrium term structure of the debt will be determined. The difficulty with this type of models is that results are usually characterized by multiple equilibria. In addition, sometimes within each set of multiple equilibria, there arises indeterminacy of relative interest rates. I use the concept of indeterminacy to characterize each set of equilibria.

When I allow for endogenous lending behavior, credit rationing will emerge endogenously. The analytical description of equilibria and the simulation results show that cases, where there is high demand for either of the short-term bonds can exist for high gross return of investment R . This is different from the results obtained with exogenous lending for two reasons: First with exogenous lending, low R entails a lower cost of liquidation and thus the equilibrium with only short-term debt arises. In chapter III, a lower R implies both a lower cost of liquidation and lower interest rates paid to international lenders at $t=1$. Thus with endogenous lending interest rates incorporate both the Keynesian concept of interest rate as the “reward of parting for liquidity” and Fisher’s view of interest rate as “impatience factor”. The tilt towards longer or shorter maturity will depend on the equilibrium interest rates. In the following chapter I look at possible equilibria under reserve requirements imposed on the bank.

1.3.3 Chapter IV

In chapter IV, I explore how imposing date- and maturity-specific reserve requirements may affect the scope of indeterminacy of different equilibria. A date- and

maturity- specific reserve requirement is a fraction of the amount borrowed by private agents that has to be deposited in a non-interest bearing account at the central bank. When the bond matures, the central bank returns the reserves.

Note that at the interior solution without reserve requirements, there was an indeterminate set of the quantities. The number of equilibrium combination of quantities was infinite. Different combinations of non-zero reserve requirements cause bifurcations, a change in number of equilibria, towards either two equilibria or a unique equilibrium. This depends on what combinations of date- and maturity- specific reserve requirements is chosen. A bifurcation towards a unique equilibrium arises if and only if the reserve requirement on the long-term debt is higher than that on the second short-term bond issued at $t=1$, which in turn is higher than that on the initial short-term bond issued at $t=0$. The intuition is that the liquidity role of reserve requirements dominates the tax role. There are two reasons for this result: First, in this model, liquidation is costly in the short-term, thus the role of liquidity provision is crucial at $t=1$. Second, I have incorporated the behavior of risk-averse international lenders, who may see in high reserve requirements an indication of the capability of borrowers to pay some of the loan. Simulation results show that for some parameter ranges, a set of date- and maturity-specific reserve requirements, which yield a unique equilibrium, can improve welfare of domestic depositors and international lenders. It is also shown that international lenders are always worse off when reserve requirements lead to a bifurcation to two equilibria. The association of bifurcation towards a unique equilibrium with the possibility of

welfare improvement implies that there might be an association between higher utility and higher determinacy.

1.3.4 Chapter V

In this chapter I analyze the vulnerability to a bank run of equilibria identified in the model I describe in chapters III and IV. A bank run follows from the unexpected realization of a bad dream, which makes it optimal for all domestic agents to withdraw from the domestic bank. If an unexpected bad dream is seen by domestic depositors at $t=1$ and the bank position signals that it is unable to pay back all the withdrawals i.e. it is in an illiquid position, a bank run will occur. This chapter addresses two questions: Does the imposition of date- and maturity-specific reserve requirements on external debt reduce vulnerability to a bank run? Do international lenders bail out the domestic banks after a bank run occurs? To address these questions, I explore whether or not the domestic banks hold an illiquidity position. If the bank's position is illiquid, the domestic depositors would run to the bank if they have a bad dream. In addition, in the case of a bank run, I look at how international lenders may deviate from their original plans of lending. In particular, I explore whether they would be willing to give any new lending to bailout the domestic banks. Finally I illustrate each case within each section with simulation results from numerical examples. Interestingly one can identify that the combination of reserve requirements that creates a bifurcation towards a unique equilibrium is always not vulnerable to a bank run.

Why would international lenders be willing to lend at $t = 1$? Their outside option is not to get back anything of their loans. Since long-term debt is paid back at $t = 2$, they

are willing to bail out the bank to retrieve the long term debt. Following the occurrence of a bank run, international lenders may find it optimal to deviate from the plan at $t=0$. Thus they re-optimize. Simulation results show that at $t = 0$, the anticipated net new lending is higher than the actual new lending at $t = 1$. The new lending will only be provided if the actual value of total debt at $t=2$ has a higher value than the anticipated value at $t=0$ of total debt at $t=2$.

In chapter VI, I provide the main conclusions of the analysis. I outline some possible extension of the environment I have developed in this dissertation. In particular, I try to assess the shortcomings of the setup I developed, and outline some possible future improvements.

CHAPTER II

EXOGENOUS LENDING

II.1. Introduction

In this chapter, I present a version of my model assuming exogenous lending, for the purpose of comparability with Chang and Velasco (2000). In addition, this chapter provides a benchmark that I will use later, after I introduce endogenous lending explicitly. In the next chapters, introducing endogenous behavior for international lenders will be a first step before looking at the effect of date- and maturity-specific reserve requirements on private flows of borrowing.

In the model described below, international lenders are exogenously willing to provide lending up to a certain limit. Chang and Velasco (2000) explain this exogenous credit limit as an assumption in tune with the literature on collateralized borrowing: in that literature, borrowing is constrained to the available amount of liquidity. However in Chang and Velasco (2000), liquidity is not modeled explicitly and the credit limit is exogenously imposed. In addition, if such credit constraint is binding, this usually implies the presence of credit rationing. Credit rationing refers to “situations where a borrower’s demand is unfulfilled, although he is willing to pay the ruling market price” (Baltensperger, 1978, pp.173). In this chapter, for comparability, a la Chang and Velasco (2000), international lending behavior is not explicitly modeled, and I impose upper limits on the borrowing by domestic borrowers.

II.2. The environment

II.2.1. Domestic depositors and domestic banks

The economy faces a two-period planning horizon with three dates: $t=0$, 1, and 2. In the domestic country, there is a continuum of agents with unit-mass. These agents are born at $t=0$. There is a single good each period, which is homogeneous across countries. This good cannot be produced: this is an endowment or pure exchange economy. At $t=0$ each depositor gets an endowment of e_0 units of the good. I assume that at $t=1$ and $t=2$, they do not receive any endowments of goods¹. At $t = 0$, depositors do not consume.

The domestic agents whom I call depositors are ex-ante identical but they may be ex-post different due to a preferences' shock that is realized at $t=1$. The distribution of this shock is known at $t=0$, and it is i.i.d. across agents. A $t=0$ with probability $\lambda \in (0,1)$, the depositors could be impatient and derive utility only from consuming goods at $t=1$. With probability $(1 - \lambda)$, they could be patient and would want to consume goods at $t=2$ only. Thus domestic depositors will not consume in both periods. The distribution is known ex-ante and is public information. However when the event is realized at $t=1$, each depositor's type realization is private information. Unlike Aizenman and Turnovsky (2002), there is no aggregate uncertainty. The only uncertainty in the borrowing country is private.

¹ With this assumption I avoid the complexity that may arise with different types of deposits while I am focusing on different types of debts. For instance, a depositor receiving a new endowment at $t=1$ would face the choice of whether or not to deposit her new endowment at the bank at $t=1$ and I rule out such a possibility. The assumption that the domestic depositors have no endowments at $t=2$ is just a normalization that is standard in the literature.

The good is perishable if not invested or consumed. There is an investment technology works as follows: one unit of the good invested at $t=0$ yields a real gross rate of return $R > 1$ at $t=2$, but produces only r units of the good if liquidated earlier at $t=1$, where $r^2 < R$.² Let k denote the amount of goods invested in this technology at $t=0$.³

Because of the uncertainty and the ex-post heterogeneity, a bank or a coalition of depositors arises endogenously. Henceforth I will use the terms “domestic bank” and “borrower” interchangeably. As is standard in the literature, due to the preferences’ shock that depositors face, they may gain from acting jointly. In fact, each depositor faces a probability λ of being of the impatient type at $t=1$. If she was to choose autarky and learns that she is impatient, liquidating the investment at $t=1$ with a lower return becomes unavoidable. Therefore, depositors find it optimal to form a bank that can provide some insurance. In this setup, banks arise endogenously⁴. Banks will offer contracts that maximize the domestic depositors expected utility inducing truth-telling self-selection. To fix ideas, I assume a logarithmic form of utility.⁵ Thus a domestic depositor’s expected utility is given by

$$U(c_1, c_2) = \lambda \ln(c_1) + (1 - \lambda) \ln(c_2) \quad (2.1)$$

² Note that a one unit of the good can be reinvested at $t=1$ yielding r units at $t=2$.

³ In Chang and Velasco (2000), the borrower has the possibility to invest in an additional technology: “international reserves” b_t with gross return equal to one. Here I abstract from such a possibility. It is important to note that in my model when $r > 1$, the rate of return on the investment technology would be higher than “international reserves” technology. The investment in k would therefore dominate the saving in “international reserves”. In fact Chang and Velasco (2000) assume $r = 0$.

⁴ One could also think of the bank as a possible contracting scheme for agents, in which agents exchange contracts to hedge against uncertainty.

⁵ Note that with the logarithmic function, the substitution effect dominates the income effect, improving greatly the tractability of the model.

where c_1 and c_2 are consumption quantities at $t=1$ and $t=2$ respectively. $\ln(c)$ is twice continuously differentiable, strictly increasing in its arguments and satisfies the Inada conditions.

II.2.2. International borrowing

All domestic agents deposit their endowments at the bank at $t=0$. In addition to receiving deposits in the amount e_0 at $t=0$, only banks can borrow from international lenders. At $t=0$, two types of international debt are available to banks: short-term and long-term. At $t=1$, banks can only borrow short-term. d_{01} is the short-term bond issued by the domestic bank at $t=0$ that matures at $t=1$, paying a gross real interest rate ρ_{01} . d_{12} is the short-term bond issued by the domestic bank at $t=1$ that matures at $t=2$, paying a gross real interest rate ρ_{12} . Finally, d_{02} is the long-term bond issued by the domestic bank at $t=0$ that matures at $t=2$ paying a gross real interest rate ρ_{02} . ρ_{01} , ρ_{02} , and ρ_{12} are exogenous.⁶ Note that in anticipation for what is coming in chapter IV, to introduce date- and maturity- specific reserve requirements, I need two distinct date-specific short-term bonds.

II.2.3. The domestic bank's problem

In this chapter, I assume that there are international lenders who are willing to lend to banks in the domestic country inelastically. However, as in Chang and Velasco (2000), I impose a borrowing constraint that works as a rule for credit rationing. The rule is as

⁶ Chang and Velasco (2000) assume $\rho_{01} = \rho_{02} = \rho_{12} = 1$.

follows: the total principal of the different types of bonds for which the banks are liable at each date must not exceed a certain limit. Let f_0 and f_1 denote the limits at $t=0$ and $t=1$, respectively. Therefore the domestic bank faces two borrowing constraints⁷:

$$d_{01} + d_{02} = f_0 \quad (2.2)$$

$$d_{12} + d_{02} = f_1 \quad (2.3)$$

I will assume, for reasons that will become clearer later, that $f_1 \geq f_0$. Notice that in Chang and Velasco (2000) $f_0 = f_1$. Banks have access to the same investment technology that depositors have. The bank will decide how much to invest long-term k at $t=0$ and how much to liquidate l at $t=1$, given R and r as explained above. Therefore the budget constraints faced by the bank at $t = 0, 1$ and 2 are given by:

$$k \leq e_0 + d_{01} + d_{02} \quad (2.4)$$

$$\lambda c_1 + \rho_{01} d_{01} \leq d_{12} + rl \quad (2.5)$$

$$(1-\lambda)c_2 + \rho_{12}d_{12} + \rho_{02}d_{02} \leq R(k-l) \quad (2.6)$$

In addition, the following incentive compatibility constraint needs to hold⁸:

⁷ In this case I assume that the borrowing constraints are binding, implying that credit is rationed. This is implicitly assumed in Chang and Velasco (2000). It means that banks could be willing to borrow arbitrarily large amounts, but they are not allowed to.

⁸ In my model I allow domestic depositors to have access to investment technology even after the bank is formed. A patient depositor has therefore the option of concealing her type and withdrawing c_1 units of $t=1$ goods from the bank. After investing this withdrawal for one period, at $t=2$, she could consume rc_1 units of the goods at $t=2$. If the patient depositor reveals her true type and waits until date $t=2$, she would consume c_2 units of $t=2$ goods. Therefore the patient depositor will have an incentive to reveal her type if and only if $c_2 \geq rc_1$. In Chang and Velasco (2000), once the bank is formed, depositors are not allowed to have access to the investment technology. Since they have a storage technology, there the incentive compatibility constraint takes the form: $c_2 \geq c_1$.

$$c_2 \geq rc_1 \quad (2.7)$$

II.3. Equilibria

The bank's problem is to choose $c_1, c_2, d_{01}, d_{02}, d_{12}, k, l$ to maximize (2.1) subject to (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7), taking as given the world real gross interest rates $\rho_{01}, \rho_{02}, \rho_{12}$ and the other parameters e_0, R and r . In Appendix A, I explain how I solve this problem. Despite the fact that interest rates are exogenous, I assume that the following perfect arbitrage condition holds.⁹

$$\rho_{01}\rho_{12} = \rho_{02} \quad (2.8)$$

I need this assumption so that I can later compare my results with the equilibria with endogenous lending.

A competitive equilibrium is defined as a set of non-negative allocations $\{\hat{c}_1, \hat{c}_2, \hat{k}, \hat{l}, \hat{d}_{01}, \hat{d}_{12}, \hat{d}_{02}\}$ for the domestic bank such that the domestic bank's problem is solved and (2.8) holds. Since I am looking at equilibria where the domestic bank is a net borrower in international markets, I dismiss cases where $d_{02} < 0, d_{12} < 0$ or $d_{01} < 0$.

Proposition 1. *If there is no early liquidation in equilibrium, i.e. $\hat{l} = 0$, then \hat{d}_{12} is strictly positive. When there is no early liquidation, then $r < \psi \leq \frac{R}{r}$, where $\psi \equiv \frac{\hat{c}_2}{\hat{c}_1}$.*

⁹ This is a version that can be comparable to “pure expectations theory (that) hypothesizes that $R_{2t}^{-1} = R_{1t}^{-1} E_t R_{1t+1}^{-1}$, which results in Lucas-Tree type Model, when utility is linear in consumption or there is no uncertainty” (Sargent, 1987, pp.105).

The proof for proposition 1 is provided in appendix A. Note that $\frac{\lambda}{1-\lambda}\psi$ is the marginal rate of substitution between consumptions across two different states: 1-being impatient or 2-being patient. It is the ratio of the marginal utility of \hat{c}_1 over the marginal utility of \hat{c}_2 . It measures how much a depositor is willing to give up of state 2 (being patient) consumption for one more unit of state 1 (being impatient) consumption. Thus proposition 1 implies that this marginal rate of substitution must be bounded. The upper bound means that for a given λ , the expected consumption of impatient depositors cannot be very high so that there is no early liquidation. The lower bound indicates that the incentive compatibility constraint is not binding. This is assumed when there is no crisis at $t=1$. In this environment, there exist three types of equilibria, which may coexist or may exist uniquely depending on the parameter values. In anticipation of what will come in the sections where I describe each type of equilibria, let me set

$$A \equiv \frac{\rho_{12}f_1 - (1-\lambda)\rho_{02}f_0}{\lambda(e_0 + f_0)} \quad (2.9)$$

$$B \equiv \frac{\rho_{12}f_1 + (\lambda\rho_{02} - \rho_{12})f_0}{\lambda(e_0 + f_0)} \quad (2.10)$$

$$C \equiv \frac{\rho_{02}f_1 - (1-\lambda)\rho_{02}f_0}{\lambda(e_0 + f_0)} \quad (2.11)$$

Note that $A > B > C$, when $\rho_{02} < \rho_{12}$ and $A < B < C$, when $\rho_{02} > \rho_{12}$. Moreover, notice that $A \neq B$ when $\rho_{01} \neq 1$. Below, I describe each one of the equilibria.

II.3.1. The interior solution

An interior solution means that the first order conditions with respect to d_{01} , d_{02} and d_{12} are zero. In this particular case $d_{01} \geq 0$, $d_{02} \geq 0$, $d_{12} > 0$. This is not a world general equilibrium analysis since the interest rates are exogenous by assumption. In this case the solution for the endogenous variables is:

$$\hat{k} = e_0 + f_0 \quad (2.12)$$

$$\hat{d}_{02} = \frac{1}{(\rho_{01} - 1)} \left\{ \frac{\lambda R(e_0 + f_0) + \rho_{01} f_0 - \rho_{12} f_1}{\rho_{12}} \right\} \quad (2.13)$$

$$\hat{d}_{01} = \frac{(\lambda \rho_{02} - \rho_{12}) f_0 + \rho_{12} f_1 - \lambda R(e_0 + f_0)}{(\rho_{02} - \rho_{12})} \quad (2.14)$$

$$\hat{d}_{12} = \frac{f_1 \rho_{02} - \lambda R(e_0 + f_0) - (1 - \lambda) \rho_{02} f_0}{\rho_{02} - \rho_{12}} \quad (2.15)$$

$$\hat{c}_1 = \frac{f_1 - \rho_{01} f_0 - (1 - \rho_{01}) \hat{d}_{02}}{\lambda} > 0 \quad (2.16)$$

$$\hat{c}_2 = \frac{R(e_0 + f_0) - \rho_{12} f_1 - (\rho_{02} - \rho_{12}) \hat{d}_{02}}{1 - \lambda} > 0 \quad (2.17)$$

Proposition 2. *Under the perfect arbitrage condition, there exists an equilibrium with an interior solution $d_{01} \geq 0$, $d_{02} \geq 0$, $d_{12} > 0$ if and only if*

(i) $A \leq R$ and $R \leq B$ when $\rho_{01} > 1$ or $\rho_{12} < \rho_{02}$

(ii) $A \geq R$ and $B \geq R$ when $\rho_{01} < 1$ or $\rho_{12} > \rho_{02}$

Proof.

Under the perfect arbitrage condition $\rho_{01}\rho_{12} = \rho_{02}$, it must be the case that $\rho_{01} > 1$ if $\rho_{12} < \rho_{02}$. I will first prove (i). Note that if $\rho_{12} < \rho_{02}$, $A < B$, and if $\rho_{12} > \rho_{02}$, $A > B$. Moreover if $\rho_{12} < \rho_{02}$, $C < B$, and if $\rho_{12} > \rho_{02}$, $C > B$. First suppose that $\rho_{02} > \rho_{12}$. If $\rho_{12} < \rho_{02}$ or $\rho_{01} > 1$, using equation (2.13), for $d_{02} \geq 0$, it must be that $R \geq A$. This means that for the domestic bank to hold positive amounts of long-term bonds at $t = 0$, the gross return on investment at $t = 2$ must be high. In addition, if $\rho_{12} < \rho_{02}$ or $\rho_{01} > 1$, using equation (2.14), for $d_{01} \geq 0$, it must be that $R \leq B$. It means that for the domestic bank to hold positive amounts of short-term bonds at $t = 0$, the gross return on investment at $t = 2$ must be high. The upper and lower bounds are consistent when $\rho_{12} < \rho_{02}$ or $\rho_{01} > 1$. Using equation (2.14) for $d_{12} > 0$, it must be $R < C$. Thus when $\rho_{01} > 1$, the interior solution exists if and only if $A \leq R \leq B < C$.¹⁰ Now I turn to prove (ii): suppose that $\rho_{01} < 1$, for $d_{02} \geq 0$ $R \leq A$, $d_{01} > 0$, $R \geq B$, and for $d_{12} > 0$, $R > C$. When $\rho_{12} > \rho_{02}$ and $\rho_{01} < 1$ the reverse applies, the interior solution exists if and only if $C < B \leq R \leq A$. \square

¹⁰ For strict interior solution, the inequalities on the bounds for R become strict inequalities.

II.3.2. The domestic bank is willing to borrow only long term debt at $t=0$

In this equilibrium, $\hat{d}_{02} = f_0$, which entails $\hat{d}_{01} = 0$ and $\hat{d}_{12} = f_1 - f_0 > 0$.¹¹ For this equilibrium to exist, it must be that $R \geq A$. The latter condition entails that when R , the gross return on investment at $t = 2$ is very high, the domestic borrower would prefer to borrow only long-term. This is because the liquidation cost at $t=1$ becomes relatively higher when R is very high. In this equilibrium, the consumptions allocation would be:

$$\hat{c}_1 = \frac{f_1 - f_0}{\lambda} \quad (2.18)$$

$$\hat{c}_2 = \frac{R(e_0 + f_0) - \rho_{12}f_1 + (\rho_{12} - \rho_{02})f_0}{(1 - \lambda)} \quad (2.19)$$

In (2.18), $f_1 > f_0$ is needed to ensure non-negative consumption at $t=1$. Notice that unlike the interior solution, in this equilibrium, the condition $R \geq A$ does not require $\hat{\rho}_{01} > 1$ or $\hat{\rho}_{02} > \hat{\rho}_{12}$. Thus in this environment when the opportunity cost of liquidating at $t=1$ is high enough, an equilibrium with only long-term borrowing at $t=0$, will arise.

II.3.3. The domestic bank is not willing to borrow long-term at $t=0$

In this equilibrium, $\hat{d}_{02} = 0$, implying $\hat{d}_{01} = f_0$ and $\hat{d}_{12} = f_1$. Again in anticipation for what is coming in chapter IV, to introduce date- and maturity- specific reserve requirements, I need two distinct date-specific short-term bonds. Note though that the setup with exogenous lending does not allow for such an interesting analysis. For instance in this equilibrium, it is not possible to generate high short-term borrowing in

¹¹ Notice that when $f_1 = f_0$, as in Chang and Velasco (2000), this is not an equilibrium with banks because having $d_{01} = d_{12} = l = 0$, entails $c_1 = 0$, yielding a utility well below the utility under autarky.

one period without generating high short-term debt in the other. This is because credit is rationed and once one of the short-term bonds is at the limit, then $d_{02} = 0$, which means that the other short-term bond would have to go to the limit too. When $\hat{d}_{02} = 0$, the consumptions at $t=1$ and $t=2$ become respectively:

$$\hat{c}_1 = \frac{f_1 - \rho_{01}f_0}{\lambda} \quad (2.20)$$

$$\hat{c}_2 = \frac{R e_0 + R f_0 - \rho_{12}f_1}{1 - \lambda} \quad (2.21)$$

Note that for $c_1 > 0$ and $c_2 > 0$, it must be the case that $f_1 - \rho_{01}f_0 > 0$ and $\rho_{12} < \frac{R(e_0 + f_0)}{f_1}$. In other words, ρ_{12} should be relatively low, so that there is a high demand for the second period short-term debt d_{12} . $R \leq B$ is a necessary condition for this equilibrium to exist. Thus when the return on investment at $t=2$ is low, the relative opportunity cost of liquidation at $t=1$ is low and short-term debt is preferred to long-term debt. Unlike the condition at the interior solution, the existence of this equilibrium does not require $\rho_{01} > 1$. If $\rho_{01} < 1$, short-term debt at $t=0$ is relatively cheap and it is obvious why this equilibrium may arise when $R \leq B$. When $\rho_{01} > 1$, short-term debt seems relatively expensive: it may not be obvious why this equilibrium may arise when there is low return on investment $R \leq B$ if the short-term debt is costly. Note that when $\rho_{01} > 1$, with the perfect arbitrage condition, I get that $\rho_{02} > \rho_{12}$. Thus both the short-term debt and the long-term seem relatively expensive. The choice of borrowing only for short-

term in this equilibrium comes from the fact that the opportunity cost of liquidation is relatively low.

II.3.4 Multiplicity of equilibria

From the above analysis, one can summarize the conditions under which multiple equilibria may be observed. As mentioned the value for R relative to a combination of parameters determines the scope for existence, uniqueness and multiplicity of equilibria. Figure 2.1 shows the possibility of existence of the three types of equilibria. In addition for existence, the non-negativity constraint of consumptions and the incentive compatibility constraints must be checked. Again the focus is on equilibria with non-negative amount of all bonds. As I mentioned earlier, this is because I am focusing on external borrowing. In the construction of figure 2.1, it is assumed that the perfect arbitrage condition holds and $f_1 > f_0$.

Two assumptions on the parameters are worthwhile discussing simultaneously: (i) $f_1 > f_0$ for all sets and (ii) whether $\rho_{12} < \rho_{02}$ or $\rho_{12} > \rho_{02}$. The assumption $f_1 > f_0$ is crucial for cases 3.2 and 3.3. This is consistent with the fact that at $t = 1$, there is a need for liquidity and therefore a higher credit limit is needed. When $\rho_{12} < \rho_{02}$ is assumed, a relatively low interest rate for the second short term debt implies that it would not be costly to get the new financing at $t=1$. The concurrence of the two assumptions $f_1 > f_0$ and $\rho_{12} < \rho_{02}$ implies the existence of multiple equilibria when $A \leq R \leq B$. In this case it is not clear whether new financing will be higher because of higher f_1 , or because of low ρ_{12} . This raises the question of what is exactly causing credit to be rationed in this

setup at $t = 1$.¹² However when $\rho_{12} > \rho_{02}$, or $\rho_{01} < 1$, there are only unique equilibria. The type of each equilibrium is dependent on the range of R in relation with other parameters. For $B \leq R \leq A$, both the short-term debt and the long-term debt at $t=0$ are relatively cheap leading the interior solution to exist uniquely. For all ρ_{01} , for low R , the opportunity cost of liquidating early is low, so at $t = 0$, only short-term debt is chosen. For high R , the opportunity cost of liquidating early is high, so at $t=0$, only long term-debt is chosen. In the next section I show some numerical example to illustrate the existence of the different types of equilibria.

Range for unique and multiple equilibria

$\rho_{01} > 1$				
$R < A$	A	$A \leq R \leq B$	B	$B < R$
Unique corner with short-term debt only (3.3) $d_{02} = 0, d_{01} = f_0, d_{12} = f_1$	Multiple Equilibria: All three cases 3.1, 3.2, 3.3 are possible		Unique corner with only long-term debt at $t=0$ (3.2) $d_{02} = f_1, d_{01} = 0, d_{12} = f_1 - f_0$	
$\rho_{01} < 1$				
$R < B$	B	$B \leq R \leq A$	A	$A < R$
Unique corner with short-term debt only (3.3) $d_{02} = 0, d_{01} = f_0, d_{12} = f_1$	Unique Interior (3.1)		Unique corner with only long-term debt at $t=0$ (3.2) $d_{02} = f_1, d_{01} = 0, d_{12} = f_1 - f_0$	

Fig.2.1. Ranges for unique and multiple equilibria

¹² See Baltensperger (1978) for a survey on different types of credit rationing.

II.4. Numerical examples from simulation

In this section I present a numerical example from simulating the above presented model with exogenous lending. This part is intended for illustration purpose of the various analytical results I have derived. I use the following parameter space:

$$e_0 = 2000, f_0 = 5000, f_1 = 10000, \rho_{01} = 1.75, \rho_{02} = 1.5, \rho_{12} = \frac{\rho_{02}}{\rho_{01}}.$$

Table 2.1. Bounds A and B

λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
A	2.602	1.8367	1.5816	1.4541	1.3776	1.3265	1.2901	1.2628	1.2415
B	7.1939	4.1327	3.1122	2.602	2.2959	2.0918	1.9461	1.8367	1.7517

with $e_0 = 2000, f_0 = 5000, f_1 = 10000, \rho_{01} = 1.75, \rho_{02} = 1.5$

I illustrate the case when $\rho_{02} > \rho_{12}$. For the other case, simulation results can be presented upon request. Table 2.1 shows that both bounds for R displayed in figure 2.1 decreases with λ . Note that this result is not specific to $\rho_{02} > \rho_{12}$. Equilibria that have negative holdings of debt are ruled out. To illustrate multiplicity or uniqueness for different values of R , I fix $\lambda=0.3$ and vary R . For the pair $(\lambda, R) = (0.3, 1.5)$, there may exist a unique equilibrium with non-negative borrowing where $d_{01} = 5000$, $d_{02} = 0$, $d_{12} = 10000$, $c_1 = 4167$, $c_2 = 2755$, $\frac{c_2}{c_1} = 0.66$. When $r = 0.7$, this equilibrium does not exist because the incentive compatibility constraint is not satisfied. When $r = 0.5$, this

equilibrium exists uniquely. This unique equilibrium is a corner with only short-term borrowing at $t=0$, and no long-term borrowing. For the pair $(\lambda, R) = (0.3, 2.6)$, the three types of equilibria may exist. Accordingly, $\frac{\hat{c}_2}{\hat{c}_1} = \{3.3, 0.55, 0.86\}$ are the respective consumption ratios for each case (3.3), (3.2), (3.1). For example, when $r = 0.5$, the three types of equilibria exist with non-negative borrowing. For the pair $(\lambda, R) = (0.3, 3.2)$, a unique equilibrium exists with only long-term borrowing at $t=0$. There $\frac{\hat{c}_2}{\hat{c}_1} = 0.91$, thus this equilibrium would exist when $r = 0.9$ but not for $r = 1.1$.

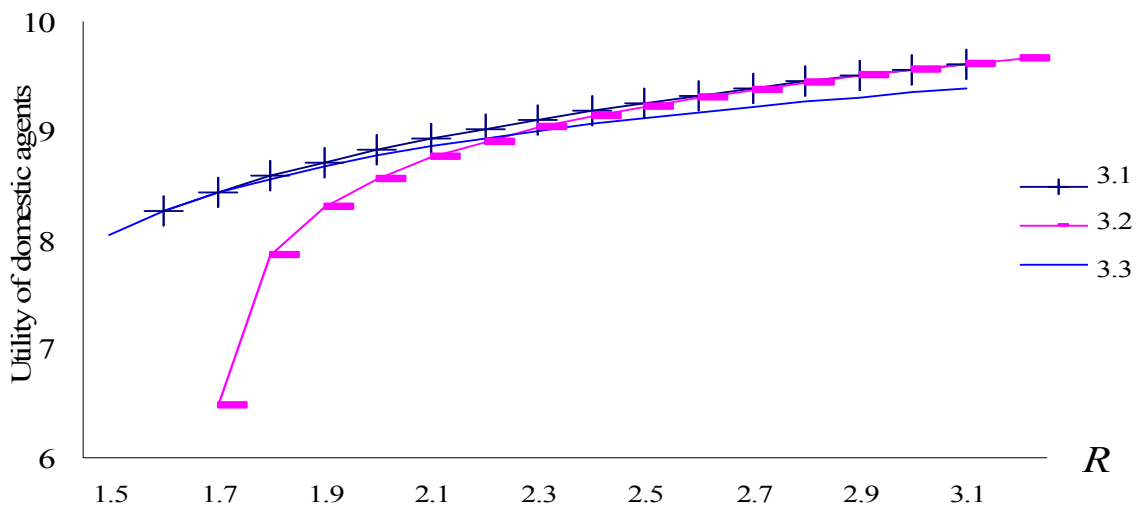


Fig.2.2. Utility of domestic agents across different types of equilibria

Now I turn to analyze how utility changes across the different sets of equilibria: 3.1, 3.2, and 3.3. Suppose $r = 0.5$, all cases illustrated in figure 2.1, with $\rho_{01} > 1$, exist. Figure 2.2 shows the expected utility of domestic agents at $t=0$ for $\lambda = 0.3$. Note that for low (high) R , the utility at the equilibrium with only short-term borrowing (case 3.3) is higher (lower) than the equilibrium with only long-term borrowing at $t=0$ (case 3.2). For all ranges of R , the equilibrium with an interior solution described in case 3.1, has the highest utility of all the three types of equilibria. This pattern is not specific to $\lambda = 0.3$.

II.5. Conclusion

I would like to highlight three issues. First, when I allow for an environment with a rich maturity structure, the scope for multiple equilibria that are locally unique increases even when credit is rationed. I have also shown that the scope of multiple equilibria and the ranking of utility across different type of equilibria depend on the opportunity cost of liquidation. Second with exogenous lending, it is impossible to generate equilibria with high demand for one of the short-term bonds without generating high demand for the other short-term bonds. Third, credit rationing is exogenous when I impose credit limits in (2.2) and (2.3). For all three issues, it is important to consider international lenders behavior explicitly before I look at the effect of reserve requirements on the composition of debt. In the next chapter I turn to the model with an endogenous lending behavior.

CHAPTER III

ENDOGENOUS LENDING

III.1. Introduction

In this chapter, I extend the model presented in chapter II to endogenize international lenders' behavior. For the purpose of the analysis, the world consists of two blocks: the high-income block and the low-income block. I assume that the high-income block is the rest of the world, and the domestic economy-which is a large open-economy- is the low-income block. In the high-income block, the international creditors are homogeneous. The low-income block corresponds to the domestic economy described in chapter II. Unlike Aizenman and Turnovsky (2002), in both economies, there is no aggregate uncertainty. The only uncertainty in the low income block is private. Both blocks are endowment (pure exchange) economies. In this chapter I assume that all agents do not expect a crisis to be possible and that they behave accordingly.

III.2. The environment with endogenous behaviors of international lenders

III.2.1. The domestic bank problem

The domestic bank and domestic depositors face a similar environment described in the chapter with exogenous lending. In particular, unlike Chang and Velasco (2000), I do not impose credit limits, since quantities and prices will be determined in equilibrium once the lenders' problem is considered explicitly. The domestic bank's problem is to choose $\{c_1, c_2, d_{01}, d_{02}, d_{12}, k, l\}$ to maximize (2.1) subject to (2.2),(2.3), (2.4) and

(2.7) given the endowment e_0 at $t=0$, the investment technology rate of return R , the costly liquidation return r and the world real gross interest rates $\rho_{01}^d, \rho_{02}^d, \rho_{12}^d$.¹³ Notice that d_{01} , d_{12} and d_{02} denote quantities demanded of each bond.

III.2.2. The lenders' problem

There is a continuum of international lenders with unit-mass. Unlike agents in the borrowing block, the agents in the lending block are ex-ante and ex-post homogeneous. They derive utility from consuming goods at both $t=1$ and $t=2$. Let c_1^* denote a representative lender's consumption of goods at $t=1$ and c_2^* be her consumption at $t=2$. Again for simplicity, I assume a logarithmic utility function. The representative lender's utility is¹⁴

$$U^* = \ln(c_1^*) + \beta^* \ln(c_2^*) \quad (3.1)$$

where β^* , the discount factor is identical across lenders. Each international lender receives endowments of goods in the amounts e_0^* , e_1^* and e_2^* at $t=0$, $t=1$ and $t=2$ respectively. For simplicity, I assume that her only investment opportunity is lending to the low-income block through the domestic banks in the amounts of s_{01}, s_{12} and s_{02} . s_{01} is the short-term loan to the domestic bank at $t=0$ and maturing at $t=1$, with a gross real interest rate of ρ_{01}^s . s_{12} is the short-term loan to the domestic bank at $t=1$ and

¹³ Refer to Appendix B.

¹⁴ For simplicity, U^* is assumed to be additively separable. Also note that the lender maximizes her lifetime utility, whereas the borrower maximizes its expected lifetime utility across two contingent states (patience and impatience.)

maturing at $t=2$ with a gross real interest rate of ρ_{12}^s . s_{02} is the long-term loan to the domestic bank at $t=0$ and maturing at $t=2$ with a gross real interest rate of ρ_{02}^s . Notice that s_{01} , s_{12} , s_{02} denote the quantity supplied of each loan. The budget constraints by a typical international lender are:

$$s_{01} + s_{02} \leq e_0^* \quad (3.2)$$

$$c_1^* + s_{12} \leq e_1^* + \rho_{01}^s s_{01} \quad (3.3)$$

$$c_2^* \leq e_2^* + \rho_{02}^s s_{02} + \rho_{12}^s s_{12} \quad (3.4)$$

The international lender's problem is therefore to choose c_1^* , c_2^* , s_{01} , s_{02} , s_{12} to maximize (3.1) subject to (3.2), (3.3), and (3.4) taking as given the endowments e_0^* , e_1^* , e_2^* and the world interest rates ρ_{01}^s , ρ_{02}^s , ρ_{12}^s . In appendix B I derive the relevant first order conditions. A summary of the players and market clearance is illustrated in figure 3.1.

III.3. Equilibria

In this economy, a competitive equilibrium is a set of interest rates $\{\hat{\rho}_{01}, \hat{\rho}_{12}, \hat{\rho}_{02}\}$, a set of international bonds' allocations $\{\hat{q}_{01}, \hat{q}_{12}, \hat{q}_{02}\}$, a set of allocations for the typical domestic bank $\{\hat{k}, \hat{c}_1, \hat{c}_2, d_{01}, d_{12}, d_{02}\}$ and a set of allocations for the typical international lender $\{\hat{c}_1^*, \hat{c}_2^*, s_{01}, s_{12}, s_{02}\}$ given $\{e_0, e_0^*, e_1^*, e_2^*, \lambda, \beta^*\}$ and the reserve requirements rates $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ such that:

- i. $\{\hat{k}, \hat{c}_1, \hat{c}_2, d_{01}, d_{12}, d_{02}\}$ solve the domestic bank problem of maximizing (2.1) subject to (2.2), (2.3) (2.4) and (2.7).
- ii. $\{\hat{c}_1^*, \hat{c}_2^*, s_{01}, s_{12}, s_{02}\}$ solve the international lender problem of maximizing (3.1) subject to (3.2), (3.3) and (3.4).
- iii. Markets clear

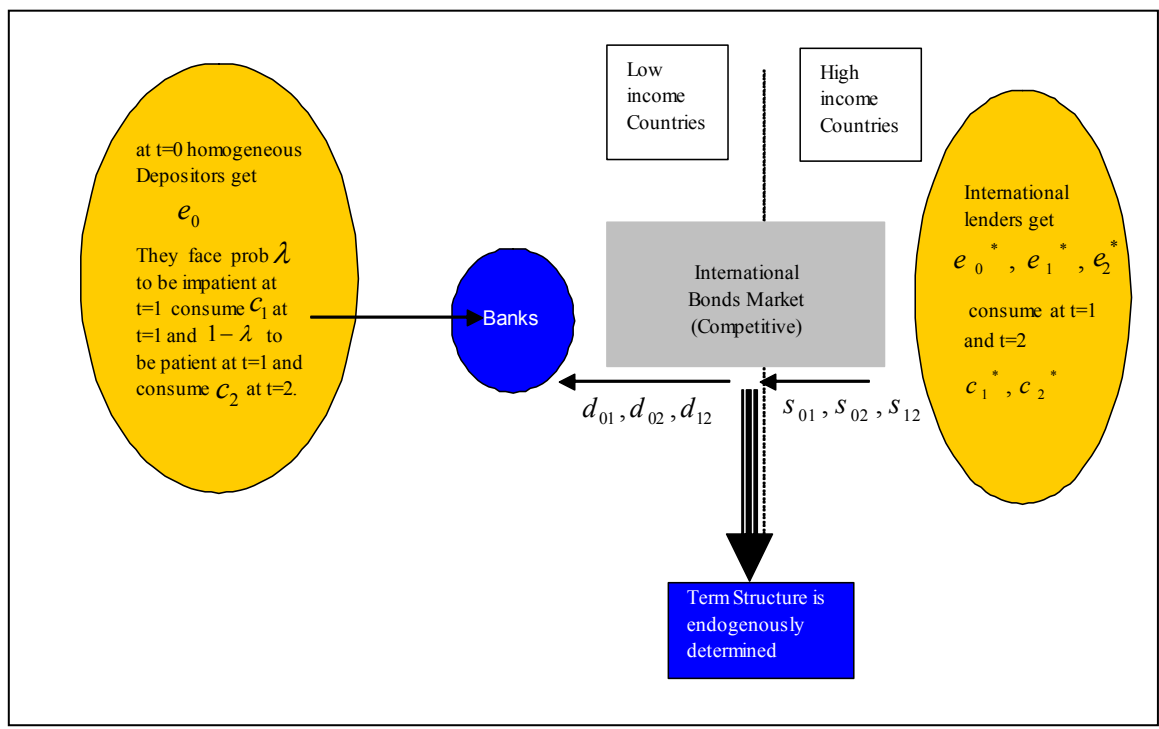


Fig.3.1. Structure of the model

The equilibrium short-term interest rate $\hat{\rho}_{01} = \rho_{01}^d = \rho_{01}^s$ clears the market for the first short-term debt so that:

$$s_{01} = d_{01} = \hat{q}_{01} \quad (3.5)$$

The equilibrium long-term interest rate $\hat{\rho}_{02} = \rho_{02}^d = \rho_{02}^s$ clears the market for the long-term debt so that:

$$s_{02} = d_{02} = \hat{q}_{02} \quad (3.6)$$

The equilibrium short-term interest rate $\hat{\rho}_{12} = \rho_{12}^d = \rho_{12}^s$ clears the market for the second short-term debt so that:

$$s_{12} = d_{12} = \hat{q}_{12} \quad (3.7)$$

III.3.1. On multiplicity and indeterminacy of equilibria

Thus in the world general equilibrium, the interest rates will be such that markets clear. Accordingly, the equilibrium term structure of the debt will be determined. The difficulty with this type of model is that the results are in general characterized by multiple equilibria. In addition, sometimes within each set of multiple equilibria, there is a tendency to find indeterminacy. Indeterminacy raises several concerns and problems. In fact equilibria that are indeterminate might be Pareto efficient or not [Kehoe and Levine (1990).] Theoretically, due to indeterminacy the task of fully ranking these equilibria becomes arduous.¹⁵ Empirically, multiple equilibria explain why economies with similar characteristics and parameters of their economic environments, face differently similar

¹⁵ Kehoe and Levine (1990) provide a thorough survey of intertemporal models with general equilibrium that have indeterminacy.

shocks. This can be exemplified by different reaction to crises despite comparable economic environments. Nevertheless, aside from the theoretical problem of Pareto ranking these equilibria, an empirical problem arises in such setups: how to set policy recommendation or fix a problem in an economy, if it is indeterminate where the economy is. This might explain perhaps why some policy recommendations have failed their purposes.

Relative prices are eventually what matters for theoretical analysis. As long as relative prices are determined, none of the problems raised above exist. Borrowing from Mas-Collell et. al.'s definition of Walrasian equilibrium, if I normalize one of the interest

rates, a normalized price vector $\left[\frac{\rho_{12}}{\rho_{01}}, \frac{\rho_{02}}{\rho_{01}} \right]$ constitutes a Walrasian equilibrium if and

only if it solves a system with 2 equations in 2 unknowns $\frac{\rho_{12}}{\rho_{01}}, \frac{\rho_{02}}{\rho_{01}}$ such that the vector

of excess demand is equal to zero, i.e. the market clearing conditions are satisfied. Mas-

Collell et al. introduce the concept of regularity to characterize such equilibrium:

An equilibrium price vector $p = (p_1, \dots, p_{L-1})$ to be regular if the $(L-1) \times (L-1)$ matrix of price effects $D\hat{z}(p)$ is nonsingular, that is has rank $(L-1)$. If every normalized equilibrium price vector is regular, I say that the economy is regular(...). Any regular (normalized) equilibrium price vector is locally isolated (or locally unique). That is there is an $\varepsilon > 0$ such that if $p' = p$, $p'_L = p_L = 1$ and $\|p' - p\| < \varepsilon$, then $z(p') \neq 0$.

Moreover if the economy is regular then the number of normalized equilibrium vector is finite. (Mas-Collell et al., 1995, pp.591)

Such a concept is helpful for characterizing local uniqueness. Using the concept of regularity or relative price determinacy allows me to compare different sets of equilibria, and to draw some comparative conclusions on the effect of imposing reserve requirements in such a model in the next chapter.

III.3.2. Description of equilibria

There exist 3 types of possible equilibria where $\hat{q}_{01} \geq 0$ $\hat{q}_{12} \geq 0$ $\hat{q}_{02} \geq 0$ and the perfect arbitrage (2.8) condition holds.⁹ Such a condition spans the relationship among the three interest rates as a sheet in the three dimensional space. Figure 3.2 displays this relationship.

I classify the three sets of equilibria according to the general behavior of the agents in the world economy: In set (a), the borrowing banks and the international lenders are at an interior solution. In set (b) the borrowing banks are at their interior solution but international lenders are willing to lend as much as possible. In set (c) The international lenders are at an interior solution but the banks are willing to borrow as much as possible. Before I proceed into explaining the cases, it is important to compare the general result with that of Chang and Velasco. Note that because of exogenous lending behavior, set (c) cannot arise along with $\rho_{01}\rho_{12} = \rho_{02}$ in Chang and Velasco (2000). I will show that set (c) is the most problematic because it exhibits a case of equilibrium with price irregularity. Note that the following always holds in equilibrium:

$$\hat{k} = e_0 + e_0^* \quad (3.8)$$

where \hat{k} is the equilibrium investment in the illiquid technology at $t=0$.

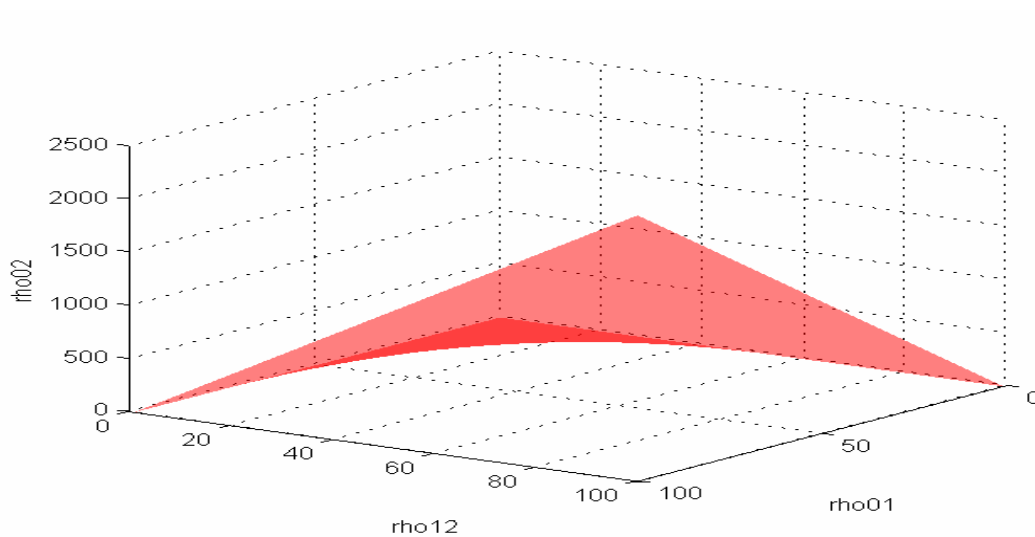


Fig.3.2. The perfect arbitrage condition

Set (a): This is what I call the standard case. Both the demands of the borrowers and supply of lenders are interior. Thus the first order conditions for both the borrower and the lender are equal to zero. Here the arbitrage condition (2.8) is derived from both the lender and borrower problems. This is a locally unique equilibrium point: The interest rates are functions of endowments and other parameters in the world economy:

The interest rates:

$$\hat{\rho}_{01} = \frac{R\beta^* e_1^*}{\lambda R e_0(1 + \beta^*) + e_2^* + R e_0^*} \quad (3.9)$$

$$\hat{\rho}_{12} = \frac{R e_0^* + e_2^* + \lambda R e_0(1 + \beta^*)}{\beta^* e_1^*} \quad (3.10)$$

$$\hat{\rho}_{02} = R \quad (3.11)$$

The above equilibrium has determinate and locally unique interest rates. Because of the perfect arbitrage condition, the quantities of debt are not determined: there are 2 equations with 3 unknown.

$$\hat{q}_{01} + \hat{q}_{02} = e_0^* \quad (3.12)$$

$$\hat{q}_{12} = \hat{\rho}_{01}\hat{q}_{01} + \frac{\lambda R e_0}{\hat{\rho}_{12}} \quad (3.13)$$

Note that the net new lending at $t=1$ $\{\hat{q}_{12} - \hat{\rho}_{01}\hat{q}_{01}\}$ and the total debt at $t=0$ are determined. The ranges for the first short-term debt, the second short-term and the long-term debt are $\hat{q}_{01} \in [0, e_0^*]$, $\hat{q}_{12} \in [\lambda\hat{\rho}_{01}e_0, (\hat{\rho}_{01}e_0^* + \lambda\hat{\rho}_{01}e_0)]$, and $\hat{q}_{02} \in [0, e_0^*]$ respectively. In this case all $\hat{c}_1, \hat{c}_2, \hat{c}_1^*, \hat{c}_2^*, \hat{k}$ have a unique solution¹⁶. Note that

¹⁶At the unique point, the depositors consumptions are: $\hat{c}_1 = \frac{R e_0 \beta^* e_1^*}{R e_0^* + e_2^* + \lambda R e_0(1 + \beta^*)}$, $\hat{c}_2 = R e_0$

The international lenders consumptions are: $\hat{c}_1^* = \frac{1}{1 + \beta^*} [e_1^* + \frac{\beta^* e_1^* (R e_0^* + e_2^*)}{R e_0^* + e_2^* + \lambda R e_0(1 + \beta^*)}]$

$\hat{c}_2^* = \frac{\beta^*}{1 + \beta^*} [e_2^* + R e_0^* + \frac{R e_0^* + e_2^* + \lambda R e_0(1 + \beta^*)}{\beta^* e_1^*}]$

considering the incentive compatibility constraint in (2.7), a sufficient condition for the

interior solution not to exist is $R < \frac{re_2^*}{\beta^* e_1^* - r[\lambda(1 + \beta^*)e_0 + e_0^*]} \equiv \mathbf{B}_a$.

Set (b): Only the borrower is at the interior solution and the lender is willing to lend

as much as possible. Here the arbitrage condition is derived from the fact that the

borrower is at the interior solution, but not the lender. The long-run interest rate is equal

to the long-run investment return rate $\hat{\rho}_{02} = R$, showing that the borrower is within the

interior solution. For the borrower $d_{01} \geq 0$, $d_{12} \geq 0$ and $d_{02} \geq 0$. The lender however

would like to lend as much as possible. In this case for the lender, $s_{01} = 0$, $s_{02} = e_0^*$

and $s_{12} \rightarrow \infty$. At equilibrium only the interest rate for the short-term debt that is issued at

$t=1$, would be high enough, i.e. higher than the consumption inter-temporal marginal rate

of substitution which is $\psi^* \equiv \frac{c_2^*}{\beta^* c_1^*}$ so that $\psi^* < \rho_{12}^s$, $\psi^* < \frac{\rho_{02}^s}{\rho_{01}^s}$. Only if the interest

rate is high enough would the lender be willing to lend as much as possible at $t=1$. Note

that in this case the lender will not lend short at $t=0$. One might wonder why not the

opposite scenario happens, i.e. why not lend only short i.e. $s_{01} = e_0^*$. The reason is that

for the perfect arbitrage condition to hold, when ρ_{12} is very high, then ρ_{02} is also high.

Correspondingly the equilibrium quantities are given by:

$$\hat{q}_{01} = 0 \tag{3.14}$$

$$\hat{q}_{02} = e_0^* \tag{3.15}$$

$$\hat{q}_{12} = \lambda \hat{\rho}_{01} e_0 \tag{3.16}$$

$$\hat{c}_1 = \hat{\rho}_{01} e_0 \quad (3.17)$$

$$\hat{c}_2 = R e_0 \quad (3.18)$$

$$\hat{c}_1^* = e_1^* - \lambda \hat{\rho}_{01} e_0 \quad (3.19)$$

$$c_2^* = e_2^* + R e_0^* + \lambda R e_0 \quad (3.20)$$

Note that this system is regular. If $\hat{\rho}_{01} = 1$, i.e. normalized, then $\hat{\rho}_{02} = \hat{\rho}_{12} = R$, there exists an equilibrium such that $\hat{d}_{01} = 0$, $\hat{d}_{02} = e_0^*$ and $\hat{d}_{12} = \lambda e_0$. Note that in terms of representation in the three-interest-rate space, this is a line on the sheet built by the perfect arbitrage condition. For this equilibrium to exist, the condition $\frac{c_2^*}{\beta^* c_1^*} < \hat{\rho}_{12}$

should hold. The latter condition doesn't hold if $R \leq \frac{e_2^*}{\beta^* e_1^* - e_0^* - (1 + \beta^*) \lambda e_0} \equiv \mathbf{B}_b$.

Set (c): Only the lender is at an interior solution but borrowers are willing to borrow as much as possible. (Each case depending on each asset): This case is characterized by the following condition:¹⁷

$$\rho_{01} \rho_{12} = \rho_{02} < R \quad (3.21)$$

The perfect arbitrage condition comes from the lender side. In this case any borrowing seems to be cheap enough and therefore, the borrower would like to borrow as much as possible. In the cases (c.1) and (c.2), the borrower demands as much as possible of only one of the short-term bonds and as much as possible of the long-term bond.

¹⁷ Note that none of the equilibria in sets (a), (b) and (c) has $\hat{\rho}_{02} > R$. This is so because if the long-term interest rate is higher than the gross return on investment, along with the perfect arbitrage condition, it would be beyond the borrower capacity to pay back the debt.

Case (c.1): At $t=0$ the borrower would like to borrow short-term within her interior set $d_{01} \geq 0$, but would like to borrow as much as possible long term $d_{02} \rightarrow \infty$. Knowing that the bank is playing in a large open economy, her high demand for long term debt would drive up the long-term interest rate and thus by the arbitrage condition, would increase the short-run interest rates. Therefore, the bank would like to make sure it will be able to pay back her first short-term debt, by rolling over the debt, and asking to borrow as much as possible at $t=1$, $d_{12} \rightarrow \infty$. On the other hand, the lender would like to limit its lending to an interior solution which bounds the high willingness of the borrower to borrow: Note that this is due to the fact that the gross interest rates are relatively low, but not too low to cut the market. In this case at equilibrium the sheet in the three dimensional space, described by the perfect arbitrage condition, has a bound such that $\hat{\rho}_{02} < R$, $\hat{\rho}_{01} \geq r$ and $\hat{\rho}_{12} < r$. The latter conditions are necessary for this equilibrium to exist: they show that the interest rate on the long-term debt and the second short-term debt would be relatively cheap, so that the demand of the bank for these bonds is very high. Notice that $\hat{\rho}_{01} \geq r$ indicates that the demand for d_{01} is interior. After markets clear, one can get the following two equalities from the bank demand for bonds and the supply of international lenders respectively:

$$\hat{q}_{12} - \hat{\rho}_{01}\hat{q}_{01} = \frac{\lambda}{(1-\lambda)R + \lambda\rho_{02}} [R e_0 + (R - \rho_{02})e_0^*] \quad (3.22)$$

$$\hat{q}_{12} - \hat{\rho}_{01}\hat{q}_{01} = \frac{\beta^* \hat{\rho}_{12} e_1^* - e_2^* - \hat{\rho}_{02} e_0^*}{\hat{\rho}_{12}(1 + \beta^*)} \quad (3.23)$$

In addition for $\hat{c}_1 > 0$ and $\hat{c}_2 > 0$, it must be that $\hat{q}_{12} > \rho_{01}\hat{q}_{01}$ or $\beta^* \hat{\rho}_{12} e_1^* - e_2^* - \hat{\rho}_{02} e_0^* > 0$. In other words if $\hat{\rho}_{01}$ is normalized to 1, or $\hat{\rho}_{12} = \hat{\rho}_{02}$, it is possible to get a normalized lower bound for $\hat{\rho}_{12}$ or $\hat{\rho}_{02}$:

$$\frac{e_2^*}{\beta^* e_1^* + e_0^*} < \hat{\rho}_{02} = \hat{\rho}_{12} \quad (3.24)$$

The lower bound for $\hat{\rho}_{12}$ reflects the supply side unwilling to buy the second short-term bond (or help a roll-over) at very low $\hat{\rho}_{12}$.

From (3.22) and (3.23), after equating the right-hand side, I get a non-linear equation with ρ_{12} and ρ_{02} . When $\hat{\rho}_{01}$ is normalized to 1, and when using the perfect arbitrage condition, one gets $\hat{\rho}_{12} = \hat{\rho}_{02}$. There the polynomial would have the following form:

$$a\hat{\rho}_{02}^2 + b\hat{\rho}_{02} + c = 0 \quad (3.25)$$

where $a = \lambda\beta^*(e_0^* + e_1^*)$, $b = \{(\beta^* e_1^* - e_0^*)(1 - \lambda)R - \lambda e_2^* - (1 + \beta^*)R\lambda(e_0^* + e_1^*)\}$

and $c = -e_2^*(1 - \lambda)R$. Here there is relative price determinacy. There are two equations with three unknown: the system is regular. However one of the equations is non-linear. Since $ac < 0$, and since $a > 0$, there is one root that is positive and one that is negative for (3.25). Therefore there is only one interest rate to solve (3.26). The normalized

system with $\hat{\rho}_{01} = 1$ has the interest rates $\hat{\rho}_{12} = \hat{\rho}_{02} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$. To compare with

the case were the lender was not at the interior solution, but the borrower was bound to

the interior solution, there the outcome was also a regular set. A sufficient condition for

this equilibrium not to exist from (3.21) and (3.24) is that $R < \frac{e_2^*}{\beta^* e_1 - e_0^*} = \mathbf{B}_{c1}$.

Case (c.2): At $t=0$, the borrower would like to borrow as much as possible of the first short-term bond $d_{01} \rightarrow \infty$ and the long-term bond $d_{02} \rightarrow \infty$ because both interest rates

are relatively cheap. In this case, the conditions (3.21), $\hat{\rho}_{01} < r < \frac{R}{r}$ and $\hat{\rho}_{12} \geq r$ hold.

$\hat{\rho}_{12} \geq r$ indicates that the demand for the second short-term borrowing is interior. Since all cases in set (c), the typical international lender is at the interior solution in the supply of all types of loans, equation (3.23) holds in this case (c.2). When markets clear, one get the following equation from the demand side:

$$\hat{q}_{12} - \hat{\rho}_{01} \hat{q}_{01} = \lambda \left[\frac{R}{\hat{\rho}_{12}} e_0 - \hat{\rho}_{01} e_0^* \right] \quad (3.26)$$

One gets the following relation between $\hat{\rho}_{02}$ and $\hat{\rho}_{12}$ so that

$$\hat{\rho}_{02} = \frac{R\lambda(e_0 + e_0^*)(1 + \beta^*) + e_2^* - \beta^* e_1^* \hat{\rho}_{12}}{e_0^*(\lambda(1 + \beta^*) - 1)} \quad (3.27)$$

This is a regular equilibrium; the first order conditions of the lender provide the perfect arbitrage condition. There are two equations with 3 unknown interest rates. If I normalize $\hat{\rho}_{01} = 1$, the perfect arbitrage condition entails $\hat{\rho}_{02} = \hat{\rho}_{12}$. Using (3.26) and (3.23), I find that equilibrium interest rates are such that:

$$\hat{\rho}_{12} = \hat{\rho}_{02} = \frac{R\lambda(e_0 + e_0^*)(1 + \beta^*) + e_2^*}{e_0^*(\lambda(1 + \beta^*) - 1) + \beta^* e_1^*} \quad (3.28)$$

For consistency with $\hat{\rho}_{02} < R$, using (3.28), the following condition should hold for the normalized system $R > \frac{e_2^*}{\beta^* e_1^* - \lambda e_0(1 + \beta^*) - e_0^*} \equiv \mathbf{B}_{c2}$. A sufficient condition for this equilibrium not to exist is $R \leq \mathbf{B}_{c2}$. To derive the latter condition, I assume $e_0^*(\lambda(1 + \beta^*) - 1) + \beta^* e_1^* > 0$. In case (c.2), when $d_{01} \rightarrow \infty$ and $d_{12} \geq 0$, the normalized system has a one relative price determinate equilibrium. $\hat{\rho}_{02}$ is a linear function of $\hat{\rho}_{12}$. (3.28) indicates that there is a monotonic relation between the interest rate on long-term debt and that born on the first short-term debt: if $\lambda(1 + \beta^*) > 1$, there is a negative relation. If $\lambda(1 + \beta^*) < 1$, there is a positive relation. The intuition is that if domestic depositors are sufficiently impatient, a higher $\hat{\rho}_{12}$ would lead to a lower $\hat{\rho}_{02}$ implying that d_{12} and d_{02} are indirectly substitutes for the domestic bank. Since at equilibrium $\hat{q}_{01} + \hat{q}_{02} = e_0^*$, a lower $\hat{\rho}_{02}$ means that the borrower will demand more of the long-term bond, but this means that in equilibrium she will demand less of the short-term bond. Thus a higher $\hat{\rho}_{12}$ would lead to a lower \hat{q}_{01} . Accordingly for low λ , in this equilibrium, the relation between d_{12} and d_{01} becomes like a relation between complements. Similarly for high λ , in this equilibrium the relation between d_{12} and d_{01} become like the relation between substitutes. I would like here to introduce the reader to the special feature of this model in anticipation of the chapter with crisis (chapter V). The new lending at $t=1$ will be complementary to d_{01} if λ is very low, because the possibility of the effect of the run by the $(1 - \lambda)$ potential runners will be low.

Case (c.3): The borrower is willing to borrow as much as possible, using all types of bonds: $d_{01} \rightarrow \infty$, $d_{02} \rightarrow \infty$ and $d_{12} \rightarrow \infty$.

Proposition 3: For the normalized $\hat{\rho}_{01} = 1$, a sufficient condition for the irregular

equilibrium (c.3) not to exist is $R < \frac{e_2^*}{e_0 \lambda (1 + \beta^*)} \equiv \mathbf{B}_{c3}$.

Proof.

Given the borrower's budget constraints, this cannot be the case unless there is some trade off between d_{01} and d_{12} which explains the bounds on d_{12} . From the borrower side,

$$D < \hat{q}_{12} - \hat{\rho}_{01} \hat{q}_{01} < E \quad (3.29)$$

$$\text{where } D \equiv \frac{\lambda \hat{\rho}_{01} [R e_0 + (R - \hat{\rho}_{02}) e_0^*]}{(1 - \lambda)R + \lambda \hat{\rho}_{02}} \text{ and } E \equiv \frac{\lambda (R - \hat{\rho}_{02}) e_0^* + \lambda R e_0}{\rho_{12}}.$$

The lender is at its interior solution so that

$$F \equiv \hat{q}_{12} - \hat{\rho}_{01} \hat{q}_{01} = \frac{\beta^*}{1 + \beta^*} e_1^* + \frac{\rho_{01} e_0^*}{1 + \beta^*} + \frac{e_2^*}{\rho_{12} (1 + \beta^*)} \quad (3.30)$$

Using (3.29) and (3.30), it must be that $D < F < E$. From $F < E$, with $\hat{\rho}_{01} = 1$, one can derive an upper bound for $\hat{\rho}_{12}$:

$$\hat{\rho}_{12} < \frac{\lambda R e_0 (1 + \beta^*) - e_2^*}{\beta^* e_1^* + e_0^* + \lambda e_0 (1 + \beta^*)} \quad (3.31)$$

Since $\hat{\rho}_{12} > 0$ must hold, the equilibrium would not exist when the upper bound in (3.31) is negative.

What does proposition 3 mean? If $\lambda(1 + \beta^*) < \frac{e_2^*}{R e_0}$, an equilibrium with $d_{01} \rightarrow \infty$

$d_{02} \rightarrow \infty$ and $d_{12} \rightarrow \infty$ would not exist. In this case, if λ is sufficiently low, there is no market equilibrium $\hat{\rho}_{12}$ that is low enough for the demand for d_{12} to be high enough. The intuition is as follows: d_{02} is very high. At $t = 2$, $\rho_{02}d_{02}$ will be paid back to international lenders. In addition, at $t = 2$, $(1 - \lambda)c_2$ goods will be paid by the bank to patient depositors. If a new borrowing at $t = 1$ is made, then an additional payment of $\rho_{12}d_{12}$ should be made to lenders. When $(1 - \lambda)$ is high, this would exceed at $t = 2$ the capacity of the bank to pay back all domestic depositors and international lenders claim.

Recall that all (c) cases satisfy (2.8). Although the bank is willing to borrow as much as possible, in general equilibrium these bonds have an upper bound. In the last of these three cases there is indeterminacy but the first two exhibit regularity. Unlike the case with exogenous lending, under the perfect arbitrage condition, equilibria with excessive demand for either of the short-term bonds separately arise with endogenous lending (see case c.1 and c.2). In general equilibrium, the interest rates are not exogenous: there will be low enough interest rates that clear the market and that are associated with an excessive quantity demanded of the short-term bonds. Intuitively, since it is a general equilibrium analysis, the total international lending in each period will not only depend on the bonds sold at that period but also on the sale of other bonds in other periods. Case (b) is the case where the borrower is in its interior solution set, therefore along with (2.8), the long-term interest rate is such that $\rho_{02} = R$. The existence of 2 equations with 3 prices renders such a set of indeterminate equilibria what is called a

regular equilibrium. Note that case (a), the locally unique point in interest rates but not in quantities, and case (b) the locally regular equilibrium has the bank behaving within the interior solution i.e. not willing to borrow as much as possible of any asset.

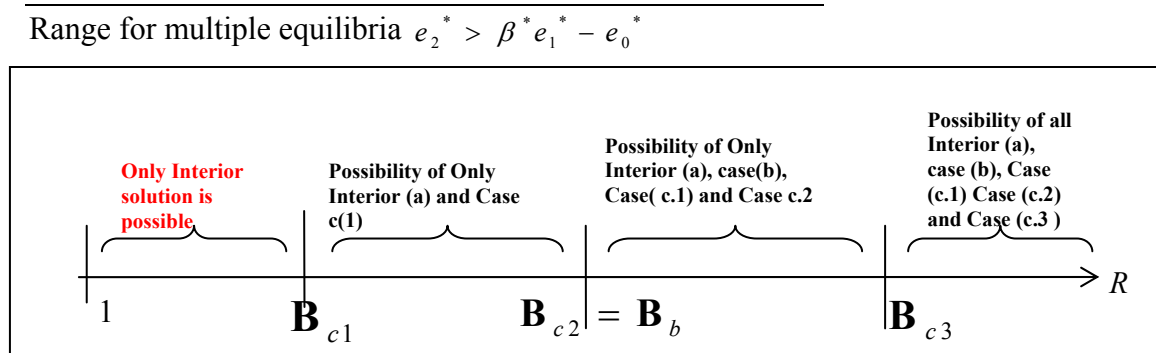


Fig.3.3. Range for multiple equilibria (i)

Figure 3.3 is constructed based on the derivation in section III.3.2 and serves only for illustration purpose. To construct figure 3.2 I assume that the conditions $e_2^* > \beta^* e_1^* - e_0^*$ and $\beta^* e_1^* > e_0^*$ hold. Note that I use the word possible when I talk about existence. The reason is that in section III.3.2 I derive only sufficient conditions for non-existence. There are more conditions to guarantee existence such as the incentive compatibility constraint. Note that the condition $B_{c2} = B_b > B_{c1}$ always holds. Note that since the endowment in the domestic country is assumed to be very low relative to the lending country, the assumption of $B_{c3} > B_{c2}$. In Figure 3.4, I

assume $e_2^* \leq \beta^* e_1^* - e_0^*$. There the region where there only the interior solution (a) exists, the only possibility is for $R < 1$, which is against the premises of this model. In the following section I illustrate these equilibria, with simulation results.

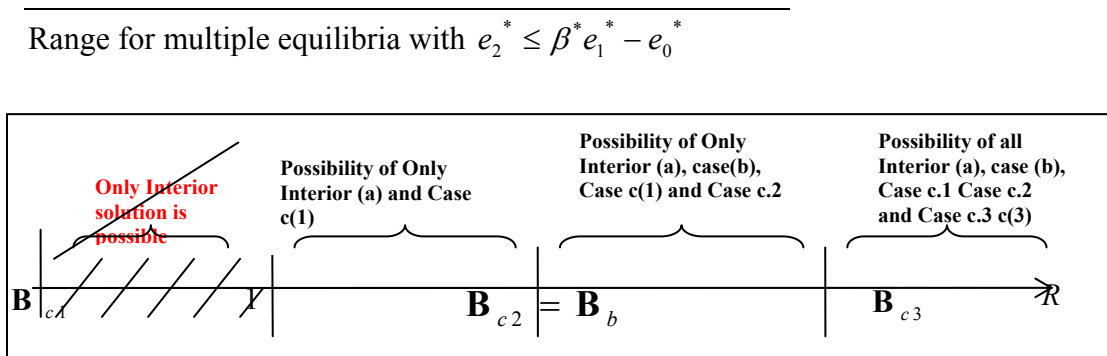


Fig.3.4. Range for multiple equilibria (ii)

III.4. Simulation results

In the following I show figures for each equilibrium case presented in section III.3.2, using the following parameter space:

$$e_0 = 2000, \beta^* = 1; e_0^* = 9500, e_1^* = 15000, e_2 = 15000 \text{ and } \lambda \in [0.1, 0.9] \text{ and}$$

$R \in [1.5, 2.3]$. The grid used was 0.1 for both the simulations over λ and R .

Figures 3.4 to 3.7 show the results for cases (a), (b), (c1) and (c2). Since c3 is an irregular set, it is impossible to pinpoint relative prices. Therefore I did not pursue any

simulation for that case. But one always can check whether the derived bound in proposition 3.1 would hold for any combination of (λ, R) given the endowments space.

Figure 3.5 shows the results for the interior solution. Because of the perfect arbitrage condition, it is clear that the line displaying the values for $\rho_{02} = R$ divides the space between ρ_{12} and ρ_{01} . Obviously in this parameter space, $\hat{\rho}_{01} < \hat{\rho}_{12}$. This is because at $t = 0$, the demand for the short-term debt is relatively elastic since there is no debt inherited. At $t = 1$, d_{01} is inherited, so the bank would need to borrow (renew its short-term debt) at $t = 1$ because of the illiquidity, described in this environment. Such a relatively inelastic demand leads to a higher interest rate. Note that $0 < \hat{\rho}_{01} < 1$ and $\hat{\rho}_{12} > 1$. In this goods' economy, the interest rate $0 < \hat{\rho}_{01} < 1$ will be accepted by international lenders because the goods are perishable. In addition international lenders do not have a storage technology. So for each unit they lend, they can retrieve $\hat{\rho}_{01}$ of the unit instead of zero. It is to be reminded that in the interior set the perfect arbitrage condition entails that quantities of the debt are not determined. So the second graph in the upper right displays second short-term debt d_{12} : "d12aa", "d12ab", "d12ac" correspond to the values of $\hat{q}_{01} = \{0, \frac{e_0^*}{2}, e_0^*\}$. Note that for $\hat{q}_{01} = e_0^*$, the slope of the \hat{q}_{12} with respect to λ and R is higher than for the other cases. In all three cases \hat{q}_{12} increases with λ and decreases with R . c_1 is lower than c_2 . For $r \leq 1$, the interior solution will always abide by the incentive compatibility constraint. It depends on the

magnitude of r, λ and R when $r > 1$. When it does not abide by the incentive compatibility constraint, then the equilibrium does not exist.

Figure 3.6 shows the results for the case (b). Because of the presence of relative price determinacy, I normalize $\hat{\rho}_{01} = 1$ and then simulate the model for this case. In case (b) $\hat{q}_{01} = 0$ and $\hat{q}_{02} = e_0^*$. There is no initial short-term borrowing. Despite that, there is still short-term borrowing at $t = 1$ $\hat{q}_{12} > 0$. So in this equilibrium \hat{q}_{12} is not a mere renewal or financing of an initial debt, it finances also impatient depositors' withdrawal. Such equilibrium is comparable to case 2.2 in exogenous lending setup of chapter II. This is so because the Diamond and Dybvig (1983) setup creates a need for liquidity financing at $t = 1$. This case exists in the parameter space chosen.

Figure 3.7 shows the results for case (c.1). This case exhibits high second short-term borrowing and high long-term borrowing with interior $\hat{q}_{01} > 0$. Note that consumptions of domestic depositors are positive only for high R and λ . In fact this equilibrium exists only for $R = 2.3$. With high λ , c_2 becomes higher than c_1 , in which case $c_2 > rc_1$ is more likely to be fulfilled. Figure 3.8 shows case (c.2). There the same analysis presented on figure 3.7 can be given.

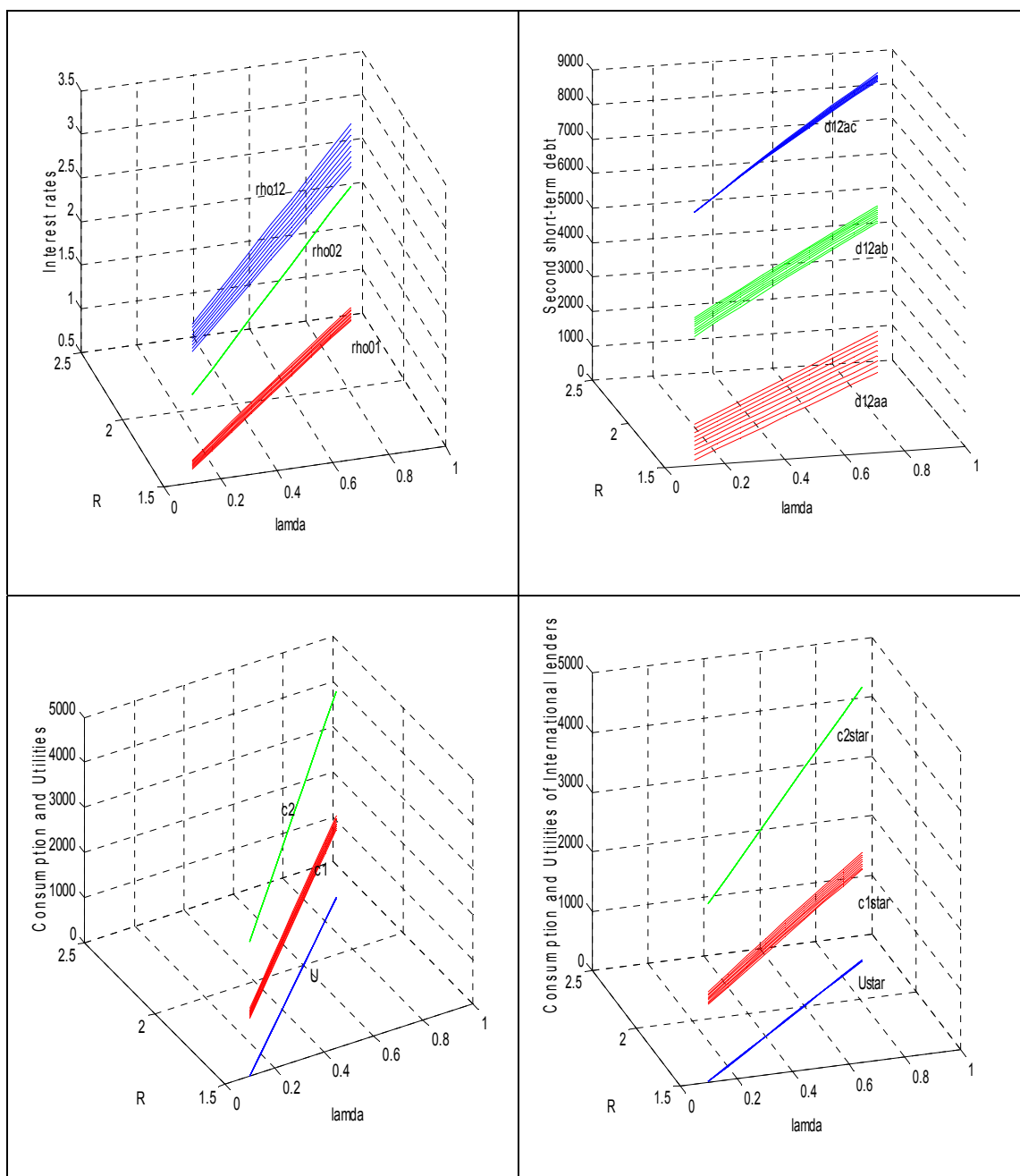


Fig.3.5. Set (a)

The graph in the upper right displays second short-term debt d_{12aa} , d_{12ab} , d_{12ac} corresponding to the values of $d_{01} = \{0, \frac{e_0^*}{2}, e_0^*\}$.

The simulation results show that Cases (c.1) and (c.2) occur for very high R . In the analysis of existence, there are more conditions to guarantee existence such as the incentive compatibility constraint so that the ranking of the possibility of equilibria may not be the same as in figures 3.1 and 3.2.

III.5. Conclusion

When I allow for endogenous lending behavior, credit rationing will come out endogenously: $\hat{q}_{01} + \hat{q}_{02} = e_0^*$ and depending on each set I have derived bounds for \hat{q}_{12} . The simulation results and the analytical description of equilibria show that cases, where there is high demand for short-term debt d_{01} or d_{12} , can exist for R higher than some bounds.(see cases c.1 and c.2) This is different from the results with exogenous lending for two reasons: First in chapter II, low R entailed a lower cost of liquidation and thus the choice of an equilibrium with only short-term debt arises. In chapter III, a lower R implies both a lower cost of liquidation and a lower $\hat{\rho}_{02} = \hat{\rho}_{01}\hat{\rho}_{12}$ paid to international lenders at $t=1$: Thus with endogenous lending interest rates incorporate both the opportunity cost of “liquidity” and the “impatience” factor. Second, in the case with exogenous lending, high demand for short-term borrowing could not be concurrent with high demand for long-term debt because the form of the credit constraints was imposed exogenously. There a high demand for short-term debt implied that $\hat{d}_{01} = f_0$ and $\hat{d}_{02} = 0$. In this chapter, in both cases c.1 and c.2, where there is high debt for one of the short-term debt, the demand for long-term debt is very high ($d_{02} \rightarrow \infty$). The tilt towards

longer or shorter maturity will depend on the equilibrium interest rates. In the following chapter I look at possible equilibria under reserve requirements imposed on the bank.

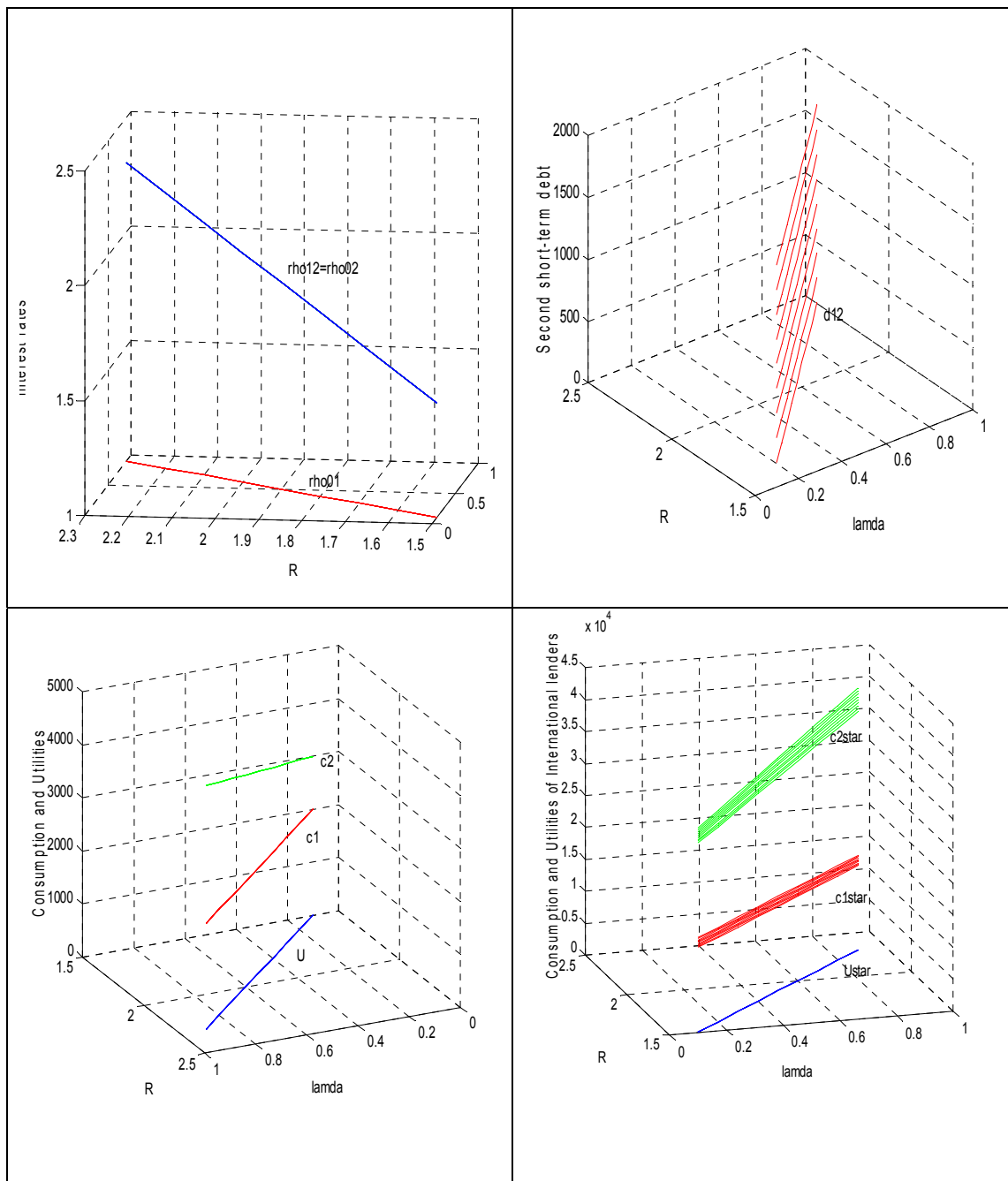


Fig.3.6. Set (b)

In this case because of lack of determinacy but the presence of relative price determinacy, I assume for the simulation $\rho_{01} = 1$. The graph in the upper right displays second short-term debt d_{12aa} , d_{12ab} , d_{12ac} corresponding to the values of $d_{01} = \{0, \frac{e_0^*}{2}, e_0^*\}$.

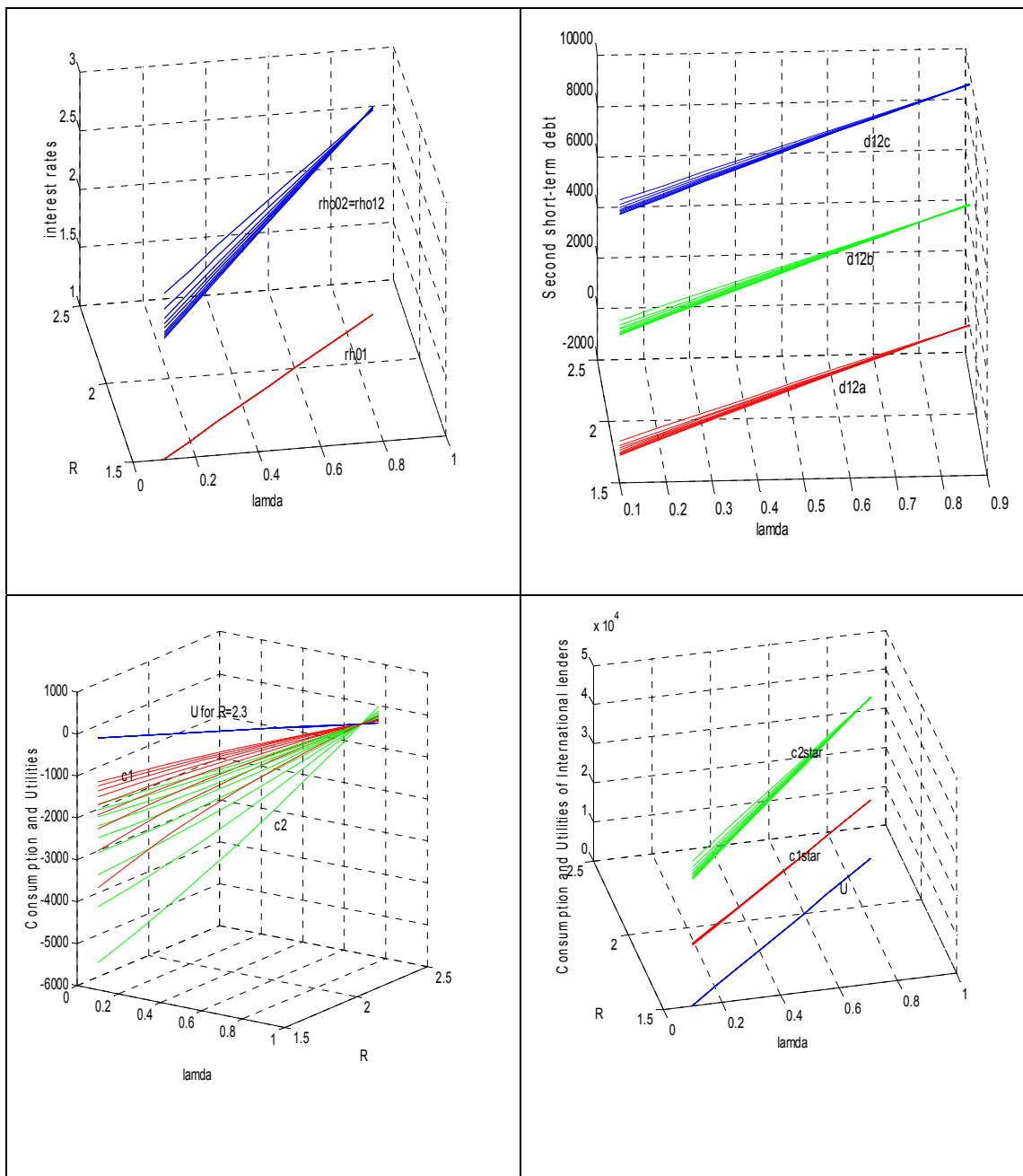


Fig.3.7. Set (c) Case c.1

In this case because of lack of determinacy but the presence of relative price determinacy, I assume for the simulation $\rho_{01} = 1$. The graph in the upper right displays second short-term debt d_{12aa} , d_{12ab} , d_{12ac}

corresponding to the values of $d_{01} = \{0, \frac{e_0^*}{2}, e_0^*\}$.

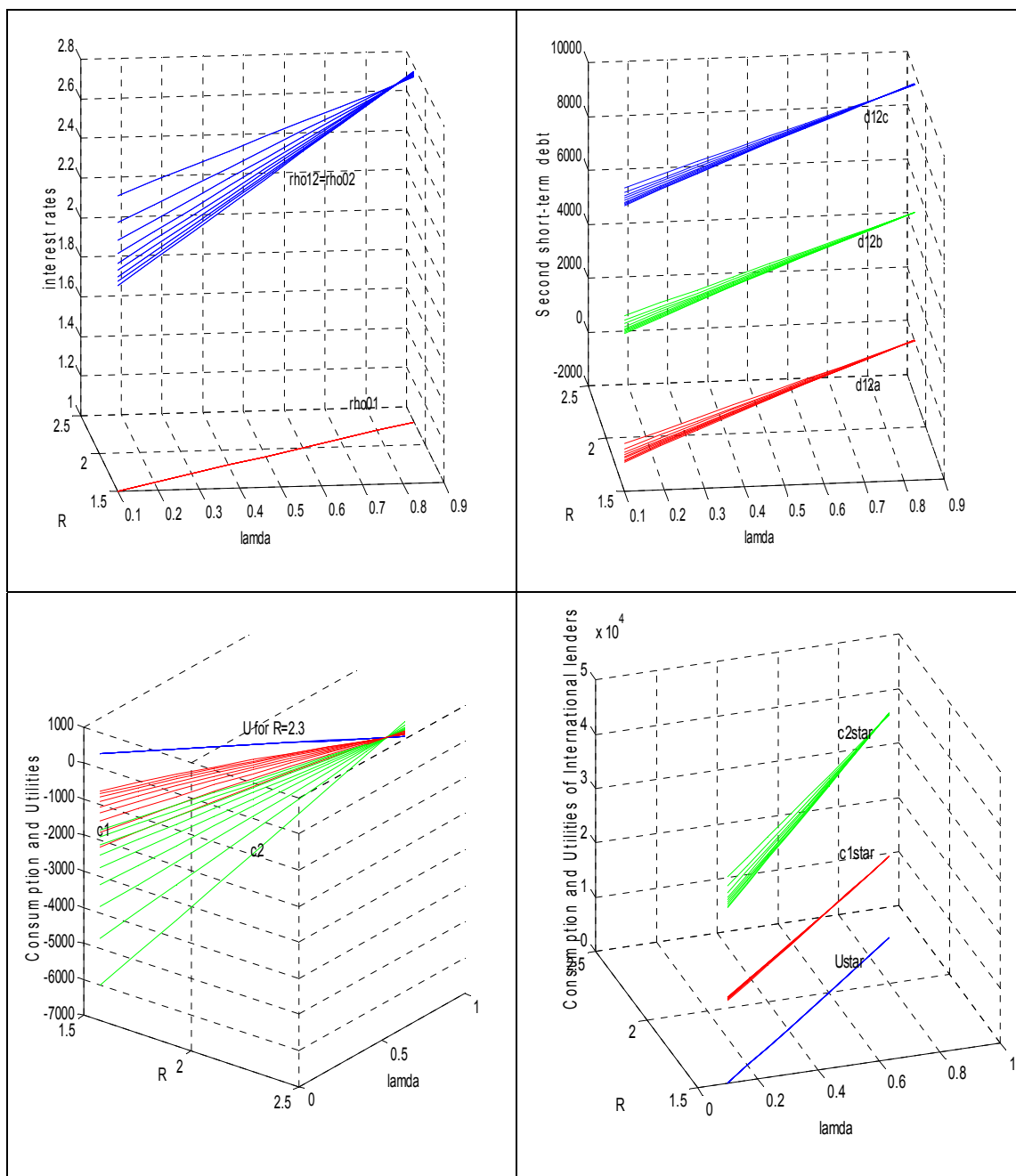


Fig.3.8. Set (c) Case c.2

CHAPTER IV

RESERVE REQUIREMENTS AND ENDOGENOUS LENDING

In this chapter I look at the effect of special type of capital controls: date- and maturity- specific reserve requirements. In this chapter, like in earlier chapters, I assume there are no bank runs. In the next chapter, I will consider the role of such type of capital controls, when a bank run is possible. In Aizenman and Turnovsky (2002), the reserve requirements can be imposed on either the lender or the borrower and they are independent of date and maturity. In this chapter, I assume that reserve requirements are imposed on the borrowing bank and are date- and maturity- specific as in the Chilean case.

IV.1. The domestic bank problem with reserve requirements

The domestic economic authority imposes the reserve-requirement rates exogenously. It allows them to be both date-specific and maturity-specific. For instance, when the bank borrows from abroad, it must deposit in the central bank a fraction θ_{01}, θ_{12} and θ_{02} on d_{01}, d_{12} and d_{02} , respectively. In other words at $t=0$, a fraction θ_{01} of the first short-term debt is placed as reserves at the central bank and will be returned to the domestic bank at $t=1$, i.e. when the debt matures. At $t=0$, a fraction θ_{02} of the long-term debt is placed as reserves at the central bank and will be returned to the bank at $t=2$. At $t=1$, a fraction θ_{12} of the second short-term debt is placed at the central bank as reserves

and will be returned to the bank at $t=2$.¹⁸ Clearly, $\theta_{01} \in [0,1]$, $\theta_{12} \in [0,1]$ and $\theta_{02} \in [0,1]$.

Then the budget constraints of the bank become:

$$k \leq e_0 + (1 - \theta_{01})d_{01} + (1 - \theta_{02})d_{02} \quad (4.1)$$

$$\lambda c_1 + \rho_{01}d_{01} \leq (1 - \theta_{12})d_{12} + rl + \theta_{01}d_{01} \quad (4.2)$$

$$(1 - \lambda)c_2 + \rho_{12}d_{12} + \rho_{02}d_{02} \leq R(k - l) + \theta_{12}d_{12} + \theta_{02}d_{02} \quad (4.3)$$

In anticipation of what the reader will find in chapter V, I would like to point out that, by looking at the budget constraints (4.1), (4.2) and (4.3), it is possible to distinguish two roles that reserve requirements may play if a bank run at $t = 1$ occurs. First, at $t=0$, a fraction θ_{01} and a fraction θ_{02} are placed as reserves for every unit of d_{01} and d_{02} , and therefore investment in k is penalized. Thus at $t=0$, the reserve requirement rate θ_{01} and θ_{02} operates like a tax. Similarly at $t=1$, θ_{12} plays the role of a “tax”. However at $t=1$, a fraction θ_{01} of the maturing short-term debt is returned as reserves and guarantees at least a partial payment of the debt to the international lenders, reducing the need for early liquidation of the long term asset. This is similar to the role of “liquidity”. Note that when θ_{12} is applied to any new issuance of debt at $t=1$, the new borrowing which is financing any liquidity shortage would need to be higher. This could precipitate a crisis if the demand for short-term borrowing is not satisfied by the supply side. At $t=2$, fractions θ_{12} and θ_{02} of the respective short-term and long-term maturing bonds are returned to

¹⁸ Note that in this setup the focus is on the case in which reserve requirements are returned to the bank when the debt matures, neither before nor after. Also it is assumed for simplicity that the reserve requirement does not earn any interest when held at the central bank. This is consistent with reality in which interest rates on reserve requirements are relatively lower than on other assets. One can think of the reserve requirement as a risk-less technology which gives back the good when the debt matures at a gross rate of return equal to one.

the domestic bank. This is again the “liquidity” role of θ_{12} and θ_{02} . Notice that for a single date- and maturity- specific reserve requirement rate, the “liquidity” role and the “tax” role work in opposite directions and at two different dates. It is therefore an interesting policy question to ask which of the two effects dominates and under which circumstances.¹⁹ In the next sections, I identify the sets of equilibria that arise with reserve requirements. Then I compare them with the sets without reserve requirements. In this chapter I assume there is no bank run.

IV.2. Equilibria

The main concern in this chapter is to look at how imposing reserve requirements on external debt change the behavior of the borrowing bank. In appendix C, I derive the relevant conditions for the borrowing bank optimization problem. In this section, I proceed by assuming logarithmic utility functions for both lenders and borrowers. In this economy, a competitive equilibrium is a set of interest rates $\{\hat{\rho}_{01}, \hat{\rho}_{12}, \hat{\rho}_{02}\}$, a set of international bonds’ allocations $\{\hat{q}_{01}, \hat{q}_{12}, \hat{q}_{02}\}$, a set of allocations for the typical domestic bank $\{\hat{k}, \hat{c}_1, \hat{c}_2, d_{01}, d_{12}, d_{02}\}$ and a set of allocations for the typical international lender $\{\hat{c}_1^*, \hat{c}_2^*, s_{01}, s_{12}, s_{02}\}$ given $\{e_0, e_0^*, e_1^*, e_2^*, \lambda, \beta^*\}$ and the reserve requirements rates $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ such that:

¹⁹ In anticipation of the analysis in chapter V, the “liquidity” role is an ex-ante preventive mechanism of a crisis. But in the event of a crisis, the “tax” role might create a worse outcome because it could make it more difficult to borrow.

i. $\{\hat{k}, \hat{c}_1, \hat{c}_2, d_{01}, d_{12}, d_{02}\}$ solve the domestic bank problem of maximizing (2.1)

subject to (4.1), (4.2) (4.3) and (2.7).

ii. $\{\hat{c}_1^*, \hat{c}_2, s_{01}, s_{12}, s_{02}\}$ solve the international lender problem of maximizing (3.1)

subject to (3.2), (3.3) and (3.4).

iii. Markets clear: $d_{01} = s_{01} = \hat{q}_{01}, d_{12} = s_{12} = \hat{q}_{12}, d_{02} = s_{02} = \hat{q}_{02}$.

Table (4.1) summarizes all possible cases with or without reserve requirements. In the remainder of this section, I will explain each set of equilibria separately. Note that (2.8) for international lenders is both the pre- and post-reserve requirement perfect arbitrage condition. For comparability purpose with the case without reserve requirement, I focus on the case where the post-reserve requirement arbitrage condition for the lender, i.e. equation (2.8) always holds. Note that under this condition, by assumption, there is no case where the borrower is at its interior solution but the lender is not. For the borrower the post-reserve requirement perfect arbitrage condition is the following:

$$\frac{(\rho_{01} - \theta_{01})}{(1 - \theta_{01})} \cdot \frac{(\rho_{12} - \theta_{12})}{(1 - \theta_{12})} = \frac{(\rho_{02} - \theta_{02})}{(1 - \theta_{02})} \quad (4.4)$$

In addition all the derived condition for the existence should be consistent with (4.4) and (2.8). The results in this section are to be compared with (a) and (c) of chapter III. Case (b) is one in which the condition (4.4) would hold but not (2.8), because the borrower is interior but not the lender: this case is not of much interest unlike (c) because it displays no indeterminacy. The listed equilibria are consistent with both (4.4) and (2.8). I will group the equilibria like before according to the general behavior of the agents in the world economy: set (a') both the borrowing banks and the international lenders are at an

interior solution. (c') The international lenders are at an interior solution but the banks are willing to borrow as much as possible. Case (c') will be divided into 6 subsets. To characterize each set I will use the concept of indeterminacy I developed in chapter III, in section III.3.1.

Set (a') Both lenders and borrowers are at their interior solutions: $s_{01} = d_{01} = \hat{q}_{01} \geq 0$, $s_{02} = d_{02} = \hat{q}_{01} \geq 0$ and $s_{12} = d_{12} = \hat{q}_{12} > 0$. There exist at most two possible equilibria depending on the combination of reserve requirements. The interest rates satisfy 3 equations: (2.8), (4.4) and the following:

$$\hat{\rho}_{02} = R(1 - \theta_{02}) + \theta_{02} \quad (4.5)$$

(4.5) is derived from the first order condition with respect to d_{02} equal to zero. Note that without reserve requirements $\hat{\rho}_{02} = R$, a higher long-term interest rate than with reserve requirements. Thus the interest rate $\hat{\rho}_{12}$ solves the quadratic equation

$$\varsigma_1 \hat{\rho}_{12}^2 + \varsigma_2 \hat{\rho}_{12} + \varsigma_3 = 0 \quad (4.6)$$

where $\varsigma_1 = \theta_{01}$, $\varsigma_2 = R[(1 - \theta_{12})(1 - \theta_{01}) - (1 - \theta_{02})] - (\theta_{02} + \theta_{01}\theta_{12})$ and $\varsigma_3 = [R(1 - \theta_{02}) + \theta_{02}]\theta_{12}$.

In order to guarantee that a unique set of interest rates arises rather than two sets of interest rate with the imposition of reserve requirement, the policymaker would implement a combination of $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ such that a unique interest rate $\hat{\rho}_{12}$ would solve equation (4.6), following the rule:

$$\varsigma_2^2 = 4\varsigma_1\varsigma_3 \quad (4.7)$$

Table 4.1.

Comparative summary between the equilibria without and with reserve requirements²⁰

Demand	Supply	Without Reserve Requirements	With Reserve Requirements
$d_{01} \geq 0$ $d_{02} \geq 0$ $d_{12} \geq 0$	$s_{01} \geq 0$ $s_{02} \geq 0$ $s_{12} \geq 0$	(a) Unique prices Indeterminate quantities	(a') 2 types of bifurcation depending on the combination reserve requirements (2 or unique sets of prices) Determinate quantities.
$d_{01} \geq 0$ $d_{02} \geq 0$ $d_{12} \geq 0$	$s_{01} = 0$ $s_{02} = e_0^*$ $s_{12} \rightarrow \infty$	(b) Relative price determinacy Determinate quantities	Non-existent
$d_{01} \geq 0$ $d_{02} \rightarrow \infty$ $d_{12} \rightarrow \infty$	$s_{01} \geq 0$ $s_{02} \geq 0$ $s_{12} \geq 0$	(c.1) Relative price determinacy, Indeterminate quantities	(c'.1) Price determinacy Determinate quantities
$d_{01} \rightarrow \infty$ $d_{02} \rightarrow \infty$ $d_{12} \geq 0$	$s_{01} \geq 0$ $s_{02} \geq 0$ $s_{12} \geq 0$	(c.2) Relative price determinate Indeterminate quantities	(c'.2) Price determinacy Determinate quantities
$d_{01} \rightarrow \infty$ $d_{02} \rightarrow \infty$ $d_{12} \rightarrow \infty$	$s_{01} \geq 0$ $s_{02} \geq 0$ $s_{12} \geq 0$	(c.3) Relative price indeterminacy Indeterminate quantities	(c'.3) Relative price determinacy Indeterminate quantities
$d_{01} = 0$ $d_{02} \rightarrow \infty$ $d_{12} \rightarrow \infty$	$s_{01} \geq 0$ $s_{02} \geq 0$ $s_{12} \geq 0$	Non-existent	(c'.4) Relative price determinacy Determinate quantities
$d_{01} = 0$ $d_{02} \geq 0$ $d_{12} \rightarrow \infty$	$s_{01} \geq 0$ $s_{02} \geq 0$ $s_{12} \geq 0$	Non-existent	(c'.5) Relative prices determinacy Determinate quantities
$d_{01} \geq 0$ $d_{02} = 0$ $d_{12} \rightarrow \infty$	$s_{01} \geq 0$ $s_{02} \geq 0$ $s_{12} \geq 0$	Non-existent	(c'.6) relative price determinate Does not exist with the condition in proposition 3

To derive analytically the implied relation between the three reserve requirements in equation (4.7) is an arduous task. However in section IV.3, I show simulation results for

²⁰ I assume for comparability that all equilibria are consistent with both (2.8) and (4.4). This is why case (b) does not have its counterpart after reserve requirement are introduced.

the interior sets. The simulation results show that for (4.7) to hold with non-negative borrowing, the combination of reserve requirements satisfies always the following condition

$$\theta_{02} > \theta_{12} > \theta_{01} \quad (4.8)$$

I will discuss later the simulation results in details. In any case with reserve requirements, unlike the case without reserve requirements, the bonds are determined by the following equations at the interior solution for the equilibrium interest rates:

$$\hat{q}_{01} = \frac{(1 - \theta_{12})}{\theta_{12} \hat{\rho}_{01}} \left[\frac{\beta^* e_1^*}{1 + \beta^*} - \frac{e_2^*}{\hat{\rho}_{12}} - \hat{\rho}_{01} e_0^* - \frac{\lambda R e_0}{\rho_{12} - \theta_{12}} \right] \quad (4.9)$$

$$\hat{q}_{02} = e_0^* - \hat{q}_{01} \quad (4.10)$$

$$\hat{q}_{12} = \frac{\hat{\rho}_{01} - \theta_{01}}{1 - \theta_{12}} \hat{q}_{01} + \frac{\lambda R e_0}{\hat{\rho}_{12} - \theta_{12}} \quad (4.11)$$

Note that at the interior solution without reserve requirements, there was an indeterminate set of the quantities. Note that for the case without reserve requirements, a unique point for interest rates exists at the interior set. The number of equilibrium was infinite because of indeterminate quantities. Thus, a combination of non-zero $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ could cause bifurcations towards two equilibria or a unique equilibrium, depending on whether the vector $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ is arbitrary or satisfy (4.7). A bifurcation is defined in this setup which is not dynamic, as a change in number of equilibria when the values of $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ change.²¹ I have identified two possibilities when

²¹ Bifurcations of equilibria are of three types “when structural parameters change, that is, in changes in the number of steady states, their stability type, and the nature of orbits near a given equilibrium.”(Azariadis

$\{\theta_{01}, \theta_{12}, \theta_{02}\}$, change from zero to some positive. If (4.7) is satisfied, then a bifurcation towards a unique equilibrium arises. In the simulation results I give an illustration of the two cases. The following figure is not derived analytically it is only to be used for the purpose of illustration. Let $\hat{q} \equiv \{\hat{q}_{01}, \hat{q}_{12}, \hat{q}_{02}\}$, and $\hat{\rho} = \{\hat{\rho}_{01}, \hat{\rho}_{12}, \hat{\rho}_{02}\}$ be the vectors of equilibrium quantities and interest rates, and $\theta \equiv \{\theta_{01}, \theta_{12}, \theta_{02}\}$ the vector of reserve requirements (structural parameters). I illustrate in figure 4.1 how different bifurcations may occur. Note that when there are no reserve requirements, there is a unique set of interest rates but indeterminate quantities. Thus the number of equilibria is infinite when there are no reserve requirements. When the set of reserve requirement changes from zero to an arbitrary $\theta \in [\theta_x, \theta_y) \cup (\theta_z, 1]$, then there is a bifurcation towards two sets of equilibria. When it changes to $\theta \in [\theta_y, \theta_z]$, a bifurcation towards a unique equilibrium occurs. When $\theta \in [\theta_y, \theta_z]$, equation (4.7) is satisfied and thus the inequality (4.8) holds.

(c')The lender is at the interior solution. The borrower is not at the interior solution: The cases (c'.1), (c'.2), and (c'.3) are the cases that are directly comparable with the sets (c.1), (c.2) and (c.3). With the imposition of reserve requirements three subsets of equilibria arises additionally in comparison with the case without reserve requirements: these are (c'.4), (c'.5) and (c'.6).

Equilibria (c'.1), (c'.2), (c'.3) and (c'.4) are characterized by the following condition:

$$\hat{\rho}_{02} < R(1 - \theta_{02}) + \theta_{02} \quad (4.12)$$

1993, pp.91) In this setup, which is not dynamic, I am interested in the first type: the change in number of equilibria.

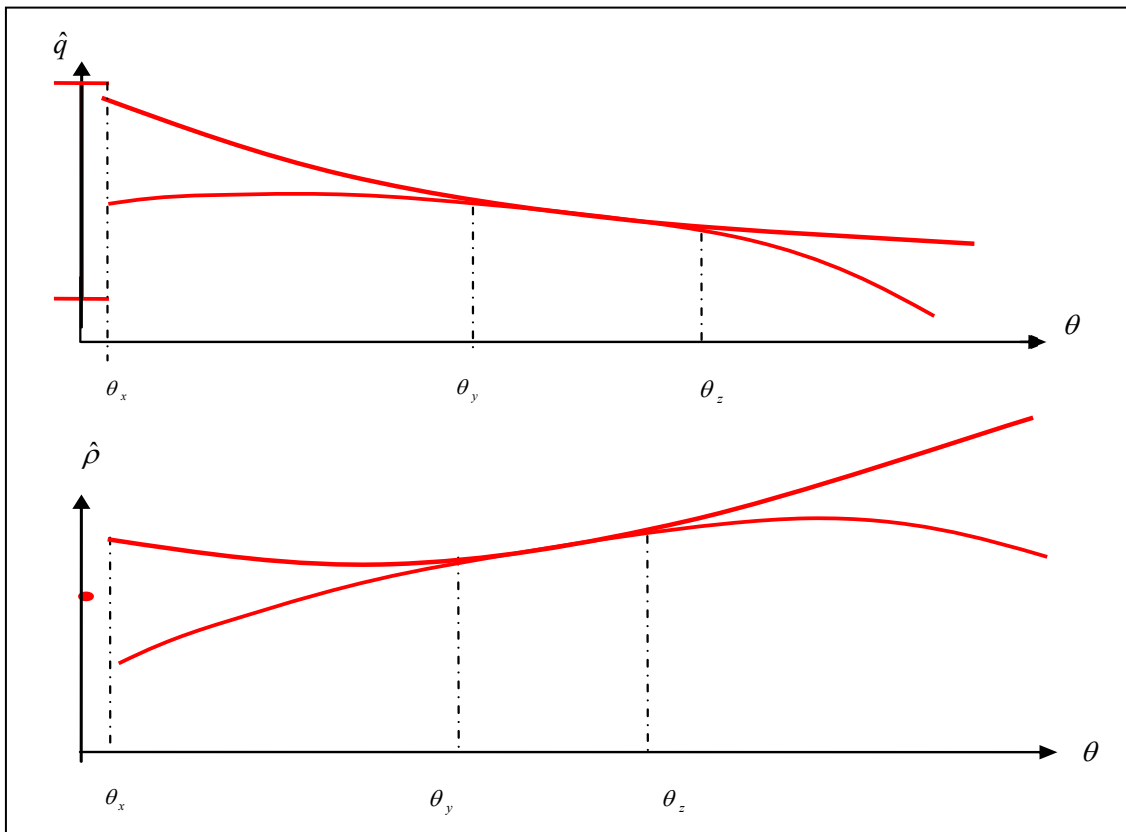


Fig.4.1. Bifurcations with reserve requirements at the interior set

Case (c'.1) There exists a set in which the bank is willing to borrow as much as possible long-term debt at $t=0$ $d_{02} \rightarrow \infty$ and short at $t=1$ $d_{12} \rightarrow \infty$ but is within her interior solution for $d_{01} \geq 0$. Here the long term interest rate is such that $\hat{\rho}_{02} < R(1 - \theta_{02}) + \theta_{02}$. This case is comparable to the case without reserve requirement (c.1) where $\hat{\rho}_{02} < R$. In addition the second short-term interest rate is relatively cheap: $\hat{\rho}_{12} < r(1 - \theta_{12}) + \theta_{12}$. One

can see that here there is more restriction on the set of possible long-term interest rate and second- short-term interest rates. Unlike the case without reserve requirements, given the normalized equilibrium sets of interest rates, bonds are determined:

$$\hat{q}_{01} = \frac{1}{\gamma_1} \left[\frac{\lambda(\rho_{01} - \theta_{01})(1 - \theta_{02}) \left[R - \frac{\rho_{02} - \theta_{02}}{1 - \theta_{02}} \right] e_0^* + R e_0 \lambda (\rho_{01} - \theta_{01})}{R(1 - \theta_{01})(1 - \theta_{12})(1 - \lambda) + (\rho_{12} - \theta_{12})(\rho_{01} - \theta_{01})\lambda} + \frac{\beta^* \rho_{12} e_1^* - e_2^* - \rho_{02} e_0^*}{\rho_{12}(1 + \beta^*)} \right] \quad (4.13)$$

where $\gamma_1 = \hat{\rho}_{01} \frac{(\hat{\rho}_{01} - \theta_{01})[(1 - \theta_{01}) - \lambda(1 - \theta_{02})]R + \lambda(\hat{\rho}_{01} - \theta_{01})(\hat{\rho}_{02} - \theta_{02})}{R(1 - \theta_{12})(1 - \theta_{01})(1 - \lambda) + (\hat{\rho}_{12} - \theta_{12})(\hat{\rho}_{01} - \theta_{01})\lambda}$. In addition the quantities

for the other bonds $\{\hat{q}_{02}, \hat{q}_{12}\}$ can be determined from (3.2) and (3.25). Considering the perfect arbitrage case from both the borrowers (4.5) and the lenders (4.4), there are two equations for relative prices, and thus they are determined. This is a similar result to the case without reserve requirements. In this case given the normalized set of interest rates, quantities are also determined unlike the case without reserve requirements.

Case (c'.2) At $t=0$, the borrower would like to borrow as much as possible of the first short-term bond $d_{01} \rightarrow \infty$ and the long-term bond $d_{02} \rightarrow \infty$ because both interest rates $\hat{\rho}_{01} < r(1 - \theta_{01}) + \theta_{01} < r$ and $\hat{\rho}_{02} < R(1 - \theta_{02}) + \theta_{02} < R$ are relatively cheap. Note also that the bounds are more restrictive than in the case without reserve requirement. Here the demand for the second short-term debt is an interior solution. $d_{12} \geq 0$. In this case also quantities are determined given the interest rates:

$$\hat{q}_{01} = \frac{1}{\gamma_2} \left\{ -[\lambda R e_0 + \lambda(R(1 - \theta_{02}) - (\hat{\rho}_{02} - \theta_{02}))e_0^*] + (\hat{\rho}_{12} - \theta_{12}) \left(\frac{\beta^* e_1^*}{1 + \beta^*} - \frac{e_2^*}{\hat{\rho}_{12}(1 + \beta^*)} - \frac{\hat{\rho}_{01} e_0^*}{1 + \beta^*} \right) \right\} \quad (4.14)$$

where $\gamma_2 = \lambda R(\theta_{02} - \theta_{01}) + (1 - \lambda) \frac{(\hat{\rho}_{01} - \theta_{01})(\hat{\rho}_{12} - \theta_{12})}{(1 - \theta_{12})} + \lambda \frac{(\hat{\rho}_{02} - \theta_{02})}{(1 - \theta_{02})} - (\hat{\rho}_{12} - \theta_{12})\hat{\rho}_{01}$. In addition

the quantities for the other bonds $\{\hat{q}_{02}, \hat{q}_{12}\}$ can be determined from (3.2) and (3.25).

Considering the perfect arbitrage case from both the borrowers (4.5) and the lenders (4.4), there are two equations for relative prices, and thus they are determined. This is a similar result to the case without reserve requirements. Unlike the case without reserve requirements, in this case given the normalized set of equilibrium interest rates, quantities are also determined.

Case (c'.3) The borrower is willing to borrow as much as possible, using all types of bonds: $d_{01} \rightarrow \infty$, $d_{02} \rightarrow \infty$ and $d_{12} \rightarrow \infty$. However, given the borrower's budget constraints, this cannot be the case unless there is some trade off between d_{01} and d_{12} which explain the bounds on d_{12} . For this case, all three interest rates are perceived by the borrower as relatively cheap because $\hat{\rho}_{02} < R(1 - \theta_{02}) + \theta_{02} < R$, $\hat{\rho}_{01} < r(1 - \theta_{01}) + \theta_{01} < r$, $\hat{\rho}_{12} < r(1 - \theta_{12}) + \theta_{12} < r$. Note that the post-reserve requirement no arbitrage condition (4.5) may or may not hold. If it does not hold, c'.3 is irregular with the only equation (4.4), however the bounds on the 3 interest rates are smaller, reducing the scope of indeterminacy. In comparison with the case without reserve requirements, I am focusing though on the cases when both (4.4) and (4.5) would hold, thus with two equations and three unknown interest rates, relative interest rates are determined. Without reserve requirements, this was an irregular set: with reserve requirements, it becomes a regular one.

I have shown in the three cases (c'.1), (c'.2), and (c'.3) that reserve requirements add determinacy to the system of interest rates and quantities of bonds when perfect arbitrage is allowed in comparison with (c.1), (c.2) and (c.3). Additional 3 cases are added when date- and maturity- specific reserve requirements are imposed. (c'.4) and (c'.5) are equilibria where at $t = 0$, only long-term borrowing occurs. (c'.6) is a case where the domestic bank borrows only short-term debt.

Additional ones:

Case (c'.4) the borrower does not want to borrow short at $t=0$. She wants to borrow long as much as possible $d_{02} \rightarrow \infty$. In addition, the bank would like to borrow as much as

possible at $t=2$ $d_{12} \rightarrow \infty$. Therefore it is found that $\frac{\hat{\rho}_{12} - \theta_{12}}{1 - \theta_{12}} \frac{\hat{\rho}_{01} - \theta_{01}}{1 - \theta_{01}} < R$ and

$\hat{\rho}_{02} < R(1 - \theta_{02}) + \theta_{02}$, which limits the space between the three interest rates. The set of prices with (4.4) and (4.6) is comparable to the above case (c'.3). The difference is in the quantities. There is also some bound for \hat{q}_{12} .

$$\hat{q}_{01} = 0 \quad (4.15)$$

$$\hat{q}_{02} = e_0^* \quad (4.16)$$

$$\hat{q}_{12} = \frac{1}{(1 + \beta^*)} [\beta^* e_1^* - \hat{\rho}_{01} e_0^* - \frac{e^*}{\hat{\rho}_{12}}] \quad (4.17)$$

$$0 < \hat{q}_{12} \leq \frac{\lambda \{R(1 - \theta_{12}) - (\hat{\rho}_{02} - \theta_{02})\}}{(\hat{\rho}_{12} - \theta_{12})} \frac{\lambda R e_0}{\hat{\rho}_{12} - \theta_{12}} \quad (4.18)$$

Case (c'.5) The borrower does not want to borrow short at $t=0$ $d_{01} = 0$, but the long term debt is interior $d_{02} \geq 0$ and at $t=1$ the bank would like to borrow as much as possible

$d_{12} \rightarrow \infty$. Because the long term debt is interior $d_{02} \geq 0$, like in case (a'), (4.6) holds so that $\hat{\rho}_{02} = R(1 - \theta_2) + \theta_{02}$. Since $\hat{\rho}_{12}\hat{\rho}_{01} = \hat{\rho}_{02}$, it is found that $\hat{\rho}_{01}\hat{\rho}_{12} = R(1 - \theta_{02}) + \theta_{02}$. This is a curve on the sheet defined by the no arbitrage condition. In addition $\hat{\rho}_{12} \leq r(1 - \theta_{12}) + \theta_{12}$ means that effectively d_{12} is relatively cheap, this explains why the bank would like to borrow as much as possible of it. At equilibrium the bonds quantities are the same as displayed in (4.16), (4.17) and (4.18).

In addition the bound on \hat{q}_{12} is such that:

$$0 < \hat{q}_{12} \leq \frac{\lambda R e_0}{\hat{\rho}_{12} - \theta_{12}} \quad (4.19)$$

Note that the higher θ_{12} , the higher the bound. This seems at first glance counterintuitive but in fact it pinpoints to the liquidity role of d_{12} for the borrower at $t=1$. Note that this might replicate cases where financial repression create liquidity crisis like South Korea in the seventies and the late 1990s. There are two equations for the three interest rates space if only the perfect arbitrage condition in the lending economy is considered. Then the system is regular. Considering that I am focusing on the cases with perfect arbitrage in both economies, the system of 3 interest rates is determinate with the presence of 3 equations. Note that the interest rates that solve these three equations could be at most 2 and are of the same magnitude as the interior case (a'). The difference from (a') is in the quantities as pointed in (4.15), (4.16) and (4.17).

Case (c'.6) The borrower is not willing to borrow long-term $d_{02} = 0$ and willing to borrow as much as possible short at $t = 1$ $d_{12} \rightarrow \infty$ because $\hat{\rho}_{12} < r(1 - \theta_{12}) + \theta_{12}$. The

demand for the first short-term debt is interior $d_{01} \geq 0$. A condition for this equilibrium to exist is that:

$$\hat{\rho}_{02} > R(1 - \theta_{02}) + \theta_{02} \quad (4.20)$$

At equilibrium market clear so that quantities are determined respectively by:

$$\hat{q}_{01} = e_0^* \quad (4.21)$$

$$\hat{q}_{02} = 0 \quad (4.22)$$

$$\hat{q}_{12} = \hat{\rho}_{01} e_0^* + \frac{1}{(1 + \beta^*)} [\beta^* e_1^* - \hat{\rho}_{01} e_0^* - \frac{e^*}{\hat{\rho}_{12}}] \quad (4.23)$$

Note that for the cases (c'.6) at equilibrium $\rho_{02} = \rho_{01} \rho_{12} > R(1 - \theta_{02}) + \theta_{02}$. In this case without post-reserve requirements no arbitrage condition, the system is irregular. With the post-reserve requirements condition (4.5) and the no arbitrage condition from the lender side (4.4), the relative interest rates are determined. It is important to see if it is within the capacity of the bank to pay back her short-term debt. Suppose one unit is borrowed, the $(1 - \theta_{01})$ will be invested in the illiquid investment so that at $t=2$: it will get a return equal to $R(1 - \theta_{01})$. The remaining reserves will be returned at $t=1$ and if reinvested in the long term asset will only get $\theta_{01}(1 - \theta_{12})r$. At $t=2$ $\theta_{01}\theta_{12}$ reserves will be returned. Therefore there is an implicit marginal return for this economy that should be higher than the cost of interest payments: $R(1 - \theta_{01}) + \theta_{01}(1 - \theta_{12})r + \theta_{01}\theta_{12} \geq \rho_{01}\rho_{12}$. This means that if the reserve requirements are set such that $R(1 - \theta_{01}) + \theta_{01}(1 - \theta_{12})r + \theta_{01}\theta_{12} \leq R(1 - \theta_{02}) + \theta_{02}$, then case (c'.6) is not anymore possible equilibrium.

Proposition 4: *When the condition $\theta_{02} \leq \frac{\theta_{01}(R-r) + \theta_{12}(r-1)}{(R-1)}$ holds, it is possible to*

rule out the case created with the imposition of reserve requirements were only short-term borrowing exists.

Note that if $\theta_{01} = \theta_{12}$, then the condition in the above proposition becomes simply $\theta_{02} \leq \theta_{12} = \theta_{01}$. In Chile the reserve requirements were maturity-specific but not date specific initially (in 1992), i.e similar to the case where $\theta_{01} = \theta_{12}$. Only in 1998 all reserve requirements on external borrowing were removed. One could argue perhaps that in Chile, the implementation was successful because the policy was targeting a tilt towards long maturity such that $\theta_{02} \leq \theta_{12} = \theta_{01}$. Note that this is in conflict with the result at the interior solution. In set (a') I have argued that simulation results show that for a bifurcation towards a unique set of interest rates, or for (4.8) to be satisfied: the simulation example show that $\theta_{02} > \theta_{01}$. I have shown in section IV.2, that date- and maturity- specific reserve requirements add determinacy to quantities of date- and maturity- specific bonds. I have shown that $\theta_{02} \leq \theta_{12} = \theta_{01}$ would mean that there are no equilibria where only short-term borrowing exists. It is important to note that in all equilibria without reserve requirements the condition $\rho_{01}\rho_{12} = \rho_{02}$ is necessary for their existence. Such a condition is the lender and the borrower perfect arbitrage condition. After the imposition of reserve requirements, the interior solution is the only equilibrium

which requires necessarily both (4.4) and (4.5) for existence. Therefore I focus on the interior solution in the simulation results of the next section and in the next chapter.

In the next section, with a numerical example, I will show that at the interior solution $\theta_{02} < \theta_{01}$ creates bifurcation towards two equilibria. I will demonstrate from the simulation results, that it is always the case that $\theta_{02} > \theta_{01}$ for a bifurcation towards a unique equilibrium. Another issue that is hardly addressed in models a la Diamond and Dybvig, is welfare comparison. The simulation results provide welfare comparisons. In the next section, I first show how bifurcation arises. Second I give a brief welfare comparison.

IV.3. Simulation results

IV.3.1. Different bifurcations with reserve requirements

For comparability with the case without reserve requirements, I use the same parameters used for the simulation results in chapter III:

$$e_0 = 2000, \beta^* = 1; e_0^* = 9500, e_1^* = 15000, e_2 = 15000 \text{ and } \lambda \in [0.1, 0.9] \text{ and } R \in [1.5, 2.3].$$

In the first experiment, I impose exogenously the three rates of reserve requirements $\theta_{01} = 0.3, \theta_{02} = 0.2$, and $\theta_{12} = 0.1$. The choice of the reserve requirements is consistent with the Chilean case. Note that interest rates that solve equation (4.6) are independent of λ , the impatience fraction. Reserve requirements stabilize all interest rates across λ . As shown analytically in section IV.2, the simulation results show that there are two positive sets of interest rates that solve equation (4.7). Figure 4.2 shows

that both post-reserve-requirement interest rate of the second short term debt are lower than the ones before reserve requirement.

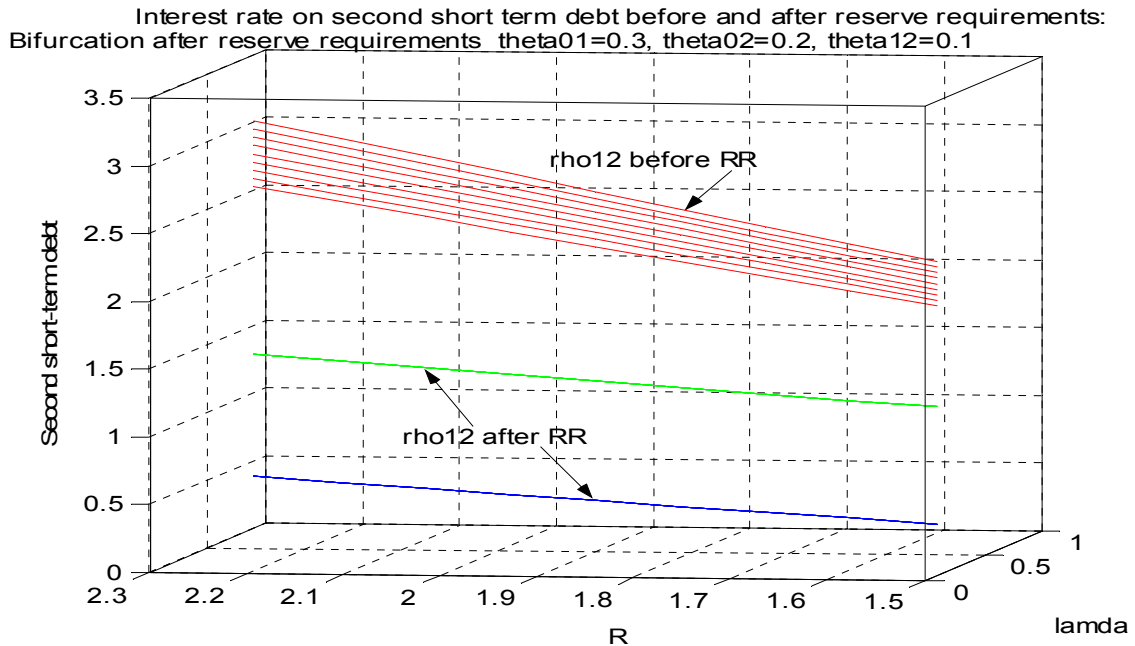


Fig.4.2. Bifurcation towards two equilibria, with reserve requirements.

Given $\theta_{01}=0.3$, $\theta_{02}=0.2$ and $\theta_{12}=0.1$. The grid used was 0.1 for both the simulations over λ and R

In the second experiment, for $\theta_{01} \in [0.05; 0.85]$, $\theta_{12} = 0.45$, I find the θ_{02} which provides bifurcation towards a unique equilibrium (i.e solves (4.7)). The simulation results show that $\theta_{02} > \theta_{01}$. Even though I display the results for $\theta_{12} = 0.15$, I have tried with other values for θ_{12} , the pattern is always $\theta_{02} > \theta_{01}$. The reason why I report the results with $\theta_{12} = 0.45$ is that for low values of θ_{12} , the borrowing in some or all of the

bonds become negative. In the case when $\theta_{12} = 0.45$, for all R and for $\theta_{01} = 0.15$, borrowing is still positive in all assets and depending on R , θ_{02} varies from 0.48 to 0.59, decreasing with an increase in R . Figure 4.3 was constructed with a grid of 0.01, and 100 repetitions. It shows for $R = 1.5$, that θ_{02} decreases with low values of θ_{01} and increases with high θ_{01} . To construct this graph I remove the case when $\theta_{02} \geq 1$ is found in the simulations because they are out of range. Such cases correspond to extreme low and extreme high θ_{01} . Observing the simulation results by looking at R increasing or θ_{01} increasing, all cases show that $\theta_{02} > \theta_{12} > \theta_{01}$. This raises the question to why $\theta_{02} > \theta_{12} > \theta_{01}$ for no bifurcation to occur. A simple answer in a model with endogenous international lending is perhaps that reserve requirements' role as a liquidity provider in this model dominates that of a tax. I will be able to explain further this intuition once I get to chapter V, where a bank run is allowed.

In the next section I look at a welfare comparison between the case with and without reserve requirements for the interior solutions. For the case with reserve requirements, I look at both the cases with bifurcation.

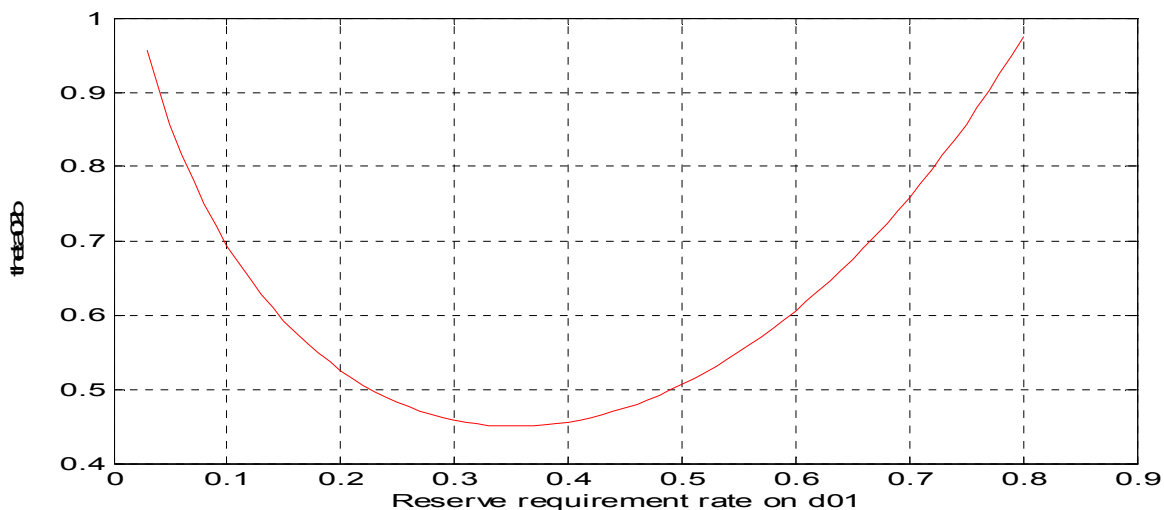


Fig.4.3. θ_{02} as a function θ_{01} for bifurcation towards a unique equilibrium
 assuming that $\theta_{12} = 0.45$ and $R = 1.5$

IV.3.2. Welfare comparison

The first panel in figure 4.4 shows the welfare comparison for the domestic economy. The red lines correspond to the case without reserve requirements. The green and the blue lines correspond to the case with reserve requirements resulting in bifurcation. For low λ , reserve requirements, with the bifurcation towards two interest rates, increase welfare for the domestic economy. These results are not considering yet the case with bank run, which will be considered in chapter V. For high λ , reserve requirements decrease welfare. In anticipation of results in chapter V, note that when λ is very low, $(1 - \lambda)$, the fraction of patient depositors, or potential runners is high. Thus when λ is low, the bank will need more resources to deal with a bank run. The second panel in

figure 4.4 shows that international lenders are always worse off with reserve requirements compared to the case without reserve requirements.

I turn to analyze the results from the case when I impose reserve requirements so that a bifurcation towards a unique equilibrium. I pursue my analysis by varying θ_{01} and finding the corresponding θ_{02} that guarantees (4.8) when $\theta_{12} = 0.45$, for all R . Like before I choose $\theta_{01} = 0.15$ where borrowing is still positive in all bonds and depending on R , θ_{02} varies from 0.48 to 0.59, decreasing with an increase in R . The left upper and lower panels show for $\lambda = 0.2$ the utility for domestic borrowers and international lenders respectively. The right upper and lower panels show for $\lambda = 0.8$ the utility for domestic borrowers and international lenders respectively. For low (high) λ , a set of date- and maturity- specific reserve requirements improve (decrease) welfare of domestic depositors. For low (high) λ , a set of date- and maturity- specific reserve requirements improve (decrease) welfare of international lenders when R is not too high (low). Note that in this model banks are competitors, and thus a single bank disregards the effect of its borrowing on aggregate borrowing.²² This results in an over-borrowing distortion. In fact it is an equilibrium result that with or without reserve requirements the total borrowing at $t=0$ is equal to e_0^* . But in particular, reserve requirements increase the second-short-term interest rate and thus decrease new borrowing at $t=1$. This indeed decreases overall borrowing across the two periods and thus reduces the described over-borrowing distortion. Such a distortion is less pronounced with high λ . Note that

²² See Auernheimer and Garcia (2000).

international lenders are always worse off, when reserve requirements lead to the bifurcation towards two equilibria. In comparison figure 4.5 shows the case with bifurcation towards a unique equilibrium: the association of bifurcation towards a unique equilibrium with the possibility of welfare improvement, imply that there might be an association between higher utility and higher determinacy.

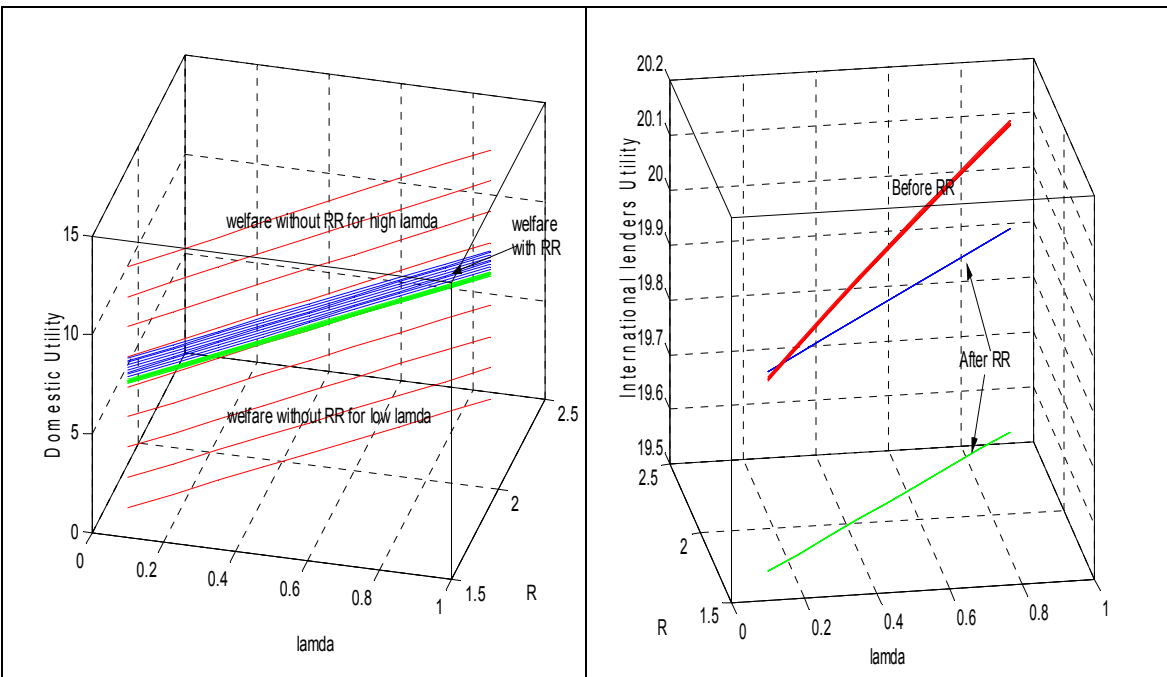


Fig.4.4. Welfare of domestic borrowers and international with and without reserve requirements (with bifurcation towards two equilibria).

Results from 0.1 grid with 10x10 repetitions in the simulation. I have assumed $\theta_{01} = 0.3$, $\theta_{02} = 0.2$ and $\theta_{12} = 0.1$

IV.4. Conclusion

In this chapter I show analytically that reserve requirements reduce the scope of indeterminacy. For the interior solution, I show with simulations what combination of date- and maturity- specific reserve requirements would create a unique equilibrium at the interior solution. Then I analyze welfare implications of different combination of date- and maturity- specific reserve requirements. In practice the implementation of reserve requirements is used as a preventive mechanism to avoid financial crises. In the next chapter I introduce in the setup I developed in chapters III and IV, the possibility of an unanticipated bank run. I analyze the role of reserve requirements in preventing crises as well as explore the ex-post crises role of international lenders in bailing out the domestic bank.

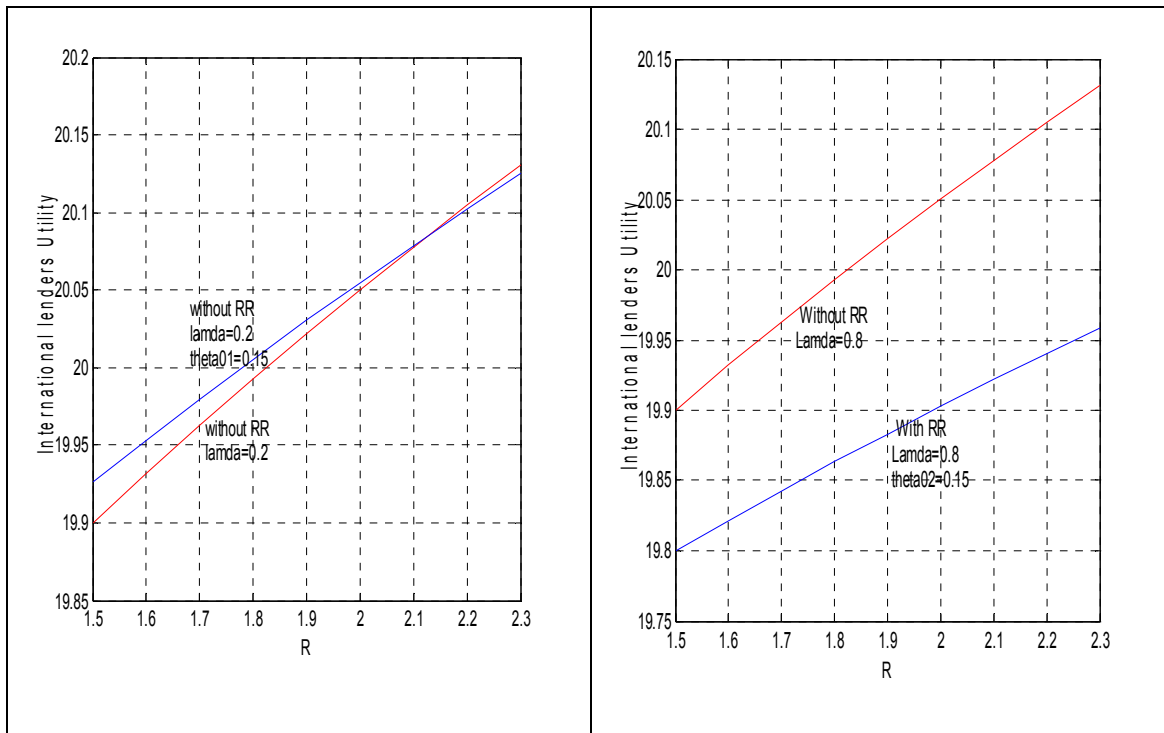


Fig 4.5. Welfare of domestic borrowers and international with and without reserve requirements (with bifurcation towards a unique equilibrium).

Results from 0.1 grid with 10x10 repetitions in the simulation

CHAPTER V
COUNTRIES ON THE VERGE OF A NERVOUS BREAKDOWN²³:

ARE THERE ANY CURES?

V.1. Introduction

In this chapter I analyze the vulnerability to a bank run of equilibria identified in the model I described in chapter III and IV. To introduce sunspots in the simplest possible way, I assume that a bad dream will unexpectedly occur at $t = 1$. If an unexpected bad dream is seen by domestic depositors at $t=1$ and the bank position signals that it is unable to pay back all the withdrawals i.e. is in an illiquid position, a bank run will occur. I focus on cases where the rule of the domestic banking system is such that, in case of a bank run, domestic depositors get paid prior to international lenders. Note that at $t=0$, international lenders and the domestic banks agree on \hat{q}_{01} , \hat{q}_{12} and \hat{q}_{02} . However, only \hat{q}_{01} and \hat{q}_{02} are actually traded at $t=0$. Only at $t=1$ would \hat{q}_{12} be actually traded. \hat{q}_{12} indicates an optimal anticipated amount at $t=0$ of new lending at $t = 1$. If new events happen such as a bank run, international lenders may find it optimal to deviate from the original decision of lending \hat{q}_{12} .

In this chapter I will first look at how a bank run arises. What is the illiquidity condition along which a bad dream creates the bank run? Second, I will see whether date- and maturity- specific reserve requirements can prevent the occurrence of a bank run. Third when a bank run occurs, I will explore the re-optimizing behavior of

²³ The first part of the title is inspired by the title of the 1988 movie: “Women on the verge of a nervous breakdown”, by Pedro Almodovar. This is because in this setup, the introduction of a bank run requires the realization of a bad dream.

international lenders in deciding whether or not to lend at $t = 1$. Thus this chapter deals with the cause of a banking crisis, the assessment of an ex-ante preventive policy, and an ex-post bailout strategy by international lenders.

V.2. The emergence of a bank run in the setup without reserve requirements

V.2.1. Defining the illiquidity condition

At $t=0$, given the decisions of $\hat{q}_{01}, \hat{q}_{12}$ and \hat{q}_{02} , international lenders and domestic depositors can observe whether the bank would be liquid or not in the case of a bank run at $t=1$. There are two possibilities: either the bank holds a liquid position or it holds an illiquid position. If the bank holds a liquid position, there is never a bank run, independently of whether or not there is a bad dream. If the bank holds an illiquid position, domestic depositors will run to the bank if and only if they see a bad dream.

Thus the illiquidity condition is as follows

$$\hat{c}_1 + \hat{\rho}_{01} \hat{q}_{01} > r\hat{k} \quad (5.1)$$

Under the rules assumed for the domestic financial system, the domestic depositors perceive the bank position as strongly illiquid when it cannot pay for all their withdrawals, even if it liquidated at $t=1$ all its investment k . A stronger illiquidity condition would therefore be $\hat{c}_1 > r\hat{k}$. Note that if $\hat{c}_1 > r\hat{k}$ holds, the illiquidity condition (5.1) must hold.

In my model the domestic depositors will run to the bank if two conditions exist:

- (i) The domestic depositors see a bad dream, in this context, the random variable takes at $t=1$ the value 1.

(ii) The illiquidity condition $\hat{c}_1 + \hat{\rho}_{01}\hat{q}_{01} > r\hat{k}$ holds.

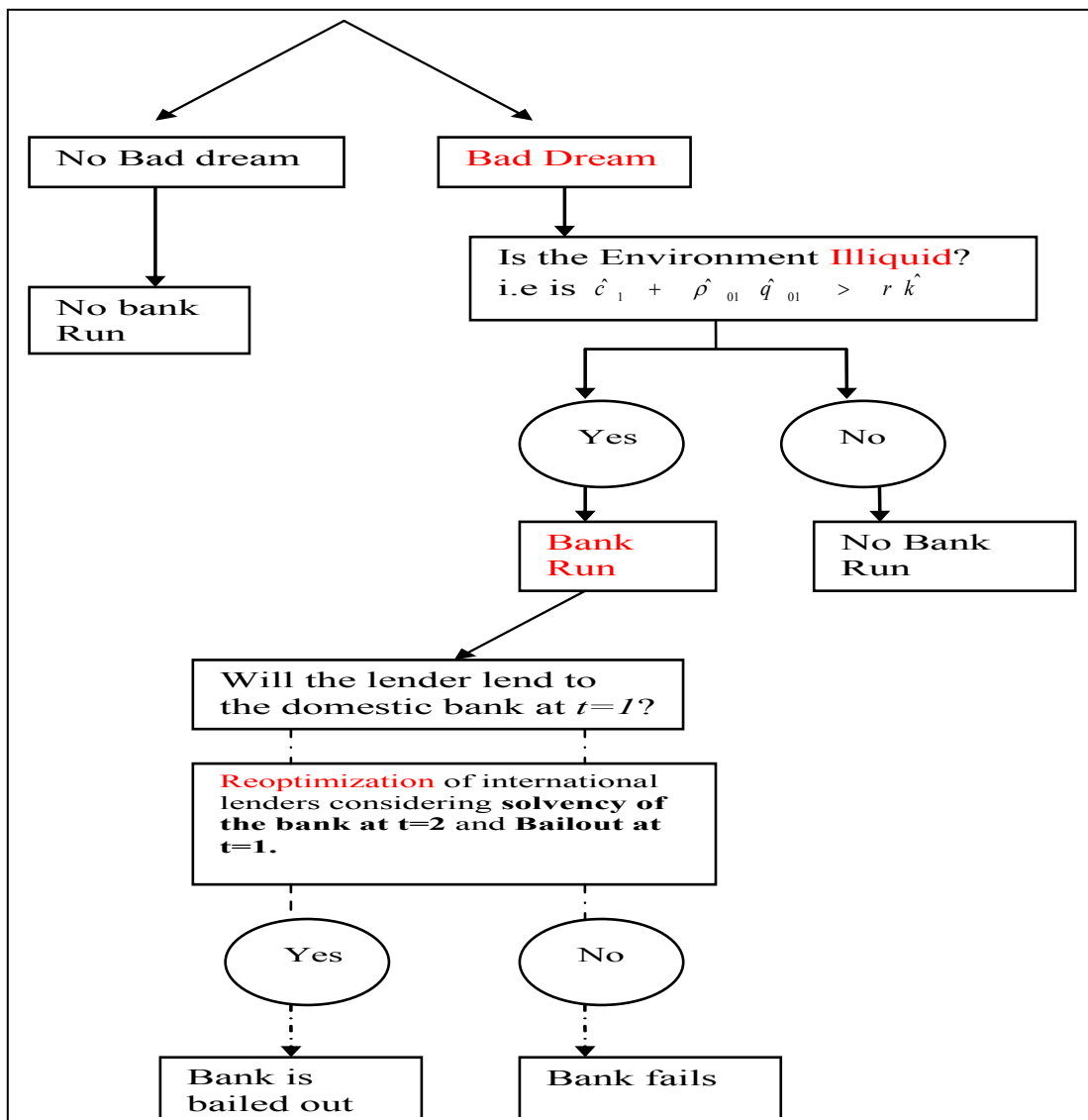


Fig 5.1. Decision tree at $t=1$

Figure 5.1 summarizes how a bank run would occur. Note that figure 5.1 will be used in section V.3.

V.2.2. Comparison of illiquidity conditions across all equilibria without reserve requirements

To look at the vulnerability to bank runs of each set of equilibria at $t = 1$, I assess (5.1) in each of the sets (a), (b) and (c) described in chapter III, with no reserve requirements. For the interior solution (a), the quantities of \hat{q}_{01} are not determined. There the illiquidity condition is the following:

$$\hat{q}_{01} > r \frac{[\lambda R e_0 (1 + \beta^*) + e_2^* + R e_0^*]}{R \beta^* e_1^*} (e_0 + e_0^*) - e_0 \quad (5.2)$$

Note that since $\hat{q}_{01} \in [0, e_0^*]$, if $\hat{q}_{01} \rightarrow 0$, this equilibrium is less likely to be vulnerable to illiquidity. If $\hat{q}_{01} \rightarrow e_0^*$, it will be more likely to be vulnerable.

For the case (b), the illiquidity condition is such that $\frac{\hat{\rho}_{01} e_0}{e_0 + e_0^*} > r$. Since this set has no

price determinacy, by normalizing $\hat{\rho}_{01} = 1$, the illiquidity condition becomes

$$\frac{e_0}{e_0 + e_0^*} > r \quad (5.3)$$

All cases in (c) have no price determinacy. Thus by normalizing $\hat{\rho}_{01} = 1$, the illiquidity condition becomes

$$\hat{q}_{01} > r(e_0 + e_0^*) + \frac{e_2^*}{\lambda \hat{\rho}_{02} (1 + \beta^*)} + \frac{e_0^*}{\hat{\rho}_{02} (1 + \beta^*)} - \frac{\beta^* e_1^*}{\lambda (1 + \beta^*)} \quad (5.4)$$

The ranking of the interior solution (a) and the sets (b) and (c), with regard to vulnerability to a bank run depends on how the bounds in (5.2), (5.3) and (5.4) are

relative to each other. This in turn is dependent on the parameters configuration. In the next section I look at simulation results for the interior solution for two reasons. The first reason is tractability. The second is the fact that two types of bifurcation may occur depending on the combinations of reserve requirements.

V.2.3. Simulation results

For comparability with the results in chapter IV and V, I use the same parameters used for the simulation results in chapter III: $e_0 = 2000$, $\beta^* = 1$; $e_0^* = 9500$, $e_1^* = 15000$, $e_2 = 15000$, $\lambda \in [0.1, 0.9]$ and $R \in [1.5, 2.3]$. The simulations were repeated over different values of λ and R . In this section I focus on the simulation at the interior solution. For $r = 1.1$, the interior solution is never vulnerable to a bank run. In table 5.1, I illustrate results for $r = 0.7$ and $r = 0.3$. It was shown in chapter III that the interior solution (a) has indeterminacy in quantities, thus it might be that the equilibrium results is such that $\hat{q}_{01} = 0$, or $\hat{q}_{01} = e_0^*$ or anything in between for the first short-term debt. For $r = 0.7$ and $r = 0.3$, when $\hat{q}_{01} = 0$, the equilibrium at the interior solution is not vulnerable to a bank run. In table 5.1, I look at the case when $\hat{q}_{01} = 0.9e_0^*$ since I am interested in looking at whether reserve requirement can reduce the scope of illiquidity and thus prevent a bank run. Table 5.1 shows the presence or not of illiquidity at the interior solution with no reserve requirements. When $r = 0.3$, for all pairs (λ, R) , the interior set with $d_{01} = 0.9e_0^*$ is vulnerable to a bank run. When $r = 0.7$, notice that for a combination of low R and low λ or a combination of high R and high λ , the interior

case with high short-term debt $d_{01} = 0.9e_0^*$ is vulnerable to bank runs. The higher R , the more relatively costly to liquidate at $t=1$. Even if λ is low, when R is low, the opportunity cost of liquidating is not that high. The environment is liquid. If λ is high, $(1-\lambda)$, the fraction of patient or potential depositors is low: the environment is liquid.

Table 5.1.
Presence of illiquidity at the interior solution

Illiquidity presence : (a) with $d_{01}=0.9e_0^*$ for $r=0.3$									
R/λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1.5	1	1	1	1	1	1	1	1	1
1.6	1	1	1	1	1	1	1	1	1
1.7	1	1	1	1	1	1	1	1	1
1.8	1	1	1	1	1	1	1	1	1
1.9	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1
2.1	1	1	1	1	1	1	1	1	1
2.2	1	1	1	1	1	1	1	1	1
2.3	1	1	1	1	1	1	1	1	1
Illiquidity presence : (a) with $d_{01}=0.9e_0^*$ for $r=0.7$									
R/λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1.5	0	0	0	0	n.e.	n.e.	n.e.	n.e.	n.e.
1.6	1	0	0	0	0	0	n.e.	n.e.	n.e.
1.7	1	1	1	0	0	0	0	n.e.	n.e.
1.8	1	1	1	1	0	0	0	0	n.e.
1.9	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	0	0	0
2.1	1	1	1	1	1	1	1	0	0
2.2	1	1	1	1	1	1	1	1	0
2.3	1	1	1	1	1	1	1	1	1

1 indicates presence of illiquidity, 0 indicates liquidity. Where $n.e.$ indicates that the equilibrium does not exist for the condition of $\hat{\rho}_{01} > r$ is not satisfied

V.3. Can reserve requirements prevent the occurrence of a bank run?

V.3.1. Illiquidity conditions with reserve requirements

In the setup with reserve requirements, at $t=1$, $\theta_{01}\hat{q}_{01}$ goods, the reserves are returned to the domestic bank by the domestic authority. The illiquidity condition takes the following form:

$$(\hat{\rho}_{01} - \theta_{01})\hat{q}_{01} + \hat{c}_1 > r\hat{k} \quad (5.5)$$

At the interior solution I compare the vulnerability to a bank run between the cases with and without reserve requirements. Note that when $\theta_{01} > 0$ and/or $\theta_{02} > 0$

$$\hat{k} = e_0 + e_0^* (1 - \theta_{02}) + (\theta_{02} - \theta_{01})\hat{q}_{01} \quad (5.6)$$

Note that at $t=0$, the reserve requirements θ_{01} and θ_{02} play a tax role since for $\theta_{01} > 0$ and $\theta_{02} > 0$, from (5.6), $\hat{k} < e_0 + e_0^*$. Thus with these reserve requirements, investment in k is lower than investment without reserve requirements. The illiquidity condition becomes

$$\hat{q}_{01} > \left\{ r - \frac{\hat{\rho}_{01} - \theta_{01}}{1 - \theta_{01}} \right\} e_0 + r e_0^* (1 - \theta_{02}) \left\{ \frac{1}{\hat{\rho}_{01} - r(\theta_{02} - \theta_{01})} \right\} \quad (5.7)$$

The liquidity role of θ_{01} and θ_{02} will be captured at $t=1$ and $t=2$ respectively.

V.3.2 Simulation results

When reserve requirements are imposed and if bifurcation towards two equilibria occurs, there will be two $\hat{\rho}_{01}$: one high corresponding to low $\hat{\rho}_{12}$ and one low corresponding to a high $\hat{\rho}_{12}$. The susceptibility to a bank run will depend also on how

\hat{q}_{01} is changing. Figure 5.2 shows the results for the case when bifurcation towards two equilibria results. To construct figure 5.2, I have imposed exogenously $\theta_{01} = 0.3$, $\theta_{02} = 0.2$ and $\theta_{12} = 0.1$. In table 5.2, the case with high (low) $\hat{\rho}_{12}$ is always (never) vulnerable to a bank run.

Table 5.2.

Illiquidity presence at the interior solution with reserve requirements (bifurcation towards 2 equilibria)

Illiquidity presence when RR (with 2 possible interest rates) with $r=0.3$

with high $\hat{\rho}_{12}$										
R / λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
1.5	1	1	1	1	1	1	1	1	1	1
1.6	1	1	1	1	1	1	1	1	1	1
1.7	1	1	1	1	1	1	1	1	1	1
1.8	1	1	1	1	1	1	1	1	1	1
1.9	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
2.1	1	1	1	1	1	1	1	1	1	1
2.2	1	1	1	1	1	1	1	1	1	1
2.3	1	1	1	1	1	1	1	1	1	1
with low $\hat{\rho}_{12}$										
R / λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
1.5	0	0	0	0	0	0	0	0	0	0
1.6	0	0	0	0	0	0	0	0	0	0
1.7	0	0	0	0	0	0	0	0	0	0
1.8	0	0	0	0	0	0	0	0	0	0
1.9	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
2.1	0	0	0	0	0	0	0	0	0	0
2.2	0	0	0	0	0	0	0	0	0	0
2.3	0	0	0	0	0	0	0	0	0	0

1 indicates presence of illiquidity, 0 indicates liquidity. It is assumed that $\theta_{01} = 0.3$, $\theta_{02} = 0.2$ and $\theta_{12} = 0.1$. This table shows the results only for $r = 0.3$. The case with $r = 0.7$ is particular: There only the result with high $\hat{\rho}_{12}$ satisfies the incentive compatibility constraint resulting in $\frac{\hat{\rho}_{12} - \theta_{12}}{1 - \theta_{12}} > r$.

In the second experiment, for $\theta_{01} \in [0.05; 0.85]$, $\theta_{12} = 0.45$, I find the θ_{02} which provides no bifurcation. As mentioned in chapter IV, the simulation results show that $\theta_{02} > \theta_{01}$. Table 5.3 show the presence or not of illiquidity for $r = 0.7$ and $r = 0.3$. The same result applies for $\lambda = 0.2$ and $\lambda = 0.2$. One can see that again with high λ , the vulnerability to a run decreases. Note that there are several combinations for each R that can prevent a run from occurring. For instance for $\lambda = 0.8$ and $R = 2.2$, the interior solution without reserve requirements is vulnerable to a run (see table 5.1). A set $\theta_{01} = 0.3$, $\theta_{02} = 0.2$ and $\theta_{12} = 0.1$, creates a bifurcation towards two equilibria: one of the emerging equilibria is always vulnerable to a bank run. From the simulation results that I used to get table 5.3, a combination of $\theta_{01} = 0.15$, $\theta_{02} = 0.48086$ and $\theta_{12} = 0.45$ guarantees no bifurcation and no vulnerability to a bank run.

For uniqueness and non-negative borrowing, one finds that $\theta_{02} > \theta_{01}$ and $\theta_{12} > \theta_{01}$. This seems to stress the role of reserve requirements as liquidity providers. If this is the case, one expects to identify that for the combinations of $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ that leads to a unique set of interest rates and no bank run, there must be a positive relation between the quantity at equilibrium of each bond and its corresponding reserve requirement. In other words for the liquidity role to dominate the tax role, \hat{q}_{01} , \hat{q}_{12} and \hat{q}_{02} should increase with higher θ_{01} , θ_{12} and θ_{02} respectively for the sets when no bank run occurs. Simulation results confirm the positive relation between \hat{q}_{01} , \hat{q}_{02} and θ_{01} , θ_{02} respectively. For these experiments θ_{12} was fixed. If for both the long-term and the short-term bonds, the liquidity role dominates, does it mean that both reserve requirements encourage more

borrowing of all bonds? The answer is no. Since the reserve requirements are date- and maturity- specific to each bond: a policy with $\theta_{02} > \theta_{01}$ would imply a tilt towards a longer maturity structure when the liquidity role dominates.

Table 5.3.
Illiquidity presence at the interior solution with reserve requirements (with bifurcation towards a unique equilibrium)

Illiquidity presence for interior with reserve requirements with $r=0.3$ (with bifurcation towards a unique set of interest rates)

R / θ_{01}	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85
1.5	1	0	0	0	0	0	0	1	1
1.6	1	0	0	0	0	0	0	1	1
1.7	1	0	0	0	0	0	0	1	1
1.8	1	0	0	0	0	0	0	1	1
1.9	1	0	0	0	0	0	0	1	1
2	1	0	0	0	0	0	0	1	1
2.1	1	0	0	0	0	0	0	1	1
2.2	1	0	0	0	0	0	0	1	1
2.3	1	0	0	0	0	0	0	1	1

Illiquidity presence for interior with reserve requirements with $r=0.7$ (with bifurcation towards a unique set of interest rates)

R / θ_{01}	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85
1.5	0	0	0	0	0	0	0	1	1
1.6	0	0	0	0	0	0	0	1	1
1.7	0	0	0	0	0	0	0	1	1
1.8	0	0	0	0	0	0	0	1	1
1.9	0	0	0	0	0	0	0	1	1
2	0	0	0	0	0	0	0	1	1
2.1	0	0	0	0	0	0	0	1	1
2.2	0	0	0	0	0	0	0	1	1
2.3	0	0	0	0	0	0	0	1	1

1 indicates presence of illiquidity, 0 indicates liquidity.

V.4. International lending after the bank run: Are international lenders “throwing good money after bad money”?

Let me go back to the case when reserve requirements are not imposed. Would the bank fail if all depositors attempt to withdraw c_1 ? Chang and Velasco (2000) answer the latter question in the positive. In my model because of endogenous lending, in the event of a bank run creditors may decide to bail out or not the bank by providing or not \hat{q}_{12} . In this section, I look at the role of international lending in bailing out the bank once a bank run happens. In the earlier section I have discussed the role of reserve requirements in preventing a bank run when a specific combination of $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ is implemented. Given such a result, in this section where I analyze the possibility of the post-bank run bailout of the bank by international lenders I do not look at the case when the combination of reserve requirements can induce a bank run. This would be a possible extension of this section in the future.

V.4.1. International re-optimization problem

At $t = 1$, when there are no reserve requirements, if a bad dream occurs, at the interior solution with high levels of short-term debt, a bank run will occur. Knowing that a new event, a bank run had occurred, international lenders would like to reconsider their decision on the anticipated \hat{q}_{12} . They will re-optimize given the bank run. They will

decide how much would be the net new lending x_{12} at $t = 1$.²⁴ Thus at $t = 1$ their new budget constraint is given by

$$c_1^* + x_{12} \leq e_1^* \quad (5.8)$$

In equilibria where the long-term lending $\hat{q}_{02} = 0$, the foreign lenders have nothing at stake to bail out the bank, therefore they will not provide any new funding $\hat{x}_{12} = 0$, in the event of a bank run. However in equilibria where $\hat{q}_{02} > 0$, it might be the case that the international lenders will provide some new lending $x_{12} \geq 0$. They would be willing to reconsider the value of \hat{q}_{02} : two possibilities may arise: First, given that their outside option is not to lend but also get nothing back of their loans, they may be willing to give up some of \hat{q}_{02} , they may accept partial payments of \hat{q}_{02} . Second, at $t = 1$ patient and impatient depositors withdraw c_1 goods: thus no domestic depositors will be able to withdraw again at $t = 2$. International lenders know that they will be the only ones paid back at $t = 2$. They may be willing to provide x_{12} only by increasing the value of their long-term loan. Therefore they will choose new value of x_{02} (in terms of goods) While re-optimizing they take the interest rates $\hat{\rho}_{02}, \hat{\rho}_{12}$ found at $t=0$ as given. Their new budget constraint at $t = 2$ is given by (5.9):

$$c_2^* \leq e_0^* + \hat{\rho}_{12}x_{12} + \hat{\rho}_{02}x_{02} \quad (5.9)$$

International lenders will not provide any new funding if they know that despite the new lending the bank would still fail or if the bank liquidate all \hat{k} to pay back all depositors.

²⁴ Note that \hat{x}_{12} would be the actual equilibrium new lending, net of the partial or full payment of $\hat{\rho}_{01}\hat{q}_{01}$.

The bank has the option of liquidating early at $t = 1$, some or all \hat{k} if she needs some liquidity. The bank will choose $l \geq 0$. The magnitude of l , needed to pay the domestic depositors will depend on the new lending provided by international lenders. Thus the bailout condition of the domestic bank by international lenders is given by (5.10):

$$\hat{c}_1 \leq x_{12} + rl \quad (5.10)$$

In addition the international lenders will only give new lending if the domestic will be able to pay back $\hat{\rho}_{12}x_{12}$ and $\hat{\rho}_{02}x_{02}$. The new solvency condition at $t = 2$ for the domestic bank is given by (5.11):

$$\hat{\rho}_{12}x_{12} + \rho_{02}x_{02} \leq R(\hat{k} - l) \quad (5.11)$$

The domestic bank does not solve any re-optimization problem since depositors, in the event of a bank run will withdraw \hat{c}_1 contracted at $t = 0$ ²⁵. The typical international lender will choose $c_1^*, c_2^*, x_{12}, x_{02}, l$ to maximize (3.1) subject to the new budget constraints (5.8) and (5.9) and the bailout and solvency conditions (5.10) and (5.11). An equilibrium is defined as a set of allocations $\{\hat{c}_1^*, \hat{c}_2^*, \hat{l}, \hat{x}_{12}, \hat{x}_{02}\}$ that maximize the international lenders problem given the parameters and the $t=0$ allocations $\{e_1^*, e_2^*, \hat{\rho}_{01}, \hat{\rho}_{02}, \hat{\rho}_{12}, r, R, \hat{k}, \hat{c}_1\}$. The solution for the optimization problem at the interior solution²⁶:

²⁵ Note that at $t=1$ when types are revealed to each individual, the role of the bank as an insurance mechanism disappears. The concerned reader may question for what reason the bank would still exist. Despite the fact that this could be an interesting research question in the future, I would like to abstract from it in the setup to avoid complexity.

²⁶ Solution for other sets of equilibria can be provided upon request.

$$\hat{l} = \frac{e_2^*}{R} + e_0^* + \frac{(r + \beta^* \hat{\rho}_{01})e_0 - \beta^* e_1^*}{r} \quad (5.12)$$

$$\hat{x}_{12} = \frac{1}{1 + \beta^*} \{ (\hat{\rho}_{01} - r)e_0 - r \frac{e_2^*}{R} - r e_0^* + \beta^* e_1^* \} \quad (5.13)$$

$$\hat{x}_{02} = e_0^* - \frac{1}{1 + \beta^*} \left(1 - \frac{r}{\hat{\rho}_{01}} \right) \left(\frac{e_2^*}{R} + e_0^* + \frac{(r + \beta^* \hat{\rho}_{01})e_0 - \beta^* e_1^*}{r} \right) \quad (5.14)$$

For $l \geq 0$, $x_{12} \geq 0$, and for the environment to be illiquid, the early liquidation r should be such that:

$$\frac{\beta^* e_1^* - \beta^* \hat{\rho}_{01} e_0}{\frac{e_2^*}{R} + e_0 + e_0^*} \leq r \leq \min \left\{ \frac{\rho_{01} e_0 + \beta^* e_1^*}{\frac{e_2^*}{R} + e_0 + e_0^*}, \frac{\hat{\rho}_{01} \hat{q}_{01} + \hat{\rho}_{01} e_0}{e_0 + e_0^*} \right\} \quad (5.15)$$

Whether $\hat{x}_{02} \geq \hat{q}_{02}$ or $\hat{x}_{02} \leq \hat{q}_{02}$ will depend on the choice of \hat{q}_{02} . For consistency with the illiquidity condition, \hat{q}_{02} will not be too high. This is so because this re-optimization problem will happen only if a bank run occurs. The next section shows simulation results.

V.4.2. Simulation results

For comparability, I use the same parameter space used in the simulation for chapter III and IV: $e_0 = 2000$, $\beta^* = 1$; $e_0^* = 9500$, $e_1^* = 15000$, $e_2 = 15000$, $\lambda \in [0.1, 0.9]$, and $R \in [1.5, 2.3]$. The grid used was 0.1 for both the simulations over λ and R . Figure 5.2 show the simulation results for the actual net new lending x_{12} vs. the anticipated net new lending $\hat{q}_{12} - \hat{\rho}_{01} \hat{q}_{01}$. The area that is struck through in black does not correspond to the interior set when $r = 0.7$ because it does not satisfy the condition $\hat{\rho}_{01} \geq r$. At the interior

solution, within the corresponding parameter space, actual lending is non-negative after the lenders re-optimize due to the occurrence of a bank run. From the figure it is clear that $t = 0$, the anticipated net new lending is higher than the actual new lending at $t = 1$. This is due to the bank run and the early liquidation of \hat{k} at $t = 1$. In fact figure 5.3 shows how the ratio of early liquidation to actual net new lending $\frac{\hat{l}}{\hat{x}_{12}}$ decreases to zero with higher R . This is because with higher R , x_{12} increases. Why do international lenders would be willing to lend at $t = 1$? As explained it is because their outside option is not to get back anything of their loans. Since long term debt is paid back at $t = 2$, they are willing to bail out the bank to retrieve the long term debt.

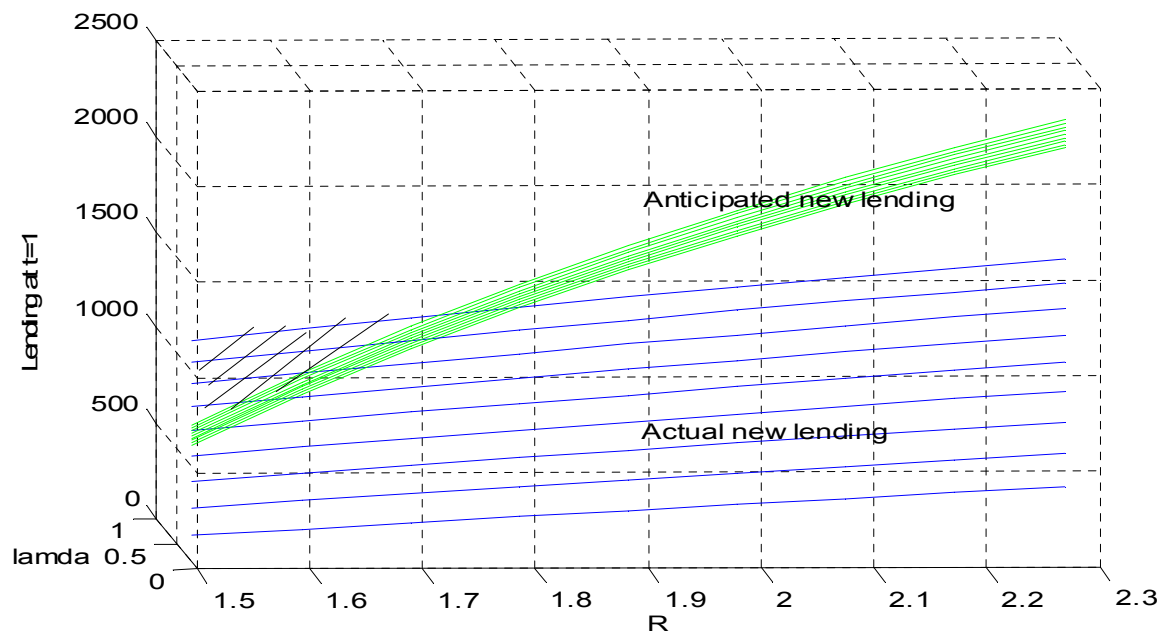


Fig 5.2. Actual vs. anticipated new lending at $t = 1$, $r = 0.7$

The long term debt at stake plays a crucial role for international lenders to lend at $t = 1$ short-term. $\frac{\hat{\rho}_{02}\hat{x}_{02} + \hat{\rho}_{12}\hat{x}_{12}}{\hat{\rho}_{02}\hat{q}_{02} + \hat{\rho}_{12}\hat{q}_{12}} \in [1.01, 1.19]$. On the figure, the lines that are struck through correspond to equilibria that are non-existent. Thus international lenders would be willing to provide the new lending only if they get back a higher value of their total loans $\hat{\rho}_{02}\hat{x}_{02} + \hat{\rho}_{12}\hat{x}_{12}$ at $t=2$ in comparison with the case without a bank run where the total indebtedness was expected to be $\hat{\rho}_{02}\hat{q}_{02} + \hat{\rho}_{12}\hat{q}_{12}$. In this model at $t=2$, the bank would still exist to liquidate $(\hat{k} - \hat{l})$ and to pay back international debt $\hat{x}_{12}, \hat{x}_{02}$. There are no domestic expenditures for the bank.

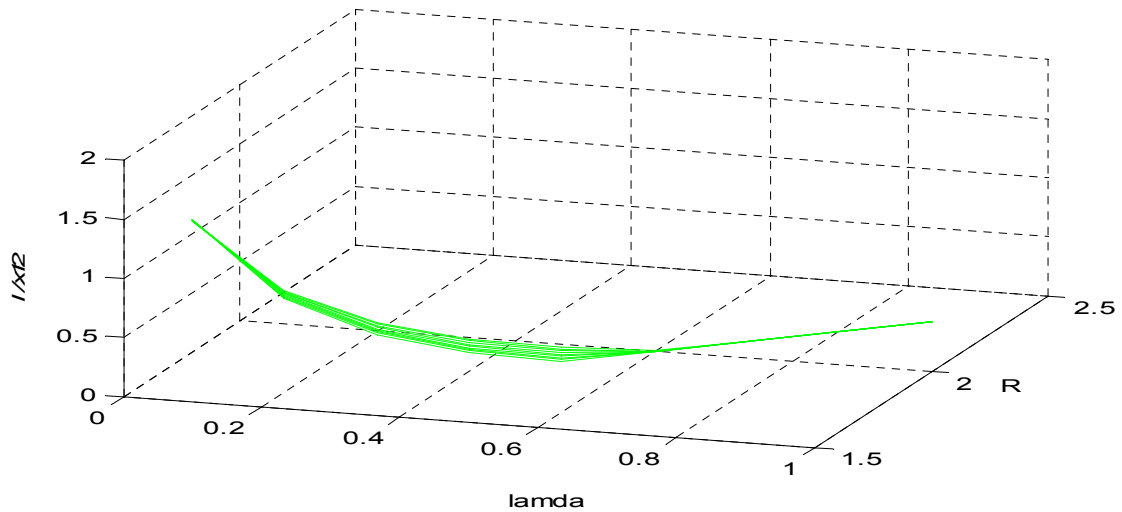


Fig 5.3. Early liquidation over new lending \hat{l} / \hat{x}_{12} at $t = 1$ for $r = 0.7$

Figure 5.4 shows how the ratio of actual vs. anticipated total debt at $t=2$ increases with R and decreases with λ . Note that for the high R , \hat{l} is zero. So all \hat{k} will be kept until $t = 2$. In other words, with values of high R international lenders will be willing to bail out the bank, by benefiting from indirectly being the sole benefactors from $R\hat{k}$ at $t = 2$: this leads to $\hat{l} = 0$. This is along the line of why international lenders would be willing to lend or to “throw good money after bad”. Results for $r = 0.3$ can be provided upon request: there for all $\lambda \in [0.1, 0.9]$ and $R \in [1.5, 2.3]$, $\hat{l} = 0$. This is because the opportunity cost of early liquidation very high.

V.5. Conclusion

I have shown in chapter IV that reserve requirements at the interior solution creates a bifurcation towards either two equilibria or unique equilibrium, depending on what combination of $\{\theta_{01}, \theta_{02}, \theta_{12}\}$ is implemented. In chapter V, simulation results show that a combination of $\{\theta_{01}, \theta_{02}, \theta_{12}\}$ leading to two equilibria has one of the equilibria vulnerable to a bank run. A specific vector $\{\theta_{01}, \theta_{02}, \theta_{12}\}$, that satisfies (4.6), creates a bifurcation towards a unique equilibrium that is not vulnerable to a bank run. Such a vector satisfies $\theta_{02} > \theta_{12} > \theta_{01}$.

For the case when there are no reserve requirements, I explore if international lenders would be willing to bail out the domestic bank from a bank run. Simulation results show that international lenders will provide new lending at $t=1$, if the return at $t=2$ will be

higher than it was anticipated at $t=0$, even if the new lending is lower than it was anticipated at $t=0$.

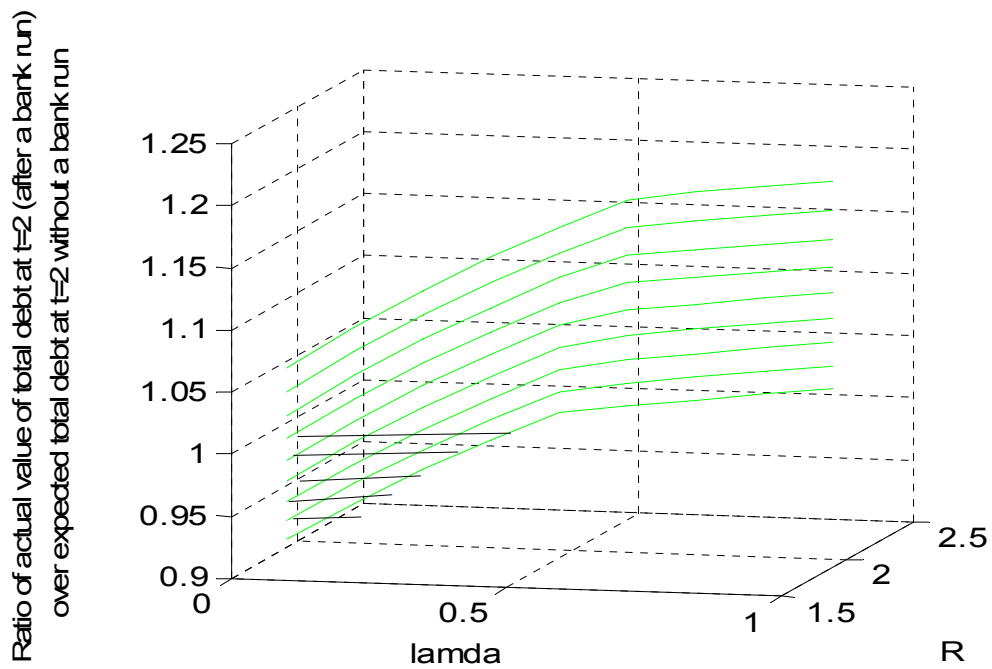


Fig 5.4. Ratio of actual total debt at $t=2$ (after bailout) over expected value of total debt (without a bank run).

This refers to $\frac{\hat{\rho}_{02}\hat{x}_{02} + \hat{\rho}_{12}\hat{x}_{12}}{\hat{\rho}_{02}\hat{q}_{02} + \hat{\rho}_{12}\hat{q}_{12}}$. It is assumed that $r = 0.7$

CHAPTER VI

CONCLUSION

In this dissertation I explored two issues: the choice of the maturity structure of external debt in an environment with liquidation cost and the role of specific type of capital controls. First I show that an environment with endogenous lending is essential to capture endogenous term structure and endogenous credit rationing. In such a rich environment the interest rate captures both the “reward for parting with liquidity” and the “impatience factor”. With endogenous behavior of lenders, multiple equilibria arise, some of which are characterized by indeterminacy.

Second, I show that date-and maturity- specific reserve requirements reduce the scope of indeterminacy. In this model reserve requirements play the role of a tax and the role of liquidity providers at different dates for each bond. The scarce literature that explored the role of these reserve requirements has identified their tax equivalent role. There, the documented benefits of such capital controls are similar to the benefits of a tax correcting for some market failure. In my setup I capture the role of tax along with the role of liquidity. In fact I identify a case in which the liquidity role of reserve requirements dominates the tax role. In this particular case, a tilt towards a longer maturity structure needs the reserve requirement rate on the long-term debt to be higher than that on the second short-term debt, which in turn should be higher than that the first short-debt. With this particular combination of reserve requirements, it is possible to achieve no vulnerability to a bank run and to get a locally unique equilibrium.

Third with regard to the post-bank-run role of international lenders, I show that international lenders may still want to provide new short-term lending to the bank after the occurrence of a bank run, in order to retrieve their long-term debt.

Both the results related to pre-crisis role of reserve requirements in preventing a bank run and the post-crisis role of international lenders are sensitive to the assumption that the setup has liquidation costs in the short-term. There are several points that may seem to be shortcomings of the model presented in this dissertation and that are fertile points for future analysis. First, when I introduced the possibility of a bank run I assumed that it is unexpected. In the future, I would like to incorporate the expectation of a possible bank run within the decision making of agents in the world economy. Second when I look at the role of international lending after a bank run, the role of a bank is not clear. Third I have not explored the optimal design of reserve requirements to decrease vulnerability despite looking at the effect of different combination. Fourth, the shock in this model is related to private information regarding consumption scheme. It would be interesting to introduce an aggregate shock. Fifth the three-date setup is a one shot-game. It would be interesting to put this game in a dynamic framework; perhaps the results on the liquidity role vs. the tax role of reserve requirements may be different. Fourth introducing money and exchange rates may seem arduous but is definitely a rich future extension.

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APPENDIX A

DOMESTIC BANK'S PROBLEM

The banks' problem is to maximize (2.1) subject to the constraints (2.4), (2.5), and (2.6), and (2.7) in chapter III. In chapter II the bank's problem has two additional constraints (2.2) and (2.3).

A Lagrangean is formed:

$$L = \lambda \ln c_1 + (1 - \lambda) \ln c_2 + \varepsilon[c_2 - rc_1] \quad (\text{A.1})$$

Using $k = e_0 + d_{01} + d_{02}$, one finds that:

$$c_1 = \frac{d_{12} + rl - \rho_{01}^d d_{01}}{\lambda} \quad (\text{A.2})$$

$$c_2 = \frac{R(e_0 + d_{01} + d_{02} - rl) - \rho_{12}^d d_{12} - \rho_{02}^d d_{02}}{1 - \lambda} \quad (\text{A.3})$$

In chapter II, two additional borrowing constraints, (2.2) and (2.3) are added to the domestic bank's problem. In this case, the domestic bank's problem reduces to choosing d_{02} to maximize the Lagrangean in (A.1). Replacing (2.2) and (2.3) in (A.2) and (A.3), one finds that:

$$c_1 = \frac{f_2 - \rho_{01}^d f_1 - (1 - \rho_{01}^d) d_{02}}{\lambda} \quad (\text{A.4})$$

$$c_2 = \frac{R(e_0 + f_1) - \rho_{12}^d f_2 - (\rho_{02}^d - \rho_{12}^d) d_{02}}{1 - \lambda} \quad (\text{A.5})$$

Accordingly, in the problem defined in chapter II, the Kuhn-Tucker condition is the following:

$$\frac{\partial L}{\partial d_{02}} d_{02} = 0 \quad (\text{A.6})$$

where

$$\frac{\partial L}{\partial d_{02}} = \frac{\rho_{01}^d - 1}{c_1} + \frac{\rho_{12}^d - \rho_{02}^d}{c_2} \quad (\text{A.7})$$

In chapter III, the domestic bank's problem is to maximize (2.1) subject to (2.4), (2.5), (2.6) and (2.7). There the Langrangean is again (A.1) where c_1 and c_2 are defined by (A.2) and (A.3). The domestic bank's problem is to choose d_{01} , d_{12} , d_{02} and l to maximize

L . Thus the Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial d_{01}} d_{01} = 0 \quad (\text{A.8})$$

$$\frac{\partial L}{\partial d_{12}} d_{12} = 0 \quad (\text{A.9})$$

$$\frac{\partial L}{\partial d_{02}} d_{02} = 0 \quad (\text{A.10})$$

$$\frac{\partial L}{\partial l} l = 0 \quad (\text{A.11})$$

Where

$$\frac{\partial L}{\partial d_{01}} = \left[\frac{\lambda}{c_1} - r\varepsilon \right] \left[-\frac{\rho_{01}^d}{\lambda} \right] + \left[\frac{(1-\lambda)}{c_2} + \varepsilon \right] \left[\frac{R}{1-\lambda} \right] \quad (\text{A.12})$$

$$\frac{\partial L}{\partial d_{02}} = \left[\frac{1-\lambda}{c_2} + \varepsilon \right] \left[\frac{R - \rho_{02}^d}{1-\lambda} \right] \quad (\text{A.13})$$

$$\frac{\partial L}{\partial d_{12}} = \left[\frac{\lambda}{c_1} - r\varepsilon \right] \left[\frac{1}{\lambda} \right] + \left[\frac{1-\lambda}{c_2} + \varepsilon \right] \left[\frac{-\rho_{12}^d}{1-\lambda} \right] \quad (\text{A.14})$$

$$\frac{\partial L}{\partial l} = \left[\frac{\lambda}{c_1} - r\varepsilon \right] \left[\frac{r}{\lambda} \right] + \left[\frac{1-\lambda}{c_2} + \varepsilon \right] \left[\frac{-R}{1-\lambda} \right] \quad (\text{A.15})$$

Proof of proposition 2.1.

Looking at the bank's problem, there are 2 conditions when a crisis is not anticipated:

- (i) The incentive compatibility constraint is binding $\varepsilon = 0$ or $\psi > r$
- (ii) There is no liquidation $l = 0$.

The condition (ii) implies that $\frac{\partial L}{\partial l} < 0$, which in turns implies $\psi \leq \frac{R}{r}$, where $\psi = \frac{c_2}{c_1}$.

It is straight forward from (2.5) that if there is no early liquidation, $d_{12} > 0$. (Q.E.D)

APPENDIX B

INTERNATIONAL LENDER'S PROBLEM

In chapter III The lenders' problem is to maximize (3.1) subject to the constraints (3.2), (3.3), and (3.4) in the text.

A Lagrangean is formed where

$$L^* = \ln c_1^* + \beta^* \ln c_2^* \quad (\text{B.1})$$

where

$$c_1^* = e_1^* + \rho_{01} s_{01} - s_{12} \quad (\text{B.2})$$

$$c_2^* = e_2^* + \rho_{02} s_{02} + \rho_{12} s_{12} \quad (\text{B.3})$$

and

$$s_{02} = e_0^* - s_{01} \quad (\text{B.4})$$

Using The international lender problem becomes to choose s_{01} and s_{12} to maximize L^* .

$$\frac{\partial L^*}{\partial s_{01}} = 0 \quad (\text{B.5})$$

$$\frac{\partial L^*}{\partial s_{12}} = 0 \quad (\text{B.6})$$

where

$$\frac{\partial L^*}{\partial s_{01}} = \frac{\rho_{01}^s}{c_1^*} - \frac{\beta^* \rho_{02}^s}{c_2^*} \quad (\text{B.7})$$

$$\frac{\partial L^*}{\partial s_{12}} = \frac{\rho_{01}^s}{c_1^*} - \frac{\beta^* \rho_{02}^s}{c_2^*} \quad (\text{B.8})$$

APPENDIX C

DOMESTIC BANK'S PROBLEM WITH RESERVE REQUIREMENTS

In chapter IV, the banks' problem is to maximize (2.1) subject to the constraints (4.1), (4.2), and (4.3), and (2.7) in chapter III. In this case, the Lagrangean (A.1) still applies.

From the budget constraint (4.1), (4.2) and (4.3), I find that:

$$c_1 = \frac{(1 - \theta_{12})d_{12} + rl + \theta_{01}d_{01} - \rho_{01}d_{01}}{\lambda} \quad (\text{C.1})$$

$$c_2 = \frac{R(e_0 + (1 - \theta_{01})d_{01} + (1 - \theta_{02})d_{02} - l) + (\theta_{12} - \rho_{12})d_{12} + (\theta_{02} - \rho_{02})d_{02}}{1 - \lambda} \quad (\text{C.2})$$

After replacing (C.1) and (C.2) in (A.1), the bank's problem becomes to choose d_{01} , d_{12} , d_{02} and l to maximize L . Thus the Kuhn-Tucker conditions are again (A.8), (A.9), (A.10) and (A.11), where

$$\frac{\partial L}{\partial d_{01}} = \left[\frac{\lambda}{c_1} - r\varepsilon \right] \left[-\frac{\rho_{01}^d - \theta_{01}}{\lambda} \right] + \left[\frac{(1 - \lambda)}{c_2} + \varepsilon \right] \left[\frac{R(1 - \theta_{01})}{1 - \lambda} \right] \quad (\text{C.3})$$

$$\frac{\partial L}{\partial d_{02}} = \left[\frac{1 - \lambda}{c_2} + \varepsilon \right] \left[\frac{R(1 - \theta_{02}) + \theta_{02} - \rho_{02}^d}{1 - \lambda} \right] \quad (\text{C.4})$$

$$\frac{\partial L}{\partial d_{12}} = \left[\frac{\lambda}{c_1} - r\varepsilon \right] \left[\frac{1 - \theta_{12}}{\lambda} \right] + \left[\frac{1 - \lambda}{c_2} + \varepsilon \right] \left[\frac{\theta_{12} - \rho_{12}^d}{1 - \lambda} \right] \quad (\text{C.5})$$

$$\frac{\partial L}{\partial l} = \left[\frac{\lambda}{c_1} - r\varepsilon \right] \left[\frac{r}{\lambda} \right] + \left[\frac{1 - \lambda}{c_2} + \varepsilon \right] \left[\frac{-R}{1 - \lambda} \right] \quad (\text{C.6})$$

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