ENERGY-OPTIMAL SCHEDULES OF REAL-TIME JOBS WITH HARD DEADLINES

A Thesis

by

JOHN V. GEORGE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2005

Major Subject: Computer Science

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Approved by:

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ABSTRACT

Energy-Optimal Schedules of Real-Time Jobs with Hard Deadlines. (August 2005)

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In this thesis, we develop algorithms that make optimal use of frequency scaling to schedule jobs with real–time requirements.

Dynamic voltage scaling is a technique used to reduce energy consumption in wide variety of systems. Reducing supply voltage results in a lower processor clock speed since the supply voltage has a proportional dependency on the clock speed of the processing system.

In hard real–time systems, unduly reducing the speed of processor could result in jobs missing their deadlines. The voltage scaling in such systems should therefore take into consideration the deadline of jobs. This thesis will address two questions: First, given a set of discrete frequency levels, we determine an energy-optimal schedule of a given set of real-time jobs. We model the problem as a network flow graph and use linear programming to solve the problem. The schedule can be used on processors with discrete frequencies (like Transmeta Efficeon Processor and AMD Turion 64 Processor).

Second, given a set of real–time jobs, we determine a set of optimal frequency levels which minimizes the energy consumption while meeting all the timing constraints. This can be used to model variable-capacity facilities in operations research, where the capacity of the facility can be controlled at a cost.

DEDICATION

To my Family and Dr. Norman L. Guinasso, Jr.

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God, Thy deeds are so perfect...

I would like to express my sincere gratitude to each and every person who has made this thesis possible.

My adviser, Dr. Riccardo Bettati, has been very patient, friendly and helpful all through the thesis. I am amazed by his brilliant ideas and approach towards problems. He has been an influence not only on my technical approach but also on the way I look at life. It is difficult to overstate my gratitude to him. Dr. Bettati, thank you so much.

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CHAPTER I

INTRODUCTION

In this thesis, we present algorithms that make optimal use of frequency scaling to schedule jobs with real–time requirements.

A job is said to have real-time requirements (or simply said to be *real–time*) if it has to finish within a certain time limit. Such jobs are often common in embedded systems where they arise from the interaction of the system with the environment. Reducing energy consumption in systems with real–time jobs has been a topic of research since over a decade. Dynamic Voltage Scaling (DVS) is a technique used to reduce energy consumption in real–time embedded systems. Energy consumption has a quadratic dependency on supply voltage and DVS aims at reducing energy consumption by reducing the processor's supply voltage in tandem with its frequency. For the case of real–time jobs, DVS makes use of the fact that there is no benefit in finishing a job earlier than its deadline. Transmeta Efficeon Processor [1] and AMD Turion 64 [2] are examples of processors that can operate at several discrete operating frequencies based on load requirements.

Variable-voltage processors and algorithms, typically based on DVS, have been developed for periodic, aperiodic, and sporadic jobs. Reducing supply voltage results in a lower processor clock speed since the supply voltage has a proportional dependency on the clock speed of the processing system.

In systems with real–time jobs, unduly reducing the speed of the processor could result in one or more jobs missing their deadline. The voltage scaling in such

This thesis follows the style of *IEEE Transactions on Computer–Aided Design of Integrated Circuits and Systems*.

systems should therefore take in to consideration timing constraints of jobs.

In this paper, we focus on generating energy efficient schedule for jobs with hard real–time requirements. The algorithms described in this thesis generate schedules *offline*, that is, before the execution of the jobs. This thesis addresses two questions:

First, given a set of discrete frequency levels and a set of jobs, we determine an energy–optimal schedule: Among all the feasible schedules possible an energy optimal schedule forms the lower bound on the energy consumption of the given job set.

We develop an algorithm to generate energy optimal schedules, and prove its optimality. That is, we show that the algorithm never fails to find an energy optimal schedule if such a schedule exists. The function to generate an optimal, feasible schedule is modelled as a minimum cost, maximum flow network flow graph problem. Since this form of network flow problems can be solved in quadratic time, our technique scales well to real–life job set.

Second, given a set of real–time jobs, we determine a set of frequency levels that minimizes the energy consumption while meeting the deadline constraints.

We evaluate the algorithms using randomly generated task sets and CNC, a real–life benchmark. We also show that the schedules generated by our algorithms consume less energy while taking polynomial time to generate such a schedule.

Note: Throughout this thesis, we assume that the energy consumption is super–linear with respect to the frequency levels of the processor, that is, $E > O(V)$. This is the case in most architectures where $E \propto V^2$, where E is the energy consumed, and f, the frequency level of the processor.

This thesis is organized as follows: In Chapter II, we give an overview on recent and current research work on DVS that other authors have proposed. The

definitions and system model used in this paper are described in Chapter III. A set of preliminary observations are stated in Chapter IV. Algorithms to generate optimal schedules are described in Chapters V and VI.

CHAPTER II

RELATED WORK

Different scheduling policies and various task models have been considered so far for Dynamic Voltage Scaling (DVS) of Real-Time Systems. DVS has been applied to both soft real-time jobs as well as hard real-time jobs.

The schemes proposed in [3] [4] [5] are based on a two-phase scheduling algorithm: Before run time, voltage settings are picked to reduce energy consumption based on workload assumptions. During run time, an online schedule adjusts voltage based on the resources that remain unused (like when jobs complete early). The authors in [4] proposes dynamic workload adaptation to take advantage of unused computation time that results from the variation of execution time of real-time tasks. A reward based approach to power aware scheduling is used to find an optimal static solution based on worst case workload. Dynamic reclamation is performed by adapting to the actual workload. The paper further proposes an aggressive speed reduction technique that can be employed to gain more energy savings by assuming a work load less than the worst case workload.

A novel slack estimation heuristic method to create an energy efficient DVS is used in [6]. The paper proposes an algorithm that estimates slack times more efficiently with a small additional overhead. The authors in [7] proposes an efficient method to handle DVS, by creating pseudo operating frequency levels between discrete frequency levels supported by the system. The energy efficient trade-off on a DVS enabled real-time system when the workload includes aperiodic jobs as well as periodic tasks is studied in [5]. The paper explores the performance of a system that consists of both periodic as well as aperiodic jobs. A composite metric Energy

$\times$ Average Response Time is used as a performance measure. A static algorithm and slack re-use scheme is also proposed in the paper.

Four voltage-scaling algorithms suitable for different system characteristics are proposed in [8]. An optimal frequency grid that minimizes the effect of discrete operating frequencies is derived in the paper. A formula for improving an existing DVS algorithm without any negative effect on the performance of the algorithm is proposed in [9].

An inter-task DVS for sporadic task model in conjunction with preemptive EDF scheduling is presented in [10]. Three voltage scaling schemes are proposed to schedule jobs with non-preemptive sections in [11]. Negative impacts on task scheduling and system wide energy can be reduced by controlling task preemption according to [12] [13]. An $O(N^2)$ algorithm, where N is the number of jobs, to obtain minimum constant speed for each job and an $O(N^3)$ to obtain the minimum constant speed for the whole job set is presented in [14].

The authors of [15] formulate DVS for a set of periodic jobs as a nonlinear optimization problem and propose an optimal off-line algorithm to solve this problem. They further propose an online dynamic algorithm to reclaim slack cycles generated by early completion of jobs. In [16], a model of dynamically variable voltage processor is presented. Static voltage scheduling problem is formulated as an integer linear programming. A set of basic theorems for power-delay optimization is also presented in the paper.

The first algorithm presented in this paper is most closely related to [17]. It proposes two offline DVS schemes to minimize the energy consumed by a processor. The first solution models DVS problem as a generalized network flow (GNF) graph and uses the generalized network simplex algorithm to solve it. A GNF model cannot scale to large task sets using processor model with large number of operating

voltages. Thus, a drawback of the solution proposed in [17] is that it cannot scale to large number of frequencies. The second scheme discussed in [17] eliminates this drawback by developing a near-optimal minimum-cost network flow model.

The second drawback of [17] is that it assumes voltage switching only at *task boundaries*. This is called *inter-task* scheduling. The authors in [18] and [19] discuss how intra-task scheduling (that is, when voltage switching can occur also during the execution of a task) always results in greater energy savings than inter-task scheduling. The method discussed in [18] though efficient, have significant impact on the portability of the application, as it requires modifications to the operating system in most cases. An improvement on the method (based on *worst-case* execution time) in [18] is proposed in [19] (based on *average-case* execution time).

In this thesis, we concentrate on developing algorithms that are simple to implement and have the added advantages of intra-task scheduling. Unlike, many of the previously discussed algorithms we do not use check points or other complex schemes that would make the algorithm complex or non-portable. We also make sure that the algorithms works independent of the number, or lack there, of frequencies or jobs.

CHAPTER III

DEFINITIONS AND SYSTEM MODEL

In the following section, we describe a reference model for real-time systems used in this thesis. According to this model, each system is characterized by three elements: (1) a workload model [20] that describes the type of applications supported by the system, (2) a processor model that describes the processing resources available to the applications, and (3) a power model that defines the power consumed when an application executes on a processing unit.

A. Workload Model

The workload model used in this thesis captures only the timing behavior of the applications in the system. Other characteristics such as functional behavior are ignored. In this paper we consider a workload that consists of a set of n jobs $\mathcal{J} = \{J_1, J_2, \ldots, J_n\}$. Each job J_i is defined by its *release time* r_i, its *deadline* d_i, and the number x_i of CPU cycles required to execute the jobs. *Release Time* of a Job J_i is the time at which it is released. *Deadline* of a Job J_i is the time at or before which J_i should complete execution. Deadline of a job is always greater than its release time. We assume that a job can start its execution at any time between its release time and deadline. The number of cycles required by a Job J_i to complete its execution is the execution cycles. We represent each job J_i by a tuple (r_i, d_i, x_i). We assume that a job can start its execution at or after its release time. We also assume that a job can be preempted at any time, and each job is independent of each other.

We call a task as *periodic task* if it executes repeatedly at regular or semi-

regular time intervals in a continuing basis. A task is called *aperiodic task* in this paper if it has no deadline and the inter arrival times between consecutive jobs vary widely. Tasks with inter arrival times that vary widely but have hard deadlines are called as *sporadic jobs*. An *executable-interval* of a Job J_i is the set of intervals occurring between r_i and d_i.

This workload model is very general in practice. For example, periodic workloads in embedded systems are often scheduled by indefinitely repeating a schedule laid out for the jobs during a so-called *hyperperiod*, which is the least common multiple of all the periods of the task set. We take a hyperperiod, H, of the jobs such that no job overlaps with any other job outside its hyperperiod. H is assumed to have n jobs.

B. Processor Model

Jobs need a processor in order to make progress towards completion. In this thesis, we assume two types of processors. We model each type of processor separately for the sake of convenience.

In the *Discrete Frequency Model*, we assume that the processor can execute at a frequency f_k if and only if $f_k \in \mathcal{F} = \{f_0, f_1 \dots f_m\}$, where $\mathcal{F}$, is the set of all the frequencies that can be used by the processor, sorted in increasing order. Thus, f_0 is the minimum frequency possible by P and f_m the maximum frequency possible by P. Throughout this thesis, we assume f_0 to be equal to 0. We call a processor that adheres to the discrete frequency model a *discrete frequency processor*.

In *Continuous Frequency Model*, we assume that the processor can execute at any frequency in the scale ranging from f_0 to f_m, where f_0 is the minimum frequency available to the processor, and f_m is the maximum frequency available to

the processor. We call a processor that adheres to the continuous frequency model a *continuous frequency processor*.

C. Power Model

According to [21], CMOS circuits have both static and dynamic power dissipation. Static power arises from bias and leakage circuits. The reduction in the use of static gates as well as efficient design of modern gates have led to negligible power consumption in static gates.

Therefore the dominant part in most designs is the dynamic power consumption.

From [21], dynamic power consumption can be defined in simple equation as follows:

$$P \cong V_{dd}^2 \times f_{clk} \times C_{eff} \ , \tag{3.1}$$

where V_{dd} is the supply voltage, f_{clk} is the clock frequency, and C_{eff} is expressed as a product of physical capacitance,C_l, and the activity weighting factor, α.

Since the energy consumption is equal to the power dissipation over time, we have

$$Energy \cong V_{dd}^2 \times f_{clk} \times C_{eff} \times t \tag{3.2}$$

where t is the time for which f_{clk} is used.

According to [18], the number of cycles, X_{cyc} executed in a processor, during time t at frequency f_{clk} is

$$X_{cyc} = f_{clk} \times t. \tag{3.3}$$

From Equation (3.3), we get

$$Energy \cong V_{dd}^2 \times C_{eff} \times X_{cyc}. \tag{3.4}$$

According to [10]

$$t_d = k\frac{V_{dd}}{(V_{dd} - V_t)^2}, \tag{3.5}$$

where t_d is the circuit delay time, k a constant, and V_t the threshold voltage and

$$f_{clk} = \frac{1}{L_d t_d}, \tag{3.6}$$

where L_d is the depth of the critical path[10].

Combining Equations (3.4), and (3.5) we get,

$$Energy \propto V_{dd}^2 \times X_{cyc}. \tag{3.7}$$

Note: Equations (3.5) and (3.6) tell us that voltage is proportional to frequency, and frequency is inversely proportional to time delay. Thus, by reducing voltage, the frequency decreases, causing time delay to increase.

D. Energy Aware Scheduling

The goal of this thesis is to develop algorithms that, given a job set $\mathcal{J}$, and a processor (discrete or continuous frequency), determine a schedule of execution of the jobs in $\mathcal{J}$ and a set of frequency allocations in order to meet the job's timing requirements (release time and deadlines) while minimizing the total energy consumption.

We say that a schedule is *valid* if it schedules all the jobs in $\mathcal{J}$ in such a way that no two jobs are scheduled at the same time on a single processor.

A valid schedule is *feasible* where all the jobs in $\mathcal{J}$ are scheduled in such a way that no job misses its deadline. Among all the feasible schedules there is one or more schedules where the total power consumption is minimal. We call such a schedule an *energy optimal schedule*. We call the sequence of frequencies at which the processor

executes the jobs in the schedule, the *frequency allocation*. An *optimal frequency* allocation gives rise to the minimum energy consumption. Therefore energy optimal schedules have optimal frequency allocations.

The rest of this thesis is about algorithms that generate energy optimal schedules.

CHAPTER IV

PRELIMINARY OBSERVATIONS

We note that the model observed in Section (III-C) is not limited to the case of $P \propto V^2$ but to any case where $P > O(V)$. A number of simple preliminary observations hold in this case. We list them as follows.

In the following observations, we substitute f for V by combining Equations (3.5) and (3.6).

Lemma 1 *[16] Given a single Job $J = (r, d, x)$, the frequency allocation f is optimal when it is constant and $f = \frac{x}{d-r}$.*

Corollary 1 *Given two jobs with identical release time, $J_1 = (r, d_1, x_1)$ and $J_2 = (r, d_2, x_2)$, $d_2 > d_1$. Let $f_1 = \frac{x_1}{d_1 - r}$ and $f_2 = \frac{x_2}{d_2 - d_1}$ be two frequencies used by j_1 and j_2 respectively. Whenever $f_2 > f_1$, the energy consumed will be minimum if f_2 equal to f_1.*

Lemma 2 *Given two jobs with identical release time, $J_1 = (r, d_1, x_1)$ and $J_2 = (r, d_2, x_2)$, $d_2 > d_1$. Let $f_1 = \frac{x_1}{d_1 - r}$ and $f_2 = \frac{x_2}{d_2 - r}$ be two frequencies used by j_1 and j_2 respectively. Whenever $f_2 < f_1$, the energy consumed will be minimum if the jobs execute at frequencies f_1 and f_2 respectively.*

Proof: Let us assume that jobs J_1 and J_2 run at frequencies other than f_1 and f_2 defined in the Lemma 2. This can be done in three ways: Without loss of generality we assume $r = 0$ in the following proof.

1. Decrease Job J_1's frequency below f_1.

Job J_1cannot run at frequency lower than f_1 since it would miss its deadline otherwise.

2. Increasing J_2's frequency above f_2.

This would increase the overall energy consumption.

3. Decrease J_2's frequency below f_2.

Let E be the energy consumed by the processor while executing at frequencies f_1 and f_2. From Equation 3.7 we have,

$$E = f_1^2 x_1 + f_2^2 x_2. \tag{4.1}$$

Substituting for x_1 and x_2 from Lemma 2 we get,

$$E = f_1^3 d_1 + f_2^3 (d_2 - d_1). \tag{4.2}$$

Assume that we decrease J_2's frequency by moving Δy cycles from interval $[d_1, d_2]$ to interval $[r_1, d_1]$. The energy consumed E' in this case is

$$E' = (f_1 + \Delta y)^3 d_1 + (f_2 - \Delta y)^3 (d_2 - d_1). \tag{4.3}$$

Since $\Delta y > 0$,

$$(\Delta y)^2 + 3f_1^2 + 3f_1 \Delta y > 0. \tag{4.4}$$

Combining Equations (4.3, 4.2, and 4.4),

$$f_1^3 d_1 + f_2^3 (d_2 - d_1) < (f_1 + \Delta y)^3 d_1 + (f_2 - \Delta y)^3 (d_2 - d_1) \tag{4.5}$$

which is equivalent to $E < E'$. This proves Lemma 2.

CHAPTER V

ENERGY OPTIMAL SCHEDULE ON DISCRETE FREQUENCY PROCESSORS

In this chapter, we define a problem to schedule a set of jobs with arbitrary release time, arbitrary deadline, and arbitrary execution time, on a processor with discrete set of frequencies, in an energy optimal manner. According to [22], an iterative flow network can be used to find a feasible cyclic schedule if such a schedule exists. According to [20], this network representation can be generalized to find schedules of jobs with arbitrary release time, arbitrary deadline, and arbitrary execution time as shown in the next section.

A. Min-Cost Max-Flow Network Flow Problem

Network flow has been used in a variety of settings to generate optimal schedules, typically in the context of generating feasible schedules or schedules that minimize other execution measures, such as makespan. In this section we describe how we formulate the problem of energy-optimal scheduling on a discrete-frequency processor as a Min-Cost Max-Flow network problem.

1. Network Flow Problem

Given a directed graph $G = (V, E)$, where V is the set of nodes, and E is the set of directed edges. An edge between any two nodes in G is represented by (u, v) where $u, v \in V$. Each edge $(u, v) \in E$ has a *capacity* $c(, u, v)$ associated with it. We say that $c(u, v) = \emptyset\,, \forall (u, v) \notin E$. Given two special nodes Source s, and sink t, the

goal is to find a *Flow* $f(u, v)$ to each edge such that,

$$
\begin{aligned}
&1. \qquad \forall\, u, v \in V, \quad f(u, v) > \emptyset \\
&2. \qquad \forall\, u, v \in V, \quad f(u, v) \leq c(u, v) \\
&3. \qquad \forall\, u, v \in V, \quad f(u, v) = -f(u, v) \\
&4. \qquad \forall\, u \in V - \{s, t\}, \quad \sum_{v \in V} f(u, v) = \emptyset.
\end{aligned}
$$

The problem is to find an allocation of flows to edges so as to maximize the total flow from Node s to Node t. In this form, the network flow problem can be solved by the Ford-Fulkerson method. Simple scheduling problems, such as preemptive scheduling of jobs with arbitrary release times[20], feasible execution times, and deadlines on a single processor can be solved using simple network flow. In the energy optimal scheduling addressed in this paper we device a solution to a more general network flow problem.

2. Min-Cost Max-Flow Network Flow Problem

In this variation of the Network Flow Problem each flow on an edge is associated with a cost function $\sigma(u, v, f(u, v))$. In the following we use a linear cost function, where

$$\sigma(u, v, f(u, v)) = C_{u,v} \times f(u, v),$$

and $C_{u,v}$ is some edge specific constant. For this simple case, the Min-Cost Max-Flow Network Flow problem can be solved using Linear Programming [23].

3. Energy Optimal Scheduling Using Min-Cost Max-Flow Network Flow

In the following, we describe how to formulate the problem of generating an energy-optimal schedule of a set of jobs as a Min-Cost Max-Flow network flow problem. We will be using the following notation to reason about frequency levels and time intervals:
We call f'_j the difference between the frequency f_j and f_{j-1}, i.e.

$$f'_j = f_j - f_{j-1}\,, \qquad j \in 1, 2, \ldots m.$$

In addition, we partition the time into intervals I_k as follows:
Sort the release times and deadlines of all the jobs in the system. All the neighboring entries then form a sequence of adjoining intervals, which we call $I_1, I_2 \ldots$. Our objective function is to minimize the energy consumption given by Equation (3.7).

The above equation can be modelled as a Min-Cost Max-Flow graph with the edge cost as a linear function of flow as shown in Figure (1). Any algorithm (like Network Simplex algorithm) that can solve a Min-Cost Max-Flow problem can be used to solve the graph. We define a flow graph, $G = (V, E)$, where V is the set of vertices, and E is the set of directed edges as follows.

· $\forall\,(u, v) \in E$, we represent flow along edge (u, v) as $\sigma(u, v)$.

· $\forall\,(u, v) \in E$, we represent the cost of flow along edge (u, v) as $\sigma(u, v)$.

The graph contains the following vertices and edges.

1. *job vertex* J_i, $\quad \forall\, J_i \in \mathcal{J}$, representing job J_i.

2. *interval vertex* I_j, $\quad \forall\, I_j \in \mathcal{I}$, representing interval I_j.

3. *frequency vertex* f_k, $\quad \forall\, f_k \in \mathcal{F}$, representing frequency f_k

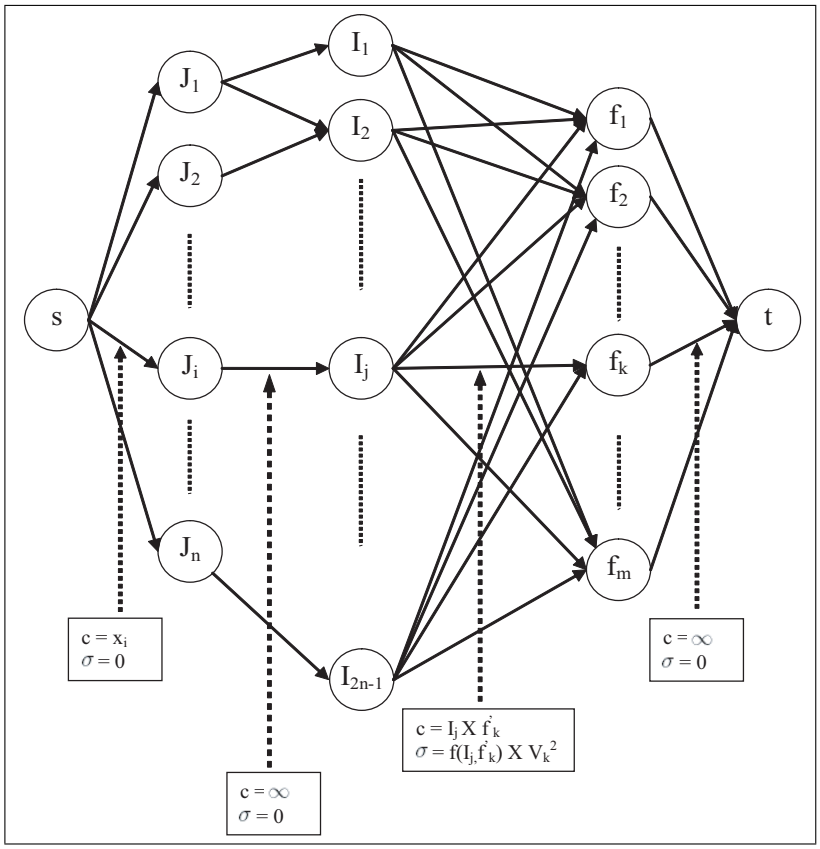


FIG. 1. Network flow graph representation

4. There exists two special vertices s and t, which represent the Source and Sink of the Flow Graph respectively.

5. $\forall\, J_i \in \mathcal{J}$, there exists a directed edge $(s, J_i),\ c(s, J_i) = x_i,\ \sigma(s, J_i) = 0$.

6. $\forall\, J_i \in \mathcal{J}$, $\forall\, I_j \in \mathcal{I}$, there exists a directed edge (J_i, I_j) if J_i can be scheduled in the interval $I_j,\ c(J_i, I_j) = \infty,\ \sigma(J_i, I_j) = 0$

7. $\forall\, I_j \in \mathcal{I}$, $\forall\, f_k \in \mathcal{F}$, there exists a directed edge $(I_j, f_k),\ c(I_j, f_k) = I_j \times f'_k,\ \sigma(I_j, f_k) = f(I_j, f_k) \times V_k^2$

8. $\forall\, f_k \in \mathcal{F}$, there exists a directed edge $(f_k, t),\ c(f_k, t) = \infty,\ \sigma(f_k, t) = 0$

Flow along each edge of the graph can be explained as follows.

$f(s, J_i)$ The flow along this arc corresponds to the number of execution cycles scheduled by the flow algorithm. This edge ensures that the maximum number of execution cycles scheduled by a job $J_i \in \mathcal{J} overall the intervals is equal to x_i$.

$f(J_i, I_j)$ This edge represents the interval over which J_i can be scheduled. The flow along this edge corresponds to the number of cycles scheduled in each interval. It ensures that a job is scheduled only during those intervals that lie between its release time and deadline.

$f(I_j, f_k)$ This cost of flow along this edge corresponds to the energy consumed to schedule the job at frequency f_k. The flow along this arc also gives the frequency sketch for each of the intervals. The objective function which minimizes the cost of maximum flow is implemented by this arc.

$f(f_k, t)$ This arc ensures that outflow from the source node is equal to inflow to the sink.

$$\sum_{i=1}^{n} x_i = \sum_{j=1}^{N} \sum_{k=1}^{m} I_j f_k. \tag{5.1}$$

Since (J_i, I_j) gives the number of cycles executed by J_i during the interval I_j, and (I_j, f_k) gives the frequency sketch along each interval I_k, the schedule can be obtained by combining these two information.

4. Algorithm to Create Min-Cost Max-Flow Graph

We now describe the steps of the algorithm depicted in Algorithm. 1

Step 1 All the release times and deadlines are sorted in increasing order and intervals are formed by partitioning the time into $2n - 1$ intervals, where n is the number of jobs.

Step 2 A graph G is defined with vertices formed by $\mathcal{J}, \mathcal{I}$, and $\mathcal{F}$. Two special nodes Source and Sink are defined as the start and end nodes of the graph.

Steps 3-4 The capacity of (I_j, f_k) is equal to the number of cycles available during I_j at f_k. Since all the cycles until frequency f_{k-1} would be used before f_k, the actual frequency available at f_k would be equal to the $f_k - f_{k-1}$. Therefore, we define $f'_k = f_k - f_{k-1}$. Note that the cost of using frequency f_k is still equal to f_k^2.

Steps 5-15 The edges of the graph G are defined.

Step 16 We run a standard Simplex algorithm [23] that would generate a maximum flow which minimizes cost along flow.

Step 17 As a result of the above step, we get the *frequency allocation* of the energy optimal schedule. The schedule is generated in the next step.

Step 18 From the energy optimal frequency allocation, compute the energy optimal schedule as follows:

Algorithm 1 Discrete-Freq-Scheduler

1: Sort r_i's and d_i's in increasing order. Let $\mathcal{I}$ be the set of non-overlapping intervals formed by the partition of time from the earliest release time to the latest deadline into at most N intervals.

2: Construct Graph $G = (V, E)$ where $V = \{s, \mathcal{J}, \mathcal{I}, \mathcal{F}, t\}$, is the set of vertices.

3: **for all** $f_k \in \mathcal{F}$ **do**

4: $\quad f_0^{'} = 0, \quad f_k^{'} = f_k - f_{k-1}$

5: **for all** $J_i \in \mathcal{J}$ **do**

6: $\quad$ Define a directed edge $(s, J_i), \quad c(s, J_i) = x_i, \quad \sigma(s, J_i) = 0$

7: **for all** $J_i \in \mathcal{J}$ **do**

8: $\quad$ **for all** $I_j \in \mathcal{I}$ **do**

9: $\quad\quad$ **if** J_i can be scheduled in the interval I_j **then**

10: $\quad\quad\quad$ Define a directed edge $(J_i, I_j), \quad c(J_i, I_j) = \infty, \quad \sigma(J_i, I_j) = 0$

11: **for all** $I_j \in \mathcal{I}$ **do**

12: $\quad$ **for all** $f_k \in \mathcal{F}$ **do**

13: $\quad\quad$ Define a directed edge $(I_j, f_k), \quad c(I_j, f_k) = I_j \times f_k, \quad \sigma(I_j, f_k) = f(I_j, f_k) \times V_k^2$

14: **for all** $f_k \in \mathcal{F}$ **do**

15: $\quad$ Define a directed edge $(f_k, t), \quad c(f_k, t) = \infty, \quad \sigma(f_k, t) = 0$

16: Solve the Min-Cost Max-Flow Graph using any existing algorithm.

17: Construct the *Frequency Allocation*

18: From *Frequency Allocation* construct the *Energy Optimal Schedule* for $\mathcal{J}$.

We observe that if there is a flow from I_x to f_y, there must be flows from I_x to all f_z, where $f_z < f_y$. This is caused by the cost function, which causes the flows from I_x to saturate first for lower frequency (and therefore lower cost) nodes. Given this, the *frequency allocation* of I_x can be obtained by dividing the flow from I_x to f_y by $f'y$, i.e. $\frac{f(I_x,f_y)}{f'_y}$. The number of cycles of each Job J_r, x_r, scheduled in the interval I_x, can be obtained by the flow from J_r to I_x, $f(J_r, I_x)$. Given the frequency allocation of an interval and the number of cycles of each job executing during that interval, we can obtain the Energy Optimal Schedule by *shaving off* parts of the interval as we allocate the jobs in the interval.

Therefore, the total time complexity of the algorithm is equal to $O(n^3)$[23]

5. Proof of Correctness

Theorem 1 *Min-Cost Max-Flow Graph produces an energy optimal schedule, if such a schedule exits.*

Proof: To prove that Min-Cost Max-Flow Graph produces an energy optimal schedule, it needs to be proven that the objective function that the graph minimizes represents the energy consumption of the processor. It also needs to be proven that the algorithm maximizes the flow through G, which in turn proves that G produces a feasible schedule whenever one exists.

According to Equation (3.7), the square of frequency times the number of execution cycles used at that frequency represents energy consumption of the system. Since more than one frequency is being used by the processor, calculating the energy consumption at each frequency level and then adding up gives:

$$E = E_1 + E_2 + \ldots E_m \tag{5.2}$$

where E_k represents the energy consumed by the processor at a frequency f_k.

$$E_k = V_k^2(x_1 + x_2 + x_3 + \ldots x_i) \tag{5.3}$$

thus, proving that the minimization function represents Equation (3.7) which in turn represents the energy consumed by a processor.

The graph created here is identical to Network flow graph explained in Section (A). The only additional constituent is the set of nodes $\mathcal{F}$, which represents the frequency used during the schedule. But, $\forall\, I_j \in \mathcal{I}, \quad I_j$ is linked to $f_k, \forall\, f_k \in \mathcal{F}$ which in turn is linked to the t. Thus, it is equivalent to I_J being linked directly to t. Thus, the graph produces an feasible schedule with minimal energy consumption, if such a schedule exists.

6. Performance

In this section we analyze the complexity of the graph as well as the algorithm to create the graph.

a. Graph Complexity

Since there are n jobs and m frequencies, the number of nodes in the graph is bounded from above as follows:
The graph will have n job nodes corresponding to each job, $2n-1$ interval nodes corresponding to each interval, and m nodes corresponding to each frequency. Since there are two special nodes corresponding to the source and the sink, the total number of nodes are bounded from above by $3n + 1 + m$.

The upper bound on the number of arcs can be estimated as follows:
The graph will have n arcs from s to J_i, $i = \{1, 2, \ldots, n\}$. In the worst case, the number of arcs from the job node to the interval node is $n(2n-1)$. The number

of arcs from the interval node to the frequency node is $m(2n-1)$. The number of arcs from the interval nodes to sink is equal to m. Thus the total number of arcs is bounded by $2n^2 + 2nm$.

b. Algorithm Complexity

The performance of the algorithm can be estimated as follows.

Step 1 In the worst case a good sorting algorithm takes $O(nlog(n))$.

Steps 2-7 Since there are $2n-1$ intervals Step (3) will take $O(n)$, and Step (5) will take $O(n)$ as well.

Steps 8-11 Since there are n jobs and $2n-1$ intervals, the worst case time complexity to define arc (J_i, I_j) will be $O(n^2)$.

Steps 12-14 Since there are m frequencies and $2n-1$ intervals, Steps (11) through (13) will take $O(nm)$.

Steps 15-16 For m frequencies, Step (14 will take $O(m)$.

Step 17 According to [23], the worst case time complexity of a Min-Cost Max-Flow Algorithm is $O(n^3)$.

Therefore the worst case time complexity of the algorithm is $O(n^3 + mn)$.

B. Linear Programming Formulation of the Discrete Frequency Processor

Based on the network flow formulation described in Section V-A, we give an equivalent Linear Programming formulation, which can be used as alternative to the Network Flow formulation. This leads to more compact representation of the problem to feed into the Linear Programming Solver. We define the following

parameters to formulate the problem in the Discrete Frequency Case:

Parameters

x_i number of execution cycles required by Job J_i

s_k start time of interval I_k

e_k end time of interval I_k

Decision Variables

x_{jk} number of cycles used during interval I_k at frequency range f'_j

y_{ik} number of cycles used by job J_i during interval I_k

Objective

$$\text{Min } E = \sum_{j=1}^{N} \sum_{k=1}^{m} V_j^2 \, x_{jk} \tag{5.4}$$

The objective function minimizes the total energy consumption of the processor based on the power model defined in Section (III-C).

Constraints

$$\forall \, J_i \in J, \quad \sum_{k=1}^{N} y_{ik} = x_i \tag{5.5}$$

Constraint 5.5 ensures that the sum of all the cycles of J_i, executed over all intervals $I_k \in \mathcal{I}$, is equal to the number of execution cycles, x_i, required, by J_i.

$$\forall \, J_i \in J, \quad \forall \, I_k \in I, \quad r_i y_{ik} \leq s_k \, y_{ik} \tag{5.6}$$

$$\forall \, I_k \in I, \quad \forall \, J_i \in J, \quad d_i y_{ik} \geq e_k \, y_{ik} \tag{5.7}$$

Constraints 5.6 and 5.7 ensure that J_i executes only during those intervals that lie between r_i and d_i.

$$\forall \, I_k \in I, \quad \sum_{i=1}^{n} y_{ik} \leq (e_k - s_k) \, f_m. \tag{5.8}$$

Constraint 5.8 ensures that no interval is assigned more execution cycles than it can hold over its entire length at the maximum frequency, f_m.

$$\forall\, I_k \in I, \quad \sum_{i=1}^{n} y_{ik} \leq \sum_{j=0}^{m} x_{jk}. \tag{5.9}$$

Constraint 5.9 ensures that the sum of execution cycles used by all the jobs in a particular interval is equal to the sum of the cycles used during all the frequency ranges f'_k at that interval.

The decision variables, x_{jk} and y_{ik}, of Linear Programming formulation defined above are non-negative real numbers. As said before, the above model can be input into a Linear Programming Solver to obtain an Energy Optimal Schedule.

CHAPTER VI

ENERGY OPTIMAL SCHEDULES ON CONTINUOUS FREQUENCY PROCESSORS

In the previous chapter we developed algorithms for energy optimal scheduling of real-time jobs on a processor capable of running at any of one or more discrete frequencies.

In this section we expand these results to the case of *continuous-frequency* processors. A *continuous-frequency* processor is capable of running at *any* frequency between a lower bound f_0 and an upper bound f_m.[1]

Continuous frequency processors can be used , for example, to model variable-capacity facilities in operations research, where the capacity of the facility can be controlled at a cost. We observed in Chapter IV that the frequency allocation on a *continuous-frequency* processor during energy-efficient operation is discrete. The additional degree of freedom added by *continuous-frequency* processors is the ability for the designer to freely choose the frequency levels at scheduling time. For static systems this means that the frequency allocation is performed at system design time.

In the context of processor scheduling, *continuous-frequency* processor scheduling therefore can for example be used to determine optimal discrete frequency settings for application specific processor realizations.

In this chapter, we define the problem to schedule, in an energy-optimal feasible

[1] In the following we assume $f_0 = zero$ and f_m to be the maximum frequency possible by the processor. In practice, any energy optimal scheduling algorithm will assign some maximum execution frequency. No feasible schedule exists if this frequency exceeds f_m.

manner, a set of jobs with arbitrary release time, arbitrary deadline, and arbitrary execution time on a single processor that can assume any frequency between f_0 and f_m.

A. Initial Approach

A possible method to estimate an optimal frequency allocation for a given job set $\mathcal{J}$ is to enumerate all the probable frequencies (based on Lemmas 1 and 2 and Corollary 1) that jobs in $\mathcal{J}$ can execute. Once this set of frequencies are known, the *continuous-frequency* problem is reduced to a *discrete-frequency* problem described in Chapter V. Unfortunately, in practice the number of possible frequencies is too large for this approach to succeed as shown in the following.

An upper bound on the number of frequencies possible by all the jobs in $\mathcal{J}$ can be calculated as follows:

- ◇ If we schedule jobs of subsets of size one of $\mathcal{J}$, we need at most $C(n, 1)$ different frequencies.

- ◇ If we schedule jobs of subsets of size two, we need at most $C(n, 2)$ additional frequencies. These frequencies occur when the execution intervals of two jobs intersect. The frequency is calculated as the sum of execution cycles of both the jobs, divided by the difference of the latest deadline job with the earliest released job among both the jobs.

- ◇ Similarly, if we schedule jobs with subsets of size n we need at most

$$\sum_{i=1}^{n} C(n, i) \tag{6.1}$$

additional frequencies.

◇ If we schedule a subset of size one first, and then schedule the remaining $n-1$ jobs in subsets of size 1 of $\mathcal{J}$, there are at most $C(n-1,1)$ additional frequencies.

◇ Similarly, there can be at most

$$\sum_{i=1}^{n-1} C(n-1,i) \tag{6.2}$$

additional frequencies if we schedule the remaining $n-1$ jobs of $\mathcal{J}$ in subsets of size n.

◇ Since there are n jobs, the maximum number of possible frequencies is at most equal to

$$n \times \sum_{i=1}^{n-1} C(n-1,i). \tag{6.3}$$

Adding up all the possible frequencies for $\mathcal{J}$, in the worst case we get,

$$\sum_{i=1}^{n} C(n,i) + n \times \sum_{i=1}^{n-1} C(n-1,i). \tag{6.4}$$

Thus, it can be seen from the above equation that frequency enumeration is exponential.

1. Continuous Frequency Problem Evaluation

In this section we evaluate the difficulty of generating an energy optimal schedule on a *continuous-frequency* processor. According to Equation (3.7), *Energy is proportional to the square of Voltage times the Number of Cycles.*

In discrete frequency model problem, the frequencies (and their respective voltages) at which a processor can be scheduled are given. Therefore, the problem to find an energy optimal schedule on a discrete frequency processor is linear. In a *continuous-frequency* model problem, voltages and the corresponding frequencies

are unknown and therefore the problem becomes non-linear and more difficult to solve.

In the next section, we solve an easier problem by assuming that all the jobs in $\mathcal{J}$ have identical release time. We then propose a solution in Section VI-C to schedule a set of jobs with arbitrary release times on a *continuous-frequency* processor.

B. Jobs with Identical Release Time

The scheduling problem addressed in this section can be formulated as follows: *Given a set of independent, preemptive jobs $\mathcal{J} = \{J_1, J_2, \ldots, J_n\}$, each with identical release time r, arbitrary deadline d_i and arbitrary execution time x_i, schedule the jobs on a single processor with continuous frequencies in an energy optimal feasible manner.*

In the following, we make use of the lemma's presented in Chapter IV to propose an algorithm to schedule a set of jobs. We call the resulting algorithm *Energy-EDF*.

1. Algorithm and Description

Energy-EDF algorithm can be described as follows:
The jobs in $\mathcal{J}$ are sorted in ascending order based on their deadline. The execution order of the jobs are in *Earliest Deadline First* (EDF) manner. We optimize each job, starting with the job having the earliest deadline to the job having the latest deadline, using a greedy approach.

We call J_k a *neighbor* of J_i if they form neighboring entries in the sorted sequence of jobs. The first job J_1 in $\mathcal{J}$ is scheduled in the interval $[r_1, d_1]$. For each job $J_i \in \mathcal{J}$, $i \in \{2, 3, \ldots, n\}$ starting with the job having the earliest deadline to

the job having the latest deadline: if the frequency of allocating J_i from $d_{PREVIOUS}$ (deadline of job $J_{PREVIOUS}$ - the *neighbor* of J_i whose $d_{PREVIOUS} < d_i$) to d_i is less than the frequency of $J_{PREVIOUS}$, $f_{PREVIOUS}$ then, J_i is scheduled in the interval $[d_{PREVIOUS}, d_i]$; Else J_i and $J_{PREVIOUS}$ are scheduled from the start time of $J_{PREVIOUS}$, $s_{PREVIOUS}$, to d_i at a frequency equal to the sum of their execution cycles divided by the total allocated execution time ($d_i - s_{PREVIOUS}$).

Algorithm 2 ENERGY-EDF (identical release time)

Require: $\mathcal{J} = \{J_1, J_2, \ldots, J_n\}$ where $J_i = (0, d_i, x_i)$

Ensure: Energy Optimal Schedule

1: Sort the jobs in $\mathcal{J}$ on d_i such that $d_1 < d_2 < \ldots < d_n$

2: Let s_i be the start time of J_i and $s_i = 0$.

3: Create empty Stack

4: **for** $i \leftarrow 1, n$ **do** **Optimize**(J_i)

The algorithm to optimize a set of jobs $\mathcal{J}$, is divided in to three functions. The main function, Algorithm 2, sorts the job set and calls *Optimize* Algorithm 3, to optimize the frequency allocation of each job. The third function, Algorithm 4 merges two jobs in to one.

The Algorithm 2 can be explained as follows.

Steps 1-3 $\mathcal{J}$ is sorted in ascending order of deadlines of jobs. A empty Stack is created for later use and the *start-time* of all the jobs are initialized to their *release time*.

Step 4 For each job, starting with the earliest deadline job to the latest deadline job, *Optimize* is called to optimize each job.

Algorithm 3 can be described as follows:

A Stack is used to store the schedule created by the algorithm. r_i is assigned the

Algorithm 3 Optimize jobs

function OPTIMIZE(job J_i)
    if Stack is empty **then**
        Schedule J_i in the interval $[0, d_i]$ at frequency $f_i = \frac{x_i}{d_i - 0}$
        Push(J_i) to Stack
    else
        $J_{PREVIOUS} \leftarrow$ Stack.Pop()
        $s_i = d_{PREVIOUS}$
        if $\frac{x_i}{d_i - s_i} > f_{PREVIOUS}$ **then**
            $J_{PREVIOUS} =$ **Merge**$(J_i, J_{PREVIOUS})$
            Optimize$(J_{PREVIOUS})$
        else
            Push($J_{PREVIOUS}$) to Stack
            Schedule J_i in the interval $[s_i, d_i]$ at frequency $f_i = \frac{x_i}{d_i - s_i}$
            Push(J_i) to Stack

Algorithm 4 Merge jobs

function MERGE($J_i, J_{PREVIOUS}$)
    Create a temporary job J_{TEMP}
    $s_{TEMP} = s_{PREVIOUS}, \quad d_{TEMP} = d_i, \quad x_{TEMP} = x_i + x_{PREVIOUS}$
    Schedule J_{TEMP} in the interval $[s_{TEMP}, d_{TEMP}]$ at frequency $f_{TEMP} = \frac{x_{TEMP}}{d_{TEMP} - s_{TEMP}}$
    Return J_{Temp}

start time of J_i in the algorithm. J_i is scheduled from $d_{PREVIOUS}$ to d_i if f_i is less than $f_{PREVIOUS}$; else both jobs are scheduled from $r_{PREVIOUS}$ to d_i.

Steps 2-4 If *Stack* is empty, then schedule J_i in the interval $[r_i, d_i]$ with frequency $f_i = \frac{x_i}{d_i - r_i}$. Push J_i to *Stack* and *Return*.

An empty stack means that the incoming job is either the first job or is obtained by merging all the jobs scheduled before the current time.

Steps 6-7 Else If there exists at least one job in the stack then, pop it out to $J_{PREVIOUS}$. Assign the start time of J_i to $d_{PREVIOUS}$.

Steps 8-10 Let $f_i = \frac{x_i}{d_i - r_i}$. If $f_i > f_{PREVIOUS}$ then, $J_{PREVIOUS}$ and J_i are scheduled from the start time of $J_{PREVIOUS}$ to d_i at frequency f_i. *Optimize* function is called recursively with $J_{PREVIOUS}$ as its argument.

The process of scheduling two jobs at an identical frequency as in Steps [8-10] is done using a function called *Merge*. This function schedules the jobs as stated above and returns the scheduled job.

Steps 12-14 If $f_i < f_{PREVIOUS}$ then, both $J_{PREVIOUS}$ and J_i are pushed to stack separately.

Once *Energy-EDF* runs to completion, the schedule can be obtained by popping the stack. Such a schedule minimizes the energy consumed by the job set, as proven below.

2. Proof of Correctness

Lemma 3 *Any job schedulable by EDF is schedulable by* Energy-EDF.

Proof: Let $J_i = (r, d_i, x_i)$ and $J_k = (r, d_k, x_k)$ be two neighboring jobs scheduled at

frequency f_i and f_k during intervals $I_i = [r, d_i]$ and $I_k = [d_i, d_k]$ using *Energy-EDF* Algorithm. The frequency can be allotted in two ways.

Case 1: $f_k \leq f_i$ In this case, *Energy-EDF* does not do any re-scheduling. It assigns J_i in the interval $[r, d_i]$ and J_k in the interval $[d_i, d_k]$. Since $d_i < d_k$, the algorithm schedules the jobs in an EDF manner.

Case 2: $f_k > f_i$ Here, *Energy-EDF* re-schedules J_i and J_k at a frequency $f = f_k = f_i = \frac{x_i + x_k}{d_k - r_i}$. Since, the frequency of execution of J_i increases from f_i to f, J_i completes its execution before d_i. J_k starts its execution at the time when J_i completes its execution. Both the jobs are still scheduled in an EDF manner.

Without loss of generality, the above case of scheduling two jobs can be extended to a set of n jobs as shown in the proof below. Thus any job schedulable by EDF is schedulable by *Energy-EDF* as well.

Lemma 4 *Given a set of n jobs, $\mathcal{J} = \{J_1, J_2, \ldots, J_n\}$ where $J_i = (r, d_i, x_i)$ sorted in the increasing order of their deadlines d_i, Energy-EDF schedules $\mathcal{J}$ in an energy optimal manner.*

Proof: *Energy-EDF* is a greedy algorithm that works on *two-job optimization* scheme. We call a scheme that can optimize the whole job set taking subsets of two jobs at a time as *two-job optimization* scheme.

Number of jobs $n = 1$

If there is only a single job J_1, Energy-EDF schedules J_1 at a frequency $f_1 = \frac{x_1}{d_1 - r_1}$. According to Lemma 1, this is the optimal frequency to schedule a single job.

Number of jobs $n = 2$

Let J_1 and J_2 be the two jobs to be scheduled by *Energy-EDF*. There are two ways in which J_1 and J_2 can be scheduled by *Energy-EDF*.

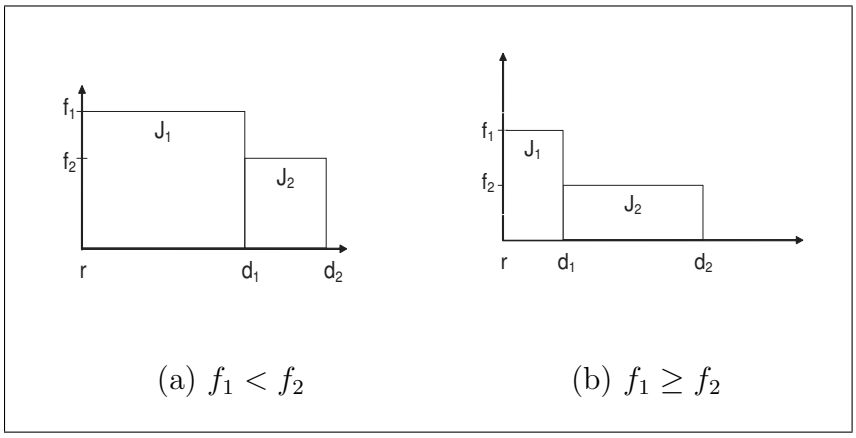


FIG. 2. Schedule when $n = 2$

$f_2 \leq f_1$**:** In this case *Energy-EDF* does not do any re-scheduling. J_1 and J_2 are scheduled at frequencies f_1 and f_2. According to Lemma 2, if $f_2 \leq f_1$, it is optimal to schedule J_1 and J_2 at frequencies f_1 and f_2 respectively. Therefore, Energy-EDF schedules the job in an energy optimal manner. This is shown in Figure 2(a).

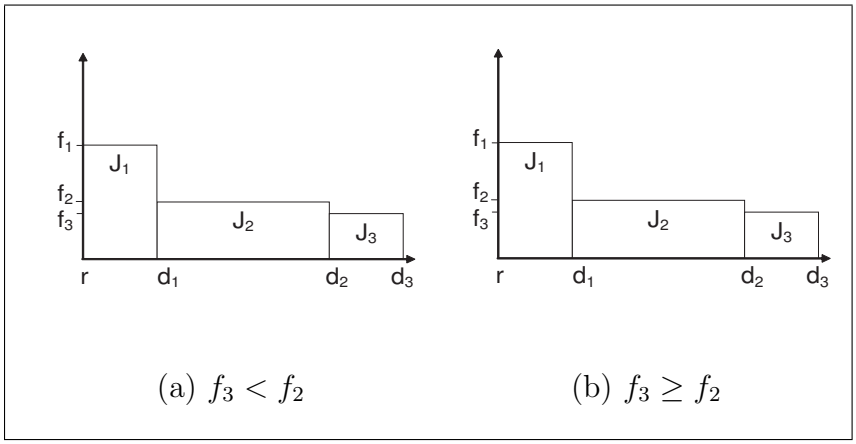
$f_2 > f_1$**:** Energy-EDF schedules the jobs J_i and J_2 at a frequency $f = \frac{x_1+x_2}{d_2-r_1}$. According to Corollary 1, the optimal frequency to schedule J_1 and J_2 if $f_2 > f_1$ is $\frac{x_1+x_2}{d_2-r_1}$. This is shown in Figure 2(b).

Thus when the number of jobs equal two, Energy-EDF generates an optimal schedule.

Number of jobs $n = 3$

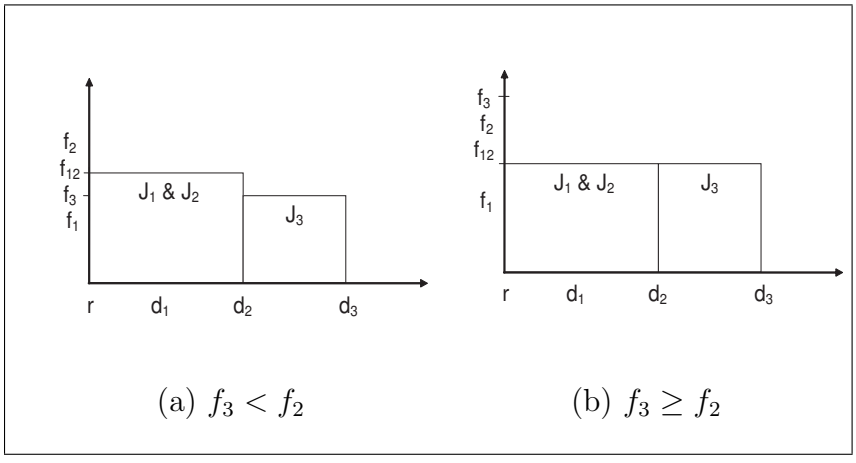
When the number of jobs equal three, Energy-EDF algorithm will first generate a schedule for J_1 and J_2 before taking J_3 into consideration. J_1 and J_2 will be scheduled in an energy optimal manner among themselves by the time J_3 is considered by Energy-EDF.

There are only two ways in which Job J_3 will have its frequency with respect to J_2.



(a) $f_3 < f_2$ (b) $f_3 \geq f_2$

FIG. 3. Schedule when $n = 3$ and $f_1 > f_2$



(a) $f_3 < f_2$ (b) $f_3 \geq f_2$

FIG. 4. Schedule when $n = 3$ and $f_2 > f_1$

$f_3 < f_2$: Jobs J_1 and J_2 can have their schedules in two ways as follows:

$f_2 < f_1$: It is already proven that J_1 and J_2 form an optimal schedule among each other. Since $f_3 < f_2$, J_3 and J_2 form an optimal schedule according to Lemma 2.This is shown in Figure 3(a)

$f_2 = f_1$: Since J_1 and J_2 *merged* together to form a single job of identical frequency, J_3 and J_1, J_2 can be scheduled using a *two-job scheduling* scheme. Since it is already proven that *Energy-EDF* generates an optimal schedule when the number of jobs equal two, the schedule is optimal in this case. This is shown in Figure 4(a)

$f_3 \geq f_2$: In this case J_3 and J_2 *merge* together to form a single job with equal frequency. Now, the three job scheduling problem is reduced to two job scheduling problem. In the pervious section we already proved that *Energy-EDF* generates an optimal schedule when the number of jobs equal two. These scenarios are depicted in Figures 3(b) and 4(b).

Assume the algorithm generates an optimal schedule for k jobs by reducing it into *two-job scheduling* problem.

Number of jobs $n = k + 1$

When there are k+1 jobs, k jobs with earliest deadline are scheduled first. The schedule will look as shown in Figure (5) or (6).

1. $f_{k+1} < f_k$

According to Lemma 2, J_{k+1} and J_k are scheduled in an energy optimal manner among themselves. Since $\{J_1, J_2, \ldots J_k\}$ already form an energy optimal schedule among themselves, the jobs in job set $\{J_1, J_2, \ldots J_{k+1}\}$ is also scheduled in an optimal manner.

2. $f_{k+1} > f_k$

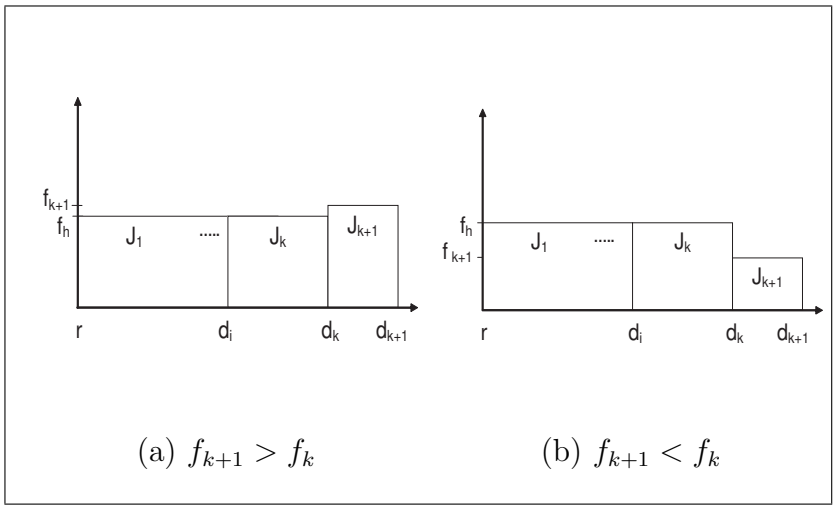


FIG. 5. Schedule when $n = k + 1$ and $f_1 \leq f_2 \leq \ldots \leq f_k$

Energy-EDF reschedules J_k and J_{k+1}, such that both the jobs are *merged* to form a single job with equal frequency, thus reducing the number of jobs to be scheduled to k. It is already proven that Energy-EDF generates an energy optimal schedule for k jobs.

Thus *Energy-EDF* generates energy optimal schedule based on a greedy scheme called *two-job optimization.*

3. Performance

The time complexity of the Algorithm 3 can be estimated as follows.

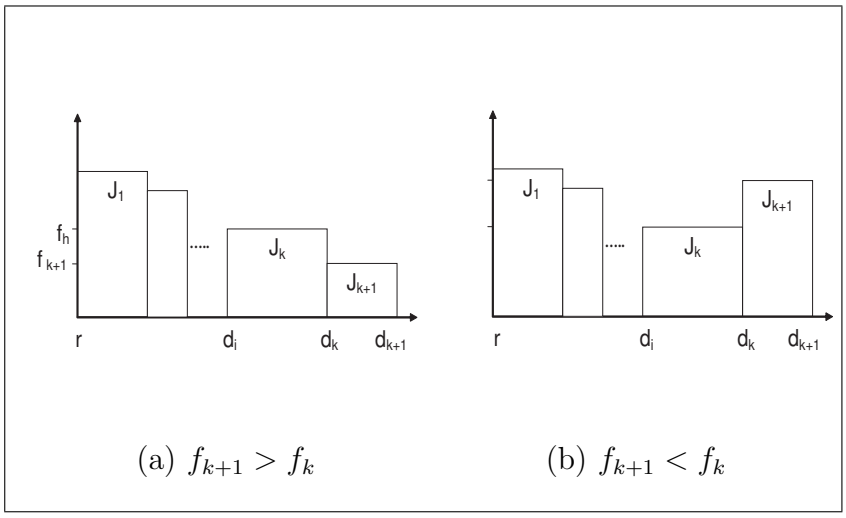
Step 1 A sorting algorithm takes $O(nlog(n))$ in the worst case.

Step 2 This step takes a constant time to execute

Step 3 Since there are n jobs, it would take $O(n)$ to complete.

The time complexity of the Algorithm 2 can be estimated as follows:

Step 1 The function is executed $O(n)$ times from Algorithm 2. In the worst case,



(a) $f_{k+1} > f_k$ (b) $f_{k+1} < f_k$

FIG. 6. Schedule when $n = k + 1$ and $f_1 \geq f_2 \geq \ldots \geq f_k$

it will be called $O(n)$ times by itself during each such call. Thus, this step takes $O(n^2)$ in the worst case.

Step 2-8 All the other steps take a constant time to execute.

Algorithm 4 takes a constant time to execute and therefore the worst case execution time of *Energy-EDF* is bounded by $O(n^2)$.

In the worst case the algorithm will take $O(n)$ extra space to store the schedule in a stack.

In the next section we first propose an algorithm based on the assumption that frequency enumeration is possible. We then propose a method to cut down the number of frequencies to polynomial and use a similar method to solve the problem of scheduling a set of jobs with arbitrary release time.

C. Jobs with Arbitrary Release Time

The scheduling problem addressed in this section can be formulated as follows: *Given, a set of independent, preemptive jobs with arbitrary release time, arbitrary*

deadline, and arbitrary execution time, deduce an algorithm to schedule the jobs on a single processor in an energy optimal feasible manner.

1. Energy Optimal Scheduling Through Extensive Frequency Enumeration

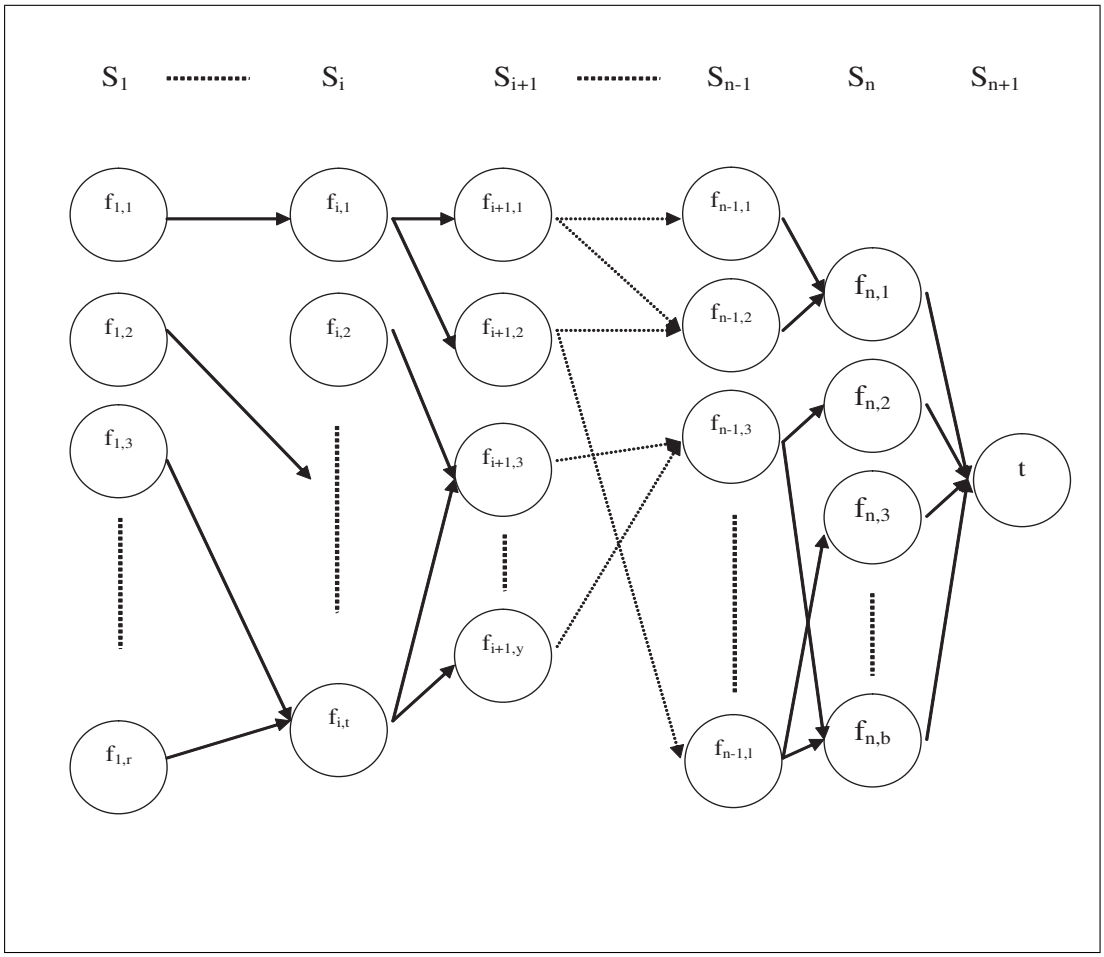


FIG. 7. Flow graph representation of frequencies

In the following we assume that all the possible frequencies for every job in $\mathcal{J}$, can be enumerated. Then, we propose a *Dynamic-Programming*(DP) based approach to obtain an optimal frequency allocation on a *continuous-frequency* processor model.

1. $F = \{f_{i,j}, t\}$, is the set of all vertices in G, where the vertices $S_i = \{f_{i,1}, f_{i,2} \ldots f_{i,k_i}\}$, are all the possible frequencies that J_i could be executed. In the following, we call S_i a stage in the networks.

2. An arc $(f_{i,j}, f_{i+1,l}) \in E$ if there exists a feasible schedule when J_i and j_{i+1} are executed at $f_{i,k}$ and $f_{i+i,l}$ respectively.

3. Cost of arc $(f_{i,j}, f_{i+1,l})$, $\sigma(f_{i,j}, f_{i+1,l})$ is $V_{i,k}^2 x_i$, where $V_{i,k}$ is the voltage corresponding to $f_{i,j}$.

4. S_{n+1} is a special node called sink t.

By definition, any path connecting a node in S_1 to t, as shown in Figure (7), represents a feasible schedule in $\mathcal{J}$. Among all the feasible schedules in $\mathcal{J}$, the energy optimal schedule is a path which minimizes the sum of all the arc costs of the path from a node in S_1 to t.

For a given feasible frequency graph, the energy optimal schedule (i.e. the energy optimal path) can be determined as follows.

Denote the energy optimal path from $f_{i,q}$ to t as $\Psi(f_{i,q})$. Then,

$$\Psi(f_{i,q}) = \begin{cases} 0, \quad if \quad i = n+1; \\ min\{\sigma(f_{i,q}, f_{i+1,r}) + \Psi(f_{i+1,r})\}, \quad if \quad i = 1, 2, \ldots, n \end{cases}$$

where node $f_{i,q} \in S_i$, and node $f_{i+1,r} \in S_{i+1}$.

The value $\sigma(f_{i,q}, f_{i+1,r})$ represents the cost of the arc from node $f_{i,q}$ to node $f_{i+1,r}$.

Though dynamic programming does give us an optimal solution for the above problem, the algorithm to generate all the possible frequencies of $\mathcal{J}$ would still have an exponential time complexity.

Since frequency enumeration is exponential and the algorithm relies on the assumption that frequency enumeration is possible, we now propose three lemmas and a theorem to cut down the number of probable frequencies to polynomial. We then use an approach similar to the one presented in this section to generate a schedule on a *continuous-frequency* processor in polynomial time.

2. Implicit Frequency Enumeration

In this section, we present a set of lemmas and a theorem that we use to reduce the number of possible frequencies to polynomial in terms of number of jobs.

Lemma 5 *[16] Given a set of jobs $\mathcal{J}$, the energy consumed will be minimum only if each interval in the interval set $\mathcal{I}$ executes at a constant frequency.*

Lemma 6 *[16] Given a set of jobs $\mathcal{J}$, the energy consumed will be minimum only if each job executes at a constant frequency.*

Lemma 7 *Given a set of jobs $\mathcal{J}$, the energy consumed will be minimum if the frequency of execution of every job in $\mathcal{J}$ is equal to or lower than the frequency of execution of all the other jobs scheduled over its executable interval.*

Proof: Assume that there is at least one other job executing with a lower frequency in the executable interval of Job J_i. We can always reschedule J_i such that it executes at a lower frequency. By extending the proof of Lemma 2 it can be seen that the energy consumed while executing a job at lower frequency is lower than the energy consumed to execute a job at higher frequency, thus proving Lemma 7.

Theorem 2 *For a schedule to be energy-optimal, it is necessary and sufficient for it to satisfy Lemmas 5, 6 and 7.*

Using the above theorem (proof given in Section VI-4), we propose a polynomial-time DP approach to calculate the optimal frequencies. We call the resulting algorithm *Cont-Freq-Scheduler*. The jobs are initially sorted in the increasing order of their release times. An initial schedule is generated by scheduling the jobs from their release time either to the release time of the next job or to it's own deadline, which ever comes earlier. Then, starting from job have the latest deadline to the job have the earliest deadline, we run an *Optimize* algorithm to obtain an energy-optimal schedule.

A minimum allowable frequency is assigned to every interval. No interval can execute at a frequency lower than the minimum allowable frequency. This minimum frequency is initialized to zero and is updated as and when new jobs are scheduled in that particular interval. A frequency is considered *relevant* if it is at least as high as the minimum allowable frequency and minimizes the energy consumption of all the interval sets.

At each state of a stage in DP, the algorithm tries to minimize the energy consumption of two sets of intervals, F_m and F_n by re-scheduling a job (having deadline equal to the end time of F_n) in F_m to F_n if the frequency of F_m is higher than F_n. Thus, the exhaustive search is reduced to a scenario of finding a relevant frequency for two sets of intervals based on a single job.

Let $\mathcal{Z}$ represent a set of intervals and $r_{\mathcal{Z}}$, $d_{\mathcal{Z}}$, $t_{\mathcal{Z}}$ represent the *start-time*, *end-time* and the length of $\mathcal{Z}$ respectively. Let $x_{\mathcal{Z}}$ equals the maximum number of cycles executable in $\mathcal{Z}$ and $f_{\mathcal{Z}}$ the frequency of $\mathcal{Z}$. We use $t_{\mathcal{Z}i}$ and $x_{\mathcal{Z}i}$ to represent the time interval from $d_{\mathcal{Z}}$ to d_i and the number of cycles in the interval $[d_i, d_Z]$ respectively. We use $\mathcal{IS}_i$ to represent the set of intervals in which J_i is scheduled initially, $\forall i, \mathcal{IS}_i = \mathcal{I}$.

Algorithm 5 Cont-Freq-Scheduler (arbitrary release time)

Require: $\mathcal{J} = (r_i, d_i, x_i)$

Ensure: Energy optimal feasible schedule

1: Sort r_i and d_i in increasing order. Let $\mathcal{I}$ be the set of non-overlapping intervals formed by $r_i's$ and $d_i's$.

2: **for all** $I_j \in \mathcal{I}$ **do**, $I_j.minima = \emptyset$

3: **if** $r_{i+1} < d_i$ **then**, schedule J_i in $\mathcal{IS}_i = \{r_i, r_{i+1}\}$

4: **else** schedule J_i in $\mathcal{IS}_i = \{r_i, d_i\}$

5: **for** $i \leftarrow n, 1$ **do**, **Optimize**$(\mathcal{IS}_i)$

A recursive method is used to solve the problem of generating an optimal schedule for a processor with continuous frequencies. The jobs are scheduled from its release time to either its own deadline or the release time of the next job, whichever comes earlier. Thus the job that is released last is scheduled from its release time to its deadline.

The algorithm uses a DP approach to create an energy optimal schedule. Algorithm starts from the job with the latest release time and goes on backward to schedule each job in an energy optimal manner. At each stage of the algorithm, the next job with the latest deadline is taken in to consideration. At the end of any stage S_i, jobs considered at or after S_i are scheduled in an energy optimal with respect to each other.

A stage in the algorithm consists of several states. In each of the state it re-schedules jobs in the interval sets being considered to minimize the energy consumption of the interval sets. The schedule at the end of each stage is shown in Figure 8. We make use of two functions to schedule the jobs optimally. The first

Algorithm 6 Optimize a given set of intervals

1: **function** OPTIMIZE(Interval Set $\mathcal{Z}$)
2: **for all** $J_i \in \mathcal{Z}$ **do**
3: Take the next job J_i, having earliest deadline after $d_{\mathcal{Z}}$
4: Calculate a relevant frequency, f_{opt}, for interval sets $\mathcal{Z}$ and $\{d_i, d_{\mathcal{Z}}\}$
5: **if** $f_{\mathcal{Z}}$ cannot be minimized **then**, goto Step [6.3]
6: **if** additional execution cycles required to schedule $\{d_{\mathcal{Z}}, d_i\}$ at $f_{opt} >$ $(x_i \in \mathcal{Z})$ **then**
7: $x_i \in \mathcal{Z}$ are rescheduled to $\{d_i, d_{\mathcal{Z}}\}$ and the $f_{\mathcal{Z}}$ and frequency of $\{d_i, d_{\mathcal{Z}}\}$ are recalculated.
8: **Optimize**($\{d_i, d_{\mathcal{Z}}\}$)
9: **else**
10: Reschedule $J_i \in \mathcal{Z}$ to $\{d_i, d_{\mathcal{Z}}\}$ such that both the interval sets execute at f_{opt}.
11: Update the minima of each interval in $\{r_{\mathcal{Z}}, d_i\}$
12: Optimize($\{d_i, d_{\mathcal{Z}}\}$)
13: **for all** $I_j \in \mathcal{Z}$ **do**
14: **if** $I_j.minima < f_{\mathcal{Z}}$ **then**, $I_j.minima = f_{\mathcal{Z}}$

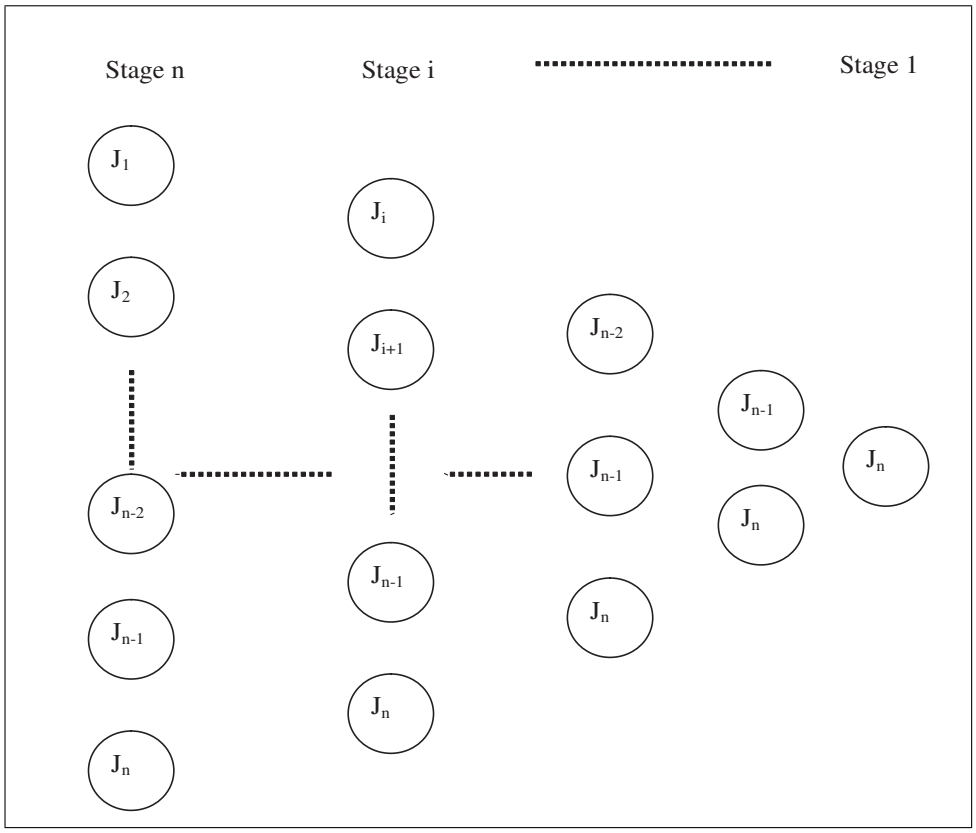


FIG. 8. Stage wise representation of Algorithm 5

algorithm starts a stage in DP, while the second algorithm creates the states to obtain an optimal energy schedule for jobs considered up to the current state.

Algorithm 5 can be described as follows.

Step 1 This step sorts the release times and deadlines of all the jobs. Non- overlapping intervals are formed by immediate release times and deadlines.

Step 2 Initialize the minimum allowable frequency of each interval as zero.

Steps 3-4 Jobs are scheduled from their release time to either their deadlines or to the release time of the next job, which ever occurs earlier.

Step 5 Starting from the job that is released last to the job that is released first, intervals of each job are passed on to Algorithm 6.

Algorithm 6 takes a set of intervals as its input and minimizes the energy consumption of jobs scheduled at or after the current interval. The steps can be explained as follows.

Step 1 An interval set $\mathcal{Z}$, is obtained in the function Optimize()

Steps 2-3 A job, J_i scheduled in $\mathcal{Z}$ and having the earliest deadline after the end time of $\mathcal{Z}$ is taken

Steps 4-5 A relevant frequency f_{opt} is the one which minimizes the energy consumption but does not go below the minimum allowable frequency of the two interval sets is calculated. The number of cycles required by $\{d_z, d_i\}$ to obtain f_{opt} is also calculated. If the frequency of J_i cannot be minimized then, we take the next such job.

Steps 6-8 At each state only J_i is re-scheduled from $\mathcal{Z}$ to $\{d_Z, d_i\}$. If x_i available in $\mathcal{Z}$ is less or equal to the required cycles at $\{d_Z, d_i\}$, then we transfer as

much of x_i available at $\mathcal{Z}$ to $\{d_Z, d_i\}$, and the frequencies are updated. Now a recursive call is made with new $\mathcal{Z} = \{d_Z, d_i\}$.

Steps 9-10 Else if, x_i available in $\mathcal{Z}$ is greater than the required cycles at $\{d_Z, d_i\}$, then we transfer to $\{d_Z, d_i\}$ as much of x_i required, and the frequencies are updated to f_opt.

Step 11 The minimum allowable frequency of each interval in $\{r_Z, d_i\}$ is updated. This can be done by considering only those jobs that have deadline at or before d_i. If the frequency thus obtained is greater than the current minimum allowable frequency, then the frequency is updated.

Step 12 Now a recursive call is made with new $\mathcal{Z} = \{r_Z, d_i\}$.

Steps 13-14 The minima of each interval in $\mathcal{Z}$ is updated with the present frequency of the intervals.

4. Proof of Correctness

Theorem 3 *Given, a set of independent, preemptive jobs with arbitrary release time, arbitrary deadline, and arbitrary execution time, the Optimal Energy Scheduler algorithm schedules the jobs on a single processor in an energy optimal manner.*

Proof: The problem of optimizing a set of n jobs with arbitrary release time, arbitrary deadline, and arbitrary execution time is approached using DP. DP produces the cheapest path for a given set. Since, in our algorithm, we do not produce all the frequencies possible by the jobs in job set, $\mathcal{J}$, we need to prove that the set of frequencies provided to the DP by our algorithm is a super set of the set of optimal frequencies. Since the algorithm is based on Theorem 2, it is sufficient to prove the correctness of the theorem.

It can be seen that Corollary 2 is false when either of Lemmas 5, 6 or 7 is false. This proves the necessary condition.

To prove the sufficiency condition, we show that Corollary 2 is true when all of Lemmas 5, 6 and 7 are true.

Assume that a feasible schedule S satisfies the criteria in Lemmas 5, 6 and 7. Moreover, assume that S is not energy-optimal. Therefore, a feasible schedule S' must exist that has lower energy consumption that S.

Since S' is an energy-optimal schedule, there should exist an interval I' in S' such that the frequency allocation f' of I' is lower than the corresponding interval I in S. Two cases arise in the way jobs are scheduled in I.

There exists some job J scheduled in I that can be feasibly re-scheduled in some other intervals of S such that the energy consumption decreases:
In this case, S does not satisfy Lemma 7.
No job scheduled in I can be re-scheduled in any other intervals of S (with/without increasing energy consumption):
If there exists no job in I that can be re-scheduled to any other intervals of S even without increasing energy consumption of S then, either intervals I and I' should have identical frequency allocation to obtain a feasible schedule, or Lemmas 5 or 6 has to be broken. If there exists a job in I that can be re-scheduled to any other interval I'' (with higher frequency allocation) of S by increasing energy consumption then; a job in I'' should be re-scheduled to I (if possible) to satisfy Lemma 5 or job J needs a frequency allocation at least equal to f to generate a feasible schedule.

The algorithm requires n stages to produce an optimal schedule for a set $\mathcal{J}$ of size n. Each stage has several states associated with it. At each state a frequency is selected based on Theorem 2. By the end of each stage, the algorithm would have produced all the possible frequencies for jobs released at or after that stage, based

on Theorem 2. Since at each state of a stage, the algorithm tries to apply Lemma 2, the set of frequencies obtained as a result is a superset of the optimal frequencies possible till that stage.

During each stage of DP, no job J_i in $\mathcal{J}$ is scheduled beyond its deadline. Thus the schedule generated by the algorithm is feasible if a feasible schedule exists.

5. Performance

The performance of the algorithm can be evaluated as follows. Each of the steps of Algorithm 5 takes the following time complexity.

Step 1 Sorting can be done in $O(nlog(n))$.

Step 2-5 In the worst case these steps will take $O(n)$.

Each of the steps in Algorithm 6 takes the following time complexity.

Step 1 The function is called $O(n)$ by Algorithm (5). In the worst case, each interval will be considered $O(n)$ times during each call. There can be a maximum of n calls to the function. Therefore in the worst case it will take $O(n^2)$.

Step 2-12 Each of the steps will execute to a maximum of $O(n)$.

Step 13-14 Since the number of intervals possible is bounded by $2n-1$, the worst case execution time of this step is bounded by $O(n)$.

Thus, the time complexity of the algorithm is $O(n^2)$.

CHAPTER VII

RESULTS

In this chapter we evaluate the performance of both the processor models presented in Chapters V and VI using various task sets. Simulation experiments were conducted to calculate the effectiveness of the proposed algorithms in saving energy.

TABLE 1. Processor models used in our experiments

Processor	Voltage	Frequency
Transmeta Crusoe	1.3	800
	1.2	667
	1.1	533
	1	400
	0.9	300
AMD K-6 IIIE	1.8	500
	1.7	450
	1.6	400
	1.5	350
	1.4	300

For brevity we refer to the algorithm described in Chapter V as *discrete* and Chapter VI as *continuous*. We use the term *base* to represent a *single-frequency* processor that executes at highest available frequency.

In this section we first compare the energy consumption of *discrete* to that of *base*. We then compare *discrete* to generalized network flow (GNF) model described

in [17]. We also compare the energy consumptions of *discrete*, *continuous* and *base* using task sets generated randomly as described later. We make a fair comparison of the models by alleviating any benefits of task timing by using a spectrum of utilization factor values. Both, GNF and *discrete* were solved using an ORMM solver [24] on a Pentium 4, 1.4 GHz machine with 512MB RAM.

The comparison of *discrete* with GNF and *base* is shown in Table 2. The processor characteristics used for comparison is shown in Table 1. The task set used in the above comparison was adopted from [17]. As can be seen from Table 2, both *discrete* and GNF outperform *base* for any value of processor utilization. Also it can be seen from Table 2 that *discrete* outperforms GNF in more than a couple of tasks. This is due to the fact that GNF is restricted to inter-task scheduling, while no such restriction applies to *discrete*. Thus in cases where it is optimal to run a task or interval at two frequencies, GNF is restricted to run at the higher of the two frequencies. Table 3 shows the percentage variation of energy consumption between *discrete*, GNF and *base*.

Time and space complexity to run both the algorithms are shown in Table 4. As can be seen from the table, the number of nodes and arcs required by GNF increases exponentially as the number of jobs increases, while that of *discrete* increases polynomially. The time required to execute GNF runs into minutes and in some cases into hours, while that of *discrete* remains less that two seconds in all the cases. Thus, *discrete* generates a schedule with lower energy consumption using a compact representation and polynomial time algorithm.

It can thus be seen that GNF, which is NP Complete is not scalable for higher number of frequencies. It also requires large number of arcs and nodes for representation and is computationally inefficient even for small number of problems.

TABLE 2. Energy consumption of *discrete*, GNF and *base*

Model	Task Set	Processor Utilization	Energy consumption		
			Discrete	GNF	Base
Crusoe	$\mathcal{T}1$	0.27	7452	7452	15548
	$\mathcal{T}2$	0.43	17406	18976	35152
	$\mathcal{T}3$	0.54	16800	16800	28392
	$\mathcal{T}4$	0.59	39104	39104	58136
	$\mathcal{T}5$	0.65	52736	52736	74360
	$\mathcal{T}6$	0.72	138751	138816	178464
	$\mathcal{T}7$	0.75	61742	61760	81120
	$\mathcal{T}8$	0.85	168831	169024	200096
	CNC	0.48	49528	51363	103336
AMD	$\mathcal{T}1$	0.27	11270	11270	18630
	$\mathcal{T}2$	0.43	25480	25480	42120
	$\mathcal{T}3$	0.54	20580	20580	34020
	$\mathcal{T}4$	0.59	42140	42140	69660
	$\mathcal{T}5$	0.65	58569	58573	89100
	$\mathcal{T}6$	0.72	155940	155992	213840
	$\mathcal{T}7$	0.75	68988	69024	97200
	$\mathcal{T}8$	0.85	195380	195333	239760
	CNC	0.48	77707	89183	128455

TABLE 3. Percentage variation of energy consumption in *discrete*, GNF and *base*

Model	Task Set	$\frac{Base-Discrete}{Base}$	$\frac{GNF-Discrete}{GNF}$	$\frac{Base-GNF}{Base}$
Crusoe	$\mathcal{T}1$	52%	0%	52%
	$\mathcal{T}2$	50%	8%	46%
	$\mathcal{T}3$	41%	0%	41%
	$\mathcal{T}4$	33%	0%	33%
	$\mathcal{T}5$	29%	0%	29%
	$\mathcal{T}6$	22%	0%	22%
	$\mathcal{T}7$	24%	0%	24%
	$\mathcal{T}8$	16%	0%	16%
	CNC	54%	4%	52%
AMD	$\mathcal{T}1$	40%	0%	40%
	$\mathcal{T}2$	40%	0%	40%
	$\mathcal{T}3$	40%	0%	40%
	$\mathcal{T}4$	40%	0%	40%
	$\mathcal{T}5$	34%	0%	34%
	$\mathcal{T}6$	27%	0%	27%
	$\mathcal{T}7$	29%	0%	29%
	$\mathcal{T}8$	19%	0%	19%
	CNC	40%	13%	31%

TABLE 4. Arcs and nodes of *discrete* versus GNF

Task Set	Discrete			GNF		
	No. of Arcs	No. of Nodes	Time [mm:ss]	No. of Arcs	No. of Nodes	Time [mm:ss]
1	66	24	0.01	164	74	0.04
2	92	28	0.01	248	88	2.15
3	65	23	0.01	158	68	0.01
4	94	30	0.01	260	100	0.01
5	140	44	0.01	404	164	0.01
6	378	90	0.01	1140	340	4.03
7	121	37	0.01	302	122	10.44
8	328	79	0.01	979	294	1.59
CNC	896	213	0.01	2138	763	1.03

TABLE 5. Energy consumption of *discrete* and *continuous* on 5-$\mathcal{T}$ set

Task set	Energy consumption	
	continuous	discrete
5-$\mathcal{T}$1	64619	126755
5-$\mathcal{T}$2	186298	253749
5-$\mathcal{T}$3	380464	380722
5-$\mathcal{T}$4	663095	663131

TABLE 6. Energy consumption of *discrete* and *continuous* on 20-$\mathcal{T}$ set

Task set	Energy consumption	
	continuous	discrete
10-$\mathcal{T}$1	179485	352040
10-$\mathcal{T}$2	517442	704816
10-$\mathcal{T}$3	1057290	1057802
10-$\mathcal{T}$4	1841150	1841303

TABLE 7. Energy consumption of *discrete* and *continuous* on 10-$\mathcal{T}$ set.

Task set	Energy consumption	
	continuous	discrete
20-$\mathcal{T}$1	29737	58432
20-$\mathcal{T}$2	85500	116863
20-$\mathcal{T}$3	174354	175295
20-$\mathcal{T}$4	306013	306103

Above, we saw that *discrete* outperforms both GNF as well as *base* in generating a schedule that consumes lower energy. In the following, we simulate a *continuous-frequency* processor from AMD K-6 processor shown in Table 1. The frequency to voltage relation is calculated from the linear relation of the voltage and frequency as shown in the Table 1. We assume that the *continuous-frequency* processor can run at any frequency lower than the maximum frequency available for the AMD processor (500 MHz).

The task sets used in the following is generated randomly. Similar methods of generating workload is already used in [19]. We generate three different kinds of task sets with four processor utilizations. A mixed class task set is created by randomly generating task periods in the range of $(1 - 1000)\mu seconds$, by dividing the range into short $(0 - 10)\mu seconds$, medium $(11 - 100)\mu seconds$, and large $(101 - 999)\mu seconds$ intervals. Two random tasks are generated in the short and large intervals while only one task is generated in the medium period range. We then simulate a homogenous workload with a shorter period range by generating tasks randomly in the period range of $(0 - 25)\mu seconds$ and $(25 - 50)\mu seconds$. The results of simulating these workloads are shown in Tables 5, 7, and 6. The percentage of variation is shown in Table 8.

It can be seen from the tables that frequency generated by *discrete* is almost equal to that of *continuous* as the processor utilization goes up (above 60%). When utilization is low (below 40%), *continuous* generates a schedule that consumes lower energy than *discrete*. This is because *discrete* cannot go below a certain minimum frequency (300 MHz in this case) assigned to it.

TABLE 8. Variation of energy consumption of *discrete*, *continuous* and *base*

Variation $= \frac{discrete - continuous}{discrete}$			
Processor Utilization	5-$\mathcal{T}$ Set	10-$\mathcal{T}$ Set	20-$\mathcal{T}$ Set
20%	49%	49%	49%
40%	27%	27%	27%
60%	0%	0%	1%
80%	0%	0%	0%

CHAPTER VIII

CONCLUSIONS AND FUTURE DIRECTIONS

In this thesis, we developed two new algorithms to solve the problem of optimal energy scheduling for hard real–time systems.

First, we developed an algorithm to schedule a set of jobs with arbitrary release time, arbitrary deadline and arbitrary execution time on a processor with given set of discrete frequencies. We use the network flow graph to solve the problem. The algorithm utilizes intra-task scheduling model, thus obtaining the minimum energy consumption possible on a given set of frequency.

Second, we developed an algorithm to schedule a set of jobs with arbitrary release time, arbitrary deadline and arbitrary execution time, on a continuous frequency processor such that the energy consumed is minimum.

We first solve a simpler version of the problem by assuming identical release time for all the jobs. We developed a polynomial time algorithm to solve the same. Later, we propose a polynomial time, dynamic programming based approach to solve the optimal energy scheduling problem.

We assume a non-linear relation of voltage to energy. Though, we assume that energy is proportional to square of voltage, our model is generalized and would work under any model with a non-linear relation of energy to voltage. We provide proofs for our claims of optimality and results to substantiate our claims. It is our belief that the results obtained are the best so far in the literature.

The continuous frequency allocation assumes that the available number of frequencies is not limited. This is acceptable in some systems where resource bandwidth is truly variable on a continuous scale. In many other cases, such as in

the design of *application-specific* processors, the number of available frequencies is bounded. For such cases, algorithms must be found that generate frequency level bounded energy-optimal schedules for *continuous-frequency* processors.

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