ANALYTICAL MODELS TO EVALUATE SYSTEM PERFORMANCE MEASURES FOR VEHICLE BASED MATERIAL-HANDLING SYSTEMS UNDER VARIOUS DISPATCHING POLICIES

A Dissertation

by

MOONSU LEE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2005

Major Subject: Industrial Engineering

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Approved as to style and content by:

Guy L. Curry
(Chair of Committee)

Brett A. Peters
(Member)

Richard M. Feldman
(Member)

Anne M. Raich
(Member)

Brett A. Peters
(Head of Department)

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ABSTRACT

Analytical Models to Evaluate System Performance Measures for Vehicle Based Material-Handling Systems under Various Dispatching Policies. (May 2005)

Moonsu Lee, B.S., Hanyang University, Korea;

M.S., Texas A&M University

Chair of Advisory Committee: Dr. Guy L. Curry

Queueing network-based approximation models were developed to evaluate the performance of fixed-route material-handling systems supporting a multiple workcenter manufacturing facility. In this research, we develop analytical models for fixed-route material-handling systems from two different perspectives: the workcenters' point of view and the transporters' point of view. The state-dependent nature of the transportation time is considered here for more accurate analytical approximation models for material-handling systems. Also, an analytical methodology is developed for analytical descriptions of the impact of several different vehicle-dispatching policies for material-handling systems. Two different types of vehicle-dispatching policies are considered. Those are workcenter-initiated vehicle dispatching rules and vehicle-initiated vehicle dispatching rules. For the workcenter-initiated vehicle dispatching rule, the Closest Transporter Allocation Rule (CTAR) was used to assign empty transporters to jobs needing to be moved between various workcenters. On the other hand, four different vehicle-initiated vehicle dispatching rules, Shortest Distance Dispatching Rule (SDR), Time Limit/Shortest Distance

Dispatching Rule (TL/SDR), First-Come First-Serve Dispatching Rule (FCFSR), Longest Distance Dispatching Rule (LDR), are used to select job requests from workcenters when a transporter is available. From the models with a queue space limit of one at each workcenter and one transporter, two different types of extensions are considered. First, the queue space limit at each workcenter is increased from one to two while the number of transporters remains at one. Second, the number of transporters in the system is also increased from one to two while maintaining the queue space limit of one at each workcenter. Finally, using a simulation approach, we modified the Nearest Neighbor (NN) heuristic dispatching procedure for multi-load transporters proposed by Tanchoco and Co (1994) and tested for a fixed-route material-handling system. The effects of our modified NN and the original NN transporter dispatching procedures on the system performance measures, such as WIP or Cycle Time were investigated and we demonstrated that the modified NN heuristic dispatching procedure performs better than the original NN procedure in terms of these system performance measures.

To my parents for their love, sacrifice, support … and everything they gave me.

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CHAPTER I

INTRODUCTION

Analytical models of manufacturing system performance are generally based on queueing network approximations. Material handling of products for transportation between production workcenters has not been considered a critical aspect of the overall system performance and, subsequently, has for the most part been ignored in analytical modeling efforts. The few papers that address material handling with analytical techniques are quite limited in their applicability due to modeling or system configuration assumptions. Also, even though many researchers have studied different dispatching policies in material-handling system using discrete-event simulation methods, the analytical approaches are quite limited so far. In this research, we develop analytical models from two different points of view, both the workcenters' point of view and the transporters' point of view.

From the workcenters' perspective, we develop a state-dependent transportation time analytical model for a fixed-route unidirectional material handling systems. For this model, we use a workcenter-initiated dispatching rule, the Closest Transporter Allocation Rule (CTAR) for transporter-job assignments. For models with the transporters' perspective, four different types of vehicle-initiated dispatching rules are analyzed for the assignment of jobs to an empty transporter located in various

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This dissertation follows the style and the format of *International Journal of Production Research*.

workcenters. These rules are: the Shortest Distance Rule (SDR), the Time Limit/Shortest Distance Rule (TL/SDR), the First-Come First-Serve Rule (FCFSR), and the Longest Distance Dispatching Rule (LDR). We develop, under these dispatching rules, analytical models of the distributions of the resulting transporter allocations as a function of the number and positions of jobs requesting service simultaneously in the system.

1.1 Motivation of the Study

This research is motivated by the fact that, in many analytical modeling approaches for material-handling systems: (1) up to date, material-handling system analysis have mainly utilized simulation as the evaluation tool; (2) the state dependent nature of the service time is not considered so that highly inaccurate analytical models have been developed; (3) no attempts to describe analytically the impact of two different types of vehicle dispatching rules (workcenter-initiated rules and vehicle-initiated rules) for material-handling systems have been made. Therefore, this study, specifically defined in the next chapters, is intended to improve on these current limitations.

The main goal for this research is to develop accurate analytical models for material-handling systems with various dispatching rules by considering the state dependency of the transportation time. To accomplish this goal, the following specific objectives will be considered: (1) design of the analytical model considering a state-dependent nature of the service time; (2) incorporate various dispatching rules (an workcenter-initiated rule and several vehicle-initiated rules) into the procedure for developing analytical models; (3) extend the limitations of analytical models and show the benefits and losses due to those extensions; (4) develop simulation models to verify the accuracy of the results of these analytical models.

The following contributions are expected this research: (1) provide a good methodology for developing and using analytical models for the evaluation of the performance of material-handling systems; (2) model and analyze the material-handling systems supporting multiple workcenters manufacturing facilities more accurately by considering the state-dependent nature of the transportation times; (3) provide a methodology for analytical descriptions of the impact of several different vehicle dispatching policies for material-handling systems.

1.2 Relation to Prior Work

Recently, facility design researchers have begun to explore alternative measures for layout performance, in particular using cycle time or work-in-process (WIP) inventory levels as a measure of the system efficiency. This work is typically based on a queueing model of the manufacturing system. Bozer and Kim (1996) develop a stochastic model, using $M_{(b)}/G/1$ and $M/G/c$ queues, that captures some of the operational characteristics of the manufacturing system. The model focuses on determining unit-load sizes based on the characteristics of the material-handling system and its response to unit-load transfer requests. They demonstrate that decreases in expected WIP are possible from the proper choice of these unit-load sizes.

Fu and Kaku (1997) present a stochastic model to evaluate changes in the expected WIP in the system for different manufacturing system layouts. This model is based on a restrictive open queueing network. Benjaafar (2002) introduces a model that relaxes some of the conditions and assumptions used by Fu and Kaku (1997). This paper focuses on selecting among alternative layout designs to minimize the expected WIP in the system. The paper uses a general $G/G/1$ queueing model approach but only considers a single product type with single unit transfers. Castillo and Peters (2002) investigate the integration of unit-load sizing decisions in conjunction with the determination of the facility layout. A simplified material transport network is modeled and exponential based queueing approximations are used. These results demonstrate that it is important to consider operational issues.

The recent paper by Johnson (2001) studies a similar material-handling problem to the one studied in this paper. His empty transporter assignment distribution under the nearest vehicle assignment rule is mathematically equivalent to our resultant distribution. However, Johnson and other authors, such as Benjaafar (2002), used state independent service time approximations (the $M/G/c$ models) that are highly inaccurate in the waiting time estimates for the dependent service time characteristics displayed by material-handling systems. The present dissertation provides the foundation for a more accurate modeling approach for the material-handling system, which is a key element of the manufacturing system.

Egbelu and Tanchoco (1984) classified vehicle dispatching rules into two categories, workcenter-initiated vehicle dispatching and vehicle-initiated vehicle dispatching. When a new job arrives at a workcenter and there is a set of idle vehicles, the new job selects a vehicle according to the workcenter-initiated dispatching rule. On the other hand, when a vehicle becomes idle and there is a set of jobs waiting to be picked up, the idle vehicle selects a job according to a vehicle-initiated dispatching rule. They evaluated and compared the performance of several workcenter-initiated dispatching rules and vehicle-initiated dispatching rules. Then, they showed that, when material flow rate is high, the system performance is more affected by vehicle-initiated dispatching rules than by workcenter-initiated dispatching rules. Egbelu (1987a) further classified vehicle-initiated dispatching rules into source-driven rules and demand-driven rules in a unit-load transportation-manufacturing system. Under a source-driven rule, an empty transporter chooses the job that has the highest priory in its workcenter queue. In the demand-driven rule, an empty transporter selects a job that has the highest demand from workcenters listed in its routing sequence. Egbelu compared the performance of demand driven dispatching rules with that of source driven dispatching rules and concluded that a demand-driven rule can out perform the best reported source driven dispatching rules from the literature.

Curry, Peters and Lee (2003) developed a state dependent analytical model for a fixed-route material-handling system with a fixed number of transporters based on the workcenter point of view. In their model, the closest transporter allocation scheme was used for vehicle dispatching. From workcenters' perspective, a job arriving at a workcenter selects an empty transporter according to the closest transporter allocation scheme. Thus, this allocation scheme works like a workcenter-initiated vehicle dispatching rule. They showed their model with state dependency is much more accurate than previous state independent analytical models in terms of the system performance measures, work-in-process (WIP) or cycle times.

1.3 Outline of the Dissertation

In Chapter II, an analytical state-dependent queueing network model for a fixed-route transportation system with a fixed number of transporters is developed from workcenters' point of view. The fixed-route assumption limits transporter flexibility on choosing the route through the facility connecting two workcenters. One-way traffic flow and fixed-track transporter systems are examples. That is, once routes of transporters are determined, those are not changed during their transportations. By the Closest Transporter Allocation Rule (CTAR), a job arrival selects the closest empty transporter among candidates from its current location. As the number of free transporters increases, there is a greater chance of a transporter being located at the needed workcenter. Therefore, the transportation times are functions of the number of available transporters at the time that a transporter is allocated to the job. By defining system states as the number of empty transporters, we can develop the approximation scheme accounting for these dependencies within a Poisson-based model and then adjust for the inter-arrival and service time generalizations using an adjustment factor. Analytical computation results are compared with simulation results and yield results in the neighborhood of 0.5%.

In Chapter III, we develop queueing approximation models of the same fixed-route transportation system with a unit-load transporter from a transporter's point of view. From a transporter's perspective, when a transporter finishes the delivery of a job to its destination in the system (termed a service), it needs to select a job to be picked up from possibly several job requests in the system according to a vehicle-initiated dispatching rule. Of course, if there is only one job request in the system, no alternatives are available and the transporter will be assigned to service that job. If there are no jobs waiting in the system when the transporter becomes empty, it waits at the current location until a request for a job movement occurs. In a transporter point of view model, the system status, i.e., the number of job requests in the system, can be checked only at the time of a job departure (service completion). If a job leaves the system empty of job movement requests, the system state remains zero until a new job movement request occurs (termed an arrival). The analytical model is based on transitions from one system state to another. From this model, we obtain steady-states probabilities and, hence, work-in-process in queue (WIP_q), and compared those with simulation results. In this chapter, four different vehicle-initiated dispatching schemes are incorporated into the analytical modeling procedures. The impact of those different dispatching control schemes is examined.

Chapter IV develops two different types of extensions from the original model of Chapter III which has a queue space limit of one at each workcenter and one transporter in the system. First, the queue space limit at each workcenter is increased to two. Second, the number of transporters in the system is increased to two. If we increase the queue space limit at each workcenter to two while the number of transporters remains at one, the system still uses a vehicle-initiated dispatching scheme because there is only one transporter in the system. However, if we add a second transporter to the system, it is now necessary to have both vehicle-initiated dispatching rules and workcenter-initiated dispatching rules components in the vehicle-job assignment control scheme. Because it is possible that a job arrival can see two empty transporters in the system and, thus, the job needs to select one of these transporters according to a workcenter-initiated dispatching rule. For both cases, simulation models are developed to verify the accuracy of our analytical models.

In Chapter V, we modified the Nearest Neighbor (NN) heuristic dispatching procedure for multi-load transporters proposed by Tanchoco and Co (1994) to dynamically reassign jobs to the available space of transporters. Using a simulation approach, the performance of the modified NN heuristic dispatching policy is compared with that of the original NN dispatching policy for two example problems.

Chapter VI summarizes conclusions of this research and discusses future work.

CHAPTER II

ANALYTICAL MODEL FOR MATERIAL-HANDLING SYSTEMS FROM THE WORKCENTERS' POINT OF VIEW*

2.1 Introduction

In this chapter, a probabilistic model of a fixed-route transportation system with a fixed number of transporters is discussed. The fixed-route assumption limits transporter flexibility on choosing the route through the facility connecting two workcenters. In this chapter, our examples are based on one-way traffic flow and fixed-track transporter systems. That is, transporters do not change their routes due to traffic and congestion on the route. Demands for transporters at each workcenter are based on the workcenter's throughput rate by product type. The recipient workcenter for the job requesting transportation is a function of the product routing sequence. The transportation delay until the allocated transporter arrives to pick up the job is developed from the steady-state distributions of the number and locations of available transporters. The Closest Transporter Allocation Rule (CTAR) results in a separate allocated transporter location distribution for each possible number of free transporters and for each demand location. These location distributions and the travel distances between workcenters yield excellent approximations to the transporter service times

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by workcenter. These times become the service times for a queueing model to estimate the queueing delay until a job receives a transporter allocation. The state-dependent nature of these service times leads to inaccuracies in the general distribution model approximations that are based on the standard Poisson model paradigm. We capture these dependencies of the state-dependent service times in the exponential queueing model and then adjust those for the general service distributions with the factor of the form:

$$\left(\frac{C_a^2 + C_s^2}{2}\right). \tag{2.1}$$

Thus, now, we develop a state-dependent multiple-server model using exponentially distributed service times for each state.

2.2 Model with the Workcenter-Initiated Dispatching Rule

An example of a fixed-track transportation system is illustrated in Figure 1. This circular transportation system is used to deliver jobs from a number of workcenters, *N*, to the various other workcenters in the plant. Flow is unidirectional along the circuit. Multiple transporters can be transversing each segment of the track simultaneously but they cannot pass except when a transporter has been offloaded to a workcenter docking site.

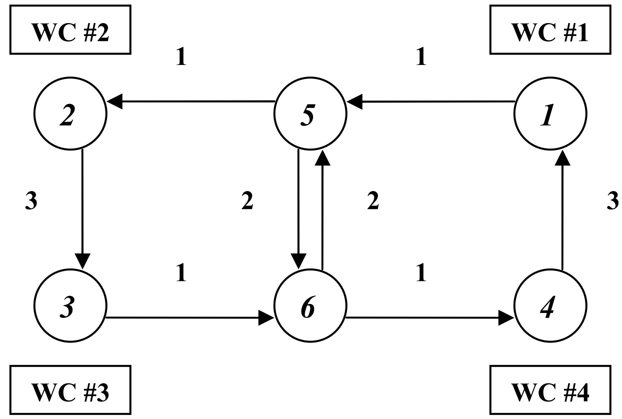

Figure 1. A fixed-route unidirectional material-handling system.

Queueing delays can occur at these off-line docking sites due to the time needed to clear a site. Queueing delays can also occur at circuit intersections in the network. Each of these locations is allocated a position with a specified transporter clearance time and the restriction that only one transporter at a time can transverse these locations. These locations are treated as single server queueing models to approximate the steady-state queueing delay that transporters encounter due to traffic on the transportation system. Demands for transportation from each workcenter to specific other workcenters occur based on the production rate at the workcenter and the routing sequences for jobs. The routing demand information is incorporated into this model via a demand rate λ_{ij} at workcenter i to be transported to workcenter j. Jobs that queue at the workcenters needing transportation are processed in first-come first-serve order.

Under Closest Transporter Allocation Rule (CTAR) control, a job ready for transportation from a workcenter selects the closest empty transporter from its current location. If an empty transporter is already located at the workcenter, then this vehicle will be allocated to the job and no empty transporter travel time will occur. If multiple (unallocated) transporters are available at different locations, then the control system will allocate the closest vehicle based on the distance the vehicles would be required to travel to reach the calling workcenter. These distances are known for a fixed-route layout and the 'closest' vehicle will be assigned to pickup the designated job. At any given time that a job becomes available at this workcenter, the available transporter placements can vary. On average then a job will see a distribution of available transporter placements. This distribution varies with the number of available vehicles and the long-term demand for transportation between workcenters. Once we compute this distribution for each possible number of available vehicles up to the maximum number of vehicles, T, the empty and loaded transporter trip times for all routes can be obtained. Then we have the mean service times as a function of the numbers of servers available at the time of the request.

Now we need to develop the system states. State *i* here is the number of jobs in the transportation subsystem at the time of the vehicle assignment. As the number of jobs in the transportation subsystem decrease, that is, the number of free transporters increases, there is a greater chance of a transporter being located at the needed workcenter. Therefore, the transportation time is the function of these numbers. Since the service (transportation) times are depend on the number of available transporters at the time that a transporter is allocated to the job, there are T service rate classes with mean rates denoted by μ_i, where the index i represents the number of jobs in the transportation subsystem, and thus, there are T – i free transporters available at the time of the vehicle assignment. When there is only one job in the transportation subsystem, this job can be on any one of the routes with mean rate μ_i, for $i \in \{1, 2, \ldots, T\}$. These possibilities, however, do not occur with equal probability. With only one job being transported, it is more likely it has a high service rate index number. To illustrate, for this job to have a mean service rate of μ_1 the job requested transportation when the transportation system was empty and, hence, was allocated the closest of T transporters. For a single job in the transportation system to have a mean service rate of μ_2, there was one other job in the transportation system when this job was assigned a transporter. Hence, there would have been only T – 1 transporters available and the closest one would have been assigned. After this assignment, the other job being transported was delivered to its destination before this new job and the result is one job currently in the transportation system with a mean service rate of μ_2. For a single job to have a mean service rate of μ_3, this job had to be assigned a transporter when there were two other jobs already being transported and then these two jobs had to be delivered at their respective destinations before the job under consideration. And so forth until all single rate possibilities have been covered (μ_1, μ_2, μ_3, …, μ_T). Conceptually, these states are increasingly unlikely due to the condition that all the other active jobs have to be delivered first. For transportation states with two-active jobs, these states are {12, 13, …, 1T, 22, …, 2T, …, TT}. These states have mean service rates denoted by a pair of rates (μ_i, μ_j). That is, there are two independent transportation processes operating each with its own service rate. States with three jobs in transit are denoted by three numbers, *ijk*, and have an associated rate three-tuple of the form (μ_i, μ_j, μ_k), etc.

To further illustrate the states and how the system transitions between them, consider a system with three transporters. There are only 14 distinct server configurations although there are an infinite number of states. The distinct transporter states are: empty (0), one busy transporter (1, 2, 3), two busy transporters (12, 13, 22, 23, 33) and all three transporters busy (123, 133, 223, 233, 333). Note that the five distinct states where all transporters are busy are repeated for each possible number of jobs in the transportation queue (0, 1, 2, …), i.e., jobs waiting for a transporter assignment. Now, consider the system transitions between above states. To enter the empty state (0), the system can be in a single busy transporter state and the job gets delivered before any new transportation requests occur. To help with this discussion, diagrams of the state transitions are given below. The diagram notation is from state to state with an arrow indicating the direction of the transition and the type of transition (denoted by the service rate or an A for the arrival of a new job transportation request). Thus, the entry into the zero state has three possibilities:

$$\textit{State } 0 : 1 \xrightarrow{\mu_1} 0,\ 2 \xrightarrow{\mu_2} 0,\ 3 \xrightarrow{\mu_3} 0$$

For the states with one busy transporter:

$$State\,1: 0 \xrightarrow{A} 1,\ 12 \xrightarrow{\mu_2} 1,\ 13 \xrightarrow{\mu_3} 1$$
$$State\ 2: 12 \xrightarrow{\mu_1} 2,\ 22 \xrightarrow{\mu_2} 2,\ 23 \xrightarrow{\mu_3} 2$$
$$State\ 3: 13 \xrightarrow{\mu_1} 3,\ 23 \xrightarrow{\mu_2} 3,\ 33 \xrightarrow{\mu_3} 3$$

For the states with two busy transporters:

$$State\,12: 1 \xrightarrow{A} 12,\ 123 \xrightarrow{\mu_3} 12$$
$$State\,13: 123 \xrightarrow{\mu_2} 13,\ 133 \xrightarrow{\mu_3} 13$$
$$State\ 22: 2 \xrightarrow{A} 22,\ 223 \xrightarrow{\mu_3} 22$$
$$State\ 23: 123 \xrightarrow{\mu_1} 23,\ 223 \xrightarrow{\mu_2} 23,\ 233 \xrightarrow{\mu_3} 23$$
$$State\ 33: 133 \xrightarrow{\mu_1} 33,\ 233 \xrightarrow{\mu_2} 33,\ 333 \xrightarrow{\mu_3} 33$$

And finally, for the states where all transporters are busy (states with Q appended indicate one job waiting in the queue for a transporter assignment):

$$State\,123: 12 \xrightarrow{A} 123,\ 123Q \xrightarrow{\mu_3} 123$$
$$State\,133: 13 \xrightarrow{A} 133,\ 133Q \xrightarrow{\mu_2} 133,\ 133Q \xrightarrow{\mu_3} 133$$
$$State\ 223: 22 \xrightarrow{A} 223,\ 223Q \xrightarrow{\mu_3} 223$$
$$State\ 233: 23 \xrightarrow{A} 233,\ 223Q \xrightarrow{\mu_1} 233,\ 233Q \xrightarrow{\mu_2} 233,\ 233Q \xrightarrow{\mu_3} 233$$
$$State\ 333: 33 \xrightarrow{A} 333,\ 233Q \xrightarrow{\mu_1} 333,\ 333Q \xrightarrow{\mu_2} 333,\ 333Q \xrightarrow{\mu_3} 333$$

There are, of course, an infinite number of states of the form 123*Q*, 123*QQ*, 123*QQQ*, etc. that the system can take on. In fact, each of the five all-busy transporter states have these multiple forms. But the major transitions have been illustrated with these 14 state-transition diagrams.

To obtain the system performance measures for our example model, such as WIP_q and WIP_{sys}, we need to compute the steady-state probabilities, P_i's, that there are exactly *i*-jobs in the system. Using the possible transition states with their associated transition rates, the steady-state flow-balance equations are written using rate matrix $\mathbf{Q}$ as $\mathbf{P}^\mathbf{T}\cdot\mathbf{Q} = \mathbf{0}^\mathbf{T}$ and $\mathbf{P}^\mathbf{T}\cdot\mathbf{1} = 1$. The transition rate matrix $\mathbf{Q}$ has the structure displayed in Figure 2.

	P_0	$\mathbf{P}_1$	$\mathbf{P}_2$	$\mathbf{P}_3$	$\mathbf{P}_4$	$\cdots$	$\mathbf{P}_{n+1}$	$\mathbf{P}_{n+2}$	$\cdots$
P_0	$\mathbf{B}_{00}$	$\mathbf{C}_{01}$							
$\mathbf{P}_1$	$\mathbf{A}_{10}$	$\mathbf{B}_{11}$	$\mathbf{C}_{12}$						
$\mathbf{P}_2$		$\mathbf{A}_{21}$	$\mathbf{B}_{22}$	$\mathbf{C}_{23}$					
$\mathbf{P}_3$			$\mathbf{A}_{32}$	$\mathbf{B}_{33}$	$\mathbf{C}_{34}$				
$\vdots$				$\ddots$	$\ddots$	$\ddots$			
$\mathbf{P}_n$					$\mathbf{A}_{n,n-1}$	$\mathbf{B}_{n,n}$	$\mathbf{C}_{n,n+1}$		
$\mathbf{P}_{n+1}$						$\mathbf{A}$	$\mathbf{B}$	$\mathbf{C}$	
$\vdots$							$\ddots$	$\ddots$	$\ddots$

Figure 2. The structure of the transition rate matrix $\mathbf{Q}$.

Now, the objective is to obtain the values for the steady-state probabilities denoted by **P**, where **P** is partitioned in the same fashion as **Q**. That is, $\mathbf{P} = (P_0|\mathbf{P}_1|\mathbf{P}_2|\cdots|\mathbf{P}_n|\cdots)$. The steady-state solution is obtained by solving the following steady-state balance equations in matrix form:

$$\mathbf{P^T} \cdot \mathbf{Q} = \mathbf{0^T}.$$

So, using the sub-matrices from partitioning, the following system of equations needs to be solved:

$$P_0 \cdot \mathbf{C}_{01} + \mathbf{P}_1 \cdot \mathbf{B}_{11} + \mathbf{P}_2 \cdot \mathbf{A}_{21} = \mathbf{0^T}$$
$$\mathbf{P}_1 \cdot \mathbf{C}_{12} + \mathbf{P}_2 \cdot \mathbf{B}_{22} + \mathbf{P}_3 \cdot \mathbf{A}_{32} = \mathbf{0^T}$$
$$\mathbf{P}_2 \cdot \mathbf{C}_{23} + \mathbf{P}_3 \cdot \mathbf{B}_{33} + \mathbf{P}_4 \cdot \mathbf{A}_{43} = \mathbf{0^T}$$
$$\vdots$$
$$\mathbf{P}_{n\text{-}2} \cdot \mathbf{C}_{n\text{-}2,n\text{-}1} + \mathbf{P}_{n\text{-}1} \cdot \mathbf{B}_{n\text{-}1,n\text{-}1} + \mathbf{P}_n \cdot \mathbf{A}_{n,n\text{-}1} = \mathbf{0^T}$$

and finally:

$$\mathbf{P}_n \cdot \mathbf{C} + \mathbf{P}_{n+1} \cdot \mathbf{B} + \mathbf{P}_{n+2} \cdot \mathbf{A} = \mathbf{0^T} \tag{2.2}$$
$$\mathbf{P}_{n+1} \cdot \mathbf{C} + \mathbf{P}_{n+2} \cdot \mathbf{B} + \mathbf{P}_{n+3} \cdot \mathbf{A} = \mathbf{0^T}$$
$$\vdots$$

Among the various solution methods for the above set of homogenous-linear equations, a back-substitution method can be used to reduce the computational work involved in obtaining these steady-state probabilities. First, the characteristic root matrix $\mathbf{R}$ is formed. Since $\mathbf{P}^{\mathbf{T}}_{k+1} = \mathbf{P}_k^{\mathbf{T}}\cdot\mathbf{R}$ for all $k \geq n\text{-}1$, equation (2.2) becomes $\mathbf{P}_n^{\mathbf{T}}\cdot(\mathbf{C} + \mathbf{R}\cdot\mathbf{B} + \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{A}) = \mathbf{0}^{\mathbf{T}}$. And, since $\mathbf{P}_n \neq \mathbf{0}$, then $\mathbf{C} + \mathbf{R}\cdot\mathbf{B} + \mathbf{R}^2\cdot\mathbf{A} = \mathbf{0}^{\mathbf{T}}$. This is equivalent to $\mathbf{R} = -(\mathbf{C} + \mathbf{R}^2\cdot\mathbf{A})\cdot\mathbf{B}^{-1}$. Using an iterative substitution scheme, $\mathbf{R}_{j+1} = -(\mathbf{C} + \mathbf{R}^2_j\cdot\mathbf{A})\cdot\mathbf{B}^{-1}$ starting with $\mathbf{R}_0 = \mathbf{0}$, the characteristic root matrix $\mathbf{R}$ is obtained. The initial conditions are solved by back substitution, yielding the set of equations:

$$\mathbf{P}_{n\text{-}2}\cdot\mathbf{C}_{n\text{-}2,n\text{-}1} + \mathbf{P}_{n\text{-}1}\cdot\mathbf{B}_{n\text{-}1,n\text{-}1} + \mathbf{R}\cdot\mathbf{P}_{n\text{-}1}\cdot\mathbf{A}_{n,n\text{-}1} = \mathbf{0}^{\mathbf{T}}.$$

Solving for $\mathbf{P}_{n\text{-}1}$, yields:

$$\mathbf{P}_{n\text{-}1} = -\mathbf{P}_{n\text{-}2}\cdot\mathbf{C}_{n\text{-}2,n\text{-}1}\cdot(\mathbf{B}_{n\text{-}1,n\text{-}1} + \mathbf{R}\cdot\mathbf{A}_{n,n\text{-}1})^{-1}.$$

Using $\mathbf{P}_{n\text{-}3}\cdot\mathbf{C}_{n\text{-}3,n\text{-}2} + \mathbf{P}_{n\text{-}2}\cdot\mathbf{B}_{n\text{-}2,n\text{-}2} + \mathbf{P}_{n\text{-}1}\cdot\mathbf{A}_{n\text{-}1,n\text{-}2} = \mathbf{0}^{\mathbf{T}}$ and substituting the above result for $\mathbf{P}_{n\text{-}1}$, then $\mathbf{P}_{n\text{-}2}$ becomes:

$$\mathbf{P}_{n\text{-}2} = -\mathbf{P}_{n\text{-}3}\cdot\mathbf{C}_{n\text{-}3,n\text{-}2}\cdot\{\mathbf{B}_{n\text{-}2,n\text{-}2} - \{\mathbf{C}_{n\text{-}2,n\text{-}1}\cdot(\mathbf{B}_{n\text{-}1,n\text{-}1} + \mathbf{R}\cdot\mathbf{A}_{n,n\text{-}1})^{-1}\}\cdot\mathbf{A}_{n\text{-}1,n\text{-}2}\}^{-1}.$$

Continuing in this manner solving for $\mathbf{P}_{n\text{-}3}$, $\mathbf{P}_{n\text{-}4}$, $\cdots$, $\mathbf{P}_2$ and then, finally:

$$\mathbf{P}_1 = -P_0 \cdot \mathbf{C}_{01} \cdot \{\mathbf{B}_{11} - \cdots \mathbf{C}_{n-2,n-1} \cdot (\mathbf{B}_{n-1,n-1} + \mathbf{R} \cdot \mathbf{A}_{n,n-1})^{-1}\} \cdot \mathbf{A}_{n-1,n-2}\}^{-1} \cdots)^{-1} \cdot \mathbf{A}_{21}\}^{-1}.$$

If the value of P_0 where known, the above scheme would yield a numerical solution for all the probabilities $\mathbf{P}_i$'s. However, the proper value for P_0 is not known. Thus, the approach is to first set $P_0 = 1$ and evaluate all $\mathbf{P}_i$'s and then, use the fact that their sum should be equal to 1 to set the correct value for P_0. Then, all of the $\mathbf{P}_i$'s are reevaluated yielding the proper values for these probabilities.

Now, to obtain the proper steady-state probabilities $\mathbf{P}_i$'s, for all $i = 1, 2, \cdots, n-1$, it is necessary to normalize all the above probabilities $\mathbf{P}_i$'s. From equation (2.2), by back substitution, where **1** is a vector of all ones, the sum of all the probabilities is:

$$P_0 1 + P_0\mathbf{P}_1 \cdot \mathbf{1} + P_0\mathbf{P}_2 \cdot \mathbf{1} + \cdots + P_0\mathbf{P}_{n-2} \cdot \mathbf{1} + P_0\mathbf{P}_{n-1} \cdot [\mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \cdots] \cdot \mathbf{1} = 1.$$

Since $\mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \cdots$ is a geometric series, this summation is $(\mathbf{I} - \mathbf{R})^{-1}$. So,

$$P_0\{1 + \mathbf{P}_1 \cdot \mathbf{1} + \mathbf{P}_2 \cdot \mathbf{1} + \cdots + \mathbf{P}_{n-2} \cdot \mathbf{1} + \mathbf{P}_{n-1} \cdot [(\mathbf{I} - \mathbf{R})^{-1}] \cdot \mathbf{1}\} = 1,$$

and

$$P_0 = \{1 + \mathbf{P}_1 \cdot \mathbf{1} + \mathbf{P}_2 \cdot \mathbf{1} + \cdots + \mathbf{P}_{n-2} \cdot \mathbf{1} + \mathbf{P}_{n-1} \cdot [(\mathbf{I} - \mathbf{R})^{-1}] \cdot \mathbf{1}\}^{-1}.$$

By normalizing all the P_i's that were previously obtained using $P_0 = 1$, the final steady-state probabilities P_i's are:

$$\mathbf{P}_1 = P_0\mathbf{P}_1,\ \mathbf{P}_2 = P_0\mathbf{P}_2,\ \cdots,\ \mathbf{P}_{n\text{-}2} = P_0\mathbf{P}_{n\text{-}2},\ \mathbf{P}_{n\text{-}1} = P_0\boldsymbol{P}_{n\text{-}1},$$

$$\text{and } \mathbf{P^T}_k = \mathbf{P}_{k\text{-}1}{}^{\mathbf{T}}\cdot\mathbf{R} \text{ for all } k \geq n.$$

To calculate work-in-process in the system $\mathrm{WIP_{sys}}$, and the number of jobs in the queue for a transporter $\mathrm{WIP_q}$ using the steady-state probabilities P_i's, the formulas are:

$$\begin{aligned}
\mathrm{WIP_{sys}} &= 0P_0 + 1\mathbf{P}_1\cdot\mathbf{1} + 2\mathbf{P}_2\cdot\mathbf{1} + \cdots + (n\text{-}1)\mathbf{P}_{n\text{-}1}\cdot\mathbf{1} + n\mathbf{P}_n\cdot\mathbf{1} + (n+1)\mathbf{P}_{n+1}\cdot\mathbf{1} + \cdots \\
&= 0P_0 + 1\mathbf{P}_1\cdot\mathbf{1} + 2\mathbf{P}_2\cdot\mathbf{1} + \cdots + \mathbf{P}_{n\text{-}1}((n\text{-}1) + n\mathbf{R} + (n+1)\mathbf{RR} + \cdots)\cdot\mathbf{1} \\
&= 0P_0 + 1\mathbf{P}_1\cdot\mathbf{1} + 2\mathbf{P}_2\cdot\mathbf{1} + \cdots + \mathbf{P}_{n\text{-}1}\cdot\mathbf{R}^{-(n-2)}((n\text{-}1)\mathbf{R}^{n-2} + n\mathbf{R}^{n-1} + (n+1)\mathbf{R}^{n} + \cdots)\cdot\mathbf{1} \\
&= 0P_0 + 1\mathbf{P}_1\cdot\mathbf{1} + 2\mathbf{P}_2\cdot\mathbf{1} + \cdots + \mathbf{P}_{n\text{-}1}\cdot\mathbf{R}^{-(n-2)}\{(\mathbf{I} + 2\mathbf{R} + 3\mathbf{R}^2 + \cdots) \\
&\qquad - (\mathbf{I} + 2\mathbf{R} + \cdots + (n\text{-}2)\mathbf{R}^{n-3})\}\ \cdot\mathbf{1} \\
&= 0P_0 + 1\mathbf{P}_1\cdot\mathbf{1} + \cdots + \mathbf{P}_{n\text{-}1}\cdot\mathbf{R}^{-(n-2)}\{(\mathbf{I} - \mathbf{R})^{-2} - (\mathbf{I} + 2\mathbf{R} + \cdots + (n\text{-}2)\mathbf{R}^{n-3})\}\ \cdot\mathbf{1},
\end{aligned}$$

$$\begin{aligned}
\mathrm{WIP_q} &= 1\mathbf{P}_{n+1}\cdot\mathbf{1} + 2\mathbf{P}_{n+2}\cdot\mathbf{1} + 3\mathbf{P}_{n+3}\cdot\mathbf{1} + \cdots \\
&= \mathbf{P}_{n+1}(\mathbf{I} + 2\mathbf{R} + 3\mathbf{R}^2 + \cdots)\cdot\mathbf{1} \\
&= \mathbf{P}_{n\text{-}1}\cdot\mathbf{R}^2((\mathbf{I} - \mathbf{R})^{-2})\ \cdot\mathbf{1}.
\end{aligned}$$

The work-in-process in the system (WIP_{sys}) is calculated by using these steady-state probabilities $\mathbf{P}_i$'s and, then by applying Little's law (Little 1961), the cycle time can be computed as:

$$CT_{sys} = \frac{WIP_{sys}}{\lambda},$$

and the queue time in the system is given by:

$$CT_q = \frac{WIP_q}{\lambda}.$$

Once the result of the analytical *M/M/T* state dependent queueing model is obtained, it can be converted to the general service time dependent queueing case using the adjustment factor equation (2.1).

2.3 Example

For illustration purposes, reconsider the previous example with 7 transporters and 4 workcenters. Using the matrix geometric technique for solving this queueing model (Neuts 1981), a system of 429 equations in 429 unknowns must be solved to get the characteristic root matrix **R**. In addition, 1001 equations must be solved to obtain the initial probabilities $P_1, \ldots, P_{n-1}$. For the seven-transporter case there are several sets of possible transition states. In the following these states are illustrated for each value of the number of jobs in the system from 1 to 7. If only one transporter is available, then the total number of all the possible states is 7. These are:

$$\{1, 2, 3, 4, 5, 6, 7\}.$$

The associated probabilities are $\mathbf{P_1} = (P_{1,1}, P_{1,2}, \ldots, P_{1,7})$.

If two transporters are available, then the total number of all the possible states is 27. These are:

$$\{(1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (3,3), (3,4), (3,5), (3,6), (3,7), (4,4), (4,5), (4,6), (4,7), (5,5), (5,6), (5,7), (6,6), (6,7), (7,7)\}.$$

The associated probabilities are $\mathbf{P_2} = (P_{2,1}, P_{2,2}, \ldots, P_{2,27})$.

If three transporters are available, then the total number of all the possible states is 75. These are:

$$\{(1,2,3), (1,2,4), (1,2,5), (1,2,6), (1,2,7), (1,3,3), (1,3,4), (1,3,5), (1,3,6), (1,3,7), \ldots, (5,5,5), (5,5,6), (5,5,7), (5,6,6), (5,6,7), (5,7,7), (6,6,6), (6,6,7), (6,7,7), (7,7,7)\}.$$

The associated probabilities are $\mathbf{P_3} = (P_{3,1}, P_{3,2}, \ldots, P_{3,75})$.

If four transporters are available, then the total number of all the possible states is 165. The states are of the form:

{(1,2,3,4), (1,2,3,5), (1,2,3,6), (1,2,3,7), (1,2,4,4), (1,2,4,5), (1,2,4,6), (1,2,4,7), … }.

The associated probabilities are $\mathbf{P_4} = (P_{4,1}, P_{4,2}, \ldots, P_{4,165})$.

If five transporters are available, then the total number of all the possible states is 297. The states are of the form:

{(1,2,3,4,5), (1,2,3,4,6), (1,2,3,4,7), (1,2,3,5,5), (1,2,3,5,6), (1,2,3,5,7), … }.

The associated probabilities are $\mathbf{P_5} = (P_{5,1}, P_{5,2}, \ldots, P_{5,297})$.

If six transporters are available, then the total number of all the possible states is 429. The states are of the form:

{(1,2,3,4,5,6), (1,2,3,4,5,7), (1,2,3,4,6,6), (1,2,3,4,6,7), (1,2,3,4,7,7), (1,2,3,5,5,6), … }.

The associated probabilities are $\mathbf{P_6} = (P_{6,1}, P_{6,2}, \ldots, P_{6,429})$.

And, finally, if seven transporters are available, then the total number of all the possible states is 429. The states are of the form:

{(1,2,3,4,5,6,7), (1,2,3,4,5,7,7), (1,2,3,4,6,6,7), (1,2,3,4,6,7,7), (1,2,3,4,7,7,7), … }.

The associated probabilities are $\mathbf{P_7} = (P_{7,1}, P_{7,2}, \ldots, P_{7,429})$.

For the 4-workcenters 7-transporters model, Table 1 lists the computational results for the mean service times as a function of the number of transporters available at the time that a job is assigned a transporter. The mean time between jobs requesting transportation is 1.5 time units.

	Value
Inter-arrival Time	1.500
1 Transporter Available Service Time	8.712
2 Transporters Available Service Time	7.506
3 Transporters Available Service Time	6.793
4 Transporters Available Service Time	6.327
5 Transporters Available Service Time	5.949
6 Transporters Available Service Time	5.698
7 Transporters Available Service Time	5.492

Table 1. The mean service times as a function of the numbers of servers available at the time of the request.

From Table 1, the inter-arrival rate $\lambda = 1/1.5 = 0.6667$, and the service rates are the inverses of the mean transportation times listed in the table. The results from the analytical model are compared with those from a 1,000,000 time-unit simulation (written in ARENA (Pegden et al. 1995) with a statistical reset at 1,000 time units and 25 replications) of this configuration. The analytical and simulation results of the $M/M/T$ state-dependent queueing model are displayed in Table 2.

	Analytical Result	Simulation Result	95% Confidence Interval	
			Min	Max
WIP_{sys}	6.9118	6.9069	6.8145	7.0430
Queue Time	2.6853	2.6777	2.5503	2.8400
Cycle Time	10.3678	10.3580	10.2260	10.5530

Table 2. The analytical and simulation results comparisons of the *M/M/T* state-dependent queueing model.

The mean cycle time error is 0.0945%, and the total WIP_{sys} error is 0.0709%. These are very acceptable results for an analytical model. From the above results, it is evident that the analytical *M/M/T* dependent queueing model is very accurate. Using this result and converting it to the state-dependent *M/G/T* model case using the adjustment factor equation (1), C_s^2 is analytically approximated to be 0.178% (C_s^2 of the simulation result is 0.179%), which yields the full system queue time of 1.584 (i.e. 2.6853×(1.18/2) = 1.584). This time compares with the result of 1.603 from the simulation model (written in MOR/DS (Curry et al. 1989)) of the full *M/G/T* system. The percentage error is approximately 1.18%. Incidentally, the error introduced by making transportation time independent assumption is 42% for this example.

2.4 Chapter Summary

In this chapter, we have developed the queueing network approximation model of a fixed-route material-handling system from the workcenters' point of view and it gives quite good results. The modeling methodology is appropriate for systems where the transporters take specific routes between various workcenters. This assumption covers circular-track systems such as the examples herein but is not limited to this particular structure. The approach is computationally tractable for small numbers of transporters, but the computational burden of the approach grows exponentially with the number of transporters supporting the system. The number of workcenters being serviced by the system does not have the same computational impact as the number of transporters. The two main analytical features of the overall model are the distributions of the location of an assigned transporter and the dependent service time model for the queueing delay waiting for a transporter assignment. The analytical model results are in excellent agreement with simulation results for the circular-track example system where the model had errors that are less than 1.2%.

CHAPTER III

ANALYTICAL MODELS FOR MATERIAL HANDLING SYSTEMS FROM THE TRANSPORTER'S POINT OF VIEW

3.1 Introduction

In Chapter II, it was shown that the results from state-dependent queueing approximation model from the workcenters' point of view are very accurate comparing to the simulation results in terms of system WIP. Now, from the transporter's point of view, we try to develop a queueing theory based analytical model of a fixed-route unidirectional material-handling system with a unit load transporter. From a transporter's perspective, when a transporter finishes the delivery of a job to its destination in the system (termed a service), it needs to select a job to be picked up from possibly several job requests in the system according to a vehicle-initiated dispatching rule. Of course, if there is only one job request in the system, no alternatives are available and the transporter will be assigned to service that job. If there are no jobs waiting in the system when the transporter becomes empty, it waits at the current location until a request for a job movement occurs. In this chapter, four different vehicle-initiated dispatching rules were considered and analytical models that can describe the impact of those dispatching polices to the system were developed and compared with simulation models.

3.2 Model with Vehicle-Initiated Dispatching Rules

In a transporter point of view model, the system status, i.e., the number of job requests in the system, can be checked only at the time of a job departure (service completion). If a job leaves the system empty of job movement requests, the system state remains zero until a new job movement request occurs (termed an arrival). The analytical model is based on transitions from one system state to another. Let $i(t)$ represent the number of job movement requests in the system at time t. We are concerned with the long-term behavior (performance) of the system and consider only the steady-state behavior of the system. In steady-state, the time aspect of the system state is no longer a concern and, thus, we use i to denote the number of active job movement requests. So, based on the above argument, the transition probabilities for state $i = 0$ are exactly same as those for the state $i = 1$. Since the inter-request times for job movement and the transportation times are both exponentially distributed and imbedded times are the times of a service completion or a job request rather than the true steady-state system status, we can say that the imbedded process is Markovian (Gross and Harris 1998). This allows us to use Markov chain theory in the analysis of the system.

Since job requests at each workcenter are functions of the factory structure and job processing sequences, there exists a state-dependent nature of the service times relative to the number of job requesting locations. So, by considering these dependencies, we can develop a very accurate approximation model for the material-handling system and routing-control being studied. Reconsider the fixed-route unidirectional material-handling system layout shown in Figure 1 of the previous chapter. For a given configuration, we model the transportation requests at each workcenter by a random selection of the destination. A different destination distribution is allowed for each workcenter, thus, no restrictions are made on the job movement characteristics of the parts being manufactured in the facility. We do, however, limit the number transportations requests at a workcenter by not allowing job movement requests to queue at the workcenters. That is, each workcenter can be occupied by only one job at a time. Thus, we are essentially modeling a single-kanban control system for each workcenter. Since we are not actually modeling the workcenters themselves, we generate transportation requests at each workcenter based on an exponentially distributed time between requests (these rates are allowed to be workcenter dependent). Thus, if there is a job waiting for a transporter at a given workcenter when the next job movement request is generated for that workcenter, then that new job arrival will be discarded. Hence, a new job arrival at each workcenter is possible only if there is not a job already waiting at that workcenter. Whenever a transporter becomes available after a transportation service completion, the number and location of jobs waiting to be picked up can vary. This is where the different priority allocation schemes impact system performance.

3.3 Dispatching Rules

3.3.1 Shortest Distance Rule (SDR)

Under the Shortest Distance Rule (SDR), an empty transporter will be assigned to the closest workcenter where a job is waiting to be picked up. If a transporter is freed at the workcenter where a job is already waiting, then the transporter selects that job and no empty transporter travel time results. If there are several jobs waiting for a transporter, then the control system will assign the empty transporter to the closest job

From / To	workcenter 1	workcenter 2	workcenter 3	workcenter 4
workcenter 1	0	2	5	4
workcenter 2	8	0	3	5
workcenter 3	5	4	0	2
workcenter 4	3	5	8	0

Table 3. The example of distance matrix between workcenters.

based on travel distances between the current empty transporter location and the job waiting locations. These distances are pre-specified for a fixed-route unidirectional layout system. To further illustrate the SDR, assume that we have the distances matrix shown in Table 3. Also, suppose that there are two jobs waiting for a transporter to be picked up (one each) at workcenter 3 and workcenter 4, and an empty transporter is located in workcenter 1. Then, since the distance (4 time units) from workcenter 1 to workcenter 4 is less than the distance from workcenter 1 to workcenter 3 (5 time units), the empty transporter is assigned to the job at workcenter 4.

3.3.2 Time Limit/Shortest Distance Rule (TL/SDR)

Under Time Limit with Shortest Distance Rule (TL/SDR), an empty transporter will be assigned to the workcenter which has a waiting job whose waiting time is greater than or equal to the pre-specified Time Limit. If there are two or more candidates that have passed the time limit, then the transporter will pick the job among these candidates only according to SDR. If there are no jobs beyond the time limit, then the SDR rule is used for all requests. To illustrate the TL/SDR allocation scheme, suppose that there are three jobs waiting for an empty transporter in the system and time limit is 0.5 minutes. Also assume that an empty transporter is freed at workcenter 1 and the waiting times of each job at workcenters 1, 3 and 4 are 0.3, 0.9 and 0.4, respectively. Then, the empty transporter is allocated to workcenter 3, because the waiting time of job at workcenter 3 is the only one that exceeds the time limit. If the waiting times of two or more jobs exceed the time limit, then an empty transporter will pick up the closest job among these jobs. For example, if the waiting times of jobs at workcenter 3 and 4 are 0.9 and 0.7, respectively, then both of these jobs exceed the time limit. Thus, the empty transporter goes to workcenter 4 because workcenter 4 is closer than workcenter 3 from the transporter’s location at workcenter 1.

3.3.3 First-Come First-Serve Rule (FCFSR)

In the system with the First-Come First-Serve Rule (FCFSR), an empty transporter selects the workcenter whose job waiting time is the longest among candidates in the system. Therefore, under this dispatching policy, job arrivals will be served according to their arrival times. Since the FCFSR control scheme does not consider the jobs' arrival location when it assign jobs to the transporter, from the transporter's point of view, empty transporter travel times may be longer than in the SDR control case.

3.3.4 Longest Distance Rule (LDR)

In the system with the Longest Distance Rule (LDR), an empty transporter selects the farthest workcenter where a job is waiting to be picked up from its current location. Since this dispatching policy tends to the have longer empty transporter travel times than either SDR or TL/SDR, it is not competitive with those two dispatching policies. Thus, it was analyzed only for the comparison purposes with the SDR and TL/SDR transporter allocation schemes.

3.4 Examples

3.4.1 Model with Shortest Distance Rule (SDR)

Now, reconsider the previous circuit network example problem in Figure 1. We assumed that all job arrivals at the four workcenters have known destination probabilities $\Pr\{R_{ij}\}$, $i, j = 1, \ldots, 4$, and the throughput rates of all workcenters are the same. Table 4 displays all 12 routes in sequential workcenters (nodes) visited. The first node and the last node denote the job generating workcenter and the job departure workcenter, respectively. New job arrivals at each job generating workcenter are independent of each other.

route /steps	1	2	3	4	5	6	7	8	9	10	11	12
1	*1*	*1*	*1*	*1*	*2*	*2*	*3*	*3*	*3*	*4*	*4*	*4*
2	*5*	*5*	*5*	*3*	*3*	*3*	*6*	*6*	*6*	*1*	*1*	*1*
3	*2*	*2*	*6*	*6*		*6*	*4*	*5*	*4*		*5*	*5*
4		*3*	*4*	*4*		*4*	*1*	*2*			*2*	*2*
5				*1*								*3*

Table 4. Routes generated for the fixed-route unidirectional material-handling problem of Figure 1 in Chapter II.

Node times are all assumed to be 0.0125 minutes and the arc times in minutes are numbers in Figure 1 times 0.1. For this four-workcenters one-transporter example, if we identify all possible states of the system in terms of the number of job requests waiting at the workcenters, there are 64 possible transition states. Each state can be represented as numbers. The first four digits represent the location of job requests as a 0 or 1 for each workcenter (0 being no request and one being a job waiting for transportation), and the fifth digit shows the transporter freed location. That is, if the third digit is 1, then this implies that there is a job request at workcenter 3. Thus, for example, if we have (1100:3), then this state representation implies that there are two job requests from workcenters 1, 2 and the current location of the empty transporter is workcenter 3. In this manner, we can detail all 64 system states as follows.

If there are no job requests in the system, then there are four such empty states. Note that the transporter can be located at any one of the four destinations. These possible states are:

{(0000:1), (0000:2), (0000:3), (0000:4)}.

If there is only one job request in the system, then the total number of all these possible states is 16. These states are:

{(1000:1), (1000:2), (1000:3), (1000:4), (0100:1), (0100:2), (0100:3), (0100:4), (0010:1), (0010:2), (0010:3), (0010:4), (0001:1), (0001:2), (0001:3), (0001:4)}.

For two job requests in the system, there are 24 possible states:

{(1100:1), (1100:2), (1100:3), (1100:4), (1010:1), (1010:2), (1010:3), (1010:4), (1001:1), (1001:2), (1001:3), (1001:4), (0110:1), (0110:2), (0110:3), (0110:4), (0101:1), (0101:2), (0101:3), (0101:4), (0011:1), (0011:2), (0011:3), (0011:4)}.

If there are three job requests in the system, this results in 16 possible states. These are:

{(1110:1), (1110:2), (1110:3), (1110:4), (1101:1), (1101:2), (1101:3), (1101:4), (1011:1), (1011:2), (1011:3), (1011:4), (0111:1), (0111:2), (0111:3), (0111:4)}.

And finally, when there are four job requests in the system, the total number of possible states is 4. These states are:

{(1111:1), (1111:2), (1111:3), (1111:4)}.

In the SDR control case, an empty transporter will pick the job waiting at the closest workcenter from its current location. So, there is little chance that the transporter will to go to the second or third closest workcenter from the freed location. To illustrate this, suppose that a transporter is freed at workcenter 3. Then, the possible system states are:

{(0000:3), (1000:3), (0100:3), (0010:3), (0001:3), (1100:3), (1010:3), (1001:3), (0110:3), (0101:3), (0011:3), (1110:3), (1101:3), (1011:3), (0111:3), (1111:3)}.

Suppose that these states occur in equal portions. Then, the probability that workcenter 3 (the closest workcenter from workcenter 3 is workcenter 3 itself) will be chosen is 8/15 and the probabilities that workcenters 4, 2 and 1 will be chosen are 4/15, 2/15 and 1/15, respectively. Note that these probabilities are proportional to the distances between job requesting workcenters and the empty transporter location. That is, the more distance between a job requesting workcenter and the empty transporter location the less probability that this workcenter will be chosen. Now, remember our assumption is that all jobs arriving at the four workcenters have their destinations in equal portion. That is, for example, the final destination of jobs arriving at workcenter 1 can be workcenter 2, 3, or 4 with equal probability. Now, there are five possible cases due to the number of job arrivals at each of the four workcenters. During the transportation service time, there can be zero, one, two, three and four job arrivals to the system (a completed part at the workcenter requesting movement to another workcenter). Thus, if there are no new job arrivals during the transportation service period, then the number of job requesting workcenters is decreasing by one, because of the service completion. For example:

One-step transition from state (1100:1) to (0100:2) or (0100:3) or (0100:4)

$\rightarrow$ Transporter picks up a job at workcenter 1 and there are no new job arrivals to the system,

One-step transition from state (1101:3) to (1100:1) or (1100:2) or (1100:3)

$\rightarrow$ Transporter picks up a job at workcenter 4 and there are no new job arrivals to the system,

One-step transition from state (1111:4) to (1110:1) or (1110:2) or (1110:3)

→ Transporter picks up a job at workcenter 4 and there are no new job arrivals to the system.

If there is exactly one job arrival during the service period, then the number of jobs requesting movement to other workcenters remains the same. However, the transporter will be at a different location and the arriving job location creates the different possible states. For example:

One-step transition from state (1100:1) to (1100:2) or (1100:3) or (1100:4)

→ Transporter picks up a job at workcenter 1 and there is a new job arrival to workcenter 1,

One-step transition from state (1100:1) to (0110:2) or (0110:3) or (0110:4)

→ Transporter picks up a job at workcenter 1 and there is a new job arrival to workcenter 3,

One-step transition from state (1100:1) to (0101:2) or (0101:3) or (0101:4)

→ Transporter picks up a job at workcenter 1 and there is a new job arrival to workcenter 4.

If there are two job arrivals at different workcenters during the service, then the number of job requesting transportation is increasing by one, because of one service completion and two new job arrivals. For example:

One-step transition from state (1100:1) to (1110:2) or (1110:3) or (1110:4)

→ Transporter picks up a job at workcenter 1 and there are two job arrivals to workcenters 1 and 3,

One-step transition from state (1100:1) to (1101:2) or (1101:3) or (1101:4)

→ Transporter picks up a job at workcenter 1 and there are two job arrivals to workcenters 1 and 4,

One-step transition from state (1100:1) to (0111:2) or (0111:3) or (0111:4)

→ Transporter picks up a job at workcenter 1 and there are two job arrivals to workcenters 3 and 4.

If there are three job arrivals at each three different workcenters during the service, then the number of job requesting workcenters is increasing by two, because of one service completion and three new job arrivals. For example:

One-step transition from state (1100:1) to (1111:2) or (1111:3) or (1111:4)

→ Transporter picks up a job at workcenter 1 and three new jobs arrive to workcenters 1, 3 and 4.

If all four workcenters have new job arrivals during the service, then the number of job requesting workcenters is increasing by three, because of one service completion and four new job arrivals. For example:

One-step transition from state (1000:1) to (1111:2) or (1111:3) or (1111:4)

→ Transporter picks up a job at workcenter 1 and four new jobs arrive at workcenters 1, 2, 3 and 4.

Now, we want to compute the average queue length, WIP_q, of the system. The first thing we have to do is to obtain steady-state probabilities, P_i's, that there are exactly i-job requests in the system. The steady-state equations relating the system states are of the form $\pi^T \mathbf{P} = \pi^T$. Figure 3 shows the general structure of the one-step transition-matrix **P** for a model with n workcenters and one transporter.

	$\mathbf{P}_0$	$\mathbf{P}_1$	$\mathbf{P}_2$	$\mathbf{P}_3$	$\mathbf{P}_4$	$\cdots$	$\mathbf{P}_n$
$\mathbf{P}_0$	A_{00}	A_{01}	A_{02}	A_{03}	A_{04}	$\cdots$	A_{0n}
$\mathbf{P}_1$	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}	$\cdots$	B_{1n}
$\mathbf{P}_2$		C_{21}	C_{22}	C_{23}	C_{24}	$\cdots$	C_{2n}
$\mathbf{P}_3$			D_{32}	D_{33}	D_{34}	$\cdots$	D_{3n}
$\mathbf{P}_4$				E_{43}	E_{44}	$\cdots$	E_{4n}
$\vdots$					$\ddots$	$\vdots$	$\vdots$
$\mathbf{P}_n$						E_{nn-1}	E_{nn}

Figure 3. The general structure of the one-step transition matrix **P**.

The blanks within the matrix denote zero matrices and all $\mathbf{A}_{ij}$, $\mathbf{B}_{ij}$, $\mathbf{C}_{ij}$, $\mathbf{D}_{ij}$ and $\mathbf{E}_{ij}$ are sub-matrices of the matrix **P** whose elements are zeros and one-step transition probabilities. For our example problem with four workcenters and one transporter, the steady-state flow-balance equations form a 64×64 generator **P** matrix with the structure shown in Figure 4.

	Prob.	$\mathbf{P_0}$	$\mathbf{P_1}$	$\mathbf{P_2}$	$\mathbf{P_3}$	$\mathbf{P_4}$
Prob.	States	(0000:1) ~ (0000:4)	(1000:1) ~ (0001:4)	(1100:1) ~ (0011:4)	(1110:1) ~ (0111:4)	(1111:1) ~ (1111:4)
$\mathbf{P_0}$	(0000:1) ~ (0000:4)	$\mathbf{A}_{00}$ (4×4)	$\mathbf{A}_{01}$ (4×16)	$\mathbf{A}_{02}$ (4×24)	$\mathbf{A}_{03}$ (4×16)	$\mathbf{A}_{04}$ (4×4)
$\mathbf{P_1}$	(1000:1) ~ (0001:4)	$\mathbf{B}_{10}$ (16×4)	$\mathbf{B}_{11}$ (16×16)	$\mathbf{B}_{12}$ (16×24)	$\mathbf{B}_{13}$ (16×16)	$\mathbf{B}_{14}$ (16×4)
$\mathbf{P_2}$	(1100:1) ~ (0011:4)		$\mathbf{C}_{21}$ (24×16)	$\mathbf{C}_{22}$ (24×24)	$\mathbf{C}_{23}$ (24×16)	$\mathbf{C}_{24}$ (24×4)
$\mathbf{P_3}$	(1110:1) ~ (0111:4)			$\mathbf{D}_{32}$ (16×24)	$\mathbf{D}_{33}$ (16×16)	$\mathbf{D}_{34}$ (16×4)
$\mathbf{P_4}$	(1111:1) ~ (1111:4)				$\mathbf{E}_{43}$ (4×16)	$\mathbf{E}_{44}$ (4×4)

Figure 4. The structure of the matrix **P** for our example problem.

Suppose that mean inter-arrival rate at workcenter *j*, λ_j, follows a Poisson distribution and let the state dependent transporter service times for each route *k* be exponentially distributed with mean $E[S_k]$. The (departure point) steady-state probability that there are exactly i-job requests in the system, P_i, $i = 0, \ldots, n$, can be computed as follows:

$$P_0 = \pi_{(0000:1)} + \pi_{(0000:2)} + \pi_{(0000:3)} + \pi_{(0000:4)},$$

$$\begin{aligned} P_1 = {} & \pi_{(1000:1)} + \pi_{(1000:2)} + \pi_{(1000:3)} + \pi_{(1000:4)} + \pi_{(0100:1)} + \pi_{(0100:2)} + \pi_{(0100:3)} \\ & + \pi_{(0100:4)} + \pi_{(0010:1)} + \pi_{(0010:2)} + \pi_{(0010:3)} + \pi_{(0010:4)} + \pi_{(0001:1)} + \pi_{(0001:2)} \\ & + \pi_{(0001:3)} + \pi_{(0001:4)}, \end{aligned}$$

$$\begin{aligned} P_2 = {} & \pi_{(1100:1)} + \pi_{(1100:2)} + \pi_{(1100:3)} + \pi_{(1100:4)} + \pi_{(1010:1)} + \pi_{(1010:2)} + \pi_{(1010:3)} \\ & + \pi_{(1010:4)} + \pi_{(1001:1)} + \pi_{(1001:2)} + \pi_{(1001:3)} + \pi_{(1001:4)} + \pi_{(0110:1)} + \pi_{(0110:2)} \\ & + \pi_{(0110:3)} + \pi_{(0110:4)} + \pi_{(0101:1)} + \pi_{(0101:2)} + \pi_{(0101:3)} + \pi_{(0101:4)} + \pi_{(0011:1)} \\ & + \pi_{(0011:2)} + \pi_{(0011:3)} + \pi_{(0011:4)}, \end{aligned}$$

$$\begin{aligned} P_3 = {} & \pi_{(1110:1)} + \pi_{(1110:2)} + \pi_{(1110:3)} + \pi_{(1110:4)} + \pi_{(1101:1)} + \pi_{(1101:2)} + \pi_{(1101:3)} \\ & + \pi_{(1101:4)} + \pi_{(1011:1)} + \pi_{(1011:2)} + \pi_{(1011:3)} + \pi_{(1011:4)} + \pi_{(0111:1)} + \pi_{(0111:2)} \\ & + \pi_{(0111:3)} + \pi_{(0111:4)}, \end{aligned}$$

$$P_4 = \pi_{(1111:1)} + \pi_{(1111:2)} + \pi_{(1111:3)} + \pi_{(1111:4)},$$

where the steady-state probabilities of state n, π_n, can be obtained from the stationary equations $\pi^T\mathbf{P} = \pi^T$ and $\pi\cdot\mathbf{1} = 1$. Now, to create the above **P** matrix, we need to compute the one-step transition probabilities between states. Since the mean inter-arrival rate at each workcenter i, λ_i, follows a Poison distribution and job arrivals at different workcenters are independent of each other, we have:

$$\Pr\{0 \text{ job arrivals during the service : route(k)}\} = \left(e^{-\left(\sum_{i=1}^{4}\lambda_i\right)\cdot E[S_k]} \right),$$

$$\Pr\{1 \text{ job arrival (at WC } i\text{) during service : route(k)}\} = \left(1 - e^{-\lambda_i \cdot E[S_k]}\right)\cdot\left(e^{-\left(\sum_{j\neq i}\lambda_j\right)\cdot E[S_k]} \right),$$

$\Pr\{2$ job arrivals (at WC *i,m*) during service : route(k)$\}$

$$= \left(1 - e^{-\lambda_i \cdot E[S_k]}\right)\cdot\left(1 - e^{-\lambda_m \cdot E[S_k]}\right)\cdot\left(e^{-\left(\sum_{j\neq i,m}\lambda_j\right)\cdot E[S_k]} \right),$$

$\Pr\{3$ job arrivals (at WC *i, m, n*) during service : route(k)$\}$

$$= \left(1 - e^{-\lambda_i \cdot E[S_k]}\right)\cdot\left(1 - e^{-\lambda_m \cdot E[S_k]}\right)\cdot\left(1 - e^{-\lambda_n \cdot E[S_k]}\right)\cdot\left(e^{-\lambda_j \cdot E[S_k]}\right),$$

$\Pr\{4$ job arrivals (at all WC's) during service : route(k)$\}$

$$= \left(1 - e^{-\lambda_1 \cdot E[S_k]}\right)\cdot\left(1 - e^{-\lambda_2 \cdot E[S_k]}\right)\cdot\left(1 - e^{-\lambda_3 \cdot E[S_k]}\right)\cdot\left(1 - e^{-\lambda_4 \cdot E[S_k]}\right).$$

For further illustration, we compute the one-step transition probabilities from (0001:1) to (1000:2) and from (1000:1) to (1100:3) in our example problem. Since the transition from (0001:1) to (1000:2) occurs when an empty transporter travels from workcenter 1 to workcenter 4 and delivers a job from workcenter 4 to workcenter 2 and there is one job arrival at workcenter 1 with no job arrivals at the other workcenters during the service time of route 1→4→2, $E[S_{142}]$, we have:

$$\Pr\{(0001:1) \rightarrow (1000:2)\} = \left(1 - e^{-\lambda_1 \cdot E[S_{142}]}\right) \cdot \left(e^{-\lambda_2 \cdot E[S_{142}]}\right) \cdot \left(e^{-\lambda_3 \cdot E[S_{142}]}\right) \cdot \left(e^{-\lambda_4 \cdot E[S_{142}]}\right) \cdot \Pr\{R_{12}\}$$

$$= \left(1 - e^{-\lambda_1 \cdot E[S_{142}]}\right) \cdot \left(e^{-E[S_{142}](\lambda_2 + \lambda_3 + \lambda_4)}\right) \cdot \Pr\{R_{12}\}\ .$$

Note that we multiplied the probability, $\Pr\{R_{12}\}$, that the destination of the job at workcenter 1 is workcenter 2, because of our assumption that all jobs arriving at the four workcenters have destination probabilities $\Pr\{R_{ij}\}$, $i, j = 1, \ldots, 4$. The one-step transition probability from (1000:1) to (1100:3) can be obtained as follows. The transition from (1000:1) to (1100:3) occurs when a transporter delivers a job from workcenter 1 to workcenter 3 and there are two job arrivals during this delivery time at workcenters 1 and 2, and no job arrivals at workcenters 3 and 4 during the service time of route 1→3, $E[S_{13}]$. In this case, no empty trip occurs because the job requesting location and the empty transporter location are the same. Thus, we have:

$$\Pr\{(1000:1) \rightarrow (1100:3)\} = \left(1 - e^{-\lambda_1 \cdot E[S_{13}]}\right) \cdot \left(1 - e^{-\lambda_2 \cdot E[S_{13}]}\right) \cdot \left(e^{-\lambda_3 \cdot E[S_{13}]}\right) \cdot \left(e^{-\lambda_4 \cdot E[S_{13}]}\right) \cdot \Pr\{R_{13}\}$$

$$= \left(1 - e^{-\lambda_1 \cdot E[S_{13}]}\right) \cdot \left(1 - e^{-\lambda_2 \cdot E[S_{13}]}\right) \cdot \left(e^{-E[S_{13}](\lambda_3 + \lambda_4)}\right) \cdot \Pr\{R_{13}\}\ .$$

From the steady-states probabilities of i-job requests in the system when a transporter is freed, P_i, $i = 0, \ldots, n$, we obtain the average number of jobs waiting in the queue, WIP_q as follows:

$$\mathrm{WIP}_q = 0 \cdot P_0 + 1 \cdot P_1 + 2 \cdot P_2 + 3 \cdot P_3 + 4 \cdot P_4\ .$$

If we want to know the average number of jobs in the system, WIP_{sys}, this information can be computed as:

$$\text{WIP}_{sys} = \text{WIP}_q + \text{ Transporter Utilization } = 0 \cdot P_0 + 2 \cdot P_1 + 3 \cdot P_2 + 4 \cdot P_3 + 5 \cdot P_4 .$$

For the four workcenters and one transporter example, we have following inter-arrival rates as shown in Table 5.

Inter-arrival rate	λ_1	λ_2	λ_3	λ_4
	0.3989	0.2382	0.1284	0.0957

Table 5. Job inter-arrival rates at each workcenter.

The analytical model result is compared with that from simulation model (written in ARENA (Pegden et al. 1995)) with a run length of 500,000 time units and a statistical reset at 30,000 time units. The analytical and simulation results are in Table 6.

	Analytical	Simulation	95%CI Min	95%CI Max	% Error
WIP_q	0.8389	0.8412	0.8376	0.8448	0.3%

Table 6. The comparison of analytical and simulation results of WIP_q for the model with the SDR control.

The percentage error between the analytical and simulation results of WIP_q is 0.3% and this is very acceptable result for an analytical model.

3.4.2 Model with Time Limit/Shortest Distance Rule (TL/SDR)

If we have a pre-specified time limit before a waiting job becomes urgent, then we need to modify the previous **P** matrix to allow for the probabilities that the transporter goes to the workcenters that are not the shortest distance from the current transporter location. Similar to the SDR control case, if a transporter becomes available and there is only one job request in the system, then the transporter obviously goes to the workcenter where the job is located. When a transporter becomes available and there are two or more job requests in the system, then the selection logic is as follows:

- Check the waiting time of each job and compare it with the given Time Limit.
- If a single job's waiting time is bigger than the given Time Limit, then priority is given to that job and the transporter goes to the workcenter where that job is located.
- If the waiting times of two or more jobs are bigger than the given Time Limit, then selection among these jobs is made according to the SDR control scheme.

For example, in SDR control, we may have the following portion of the **P** matrix:

From/To	⋯	(1000:2)	(1000:3)	(1000:4)	⋯	(0100:2)	(0100:3)	(0100:4)
⋮				⋮				
(1100:1)	⋯	*0*	*0*	*0*	⋯	0.2905	0.2425	0.2569
(1100:4)	⋯	*0*	*0*	*0*	⋯	0.2407	0.2009	0.2129

Note that, under the SDR control scheme, the one-step state transition from (1100:1) to (1000:2) cannot occur because an empty transporter will pick up the job at workcenter 1. Thus, as we can see in the above table, those transition probabilities are all zeros. However, under the TL/SDR control scheme, some portion of entities that are located in workcenters which are never chosen in the SDR control scheme is picked up and that portion changes according to the time limit value. Therefore, those transition probabilities may not necessarily be zero. Thus, in the time limit model, the previous portion of the **P** matrix would become:

From/To	⋯	(1000:2)	(1000:3)	(1000:4)	⋯	(0100:2)	(0100:3)	(0100:4)
⋮				⋮				
(1100:1)	⋯	*0.0705*	*0.0125*	*0.0269*	⋯	0.2123	0.2209	0.2076
(1100:4)	⋯	*0.0407*	*0.0009*	*0.0129*	⋯	0.2007	0.1751	0.1984

To further illustrate the difference between the SDR control scheme and the TL/SDR control scheme, consider all possible one-step transition cases from (1100:3). These yield the following four possible cases:

From (1100:3)

- An empty transporter from workcenter 3 picks a job at workcenter 2 by the SDR control and follows a routing sequence according to the job type, i.e., route 4 (workcenter 2 $\rightarrow$ 3 $\rightarrow$ 6 $\rightarrow$ 4 $\rightarrow$ 1) or route 5 (workcenter 2 $\rightarrow$ 3) or route 6 (workcenter 2 $\rightarrow$ 3 $\rightarrow$ 6 $\rightarrow$ 4), and there are no arrivals during that service period.

States	(1000:1)	(1000:3)	(1000:4)	(0100:2)	(0100:3)	(0100:4)
Prob.	1-①	1-②	1-③	1-❶	1-❷	1-❸

For 1-①: $\Pr\{(1100:3) \rightarrow (1000:1)\} = \left(e^{-E[S_{321}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{21}\}\right) \cdot \Pr\{SDR_{(1100:3)}\}$

For 1-②: $\Pr\{(1100:3) \rightarrow (1000:3)\} = \left(e^{-E[S_{323}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{23}\}\right) \cdot \Pr\{SDR_{(1100:3)}\}$

For 1-③: $\Pr\{(1100:3) \rightarrow (1000:4)\} = \left(e^{-E[S_{324}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{24}\}\right) \cdot \Pr\{SDR_{\{1100:3)}\}$

For 1-❶: $\Pr\{(1100:3) \rightarrow (0100:2)\} = \left(e^{-E[S_{312}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{12}\}\right) \cdot \Pr\{nonSDR_{\{1100:3)}\}$

For 1-❷:

$\Pr\{(1100:3) \rightarrow (0100:3)\} = \left(e^{-E[S_{313}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{13}\}\right) \cdot \Pr\{nonSDR_{(1100:3)}\}$

For 1-❸: $\Pr\{(1100:3) \rightarrow (0100:4)\} = \left(e^{-E[S_{314}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{14}\}\right) \cdot \Pr\{nonSDR_{(1100:3)}\}$

- An empty transporter from workcenter 3 picks up a job at workcenter 2 and follows a routing sequence according to the job type, i.e., route 4 or 5 or 6, and there is one job arrival during the service period.

States	(1100:1)	(1100:2)	(1100:3)	(1100:4)	(1010:1)	(1010:3)
Prob.	2-①	2-❶	2-②	2-③	2-④	2-⑤
States	(1010:4)	(1001:1)	(1001:3)	(1001:4)	(0110:2)	(0110:3)
Prob.	2-⑥	2-⑦	2-⑧	2-⑨	2-❷	2-❸
States	(0110:4)	(0101:2)	(0101:3)	(0101:4)		
Prob.	2-❹	2-❺	2-❻	2-❼		

For 2-①:

$$\Pr\{(1100:3)\rightarrow(1100:1)\}=\left(1-e^{-\lambda_2\cdot E[S_{321}]}\right)\cdot\left(e^{-E[S_{321}](\lambda_1+\lambda_3+\lambda_4)}\right)\cdot\Pr\{R_{21}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

For 2-②:

$$\Pr\{(1100:3)\rightarrow(1100:3)\}=\left(1-e^{-\lambda_2\cdot E[S_{323}]}\right)\cdot\left(e^{-E[S_{323}](\lambda_1+\lambda_3+\lambda_4)}\right)\cdot\Pr\{R_{23}\}\cdot\Pr\{SDR_{(1100:3)}\}$$
$$+\left(1-e^{-\lambda_1\cdot E[S_{313}]}\right)\cdot\left(e^{-E[S_{313}](\lambda_2+\lambda_3+\lambda_4)}\right)\cdot\Pr\{R_{13}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

For 2-③:

$$\Pr\{(1100:3)\rightarrow(1100:4)\}=\left(1-e^{-\lambda_2\cdot E[S_{324}]}\right)\cdot\left(e^{-E[S_{324}](\lambda_1+\lambda_3+\lambda_4)}\right)\cdot\Pr\{R_{24}\}\cdot\Pr\{SDR_{(1100:3)}\}$$
$$+\left(1-e^{-\lambda_1\cdot E[S_{314}]}\right)\cdot\left(e^{-E[S_{314}](\lambda_2+\lambda_3+\lambda_4)}\right)\cdot\Pr\{R_{14}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

For 2-④:

$$\Pr\{(1100:3)\rightarrow(1010:1)\}=\left(1-e^{-\lambda_3\cdot E[S_{321}]}\right)\cdot\left(e^{-E[S_{321}](\lambda_1+\lambda_2+\lambda_4)}\right)\cdot\Pr\{R_{21}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

$$\vdots$$

For 2-⑨:

$$\Pr\{(1100:3)\rightarrow(1001:4)\}=\left(1-e^{-\lambda_4\cdot E[S_{324}]}\right)\cdot\left(e^{-E[S_{324}](\lambda_1+\lambda_2+\lambda_3)}\right)\cdot\Pr\{R_{24}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

For 2-❶:

$$\Pr\{(1100:3)\rightarrow(1100:2)\}=\left(1-e^{-\lambda_1\cdot E[S_{312}]}\right)\cdot\left(e^{-E[S_{312}](\lambda_2+\lambda_3+\lambda_4)}\right)\cdot\Pr\{R_{12}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

$$\vdots$$

For 2-❼:

$$\Pr\{(1100:3)\rightarrow(0101:4)\}=\left(1-e^{-\lambda_4\cdot E[S_{314}]}\right)\cdot\left(e^{-E[S_{314}](\lambda_1+\lambda_2+\lambda_3)}\right)\cdot\Pr\{R_{14}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

- An empty transporter from workcenter 3 picks up a job at workcenter 2 and follows a routing sequence according to the job type, i.e., route 4 or 5 or 6, and there are two job arrivals during the service period.

States	(1110:1)	(1110:2)	(1110:3)	(1110:4)	(1101:1)	(1101:2)
Prob.	3-①	3-❶	3-②	3-③	3-④	3-❷
States	(1101:3)	(1101:4)	(1011:1)	(1011:3)	(1011:4)	(0111:2)
Prob.	3-⑤	3-⑥	3-⑦	3-⑧	3-⑨	2-❸
States	(0111:3)	(0111:4)				
Prob.	2-❹	2-❺				

For 3-①: $\Pr\{(1100:3)\rightarrow(1110:1)\}$

$$=\left(1-e^{-\lambda_2\cdot E[S_{321}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{321}]}\right)\cdot\left(e^{-E[S_{321}](\lambda_1+\lambda_4)}\right)\cdot\Pr\{R_{21}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

For 3-②: $\Pr\{(1100:3)\rightarrow(1110:3)\}$

$$=\left(1-e^{-\lambda_2\cdot E[S_{323}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{323}]}\right)\cdot\left(e^{-E[S_{323}](\lambda_1+\lambda_4)}\right)\cdot\Pr\{R_{23}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

$$+\left(1-e^{-\lambda_1\cdot E[S_{313}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{313}]}\right)\cdot\left(e^{-E[S_{313}](\lambda_2+\lambda_4)}\right)\cdot\Pr\{R_{13}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

For 3-③: $\Pr\{(1100:3)\rightarrow(1110:4)\}$

$$=\left(1-e^{-\lambda_2\cdot E[S_{324}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{324}]}\right)\cdot\left(e^{-E[S_{324}](\lambda_1+\lambda_4)}\right)\cdot\Pr\{R_{24}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

$$+\left(1-e^{-\lambda_1\cdot E[S_{314}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{314}]}\right)\cdot\left(e^{-E[S_{314}](\lambda_2+\lambda_4)}\right)\cdot\Pr\{R_{14}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

For 3-④: $\Pr\{(1100:3)\rightarrow(1101:1)\}$

$$=\left(1-e^{-\lambda_2\cdot E[S_{321}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{321}]}\right)\cdot\left(e^{-E[S_{321}](\lambda_1+\lambda_3)}\right)\cdot\Pr\{R_{21}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

For 3-⑤: $\Pr\{(1100:3)\rightarrow(1101:3)\}$

$$=\left(1-e^{-\lambda_2\cdot E[S_{323}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{323}]}\right)\cdot\left(e^{-E[S_{323}](\lambda_1+\lambda_3)}\right)\cdot\Pr\{R_{23}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

$$+\left(1-e^{-\lambda_1\cdot E[S_{313}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{313}]}\right)\cdot\left(e^{-E[S_{313}](\lambda_2+\lambda_3)}\right)\cdot\Pr\{R_{13}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

For 3-⑥: $\Pr\{(1100:3)\rightarrow(1101:4)\}$

$$=\left(1-e^{-\lambda_2\cdot E[S_{324}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{324}]}\right)\cdot\left(e^{-E[S_{324}](\lambda_1+\lambda_3)}\right)\cdot\Pr\{R_{24}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

$$+\left(1-e^{-\lambda_1\cdot E[S_{314}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{314}]}\right)\cdot\left(e^{-E[S_{314}](\lambda_2+\lambda_3)}\right)\cdot\Pr\{R_{14}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

$$\vdots$$

For 3-⑨: $\Pr\{(1100:3)\rightarrow(1011:4)\}$

$$=\left(1-e^{-\lambda_3\cdot E[S_{324}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{324}]}\right)\cdot\left(e^{-E[S_{324}](\lambda_1+\lambda_2)}\right)\cdot\Pr\{R_{24}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

For 3-❶: $\Pr\{(1100:3)\rightarrow(1110:2)\}$

$$=\left(1-e^{-\lambda_1\cdot E[S_{312}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{312}]}\right)\cdot\left(e^{-E[S_{312}](\lambda_2+\lambda_4)}\right)\cdot\Pr\{R_{12}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

$$\vdots$$

For 3-❺: $\Pr\{(1100:3)\rightarrow(0111:4)\}$

$$=\left(1-e^{-\lambda_3\cdot E[S_{314}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{314}]}\right)\cdot\left(e^{-E[S_{314}](\lambda_1+\lambda_2)}\right)\cdot\Pr\{R_{14}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

- An empty transporter from workcenter 3 picks up a job at workcenter 2 and follows a routing sequence according to the job type, i.e., route 4 or 5 or 6, and there are three job arrivals during the service.

States	(1111:1)	(1111:2)	(1111:3)	(1111:4)
Prob.	4-①	4-❶	4-②	4-③

For 4-①: $\Pr\{(1100:3)\to(1111:1)\}$

$$=\left(1-e^{-\lambda_2\cdot E[S_{321}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{321}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{321}]}\right)\cdot\left(e^{-\lambda_1\cdot E[S_{321}]}\right)\cdot\Pr\{R_{21}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

For 4-②: $\Pr\{(1100:3)\to(1111:3)\}$

$$=\left(1-e^{-\lambda_2\cdot E[S_{323}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{323}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{323}]}\right)\cdot\left(e^{-\lambda_1\cdot E[S_{323}]}\right)\cdot\Pr\{R_{23}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

$$+\left(1-e^{-\lambda_1\cdot E[S_{313}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{313}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{313}]}\right)\cdot\left(e^{-\lambda_2\cdot E[S_{313}]}\right)\cdot\Pr\{R_{13}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

For 4-③: $\Pr\{(1100:3)\to(1111:4)\}$

$$=\left(1-e^{-\lambda_2\cdot E[S_{324}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{324}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{324}]}\right)\cdot\left(e^{-\lambda_1\cdot E[S_{324}]}\right)\cdot\Pr\{R_{24}\}\cdot\Pr\{SDR_{(1100:3)}\}$$

$$+\left(1-e^{-\lambda_1\cdot E[S_{314}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{314}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{314}]}\right)\cdot\left(e^{-\lambda_2\cdot E[S_{314}]}\right)\cdot\Pr\{R_{14}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

For 4-❶: $\Pr\{(1100:3)\to(1111:2)\}$

$$=\left(1-e^{-\lambda_1\cdot E[S_{312}]}\right)\cdot\left(1-e^{-\lambda_3\cdot E[S_{312}]}\right)\cdot\left(1-e^{-\lambda_4\cdot E[S_{312}]}\right)\cdot\left(e^{-\lambda_2\cdot E[S_{312}]}\right)\cdot\Pr\{R_{12}\}\cdot\Pr\{nonSDR_{(1100:3)}\}$$

For this case, $\Pr\{SDR_{(1100:3)}\}$ is the probability that an empty transporter goes to the workcenter 2 from workcenter 3 (SDR choice) and $\Pr\{nonSDR_{(1100:3)}\}$ is the probability that an empty transporter goes to the workcenter 1 from workcenter 3 (non-SDR choice) where there are two job requests from workcenters 1 and 2 in the system. Under the SDR control scheme or the TL/SDR control scheme with a Time Limit of 0, we have Pr$\{SDR_{(1100:3)}\}$ = 1 and Pr$\{nonSDR_{(1100:3)}\}$ = 0 for all system states. Therefore, all one-step transition probabilities of numbers with black circles should be zeros. On the other hand, under the TL/SDR control with non-zero Time Limits, we have Pr$\{nonSDR_{(1100:3)}\} \neq 0$. Thus, those zero probabilities in SDR control cases do not remain zero and those will change according to the Time Limit. That is, if the Time Limit is not zero, then priority is given to the workcenter that has a job which has waited more than the Time Limit. Note that this workcenter may not be the closest one from an empty transporter. Thus, there exist cases where an empty transporter goes to a non-SDR choice and this implies that the one-step transition probabilities associated with those cases will not be zero. Thus, with different Time Limits, we may have different **P** matrices.

To model the probabilities Pr$\{SDR\}$ and Pr$\{nonSDR\}$, we first performed several preliminary simulation runs for a set of Time Limit points and obtained the solid-line curve in Figure 5. This curve represents the change of total portion of the non-SDR choices with-respect-to the Time Limit value when there were two or more job requests in the system. If the Time Limit is zero, then the model is the same as the SDR control model. Thus, the portion of the non-SDR choices will be zero. As the Time Limit value increases from zero, an empty transporter begins to be allocated to the workcenter that is not the closest one from its current location, thus the proportion the curve is increasing. After the certain Time Limit point (the peak point), the total portion of non-SDR choices would start decreasing. That is, if the Time Limit becomes larger after the peak point, then the number of jobs whose waiting time is greater than the Time Limit will decrease. Thus, the number of non-SDR choices also decreasing to zero as we can see in curve from Figure 5.

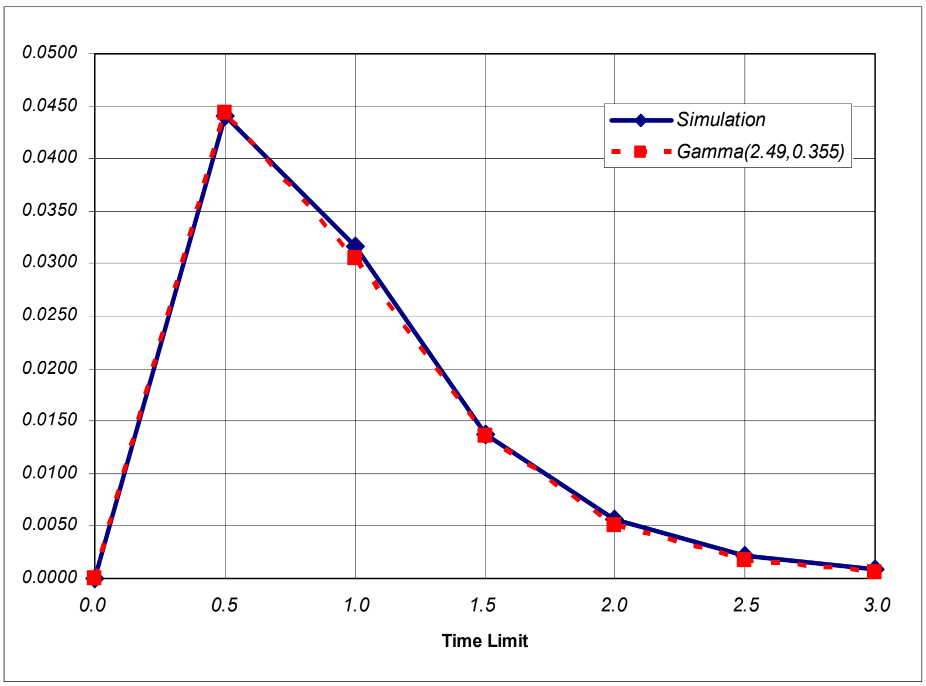

Figure 5. Plots of the non-SDR choice portion for all decisions made during simulation runs as compared to a Gamma distribution estimate (best fit).

As the Time Limit becomes very large, this total portion of the non-SDR choices again approaches zero, which implies that the model returns to the SDR control model.

From above Figure 5, the solid line is the simulation result and the dotted line is Gamma distribution best fit for the simulation curve which is specific to a particular configuration. From these Gamma distribution data, we can compute the sets of proper Pr{*SDR*}'s and Pr{*nonSDR*}'s for all possible Time Limit values. Thus, using the Gamma distribution approximation, it is possible to incorporate Time Limits into the transition probability matrix **P**.

To check whether the shape of the non-SDR choice curve shown in Figure 5 is affected by the traffic intensity, we differentiated the traffic intensity by changing arrival rates to the system and obtained the simulation results of Figures 6 and 7.

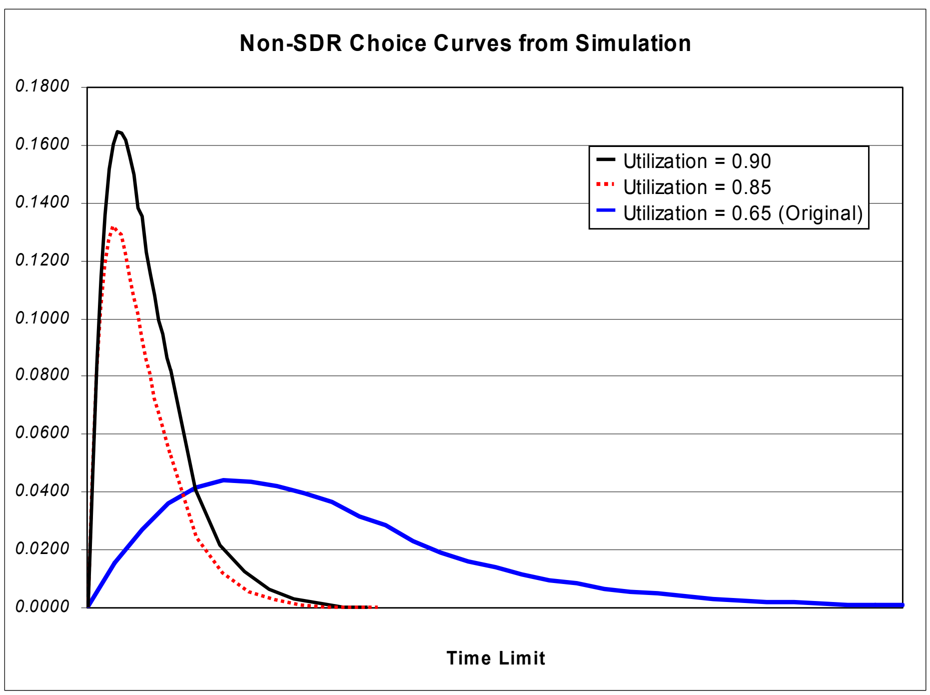

Figure 6. Simulation of non-SDR choice curves for different traffic intensities.

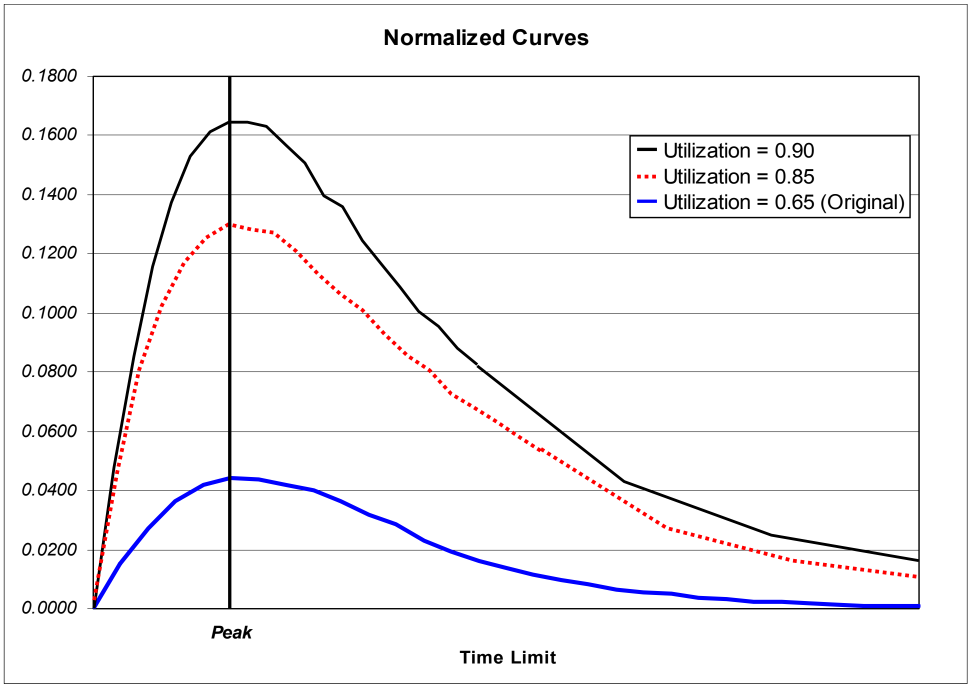

Figure 7. Simulation of non-SDR choice curves normalized with respect to their peak points for different traffic intensities.

Figure 6 shows three non-SDR simulation choice curves with different traffic intensity cases. In Figure 7, we normalized those simulation curves at the peak point. As can be seen in Figure 7, even though these simulation curves are different with respect to the traffic intensities of the system, these curves are very similar in their shapes. The best fit distributions for the non-SDR choice simulations curves are Gamma distributions with resulting parameters, α and β, that are shown in Table 7.

	α	β
Utilization = 0.65 (Original)	2.490	0.355
Utilization = 0.90	2.850	0.041
Utilization = 0.85	3.000	0.039

Table 7. Gamma distribution (best fit) parameters for different traffic intensity cases.

Using the Gamma distribution modified **P** matrices for each Time Limit value, we compare analytical estimates from those obtained from simulation for various Time Limit values the following table. Simulation results using ARENA are for a run length of 500,000 time units with a statistical reset at 30,000 time units. As we can see from Table 8, the analytical WIP_q errors are less than ±1.0% for the five different Time Limit values analyzed (note that when the Time Limit is set to 0, the policy is the SDR control scheme).

Time Limit	Analytical WIP_q	Simulation WIP_q	95%CI Min	95%CI Max	% Error
0.0	0.8389	0.8412	0.8376	0.8448	–0.3
0.5	0.8586	0.8546	0.8518	0.8574	+0.5
1.5	0.8452	0.8447	0.8401	0.8493	+0.1
2.5	0.8401	0.8465	0.8436	0.8495	–0.7
3.0	0.8394	0.8434	0.8405	0.8463	–0.5

Table 8. The comparison of analytical and simulation results of WIP_q for the model with the TL/SDR control scheme.

3.4.2.1 Model with an Outlier of Length 1

Now, consider the case that there is an outlier in the system. An outlier is a workcenter (node in the network) that is so far from every other workcenter that it is never the closest workcenter when more than one workcenter has jobs awaiting transportation. As we can see in Figure 8, workcenter 7 is an outlier connected to node 4. In this model, a new job arrives at workcenter 7 and all assumptions remain the same as the previous non-outlier example.

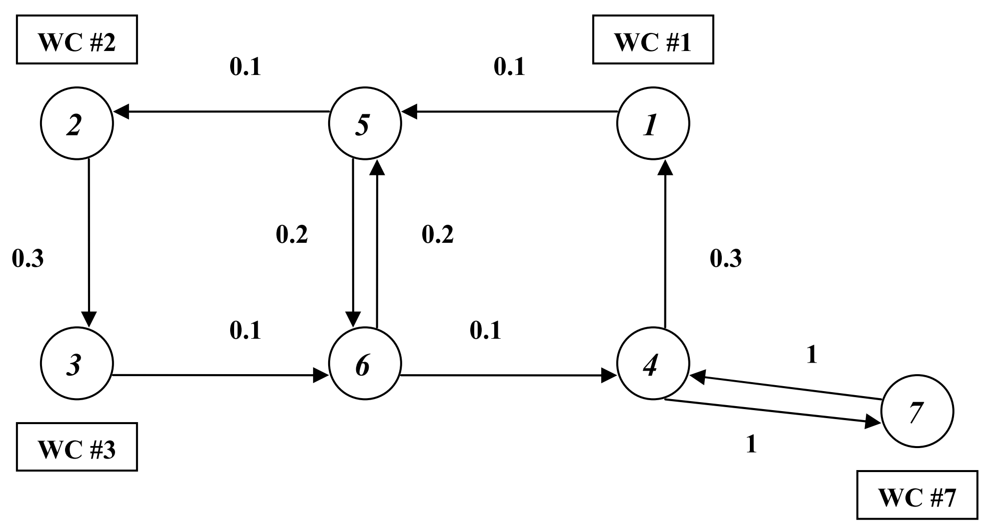

Figure 8. A fixed-route unidirectional material-handling system with an outlier (workcenter 7).

Table 9 displays the 12 routes for this model. Note that only routes 10, 11 and 12 are different from previous model with no outlier. That is, route 10, 11 and 12 start at workcenter 7 instead of node 4.

route/ steps	1	2	3	4	5	6	7	8	9	10	11	12
1	*1*	*1*	*1*	*1*	*2*	*2*	*3*	*3*	*3*	*7*	*7*	*7*
2	*5*	*5*	*5*	*3*	*3*	*3*	*6*	*6*	*6*	*4*	*4*	*4*
3	*2*	*2*	*6*	*6*		*6*	*4*	*5*	*4*	*1*	*1*	*1*
4		*3*	*4*	*4*		*4*	*1*	*2*			*5*	*5*
5				*1*							*2*	*2*
6												*3*

Table 9. Routes generated for the example problem with an outlier of Figure 8.

Due to the structure of this model, under the SDR control scheme, a new job arrival at workcenter 7 can be picked up only if there is no other job request in the system. Thus, if there are two or more job requests in the system when a transporter is available, then workcenter 7 would never be chosen under SDR control. However, under the TL/SDR control scheme, by changing the Time Limit, an empty transporter can be sent to workcenter 7 when there are two or more jobs in the system. For this outlier model, from eleven simulation runs with several Time Limit values, we obtain via simulation the dashed curve in Figure 9. Again the best fit for this simulation curve (dashed line) is a Gamma distribution (solid line). Thus, we can compute sets of proper Pr{*SDR*}'s and Pr{*nonSDR*}'s for all possible Time Limits from above Gamma distribution curve fit and, from those, new **P** matrices with all possible Time Limits for the outlier model can be obtained.

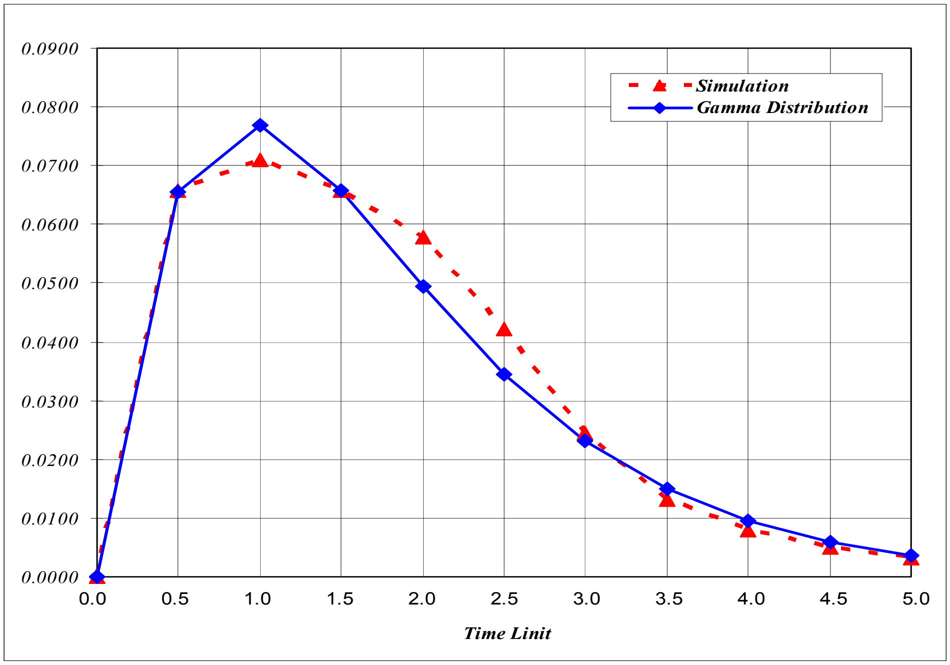

Figure 9. Plots of total non-SDR choice portion of all decisions made during simulation runs for an outlier model and Gamma distribution curve (best fit).

Using the modified **P** matrix considering Time Limits, we have the following analytical WIP_q's according to Time Limit values as shown in Table 10. As the previous case, the simulation model is written in ARENA (Pegden et al. 1995) with the same configuration with analytical model and it has a run length of 500,000 time units and a statistical reset at 30,000 time units. Again, all percentage errors between analytical and simulation WIP_q's for different Time Limits are less than ±1%.

Time Limit	Analytical WIP_q	Simulation WIP_q	95%CI Min	95%CI Max	% Error
0.0	1.1105	1.1199	1.1151	1.1247	−0.8
0.5	1.1142	1.1129	1.1065	1.1193	+0.1
1.5	1.1155	1.1105	1.1037	1.1173	+0.4
2.5	1.1118	1.1095	1.1033	1.1156	+0.2
3.0	1.1110	1.1151	1.1102	1.1200	−0.4

Table 10. The comparison of analytical and simulation results of WIP_q for the outlier model (length 1).

3.4.2.2 Model with an Outlier of Length 0.5

At this time, suppose that, in Figure 8, the distance from workcenter 4 to workcenter 7 (outlier) is 0.5 instead of 1. Then, workcenter 7 is not an absolute outlier. That is, even in SDR control, an empty transporter would go to the workcenter 7 in some cases. For example in Figure 8, since the distance from workcenter 4 to workcenter 3 is bigger than the distance from workcenter 4 to workcenter 7, if an empty transporter is located at workcenter 4 and there are two job requests from workcenters 3 and 7, then an empty transporter will go to workcenter 7. For this small outlier model, from eleven simulation runs with different Time Limit values, we obtain the simulation curve (dashed line) of Figure 10.

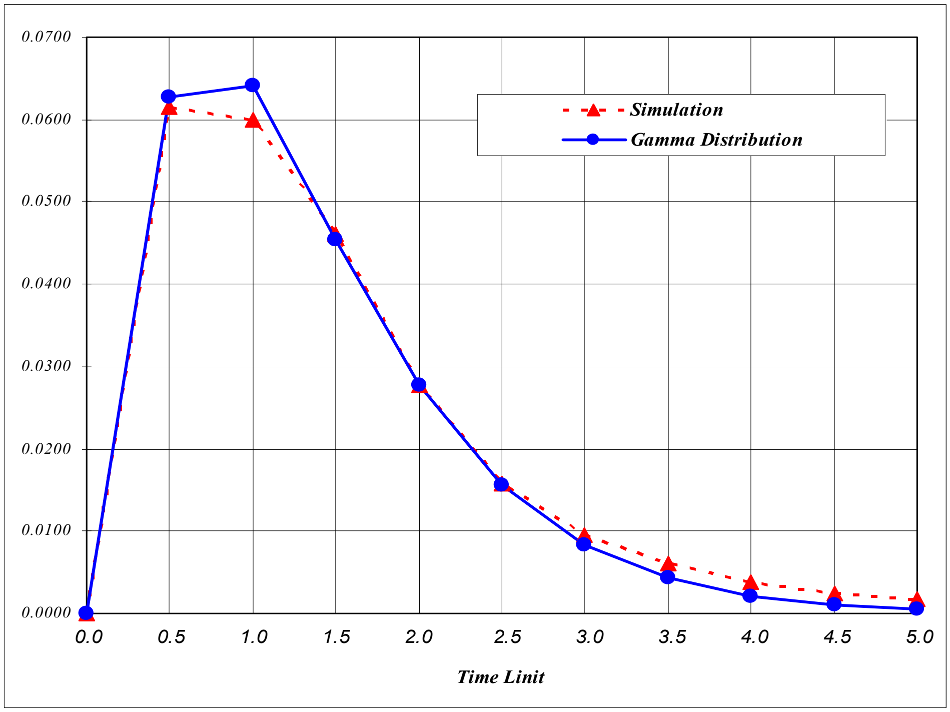

Figure 10. Plots of total non-SDR choice portion of all decisions made during the simulation runs for a model with an outlier length 0.5 and Gamma distribution curve which is the best fit.

Again, by finding the best fit for this simulation curve (dashed line) is a Gamma distribution (solid line). From Gamma distribution above, sets of proper Pr{*SDR*}'s and Pr{*nonSDR*}'s are available and the **P** matrices modified for all possible Time Limit values for an small outlier model can be obtained. Using the modified **P** matrix considering these Time Limits, a comparison between the analytical and simulation results are shown in Table 11.

Time Limit	Analytical WIP_q	Simulation WIP_q	95%CI Min	95%CI Max	% Error
0.0	1.0124	1.0056	1.0004	1.0108	+0.7
0.5	1.0173	1.0008	0.9946	1.0070	+1.6
1.5	1.0150	0.9991	0.9951	1.0031	+1.6
2.5	1.0129	0.9993	0.9951	1.0035	+1.3
3.0	1.0126	1.0007	0.9966	1.0048	+1.2

Table 11. The comparison of analytical and simulation results of WIP_q for the outlier (length 0.5) model.

Again, the simulation model is written in ARENA (Pegden et al. 1995) and it has a run length of 500,000 time units with a statistical reset at 30,000 time units. Table 11 shows that all percentage errors are less than ±2% between the analytical and simulation WIP_q's for different Time Limit values.

3.4.3 Model with First-Come First-Serve Rule (FCFSR)

Now, assume a First-Come First-Serve Rule (FCFSR) for empty transporter allocations. In the FCFSR control scheme case, job selections depend only on the arrival sequence of the available jobs and not on the location of these jobs. That is, an empty transporter will give the highest priority to the job whose arrival time is earlier than any other candidates in the system no matter how far away the job's location is from the current location of the empty transporter. Thus, jobs in the system will be served by an empty transporter in the FCFS manner. To obtain the steady-state probabilities that there are n job requests in the system under the FCFSR control scheme, it is necessary to modify the **P** matrix from previous section to allow for the empty transporter to select job requests in the system according to their waiting times (or arrival order). Suppose that we have only two workcenters, i and j, in the system and the job arrival rate λ_i at workcenter i is greater than the job arrival rate λ_j at workcenter j ($\lambda_i > \lambda_j$). Then, on average, it is likely that a job arrival at the workcenter i will wait longer for an empty transporter than a job arrival at workcenter j. Thus, the probability that an empty transporter will go to workcenter i is bigger than the probability that an empty transporter will go to workcenter j because the job selection only depends on the jobs' waiting time. When there are two job requests from workcenters i and j in the system, we approximate the probability that an empty transporter will go to workcenter i by $\lambda_i / (\lambda_i + \lambda_j)$. More generally, the probability, $\Pr\{WC^i_{\Re}\}$, that the empty transporter will select workcenter i among the set of job requesting workcenters, $\Re$, can be approximated by:

$$\Pr\{WC^i_{\Re}\} = \frac{\lambda_i}{\sum_{k \in \Re} \lambda_k} \quad \text{for } i \in \Re, \tag{3.1}$$

where $\Re$ = set of all the locations of job requests when a transporter becomes available. Since this probability, $\Pr\{WC^i_{\Re}\}$, is only for the empty trip decision for the transporter that became available after the previous service, it is basically the same as the probability of an empty trip to workcenter *i* which is given by Castillo and Peters (2002):

$$q_{im} = \frac{U_i^{(I)} U_m^{(O)} / U_t}{U_m^{(O)}} = \frac{U_i^{(I)}}{U_t}, \qquad (3.2)$$

where $U_i^{(I)} U_m^{(O)} / U_t$ is the rate of empty trips from workcenter *i* to workcenter *m*, $U_m^{(O)}$ represents the empty material-handling device requirement at workcenter *m*, $U_i^{(I)}$ denotes the arrival rate to workcenter *i* in unit loads per time unit and U_t denotes the total arrival rate to the material-handling system in unit loads per time unit. Castillo and Peters used this probability to compute the expected empty trip time and its second moment. Note that the probability q_{im} in Equation (3.2) does not depend on workcenter *m*. Therefore, as we can see in above equations (3.1) and (3.2), $\Pr\{WC_{\Re}^{i}\}$ and q_{im} are essentially the same probabilities. This approximation technique is referred to as factoring and we can estimate the rate of empty trips between workcenters from this approximation (Egbelu 1987b, Bakkalbasi and McGinnis 1988). To illustrate the usage of this probability, $\Pr\{WC_{\Re}^{i}\}$, consider the state example of (1100:3) case.

If an empty transporter is freed at workcenter 3 and there are job requests from workcenters 1 and 2 in the system at that time, state (1100:3), then the transporter will select the different workcenters with probabilities, $\Pr\{WC_{\Re}^{i}\}$'s, as shown in Equation (3.1) for $\Re$ = set of all the locations of the job requests when a transporter becomes available and $i \in \Re$, where:

$$\Pr\{WC_{\Re}^{i}\} = \frac{\lambda_i}{\sum_{k \in \Re} \lambda_k} = \frac{\lambda_i}{\lambda_1 + \lambda_2} \;, \quad \text{for } i \in \Re = \{1, 2\}.$$

For state (1100:3), we have $\Re = \{1, 2\}$ because there are only two job requests from workcenters 1 and 2 in the system when a transporter is freed at workcenter 3. If we consider the state case of (1100:3) and there are no job arrivals in the system during the service, then we have the following portion of all one-step transition cases from (1100:3). Note that, from state (1100:3), an empty transporter has two choices, workcenter 1 and 2, when it becomes available at workcenter 3. If the empty transporter chooses workcenter 2 with the probability, $\Pr\{WC_{\{1,2\}}^{2}\}$, then we have the cases ①, ② and ③ in the following table depending on the job type, i.e., either route 4 ($2 \to 3 \to 6 \to 4 \to 1$) or route 5 ($2 \to 3$) or route 6 ($2 \to 3 \to 6 \to 4$). On the other hand, if the empty transporter selects workcenter 1 with the probability, $\Pr\{WC_{\{1,2\}}^{1}\}$, then we have the cases ④, ⑤ and ⑥ in the following table depending on the job type, i.e., either route 1 ($1 \to 5 \to 2$) or route 2 ($1 \to 5 \to 2 \to 3$) or route 3 ($1 \to 5 \to 6 \to 4$). Again, $\Pr\{(1100{:}3) \to (1000{:}3)\}$ is the probability that an empty transporter from workcenter 3 will select the workcenter 2 to pick a job up and then finish its service at workcenter 3 with no new job arrivals in the system and Pr{R_{ij}} is the probability that the destination of the job at workcenter *i* is workcenter *j*, for *i, j* = 1, …, 4.

States	(1000:1)	(1000:3)	(1000:4)	(0100:2)	(0100:3)	(0100:4)
Prob.	①	②	③	④	⑤	⑥

For ①: $\Pr\{(1100:3) \to (1000:1)\} = \left(e^{-E[S_{321}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{21}\}\right) \cdot \Pr\{WC^2_{\{1,2\}}\}$

For ②: $\Pr\{(1100:3) \to (1000:3)\} = \left(e^{-E[S_{323}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{23}\}\right) \cdot \Pr\{WC^2_{\{1,2\}}\}$

For ③: $\Pr\{(1100:3) \to (1000:4)\} = \left(e^{-E[S_{324}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{24}\}\right) \cdot \Pr\{WC^2_{\{1,2\}}\}$

For ④: $\Pr\{(1100:3) \to (0100:2)\} = \left(e^{-E[S_{312}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{12}\}\right) \cdot \Pr\{WC^1_{\{1,2\}}\}$

For ⑤: $\Pr\{(1100:3) \to (0100:3)\} = \left(e^{-E[S_{313}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{13}\}\right) \cdot \Pr\{WC^1_{\{1,2\}}\}$

For ⑥: $\Pr\{(1100:3) \to (0100:4)\} = \left(e^{-E[S_{314}](\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \cdot \Pr\{R_{14}\}\right) \cdot \Pr\{WC^1_{\{1,2\}}\}$.

Once the generator matrix **P** has been obtained using the above probabilities, the steady-states probabilities and the work-in-process WIP_q can be computed. For our example model described in Figure 1, we have the results of Table 12. Here, the analytical model result is compared with that from a simulation model using the same configuration with a run length of 500,000 time units and a statistical reset at 30,000 time units.

	Analytical	Simulation	95%CI Min	95%CI Max	% Error
WIP_q	0.8829	0.8654	0.8611	0.8697	2.0 %

Table 12. The comparison of analytical and simulation WIP_q results for the example model using the FCFSR control scheme.

Table 12 shows the excellent agreement between the analytical WIP_q and the simulation WIP_q. The percentage WIP_q error of the analytical model is 2.0%.

3.4.4 Model with Longest Distance Rule (LDR)

To develop another control scheme, assume that we have the Longest Distance Rule (LDR) for transporter allocations. In the LDR control case, an empty transporter will pick the job waiting at the farthest workcenter from its current location. That is, an empty transporter will give the highest priority to the farthest workcenter from its current location. Thus, contrary to the SDR control scheme, the probability that a certain workcenter will be selected by an empty transporter is positively related to the distance between job requesting workcenters and the empty transporter location. That is, the more distance there is between the job requesting workcenters and the empty transporter location the higher the probability that those workcenters will be chosen. To illustrate this, reconsider the previous case when a transporter is freed at workcenter 3. Then we have the following possible system states, and suppose that these states occur in equal portions:

{(0000:3), (1000:3), (0100:3), (0010:3), (0001:3), (1100:3), (1010:3), (1001:3), (0110:3), (0101:3), (0011:3), (1110:3), (1101:3), (1011:3), (0111:3), (1111:3)}.

In an equally likely scenario, the probability that workcenter 3 will be chosen is 1/15 and the probabilities that workcenters 4, 2 and 1 will be chosen are 2/15, 4/15 and 8/15, respectively. Now, to develop the proper generator **P** matrix for the LDR control case, we need to have a proper set of Pr{*LDR*}*'s* and Pr{*nonLDR*}*'s*; where Pr{*LDR*} is the probability that an empty transporter goes to the farthest workcenter from its freed workcenter and Pr{*nonLDR*} is the probability that an empty transporter goes to the a workcenter that is not the farthest workcenter from its freed workcenter. Therefore, under the LDR control scheme, we have Pr{*LDR*} = 1 and Pr{*nonLDR*} = 0 for all system states. To illustrate this, reconsider the following portion of all one-step transition cases from (1100:3):

If an empty transporter is freed at workcenter 3 and there are two job requests from workcenters 1 and 2 in the system at that time, state (1100:3), then the transporter picks a job at workcenter 1 and follows a routing sequence according to the job type, i.e., either route 1 ($1 \rightarrow 5 \rightarrow 2$) or route 2 ($1 \rightarrow 5 \rightarrow 2 \rightarrow 3$) or route 3 ($1 \rightarrow 5 \rightarrow 6 \rightarrow 4$). The following table shows all possible transition states from state (1100:3) when there is exactly one job arrival during the service time. For example, if the job picked up at workcenter 1 has its destination as workcenter 2 and there is one job arrival to workcenter 1 during the service time, then Pr{(1100:3)→(1100:2)} is the transition probability for that case, i.e., case 2-① below.

States	(1100:1)	(1100:2)	(1100:3)	(1100:4)	(0110:2)	(0110:3)
Prob.	0	2-①	2-②	2-③	2-④	2-⑤
States	(0110:4)	(0101:2)	(0101:3)	(0101:4)	(1010:1)	(1010:3)
Prob.	2-⑥	2-⑦	2-⑧	2-⑨	0	0
States	(1010:4)	(1001:1)	(1001:3)	(1001:4)		
Prob.	0	0	0	0		

For 2-①: $\Pr\{(1100:3) \to (1100:2)\} = \left(1 - e^{-\lambda_1 \cdot E[S_{312}]}\right) \cdot \left(e^{-E[S_{312}](\lambda_2+\lambda_3+\lambda_4)}\right) \cdot \Pr\{R_{12}\} \cdot 1$

For 2-②: $\Pr\{(1100:3) \to (1100:3)\} = [\left(1 - e^{-\lambda_1 \cdot E[S_{313}]}\right) \cdot \left(e^{-E[S_{313}](\lambda_2+\lambda_3+\lambda_4)}\right) \cdot \Pr\{R_{13}\} \cdot 1]$

$$+ [\left(1 - e^{-\lambda_2 \cdot E[S_{323}]}\right) \cdot \left(e^{-E[S_{323}](\lambda_1+\lambda_3+\lambda_4)}\right) \cdot \Pr\{R_{23}\} \cdot 0]$$

For 2-③: $\Pr\{(1100:3) \to (1100:4)\} = [\left(1 - e^{-\lambda_1 \cdot E[S_{314}]}\right) \cdot \left(e^{-E[S_{314}](\lambda_2+\lambda_3+\lambda_4)}\right) \cdot \Pr\{R_{14}\} \cdot 1]$

$$+ [\left(1 - e^{-\lambda_2 \cdot E[S_{324}]}\right) \cdot \left(e^{-E[S_{324}](\lambda_1+\lambda_3+\lambda_4)}\right) \cdot \Pr\{R_{24}\} \cdot 0]$$

For 2-④: $\Pr\{(1100:3) \to (0110:2)\} = \left(1 - e^{-\lambda_3 \cdot E[S_{312}]}\right) \cdot \left(e^{-E[S_{312}](\lambda_1+\lambda_2+\lambda_4)}\right) \cdot \Pr\{R_{12}\} \cdot 1$

$$\vdots$$

For 2-⑨: $\Pr\{(1100:3) \to (1001:4)\} = \left(1 - e^{-\lambda_4 \cdot E[S_{314}]}\right) \cdot \left(e^{-E[S_{314}](\lambda_1+\lambda_2+\lambda_3)}\right) \cdot \Pr\{R_{14}\} \cdot 1$

where $\Pr\{R_{ij}\}$ is the probability that the destination of the job at workcenter *i* is workcenter *j*, for *i, j* = 1, …, 4. Note that the transition probability from (1100:3) to (1100:1) is zero, i.e., Pr{(1100:3) → (1100:1)} = 0, because the job picked up at workcenter 1 cannot have its destination as workcenter 1. The transition probability, Pr{(1100:3) $\rightarrow$ (1010:1)}, also should be zero, because of the job waiting at workcenter 2.

Once we have the proper generator matrix **P**, we can compute the steady-state probabilities and the work-in-process WIP_q. For the four workcenters and one transporter example, we have the results of Table 13. Here the analytical model result is compared with that from a simulation model using the same configuration with a run length of 500,000 time units and a statistical reset at 30,000 time units.

	Analytical	Simulation	95%CI Min	95%CI Max	% Error
WIP_q	0.9349	0.9158	0.9111	0.9205	+2.0 %

Table 13. The comparison of analytical and simulation results of WIP_q for the model with the LDR control scheme.

As is clear from the results in Table 13, the analytical model yields a very acceptable percentage WIP_q error of 2.0%. For the same problem, if we compare the analytical WIP_q results from the SDR control scheme, the FCFSR control scheme, and the LDR control scheme, we see that the WIP_q for the SDR control case is smaller than either the WIP_q for the FCFSR control case or the WIP_q for the LDR control case. From these results, we conclude that the SDR control scheme is the best one among these three different dispatching policies, as one would expect. The results are shown in Table 14.

	SDR control	FCFSR control	LDR control
WIP_q	0.8389	0.8829	0.9349

Table 14. The analytical results comparison between the SDR control scheme WIP_q, the FCFSR control scheme WIP_q, and the LDR control scheme WIP_q.

From the analytical WIP_q results from the TL/SDR control scheme in Table 8, all WIP_q values lie between WIP_q from the SDR control case (0.8389), and WIP_q from the FCFSR control case (0.8829). Thus, we also can say that, among these four vehicle-dispatching rules for our example system, the SDR control scheme is the best one, the TL/SDR control scheme is the second best, the FCFSR control scheme is the third best, and the least efficient approach is the LDR control scheme in terms of the WIP_q. Now, we have the following theorem.

Theorem: With the situations such that there is no locking phenomenon in the system, the best transporter dispatching rule in terms of WIP_q for an *M/G/*1 model is SDR**.**

Proof:

The proof is shown in Appendix A.

3.5 Chapter Summary

In this chapter, we developed a queueing approximation model for a fixed-route unidirectional material-handling system from the transporter's point of view and investigated the effects of different vehicle dispatching rules. The analyzed models incorporated four different dispatching rules: the Shortest Distance Rule (SDR), the Time Limit/Shortest Distance Rule (TL/SDR), the First-Come First-Serve Rule (FCFSR) and the Longest Distance Rule (LDR). Comparisons were made between these four control schemes for several example problems. The analytical model results are in excellent agreement with simulation results for all example systems studied with analytical model errors that were less than or equal to ± 2.0%. These results also show that, for our example models, the SDR control scheme is the best one, and the TL/SDR control scheme is the second best, the FCFSR control scheme is the third best, and the least efficient dispatching policy is the LDR control scheme in terms of the system performance measure WIP_q. Although we don't have a method at this time for predicting the parameter values of the Gamma distribution adjustments for the TL/SDR control scheme, the analytical models using these Gamma adjustments yield very good estimates for the system performance measures.

The queueing approximation models developed in this chapter have queue space limits set at one for each workcenter and there is only one transporter in the system. With this system configuration, we have 64 system states. If we increase the queue space from one to two, three, and four, then number of states for the analytical models becomes 325, 1,025, and 2,341, respectively. On the other hand, if we increase the number of transporters from one to two and three, then number of states for the analytical models becomes 186 and 420, respectively. Since a lot of the computational complexity arises from this rapid growth of the number of system states as the queue space length or number of transporters increase, future research is needed to develop efficient dependent service time queueing approximations for these systems. In the next chapter, we try to generalize our model by developing analytical models for two different situations. First, we will allow the queue space size at each workcenter of two. Second, we will increase the number of transporters in the system to two.

CHAPTER IV

ANALYTICAL MODELS FOR MATERIAL-HANDLING SYSTEMS WITH TWO DIFFERENT KINDS OF EXTENSIONS

4.1 Introduction

In this chapter, two different extended models of our previous basic models with the queue space limit of one at each workcenter and one transporter were developed. First, we develop an analytical model with the queue space limit of two at each workcenter. Second, we allow two transporters in the system. Thus, for the example problem studied, total system sizes can be up to nine and six jobs for the first model and the second model, respectively. In the model with queue space limit of two at each workcenter, when a job arrival occurs at a workcenter and there is already one waiting job at the workcenter, then that job joins the queue and additional job arrivals to the workcenter will be discarded due to the queue length limit of two. Since this model has a single transporter, only vehicle-initiated vehicle dispatching rules can be used. In the model with two transporters, however, both workcenter-initiated dispatching rules and vehicle-initiated dispatching rules are required for transporter-job assignment decisions. That is, by the workcenter-initiated vehicle dispatching control scheme, a job arrival selects the transporter when there are two empty transporters available in the system and, by the vehicle-initiated vehicle dispatching control scheme, the empty transporter chooses the job requests when there are two or more job requests in the system.

4.2 Model with the Queue Space Limit of Two at Each Workcenter

Now, in this section, we develop a model for a fixed-route unidirectional material-handling system with the queue space limit of two at each workcenter. That is, we allow two queue spaces at each workcenter and, thus, for the example problem the total system size can be up to nine jobs including the job currently being serviced. When job arrival sees the system empty, then this job is assigned to an empty transporter. If a job arrival at a workcenter sees one job waiting in the queue of the workcenter, then that job joins the queue and since the queue length at the workcenter is now reached its maximum capacity, two, with this new arrival, no more jobs are allowed at the workcenter until the first job in that workcenter queue is assigned to an empty transporter. Reconsider the previous circuit network example problem in Figure 1 in the previous chapter. All assumptions here are the same as the previous model except the allowed queue space size is now two at each workcenter. We have four workcenters and one transporter in the system. If we identify all possible states of the system in terms of the number of jobs in the system, there are a total of 325 possible system states. Each state can be represented as numbers. The first four digits represent the number of jobs waiting as 0, 1 or 2 for each workstation (0 being no job, one being a one job and two being two jobs waiting for transportation at that workcenter), and the fifth digit shows the arrival location of the job currently being serviced. That is, if the second digit is 2 and the fifth digit is 3, then this implies that there are two jobs waiting for an empty transporter at workcenter 2 and the job that is currently being serviced was picked up at workcenter 3. Thus, for example, if we have (1020:4), then this state representation implies that there are three jobs waiting at the system queues (one is at workcenter 1 and two is at workcenter 3) and the pick-up location of the job that is currently being serviced was workcenter 4. In this manner, all 325 system states can be obtained as follows:

If there are no jobs in the system, then we have only one state. This state is:

{(0000:0)}.

If there is only one job in the system, then there are four such system states. Note that the job currently being serviced can be picked up at any one of the four workcenters. These possible states are:

{(0000:1), (0000:2), (0000:3), (0000:4)}.

If there are two jobs (one is being serviced and one is being waiting for an empty transporter) in the system, then the total number of all these possible states is 16. These states are:

{(1000:1), (1000:2), (1000:3), (1000:4), (0100:1), (0100:2), (0100:3), (0100:4), (0010:1), (0010:2), (0010:3), (0010:4), (0001:1), (0001:2), (0001:3), (0001:4)}.

For thee jobs (one is being serviced and two are being waiting for an empty transporter) in the system, there are 40 possible states:

{(2000:1), (2000:2), (2000:3), (2000:4), (0200:1), (0200:2), (0200:3), (0200:4), (0020:1), (0020:2), (0020:3), (0020:4), (1100:1), (1100:2), (1100:3), (1100:4), (1010:1), (1010:2), (1010:3), (1010:4), (1001:1), (1001:2), (1001:3), (1001:4), (0110:1), (0110:2), (0110:3), (0110:4), (0101:1), (0101:2), (0101:3), (0101:4), (0011:1), (0011:2), (0011:3), (0011:4) (0002:1), (0002:2), (0002:3), (0002:4)}.

If there are four jobs (one is being serviced and three are being waiting for an empty transporter) in the system, this results in 64 possible states. These are:

{(2100:1), (2100:2), (2100:3), (2100:4), (2010:1), (2010:2), (2010:3), (2010:4), (2001:1), (2001:2), (2001:3), (2001:4), (1200:1), (1200:2), (1200:3), (1200:4), (1020:1), (1020:2), (1020:3), (1020:4), (1002:1), (1002:2), (1002:3), (1002:4), (0210:1), (0210:2), (0210:3), (0210:4), (0201:1), (0201:2), (0201:3), (0201:4), (0120:1), (0120:2), (0120:3), (0120:4), (0102:1), (0102:2), (0102:3), (0102:4), (1110:1), (1110:2), (1110:3), (1110:4), (1101:1), (1101:2), (1101:3), (1101:4), (1011:1), (1011:2), (1011:3), (1011:4), (0111:1), (0111:2), (0111:3), (0111:4), (0021:1), (0021:2), (0021:3), (0021:4), (0012:1), (0012:2), (0012:3), (0012:4)}.

If there are five jobs (one is being serviced and four are being waiting for an empty transporter) in the system, this results in 76 possible states. These are:

{(2200:1), (2200:2), (2200:3), (2200:4), (2020:1), (2020:2), (2020:3), (2020:4), (2110:1), (2110:2), (2110:3), (2110:4), (2101:1), (2101:2), (2101:3), (2101:4), (2011:1), (2011:2), (2011:3), (2011:4), (1210:1), (1210:2), (1210:3), (1210:4), (1201:1), (1201:2), (1201:3), (1201:4), (1120:1), (1120:2), (1120:3), (1120:4), (1102:1), (1102:2), (1102:3), (1102:4), (1021:1), (1021:2), (1021:3), (1021:4), (1012:1), (1012:2), (1012:3), (1012:4), (2002:1), (2002:2), (2002:3), (2002:4), (0220:1), (0220:2), (0220:3), (0220:4), (0211:1), (0211:2), (0211:3), (0211:4), (0121:1), (0121:2), (0121:3), (0121:4), (0112:1), (0112:2), (0112:3), (0112:4), (0202:1), (0202:2), (0202:3), (0202:4), (1111:1), (1111:2), (1111:3), (1111:4), (0022:1), (0022:2), (0022:3), (0022:4)}.

If there are six jobs (one is being serviced and five are being waiting for an empty transporter) in the system, this results in 64 possible states. These are:

{(2210:1), (2210:2), (2210:3), (2210:4), (2201:1), (2201:2), (2201:3), (2201:4), (2120:1), (2120:2), (2120:3), (2120:4), (2111:1), (2111:2), (2111:3), (2111:4), (2102:1), (2102:2), (2102:3), (2102:4), (2021:1), (2021:2), (2021:3), (2021:4), (2012:1), (2012:2), (2012:3), (2012:4), (1220:1), (1220:2), (1220:3), (1220:4), (1211:1), (1211:2), (1211:3), (1211:4), (1202:1), (1202:2), (1202:3), (1202:4), (1121:1), (1121:2), (1121:3), (1121:4), (1112:1), (1112:2), (1112:3), (1112:4),

(1022:1), (1022:2), (1022:3), (1022:4), (0221:1), (0221:2), (0221:3), (0221:4),

(0212:1), (0212:2), (0212:3), (0212:4), (0122:1), (0122:2), (0122:3), (0122:4)}.

For seven jobs (one is being serviced and six are being waiting for an empty transporter) in the system, there are 40 possible states:

{(2220:1), (2220:2), (2220:3), (2220:4), (2211:1), (2211:2), (2211:3), (2211:4),

(2202:1), (2202:2), (2202:3), (2202:4), (2112:1), (2112:2), (2112:3), (2112:4),

(2022:1), (2022:2), (2022:3), (2022:4), (1122:1), (1122:2), (1122:3), (1122:4),

(1221:1), (1221:2), (1221:3), (1221:4), (2121:1), (2121:2), (2121:3), (2121:4),

(1212:1), (1212:2), (1212:3), (1212:4), (0222:1), (0222:2), (0222:3), (0222:4)}.

For eight jobs (one is being serviced and seven are being waiting for an empty transporter) in the system, there are 16 possible states:

{(2221:1), (2221:2), (2221:3), (2221:4), (2212:1), (2212:2), (2212:3), (2212:4),

(2122:1), (2122:2), (2122:3), (2122:4), (1222:1), (1222:2), (1222:3), (1222:4)}.

And finally, when there are nine jobs in the system, the total number of possible states is 4. These states are:

{(2222:1), (2222:2), (2222:3), (2222:4)}.

Here, we assume the SDR control scheme for empty transporter dispatching. Then, an empty transporter will pick the job waiting at the closest workcenter from its current location. State transitions occur when there are new job arrivals or job service completions in the system. For illustration of the transitions between states, let's consider the following example case of state (2111:4) shown in Figure 11.

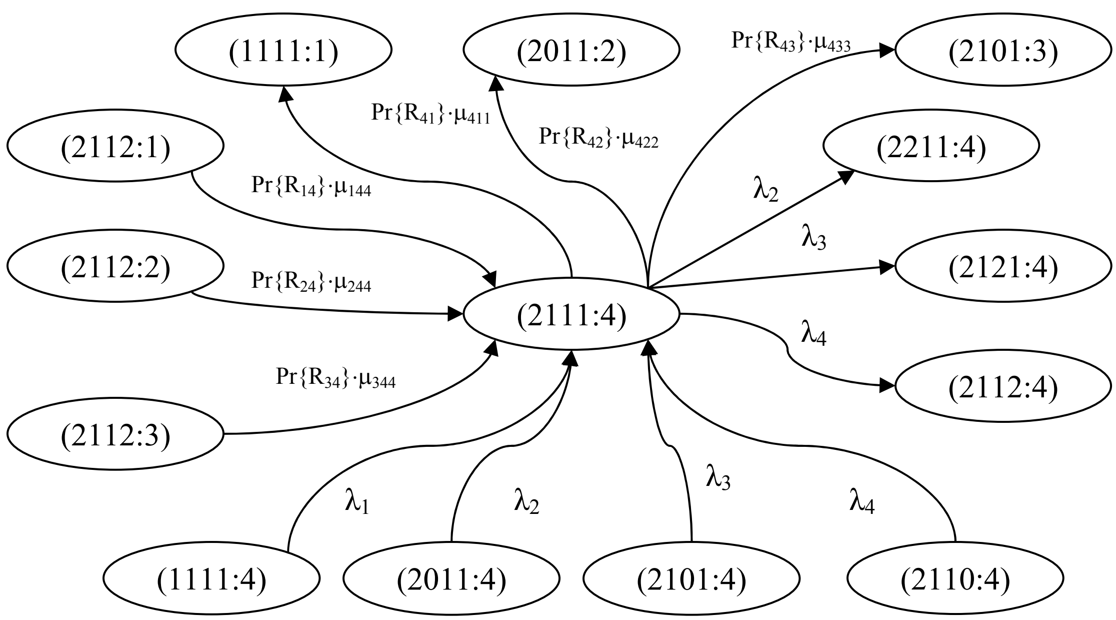

Figure 11. The diagram of all possible state transitions for state (2111:4).

Note that the fraction, $\Pr\{R_{ij}\}$, is the probability that the job arrival at workcenter *i* has its destination as workcenter *j*. Thus, we have:

$$\sum_{j \in \Im} \Pr\{R_{ij}\} = 1 \quad \text{for } i = 1,2,3,4, \text{ and } \Im = \{k \mid k \neq i,\ k = 1,2,3,4\},$$

where the service rate μ_{ijk} is the reciprocal of the service time from workcenter i to workcenter k via workcenter j. That is:

$$\mu_{ijk} = \frac{1}{E[S_{ijk}]} \quad \text{for } i, j, k = 1,2,3,4.$$

As we can see in above Figure 9, if there is a job arrival in the system state transition can occur to state (2111:4) from (1111:4), (2011:4), (2101:4), and (2110:4) with rates λ_1, λ_2, λ_3, and λ_4, respectively. With arrival rates λ_2, λ_3, and λ_4, system states can go from (2111:4) to (2211:4), (2121:4), and (2112:4). Note that since the queue space limit is two at each workcenter, the state (3111:4) or (1301:4) cannot exist. If the service of the job picked up at workcenter 1 is finished, then the all possible locations for an empty transporter are workcenter 2, 3, and 4 with probabilities $\Pr\{R_{12}\}$, $\Pr\{R_{13}\}$, and $\Pr\{R_{14}\}$, respectively. Thus, the transition from state (2112:1) to (2111:4) will occur with rate $\Pr\{R_{14}\}\cdot\mu_{144}$ when a transporter finishes its job at workcenter 4 and picks a job up at that workstation due to the SDR control scheme. Similarly, we can have state transitions from states (2112:2) and (2112:3) to state (2111:4) with rates $\Pr\{\pi_{24}\}\cdot\mu_{244}$ and $\Pr\{R_{34}\}\cdot\mu_{344}$, respectively. The system state (2111:4) can be changed to state (1111:1) when the job currently being serviced (it was picked up at workcenter 4) departs the system at workcenter 1 with the probability Pr{R_{41}}, and the transporter picks a job up at that workcenter. In this case, since the transporter goes from workcenter 4 to workcenter 1 and then no empty transporter time is required, the rate will be μ_{411}. Thus, the transition can occur from state (2111:4) to state (1111:1) with rate Pr{R_{41}}$\cdot\mu_{411}$. Similarly, the state transitions from state (2111:4) to states (2011:2) and (2101:3) with rates Pr{R_{42}}$\cdot\mu_{422}$ and Pr{R_{34}}$\cdot\mu_{433}$, can occur. Now, suppose we want to compute the average queue length, WIP_q, and the average system size, WIP_{sys}. Then we need the steady-state probabilities, P_i's, that there are exactly *i*-jobs in the system. To obtain those steady-state probabilities, we develop the generator matrix **Q**. Then the steady-state equations

	$\mathbf{P}_0$	$\mathbf{P}_1$	$\mathbf{P}_2$	$\mathbf{P}_3$	$\mathbf{P}_4$	…	$\mathbf{P}_n$
$\mathbf{P}_0$	A_{00}	A_{01}					
$\mathbf{P}_1$	B_{10}	B_{11}	B_{12}				
$\mathbf{P}_2$		C_{21}	C_{22}	C_{23}			
$\mathbf{P}_3$			D_{32}	D_{33}	D_{34}		
$\mathbf{P}_4$				E_{43}	E_{44}	$\ddots$	
$\vdots$					$\ddots$	$\ddots$	
$\mathbf{P}_n$						Z_{nn-1}	Z_{nn}

Figure 12. The general structure of a generator matrix **Q** for *n* workcenters.

relating the system states are of the form $\mathbf{P}^T \cdot \mathbf{Q} = \mathbf{0}^T$. The generator matrix $\mathbf{Q}$ for a model with n workcenters with one transporter has the general structure shown in Figure 12.

		$\mathbf{P_0}$ (0000:0)	$\mathbf{P_1}$ (0000:1) ~ (0000:4)	$\mathbf{P_2}$ (1000:1) ~ (0001:4)	$\mathbf{P_3}$ (2000:1) ~ (0002:4)	$\mathbf{P_4}$ (2100:1) ~ (0012:4)	$\mathbf{P_5}$ (2200:1) ~ (0022:4)	$\mathbf{P_6}$ (2210:1) ~ (0122:4)	$\mathbf{P_7}$ (2220:1) ~ (0222:4)	$\mathbf{P_8}$ (2221:1) ~ (1222:4)	$\mathbf{P_9}$ (2222:1) ~ (2222:4)
$\mathbf{P_0}$	(0000:0)	$\mathbf{A_{00}}$ (1×1)	$\mathbf{B_{01}}$ (1×4)								
$\mathbf{P_1}$	(0000:1) ~ (0000:4)	$\mathbf{A_{10}}$ (4×1)	$\mathbf{B_{11}}$ (4×4)	$\mathbf{C_{12}}$ (4×16)							
$\mathbf{P_2}$	(1000:1) ~ (0001:4)		$\mathbf{B_{21}}$ (16×4)	$\mathbf{C_{22}}$ (16×16)	$\mathbf{D_{23}}$ (16×40)						
$\mathbf{P_3}$	(2000:1) ~ (0002:4)			$\mathbf{C_{32}}$ (40×16)	$\mathbf{D_{33}}$ (40×40)	$\mathbf{E_{34}}$ (40×64)					
$\mathbf{P_4}$	(2100:1) ~ (0012:4)				$\mathbf{D_{43}}$ (64×40)	$\mathbf{E_{44}}$ (64×64)	$\mathbf{F_{45}}$ (64×76)				
$\mathbf{P_5}$	(2200:1) ~ (0022:4)					$\mathbf{E_{54}}$ (76×64)	$\mathbf{F_{55}}$ (76×76)	$\mathbf{G_{56}}$ (76×64)			
$\mathbf{P_6}$	(2210:1) ~ (0122:4)						$\mathbf{F_{65}}$ (64×76)	$\mathbf{G_{66}}$ (64×64)	$\mathbf{H_{67}}$ (64×40)		
$\mathbf{P_7}$	(2220:1) ~ (0222:4)							$\mathbf{G_{76}}$ (40×64)	$\mathbf{H_{77}}$ (40×40)	$\mathbf{I_{78}}$ (40×16)	
$\mathbf{P_8}$	(2221:1) ~ (1222:4)								$\mathbf{H_{87}}$ (16×40)	$\mathbf{I_{88}}$ (16×16)	$\mathbf{J_{89}}$ (16×4)
$\mathbf{P_9}$	(2222:1) ~ (2222:4)									$\mathbf{I_{98}}$ (4×16)	$\mathbf{J_{99}}$ (4×4)

Figure 13. The structure of a generator matrix $\mathbf{Q}$ (325×325) for the example problem with a maximum queue length of two.

The blanks within the matrix denote zero matrices and all $\mathbf{A}_{ij}$, $\mathbf{B}_{ij}$, $\mathbf{C}_{ij}$, $\mathbf{D}_{ij}$, ..., $\mathbf{Z}_{ij}$'s are sub-matrices of the matrix $\mathbf{Q}$ whose elements are zeros and the transition rates between states. For our example problem with one transporter and four workcenters whose queue space limits of two, the steady-state flow-balance equations have a 325×325 generator $\mathbf{Q}$ matrix with the structure shown in Figure 13. Then, from the system $\mathbf{P}^T \cdot \mathbf{Q} = \mathbf{0}^T$, we have the following system of equations to be solved:

$$P_0 \cdot A_{00} + \mathbf{P_1}^T \cdot \mathbf{A_{10}} = 0$$

$$P_0 \cdot \mathbf{B_{01}} + \mathbf{P_1}^T \cdot \mathbf{B}_{11} + \mathbf{P_2}^T \cdot \mathbf{B_{21}} = \mathbf{0}^T,$$

$$\mathbf{P_1}^T \cdot \mathbf{C}_{12} + \mathbf{P_2}^T \cdot \mathbf{C}_{22} + \mathbf{P_3}^T \cdot \mathbf{C}_{32} = \mathbf{0}^T,$$

$$\mathbf{P_2}^T \cdot \mathbf{D_{23}} + \mathbf{P_3}^T \cdot \mathbf{D_{33}} + \mathbf{P_4}^T \cdot \mathbf{D_{43}} = \mathbf{0}^T,$$

$$\mathbf{P_3}^T \cdot \mathbf{E_{34}} + \mathbf{P_4}^T \cdot \mathbf{E_{44}} + \mathbf{P_5}^T \cdot \mathbf{E_{54}} = \mathbf{0}^T,$$

$$\mathbf{P_4}^T \cdot \mathbf{F_{45}} + \mathbf{P_5}^T \cdot \mathbf{F_{55}} + \mathbf{P_6}^T \cdot \mathbf{F_{65}} = \mathbf{0}^T,$$

$$\mathbf{P_5}^T \cdot \mathbf{G_{56}} + \mathbf{P_6}^T \cdot \mathbf{G_{66}} + \mathbf{P_7}^T \cdot \mathbf{G_{76}} = \mathbf{0}^T,$$

$$\mathbf{P_6}^T \cdot \mathbf{H_{67}} + \mathbf{P_7}^T \cdot \mathbf{H_{77}} + \mathbf{P_8}^T \cdot \mathbf{H_{87}} = \mathbf{0}^T,$$

$$\mathbf{P_7}^T \cdot \mathbf{I_{78}} + \mathbf{P_8}^T \cdot \mathbf{I_{88}} + \mathbf{P_9}^T \cdot \mathbf{I_{98}} = \mathbf{0}^T,$$

$$\mathbf{P_8}^T \cdot \mathbf{J_{89}} + \mathbf{P_9}^T \cdot \mathbf{J_{99}} = \mathbf{0}^T,$$

and

$$P_0 \cdot \mathbf{1_{1\times1}} + \mathbf{P_1}^T \cdot \mathbf{1_{1\times4}} + \mathbf{P_2}^T \cdot \mathbf{1_{1\times16}} + \cdots + \mathbf{P_8}^T \cdot \mathbf{1_{1\times16}} + \mathbf{P_9}^T \cdot \mathbf{1_{1\times4}} = 1.$$

Note that the last one of above equations is the norming equation and can be used to obtain P_0. The first one of above equations is ignored because, for a finite irreducible Markov system, we always have one redundant equation (Feldman and Valdez-Flores 1996). Using successive substitution, we obtain:

$$\mathbf{P}_1^T = -P_0 \cdot \mathbf{B}_{01} \cdot [\mathbf{B}_{11} - \mathbf{C}_{12} \cdot [\mathbf{C}_{22} - \mathbf{D}_{23} \cdot [\mathbf{D}_{33} - \mathbf{E}_{34} \cdot [\cdots]^{-1} \cdot \mathbf{D}_{43}]^{-1} \cdot \mathbf{C}_{32}]^{-1} \cdot \mathbf{B}_{21}]^{-1},$$

$$\mathbf{P}_2^T = -\mathbf{P}_1^T \cdot \mathbf{C}_{12} \cdot [\mathbf{C}_{22} - \mathbf{D}_{23} \cdot [\mathbf{D}_{33} - \mathbf{E}_{34} \cdot [\mathbf{E}_{44} - \mathbf{F}_{45} \cdot [\cdots]^{-1} \cdot \mathbf{E}_{54}]^{-1} \cdot \mathbf{D}_{43}]^{-1} \cdot \mathbf{C}_{32}]^{-1},$$

$$\mathbf{P}_3^T = -\mathbf{P}_2^T \cdot \mathbf{D}_{23} \cdot [\mathbf{D}_{33} - \mathbf{E}_{34} \cdot [\mathbf{E}_{44} - \mathbf{F}_{45} \cdot [\mathbf{F}_{55} - \mathbf{G}_{56} \cdot [\cdots]^{-1} \cdot \mathbf{F}_{65}]^{-1} \cdot \mathbf{E}_{54}]^{-1} \cdot \mathbf{D}_{43}]^{-1},$$

$$\mathbf{P}_4^T = -\mathbf{P}_3^T \cdot \mathbf{E}_{34} \cdot [\mathbf{E}_{44} - \mathbf{F}_{45} \cdot [\mathbf{F}_{55} - \mathbf{G}_{56} \cdot [\mathbf{G}_{66} - \mathbf{H}_{67} \cdot [\cdots]^{-1} \cdot \mathbf{G}_{76}]^{-1} \cdot \mathbf{F}_{65}]^{-1} \cdot \mathbf{E}_{54}]^{-1},$$

$$\mathbf{P}_5^T = -\mathbf{P}_4^T \cdot \mathbf{F}_{45} \cdot [\mathbf{F}_{55} - \mathbf{G}_{56} \cdot [\mathbf{G}_{66} - \mathbf{H}_{67} \cdot [\mathbf{H}_{77} - \mathbf{I}_{78} \cdot [\cdots]^{-1} \cdot \mathbf{H}_{87}]^{-1} \cdot \mathbf{G}_{76}]^{-1} \cdot \mathbf{F}_{65}]^{-1},$$

$$\mathbf{P}_6^T = -\mathbf{P}_5^T \cdot \mathbf{G}_{56} \cdot [\mathbf{G}_{66} - \mathbf{H}_{67} \cdot [\mathbf{H}_{77} - \mathbf{I}_{78} \cdot [\mathbf{I}_{88} - \mathbf{J}_{89} \cdot [\mathbf{J}_{99}]^{-1} \cdot \mathbf{I}_{98}]^{-1} \cdot \mathbf{H}_{87}]^{-1} \cdot \mathbf{G}_{76}]^{-1},$$

$$\mathbf{P}_7^T = -\mathbf{P}_6^T \cdot \mathbf{H}_{67} \cdot [\mathbf{H}_{77} - \mathbf{I}_{78} \cdot [\mathbf{I}_{88} - \mathbf{J}_{89} \cdot [\mathbf{J}_{99}]^{-1} \cdot \mathbf{I}_{98}]^{-1} \cdot \mathbf{H}_{87}]^{-1},$$

$$\mathbf{P}_8^T = -\mathbf{P}_7^T \cdot \mathbf{I}_{78} \cdot [\mathbf{I}_{88} - \mathbf{J}_{89} \cdot [\mathbf{I}_{99}]^{-1} \cdot \mathbf{I}_{98}]^{-1},$$

$$\mathbf{P}_9^T = -\mathbf{P}_8^T \cdot \mathbf{J}_{89} \cdot [\mathbf{J}_{99}]^{-1}.$$

And, to get P_0, the norming equation was used and we finally have the following:

$$P_0 = 1 \Big/ \left\{1 - \mathbf{B}_{01} \cdot [\mathbf{B}_{11} \cdot [\cdots]^{-1} \cdot \mathbf{B}_{21}]^{-1} + \cdots - \mathbf{B}_{01} \cdot [\mathbf{B}_{11} \cdot [\cdots]^{-1} \cdot \mathbf{B}_{21}]^{-1} \cdots \cdot \mathbf{J}_{89} \cdot [\mathbf{J}_{99}]^{-1}\right\}.$$

Once the steady-state probabilities are obtained from the above system of equations, using those steady-state probabilities of *i* jobs in the system, P_i, $i = 0, \ldots, 9$, we can get the average number of jobs in the system, WIP_{sys}, as follows:

$$\mathrm{WIP}_{sys} = 0 \cdot P_0 + 1 \cdot P_1 + 2 \cdot P_2 + 3 \cdot P_3 + 4 \cdot P_4 + 5 \cdot P_5 + 6 \cdot P_6 + 7 \cdot P_7 + 8 \cdot P_8 + 9 \cdot P_9 .$$

If we want to know the average number of jobs waiting in the system queue, WIP_q, then it can be computed as follows:

$$\mathrm{WIP}_q = 1 \cdot P_2 + 2 \cdot P_3 + 3 \cdot P_4 + 4 \cdot P_5 + 5 \cdot P_6 + 6 \cdot P_7 + 7 \cdot P_8 + 8 \cdot P_9 .$$

For the four workstations and one transporter example with the queue limit of two at each workcenter, i.e., the total queue limit in the system of eight, the analytical model result is compared in Table 15 with that from a simulation model (written in ARENA (Pegden et al. 1995)) with a run length of 500,000 time units and a statistical reset at 30,000 time units. As we can see, the percentage error between the analytical and simulation results of WIP_{sys} is 1.9% and the percentage error between the analytical and simulation results of WIP_q is 2.9%. These are very acceptable results for an analytical model.

	Analytical	Simulation	95%CI Min	95%CI Max	% Error
WIP_q	0.9671	0.9387	0.9230	0.9474	2.9%
WIP_{sys}	1.7251	1.7580	1.7476	1.7684	1.9%

Table 15. The comparison of analytical and simulation results of WIP_q and WIP_{sys} for the model with the SDR control scheme

4.3 Lost Arrivals to the System Due to the Queue Space Limit

As we have seen in the previous section, we can successfully develop analytical model for the system with the queue space limit of two at each workcenter. With a specific configuration in our example model, by increasing the queue space of all four workcenters by one, we can decrease the percentage of the total lost arrivals to the system due to the queue space limit to 6.84% from 21.46% in the case of the queue space limit of one. If we increase the limit of the queue space at each workcenter up to three, four, five, and six, then the percentage of the total lost arrivals becomes 3.3%, 1.75%, 0.98%, and 0.56%, respectively. This decrement of the percentage of the total lost job arrivals to the system according to the queue space limit at each workcenter is shown in Figure 14. However, the state-space sizes of the case of the queue space of three and four are $1{,}025^2 = 1{,}050{,}625$ and $2{,}341^2 = 5{,}480{,}281$, respectively. Thus, the state-space size of the case of the queue space of two will be increased by almost 10 times when we have the queue space limit of three at each workcenter. Moreover, if we increase the limit of queue space at each workcenter to four, then the state-space

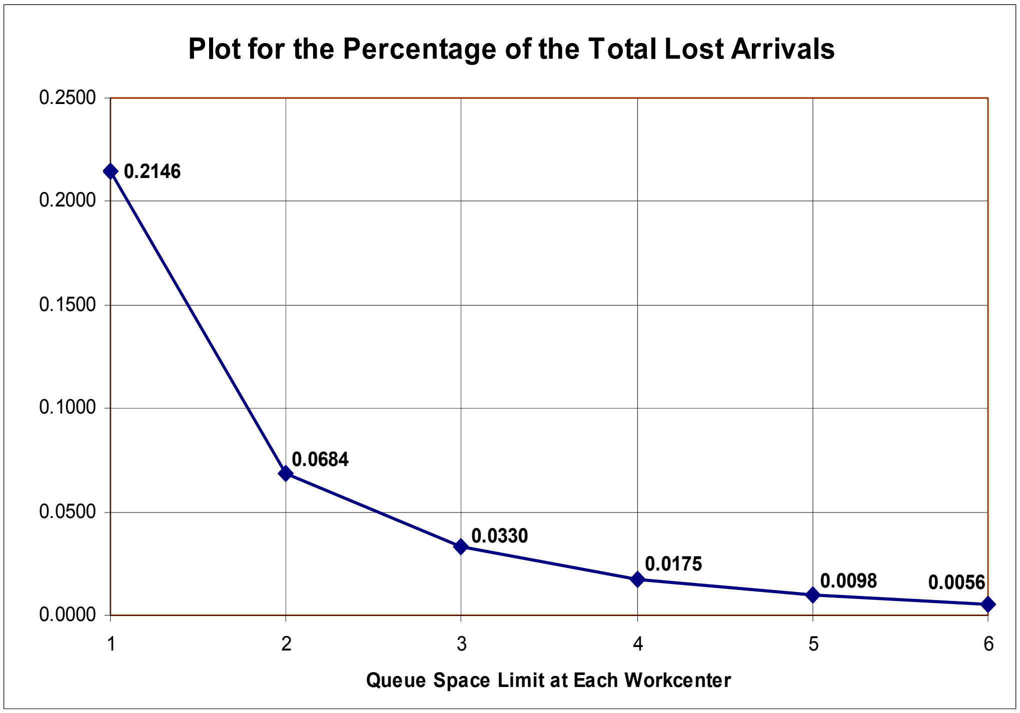

Figure 14. Plots for the percentage of the total lost arrivals to the system when we increase all queue space limits in equal portion.

size will be increased up to almost 50 times. Due to this fast growth of the computational difficulty, we cannot increase the queue space limit infinitely in the analytical model. Therefore, we want to find the queue space limit that has both less percentage of the lost arrivals and state-space size. For our material-handling system model, all four workcenters have external arrivals and these external arrival rates are not necessarily the same. Since we set the rate of the external arrivals at each workcenter to be different for our example models, it can be expected that the amount of lost arrivals would be different at each workcenter. In fact, the amount of lost arrivals at each workcenter is positively related to the arrival rates to the workcenter. The larger the arrival rate to the workcenter, the more lost arrivals at the workcenter. That is, since we have $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ for our examples, the lost arrivals at workcenter 1 will be the largest, and the lost arrivals at workcenter 2 will be the next largest, etc. Thus, we examine this percentage of lost arrivals at the system by workcenter. That is, we compute the level-changes of the percentage of the lost arrivals at the workcenter level. Then, we have the following Figure 15.

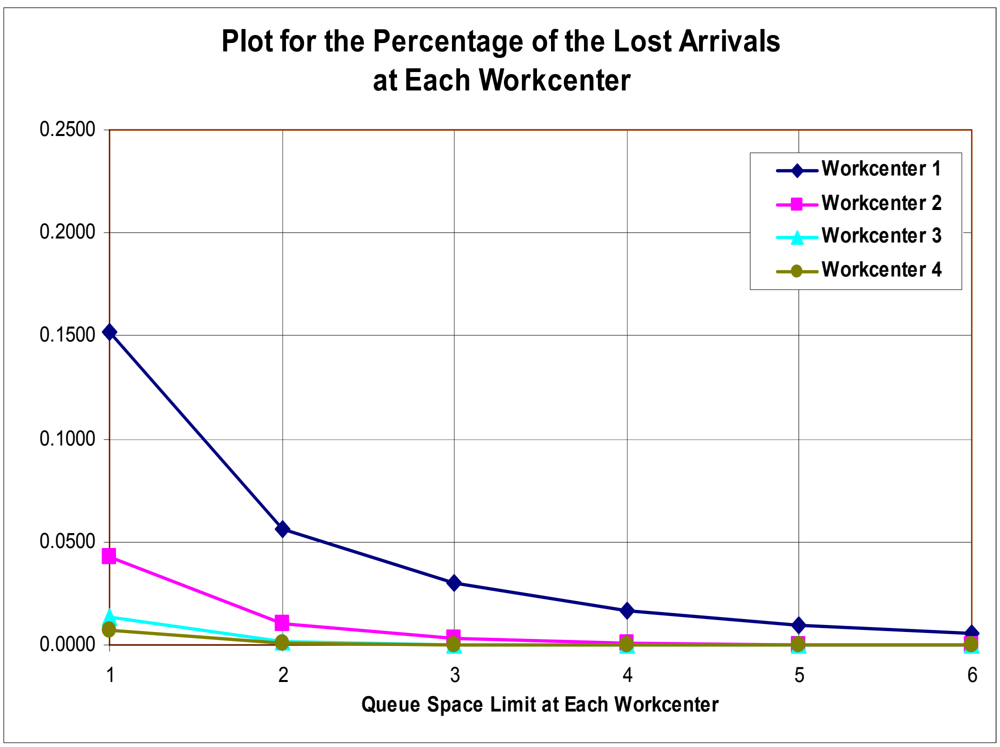

Figure 15. Plots for the percentage of the lost arrivals at each workcenter when we increase all queue space limits in equal portion.

As we can see in Figure 15, the percentage of the lost arrivals at workcenter 1 is bigger than those at any other workcenters in the system as we suspected. Actually, it dominates the percentages of the lost arrivals at all other workcenters. Note that, when the queue space limits at workcenter 2, 3, and 4 exceed four, no more improvement can be achieved by increasing the queue spaces because the percentage of lost arrivals at those workcenters are already almost zero. Therefore, instead of increasing the queue space limits of all workcenters by equal amount, if we allow more queue spaces for workcenter 1 than for all other workcenters, then we may have better results.

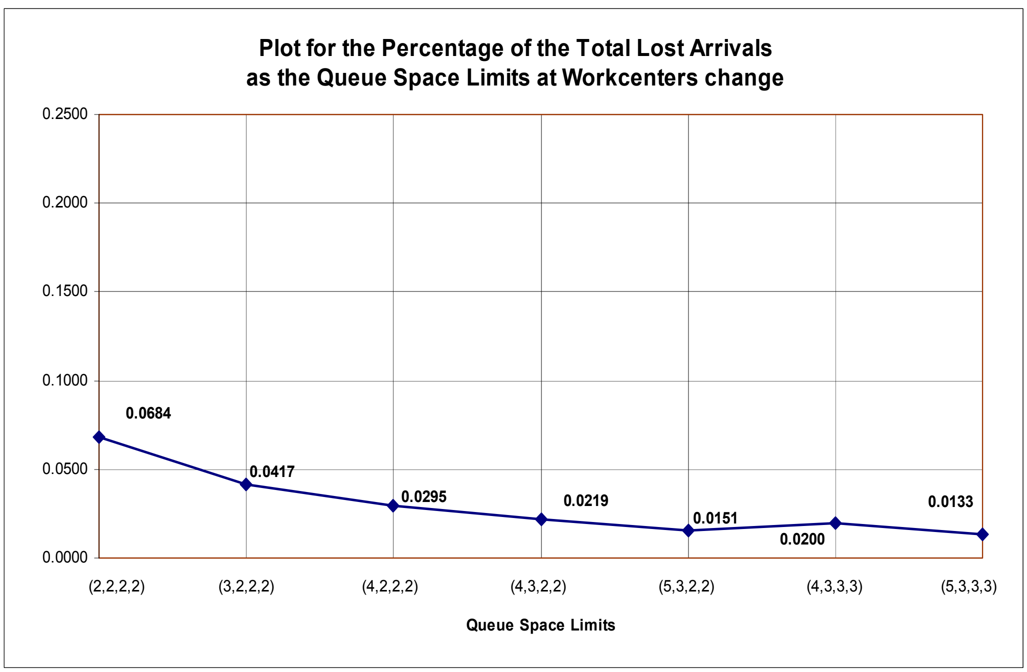

Figure 16. Plots for the percentage of the total lost arrivals to the system when we increase queue space limits individually.

Figure 16 shows the percentages of the total lost arrivals to the system whose queue space limits are increased individually. Note that, in the bottom of Figure 16, (*i, j, k, m*) denotes the queue space limits of workcenter *i*, *j*, *k*, and *m*. As is clear from the results in Figure 14 and Figure 16, by allowing more queue spaces for workcenters whose arrival rates are higher than other workcenters, we can reduce the percentage of the total lost arrivals to the system while the state-space size of system is maintained less than the that from the case of the equal amount queue space increment at each workcenter. For example, the percentage of the lost arrivals to the system with the queue space limit of three at each workcenter, 3.3%, is bigger than the percentage of the lost arrivals to the system of (4,2,2,2) case, 2.9%. Moreover, the state-space size of (4,2,2,2) case is 541^2 = 292,681, whereas the state-space size of (3,3,3,3) is $1{,}025^2$ = 1,050,625. Table 16 shows the size of the state-space for each queue space limit case.

	(2,2,2,2)	(3,2,2,2)	(4,2,2,2)	(4,3,2,2)	(5,3,2,2)
State-Space Size	105,625	187,489	292,681	514,089	734,449

Table 16. The state-space sizes for different queue space limit cases.

By allowing different queue space at each workcenter, we also can minimize the differences of the lost arrivals between workcenters. In Figure 17, we can see that the difference of the percentage of the lost arrivals between workcenters becomes very small at (4,2,2,2) and (5,3,2,2). Therefore, this can be another benefit from individual increments of the queue space limits at each workcenter.

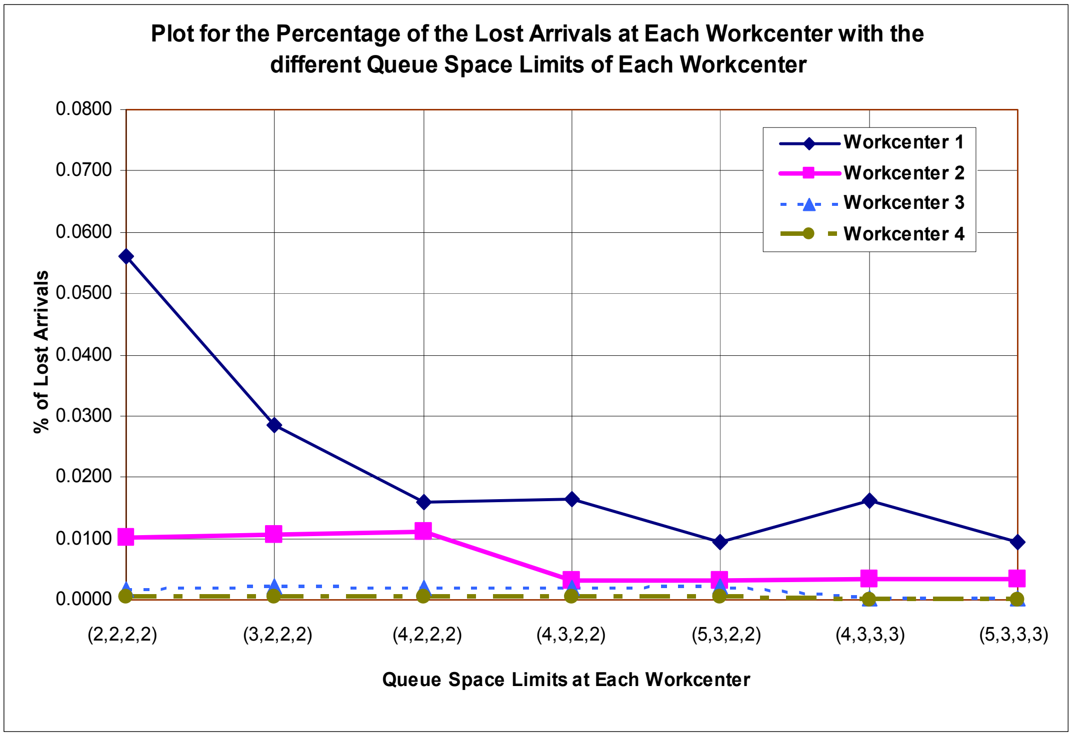

Figure 17. Plots for the percentage of the lost arrivals to each workcenter when we increase queue space limits individually.

4.4 Model with Two Transporters in the System

In this section, we develop an analytical model for a fixed-route material-handling system with two transporters. That is, we add one more transporter to the system while the queue space limit at each workcenter remains to be one. When a job arrival sees the system empty, then that job sees two empty transporters located at arbitrary workcenters and it has to select one of them according to the workcenter-initiated vehicle dispatching policies. On the other hand, when the transporter is available after the previous service and there are two or more job requests in the system, it chooses the job request according to the vehicle-initiated vehicle dispatching policies. Therefore, the system uses both workcenter-initiated dispatching rules and vehicle-initiated dispatching rules. The previous circuit network example problem in Figure 1 was used again in this section. All assumptions here are the same as the previous section except the number of transporters in the system. That is, we have the queue space limit of one at each workcenter and two transporters in the system. If we identify all possible states of the system in terms of the number of jobs in the system, there are total 186 possible transition states. Each state can be represented as numbers. The first four digits represent the location of job request in the system as a 0 or 1 for each workstation (0 being no job request, 1 being a job request for transportation), and the fifth and sixth digits show the locations of empty transporters, and finally, the seventh and eighth digits denote the job pick-up locations that are currently being serviced. Thus, for example, if we have (1010:00:34) as the system state, then this state representation implies that there are two job requests in the system (one is at workcenter 1 and one is at workcenter 3) and the pick-up locations of jobs that are currently being serviced were workcenter 3 and 4. Note that, since both transporters are currently working, there are no empty transporters in the system. In this manner, we can define all 186 system states.

For this system, we assume SDR control scheme for the vehicle-initiated dispatching rule and Closest Transporter Allocation Rule (CTAR) control for the workcenter-initiated dispatching rule. Under SDR control scheme, when the transporter becomes available after the previous service, an empty transporter selects the job waiting at the closest workcenter from its current workcenter. Under CTAR control scheme, a job arrival chooses the empty transporter whose location is the closest one from the current job arrival workcenter. To illustrate the difference between the selection schemes under SDR control and under CTAR control in our model, consider the following two cases in Figure 18 for the previous unidirectional circuit network example in Figure 1.

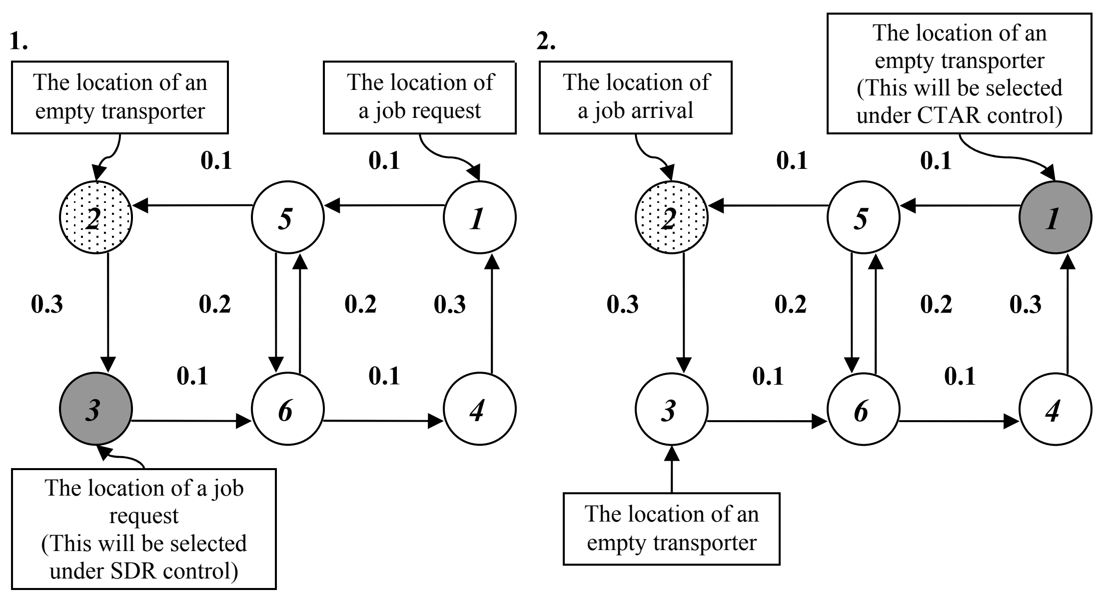

Figure 18. The difference between selection schemes under SDR control and under CTAR control for our unidirectional circuit network layout example.

Assume that we have the first situation shown in Figure 18. That is, when the transporter becomes available at workcenter 2 after the previous service, there are two job requests from workcenter 1 and workcenter 3. Then, by SDR control scheme, the empty transporter will go from workcenter 2 to workcenter 3, i.e., the workcenter 3 will be selected, because the distance from workcenter 2 to workcenter 3 is shorter than the distance from workcenter 2 to workcenter 1. On the other hand, suppose that we have the second situation shown in Figure 18. That is, a job arrival to workcenter 2 sees two empty transporters waiting at workcenter 1 and workcenter 3. Then, by CTAR control scheme, the job arrival to workcenter 2 will be picked up by the empty transporter from workcenter 1, i.e., the workcenter 1 will be selected, because the distance from workcenter 1 to workcenter 2 is shorter than the distance from workcenter 3 to workcenter 2.

State transitions occur when there are new job arrivals or job service completions in the system. For illustration of the transitions between states, let's consider the following example case of state (1010:00:34) shown in Figure 19. Note that the probability that the job arrival at workcenter i has its destination as workcenter j, $\Pr\{R_{ij}\}$, and the service rate μ_{ijk}, the reciprocal of the service time from workcenter i to workcenter k via workcenter j, are defined as in the previous section.

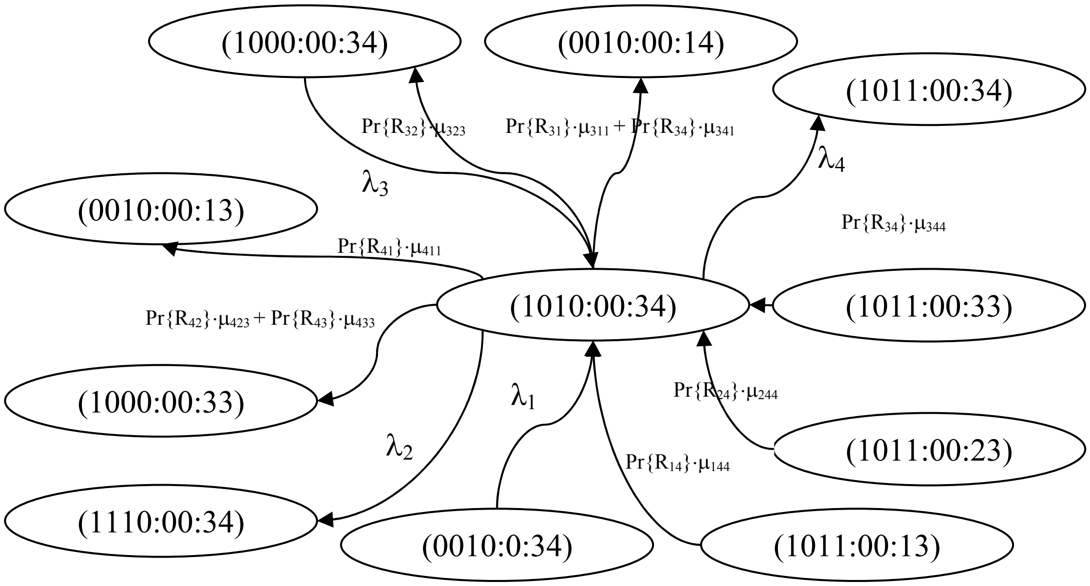

Figure 19. The diagram of all possible state transitions for state (1010:0:34).

As we can see in Figure 19, if there is a job arrival in the system, a state transition can occur from state (1010:00:34) to (1110:00:34) and (1011:00:34) with rates λ_2 and λ_4, respectively. Now, consider the state transition from (1010:00:34) to (1000:00:33). When the service of the job picked up at workcenter 4 is finished, the locations of the empty transporter will be either workcenter 2 or workcenter 3 depending on the job types. Since the empty transporter selects job requests from workcenter 3 in both cases because the SDR control was used, routes will be either $4 \rightarrow 2 \rightarrow 3$ or $4 \rightarrow 3 \rightarrow 3$ depending on the job types and the associated rate for this transition will be $\Pr\{R_{42}\}\cdot\mu_{423} + \Pr\{R_{43}\}\cdot\mu_{433}$. The case that the service of the job picked up at workcenter 3 is finished first will not happen in the transition, because, if then, we need to have 4 in the sixth or the seventh digits of resultant system state (1000:00:33). The transition from (1010:00:34) to (1000:00:34) occurs when the service of the job picked up at workcenter 3 is finished at workcenter 2, and an empty transporter selects job request from workcenter 3 due to SDR control. Thus, route will be $3 \rightarrow 2 \rightarrow 3$ and the associated transition rate will be $\Pr\{R_{32}\}\cdot\mu_{323}$. The case that the service of the job picked up at workcenter 4 is finished first will not happen in this transition, because, if then, we cannot have 4 in the sixth or the seventh digits of resultant system state (1000:00:34).

Now, to compute the average queue length, WIP_q, we need the steady-state probabilities, P_i's, that there are exactly i-jobs in the system. To obtain those steady-state probabilities, we need to develop the generator matrix **Q**. Then the steady-state equations relating the system states are of the form $\mathbf{P}^T \cdot \mathbf{Q} = \mathbf{0}^T$. For our example problem with four workcenters and two transporters, the steady-state flow-balance equations have a 186×186 generator **Q** matrix with the structure shown in Figure 20. The blanks within the matrix denote zero matrices and all $\mathbf{A}_{ij}$, $\mathbf{B}_{ij}$, $\mathbf{C}_{ij}$, $\mathbf{D}_{ij}$, ..., $\mathbf{G}_{ij}$'s are sub-matrices of the matrix **Q** whose elements are zeros and the transition rates between system states.

		$\mathbf{P_0}$ (0000:11:0) ~ (0000:44:0)	$\mathbf{P_1}$ (0000:1:1) ~ (0000:4:4)	$\mathbf{P_2}$ (0000:0:11) ~ (0000:0:44)	$\mathbf{P_3}$ (1000:0:11) ~ (0001:0:44)	$\mathbf{P_4}$ (1100:0:11) ~ (0011:0:44)	$\mathbf{P_5}$ (1110:0:11) ~ (0111:0:44)	$\mathbf{P_6}$ (1111:0:11) ~ (1111:0:44)
$\mathbf{P_0}$	(0000:11:0) ~ (0000:44:0)	$\mathbf{A_{00}}$ (10×10)	$\mathbf{B_{01}}$ (10×16)					
$\mathbf{P_1}$	(0000:1:1) ~ (0000:4:4)	$\mathbf{A_{10}}$ (16×10)	$\mathbf{B_{11}}$ (16×16)	$\mathbf{C_{12}}$ (16×10)				
$\mathbf{P_2}$	(0000:0:11) ~ (0000:0:44)		$\mathbf{B_{21}}$ (10×16)	$\mathbf{C_{22}}$ (10×10)	$\mathbf{D_{23}}$ (10×40)			
$\mathbf{P_3}$	(1000:0:11) ~ (0001:0:44)			$\mathbf{C_{32}}$ (40×10)	$\mathbf{D_{33}}$ (40×40)	$\mathbf{E_{34}}$ (40×60)		
$\mathbf{P_4}$	(1100:0:11) ~ (0011:0:44)				$\mathbf{D_{43}}$ (60×40)	$\mathbf{E_{44}}$ (60×60)	$\mathbf{F_{45}}$ (60×40)	
$\mathbf{P_5}$	(1110:0:11) ~ (0111:0:44)					$\mathbf{E_{54}}$ (40×60)	$\mathbf{F_{55}}$ (40×40)	$\mathbf{G_{56}}$ (40×10)
$\mathbf{P_6}$	(1111:0:11) ~ (1111:0:44)						$\mathbf{F_{65}}$ (10×40)	$\mathbf{G_{66}}$ (10×10)

Figure 20. The structure of a generator matrix **Q** (186×186) for the example problem with two transporters in the system.

Then, from the system $\mathbf{P}^T \cdot \mathbf{Q} = \mathbf{0}^T$, we have following systems of equations to be solved:

$$\mathbf{P_0^T} \cdot \mathbf{A_{00}} + \mathbf{P_1^T} \cdot \mathbf{A_{10}} = \mathbf{0^T},$$

$$\mathbf{P_0^T} \cdot \mathbf{B_{01}} + \mathbf{P_1^T} \cdot \mathbf{B_{11}} + \mathbf{P_2^T} \cdot \mathbf{B_{21}} = \mathbf{0^T},$$

$$\mathbf{P_1}^T \cdot \mathbf{C_{12}} + \mathbf{P_2}^T \cdot \mathbf{C_{22}} + \mathbf{P_3}^T \cdot \mathbf{C_{32}} = \mathbf{0}^T,$$

$$\mathbf{P_2}^T \cdot \mathbf{D_{23}} + \mathbf{P_3}^T \cdot \mathbf{D_{33}} + \mathbf{P_4}^T \cdot \mathbf{D_{43}} = \mathbf{0}^T,$$

$$\mathbf{P_3}^T \cdot \mathbf{E_{34}} + \mathbf{P_4}^T \cdot \mathbf{E_{44}} + \mathbf{P_5}^T \cdot \mathbf{E_{54}} = \mathbf{0}^T,$$

$$\mathbf{P_4}^T \cdot \mathbf{F_{45}} + \mathbf{P_5}^T \cdot \mathbf{F_{55}} + \mathbf{P_6}^T \cdot \mathbf{F_{65}} = \mathbf{0}^T,$$

$$\mathbf{P_5}^T \cdot \mathbf{G_{56}} + \mathbf{P_6}^T \cdot \mathbf{G_{66}} = \mathbf{0}^T,$$

and

$$\mathbf{P_0} \cdot \mathbf{1_{1\times10}} + \mathbf{P_1}^{\mathbf{T}} \cdot \mathbf{1_{1\times16}} + \mathbf{P_2}^{\mathbf{T}} \cdot \mathbf{1_{1\times40}} + \cdots + \mathbf{P_5}^{\mathbf{T}} \cdot \mathbf{1_{1\times40}} + \mathbf{P_6}^{\mathbf{T}} \cdot \mathbf{1_{1\times10}} = \mathbf{1}.$$

Note that the last equation is the norming equation which can be used to obtain P_0. Since we always have one redundant equation for a finite irreducible Markov system (Feldman and Valdez-Flores 1996), the first one of above equations is ignored. Using successive substitutions and the norming equation, the steady-state probabilities are obtained from above systems of equations. Then, using those steady-state probabilities of i-jobs in the system, P_i, $i = 0, \ldots, 6$, we can compute the average number of jobs waiting in the queue, WIP_q, as follows:

$$\mathrm{WIP}_q = \sum_{n=3}^{6} (n-2) \cdot P_n = 1 \cdot \mathbf{P_3^T} \cdot \mathbf{1_{1\times40}} + 2 \cdot \mathbf{P_4^T} \cdot \mathbf{1_{1\times60}} + 3 \cdot \mathbf{P_5^T} \cdot \mathbf{1_{1\times40}} + 4 \cdot \mathbf{P_6^T} \cdot \mathbf{1_{1\times10}}.$$

If we want to know the average number of jobs in the system, WIP_{sys}, it can be obtained from:

$$\mathrm{WIP}_{sys} = \sum_{n=0}^{6} n \cdot P_n = 0 \cdot \mathbf{P}_\mathbf{0}^\mathbf{T} \cdot \mathbf{1}_{\mathbf{1\times10}} + 1 \cdot \mathbf{P}_\mathbf{1}^\mathbf{T} \cdot \mathbf{1}_{\mathbf{1\times16}} + 2 \cdot \mathbf{P}_\mathbf{2}^\mathbf{T} \cdot \mathbf{1}_{\mathbf{1\times10}} + \cdots + 5 \cdot \mathbf{P}_\mathbf{5}^\mathbf{T} \cdot \mathbf{1}_{\mathbf{1\times40}} + 6 \cdot \mathbf{P}_\mathbf{6}^\mathbf{T} \cdot \mathbf{1}_{\mathbf{1\times10}} .$$

For the four workstations and two transporters example with the queue limit of one at each workcenter, i.e., the total system size can be up to six, the analytical model result of WIP_q is compared in Table 17 with that from a simulation model (written in ARENA (Pegden et al. 1995)) with a run length of 500,000 time units and a statistical reset at 30,000 time units.

	Analytical	Simulation	95%CI Min	95%CI Max	% Error
WIP_q	0.3997	0.4026	0.4005	0.4047	0.7%

Table 17. The comparison of analytical and simulation results of WIP_q for the model with both SDR control and CTAR control schemes

As the table shows, the percentage error between the analytical and simulation results of WIP_q is 0.7%. Again, this is very acceptable result for an analytical model.

4.5 Chapter Summary

In this chapter, we tried to extend our original analytical model in two ways: for the model in Section 4.2, we increased the queue space limit at each workcenter to two, and, for the model in Section 4.4, we added one more transporter to the system so that there are total of two transporters in the system. By increasing the queue space limit at each workcenter, we can decrease lost arrivals to the system. However, due to the fast growth in the computational difficulty, we cannot increase the queue space limit infinitely. As shown in Section 4.3, given the same transporters' service rates, if we allow more queue spaces for workcenters whose arrival rates are higher than other workcenters and hold the lower rates at lower limits instead of increasing all queue space limits in same amount, then we can obtain better results with fewer computational difficulties. As we seen in Section 4.4, when we have two or more transporters in the system, the system should have both workcenter-initiated dispatching rules and vehicle-initiated dispatching rules for transporter-job assignments because of the possibilities that a job arrival sees two or more empty transporters in the system.

CHAPTER V

MODIFIED NEAREST NEIGHBOR (NN) HEURISTIC VEHICLE DISPATCHING PROCEDURE FOR MATERIAL-HANDLING SYSTEMS WITH MULTI-LOAD TRANSPORTERS

5.1 Introduction

In this chapter, we developed a revised dispatching policy for multi-load transporters from a dispatching policy originally proposed by Tanchoco and Co (1994), and attempted to investigate the effects of those two different dispatching policies on the system performance measures, such as WIP or cycle time. A multi-load transporter can pick up additional loads while transporting a previously assigned job. Therefore, by using multi-load transporters, we can reduce the transporter's empty trip time as well as the total distance traveled. Also, the traffic congestion and control complexity could be reduced.

We need to determine the appropriate number of transporters to satisfy the material-handling requirements in the system. Using a large number of transporters in the system, it is true that we can meet the high volume of job transportation requirements. However, we will have more traffic congestion and, therefore, need to have a complex control system to avoid transporter collisions and deadlock problems. Instead of having a large number of transporters, we might use a small number of multi-load transporters to meet the same level of job transportation requirements. By doing so, we will reduce the deadhead or unproductive time of transporters and the total distance traveled as well as making a smaller fleet size (number of transporters) possible (Bilge and Tanchoco 1997). Even though multi-load transporters can provide many benefits over single-load transporters, research on multi-load transporters is quite limited so far. Using a simulation approach, Ozden (1988) studied the interaction between design parameters such as the carrying capacity of AGVs, the number of AGVs, the queue capacity at each workcenter and the total number of pallets. In his simulation study, he observed that the throughput rate of the system during a constant period of time behaves in a concave fashion as a function of these design parameters. Also, he demonstrates that, by increasing the load-carrying capacity of the transporter and the buffer size at each workcenter to two, a 50% reduction in the fleet size can be achieved.

Bilge and Tanchoco (1997) showed the benefits of multi-load transporters over unit-load transporters using simulation. In their study, two different types of transporter dispatching strategies, variable-path dispatching and fixed-path dispatching schemes were examined. In the variable-path dispatching policy, a multi-load transporter can change its original path to the destination so as to pick up additional loads. On the other hand, under the fixed-path dispatching policy, new load picking points should lie on the original path of the transporter. Then, they showed that the variable-path dispatching policy is more advantageous in preventing gridlock. After their simulation experiments, they concluded that, for a system with high transportation demand, multi-load transporters increase the system throughput. In addition, they state that a two-load transporter system is not as sensitive to the guide-path layout design as a single-load transporter system. Nayyar and Khator (1993) studied the operational control issues of multi-load AGVs using simulation. They compared the performances of multi-load transporters and single-load transporters under several different dispatching rules and concluded that multi-load transporters outperform single-load transporters under several conditions. They observed that, with larger number of vehicles, the performance of multi-load vehicles was lower than that with unit load vehicles as low levels of shop loading. According to them, this is because the loaded travel time in case of unit load vehicles is higher. Co and Tanchoco (1991) mentioned that the performance of the dispatching rules is highly depend on the guide path layout, the fleet size and the transport patterns in the network. Tanchoco and Co (1994) proposed transporter dispatching control schemes for multi-load transporters. In their study, they developed the simple Nearest Neighbor (NN) heuristic dispatching procedure for multi-load transporters. We modify their NN heuristic procedure to incorporate dynamic reallocation features for the transporter's reservation space into multi-load transporter dispatching. That is, we include additional steps to their original NN procedure to reevaluate the system status periodically and then reassign jobs that have yet to be picked up to the transporter.

5.2 Nearest Neighbor (NN) Transporter Dispatching Procedures

In this section, the Nearest Neighbor (NN) heuristic dispatching procedure for multi-load transporters (Tanchoco and Co 1994) is discussed. According to this procedure, once a task (pickup or delivery) is assigned to a transporter, a position on the vehicle is reserved and will not be made available for other tasks until the corresponding load has been delivered. Therefore, it is possible that a vehicle can be unnecessarily reserved for some period of time. The NN procedure is summarized in the following section.

The original NN heuristic transporter dispatching procedure (Tanchoco and Co 1994)**.** We start with the information of the transporter's current "Job List" where the set of jobs, **Q**, is a set of active job requesting workcenters and destination workcenters of currently onboard jobs. That is, **Q** = {WC_1(p), WC_2(p), …, WC_n(p), WC_{n+1}(d), WC_{n+2}(d), …, WC_{n+m}(d)} where WC_k(p) denotes the workcenter where a load is waiting to be picked up and WC_k(d) denotes the destination workcenter of a load that is currently being carried by the transporter. Also, the current location of the transporter, CL, is known. Note that WC_1(d), …, WC_n(d) are not included in **Q** because those workcenters cannot be visited before WC_1(p), …, WC_n(p).

- Initialize the transporter location marker T = CL set C = the available capacity of the transporter.
- Sort the jobs list, **Q** according to the distance from the transporter location marker, T. At this point, the transporter checks whether a high-priority job exists or not. If there is a high-priority job (for example, its waiting time is greater than or equal to the pre-specified Time Limit), then this job is assigned. Otherwise,

select the first workcenter j, WC_j(p or d) in the sorted **Q**. That is, workcenter j is the closest workcenter from the transporter's current location.

- Remove WC_j(p or d) from **Q**. and update C. If the selected workcenter j is a job pickup workcenter, then add the destination workcenter of the job, WC_q(d), into **Q**. Set T = workcenter j and update **Q** if additional pickup jobs have arrived during this service time.
- Repeat Step 2 and Step 3 until **Q** becomes empty. (At this point, transporter will wait until the arrival of the next transport request.)

Since the position of the transporter is changed dynamically, it is possible to obtain a better system performance by reevaluating the system status (e.g., locations of jobs waiting to be transported and the transporter) and reassigning tasks to the transporter periodically. To illustrate the logic of our revised NN procedure, consider the following situation. When a transporter selects the closest job (pickup or delivery) from its current location, the transporter starts to move from its current location to the location of the selected job. During the transporter's travel, if a job arrival occurs at a closer location than the original destination of the transporter, the transporter stops at that location instead of the location of the originally assigned job and picks up the job at that location. That is, we reassign the new job to the transporter. Depending on the system guide path layout, if the location of a workcenter is not the closest one from any other workcenters (we call the workcenter as an outlier), some jobs associated with the workcenter cannot be delivered or picked up for long period of time. To avoid these situations, we use the Time Limit concept. That is, the transporter will be assigned to the job whose waiting time is greater than or equal to the pre-specified Time Limit. The following is the summarization of our revised NN procedure.

Our revised NN heuristic transporter dispatching procedure. Start with the job list, **Q**, and the current location of the transporter, CL.

- Initialize the transporter location marker T = CL and set C = the available capacity of the transporter.
- Sort the jobs list, **Q**, according to the distance from the transporter location marker, T. At this point, the transporter checks whether a high-priority job exists or not. If there is a high-priority job (for example, its waiting time is greater than or equal to the pre-specified Time Limit), then select that job. Otherwise, select the first workcenter *j*, WC_j(p or d) in the sorted **Q**. That is, the closest workcenter from the transporter's current location, T.
- Temporarily remove WC_j(p or d) from **Q**. During its route to workcenter *j* from T, the transporter updates **Q** with new information (due to new arrivals during this route time). That is, add WC_j(p or d) back into **Q** and update **Q** if additional pickup jobs have arrived to the system during its route to the current location. Update T.
- Perform Step 2 and Step 3 until the transporter reaches at the final destination.

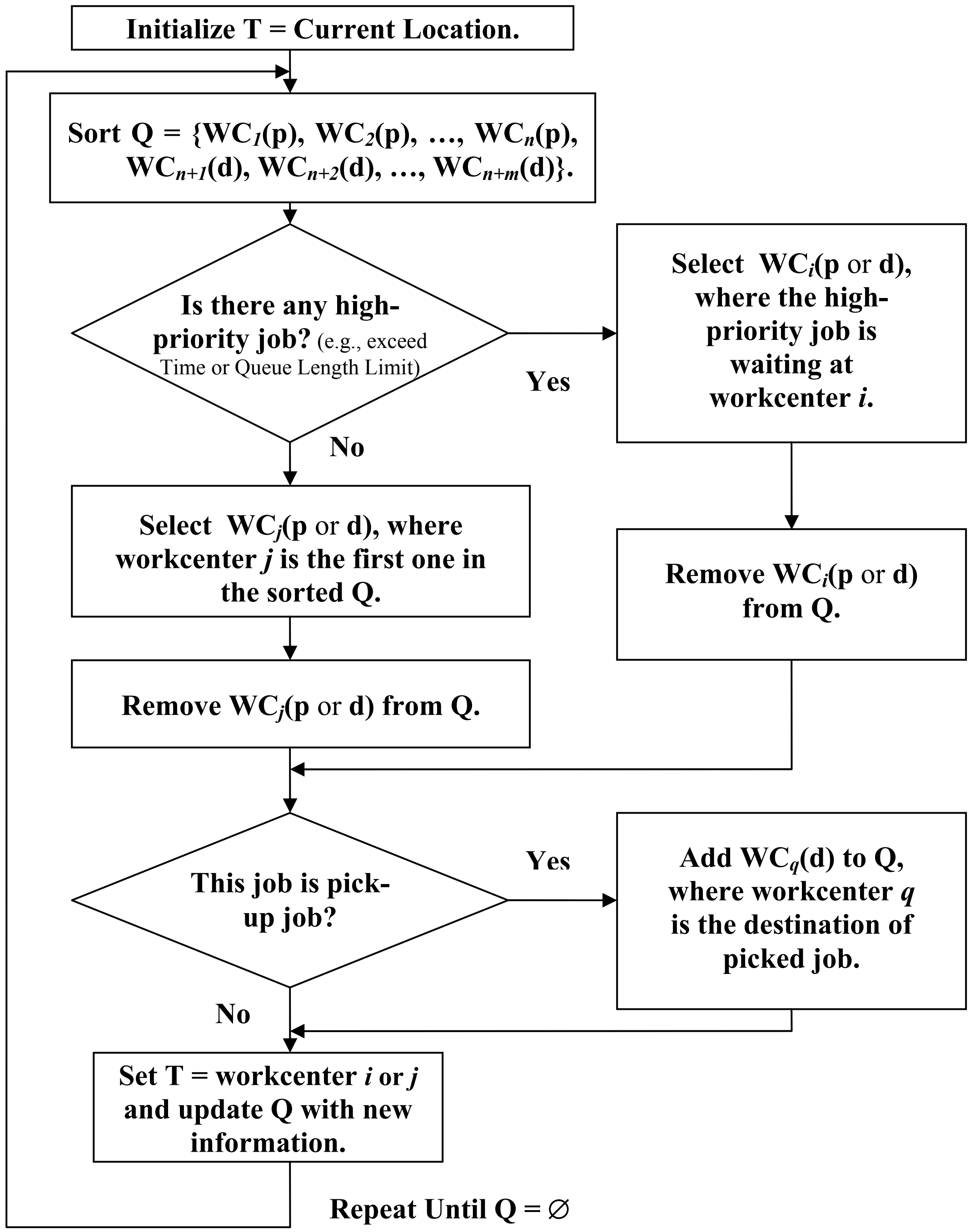

Figure 21. Flow diagram of the original NN heuristic dispatching procedure.

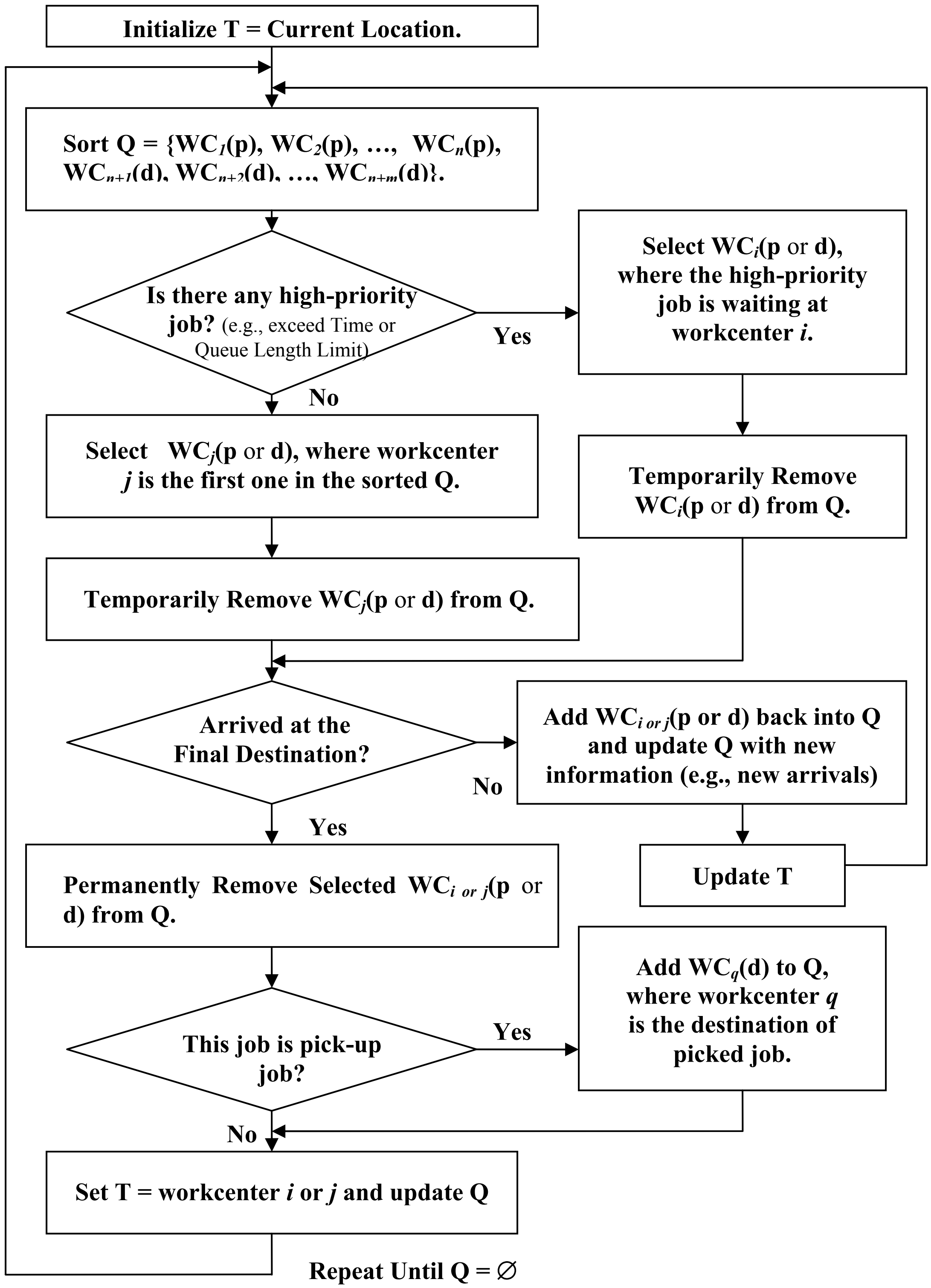

Figure 22. Flow diagram of the modified NN heuristic dispatching procedure.

- At workcenter k, permanently remove WC_k(p or d) from the sorted set **Q**. If the selected workcenter k is a job pickup workcenter, then add the destination workcenter of the job, WC_q(d), into **Q**. Set T = workcenter k. Update **Q** and C.
- Repeat from Step 2 to Step 5 until **Q** becomes empty.

In Figure 21 and Figure 22, these basic implementation procedures are summarized in flow diagrams for both dispatching policies.

5.3 Example Models

To illustrate the differences between the original and the modified NN procedures, consider the small example problem shown in Figure 23.

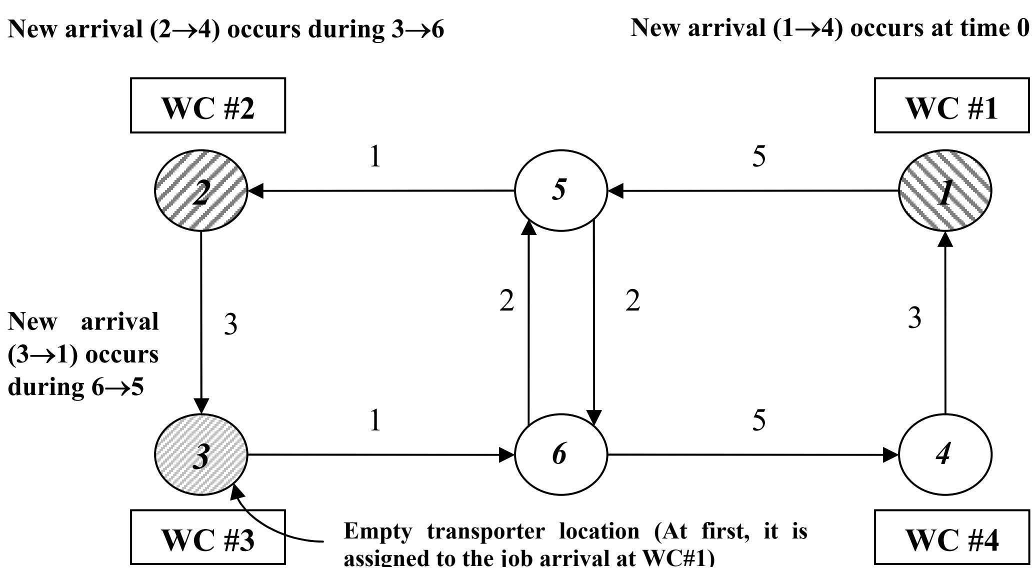

Figure 23. The example layout configuration for the fixed-route unidirectional material-handling system.

Assume that we have a small material-handling system layout as shown in Figure 21 and there is one two-load transporter in the system. The numbers on arcs denote the distances between nodes.

Now, suppose we have the situation described in Figure 21. At time 0, a new job arrival occurs at workcenter 1 with destination workcenter 4, and suppose the job sees an empty transporter located at workcenter 3. Then, the job is assigned to the transporter and the transporter starts to move to workcenter 1. During the transporter's empty trip from workcenter 3 to Node 6 to pick the job at workcenter 1, another job arrival occurs at workcenter 2. Then, another job arrival occurs at workcenter 3 before the transporter reaches workcenter 2. Then, we have the following results from two different dispatching policies.

Original NN heuristic dispatching procedure for our example problem:

- Initialize T = Workcenter 3 and set the transporter's available capacity, C, to 2.
- New arrival occurs at Workcenter 1. Thus, we have **Q** = $\{WC_1(p)\}$.
- Select and remove $WC_1(p)$ from **Q**. That is, assign this job to the transporter. Set C = 2 – 1 = 1. Since this is a pickup job, add the destination of this job, $WC_4(d)$, into **Q**. Now, we have **Q** = $\{WC_4(d)\}$.
- New arrival occurs at Workcenter 2 during route Workcenter 3 $\rightarrow$ Node 6. Update **Q** = $\{WC_2(p), WC_4(d)\}$ and set T = Node 6.

- Sort **Q** according to the distances from Node 6.

  Select and remove $WC_2(p)$ from **Q** because the distance between Workcenter 2 and Node 6 is shorter than the distance between Workcenter 4 and Node 6.

  Set C = 1 – 1 = 0 and add the destination of the job picked up at Workcenter 2, $WC_4(d)$, into **Q**. Now, we have **Q** = {$WC_4(d)$, $WC_4(d)$}.

- New arrival occurs at Workcenter 3 during route Node 6 → Node 5.

  Update **Q** = {$WC_3(p)$, $WC_4(d)$, $WC_4(d)$}. But, since two jobs are assigned to the transporter (C = 0), the transporter cannot pick up a new job.

  Go to the shortest delivery point, Workcenter 4, to drop-off loads.

- Set T = Workcenter 4 and delete two $WC_4(d)$s from **Q**. Set C = 0 + 2 = 2

  Thus, we have **Q** = {$WC_3(p)$}.

- Select and remove $WC_3(p)$ from **Q** and set C = 2 – 1 = 1. That is, assign this job to the transporter. Since this is a pickup job, add the destination of this job, $WC_1(d)$, into **Q**. Thus, we have **Q** = {$WC_1(d)$}. Set T = Workcenter 3.

- Select and remove $WC_1(d)$ from **Q** and set C = 1 + 1 = 2. Set T = Workcenter 1.

  Since we have **Q** = ∅, Stop.

Modified NN heuristic dispatching procedure for our example problem:

- Initialize T = Workcenter 3 and set C = 2.
- New arrival occurs at Workcenter 1. Thus, we have **Q** = {$WC_1(p)$}.

- Select and *temporarily* remove $WC_1(p)$ from **Q**.

  Do not assign this job to the transporter at this point.

- New arrival occurs at Workcenter 2 during route Workcenter 3 → Node 6. Add the *temporarily* removed $WC_1(p)$ back into **Q**. Set T = Node 6.

  Thus, we have **Q** = {$WC_1(p)$, $WC_2(p)$}.

  Sort **Q** according to the distances from Node 6.

  Select and *temporarily* remove $WC_2(p)$ from **Q** because the distance between Workcenter 2 and Node 6 is shorter than the distance between Workcenter 1 and Node 6.

- New arrival occurs at Workcenter 3 during route Node 6 → Node 5.

  Add the *temporarily* removed $WC_2(p)$ back into **Q**. Set T = Node 5. Thus we have **Q** = {$WC_1(p)$, $WC_2(p)$, $WC_3(p)$}.

  Sort **Q** according to the distances from Node 5.

  Select and *temporarily* remove $WC_2(p)$ from **Q** because the distance between Workcenter 2 and Node 5 is the shortest one.

- At Workcenter 2, *permanently* remove $WC_2(p)$. **That is, assign this job to the transporter.** Set T = Workcenter 2 and add the destination of the job picked at Workcenter 2, $WC_4(d)$, into **Q**. Thus, we have **Q** = {$WC_1(p)$,$WC_3(p)$,$WC_4(d)$}.

  Set C = 2 – 1 = 1.

- At Workcenter 3, *permanently* remove $WC_3(p)$ from **Q**. Set T = Workcenter 3 and add the destination of the job picked at Workcenter 3, $WC_1(d)$, into **Q**. Thus, we have **Q** = {$WC_1(p)$,$WC_1(d)$,$WC_4(d)$}. Set C = 1 – 1 = 0.

- Since two jobs are assigned to the transporter, go to the shortest delivery point, Workcenter 4. Set T = Workcenter 4 and delete $WC_4(d)$ from **Q**.

  Now, we have **Q** = {$WC_1(p)$, $WC_1(d)$}. Set C = 0 + 1 = 1.

- At Workcenter 1, *permanently* remove $WC_1(p)$ and $WC_1(d)$.

  Set T = Workcenter 1 and set C = 1 + 1 – 1 = 1.

  Add the destination of the job picked at Workcenter 1, $WC_4(d)$, into **Q**.

  Thus, we have **Q** = {$WC_4(d)$}.

- Select and *permanently* remove $WC_4(d)$ from **Q**. Set T = Workcenter 4 and set C = 1 + 1 = 2. Since we have **Q** = ∅, Stop.

Resulting tours and total distances traveled from both procedures are summarized in Table 18. As we can see, the number of steps required to complete the trip for the modified NN procedure is smaller than that in original NN procedure. Also, the total distance traveled has decreased. Note that, in modified NN procedure, pick-up and drop-off jobs can be done at the same time in workcenter 1 (see ❶ in Table 18).

	Steps of Trip	Distance
Original NN	3→6→5→**2**→3→6→**4**→**1**→5→2→**3**→6→**4**→**1**	34
Modified NN	3→6→5→**2**→**3**→6→**4**→**❶**→5→6→**4**	28

Table 18. The comparison of the number of steps and distances traveled for the original NN heuristic procedure and the modified NN heuristic procedure.

5.4 Simulation Results

To compare these two procedures more fully, the AutoMod 11.1 (Banks 2004) simulation software is used to evaluate the system performance under two different vehicle dispatching procedures described in the precious section. The capacity of transporter assumed to be two and there is only one transporter in the system. Now, reconsider the circuit network example layout shown in Figure 23. We assumed that job arrivals to each workcenter are exponentially distributed with mean inter-arrival time units of 15 and the throughput rates of all workcenters are the same. Table 19 displays all 12 routes in sequential workcenters (nodes) visited. The first node and the last node denote the job generating workcenter and the job departure workcenter, respectively. In addition, new job arrivals at each job generating workcenter are independent of each other. We use pre-specified Time Limit whose value is 3.3 time units to give priority to the qualified jobs.

route/ steps	1	2	3	4	5	6	7	8	9	10	11	12
1	*1*	*1*	*1*	*2*	*2*	*2*	*3*	*3*	*3*	*4*	*4*	*4*
2	*5*	*5*	*5*	*3*	*3*	*3*	*6*	*6*	*6*	*1*	*1*	*1*
3	*2*	*2*	*6*	*6*		*6*	*4*	*5*	*4*		*5*	*5*
4		*3*	*4*	*4*		*4*	*1*	*2*			*2*	*2*
5				*1*								*3*

Table 19. Routes generated for the fixed-route unidirectional material-handling problem of Figure 23.

Simulation models were developed for the two different dispatching policies and the results are shown in Table 20. Here, the simulation results under the original NN dispatching procedure are compared with those from the revised procedure. The run length is 1,000,000 time units with a statistical reset at 100,000 time units.

	WIP_{sys}	Cycle Time	Transporter Utilization
Original NN	1.59	1.991	0.753
Modified NN	1.35	1.667	0.727

Table 20. The comparison of simulation results of the original NN heuristic procedure and the modified NN heuristic procedure for a non-outlier example model in Figure 23.

As we can see in Table 20, WIP_{sys} and cycle time from modified NN procedure are less than those from original NN procedure. Thus, from the results shown in Table 20, we can conclude that the modified NN heuristic dispatching procedure performs better than the original NN procedure for the example layout problem in Figure 23.

Now, consider another example system that has an outlier. As mentioned earlier, an outlier is a workcenter (node in the network) that is so far from every other workcenter that it is never the closest workcenter when more than one workcenter has jobs awaiting transportation. From the system layout configuration shown in Figure 24, we can see that workcenter 1 is an outlier in this example. Again, all external job arrivals at each workcenter are assumed to be exponentially distributed with mean inter-arrival time units of 15.

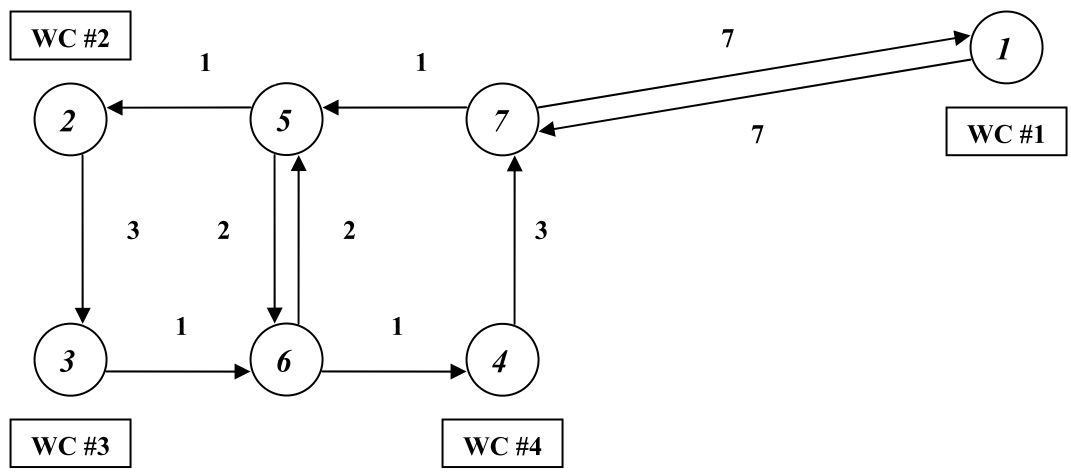

Figure 24. The layout configuration for the system with an outlier.

Table 21 displays the 12 routes for this outlier model. Due to the structure of this model, if we don't have the priority option such as Time Limit, a new job arrival at workcenter 1 cannot be picked up if there are two or more job requests in the system and a transporter with available capacity is located at different workcenters. We use the pre-specified Time Limit to send transporters to workcenter 1 to prevent a long queue length at that workcenter. Again, the pre-specified Time Limit is set to be 3.3 time units.

route/ steps	1	2	3	4	5	6	7	8	9	10	11	12
1	*1*	*1*	*1*	*2*	*2*	*2*	*3*	*3*	*3*	*4*	*4*	*4*
2	*7*	*7*	*7*	*3*	*3*	*3*	*6*	*6*	*6*	*7*	*7*	*7*
3	*5*	*5*	*5*	*6*		*6*	*4*	*5*	*4*	*1*	*5*	*5*
4	*2*	*2*	*6*	*4*		*4*	*7*	*2*			*2*	*2*
5		*3*	*4*	*7*			*1*					*3*
6				*1*								

Table 21. Routes generated for the example problem with an outlier of Figure 24.

From the simulation runs, we obtained the results shown in Table 22. Again, the system performance measures under the revised NN dispatching procedure are compared with those under original procedure. Again, the run length is 1,000,000 time units with a statistical reset at 100,000 time units.

	WIP_{sys}	Cycle Time	Transporter Utilization
Original NN	2.46	3.075	0.843
Modified NN	1.97	2.466	0.817

Table 22. The comparison of simulation results of the original NN heuristic procedure and the modified NN heuristic procedure for an outlier example model in Figure 24.

As we can see in Table 22, WIP_{sys} and cycle time under the modified NN heuristic dispatching procedure are smaller than those under the original NN heuristic procedure. Thus, we can say that the modified NN heuristic dispatching procedure performs better than the original NN procedure for the outlier example layout problem also.

Throughout this chapter, we assumed that the transporter capacity was two jobs. Now, consider the question as to how the system performance varies with respect to transporter capacity? This question is analyzed for both control procedures using the outlier example shown in Figure 24. The graphs shown in Figures 25 ~ 27 display the system performance measures for a multi-load transporter as the transporter capacity increases. In Figures 25, 26 and 27, cycle time, system WIP and transporter utilization, respectively, are compared for both control procedures. In Figures 25 ~ 27, two important observations should be noted. First, after the transporter capacity reaches five, additional capacity of the transporter doesn't improve the system performance with the same job inter-arrival rates. Second, when the transporter capacity exceeds six, the modified NN dispatching procedure and the original NN procedure exhibit almost the same performance. That is, if a transporter has a very large capacity, then there is little difference between the modified NN procedure and the original NN procedure, because most jobs will be picked up and therefore, the dynamic selection procedure of the modified NN procedure is nullified.

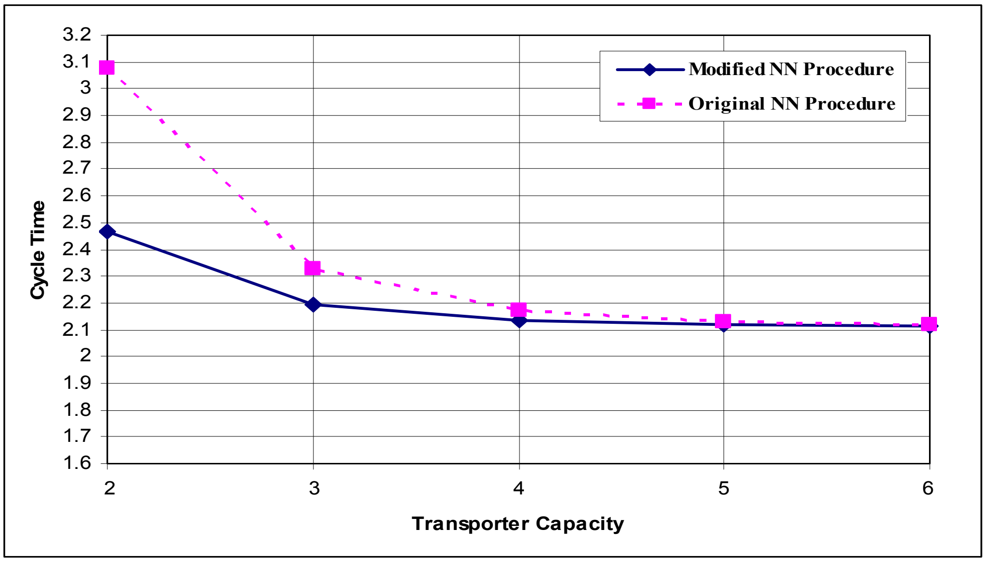

Figure 25. Plots for cycle times under both dispatching procedures for the example system with an outlier.

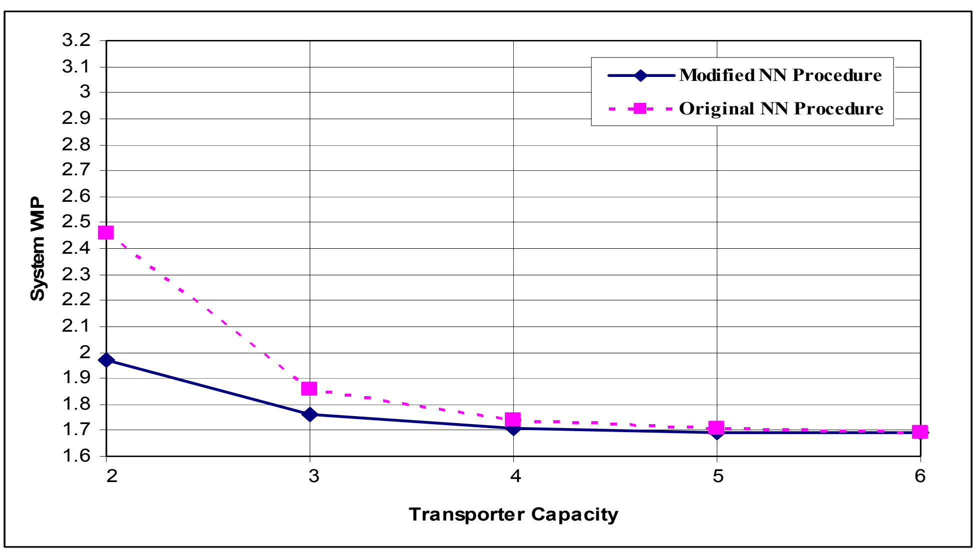

Figure 26. Plots for system WIPs under both dispatching procedures for the example system with an outlier.

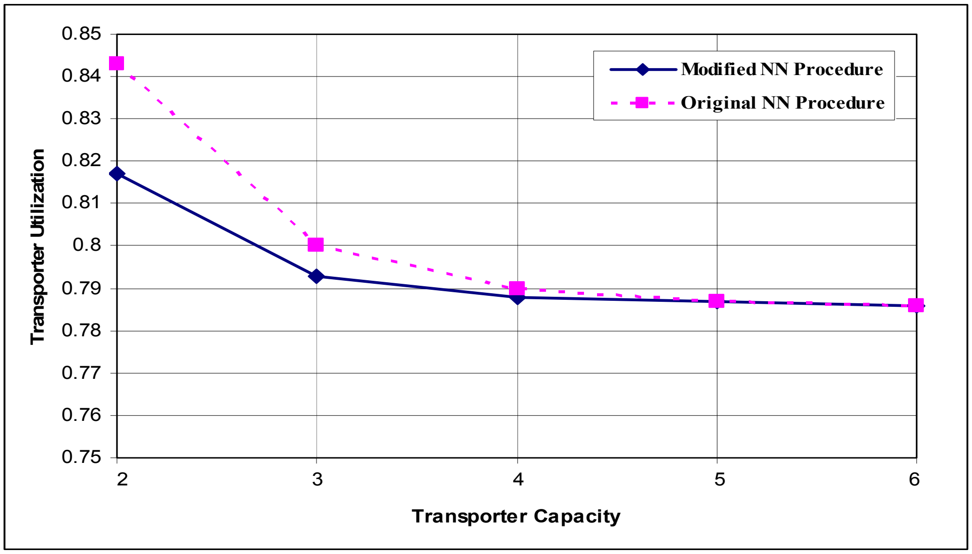

Figure 27. Plots for transporter utilizations under both dispatching procedures for the example system with an outlier.

5.5 Chapter Summary

The study of this chapter shows that incorporating periodic reevaluation of the current material-handling demands into heuristic transporter dispatching procedures can improve the performance of systems with multi-load transporters. By reevaluating system status and reallocating transporter reservation space periodically, we can reduce the unnecessarily reserved transporter capacities and increase the system responsiveness to a dynamic environment. Using a simulation approach, the potential benefits of dynamic reallocation of transporter reservation space is illustrated. As previously stated, even though multi-load transporters can provide many potential benefits over single-load transporters, it seems that extensive research on this area has not been done, especially using analytical modeling approaches. Therefore, further study of these systems with multi-load transporters using an analytical modeling approach should be conducted.

CHAPTER VI

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

In this dissertation, analytical models for material-handling systems with multiple workcenters from both workcenters' perspective and transporters' perspective are considered. In Chapter II, a queueing approximation model is developed from a workcenters' point of view. A job arrival selects an empty transporter by the CTAR to minimize the empty travel time. To develop more accurate analytical models, in Chapter II, a state-dependent transportation time approach was developed. Since the transportation time is the function of system states, the numbers of jobs in the transportation subsystem at the time of the vehicle assignment, we need to consider the state-dependent nature of the transportation times. By considering these state dependencies, we could decrease the percentage error of the performance measures of the system, such as WIP and/or cycle time, by almost 40% compared to results from most previous research.

The main conclusion from Chapter II is that the standard queueing network decomposition approach (Johnson 2001, Benjaafar 2002) must be extended to incorporate dependent queueing node approximations to adequately capture the behavior of the material-handling system. The approach taken in Chapter II to accommodate this strong service time dependency is the development of a Poisson-based model incorporating the service time dependencies and then to generalize to non-Poisson systems via a typical adjustment factor.

In Chapter III, queueing theory based analytical models for material-handling systems from the transporter's point of view were developed. From the transporter's perspective, an empty transporter selects a job request by four different vehicle-initiated vehicle dispatching rules. The guidance for developing different generator matrices according to different vehicle-initiated vehicle dispatching rules was investigated in the chapter. For the analytical description of impacts of these dispatching rules, we also developed different generator matrices and, using those generator matrices, we were able to obtain very accurate analytical results for those four different dispatching policies for our example problems. We found that the best dispatching policy is the SDR control scheme and the least efficient dispatching policy is the LDR control scheme in terms of the system performance measure WIP_q. TL/SDR and FCFSR control schemes lie in between those two methods. In this chapter, it was also shown that, with the situations such that there is no locking phenomenon in the system, the best transporter dispatching rule in terms of WIP_q for an *M/G*/1 model is SDR.

In Chapter IV, we extended our models developed in Chapter III in two different ways: the extension of the queue length at each workcenter and the extension of the number of transporters in the system. Again, by considering state-dependent nature of the transportation time, we can obtain very accurate analytical results in both cases. The more we increase the queue space at each workcenter, the fewer the lost arrivals to the system. However as we increase the queue space, the computational burden increases very quickly. Therefore, by differentiating queue space additions according to arrival rates at workcenters, we can reduce lost arrivals to the system with smaller state-space size. When we have two or more transporters in the system, we need both workcenter-initiated dispatching rules and vehicle-initiated dispatching rules. That is, a job arrival which sees two or more empty transporters needs to select an empty transporter according to a workcenter-initiated dispatching rule and an empty transporter which sees two or more job requests has to choose a job request according to a vehicle-initiated dispatching rule.

In Chapter V, from Nearest Neighbor (NN) dispatching policy proposed by Tanchoco and Co (1994), we developed a revised dispatching policy and compared the system performance measures, such as WIP or cycle time from both dispatching procedures. From simulation experiments for two example problems, we conclude that the modified NN heuristic dispatching procedure performs better than the original NN procedure. That is, we can reduce the unnecessarily reserved transporter capacities and increase the system responsiveness to a dynamic environment by periodic reevaluation of the system status and reallocation of transporter reservation space.

6.2 Future Work

When we developed the queueing approximation model in Chapter II, the system exhibits fast growth in the computation burden as the number of transporters increases. Thus, there is certainly a need for research into efficient dependent service time approximations in the spirit of the Pollaczek-Khintchine (Gross and Harris 1998) and Allen-Cunneen (Allen 1990: 341) formulas. The queueing approximation models developed in Chapter III have queue space limits set at one for each workcenter and there is only one transporter in the system. If we increase the queue space or the number of transporters for the system, then the number of states of the resulting analytical models increase very quickly. Therefore, since this rapid growth of the number of system states as the queue space length or number of transporters increase causes rapid increase in the computational complexity, future research is needed to develop efficient dependent service time queueing approximations for these systems.

To obtain the parameter values of the Gamma distribution adjustments for the TL/SDR control scheme of Chapter III, we performed preliminary simulation runs and used results from those runs. Since, we don't have a method at this time for predicting the parameter values of the Gamma distribution adjustments for the TL/SDR control scheme, further research is needed to develop an analytical methodology for predicting the parameter values of the Gamma distribution adjustments for the TL/SDR control scheme. However, since the analytical models using these Gamma adjustments yield very good estimates for the system performance measures, this study is a first step in analytically describing the impact of TL/SDR control schemes for material-handling systems.

As stated in Chapter V, in spite of many potential benefits of multi-load transporters, research on this area using analytical modeling approach is quite limited. Therefore, further study of these systems with multi-load transporters using an analytical modeling approach is needed. Also, more intelligent and efficient dispatching policies for multi-load transporters might be developed.

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APPENDIX A

THE PROOF OF THE THEOREM SHOWN IN CHAPTER III

Theorem: With the situations such that there is no locking phenomenon in the system, the best transporter dispatching rule in terms of WIP_q for an *M/G/*1 model is SDR**.**

Proof:

P.1 Assumptions

Since a transporter becomes available to be assigned when it is released from the previous service at the job delivery point, the transporter initiated dispatching rules, like SDR, can be greatly affected by the facility layout and the location of workcenters. Therefore, a locking phenomenon can occur when we use the SDR control scheme. According to Egbelu and Tanchoco (1984), if the locations of some workcenters in the system are not the closest workcenter from any other workcenters in the system (we call these workcenters outliers), then those workcenters will seldom be selected for picking up jobs by an empty transporter. Thus, the number of jobs waiting at those workcenters will increase and can eventually reach its maximum queue capacity. As the result, job delivery to those workcenters becomes impossible. Egbelu and Tanchoco called this phenomenon the locking phenomenon.

Our original model under the SDR control scheme shown in Section 3.3.1 has no outliers. Moreover, we assumed that the queue space at each workcenter is limited to one. Therefore, we don’t have the locking phenomenon mentioned above and we don't need any assumptions to process the proof shown in Section P.2. However, if we generalize the model by including an outlier in our model as described in Section 3.5.1 and allow an infinite queue space limit at each workcenter, then we need to check the following two conditions. Let λ_i be the external rates at the workcenter *i* and P_{ij} be the probability that a job at the workcenter *i* goes to the workcenter *j*. For example, if jobs at the workcenter 1 go to the workcenter 2, 3 or 4 with equal probabilities, then we have $P_{12} = P_{13} = P_{14} = 1/3$. Also, let $E[ETT_{ij}]$ be the average empty transporter travel time from workcenter *i* to workcenter *j* and let $E[LTT_{ij}]$ be the average loaded transporter travel time form workcenter *i* and workcenter *j*. Note that we have $E[ETT_{jj}] = 0$ *for* $j = 1,\ldots,4$. Then, to insure the nonexistence of the locking phenomenon in the system with an outlier and the infinite queue space at each workcenter, we need to have either one of the following two relationships for some outlier workcenter *j*:

$$\sum_{k=1}^{4}\alpha_{kj}\left(\sum_{i\neq k}^{4}\lambda_i P_{ik}\right)=\alpha_{1j}\left(\sum_{i\neq 1}^{4}\lambda_i P_{i1}\right)+\alpha_{2j}\left(\sum_{i\neq 2}^{4}\lambda_i P_{i2}\right)+\alpha_{3j}\left(\sum_{i\neq 3}^{4}\lambda_i P_{i3}\right)+\alpha_{4j}\left(\sum_{i\neq 4}^{4}\lambda_i P_{i4}\right)\geq\lambda_j\,, \tag{A.1}$$

$$\begin{aligned}1-\rho=1-\left(\sum_{i=1}^{4}\lambda_i\right)E[T]>\lambda_j\Bigg\{&\left(\frac{\sum_{i\neq 1}^{4}\lambda_i P_{i1}}{\sum_{i=1}^{4}\lambda_i}\right)\left(E[ETT_{1j}]+\sum_{k\neq j}^{4}E[LTT_{jk}]P_{jk}\right)+\left(\frac{\sum_{i\neq 2}^{4}\lambda_i P_{i2}}{\sum_{i=1}^{4}\lambda_i}\right)\left(E[ETT_{2j}]+\sum_{k\neq j}^{4}E[LTT_{jk}]P_{jk}\right)\\&+\left(\frac{\sum_{i\neq 3}^{4}\lambda_i P_{i3}}{\sum_{i=1}^{4}\lambda_i}\right)\left(E[ETT_{3j}]+\sum_{k\neq j}^{4}E[LTT_{jk}]P_{jk}\right)+\left(\frac{\sum_{i\neq 4}^{4}\lambda_i P_{i4}}{\sum_{i=1}^{4}\lambda_i}\right)\left(E[ETT_{4j}]+\sum_{k\neq j}^{4}E[LTT_{jk}]P_{jk}\right)\Bigg\}\,.\end{aligned} \tag{A.2}$$

In inequality (A.1), α_{kj}'s are the probabilities that an empty transporter will go to the workcenter j from the workcenter k and consists of two components, $\alpha_{kj}^{\mathbf{1}}$ and $\alpha_{kj}^{\mathbf{2}}$. That is, $\alpha_{kj} = \alpha_{kj}^{\mathbf{1}} + \alpha_{kj}^{\mathbf{2}}$, where $\alpha_{kj}^{\mathbf{1}}$ is the probabilities that an empty transporter will go to the workcenter j from the workcenter k when there is only one job request in the system and $\alpha_{kj}^{\mathbf{2}}$ denotes the same probability when there are two or more job requests in the system. And $\sum_{i\neq k}^{4} \lambda_i P_{ik}$ denotes the probability of the external arrivals at all workcenters (except workcenter k) in the system that arrive at the workcenter k. Note that if workcenter 4 is an outlier in the system under the SDR control scheme, then we have $\alpha_{14}^{\mathbf{2}} = \alpha_{24}^{\mathbf{2}} = \alpha_{34}^{\mathbf{2}} = 0$ and $\alpha_{44}^{\mathbf{2}} \neq 0$. Therefore, for the right hand side of (A.1), we have:

$$\alpha_{1j}^{1}\left(\sum_{i\neq 1}^{4}\lambda_i P_{i1}\right) + \alpha_{2j}^{1}\left(\sum_{i\neq 2}^{4}\lambda_i P_{i2}\right) + \alpha_{3j}^{1}\left(\sum_{i\neq 3}^{4}\lambda_i P_{i3}\right) + (\alpha_{4j}^{1} + \alpha_{4j}^{2})\left(\sum_{i\neq 4}^{4}\lambda_i P_{i4}\right).$$

Note that since the job assignments of empty transporters are determined by the transporter dispatching rules, these α_{kj}'s are heavily depend on the various transporter dispatching rules. The right hand side of inequality (A.2) is the transporter idle fraction, i.e., one minus the transporter utilization, and the left hand side of inequality (A.2) is the transporter service time generated utilization for the outlier workcenter j. We can rewrite inequality (A.2) as follows:

$$1-\rho > \lambda_j E[\textit{Time for Servicing Outlier } j].$$

If inequality (A.1) is valid, this insures that the internal job arrival rate into the outlier workcenter *j* (the empty transporter available rate at the workcenter *j*) is greater than or equal to the external job arrival rates at the outlier workcenter *j*. Furthermore, by inequality (A.2), we insure that the transporter idle fraction can cover the percentage of the transporter time required to serve jobs arrived at the outlier workcenter *j*. Now, suppose that neither inequality is true, then,

$$\lambda_j - \sum_{k=1}^{4} \alpha_{kj} \left(\sum_{i \neq k}^{4} \lambda_i P_{ik} \right) > 0 \tag{A.3}$$

and

$$1-\rho \le \lambda_j E[\textit{Time for Servicing Outlier } j], \text{ for some outlier workcenter } j. \tag{A.4}$$

Note that inequality (A.3) implies that the external arrival rate of jobs at the outlier workcenter *j* is bigger than the internal arrival rate of jobs at the outlier workcenter *j*, where the internal job arrival rate outlier workcenter *j* results in empty transporters being available at the outlier workcenter *j*. Therefore, the transporters that become available at the outlier workcenter *j* cannot satisfy the external job arrivals at that workcenter *j*. By multiplying $E[\textit{Time for Servicing Outlier } j] > 0$ on both sides of above inequality (A.3), we obtain, for some outlier workcenter *j*,

$$\left\{\lambda_j - \sum_{k=1}^{4}\alpha_{kj}\left(\sum_{i\neq k}^{4}\lambda_i P_{ik}\right)\right\} E[\mathit{Time\ for\ Servicing\ Outlier\ j}] > 0. \qquad \text{(A.5)}$$

Inequality (A.5) implies that the required net transporter service time generated utilization for the outlier workcenter j is positive and that must be taken care of to avoid a system explosion. By subtracting:

$$\sum_{k=1}^{4}\alpha_{kj}\left(\sum_{i\neq k}^{4}\lambda_i P_{ik}\right) E[\mathit{Time\ for\ Servicing\ Outlier\ j}]$$

from the right hand side of inequality (A.4), we can have the following two possible cases. First, if we have:

$$\left\{\lambda_j - \sum_{k=1}^{4}\alpha_{kj}\left(\sum_{i\neq k}^{4}\lambda_i P_{ik}\right)\right\} E[\mathit{Time\ for\ Servicing\ Outlier\ j}] \geq 1-\rho \ , \qquad \text{(A.6)}$$

then the queue length of the outlier workcenter j increases and reaches its maximum capacity because the transporter idle time proportion cannot satisfy the positive net demands for empty transporters from the outlier workcenter j. Thus, the system will eventually become unstable. Second, if we have:

$$\left\{\lambda_j - \sum_{k=1}^{4}\alpha_{kj}\left(\sum_{i\neq k}^{4}\lambda_i P_{ik}\right)\right\} E[\mathit{Time\ for\ Servicing\ Outlier\ j}] < 1-\rho \ ,$$

then, by inequality (A.4) and (A.5), we can get the following inequality:

$$\lambda_j E[\mathit{Time\ for\ Servicing\ Outlier\ j}] \geq 1-\rho$$

$$> \left\{\lambda_j - \sum_{k=1}^{4}\alpha_{kj}\left(\sum_{i\neq k}^{4}\lambda_i P_{ik}\right)\right\} E[\mathit{Time\ for\ Servicing\ Outlier\ j}] > 0. \quad \text{(A.7)}$$

In this case, since the transporter idle fraction, $1-\rho$, can normally cover the positive net demands for empty transporters from the outlier workcenter *j*, the system will not be unstable. However, some realizations might be unstable due to the locking phenomenon when the transporter idle fraction become very close to $\lambda_j E[$*Time for Servicing Outlier j*]. If the internal job arrival rate at the outlier workcenter *j* decreases to almost zero, then the right hand side of inequality (A.7) becomes $\lambda_j E[$*Time for Servicing Outlier j*]. Thus, $1-\rho$ will be squeezed down to $\lambda_j E[$*Time for Servicing Outlier j*]. Therefore, inequalities (A.1) and (A.2) are upper bounds and inequality (A.6) is a lower bound for the nonexistence of the locking phenomenon. And inequality (A.7) is the area lies in between those upper and lower bounds.

P.2 Proof

From the result of the previous section, we can assume that the situation is such that no locking phenomenon occurs. That is, we can select the system parameters that put us in the proper regions. Now, under the SDR control scheme, we can get the minimum value of the empty transporter travel time because an empty transporter will select the closest workcenter from its current location as its destination. Thus, we can say that, in the long run, the average empty transporter travel time under the SDR control scheme, $E[ETT_{SDR}]$, is always less than or equal to the average empty transporter travel time under any other dispatching rules, $E[ETT_{NONSDR}]$. That is, $E[ETT_{SDR}] \leq E[ETT_{NONSDR}]$ for all non-SDR control schemes. Note that the expected service time, $E[T]$, has three components, that is,

$$E[T] = E[ETT] + E[LTT] + E[PDT],$$

where $E[LTT]$ is the average loaded transporter travel time and $E[PDT]$ is the average job pick-up/drop-off time, and both $E[LTT]$ and $E[PDT]$ have no relationship with the specific transporter dispatching rules. If $E[ETT]$ increases while the other factors, $E[LTT]$ and $E[PDT]$, remain the same, then $E[T]$ will also increase. Thus, the followings are true:

$$E[T_{SDR}] \leq E[T_{NONSDR}]$$

or

$$\mu_{SDR} \geq \mu_{NONSDR}$$

or

$$\rho_{SDR} \leq \rho_{NONSDR}\text{, for all non-SDR control schemes.} \tag{A.8}$$

Now, suppose that the SDR control scheme is not always the best dispatching rule in terms of WIP_q. Then we have $\text{WIP}_q^{SDR} > \text{WIP}_q^{NONSDR}$ for some non-SDR control schemes. Therefore, for an *M/G*/1 model, we can get the followings:

$$\text{WIP}_q^{SDR} = \frac{\rho_{SDR}^2 + \lambda^2 \cdot \sigma_s^2}{2(1-\rho_{SDR})} > \frac{\rho_{NONSDR}^2 + \lambda^2 \cdot \sigma_s^2}{2(1-\rho_{NONSDR})} = \text{WIP}_q^{NONSDR},$$

$$2\rho_{SDR}^2 \cdot (1-\rho_{NONSDR}) > 2\rho_{NONSDR}^2 \cdot (1-\rho_{SDR}),$$

$$\rho_{SDR}^2 - \rho_{SDR}^2 \cdot \rho_{NONSDR} - \rho_{NONSDR}^2 + \rho_{NONSDR}^2 \cdot \rho_{SDR} > 0,$$

$$(\rho_{SDR} + \rho_{NONSDR}) \cdot (\rho_{SDR} - \rho_{NONSDR}) - \rho_{SDR} \cdot \rho_{NONSDR}(\rho_{SDR} - \rho_{NONSDR}) > 0,$$

$$(\rho_{SDR} - \rho_{NONSDR}) \cdot (\rho_{SDR} + \rho_{NONSDR} - \rho_{SDR} \cdot \rho_{NONSDR}) > 0,$$

$$(\rho_{SDR} - \rho_{NONSDR}) \cdot \{\rho_{SDR} \cdot (1-\rho_{NONSDR}) + \rho_{NONSDR}\} > 0.$$

Since the second portion of the above equation, $\{\rho_{SDR} \cdot (1-\rho_{NONSDR}) + \rho_{NONSDR}\}$, is always positive, we have finally the following result:

$$(\rho_{SDR} - \rho_{NONSDR}) > 0$$

or

$$\rho_{SDR} > \rho_{NONSDR}\,, \text{ for some non-SDR control schemes.} \tag{A.9}$$

But inequality (A.9) contradicts with inequality (A.8). Hence, we can say that $\mathrm{WIP}_q^{SDR} \leq \mathrm{WIP}_q^{NONSDR}$ for all non-SDR control schemes for an *M/G/*1 model with the assumption that there is no locking phenomenon in the system. This completes the proof. ■

VITA

Name: Moonsu Lee

Permanent Address: Samho Garden APT. #7−903, Banpo−Dong, Socho−Gu,

Seoul, Korea, 137−041

Educational Background: B.S. 1994. Department of Industrial Engineering

Hanyang University

M.S. 2001. Department of Industrial Engineering

Texas A&M University