Staying the Course or Rolling the Dice:
Time Horizon's Effect on the Propensity to Take Risk

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Abstract

This paper analyzes a model of dynamic decision making under uncertainty, where in each period the decision maker (DM) is faced with a choice between accepting a known level of benefit inherited from the last period ("staying the course") and drawing from a random benefit distribution ("rolling the dice"). It shows that the DM's propensity to roll the dice decreases as it gets closer to the ending period, therefore providing a justification for the conventional wisdom that the young would take more risk than the old. Other results are also derived and compared with those from the standard model of decision making under uncertainty. These include that it is possible for a risk averse DM with an inherited benefit level greater than the mean of the random benefit distribution to prefer rolling the dice to staying the course, and that it is also possible for a risk averse DM to be more prone to risk taking as the underlying benefit distribution becomes riskier.

Keywords: Time horizon; Risk taking; Dynamic decision making

1. Introduction

Do the young take more risk than the old? The conventional wisdom says yes because, intuitively, the young have a longer time horizon to recover from any bad outcome of risk taking. Economists have examined this conventional wisdom within models of both financial and non-financial risk taking. However, the existing theoretical results and empirical evidence either are inconclusive or point to the opposite of the conventional wisdom.

Liu and Rettenmaier (2007) find that as an exogenous mortality risk increases, which can be interpreted as a shorter horizon, the individual will take more other mortality risk, as well as more financial risk. This theoretical result is the opposite of the conventional wisdom that older people – who have higher mortality risk due to their age – would have a lower propensity to take risk. However, the result of Liu and Rettenmaier is consistent with the empirical evidence. For example, Dow et al. (1999) find that the United Nations' mortality-reducing Expanded Programme on Immunization (EPI) positively affects newborn birthweight (an indicator of the mother's investment in the baby), and hence reduces mortality risks associated with low birthweight. Ganz (2000) finds that the higher the chances an individual may die from neighborhood violence, the more likely he or she will become a smoker with a higher risk of having lung cancer or other diseases associated with smoking. Picone et al. (2004) find that individuals with higher expected longevity are more likely to undergo cancer screening that reduces the mortality risk caused by cancer. [1]

[1]In the same vein, the Liu and Rettenmaier result provides an economic rationale for some folklore stories that soldiers are more likely to engage in gambling and risky personal behavior during wartime and that terminal illness diagnoses often trigger new hobbies for a dangerous sport or activity. Further, the Liu and Rettenmaier result is also consistent with the related theoretical finding of Eeckhoudt and Hammitt (2001) that as a background mortality risk increases, the marginal willingness to pay for the reduction in an independent mortality risk will decrease.

The conventional wisdom has also been examined within the context of financial risk taking. In (dynamic) portfolio choice models, an individual's propensity to take risk is naturally measured by the portion of his savings invested in risky stocks, as opposed to (relatively) riskless bonds. Therefore, the question becomes whether the portion of stocks in the individual's investment portfolio will become smaller as he grows older. While this portfolio path is the one recommended by almost all financial advisors, it has received very limited support from the economics literature. Using a benchmark investment model with many simplifying assumptions,[2] Samuelson (1969) and Merton (1969) show that the optimal weight on stocks in one's portfolio is the same regardless of the length of remaining life. From there, different, and sometimes opposing, results are obtained by relaxing various assumptions made by Samuelson and Merton.[3] Perhaps the most critical argument against the conventional wisdom, as it is represented in the framework of dynamic portfolio decision, is the empirical evidence that younger investors actually hold a smaller portion of risky assets in their portfolios (Guiso et al. 1996 and Ameriks and Zeldes 2001).

In this paper, we propose and analyze a non-financial model of dynamic decision making under uncertainty in order to provide a clear-cut justification, which has been lacking in both financial and non-financial risk taking models, for the conventional wisdom that the young would take more risk than the old. Specifically, suppose that at the beginning of each period a decision maker (DM) is faced with a choice between accepting a known level of benefit inherited

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[2] These assumptions include that stock returns in different periods are i.i.d., that there is no labor income or non-tradable assets, that lifetime preferences are additively separable in time, and that the utility function in each period displays CRRA.

[3] Bodie et al. (1992) and Jagannathan and Kocherlakota (1996) introduce riskless labor income and show that the weight on stocks should decrease over time. On the other hand, Bommier and Rochet (2006), by allowing consumption in different periods to be substitutable, show that the weight on stocks should increase over time. Somewhere in between, and by relaxing the CRRA assumption on the utility function, Gollier and Zeckhauser (2002) find necessary and sufficient conditions for younger investors to take more financial risk.

from the last period ("staying the course") and drawing from a random benefit distribution ("rolling the dice"). Under the staying the course alternative, both the current-period consumption and the inherited benefit level in the next period equal the inherited level of benefit in this period. By taking the rolling the dice alternative, on the other hand, the DM draws a benefit level from a random distribution, and at the same time incurs a search cost c and gives up the inherited benefit level. Therefore, the current-period consumption is equal to the drawn level of benefit minus c, and the inherited benefit level in the next period is equal to the drawn level of benefit itself. The choice is made in every period with the final decision being made in a predetermined ending period.

This model of dynamic decision making under uncertainty captures some common features of many real-life decision problems such as career choice, location choice, entrepreneurial exploitation and formation of an exclusive relationship, where the DM is faced with a choice in every period between a status quo with (relative) certainty and an uncertain alternative drawn from a distribution. For example, when a worker considers his career at any point in life, his choice is between staying with an existing job and searching for a new job. The existing and the new job each yield a stream of benefit (income) flows. The difference between them is that the benefit level of the existing job is known whereas the benefit level of the new job is not fully known at the time of making choice. As another example, a person in an exclusive relationship can choose to be contended with the current relationship or to search for a new one. The present and the new relationship each yield a stream of benefit flows. Again, the benefit level provided by the current relationship is known whereas the benefit level provided by the new relationship is not fully known at the time of making the choice.

For each period, a critical benefit level is defined as the inherited benefit level with which the DM would be indifferent between staying the course and rolling the dice. The DM prefers rolling the dice to staying the course if the critical benefit level for a period is larger than the actual inherited benefit level in that period. In this sense, the critical benefit level in a period measures the DM's propensity to take risk (to roll the dice) in that period.

We find that the DM's propensity to take risk, as measured by the critical benefit level, decreases over time. This finding thus provides a justification for the conventional wisdom that the young would take more risk than the old. The reason for this is that the young have ahead of them a larger number of periodic opportunities of drawing from a random benefit distribution with an option to stay the course each time. A younger DM is more likely to choose the rolling the dice alternative because a good outcome from rolling the dice can be kept for a longer time while a bad outcome can potentially be erased in the next period by rolling the dice again.

The main difference between the dynamic portfolio choice model and the model here is that in the portfolio model, a DM does not have the option to stay the course (i.e., to lock in a good portfolio performance realized in the previous period), and is forced to face the uncertainty of the stock market in each and every period. On the other hand, the main difference between the Liu and Rettenmaier (2007) model – which can be used to explain the empirical evidence on the time horizon's effect on non-financial risk taking that is opposite of the conventional wisdom – and the model here is that in the Liu and Rettenmaier model, a DM does not have repeated future opportunities of drawing from a random benefit distribution, and is forced to accept the outcome from risk taking at a given time in life, whether it is good or bad, for his remaining life.

The dynamic model of risk taking in this paper also generates some other interesting findings, which stand in sharp contrast to those from the standard model of decision making under uncertainty. First, even for a risk averse DM, the critical benefit level in a period, except for the ending period, may be larger than the mean of the random benefit distribution from which the DM draws. In other words, it is possible for a risk averse DM with an inherited benefit level greater than the mean of the random benefit distribution to prefer rolling the dice to staying the course. This is due to the future opportunities of drawing from the random benefit distribution with the option to stay the course, which could erase a bad draw, but make a good draw persist.

Second, it is also possible that, even for a risk averse DM, the critical benefit level in a period, except for the ending period, increases as the riskiness of the random benefit distribution increases. Because the critical benefit level in a period indicates the propensity to take risk in that period, this means that a risk averse DM may be more prone to risk taking as the underlying distribution gets riskier. The intuition behind this result is that even though a riskier underlying distribution reduces a risk averse DM's expected utility from the current-period consumption under the rolling the dice alternative, it increases the future expected utilities from rolling the dice now because good draws that you want to keep become better, and bad draws, although becoming worse, could be erased by drawing again in the next period.

Our model is related to the multi-armed bandit problems in applied mathematics. The multi-armed bandit problems are a large class of problems that analyze sequential choices among a number of options. Different restrictions are made in these problems in order to gain insight into some specific aspects of dynamic decision making (Basu et al. 1990 and Jun 2004). The restrictions made in our model are that the choice is always between staying with the sure status quo benefit and drawing from a fixed random benefit distribution, and that the benefit drawn in one period becomes the status quo benefit in the next period. These special restrictions, based on some common features of a few real-world dynamic decision problems, are responsible for the clear-cut prediction in our model with respect to the time horizon's effect on the propensity to take risk.

Our model is also related to the job search/shopping problems in labor economics. While our model is similar to a feature in the job search/shopping problems (e.g., Stigler 1961, McCall 1970, Mortensen 1970 and Johnson 1978),[4] namely the choice is between the status quo with a known benefit level and an alternative whose benefits are uncertain, our model adds the dimension of time horizon to the labor market search/shopping model. As the result, we are able to generate the results in this paper that are not the focus of the literature on job search.[5]

The paper is organized as follows. In the next section, we present a model of periodic decisions between staying the course and rolling the dice, and use it to examine the horizon's effect on the propensity to roll the dice. The main result obtained in this section provides a justification for the conventional wisdom that a DM with a longer horizon ahead would be more prone to risk taking. Then in Section 3, we draw further implications of the model. We show that it is possible for a risk averse DM with an inherited benefit level greater than the mean of the random benefit distribution to prefer rolling the dice to staying the course, and that it is also possible for a risk averse DM to be more prone to risk taking as the underlying distribution gets riskier. Finally in Section 4, we provide a brief conclusion.

2. A Model of Periodic Decisions between Staying the Course and Rolling the Dice

Consider a DM who is in a dynamic decision process choosing a job in every period. A job is characterized by a stream of constant benefit flows. Therefore, a job can be represented by

[4] See also Van Den Berg (1990) and Rogerson, Shimer and Wright (2005).

[5] Cubas and Silos (2017) analyze why higher mean wages occur in industries with higher wage volatility. Their model allows them to identify the degree to which these higher expected wages result from either selection or from a risk premium. In the model, individuals choose their industry when they enter the labor market, but they must stay in that industry of the remainder of their careers. Our model, in contrast, allows workers to re-draw from an uncertain distribution in each period to focus on how workers' willingness to draw from the distribution changes as they age.

a certain level of benefit. At the beginning of an arbitrary period t, the DM inherits a level of benefit carried over from period t - 1. The inherited benefit level at the beginning of period t is denoted B_t^*.

The DM's choice at the beginning of period t is between "staying the course" and "rolling the dice". Under staying the course, he sticks to the inherited benefit level B_t^*. As a result, his consumption in period t, C_t, equals B_t^*.[6] In addition, the inherited benefit level in period $t + 1$, B_{t+1}^*, would also equal B_t^*.

Under rolling the dice, the DM draws a benefit level from a nondegenerate random distribution with a CDF $F(B)$ that is defined on a bounded interval $[a, b]$, and at the same time incurs a search cost $c \geq 0$ and gives up the inherited benefit level. Therefore, the period t consumption, C_t, is equal to the drawn level of benefit minus c, and the inherited benefit level in period $t+1$, B_{t+1}^*, is equal to the drawn level of benefit itself. Note that a positive c diminishes the attractiveness of the "rolling the dice" option not only by adding an instant switching cost but also by making getting out of a bad outcome from rolling the dice more costly in the next period.

This decision process repeats in the following periods $t + 1, t + 2, \cdots$, until the predetermined ending period T. For comparability across periods, c and $F(B)$ are both assumed to be the same in every period.

[6] For simplicity, we assume that the benefit from a job or a partner in one period cannot be transferred to another period. This is a reasonable assumption for benefits from a partner. This assumption would also be reasonable for benefits from a job if the benefits are nonmonetary in nature such as on-job amenities or job satisfaction. For monetary benefits from a job, on the other hand, this assumption amounts to assuming that there exists no functional capital market that one can use to smooth consumption across different periods.

For the given inherited benefit level B_t^*, the goal for the dynamic decision making starting at the beginning of period *t* is to maximize the expected present value of current and future utilities,

$$(1) \qquad E\left(\sum_{s=t}^{T}\beta^{s-t}u(C_s)\right),$$

where $u(C)$ is an increasing Von Neumann-Morgenstern utility function, and $\beta > 0$ is the discount factor. Associated with each level of B_t^* is a maximal value of (1) that results from the DM optimally making the choice between staying the course and rolling the dice for periods *t*, *t* + 1, $\cdots$, and *T*. The concept defined below is important in organizing our reasoning in the proof of our main result.

<u>Definition 1.</u> *For s = t, t + 1,* $\cdots$*, and T,* $M_s\left(B_s^*\right)$ *is defined as the maximal value of* $E\left(\sum_{j=s}^{T}\beta^{j-s}u(C_j)\right)$ *that results from the DM optimally making the choice between staying the course and rolling the dice for periods s, s + 1,* $\cdots$*, and T, given the period-s inherited benefit level* B_s^*.

Note that $M_t(\cdot)$, $M_{t+1}(\cdot)$, $\cdots$, and $M_T(\cdot)$ are different value functions defined for different periods. By definition, $M_s\left(B_s^*\right)$ is non-decreasing in B_s^*, for all $s = t, t + 1, \cdots,$ and T.

Obviously, the larger the inherited benefit level B_t^*, the better the staying the course strategy in period *t*, relative to the rolling the dice strategy. This is because, with a larger B_t^*, it is less likely to draw a benefit level from the random distribution $F(B)$ that is higher than B_t^*. Based on this observation, we have the following definition.

Definition 2. *The critical benefit level in period t, denoted $\bar{B}_t$, is the inherited benefit level that makes the DM indifferent between staying the course and rolling the dice in period t.*

Specifically, for the last period T, $\bar{B}_T$ is given by

(2) $$u\left(\bar{B}_T\right) = \int_a^b u(B-c)dF(B),$$

and for $t < T$, $\bar{B}_t$ is determined by

(3) $$u(\bar{B}_t) + \beta M_{t+1}\left(\bar{B}_t\right) = \int_a^b u(B-c)dF(B) + \beta \int_a^b M_{t+1}(B)dF(B).$$

The LHS of (2) (or (3)) is the expected present value of utilities over the remaining life from staying the course in period T (or $t < T$) when the inherited benefit level is $\bar{B}_T$ (or $\bar{B}_t$), whereas the corresponding RHS is the expected present value of utilities over the remaining life from rolling the dice in that period.

Given the specific functional forms for $u(C)$ and $F(B)$, (2) and (3) can be used to calculate the critical benefit level in periods t, $t+1$, $\cdots$, and T, in a backward fashion. We present an illustrative example of how to calculate the critical benefit levels in different periods in the appendix. Some results in the example are used in the proof of Proposition 2 in the next section.

The choice between staying the course and rolling the dice in period t ($t \le T$) boils down to a comparison of the critical benefit level $\bar{B}_t$ and the inherited benefit level B_t^*: the DM would opt for rolling the dice if and only if $B_t^* < \bar{B}_t$. Because the larger $\bar{B}_t$ is, the more likely that $B_t^* < \bar{B}_t$ (and the more likely that the DM is willing to roll the dice), the critical benefit level $\bar{B}_t$ can be viewed as a measure of the DM's propensity to take risk in period t.

The main result of the paper is the following proposition.

Proposition 1. The critical benefit level decreases as the DM gets closer to the ending period. That is, $\overline{B}_t > \overline{B}_{t+1} > \cdots > \overline{B}_T$.

Proof: We utilize the following method of proof: (i) show that $\overline{B}_{T-1} > \overline{B}_T$, and (ii) show that for any $t \le T-2$, $\overline{B}_t > \overline{B}_{t+1}$ if $\overline{B}_{t+1} > \cdots > \overline{B}_T$.

(i) $\overline{B}_{T-1} > \overline{B}_T$.

By the definition of $\overline{B}_{T-1}$, $\overline{B}_{T-1} > \overline{B}_T$ is equivalent to that the DM is better off rolling the dice than staying the course in period $T-1$ when his inherited benefit level in period $T-1$ is $\overline{B}_T$, or

$$(4) \qquad u(\overline{B}_T) + \beta u\left(\overline{B}_T\right) < \int_a^b u(B-c)dF(B) + \beta\int_a^b M_T(B)dF(B).$$

The LHS of inequality (4) is the present value of remaining lifetime utilities from staying the course in period $T-1$ (with $\overline{B}_T$ being the inherited benefit level), and the RHS is the expected present value of remaining lifetime utilities from rolling the dice.

From (2), inequality (4) is equivalent to

$$(5) \qquad u\left(\overline{B}_T\right) < \int_a^b M_T(B)dF(B).$$

Note that (5) always holds because, by the definitions of $M_T(B)$ and $\overline{B}_T$, $M_T(B) = u(B)$ for $B \ge \overline{B}_T$ and $M_T(B) = u(\overline{B}_T)$ for $B \le \overline{B}_T$.[7] Therefore, $\overline{B}_{T-1} > \overline{B}_T$.

(ii) For any $t \le T-2$, $\overline{B}_t > \overline{B}_{t+1}$ if $\overline{B}_{t+1} > \cdots > \overline{B}_T$.

To show $\overline{B}_t > \overline{B}_{t+1}$, it is equivalent to show that the DM is better off rolling the dice than staying the course in period t when his inherited benefit level in period t is $\overline{B}_{t+1}$, or

[7] Note that the strict inequality in (5) holds even if $c = 0$ as long as the random benefit distribution is nondegenerate.

(6) $$u(\overline{B}_{t+1})+\beta M_{t+1}\left(\overline{B}_{t+1}\right)<\int_a^b u(B-c)dF(B)+\beta\int_a^b M_{t+1}(B)dF(B).$$

The LHS of inequality (6) is the expected present value of remaining lifetime utilities from staying the course in period t (with $\overline{B}_{t+1}$ being the inherited benefit level), and the RHS is the expected present value of remaining lifetime utilities from rolling the dice.

Letting t be replaced with $t + 1$ in (3), we have

(7) $$u(\overline{B}_{t+1})+\beta M_{t+2}\left(\overline{B}_{t+1}\right)=\int_a^b u(B-c)dF(B)+\beta\int_a^b M_{t+2}(B)dF(B).$$

Using (7), inequality (6) is equivalent to

(8) $$M_{t+1}(\overline{B}_{t+1})-M_{t+2}\left(\overline{B}_{t+1}\right)<\int_a^b\left[M_{t+1}(B)-M_{t+2}(B)\right]dF(B)$$

Note that, under the given condition $\overline{B}_{t+1}>\cdots>\overline{B}_T$,

$$\begin{aligned}M_{t+1}\left(\overline{B}_{t+1}\right)&=u\left(\overline{B}_{t+1}\right)+\beta u\left(\overline{B}_{t+1}\right)+\cdots+\beta^{T-t-1}u\left(\overline{B}_{t+1}\right)\\M_{t+2}\left(\overline{B}_{t+1}\right)&=u\left(\overline{B}_{t+1}\right)+\beta u\left(\overline{B}_{t+1}\right)+\cdots+\beta^{T-t-2}u\left(\overline{B}_{t+1}\right).\end{aligned}$$

Therefore, the LHS of (8) is

(9) $$M_{t+1}(\overline{B}_{t+1})-M_{t+2}\left(\overline{B}_{t+1}\right)=\beta^{T-t-1}u\left(\overline{B}_{t+1}\right).$$

To evaluate the RHS of (8), first spell out $M_{t+1}(B)$ and $M_{t+2}(B)$ as follows:

(10) $$M_{t+1}(B)=\begin{matrix}u(B)+\cdots+\beta^{T-t-1}u(B) & B\geq\overline{B}_{t+1}\\ u(\overline{B}_{t+1})+\cdots+\beta^{T-t-1}u(\overline{B}_{t+1}) & B\leq\overline{B}_{t+1}\end{matrix}$$

and

(11) $$M_{t+2}(B)=\begin{matrix}u(B)+\cdots+\beta^{T-t-2}u(B) & B\geq\overline{B}_{t+2}\\ u(\overline{B}_{t+2})+\cdots+\beta^{T-t-2}u(\overline{B}_{t+2}) & B\leq\overline{B}_{t+2}\end{matrix}$$

Note that the condition $\overline{B}_{t+1}>\cdots>\overline{B}_T$ is used in obtaining (10) and (11). Based on (10) and (11), and noting that $\overline{B}_{t+1}>\overline{B}_{t+2}$, one has

$$M_{t+1}(B) - M_{t+2}(B) \begin{matrix} = \beta^{T-t-1}u(B) & B \geq \overline{B}_{t+1} \\ > \beta^{T-t-1}u(\overline{B}_{t+1}) & B \leq \overline{B}_{t+1} \end{matrix}.$$

Therefore the RHS of (8) is

(12) $$\int_a^b \left[M_{t+1}(B) - M_{t+2}(B)\right] dF(B) > \beta^{T-t-1}u(\overline{B}_{t+1}).$$

From (9) and (12), inequality (8) holds. Thus, $\overline{B}_t > \overline{B}_{t+1}$. Q.E.D.

Because the critical benefit level in a period indicates the propensity to take risk in that period, Proposition 1 says that the DM is more prone to risk-taking when further away from the ending period. This result is therefore supportive of the conventional wisdom that the young take more risk than the old, in contrast to the inconclusive results about the time horizon's effect on risk taking obtained in previous models of financial and non-financial risk taking. The intuition behind the unambiguous horizon effect in our model is that when young, any good outcome from taking risk can be sustained for a longer time (due to the option to stay the course), while any bad outcome can be potentially corrected in the next period by drawing again (due to the periodic opportunities of rolling the dice, i.e., drawing from a random benefit distribution).

Compared to our model of periodic opportunities of rolling the dice with an option to stay the course, the dynamic portfolio model has the periodic opportunities of rolling the dice but not the option to stay the course. Investors cannot lock in a good portfolio performance realized in the previous period. Therefore, this paper is another example where incorporating the option value of flexibility has important implications for dynamic decision making.[8] The Liu and

[8] Earlier examples of the importance of incorporating the option value of flexibility into project evaluation include Arrow and Fisher (1974), Henry (1974), Dixit and Pindyck (1994), Gollier et al. (2000), Gollier and Treich (2003),

Rettenmaier (2007) model, on the other hand, does not have the periodic opportunities of rolling the dice. In their analysis, the decision whether to take a risk is made in a "now or never" situation where the outcome from taking the risk, be it good or bad, will follow the DM to the end of his life. These modeling differences are basically the reason why the conventional wisdom unambiguously emerges from the model in this paper but not from the other two models.

3. Other Results from the Model

By the definition of the critical benefit level, when inheriting a benefit level equal to the critical benefit level in the same period, the DM is indifferent between staying the course and rolling the dice. Therefore, the critical benefit level in a period can be viewed as the "certainty equivalent" for rolling the dice in that period. Indeed, it is straightforward to see that the critical benefit level in the ending period is identical to the conventional concept of certainty equivalent. However, the critical benefit level in our model differs in important ways from the certainty equivalent that is developed to evaluate a random outcome under risk evaluation in a static setting. As Proposition 1 indicates, for example, the critical benefit level increases as one gets further away from the ending period.

In this section, we present two differences between the critical benefit level and the traditional certainty equivalent.[9] For this purpose, assume that the DM is risk averse, i.e., $u'' < 0$. From the classic works of Arrow (1971), Pratt (1964) and Rothschild and Stiglitz (1970), two main results about risk aversion's impacts on the certainty equivalent are well known: (a) the

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and Eeckhoudt et al. (2005). However, our paper seems to be the first to relate the option value to the horizon effect on risk taking.

[9] In the sense that results from a single-period decision-making model can be reversed by extending the model to a multi-period one, the comparative analysis here between the traditional certainty equivalent (which is based on a single-period model) and the critical benefit level in this paper (which is based on a multi-period model) is similar in spirit to that in the self-protection literature between one-period and two-period models (i.e., between contemporary and advance prevention). See, for example, Dionne and Eeckhoudt (1985), Eeckhoudt and Gollier (2005), Liu et al. (2009), Menegatti (2009), Eeckhoudt et al. (2012), Wang et al. (2015), Crainich et al. (2016), Denuit et al. (2016), Peter (2017), and Menegatti (2018).

certainty equivalent for a random distribution is smaller than the mean of the distribution; (b) a change in the distribution that represents an increase in risk (in the sense of Rothschild and Stiglitz) will cause the certainty equivalent to become smaller. In contrast, similar results do not hold for the critical benefit level in our model, as is formally stated in the following proposition.

<u>Proposition 2.</u> *Except for the ending period T, it is possible that the critical benefit level in a period is larger than the mean of the random benefit distribution F(B); and it is also possible that the critical benefit level increases as F(B) changes into another, more risky distribution.*

<u>Proof:</u> Examples are used to illustrate these possible situations. Specifically, we use the explicit functional forms for $u(C)$ and $F(B)$ as specified in the appendix. From (A1),

$$\overline{B}_T = \left[(B_0+\varepsilon)(B_0-\varepsilon)\right]^{1/2} = \left(B_0^2-\varepsilon^2\right)^{1/2}, \tag{13}$$

and from (A2),

$$\begin{aligned}&\left(1+\beta\right)\ln(\overline{B}_{T-1})\\ &=\ln(\overline{B}_T)+\frac{1}{2}\beta\ln(B_0+\varepsilon)+\frac{1}{2}\beta\ln(\overline{B}_T).\end{aligned}$$

So

$$\overline{B}_{T-1} = (B_0+\varepsilon)^{\frac{2+3\beta}{4(1+\beta)}}(B_0-\varepsilon)^{\frac{2+\beta}{4(1+\beta)}}. \tag{14}$$

From (14), $\lim_{\varepsilon\to 0}\overline{B}_{T-1} = B_0$, and $\overline{B}_{T-1}$ increases as ε increases whenever $\varepsilon < \frac{\beta}{2(1+\beta)}B_0$. So it is possible (when ε is sufficiently small in this case) for $\overline{B}_{T-1}$ to be larger than the mean of the distribution $F(B)$, which is B_0. Because an increase in ε represents a Rothschild-Stiglitz increase in risk, it is also possible for $\overline{B}_{T-1}$ to increase as the distribution $F(B)$ becomes riskier. (Note, on the other hand, that $\overline{B}_T$ given by (13) – which is identical to the traditional certainty equivalent – is always smaller than B_0, and always decreases as ε increases.) Q.E.D.

Because a DM would prefer drawing from a random benefit distribution to settling on a non-random inherited benefit level in a period if and only if the critical benefit level is larger than the inherited benefit level in that period, the first part of Proposition 2 implies that it is possible for a risk averse individual to prefer drawing from a random distribution to getting the mean of the distribution with certainty. In addition, the second part of Proposition 2 implies that the rolling the dice alternative may become more attractive as the underlying distribution gets riskier. Therefore, Proposition 2 predicts a puzzling tendency in choice behavior that, in their job search or other explorations, otherwise risk averse individuals may be enticed by uncertainty, rather than repelled by it, in the sense that they would prefer a job with an uncertain level of benefits to a job with a known, but average level of benefits. Moreover, it may be the case that risk averse individuals may be more attracted to an unknown job when the sampling pool becomes more diverse.[10]

The reason behind the results in Proposition 2 is the option to stay the course in the next period (i.e., the option to lock in the outcome from the choice in the current period), which makes the "rolling the dice" alternative more attractive than in the case where such an option does not exist. Basically, this option lets a good draw persist while not hindering the ability to erase a bad draw by drawing again. Moreover, while a riskier underlying distribution reduces the current-period expected utility from "rolling the dice", it increases the future expected

[10] There exist other scenarios in which increased variability may be desirable. For example, Chevalier and Ellison (1997) suggest that mutual fund managers who lag behind their peers may increase the riskiness of their portfolio in order to outperform competitors; Tsetlin et al. (2004) establish that contestants in relatively weak positions can increase the probability of winning by taking more risk in their next performance; Karelaia and Hogarth (2010) provide experimental evidence that, in tournaments, additional uncertainty encourages entry by relatively low-skilled contestants but not relatively high-skilled contestants.

utilities from "rolling the dice" now because good draws that you want to keep become better and bad draws, although becoming worse, could be erased by drawing again in the next period.

Note that Proposition 1 in the last section does not need risk aversion to be true. In contrast, we assume risk aversion for Proposition 2. This is because the results stated in Proposition 2 – that the DM may prefer drawing from a random benefit distribution even though the mean of the random distribution is smaller than the sure status quo benefit, and that the DM may be more likely to draw from a random benefit distribution as it becomes more risky in the Rothschild and Stiglitz sense – are only interesting when the DM is risk averse. Moreover, it is relatively easy to see from the appendix that when $u(C) = \ln C$ is replaced with $u(C) = C$, the critical benefit levels $\overline{B}_t, \overline{B}_{t+1}, \cdots, \overline{B}_T$ all become larger, suggesting that compared to a risk neutral DM, a risk averse DM is less likely to choose the "rolling the dice" option.

It is therefore possible to make an evolution-based argument for the young being more prone to risk taking than the old. As Proposition 1 establishes, a rational DM should "roll the dice" more when young (for reasons unrelated to risk aversion). One way to implement this optimal strategy in agents with limited cognitive abilities – through the force of evolution – is to impart the agents with age-dependent risk aversion: less risk averse when young and more risk averse when old.

4. Conclusion

According to the conventional wisdom, the young would take more risk than the old. However, existing theoretical results and empirical evidence on this issue either are inconclusive or point to the opposite of the conventional wisdom.

In this paper, we present a model of periodic risk taking where in each period the decision maker (DM) is faced with a choice between "staying the course" and "rolling the dice". The main finding of the paper is that the DM's propensity to roll the dice – as measured by the critical benefit level defined in the paper – decreases as the DM gets closer to the ending period, therefore providing a justification for the conventional wisdom that the young would take more risk than the old.

The critical benefit level is analogous to the concept of certainty equivalent. Indeed, the critical benefit level for the ending period is identical to the certainty equivalent. However, the critical benefit level differs from the certainty equivalent in important ways. In contrast to the well-known results about risk aversion's effects on the certainty equivalent, that the certainty equivalent is smaller than the mean of the benefit distribution and that the certainty equivalent decreases as the underlying benefit distribution becomes riskier, we show that, for risk averse DMs, it is possible that the critical benefit level in a period is larger than the mean of the random benefit distribution, and it is also possible that the critical benefit level increases as the random benefit distribution becomes riskier.

In order to simplify the technical analysis and to obtain clear-cut theoretical results, we have made some strong assumptions in our model which may seem unrealistic for the real-world situations to which our model is intended to be applied. One such assumption is that both the nonrandom status quo benefit level and the random benefit distribution are exogenous to the DM's effort, but in a real-world situation where a worker is weighing the choice between staying put and finding a new job, he can enhance both options by investing in human capital formation. Another assumption is, again using the labor market example, that once a worker has a job, he can hold on to it as long as he wants, but with few exceptions workers typically do not enjoy guaranteed lifetime employment. We have two remarks regarding these strong modeling assumptions. First, we only intend to use our model to capture some aspects, not all the aspects, of the labor market mobility (or any other potential application of our model mentioned earlier). Second, more realistic assumptions – which are harder to deal with analytically – would not systematically change the relative attractiveness of the two options available at any given time point (rolling the dice vs. staying the course), and therefore are unlikely to imply big changes to our main findings. For example, skill-improving effort can be made either for one's current job or for a potential new job. As a result, it is unclear how incorporating endogenous capital formation would affect the choice between "rolling the dice" and "staying the course". Similarly, we can more realistically assume that one's current job may become unavailable with a positive probability in the next period, and it may seem that this would make the "staying the course" option less attractive relative to the "rolling the dice" option, since the current job might be gone a year later anyway. However, if we apply this more realistic assumption about job duration consistently, then any good outcome from "rolling the dice" might also be unavailable next year. Therefore, the more realistic assumption about job duration does not seem to affect the relative attractiveness of the two options in a systemic way.

Because our model captures some common features in a range of dynamic decision problems including career choice, location choice, entrepreneurial exploitation and formation of an exclusive relationship, future empirical or experimental investigations into the time horizon's effect on risk taking may examine decision making in those areas. Indeed, some have incorporated the horizon effect in the context of job search for unemployed workers. Hairault, et al. (2012) and Hairault et al. (2010) note that because older workers have shorter horizons, among other factors, they are less likely to search while receiving unemployment benefits. Moreover, Antonovics and Golan (2012) find evidence that workers' level of experimentation initially increases over time, and then eventually decreases. The second portion of their experimentation-age profile is consistent with the theoretical prediction of our model. Another avenue for future research is to introduce unemployment insurance into the model here and look at the effect of time horizon on the value of such insurance. Based on our finding that the propensity to try a new job decreases over time, it is plausible that the value of unemployment insurance increases as one ages, everything else the same.

Appendix: An Example of Calculating the Critical Benefit Levels $\overline{B}_t, \overline{B}_{t+1}, \cdots, \overline{B}_T$

Let $u(C) = \ln C$ and *F*(*B*) be the CDF of a Bernoulli distribution: $B = B_0 + \varepsilon$ or $B = B_0 - \varepsilon$, each with probability ½, where $B_0 > \varepsilon > 0$. For simplicity, assume that the search cost is zero or $c = 0$.

In period *T*, $\overline{B}_T$ is determined by, according to (2), $\ln \overline{B}_T = \frac{1}{2}\ln(B_0+\varepsilon) + \frac{1}{2}\ln(B_0-\varepsilon)$. Therefore,

(A1) $$\overline{B}_T = \left[(B_0+\varepsilon)(B_0-\varepsilon)\right]^{1/2}.$$

In any period $t < T$, $\overline{B}_t$ is theoretically determined by (3). However, (3) is not readily operational because the functional form of M_{t+1} is not explicitly given. For the special *u*(*C*) and *F*(*B*) given in this example, we can provide an operational method of calculating $\overline{B}_t$ for $t < T$. First, according to Proposition 1, $\overline{B}_t > \overline{B}_{t+1} > \cdots > \overline{B}_T$. So if the DM inherits $\overline{B}_t$ in period *t*+1, he would follow the "staying the course" strategy and lock in $\overline{B}_t$ for all the periods *t*+1, …, *T* . Therefore,

$$M_{t+1}\left(\overline{B}_t\right) = \ln(\overline{B}_t)\left(1+\beta+\cdots+\beta^{T-t-1}\right).$$

Further, based on the special form of *F*(*B*),

$$\int_a^b M_{t+1}(B)dF(B) = \frac{1}{2}\left(1+\beta+\cdots+\beta^{T-t-1}\right)\ln(B_0+\varepsilon) + \frac{1}{2}\left(1+\beta+\cdots+\beta^{T-t-1}\right)\ln(\overline{B}_{t+1})$$

Substituting the above two expressions into (3), we have,

(A2) $$\begin{aligned}&\ln(\overline{B}_t)\left(1+\beta+\cdots+\beta^{T-t}\right)\\ &= \ln(\overline{B}_T) + \frac{1}{2}\beta\left(1+\beta+\cdots+\beta^{T-t-1}\right)\ln(B_0+\varepsilon) + \frac{1}{2}\beta\left(1+\beta+\cdots+\beta^{T-t-1}\right)\ln(\overline{B}_{t+1}).\end{aligned}$$

From (A1) and (A2), the critical benefit levels in all periods can be obtained in the backward fashion for the special *u*(*C*) and *F*(*B*) given in this example.

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