# Optimal Regional Insurance Provision: Do Federal Transfers Complement Local Debt?*

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**Abstract**

We study the design and implementation of optimal regional insurance provision against privately observable shocks to the degree of intergenerational externality (DIE) induced by, or the degree of technological progress (DTP) for producing, intergenerational public goods (IPGs). Federal transfers provide interregional insurance while local debt provides intergenerational insurance. If regions have autonomy in the choice of local debt issuance, the optimal allocation of federal transfers that induces truth-telling requires that regions issuing higher debt receive more transfers under complementarity but less transfers under substitutability. In the case of shocks to the DIE, federal transfers and local debt are complementary in implementing the asymmetric-information optimum; in the case of shocks to the DTP, they are complementary with observable output of the IPGs, but are substitutive with observable expenditure on the IPGs.

**Keywords:** Intergovernmental grants; interregional insurance; regional public debt; durable public goods; intergenerational externality; asymmetric information.

**JEL Codes:** H41; H74; H77; D82.

# 1 Introduction

Intergovernmental grants implemented by the central government of a federal fiscal system are justified on the grounds that they internalize interregional spillovers generated

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by local public goods provision (Oates, 1972) or inter-jurisdictional migrations,[1] redistribute income between regions,[2] and serve as a risk-sharing device against region-specific shocks.[3] As shown by Sala-i-Martin and Sachs (1992), even policies aimed at redistribution may have an effect on the degree of interregional risk sharing. Indeed, there is empirical evidence showing that fiscal transfers from the federal government provide substantial insurance against regional economic fluctuations in the United States, Canada, Japan, Norway, and so on.[4] Even in the presence of complete markets, Farhi and Werning (2017) provide a rationale for government intervention in terms of public risk sharing.

In addition, regional public debt serves as a public contract for sharing risks between generations or over lifecycle in a given region.[5] Since present generations are imperfectly altruistic (e.g., Altonji, Hayashi and Kotlikoff, 1992, 1997), however, the design of optimal public debt that takes into account possible intergenerational conflicts turns out to be a nontrivial task (e.g., Rangel, 2003, 2005; Huber and Runkel, 2008; Dai, Liu and Tian, 2019b).

Given the insurance role played by both federal transfers and local public debt, the following questions arise. How would these two options of insurance provision behave when jointly designed by the central government? Under decentralized debt decisions, how would the interregional insurance provided by the central government interact with the intergenerational insurance provided locally? More specifically, shall they exhibit complementarity or substitutability in the course of implementation? Indeed, whether federal grants and local debt are complementary or substitutive matters greatly in regional insurance design. Specifically, if these two insurance schemes are complementary, then using both of them jointly for regional insurance is justified on efficiency grounds; if they are substitutive, on the other hand, then efficiency considerations require using either federal grants or local debt but not both of them simultaneously in the provision of regional insurance.[6]

To the best of our knowledge, these issues have not been explored in the literature. The goal of this investigation is to address these questions via tackling the optimal design and implementation of risk-sharing contracts consisting of both intergovernmental grants and regional public debt along the space and time dimensions, respectively.

We consider a country that consists of a central government and many sub-national

[1]See, e.g., Hercowitz and Pines (1991); Cremer, Marchand and Pestieau (1997); Figuières and Hindriks (2002); Breuillé and Gary-Bobo (2007); and Dai, Liu and Tian (2019a).

[2]See, e.g., Cremer and Pestieau (1997); Raff and Wilson (1997); Cornes and Silva (2000); and Bordignon, Manasse and Tabellini (2001).

[3]See, e.g., Persson and Tabellini (1996a, 1996b), Bucovetsky (1998), Lockwood (1999), Cornes and Silva (2000), and Jüßen (2006).

[4]See, e.g., Atkeson and Bayoumi (1993); Asdrubali, Sørensen and Yosha (1996); Mélitz and Zumer (1999); Athanasoulis and van Wincoop (2001); Kalemli-Ozcan, Sørensen and Yosha (2003); Borge and Matsen (2004); Evers (2015).

[5]In an infinite-horizon economy where individuals face uninsurable risks to their human capital accumulation, Gottardi, Kajii and Nakajima (2015) show that the benefits of government debt increase with the magnitude of risks and the degree of risk aversion.

[6]In particular, identifying the case with policy substitutability creates a sort of policy flexibility for regional insurance provision, i.e., grant and debt can be used simultaneously while targeting alternative policy goals. For instance, federal transfers are used for interregional income redistribution or interregional externality correction, while local debt is used for regional insurance provision. Or, federal transfers are used mainly for regional insurance provision, while local debt is strictly constrained to defuse local government debt bomb (e.g., *The Economist*, 2015).

governments located in geographically decentralized regions. Throughout, the center is in charge of revenue transfers across regions whereas local governments are responsible for collecting taxes used for the provision of local public goods. Each region is populated by a continuum of identical residents who live for one period only. We focus on an economy that lasts for two periods, thus enabling us to incorporate intergenerational concerns into the current setting, while retaining simplicity and tractability. The current generation chooses how much debt to pass to the future generation and how much to invest in intergenerational public goods (IPGs), such as basic science, environmental protection and public capital. Initially, as a centralization benchmark to which we refer, we let the center jointly determine the amount of public debt a region can issue as well as the transfers it can receive. We then move to the more realistic situation with decentralized leadership in which local governments are allowed to have the autonomy in choosing the level of regional public debt.

Regions are assumed to be ex ante identical but are subject to stochastic shocks to either the degree of intergenerational externality induced by, or the degree of technological progress for producing, the IPGs. In this context, while regional heterogeneity in shock realizations creates a natural role for interregional insurance represented by transfers from the center to the regions, a potential role of intergenerational insurance played by local public debt is also easy to understand because both types of shocks primarily affect the future generations. Firstly, the present generation incurs the cost of IPG investment that generates a positive externality on the future generation. Secondly, it is well recognized that the progress made in fields like basic science, space exploration and environmental protection benefits from standing on the shoulder of giants, and hence a high degree of technological progress to be realized in the future appeals to R&D investments in the present.

As is customary in the fiscal federalism literature, regional governments are better informed about the shocks than the federal government.[7] As such, intergovernmental grants and regional public debt form the risk-sharing contracts designed by the center, taking into account the fiscal budget balance, participation and truth-telling constraints. From solving the mechanism design problem facing the center, conducting the comparison with the full-information optimum (or the first-best allocation), and implementing the optimal allocation through decentralized regional debt decisions, we obtain the following four results, regardless of the source of shocks.

First, the intertemporal allocation is not distorted, i.e., the intertemporal rate of substitution between current and future consumption equals the intertemporal rate of transformation, only at the bottom and top types, and full insurance is not achievable for all types. Since the informational asymmetry between the center and regions prevents complete public insurance from happening in this setting, we somehow provide a rationale for the usefulness of private insurance. Second, if the intergovernmental grant received by the bottom type is distorted upward (or is large), then its public debt must be distorted downward (or be low), and vice versa; meanwhile, the direction of distortion is qualitatively reversed between the bottom and top types. Overall, these two results characterize the asymmetric-information welfare optimum.

Third, for all but the bottom and top types, to truthfully implement the welfare optimum through decentralized regional debt decisions, the intergovernmental grant scheme

[7] See, for example, Oates (1972); Bucovetsky, Marchand and Pestieau (1998); Lockwood (1999); Cornes and Silva (2000, 2002); Huber and Runkel (2008).

enforced by the center must be a nonlinear, almost everywhere differentiable and monotonic function of local debt. Since the intertemporal allocation is distorted for these types in the asymmetric-information optimum, the amounts of public debt allocated to these regions are different than the ones established by maximizing their respective regional goals. As a result, if borrowing decisions are decentralized to the regional governments, the grant scheme enforced by the center must depend on regional debt such that regions have incentives to truthfully reveal their types. And fourth, for the bottom and top types, the grant scheme that decentralizes the welfare optimum is, however, independent of the regional public debt. The reason is that the intertemporal allocations desired by regions of the top and bottom types are not distorted in the asymmetric-information welfare optimum, and hence their truth-telling can be guaranteed by directly setting the grants established in the asymmetric-information optimum. These two results show the key features of the implementation scheme over the entire type distribution.

Moreover, when considering the source of shocks, the relationship between these two insurance-provision instruments in the course of implementing welfare optimum is characterized as follows. When regions differ in the degree of intergenerational externality, they are complementary in insurance provision. The immediate implication is that it is socially optimal to use both insurance schemes simultaneously when facing this sort of shocks. When regions differ in the degree of technological progress for producing the IPGs, they are complementary if it is the physical output of public goods that is observable, whereas they are substitutive if it is the regional expenditure on public goods that is observable. For example, the physical output of some IPGs such as parks, public schools and highways is observable, whereas that of other IPGs such as environmental protection, basic science and R&D is unobservable, at least in the short run, by the center who is in general not involved in the process of producing these public goods. Consequently, it is not always socially beneficial to adopt both insurance schemes in insuring against shocks to the degree of technological progress. Whether it is the input or the output of IPGs that is observable makes a great difference in determining whether the insurance provided by federal grants and the insurance provided by local debt should be used jointly or in isolation. In terms of identifying the effect of alternative observability on the implementation of information-constrained optima, this finding contributes to the public finance and regional science literature. In addition, if the intergenerational conflict induced by local debt is the dominant issue in an economy, then the case of substitutability shows the social desirability of tightening local borrowing to the lowest possible level while relying on intergovernmental grants enforced by the center. These results characterize the connections between these public risk-sharing schemes and the underlying environment, thus helping us understand how they should be adopted in real-world federations.

The present study is related to the literature that examines theoretically the design of regional insurance provision in a federation, such as Persson and Tabellini (1996a, 1996b), Bucovetsky (1998), Lockwood (1999), and Cornes and Silva (2000). A comparison with these studies reveals three distinctive features of our study. Firstly, rather than adopting a one-period static setting, we consider a two-period setting that allows for taking into account intergenerational concerns and a natural role for public debt. Secondly, the sources of shocks considered in the paper as well as the informational asymmetry between the center and the regions are novel for analyzing optimal regional insurance provision. Thirdly, instead of studying the local debt and the federal grants as unrelated policy variables, we investigate the *joint design* of these two risk-sharing schemes along two dimensions, namely intergovernmental grants along the interregional dimension and public debt along the intertemporal/intergenerational dimension, and further analyze their interaction in the course of optimal decentralization. For these features, our paper extends and complements the existing literature.

The remainder of the paper is organized as follows. Section 2 describes the economic environment. Section 3 derives the welfare optimum and discusses its implementation when regions differ in the degree of intergenerational externality. Section 4 derives the welfare optimum and discusses its implementation when regions differ in the degree of technological progress for producing local IPGs. Section 5 concludes. Proofs are relegated to Appendix A.

# 2 Environment

We consider a two-period economy of a federation consisting of a federal government (also referred to as the center) and $n$ regions, each of which is inhabited by a representative immobile resident in each period.[8] That is, each resident lives for one period only. The social welfare of region $i$, for $i = 1, 2, ..., n$, is given by

$$\underbrace{u_1(c_1^i) + g_1(G_1^i)}_{\text{utility of generation 1}} + \underbrace{u_2(c_2^i) + g_2(\theta^i G_1^i + G_2^i)}_{\text{utility of generation 2}}, \tag{1}$$

in which $c_1^i$ and $c_2^i$ are private consumptions, $G_1^i$ and $G_2^i$ are public goods, and $\theta^i \in (0, 1]$ is a parameter measuring the degree of intergenerational externality of the IPG, $G_1^i$.[9] All four functions in (1) are strictly increasing, strictly concave and satisfy the usual Inada conditions.[10]

The representative resident of generation $t$, for $t = 1, 2$, in region $i$ has private budget constraint $c_t^i + \tau_t^i = y_t$, where $y_t$ is the commonly given income across all regions.[11] The lump sum tax $\tau_t^i$ is collected by the local government to finance the provision of local public goods. In period 1, it receives a transfer $z^i$ from the center and issues debt $b^i$. If $z^i < 0$, then the local government has to pay a tax to the center. Debt plus interest has to be repayed in period 2, taking as given the common interest rate $r > 0$.[12] The fiscal budget constraints of region $i$ in periods 1 and 2 can be written as $G_1^i = \tau_1^i + b^i + z^i$ and $\mathcal{G}_2^i = \tau_2^i - (1 + r)b^i$, respectively. We let $\mathcal{G}_2^i = \xi^i G_2^i$, in which the parameter $\xi^i \in (0, 1]$ measures the per unit cost of period-2 public goods provision. The case of $\xi^i < 1$ captures the effect of technological progress, which as argued by Rangel (2005) is important for IPGs such

[8] We leave the interesting case with horizontal fiscal externalities induced by cross-region labor mobility to our future research.

[9] IPG is a kind of public good produced in generation 1 and still (partially) usable in generation 2 (Rangel, 2005).

[10] Also, note that the preference specification encompasses the special case with $u_2(c_2^i)+g_2(\theta^i G_1^i+G_2^i) \equiv \beta[u_1(c_2^i)+g_1(\theta^i G_1^i+G_2^i)]$, in which $\beta > 0$ is a social discount factor that can be interpreted as a political-economy parameter reflecting the degree to which regional governments take into account the welfare of future generations.

[11] Instead of considering income heterogeneity across regions, which actually has been well studied in terms of optimal intergovernmental grants, we shall consider some novel dimensions of cross-region heterogeneity. Also, one can interpret our model as restricting attention to regions of similar personal incomes, such as California and Texas in the United States, or Jiangsu and Zhejiang provinces in China.

[12] Assuming that there is a common capital market within a federation, there is a unique rental price level of capital such that arbitrage opportunities are eliminated.

as infrastructure, space exploration and environmental capital. In addition, as shown by Maskin and Riley (1985), it generally makes a difference in the implementation process whether the expenditure $\mathcal{G}_2^i$ or the physical output $G_2^i$ is observable to the mechanism designer. If it is the expenditure that is observable, then we need to express the output as a function of expenditure, namely $G_2^i = \mathcal{G}_2^i/\xi^i \equiv \rho^i \mathcal{G}_2^i$.

For expositional convenience, the region index $i$ is suppressed in the remainder of the set-up. Combining the private budget constraints with the public budget constraints and applying them to equation (1), a region's social welfare maximization problem is given by

$$\begin{aligned}\max_{c_1,c_2} \quad & u_1(c_1) + g_1(y_1 + b + z - c_1) \\ & + u_2(c_2) + g_2\left(\theta(y_1 + b + z - c_1) + \rho[y_2 - b(1+r) - c_2]\right),\end{aligned} \tag{2}$$

in which $\rho \geq 1$. Note that in problem (2), choosing $c_1$ and $c_2$ is equivalent to choosing $\tau_1$ and $\tau_2$. The first-order conditions are thus written as

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + G_2) \text{ and } u_2'(c_2) = \rho g_2'(\theta G_1 + G_2), \tag{3}$$

which represent the Samuelson conditions for the optimal provision of public goods.

We allow regions to differ in two dimensions in terms of privately observable shocks: the degree of intergenerational externality measured by $\theta$ and the degree of technological progress for producing IPGs measured by $\xi$ (or, equivalently, by $\rho$). Interpreted as a measure of the quality or durability of local IPGs, it is reasonable to assume that $\theta$ is privately observable by local governments. We argue from the following two perspectives. Firstly, the quality of the physical output of some IPGs, such as basic science, local environmental protection and R&D, is objectively unobservable, at least in the short run, by the center who is in general not involved in the process of producing these public goods. Secondly, the local politicians have subjective incentives to hide/misreport such information for the sake of either getting more transfers, getting personal promotions, or avoiding punishments. For example, local politicians in China may get promoted to higher levels because of doing a good job in public infrastructure investment or establishing a business friendly environment, or may get punished for being responsible for *tofu-dreg projects*[13] in the provision of local IPGs, such as public schools, bridges and dams, that end up in very low quality or even tragedies. As for $\xi$ or $\rho$, namely the per unit cost of period-2 public goods provision, it is usually assumed to be the private information of local governments in the relevant literature, such as Boadway, Horiba and Jha (1999), Lockwood (1999), and Cornes and Silva (2002).

As is well known, it is analytically intractable to obtain interesting results in the presence of multidimensional private information (e.g., Rochet and Choné, 1998; Armstrong and Rochet, 1999). We thus consider two separate cases with the first one featuring privately observable shocks to the degree of intergenerational externality (DIE) and the second one featuring privately observable shocks to the degree of technological progress (DTP).[14] The random variables are assumed to be continuously distributed in intervals

[13]This is a well-known phrase coined by Zhu Rongji, the former premier of the People's Republic of China, on a visit to Jiujiang City, Jiangxi Province to describe a jerry-built dam.

[14]In Appendix B, we discuss a more general case with both parameters being unobservable by the center but there being a certain functional relationship between these two parameters, thereby reducing a multi-dimensional screening problem to a one-dimensional screening problem. That is, for the sake of obtaining

$[\underline{\theta}, \bar{\theta}] \equiv \Theta$ and $[\underline{\xi}, \bar{\xi}] \equiv \Xi$ (or $[\underline{\rho}, \bar{\rho}] \equiv \Upsilon$), and also are identically and independently distributed across regions. We denote by $f = F' > 0$ and $F$, respectively, the density and distribution functions, which are common knowledge throughout.

# 3 Welfare Optimum and Implementation when Regions Differ in Intergenerational Externality

In this section, we focus on the optimal provision of regional insurance against shocks to the degree of intergenerational externality $\theta^i$. For this purpose, we assume that all regions have the same degree of technological progress, and let $\rho^i$ (or $\xi^i$) $= 1$ for all $i$ without loss of generality. We introduce first the problem of the center. Then, we proceed to derive welfare optimum in cases of complete and asymmetric information between the center and regions, and discuss the implementation issue.

## 3.1 The Problem of the Center

The center is responsible for determining regional debt and cross-region transfers as non-market insurances against shocks on intergenerational externality. Assuming it treats all regions equally and the realization of shocks can be privately observed by each region, it thus maximizes the expectation of the value function (2) of any region, subject to fiscal budget balance, incentive-compatibility and participation constraints.

We follow the mechanism design approach and apply the direct revelation principle. The center offers each region $i$ a contract stipulating the federal transfer and the region's debt conditional only on its report of type $\theta^i$. The reported type also belongs to set $\Theta$. Since all regions are ex ante identical, the insurance contract can be thought of being signed between the principal (the center) and an agent (a region) whose type $\theta$ belongs to set $\Theta$ following the distribution $F$.[15]

The timing of the underlying game is given as follows:

- Shock occurs, i.e., nature moves first.
- Local governments privately observe shock realizations.
- The federal government offers the contract menu, $\{b(\theta), z(\theta)\}$, for all $\theta \in \Theta$.
- The local governments simultaneously pick a contract (or equivalently report their types), and the game ends.

some clear-cut results, there is multidimensional heterogeneity while maintaining one-dimensional private information. Two main results are obtained. First, it remains true that the informational asymmetry between the center and regions prevent complete public insurance from happening. Second, federal transfers and local debt exhibit policy complementarity in terms of truthfully implementing the welfare optimum, regardless of whether it is the physical output of or the expenditure on the IPGs that is observable by the center.

[15] In fact, the center offers each region $i$ a contract stipulating the federal transfer and the region's debt conditional only on its report of type $\theta^i$, denoted by $\hat{\theta}^i$, i.e., $b^i = b(\hat{\theta}^i)$ and $z^i = z(\hat{\theta}^i)$. To formulate the constraints facing the mechanism designer, we consider the limiting case with the number of regions being large, i.e., $n \to \infty$. Making use of the weak law of large numbers, the empirical distributions of $b^i$ and $z^i$ across regions approximate the theoretical distributions generated by $b^i = b(\hat{\theta}^i)$, $z^i = z(\hat{\theta}^i)$ and $F$.

We write the value function generated by the maximization problem (2) as $V(b, z, \theta)$. As all regions are ex ante identical, the objective of the center can be written as:

$$EU = \int_{\underline{\theta}}^{\bar{\theta}} V(b(\theta), z(\theta), \theta) f(\theta) d\theta. \tag{4}$$

The truth-telling constraints require that any region with shock realization $\theta$ prefers to report $\theta$ rather than some $\theta'$; formally

$$V(b(\theta), z(\theta), \theta) \geq V(b(\theta'), z(\theta'), \theta) \;\; \forall \theta' \neq \theta, \;\theta', \theta \in \Theta. \tag{5}$$

The participation constraint for any region specifies that it would like to participate in the federation and receive some interregional insurance via the grant system rather than secede. Following the arguments of Lockwood (1999), we impose the ex ante participation constraint:[16]

$$EU \geq \int_{\underline{\theta}}^{\bar{\theta}} \max_{b(\theta)} V(b(\theta), 0, \theta) f(\theta) d\theta, \tag{6}$$

where $EU$ is given by (4). Since the center can always replicate by setting $z \equiv 0$ what any region could get by seceding, we are sure that constraint (6) will never bind and can be safely ignored in the following analysis.

The federal budget balance constraint for large $n$ reads as

$$\int_{\underline{\theta}}^{\bar{\theta}} z(\theta) f(\theta) d\theta \leq 0, \tag{7}$$

which implies that in aggregate transfers must sum to at most zero.

The problem facing the center is thus to choose $\{b(\theta), z(\theta)\}_{\theta \in \Theta}$ to maximize (4) subject to (5) and (7). As is common in the mechanism design literature, we let $b$ and $z$ be piecewise continuously differentiable functions, and let $b(\theta)$ be everywhere continuous.

## 3.2 Welfare Optimum

As a standard benchmark result to which we can refer, we start our analysis by deriving the full-information (first-best) allocation that maximizes (4) subject to (7) only. We index the first-best optimum by the superscript $^{FB}$.

**Lemma 3.1** *In the full-information case, the welfare optimum $\{b^{FB}(\theta), z^{FB}(\theta)\}_{\theta \in \Theta}$ satisfies:*

*(i) The intertemporal rate of substitution between current and future public goods consumption equals intertemporal rate of transformation, namely*

$$\frac{g_1'(G_1^{FB}(\theta))}{g_2'(\theta G_1^{FB}(\theta) + G_2^{FB}(\theta))} = 1 + r - \theta \;\; \text{for any } \theta \in \Theta.$$

*(ii) Full insurance is achievable, namely*

$$V_z\left(b^{FB}(\theta), z^{FB}(\theta), \theta\right) = \gamma \;\; \text{for any } \theta \in \Theta,$$

*in which $\gamma > 0$ denotes the Lagrangian multiplier on the budget constraint (7).*

[16] Indeed, the ex post participation constraint is usually too strong to be fulfilled.

**Proof.** Straightforward and omitted. ■

Part (i) yields that the intertemporal allocation of any type of regions is not distorted in the first-best optimum. Part (ii) gives the standard insurance condition which states that the consumption of period-2 public goods is the same regardless of the shock realization on the degree of intergenerational spillovers.

We now turn to the more interesting case with asymmetric information between the center and regions. In this case, the realization of the random variable measuring the degree of intergenerational externality is private information so that regions of one type could mimic regions of another type in order to obtain (more) insurance transfers. We now index the second-best allocation by the superscript $^*$.

We shall need the following assumption:

**Assumption 3.1** $-\theta G_1 g_2'' \leq g_2'$ *for all* $\theta \in (\underline{\theta}, \bar{\theta})$*, namely the absolute value of the elasticity of generation 2's marginal utility from* $G_1$ *is no greater than one for all but the endpoints of the type distribution.*

This is a technical restriction imposed on generation 2's preference on public goods. It is easy to verify that this assumption is satisfied for log and power utility functions.

**Proposition 3.1** *In the asymmetric-information case without bunching, namely* $\dot{b}(\theta) > 0$*, the welfare optimum* $\{b^*(\theta), z^*(\theta)\}_{\theta \in \Theta}$ *satisfies:*

*(i) Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we have:*

$$\frac{g_1'(G_1^*(\theta))}{g_2'(\theta G_1^*(\theta) + G_2^*(\theta))} \begin{cases} = \ 1 + r - \theta & \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < \ 1 + r - \theta & \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

*(ii) Suppose Assumption 3.1 holds. Let* $\mu_1(\theta) > 0$ *be the Lagrangian multiplier on the value constraint* $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$ *of any type-*$\theta$ *region who is reporting truthfully, then we have:*

$$V_z\left(b^*(\theta), z^*(\theta), \theta\right) \begin{cases} = \ \gamma/\mu_1(\theta) & \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < \ \gamma/\mu_1(\theta) & \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

**Proof.** See Appendix A. ■

The key finding of this proposition is the following. First, the intertemporal allocation under asymmetric information is not distorted only at the endpoints of shock distribution, that is, for regions of the highest and the lowest degrees of intergenerational externality. Second, given that the multiplier $\mu(\theta)$ is type-dependent, there is incomplete insurance under asymmetric information.

As informational friction is what we focus on in this study, it is interesting to identify the effect of asymmetric information between the center and regions on optimal debt and intergovernmental grants policies. To this end, it is worthwhile providing a detailed characterization of the Lagrangian multiplier $\mu_1(\theta)$ after comparing Lemma 3.1 and Proposition 3.1.

**Lemma 3.2** *For the current economic environment, the following statements are true.*

*(i) If $\mu_1(\theta)$ is decreasing in $\theta$, then there exists some $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $\mu_1(\theta) > 1$ for $\theta \in [\underline{\theta}, \tilde{\theta})$, $\mu_1(\theta) = 1$ for $\theta = \tilde{\theta}$, and $\mu_1(\theta) < 1$ for $\theta \in (\tilde{\theta}, \bar{\theta}]$.*

*(ii) If $\mu_1(\theta)$ is increasing in $\theta$, then there exists some $\breve{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $\mu_1(\theta) < 1$ for $\theta \in [\underline{\theta}, \breve{\theta})$, $\mu_1(\theta) = 1$ for $\theta = \breve{\theta}$, and $\mu_1(\theta) > 1$ for $\theta \in (\breve{\theta}, \bar{\theta}]$.*

**Proof.** See Appendix A. ■

Although $\mu(\theta)$ could be a complex nonlinear function of $\theta$, here we focus on the case of monotonicity for obtaining some clear-cut results. Indeed, applying Lemma 3.2, the following proposition is established.

**Proposition 3.2** *Under Assumption 3.1, the following statements are true.*

*(i) If $\mu_1(\theta)$ is decreasing in $\theta$, then (i-a) $z^*(\theta) > z^{FB}(\theta)$ for all $\theta \in [\underline{\theta}, \tilde{\theta}]$; (i-b) $z^*(\bar{\theta}) < z^{FB}(\bar{\theta})$; and (i-c) $b^*(\underline{\theta}) < b^{FB}(\underline{\theta})$ and $b^*(\bar{\theta}) > b^{FB}(\bar{\theta})$.*

*(ii) If $\mu_1(\theta)$ is increasing in $\theta$, then (ii-a) $z^*(\theta) > z^{FB}(\theta)$ for all $\theta \in [\breve{\theta}, \bar{\theta}]$; (ii-b) $z^*(\underline{\theta}) < z^{FB}(\underline{\theta})$; and (ii-c) $b^*(\underline{\theta}) > b^{FB}(\underline{\theta})$ and $b^*(\bar{\theta}) < b^{FB}(\bar{\theta})$.*

**Proof.** See Appendix A. ■

If $\mu_1(\theta)$ is decreasing in $\theta$, as considered in claim (i-a), then the shadow prices of the value constraint under truth-telling are larger for lower types of regions (i.e., regions of lower degrees of intergenerational externality) than for higher types. As a result, it is more likely to be that lower types *extract* the information rent and receive more transfers under asymmetric information than they would do under full information. Claim (ii-a) can be interpreted in a similar way. Regardless of whether $\mu_1(\theta)$ is decreasing or increasing in $\theta$, the optimal allocation under asymmetric information is distorted at both top and bottom types, and importantly, the distortion is qualitatively reversed between the top and bottom of shock distribution.

Moreover, if the shadow prices of the value constraint under truth-telling are larger for lower types than for higher types, then the top type receives less transfers while issues more debt, and the bottom type receives more transfers while issues less debt than they would exhibit, respectively, in the full-information optimum. The intuition for this result is the following. Recall first that the intertemporal rate of transformation is the rate at which savings in the first period can be transformed into consumption in the second period, and an increase in which implies an increase in the opportunity cost of borrowing. Note that the positive intergenerational spillovers induced by IPG investment partly offset the negative intergenerational externality caused by borrowing, the bottom-type regions have the largest opportunity cost of borrowing, formally $1+r-\underline{\theta} > 1+r-\theta$ for any $\theta > \underline{\theta}$, which is public knowledge. In consequence, when $\mu_1(\theta)$ is decreasing in $\theta$, imposing an upward distortion on transfers while a downward distortion on debt provides appropriate incentives such that the bottom-type regions reveal their type truthfully under asymmetric information. The other case, in which the shadow prices of the value constraint under truth-telling are larger for higher types than for lower types, can be analyzed analogously.

## 3.3 Implementation

We have established the welfare optimum under both complete and asymmetric information in the previous subsection, we now proceed to consider how to implement it via regionally-decentralized debt decisions. That is, both regions choose a level of public debt to maximize their regional welfare, taking as given the intergovernmental grants scheme provided by the center. Formally, the maximization problem of regions of type-$\theta$ is

$$\max_{b(\theta)} \; V(b(\theta), z(\theta), \theta)$$

for any given $z(\theta)$. Rewriting private consumptions as $c_1 = \tilde{\phi}(b(\theta), z(\theta), \theta)$ and $c_2 = \tilde{\psi}(b(\theta), z(\theta), \theta)$ and applying the Envelope Theorem, the first-order condition is thus written as

$$\frac{g_1'\left(y_1 + b(\theta) + z(\theta) - \tilde{\phi}(b(\theta), z(\theta), \theta)\right)}{g_2'\left(\theta\left(y_1 + b(\theta) + z(\theta) - \tilde{\phi}(b(\theta), z(\theta), \theta)\right) + y_2 - (1+r)b(\theta) - \tilde{\psi}(b(\theta), z(\theta), \theta)\right)} \qquad (8)$$
$$= 1 + r - \theta,$$

showing that the intertemporal rate of substitution must be equal to the intertemporal rate of transformation at the regional welfare optimum.

Making use of (8) and Lemma 3.1, we immediately have the result: The full-information optimum is attained by simply setting $z(\theta) = z^{FB}(\theta)$ for all $\theta \in \Theta$. The reason is that the center can observe the type of each region and also the full-information optimum does not distort the intertemporal allocation desired by each region.

Under asymmetric information, the center must design intergovernmental grants scheme that guarantees incentive compatibility for all regions. It follows from Proposition 3.1 that the intertemporal allocation of regions of all but top and bottom types is distorted, so the asymmetric-information optimum can no longer be implemented through decentralized debt decisions characterized by (8) with the center simply setting $z(\theta) = z^*(\theta)$. Indeed, we have established the following proposition.

**Proposition 3.3** *Suppose the second-order sufficient condition for incentive compatibility is not binding, namely $\dot{b}(\theta) > 0$. The grant scheme $z^*(b)$ that decentralizes the asymmetric-information optimum $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ is a nonlinear nondecreasing function of $b$, almost everywhere differentiable, with the slope*

$$\frac{dz^*}{db}\begin{cases} = \; 0 & \text{for } b \in \{b^*(\underline{\theta}), b^*(\bar{\theta})\}; \\ > \; 0 & \text{for } b \in (b^*(\underline{\theta}), b^*(\bar{\theta})). \end{cases}$$

**Proof.** See Appendix A. ■

By Proposition 3.1 we see that the intertemporal allocation under asymmetric information is not distorted at the endpoints of type distribution, thus the levels of local debt desired by regions at the bottom and top of type distribution, namely $b^*(\underline{\theta})$ and $b^*(\bar{\theta})$, can be realized by simply setting $z^*(\underline{\theta})$ and $z^*(\bar{\theta})$, respectively. This explains why we have $dz^*/db = 0$ at the endpoints of type distribution in Proposition 3.3. For regions of types between $\underline{\theta}$ and $\bar{\theta}$, their intertemporal allocations are indeed distorted with respect to the first-best. As such, if regions of higher degrees of intergenerational spillovers are allowed to issue more debt than regions of lower degrees, then the grant scheme that decentralizes the asymmetric-information optimum should be designed such that the former regions get strictly more grants than the latter ones. Moreover, since we have $V_z > 0$, this grant scheme guarantees that regions with high degrees of intergenerational spillovers shall not mimic regions of low degrees.

The implication for the optimal funding structure of IPGs when facing this kind of shocks is the following: more (respectively less) federal transfers plus more (respectively less) local borrowing for regions of higher (respectively lower) degrees of intergenerational externality induced by these IPGs. Therefore, to guarantee incentive compatibility when the leadership of local borrowing is decentralized to each region, federal transfers and local debt exhibit *complementarity* in the case of shocks to intergenerational externality.

# 4 Welfare Optimum and Implementation when Regions Differ in Technological Progress

To analyze the optimal regional insurance provision when regions differ in shocks to the degree of technological progress, we assume that all regions have the same degree of intergenerational externality, denoted $\theta$. $\rho^i$ (or $\xi^i$) is a random variable the realization of which is region $i$'s private information. As shown by Maskin and Riley (1985) and Lockwood (1999), it generally makes a difference in the implementation process whether the expenditure $\mathcal{G}_2^i$ or the physical output $G_2^i$ is observable to the mechanism designer. So we shall discuss both possibilities.

## 4.1 The Case with Observable Expenditure on Public Goods

### 4.1.1 Welfare Optimum

The first-order conditions of problem (2) are now written as

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + \rho \mathcal{G}_2) \ \text{ and } \ u_2'(c_2) = \rho g_2'(\theta G_1 + \rho \mathcal{G}_2), \tag{9}$$

and the corresponding regional value function is written as $V(b, z, \rho)$.

Applying the Envelope Theorem, the first-best allocation can be characterized as stated in Lemma 4.1. The proof is straightforward and omitted.

**Lemma 4.1** *In the full-information case, the welfare optimum* $\{b^{FB}(\rho), z^{FB}(\rho)\}_{\rho \in \Upsilon}$ *satisfies:*

*(i) The intertemporal rate of substitution between current and future public goods consumption equals intertemporal rate of transformation, namely*

$$\frac{g_1'(G_1^{FB}(\rho))}{g_2'(\theta G_1^{FB}(\rho) + \rho \mathcal{G}_2^{FB}(\rho))} = \rho(1+r) - \theta \ \ \text{for any } \rho \in \Upsilon.$$

*(ii) Full insurance is achievable, namely*

$$V_z\left(b^{FB}(\rho), z^{FB}(\rho), \rho\right) = \gamma \ \ \text{for any } \rho \in \Upsilon,$$

*in which* $\gamma > 0$ *denotes the Lagrangian multiplier on the budget constraint* $\int_{\underline{\rho}}^{\bar{\rho}} z(\rho) f(\rho) d\rho \leq 0$.

Part (i) yields that the intertemporal allocation of any type of regions is not distorted in the first-best optimum. Part (ii) gives the standard insurance condition.

To derive the asymmetric-information optimum, we shall need the following assumption:

**Assumption 4.1** $-\rho\mathcal{G}_2 g_2'' \leq g_2'$ *for all* $\rho \in (\underline{\rho}, \bar{\rho})$*, namely the absolute value of the elasticity of marginal utility from consuming public good* $G_2 = \rho\mathcal{G}_2$ *is no greater than one for all but the endpoints of the type distribution.*

This is a technical restriction that is quite similar to Assumption 3.1. Under Assumption 4.1, the center is thought of as solving the following maximization problem:

$$\max \int_{\underline{\rho}}^{\bar{\rho}} v(\rho) f(\rho) d\rho$$

$$\begin{aligned} s.t. \quad v(\rho) &= V(b(\rho), z(\rho), \rho); \\ \int_{\underline{\rho}}^{\bar{\rho}} z(\rho) f(\rho) d\rho &\leq 0; \\ \dot{v}(\rho) &= g_2' \left(\theta\phi(b(\rho), z(\rho), \rho) + \rho\psi(b(\rho), z(\rho), \rho)\right) \psi(b(\rho), z(\rho), \rho); \\ \dot{b}(\rho) &\leq 0 \end{aligned} \tag{10}$$

in which we rewrite public goods expenditures as $\phi(b(\rho), z(\rho), \rho) = G_1(\rho)$ and $\psi(b(\rho), z(\rho), \rho) = \mathcal{G}_2(\rho)$, the first equality constraint gives the value function of regions of type-$\rho$ when they are telling the truth, the second one is the fiscal budget constraint under pure intergovernmental grants, the third one is the first-order necessary condition for incentive compatibility, and the last one is the second-order sufficient condition for incentive compatibility.[17]

Indeed, solving problem (10) gives the following proposition.

**Proposition 4.1** *In the asymmetric-information case without bunching, the welfare optimum* $\{b^*(\rho), z^*(\rho)\}_{\rho\in\Upsilon}$ *satisfies:*

*(i) Suppose Assumption 4.1 holds. Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we have:*

$$\frac{g_1'(G_1^*(\rho))}{g_2'(\theta G_1^*(\rho) + \rho\mathcal{G}_2^*(\rho))} \begin{cases} = \ \rho(1+r) - \theta & \text{for } \rho \in \{\underline{\rho}, \bar{\rho}\}; \\ > \ \rho(1+r) - \theta & \text{for } \rho \in (\underline{\rho}, \bar{\rho}). \end{cases}$$

*(ii) Let* $\mu_1(\rho) > 0$ *be the Lagrangian multiplier on the value constraint* $v(\rho) \equiv V(b(\rho), z(\rho), \rho)$ *of any type-*$\rho$ *region who is reporting truthfully, then we have:*

$$V_z\left(b^*(\rho), z^*(\rho), \rho\right) \begin{cases} = \ \gamma/\mu_1(\rho) & \text{for } \rho \in \{\underline{\rho}, \bar{\rho}\}; \\ > \ \gamma/\mu_1(\rho) & \text{for } \rho \in (\underline{\rho}, \bar{\rho}). \end{cases}$$

[17]The derivation of this monotonicity constraint is given in the proof of Proposition 4.1.

**Proof.** See Appendix A. ■

As shown in Proposition 3.1, the intertemporal allocation under asymmetric information is not distorted only at the endpoints of type distribution, and also there is incomplete insurance.

**Proposition 4.2** *For the current economic environment, the following statements are true.*

*(i) If $\mu_1(\rho)$ is decreasing in $\rho$, then (i-a) there exists some $\breve{\rho} \in (\underline{\rho}, \bar{\rho})$ such that $z^*(\rho) < z^{FB}(\rho)$ for all $\rho \in [\breve{\rho}, \bar{\rho}]$; (i-b) $z^*(\underline{\rho}) > z^{FB}(\underline{\rho})$; and (i-c) $b^*(\underline{\rho}) < b^{FB}(\underline{\rho})$ and $b^*(\bar{\rho}) > b^{FB}(\bar{\rho})$.*

*(ii) If $\mu_1(\rho)$ is increasing in $\rho$, then (ii-a) there exists some $\tilde{\rho} \in (\underline{\rho}, \bar{\rho})$ such that $z^*(\rho) < z^{FB}(\rho)$ for all $\rho \in [\underline{\rho}, \tilde{\rho}]$; (ii-b) $z^*(\bar{\rho}) > z^{FB}(\bar{\rho})$; and (ii-c) $b^*(\underline{\rho}) > b^{FB}(\underline{\rho})$ and $b^*(\bar{\rho}) < b^{FB}(\bar{\rho})$.*

**Proof.** See Appendix A. ■

If $\mu_1(\rho)$ is decreasing in $\rho$, then the shadow prices of the value constraint under truth-telling are larger for lower types of regions than for higher types. As a result, it is more likely to be that higher types *incur* the information rent and receive less transfers under asymmetric information than they would do under full information. Claim (ii-a) can be interpreted in a similar way.

Regardless of whether $\mu_1(\rho)$ is decreasing or increasing in $\rho$, the optimal allocation under asymmetric information is distorted at both top and bottom of the shock distribution, and importantly, the distortion is qualitatively reversed between the top and bottom. For example, if the shadow prices of the value constraint under truth-telling are larger for lower types than for higher types, then the top type receives less transfers while issues more debt, and the bottom type receives more transfers while issues less debt than they would exhibit, respectively, in the full-information optimum.

### 4.1.2 Implementation

To implement the welfare optimum via regionally-decentralized debt decisions, we solve first the maximization problem of regions of type-$\rho$:

$$\max_{b(\rho)} \; V(b(\rho), z(\rho), \rho)$$

for any given $z(\rho)$. Rewriting private consumptions as $c_1 = \tilde{\phi}(b(\rho), z(\rho), \rho)$ and $c_2 = \tilde{\psi}(b(\rho), z(\rho), \rho)$ and applying the Envelope Theorem, the first-order condition is thus written as

$$g_1'\left(y_1 + b(\rho) + z(\rho) - \tilde{\phi}(b(\rho), z(\rho), \rho)\right) = [\rho(1+r) - \theta]\times$$
$$g_2'\left(\theta\left(y_1 + b(\rho) + z(\rho) - \tilde{\phi}(b(\rho), z(\rho), \rho)\right) + \rho\left[y_2 - (1+r)b(\rho) - \tilde{\psi}(b(\rho), z(\rho), \rho)\right]\right). \tag{11}$$

Making use of (11) and Lemma 4.1, we immediately have the following result: The full-information optimum is attained by simply setting $z(\rho) = z^{FB}(\rho)$ for all $\rho \in \Upsilon$.

Under asymmetric information, we obtain the following implementation scheme.

**Proposition 4.3** *Suppose Assumption 4.1 holds and the second-order sufficient condition for incentive compatibility is not binding, namely $\dot{b}(\rho) < 0$. The grant scheme $z^*(b)$ that decentralizes the asymmetric-information optimum $\{b^*(\rho), z^*(\rho)\}_{\rho\in\Upsilon}$ is a nonlinear non-increasing function of $b$, almost everywhere differentiable, with the slope*

$$\frac{dz^*}{db} \begin{cases} = 0 & \text{for } b \in \{b^*(\underline{\rho}), b^*(\bar{\rho})\}; \\ < 0 & \text{for } b \in (b^*(\bar{\rho}), b^*(\underline{\rho})). \end{cases}$$

**Proof.** See Appendix A. ■

The intuition for zero slope at the endpoints of type distribution is the same as that stated for Proposition 3.3. For regions of other types, we have nonzero slopes. In particular, if regions of lower degrees of technological progress (namely smaller $\rho$) are allowed to issue more debt than regions of higher degrees, then the grant scheme that decentralizes the asymmetric-information optimum should be designed such that the former regions get strictly less grants than the latter ones. As we have $V_z > 0$, this grant scheme guarantees that regions with high degrees of technological progress shall not mimic regions of low degrees.

The implication for the optimal funding structure of IPGs with the physical output being unobservable by the center who is in general not involved in the production process, such as local environmental protection, basic science and R&D, is the following: more federal transfers plus less local borrowing for regions of higher degrees of technological process for producing these IPGs, while less federal transfers plus more local borrowing for regions of lower degrees of technological process for producing these IPGs. In consequence, when facing this kind of shocks, federal transfers and local debt exhibit *substitutability* in terms of regional insurance provision.

## 4.2 The Case with Observable Physical Output of Public Goods

### 4.2.1 Welfare Optimum

Now, the value function of regions of type-$\xi$ reads as follows:

$$\begin{aligned} V(b, z, \xi) \equiv \max_{G_1, G_2} \; & u_1(y_1 + b + z - G_1) + g_1(G_1) \\ & + u_2(y_2 - b(1+r) - \xi G_2) + g_2(\theta G_1 + G_2). \end{aligned} \tag{12}$$

The first-order conditions are:

$$\begin{aligned} u_1'(y_1 + b + z - G_1) &= g_1'(G_1) + \theta g_2'(\theta G_1 + G_2) \quad \text{and} \\ \xi u_2'(y_2 - b(1+r) - \xi G_2) &= g_2'(\theta G_1 + G_2). \end{aligned} \tag{13}$$

We still give first the following benchmark result.

**Lemma 4.2** *In the full-information case, the welfare optimum $\{b^{FB}(\xi), z^{FB}(\xi)\}_{\xi\in\Xi}$ satisfies:*

*(i) The intertemporal rate of substitution between current and future private goods consumption equals intertemporal rate of transformation, namely*

$$\frac{u_1'(c_1^{FB}(\xi))}{u_2'(c_2^{FB}(\xi))} = 1 + r \quad \text{for any } \xi \in \Xi.$$

*(ii) Full insurance is achievable, namely*

$$u_1'\left(c_1^{FB}(\xi)\right) = \gamma \quad \textit{for any } \xi \in \Xi,$$

*in which* $\gamma > 0$ *denotes the Lagrangian multiplier on the budget constraint* $\int_{\underline{\xi}}^{\bar{\xi}} z(\xi)f(\xi)d\xi \leq 0$.

Under asymmetric information, the center takes into account truth-telling constraints and solves the following program:

$$\begin{aligned} &\max \int_{\underline{\xi}}^{\bar{\xi}} v(\xi)f(\xi)d\xi \\ s.t. \quad v(\xi) &= V(b(\xi), z(\xi), \xi); \\ \int_{\underline{\xi}}^{\bar{\xi}} z(\xi)f(\xi)d\xi &\leq 0; \\ \dot{v}(\xi) &= -u_2'\left(y_2 - b(\xi)(1+r) - \xi\psi(b(\xi), z(\xi), \xi)\right)\psi(b(\xi), z(\xi), \xi); \\ \dot{b}(\xi) &\leq 0 \end{aligned} \tag{14}$$

in which $\psi(b(\xi), z(\xi), \xi) = G_2(\xi)$ and the constraints can be similarly interpreted as those in program (10).

Moreover, to derive the welfare optimum under asymmetric information, we need the following technical assumption.

**Assumption 4.2** $-G_2 g_2'' \leq g_2'$ *for all* $\xi \in (\underline{\xi}, \bar{\xi})$*, namely the absolute value of the elasticity of marginal utility from* $G_2$ *for generation 2 is no greater than one for all but the endpoints of the type distribution.*

Indeed, solving problem (14) gives the following proposition.

**Proposition 4.4** *In the asymmetric-information case without bunching, the welfare optimum* $\{b^*(\xi), z^*(\xi)\}_{\xi\in\Xi}$ *satisfies:*

*(i) Suppose Assumption 4.2 holds. Concerning the relationship between the intertemporal rate of substitution between current and future private goods consumption and the intertemporal rate of transformation, we have:*

$$\frac{u_1'(c_1^*(\xi))}{u_2'(c_2^*(\xi))} \begin{cases} = \ 1+r & \textit{for } \xi \in \{\underline{\xi}, \bar{\xi}\}; \\ < \ 1+r & \textit{for } \xi \in (\underline{\xi}, \bar{\xi}). \end{cases}$$

*(ii) Let* $\mu_1(\xi) > 0$ *be the Lagrangian multiplier on the value constraint* $v(\xi) \equiv V(b(\xi), z(\xi), \xi)$ *of any type-*$\xi$ *region who is reporting truthfully, then we have:*

$$u_1'\left(c_1^*(\xi)\right) \begin{cases} = \ \gamma/\mu_1(\xi) & \textit{for } \xi \in \{\underline{\xi}, \bar{\xi}\}; \\ < \ \gamma/\mu_1(\xi) & \textit{for } \xi \in (\underline{\xi}, \bar{\xi}). \end{cases}$$

**Proof.** See Appendix A. ■

As before, due to the informational constraint, the intertemporal allocation is not distorted only at the endpoints of shock distribution and there is incomplete insurance. Also, a comparison of the first-best allocation and the asymmetric-information optimum leads to the following proposition.

**Proposition 4.5** *For the current economic environment, the following statements are true.*

*(i) If $\mu_1(\xi)$ is decreasing in $\xi$, then (i-a) there exists some $\tilde{\xi} \in (\underline{\xi}, \bar{\xi})$ such that $z^*(\xi) > z^{FB}(\xi)$ for all $\xi \in [\underline{\xi}, \tilde{\xi}]$; (i-b) $z^*(\bar{\xi}) < z^{FB}(\bar{\xi})$; and (i-c) $b^*(\underline{\xi}) < b^{FB}(\underline{\xi})$ and $b^*(\bar{\xi}) > b^{FB}(\bar{\xi})$ whenever $g_1''/g_2'' \leq \rho\theta(1+r)$.*

*(ii) If $\mu_1(\xi)$ is increasing in $\xi$, then (ii-a) there exists some $\breve{\xi} \in (\underline{\xi}, \bar{\xi})$ such that $z^*(\xi) > z^{FB}(\xi)$ for all $\xi \in [\breve{\xi}, \bar{\xi}]$; (ii-b) $z^*(\underline{\xi}) < z^{FB}(\underline{\xi})$; and (ii-c) $b^*(\underline{\xi}) > b^{FB}(\underline{\xi})$ and $b^*(\bar{\xi}) < b^{FB}(\bar{\xi})$ whenever $g_1''/g_2'' \leq \rho\theta(1+r)$.*

**Proof.** See Appendix A. ∎

If $\mu_1(\xi)$ is decreasing in $\xi$, then the shadow prices of the value constraint under truth-telling are larger for lower types of regions than for higher types. As a result, it is more likely to be that the lower types *extract* the information rent and receive larger grants under asymmetric information than they would under full information. Claim (ii-a) can be interpreted in a similar way.

Regardless of whether $\mu_1(\xi)$ is decreasing or increasing in $\xi$, the optimal allocation under asymmetric information is distorted at both top and bottom of the shock distribution, and importantly, the distortion is qualitatively reversed between the top and bottom. For example, in the case of part (i), the top type receives smaller grants and issues more debt, whereas the bottom type receives larger grants and issues less debt than they would exhibit, respectively, in the full-information optimum.

In particular, if $g_1(\cdot) = \ln G_1$ and $g_2(\cdot) = \ln(\theta G_1 + \xi G_2)$, then $g_1''/g_2'' \leq \rho\theta(1+r)$ implies that $G_2/G_1 \leq [\sqrt{\rho\theta(1+r)} - \theta]\rho$ with $\sqrt{\rho\theta(1+r)} > \theta$; if $g_1(\cdot) = G_1^\alpha$ and $g_2(\cdot) = (\theta G_1 + \xi G_2)^\alpha$ for some parameter $\alpha \in (0,1)$, then $g_1''/g_2'' \leq \rho\theta(1+r)$ implies that $G_2/G_1 \leq \{[\rho\theta(1+r)]^{1/(2-\alpha)} - \theta\}\rho$ with $[\rho\theta(1+r)]^{1/(2-\alpha)} > \theta$. That is, under log or power utility functions of public goods consumption, the sufficient condition for claims (i-c) and (ii-c) to hold is that the growth rate of local public goods provision must be bounded above.

### 4.2.2 Implementation

To implement the welfare optimum via regionally-decentralized debt decisions, we solve first the maximization problem of regions of type-$\xi$:

$$\max_{b(\xi)} V(b(\xi), z(\xi), \xi)$$

for any given $z(\xi)$. Rewriting public goods consumptions as $G_1 = \phi(b(\xi), z(\xi), \xi)$ and $G_2 = \psi(b(\xi), z(\xi), \xi)$ and applying the Envelope Theorem, the first-order condition is thus written as

$$\begin{aligned} &u_1'\left(y_1 + b(\xi) + z(\xi) - \phi(b(\xi), z(\xi), \xi)\right) \\ &= (1+r)u_2'\left(y_2 - (1+r)b(\xi) - \xi\psi(b(\xi), z(\xi), \xi)\right). \end{aligned} \tag{15}$$

Making use of (15) and Lemma 4.2, we immediately have the result: The full-information optimum is attained by simply setting $z(\xi) = z^{FB}(\xi)$ for all $\xi \in \Xi$.

Under asymmetric information, we obtain the following implementation scheme.

**Proposition 4.6** *Suppose Assumption 4.2 holds and the second-order sufficient condition for incentive compatibility is not binding, namely $\dot{b}(\xi) < 0$. Then, the grant scheme $z^*(b)$*

*that decentralizes the asymmetric-information optimum* $\{b^*(\xi), z^*(\xi)\}_{\xi\in\Xi}$ *is a nonlinear nondecreasing function of* $b$*, almost everywhere differentiable, with the slope*

$$\frac{dz^*}{db}\begin{cases} = \ 0 & \textit{for } b \in \{b^*(\underline{\xi}), b^*(\bar{\xi})\}; \\ > \ 0 & \textit{for } b \in (b^*(\bar{\xi}), b^*(\underline{\xi})). \end{cases}$$

**Proof.** See Appendix A. ■

The intuition for zero slope at the endpoints of type distribution is the same as that stated for Proposition 3.3. For regions of other types, we have nonzero slopes. In particular, if regions of higher degrees of technological progress (namely smaller $\xi$) are allowed to issue more debt than regions of lower degrees, then the grant scheme that decentralizes the asymmetric-information optimum should be designed such that the former regions get strictly more grants than do the latter ones. As $V_z > 0$, this grant scheme guarantees that regions with high degrees of technological progress shall not mimic regions of low degrees.

The implication for the optimal funding structure of IPGs with observable physical output, such as parks, public schools and highways, is the following: more (respectively less) federal transfers plus more (respectively less) local borrowing for regions of higher (respectively lower) degrees of technological process for producing these IPGs. As such, when facing this kind of shocks and when each region has the autonomy in choosing the level of its public debt, federal transfers and local debt exhibit *complementarity* in terms of regional insurance provision.

# 5 Conclusion

This paper aims to study theoretically the design and implementation of optimal insurance provision to sub-national regions against privately observable shocks. We consider two types of shocks to regional economies, one of which is to the degree of intergenerational spillovers induced by IPGs and the other is to the degree of technological progress for producing the IPGs.

We focus on the joint design of two widely-adopted public risk-sharing schemes, namely intergovernmental grants that provide *cross-region insurance* along the space dimension and public debt that provides *cross-generation insurance* along the time dimension. To the best of our knowledge, this paper is the first attempt towards the joint design of these two risk-sharing schemes and the formal analysis of their interaction in the course of implementing welfare optimum in the literature of regional insurance provision within federations.

In the welfare optimum, we have the following two predictions. First, the informational asymmetries considered here preclude the availability of complete public insurance under risk-averse individual preferences. This is consistent with the empirical evidence that public insurance schemes need to be complemented by private ones. Second, if the intergovernmental grant received by the bottom type is distorted upward (or is large), then its public debt must be distorted downward (or be low), and vice versa; meanwhile, the direction of distortion is qualitatively reversed between the bottom and top types.

To decentralize truthfully the welfare optimum, we have the following three predictions. First, for the top and bottom types of regions, the intergovernmental grant scheme that decentralizes the asymmetric-information optimum turns out to be independent of regional public debt, regardless of the source of shocks and whether the expenditure on or the output of public goods is observable when regions differ in the degree of technological progress. Second, for all other types of regions when they differ in the degree of intergenerational externality, regional debt is complementary to the grant scheme that decentralizes the welfare optimum. That is, intergenerational insurance and interregional insurance exhibit complementarity. Third, for all other types of regions when they differ in the degree of technological progress, regional debt is complementary to the grant scheme that decentralizes the welfare optimum if only the physical output of public goods is observable; otherwise, regional debt is substitutive to the grant scheme. Therefore, it is worthwhile distinguishing the case of observable input to that of observable output in terms of regional insurance provision.

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# Appendix A: Proofs

**Proof of Proposition 3.1.** We shall complete the proof in four steps.

**Step 1:** We define the value function to a type-$\theta$ region when it is truth-telling as

$$v(\theta) \equiv V(b(\theta), z(\theta), \theta). \tag{16}$$

Applying Envelope Theorem to (2), we get the following first-order necessary condition for the truth-telling constraints (5) to be satisfied:

$$\dot{v}(\theta) = g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta))\phi(b(\theta), z(\theta), \theta), \tag{17}$$

in which $G_1(\theta) \equiv \phi(b(\theta), z(\theta), \theta)$ and $G_2(\theta) \equiv \psi(b(\theta), z(\theta), \theta)$.

We now derive the second-order sufficient condition for incentive compatibility. After some algebra, the local second-order condition of (5) can be written as

$$\dot{b}(\theta) \cdot V_z(b(\theta), z(\theta), \theta) \cdot \frac{\partial}{\partial\tilde{\theta}} \left( \frac{V_b(b(\theta), z(\theta), \tilde{\theta})}{V_z(b(\theta), z(\theta), \tilde{\theta})} \right) \Bigg|_{\tilde{\theta}=\theta} \geq 0.$$

Noting that $V_z(\cdot) = g_1' + \theta g_2' > 0$ and the Spence-Mirrlees property reads as

$$\frac{\partial}{\partial\theta} \left( \frac{V_b}{V_z} \right) = \frac{(1+r)[(g_2')^2 - G_1 g_1' g_2'']}{(g_1' + \theta g_2')^2} > 0,$$

we thus must have

$$\dot{b}(\theta) \geq 0, \tag{18}$$

which gives the desired monotonicity constraint. It is easy to verify that the local second-order condition also implies global optimality of the truth-telling strategy with the help of the above Spence-Mirrlees property.

We can equivalently rewrite (18) as

$$\dot{b}(\theta) = \beta(\theta), \quad \beta(\theta) \geq 0. \tag{19}$$

The problem of the center is therefore to choose piecewise continuous control variables $b(\theta)$ and $z(\theta)$ to maximize

$$\int_{\underline{\theta}}^{\bar{\theta}} v(\theta) f(\theta) d\theta$$

subject to constraints (7), (16), (17) and (19).

**Step 2:** To solve the optimal control problem with integral and inequality constraints, we write the generalized Hamiltonian as:

$$\begin{aligned}\mathcal{H} &= v(\theta)f(\theta) + \mu_1(\theta)[V(b(\theta), z(\theta), \theta) - v(\theta)]f(\theta) + \mu_2(\theta)\beta(\theta) - \gamma z(\theta)f(\theta) \\ &\quad + \eta_1(\theta)g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta))\phi(b(\theta), z(\theta), \theta) + \eta_2(\theta)\beta(\theta),\end{aligned}$$

where $\mu_1(\theta)$, $\mu_2(\theta)$ and $\gamma$ are non-negative Lagrangian multipliers, and $\eta_1(\theta)$ and $\eta_2(\theta)$ are co-state variables. The first-order necessary conditions for a solution to the optimal control problem can now be stated as the state equations (17) and (19), plus

$$\mathcal{H}_z = \mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta) + \eta_1(\theta)[g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z] = 0, \tag{20}$$

$$\mathcal{H}_\beta \;=\; \mu_2(\theta) + \eta_2(\theta) = 0, \tag{21}$$

and

$$\dot{\eta}_1(\theta) \;=\; -\mathcal{H}_v \;=\; [\mu_1(\theta) - 1]f(\theta), \tag{22}$$

$$\dot{\eta}_2(\theta) \;=\; -\mathcal{H}_b \;=\; -\mu_1(\theta)[g_1' - (1+r-\theta)g_2']f(\theta) - \eta_1(\theta)[g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b]. \tag{23}$$

In addition, we have the following transversality conditions:

$$\eta_1(\theta) \;=\; \eta_2(\theta) \;=\; 0 \;\text{ for } \forall\theta \in \{\underline{\theta}, \bar{\theta}\}. \tag{24}$$

**Step 3:** Using (3) and the assumption that $\rho = 1$, we can write private consumptions as functions of debt, transfers and the degree of intergenerational spillovers: $c_1 \equiv \tilde{\phi}(b,z,\theta)$ and $c_2 \equiv \tilde{\psi}(b,z,\theta)$. Applying Implicit Function Theorem to (3), we have these partial derivatives:

$$\begin{aligned}\tilde{\phi}_b(b,z,\theta) &= \frac{g_1''(u_2''+g_2'') - (1+r-\theta)\theta u_2'' g_2''}{\Sigma},\\ \tilde{\psi}_b(b,z,\theta) &= \frac{\theta u_1'' g_2'' - (1+r)(u_1''+g_1'')g_2''}{\Sigma};\end{aligned} \tag{25}$$

and

$$\tilde{\phi}_z(b,z,\theta) = \frac{g_1''(u_2''+g_2'') + \theta^2 u_2'' g_2''}{\Sigma},\;\; \tilde{\psi}_z(b,z,\theta) = \frac{\theta u_1'' g_2''}{\Sigma}; \tag{26}$$

with $\Sigma \equiv (u_1''+g_1'')(u_2''+g_2'') + \theta^2 u_2'' g_2'' > 0$. Using $\phi(b,z,\theta) = y_1 + b + z - \tilde{\phi}(b,z,\theta)$, $\psi(b,z,\theta) = y_2 - b(1+r) - \tilde{\psi}(b,z,\theta)$, (25) and (26), we obtain

$$\begin{aligned}\phi_b &= \frac{u_1''(u_2''+g_2'') + \theta^2 u_2'' g_2'' + (1+r-\theta)\theta u_2'' g_2''}{\Sigma} > 0,\\ \psi_b &= -\frac{\theta u_1'' g_2'' + (1+r)[(u_1''+g_1'')u_2'' + \theta^2 u_2'' g_2'']}{\Sigma} < 0;\end{aligned} \tag{27}$$

and

$$\phi_z = \frac{u_1''(u_2''+g_2'')}{\Sigma} > 0,\;\; \psi_z = -\frac{\theta u_1'' g_2''}{\Sigma} < 0. \tag{28}$$

Using (27) and (28), we get

$$\theta\phi_z + \psi_z = \frac{\theta u_1'' u_2''}{\Sigma} > 0,\;\; \theta\phi_b + \psi_b = -\frac{(1+r-\theta)u_1'' u_2'' + (1+r)g_1'' u_2''}{\Sigma} < 0. \tag{29}$$

Using (29) and (28) gives

$$g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z = \frac{(\theta G_1 g_2'' + g_2')u_1'' u_2'' + u_1'' g_2'' g_2'}{\Sigma} > 0 \tag{30}$$

under Assumption 3.1. Also, it is immediate from (29) and (27) that

$$g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b > 0. \tag{31}$$

**Step 4:** Since we are interested in the case without bunching, the monotonicity constraint (18) must be $\dot{b}(\theta) > 0$, and hence $\mu_2(\theta) = 0$ for all $\theta \in \Theta$ based on the complementary slackness conditions. By (21), we must have $\eta_2(\theta) = 0$ everywhere, yielding $\dot{\eta}_2 \equiv 0$. Consequently, we get from (23) and (31) that

$$\mu_1(\theta)[g_1' - (1+r-\theta)g_2']f(\theta) \;=\; \underbrace{-\eta_1(\theta)[g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b]}_{\leq 0},$$

by which combined with (24) and $\mu_1(\theta)f(\theta) > 0$ for all $\theta \in \Theta$ we have established the result in part (i).

Moreover, using (20) and (30) gives rise to

$$\mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta) \;=\; \underbrace{-\eta_1(\theta)[g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z]}_{\leq 0}$$

under Assumption 3.1. This combined with (24) and $\mu_1(\theta)f(\theta) > 0$ for all $\theta \in \Theta$ completes the proof of part (ii). ■

**Proof of Lemma 3.2.** It follows from (22) and (24) that

$$\int_{\underline{\theta}}^{\bar{\theta}} [\mu_1(\theta) - 1]f(\theta)d\theta \;=\; \eta_1(\bar{\theta}) - \eta_1(\underline{\theta}) \;=\; 0. \tag{32}$$

By (20), $\mu_1(\theta)$ must be everywhere continuous. Therefore, if $\mu_1(\theta)$ is decreasing in $\theta$, then (32) implies that $\mu_1(\theta) - 1$ is first positive and then negative as $\theta$ increases, and that the application of Intermediate Value Theorem yields that there must be some $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $\mu_1(\tilde{\theta}) = 1$, as desired in part (i). The proof of part (ii) can be analogously done. ■

**Proof of Proposition 3.2.** Here we just need to show the proof of part (i) because that of part (ii) is similar. We have by applying $\rho = 1$ and Envelope Theorem to (2) that $V_z = g_1'(\phi(b, z, \theta)) + \theta g_2'(\theta\phi(b, z, \theta) + \psi(b, z, \theta))$. Using this, (28) and (29) gives

$$V_{zz}(b, z, \theta) \;=\; g_1''\phi_z + \theta g_2''(\theta\phi_z + \psi_z) < 0,$$

which combined with Lemma 3.2 produces the the desired results (i-a) and (i-b).

We now proceed to prove result (i-c). It follows from Lemma 3.1 and Proposition 3.1 that the optimal debt policy is a solution to the equation

$$g_1'(\phi(b, z, \theta)) \;=\; (1 + r - \theta)g_2'(\theta\phi(b, z, \theta) + \psi(b, z, \theta)) \tag{33}$$

for any $\theta \in \{\underline{\theta}, \bar{\theta}\}$. Differentiating both sides of equation (33) with respect to $z$ and rearranging the algebra reveal that

$$[g_1''\phi_b - (1 + r - \theta)g_2''(\theta\phi_b + \psi_b)]\frac{db}{dz} = [(1 + r - \theta)\theta g_2'' - g_1'']\phi_z + (1 + r - \theta)g_2''\psi_z.$$

Using (27) and (29) shows that $g_1''\phi_b - (1 + r - \theta)g_2''(\theta\phi_b + \psi_b) < 0$. Differentiating both sides of equation (33) with respect to $G_1$ reveals that $(1+r-\theta)\theta g_2'' = g_1''$. Moreover, using (28) leads us to that $(1 + r - \theta)g_2''\psi_z > 0$. In consequence, we must have $db/dz < 0$ for any $\theta \in \{\underline{\theta}, \bar{\theta}\}$. This combined with results (i-a) and (i-b) completes the proof. ■

**Proof of Proposition 3.3.** By (5) and applying Envelope Theorem to (2), the first-order condition for incentive compatibility can be written as:

$$(g_1' + \theta g_2')\,\frac{dz}{d\theta} \;=\; [(1 + r - \theta)g_2' - g_1']\,\frac{db}{d\theta},$$

by which we arrive at

$$\frac{dz}{db} \;=\; \frac{dz}{d\theta}\frac{d\theta}{db} \;=\; \frac{(1 + r - \theta)g_2' - g_1'}{g_1' + \theta g_2'}. \tag{34}$$

It follows from (20) and (23) that

$$g_1' + \theta g_2' \;=\; \frac{\gamma}{\mu_1(\theta)} - \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}[g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z] \tag{35}$$

and

$$(1 + r - \theta)g_2' - g_1' \;=\; \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}[g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b] \tag{36}$$

whenever there is no bunching. Plugging (35) and (36) in (34) results in

$$\frac{dz}{db} \;=\; \frac{\eta_1(\theta(b))[g_2''(\theta(b)\phi_b + \psi_b)\phi + g_2'\phi_b]}{\gamma f(\theta(b)) - \eta_1(\theta(b))[g_2''(\theta(b)\phi_z + \psi_z)\phi + g_2'\phi_z]}$$

in which $\theta(b)$ is the inverse of $b(\theta)$, which exists given that $\dot{b}(\theta) > 0$. As is obvious, $dz/db$ satisfies the property required. ■

**Proof of Proposition 4.1.** We shall complete the proof in three steps.

**Step 1:** Applying Envelope Theorem to the value function $V(b, z, \rho)$ and simplifying the algebra, we obtain the Spence-Mirrlees property:

$$\frac{\partial}{\partial \rho}\left[\frac{V_b(b, z, \rho)}{V_z(b, z, \rho)}\right] = -(1 + r)\frac{\theta(g_2')^2 + g_1'[g_2' + \rho\mathcal{G}_2 g_2'']}{(g_1' + \theta g_2')^2} \;<\; 0$$

under Assumption 4.1. Noting that $V_z(\cdot) = g_1' + \theta g_2' > 0$, the second-order condition for incentive compatibility can be written as

$$\dot{b}(\rho) \cdot V_z(b(\rho), z(\rho), \rho) \cdot \frac{\partial}{\partial \tilde{\rho}}\left(\frac{V_b(b(\rho), z(\rho), \tilde{\rho})}{V_z(b(\rho), z(\rho), \tilde{\rho})}\right)\Bigg|_{\tilde{\rho}=\rho} \geq 0,$$

which leads to $\dot{b}(\rho) \leq 0$ under Assumption 4.1, as desired in (10). Let's equivalently rewrite this monotonicity constraint as $\dot{b}(\rho) = \beta(\rho)$ and $\beta(\rho) \leq 0$, then the Hamiltonian of the optimal control problem (10) is given by

$$\begin{aligned}\mathcal{H} \;&= v(\rho)f(\rho) + \mu_1(\rho)[V(b(\rho), z(\rho), \rho) - v(\rho)]f(\rho) - \mu_2(\rho)\beta(\rho) - \gamma z(\rho)f(\rho)\\ &\quad + \eta_1(\rho)g_2'(\theta\phi(b(\rho), z(\rho), \rho) + \rho\psi(b(\rho), z(\rho), \rho))\psi(b(\rho), z(\rho), \rho) + \eta_2(\rho)\beta(\rho),\end{aligned}$$

where $\phi(b(\rho), z(\rho), \rho) = G_1(\rho)$, $\psi(b(\rho), z(\rho), \rho) = \mathcal{G}_2(\rho)$, $\mu_1(\rho)$, $\mu_2(\rho)$ and $\gamma$ are non-negative Lagrangian multipliers, and $\eta_1(\rho)$ and $\eta_2(\rho)$ are co-state variables. The first-order necessary conditions are given by

$$\mathcal{H}_z \;=\; \mu_1(\rho)V_z(b(\rho), z(\rho), \rho)f(\rho) - \gamma f(\rho) + \eta_1(\rho)[g_2''(\theta\phi_z + \rho\psi_z)\psi + g_2'\psi_z] = 0, \tag{37}$$

$$\mathcal{H}_\beta \;=\; -\mu_2(\rho) + \eta_2(\rho) = 0, \tag{38}$$

and

$$\dot{\eta}_1(\rho) \;=\; -\mathcal{H}_v \;=\; [\mu_1(\rho) - 1]f(\rho), \tag{39}$$

$$\dot{\eta}_2(\rho) \;=\; -\mathcal{H}_b \;=\; -\mu_1(\rho)\{g_1' - [\rho(1+r) - \theta]g_2'\}f(\rho) - \eta_1(\rho)[g_2''(\theta\phi_b + \rho\psi_b)\psi + g_2'\psi_b]. \tag{40}$$

In addition, we have the following transversality conditions:

$$\eta_1(\rho) \;=\; \eta_2(\rho) \;=\; 0 \;\text{ for } \forall\rho \in \{\underline{\rho}, \bar{\rho}\}. \tag{41}$$

**Step 2:** Applying Implicit Function Theorem to (9) gives rise to:

$$\phi_b = \frac{u_1''(u_2''+\rho^2 g_2'')+\rho\theta(1+r)u_2''g_2''}{M} > 0,\ \ \phi_z = \frac{u_1''(u_1''+\rho^2 g_2'')}{M} > 0; \tag{42}$$

and

$$\psi_b = -\frac{(1+r)[(u_1''+g_1'')u_2''+\theta^2 u_2''g_2'']+\rho\theta u_1''g_2''}{M} < 0,\quad \psi_z = -\frac{\rho\theta u_1''g_2''}{M} < 0 \tag{43}$$

in which $M \equiv (u_1''+g_1'')(u_2''+\rho^2 g_2'')+\theta^2 u_2''g_2'' > 0$. Making use of (42) and (43), we have

$$g_2''(\theta\phi_z+\rho\psi_z)\psi + g_2'\psi_z \ < \ 0 \tag{44}$$

given that

$$\theta\phi_z+\rho\psi_z \ = \ \frac{\theta u_1''u_2''}{M} \ > \ 0. \tag{45}$$

In addition, we get by (42), (43) and

$$\theta\phi_b+\rho\psi_b \ = \ -\frac{[\rho(1+r)-\theta]u_1''u_2''+\rho(1+r)u_2''g_1''}{M} \ < \ 0 \tag{46}$$

that

$$\begin{aligned} & g_2''(\theta\phi_b+\rho\psi_b)\psi + g_2'\psi_b \\ = \ & -\frac{[\rho\mathcal{G}_2 g_2''+g_2'](1+r)(u_1''+g_1'')u_2''-\theta\mathcal{G}_2 u_1''u_2''g_2''+[(1+r)\theta u_2''+\rho u_1'']\theta g_2'g_2''}{M} < 0 \end{aligned} \tag{47}$$

under Assumption 4.1.

**Step 3:** Since we focus on the case without bunching, we must have $\mu_2(\rho)=0$ for all $\rho\in\Upsilon$. By (38), we have $\eta_2(\rho)=0$ everywhere, implying that $\dot\eta_2\equiv 0$. Applying this, (41) and (47) to (40) yields the desired assertion in part (i). Finally, applying (44), (41) and $\mu_1(\rho)f(\rho)>0$ to (37) produces the desired assertion in part (ii). ■

**Proof of Proposition 4.2.** Using (39), the proof is quite similar to that of Proposition 3.2. Here we just need to show the following. Firstly, using (42) and (45) reveals that $V_{zz}=g_1''\phi_z+\theta g_2''(\theta\phi_z+\rho\psi_z)<0$ for all $\rho\in\Upsilon$. Secondly, by differentiating both sides of equation $g_1'=[\rho(1+r)-\theta]g_2'$ with respect to $z$, we obtain

$$\underbrace{[g_1''\phi_b-[\rho(1+r)-\theta]g_2''(\theta\phi_b+\rho\psi_b)]}_{<0}\frac{db}{dz} \ = \ [\rho(1+r)-\theta]g_2''(\theta\phi_z+\rho\psi_z)-g_1''\phi_z$$

under (42) and (46). As we get from (42) and (45) that

$$[\rho(1+r)-\theta]g_2''(\theta\phi_z+\rho\psi_z)-g_1''\phi_z \ = \ -\frac{\rho^2 u_1''g_1''g_2''}{M} > 0,$$

we thus have $db/dz<0$ at the welfare optimum. ■

**Proof of Proposition 4.3.** The key for a grant scheme to decentralize the asymmetric-information optimum is that it takes into account the incentive-compatibility constraint. First, making use of the first-order necessary condition for incentive compatibility, we get

$$\frac{dz}{db} = \frac{dz}{d\rho}\frac{d\rho}{db} = -\frac{V_b}{V_z}.$$

As the monotonicity constraint is assumed to be not binding, we get from (37) and (40) that

$$\frac{dz}{db} = \frac{\eta_1(\rho(b))\left[g_2''(\theta\phi_b + \rho(b)\psi_b)\psi + g_2'\psi_b\right]}{\gamma f(\rho(b)) - \eta_1(\rho(b))\left[g_2''(\theta\phi_z + \rho(b)\psi_z)\psi + g_2'\psi_z\right]},$$

in which $\rho(b)$ denotes the inverse of $b(\rho)$. Secondly, making use of (41), (44) and (47), the proof is immediately complete. ■

**Proof of Proposition 4.4.** We shall complete the proof in two steps.

**Step 1:** As before, the Hamiltonian of the optimal control problem (14) is given by

$$\begin{aligned}\mathcal{H} &= v(\xi)f(\xi) + \mu_1(\xi)[V(b(\xi), z(\xi), \xi) - v(\xi)]f(\xi) - \mu_2(\xi)\beta(\xi) - \gamma z(\xi)f(\xi) \\ &\quad - \eta_1(\xi)u_2'\left(y_2 - (1+r)b(\xi) - \xi\psi(b(\xi), z(\xi), \xi)\right)\psi(b(\xi), z(\xi), \xi) + \eta_2(\xi)\beta(\xi).\end{aligned}$$

The first-order necessary conditions are given by

$$\mathcal{H}_z = \mu_1(\xi)V_z(b(\xi), z(\xi), \xi)f(\xi) - \gamma f(\xi) - \eta_1(\xi)(-\xi u_2''\psi_z\psi + u_2'\psi_z) = 0, \tag{48}$$

$$\mathcal{H}_\beta = -\mu_2(\xi) + \eta_2(\xi) = 0, \tag{49}$$

and

$$\dot{\eta}_1(\xi) = -\mathcal{H}_v = [\mu_1(\xi) - 1]f(\xi), \tag{50}$$

$$\dot{\eta}_2(\xi) = -\mathcal{H}_b = -\mu_1(\xi)[u_1' - (1+r)u_2']f(\xi) + \eta_1(\xi)[-(1+r)u_2''\psi - \xi u_2''\psi_b\psi + u_2'\psi_b]. \tag{51}$$

In addition, we have the following transversality conditions:

$$\eta_1(\xi) = \eta_2(\xi) = 0 \ \text{ for } \forall\xi \in \{\underline{\xi}, \bar{\xi}\}. \tag{52}$$

**Step 2:** Applying Implicit Function Theorem to (13) gives rise to:

$$\phi_b = \frac{u_1''(\xi^2 u_2'' + g_2'') + \xi\theta(1+r)u_2''g_2''}{Q} > 0, \ \phi_z = \frac{u_1''(\xi^2 u_2'' + g_2'')}{Q} > 0; \tag{53}$$

and

$$\psi_b = -\frac{\xi(1+r)(u_1'' + g_1'' + \theta^2 g_2'')u_2'' + \theta u_1''g_2''}{Q} < 0, \quad \psi_z = -\frac{\theta u_1''g_2''}{Q} < 0 \tag{54}$$

in which $Q \equiv (u_1'' + g_1'')(\xi^2 u_2'' + g_2'') + \xi^2\theta^2 u_2''g_2'' > 0$. Now, applying (52) and (54) to (48) gives the desired assertion in part (ii). Moreover, using (54) again reveals that

$$\begin{aligned}&-(1+r)u_2''\psi - \xi u_2''\psi_b\psi + u_2'\psi_b = \\ &-\frac{(1+r)(u_1'' + g_1'')u_2''(g_2''\psi + g_2') + \theta u_1''g_2''(u_2' - \xi\psi u_2'') + \theta^2(1+r)u_1''u_2''g_2'}{Q} < 0\end{aligned} \tag{55}$$

under Assumption 4.2. In the case of no bunching, applying (49), (52) and (55) to (51) produces the desired assertion in part (i). ■

**Proof of Proposition 4.5.** The proof follows from using (50), (52), Lemma 4.2 and Proposition 4.4. Here we just give by using (53), (54) and $u_1' = (1+r)u_2'$ evaluated at the welfare optimum that

$$V_{zz} = u_1''(1-\phi_z) = \frac{u_1''g_1''(\xi^2 u_2'' + g_2'') + \theta^2\xi^2 u_1''u_2''g_2''}{Q} < 0$$

and

$$[(1-\phi_b)u_1'' + (1+r)(1+r+\xi\psi_b)u_2'']\frac{db}{dz} \;=\; -\xi(1+r)u_2''\psi_z - (1-\phi_z)u_1''$$

in which

$$\begin{aligned}&(1-\phi_b)u_1'' + (1+r)(1+r+\xi\psi_b)u_2''\\ =\;& \frac{u_1''g_1''(\xi^2 u_2'' + g_2'') + (1+r-\theta\xi)^2 u_1''u_2''g_2'' + (1+r)^2 u_2''g_1''g_2''}{Q} < 0\end{aligned}$$

and

$$\begin{aligned}&-\xi(1+r)u_2''\psi_z - (1-\phi_z)u_1''\\ =\;& -\frac{\xi u_1''u_2''[\xi g_1'' - (1+r)\theta g_2''] + \theta^2\xi^2 u_1''u_2''g_2'' + u_1''g_1''g_2''}{Q} > 0\end{aligned}$$

whenever $g_1'' \le \rho\theta(1+r)g_2''$ holds. ■

**Proof of Proposition 4.6.** As we focus on the case of no bunching, we have by using the first-order necessary condition for incentive compatibility, (48) and (51) that

$$\frac{dz^*}{db} \;=\; \frac{-\eta_1(\xi(b))[-(1+r)u_2''\psi - \xi(b)u_2''\psi_b\psi + u_2'\psi_b]}{\gamma f(\xi(b)) + \eta_1(\xi(b))[-\xi(b)u_2''\psi_z\psi + u_2'\psi_z]},$$

in which $\xi(b)$ denotes the inverse of $b(\xi)$ under the assumption of $\dot{b}(\xi) < 0$. By using (55) and (52), we see that $dz^*/db$ satisfies the property required. ■

# Appendix B: Discussion on Multidimensional Heterogeneity

Following Lockwood (1999), in the text we focus on the analysis of one-dimensional unobserved heterogeneity, which is either the degree of intergenerational externality induced by IPGs, denoted by parameter $\theta$, or the degree of technological progress for producing the IPGs, denoted by parameter $\rho$ (or equivalently $\xi$). Here we attempt to analyze the case with multidimensional heterogeneity, i.e., regions differ in both the degree of intergenerational externality and the degree of technological progress. Nevertheless, for the sake of obtaining some interesting theoretical results, we need to impose the following restriction:

**Assumption 5.1** *Let* $\xi \equiv \Psi(\theta)$ *and* $\rho = 1/\xi = 1/\Psi(\theta) \equiv \Phi(\theta)$*, in which* $\Psi(\cdot)$ *is a continuously differentiable function satisfying* $\Psi'(\cdot) > 0$*.*

Since both the degree of intergenerational externality and the degree of technological progress are closely related to the IPGs, by Assumption 5.1 we mean that there is a publicly observable functional relationship that governs these two parameters. In particular, $\Psi'(\theta) > 0$ means that a higher degree of intergenerational externality induced by IPGs leads to a higher per unit cost of producing the IPGs. Intuitively, we assume that if a public good is of higher quality, durability or intergenerational spillovers, then the per unit cost of producing it tends to be higher. For example, a study of the construction costs of high-speed railways in China by Ollivier, Sondhi and Zhou (2014) shows that the weighted average unit cost for a passenger-dedicated line was RMB 129 million per km for a 350 km/h project and RMB 87 million per km for a 250 km/h project. As we could reasonably expect that more and more passengers in the future are willing to take trains of higher speeds, we might roughly interpret that a 350 km/h project generates higher intergenerational spillovers than a 250 km/h project, somehow justifying Assumption 5.1. Also, Assumption 5.1 helps us to consider the case with multidimensional heterogeneity but with one-dimensional asymmetric information.

As in Section 4, whenever regions face shocks to the degree of technological process for producing public goods, we need to distinguish the case with observable expenditure on public goods to the case with observable physical output of public goods.

## I: The Case with Observable Expenditure on Public Goods

Applying Assumption 5.1, the FOCs given by equation (9) can be rewritten as:

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + \Phi(\theta)\mathcal{G}_2) \quad \text{and} \quad u_2'(c_2) = \Phi(\theta) g_2'(\theta G_1 + \Phi(\theta)\mathcal{G}_2). \tag{56}$$

As in the text, let the value function be written as $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$. Applying Assumption 5.1 and Envelope Theorem to (2) produces the following first-order necessary condition for incentive compatibility:

$$\begin{aligned} \dot{v}(\theta) \; &= g_2'\left(\theta\phi(b(\theta), z(\theta), \theta) + \Phi(\theta)\psi(b(\theta), z(\theta), \theta)\right) \\ &\quad \times \left[\phi(b(\theta), z(\theta), \theta) + \Phi'(\theta)\psi(b(\theta), z(\theta), \theta)\right], \end{aligned} \tag{57}$$

in which $G_1(\theta) \equiv \phi(b(\theta), z(\theta), \theta)$ and $\mathcal{G}_2(\theta) \equiv \psi(b(\theta), z(\theta), \theta)$. Applying Envelope Theorem to the value function $V(b, z, \theta)$ again gives us $V_b(b, z, \theta) = g_1' + g_2' \cdot [\theta - \Phi(\theta)(1 + r)]$ and $V_z(b, z, \theta) = g_1' + \theta g_2' > 0$ for all $\theta \in \Theta$, by which we can obtain:

**Lemma 5.1** *Under Assumption 5.1, if $G_1(\theta)/\mathcal{G}_2(\theta) \geq -\Phi'(\theta)$, then the global optimality of truth-telling strategy is guaranteed by the second-order condition $\dot{b}(\theta) \geq 0$ for all $\theta \in \Theta$.*

**Proof.** Since by equation (2) and Assumption 5.1 we get

$$\begin{aligned}\frac{\partial}{\partial\theta}\left[\frac{V_b(b,z,\theta)}{V_z(b,z,\theta)}\right] &= \frac{1+r}{(g_1'+\theta g_2')^2} \\ &\times\left\{\underbrace{[\Phi(\theta)-\theta\Phi'(\theta)](g_2')^2-\Phi'(\theta)g_1'g_2'}_{>0}-\underbrace{\Phi(\theta)g_1'g_2''}_{<0}[G_1+\Phi'(\theta)\mathcal{G}_2]\right\},\end{aligned}$$

we thus have

$$\frac{\partial}{\partial\theta}\left[\frac{V_b(b,z,\theta)}{V_z(b,z,\theta)}\right] > 0 \tag{58}$$

whenever $G_1+\Phi'(\theta)\mathcal{G}_2 \geq 0$. Condition (58) thus guarantees the Spence-Mirrlees property. The second-order condition for incentive compatibility can be expressed as:

$$\dot{b}(\theta)\cdot V_z(b(\theta),z(\theta),\theta)\cdot\frac{\partial}{\partial\tilde{\theta}}\left(\frac{V_b(b(\theta),z(\theta),\tilde{\theta})}{V_z(b(\theta),z(\theta),\tilde{\theta})}\right)\Bigg|_{\tilde{\theta}=\theta} \geq 0,$$

which combined with the fact $V_z > 0$ and Spence-Mirrlees property (58) reveals that $\dot{b}(\theta) \geq 0$ must hold. Applying the standard argument given on page 143 of Laffont and Martimort (2002), the proof is then complete. ■

Lemma 5.1 states that truth-telling calls for a regional debt allocation which is non-decreasing in the degree of intergenerational externality. Condition $G_1(\theta)/\mathcal{G}_2(\theta) \geq -\Phi'(\theta)$ means that the ratio of period-1 public goods expenditure to period-2 public goods expenditure is greater than some lower bound. For later use, we give

**Assumption 5.2** *$G_1(\theta)/\mathcal{G}_2(\theta) \geq -\Phi'(\theta)$ for all $\theta \in \Theta$.*

Now, by safely replacing the global incentive-compatibility condition (5) by (57) and $\dot{b}(\theta) \geq 0$ established in Lemma 5.1, the optimization problem facing the center is formalized as:

$$\max\int_{\underline{\theta}}^{\bar{\theta}} v(\theta)f(\theta)d\theta$$

$$\begin{aligned} s.t.\quad v(\theta) &= V(b(\theta),z(\theta),\theta); \\ \int_{\underline{\theta}}^{\bar{\theta}} z(\theta)f(\theta)d\theta &\leq 0; \\ \dot{v}(\theta) &= g_2'\left(\theta\phi(b(\theta),z(\theta),\theta)+\Phi(\theta)\psi(b(\theta),z(\theta),\theta)\right) \\ &\quad\times\left[\phi(b(\theta),z(\theta),\theta)+\Phi'(\theta)\psi(b(\theta),z(\theta),\theta)\right]; \\ \dot{b}(\theta) &\geq 0. \end{aligned}$$

By solving this problem we arrive at the following proposition:

**Proposition 5.1** *Suppose Assumptions 5.1 and 5.2 hold. In the asymmetric-information case without bunching, the welfare optimum $\{b^*(\theta),z^*(\theta)\}_{\theta\in\Theta}$ satisfies:*

*(i) Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we have:*

$$\frac{g_1'(G_1^*(\theta))}{g_2'(\theta G_1^*(\theta)+\Phi(\theta)\mathcal{G}_2^*(\theta))}\begin{cases} = \ \Phi(\theta)(1+r)-\theta & \text{for } \theta\in\{\underline{\theta},\bar{\theta}\}; \\ < \ \Phi(\theta)(1+r)-\theta & \text{for } \theta\in(\underline{\theta},\bar{\theta}). \end{cases}$$

*(ii) Let $\mu_1(\theta)>0$ be the Lagrangian multiplier on the value constraint $v(\theta)\equiv V(b(\theta),z(\theta),\theta)$ of any type-$\theta$ region who is reporting truthfully, we have:*

- *$V_z\left(b^*(\theta),z^*(\theta),\theta\right)=\gamma/\mu_1(\theta)$ for $\theta\in\{\underline{\theta},\bar{\theta}\}$;*
- *If the ratio $G_1(\theta)/\mathcal{G}_2(\theta)$ is sufficiently close to $-\Phi'(\theta)$, then*

  $$V_z\left(b^*(\theta),z^*(\theta),\theta\right)<\gamma/\mu_1(\theta)$$

  *for $\theta\in(\underline{\theta},\bar{\theta})$;*
- *If the ratio $G_1(\theta)/\mathcal{G}_2(\theta)$ is sufficiently larger than $-\Phi'(\theta)$, then we have for any $\theta\in(\underline{\theta},\bar{\theta})$ that:*

  $$V_z\left(b^*(\theta),z^*(\theta),\theta\right)\begin{cases} < \ \gamma/\mu_1(\theta) & \text{for } |\varepsilon_{g_2',\theta\phi+\Phi(\theta)\psi}|\cdot\varepsilon_{\theta\phi+\Phi(\theta)\psi,z}<\varepsilon_{\phi+\Phi'(\theta)\psi,z}, \\ = \ \gamma/\mu_1(\theta) & \text{for } |\varepsilon_{g_2',\theta\phi+\Phi(\theta)\psi}|\cdot\varepsilon_{\theta\phi+\Phi(\theta)\psi,z}=\varepsilon_{\phi+\Phi'(\theta)\psi,z}, \\ > \ \gamma/\mu_1(\theta) & \text{for } |\varepsilon_{g_2',\theta\phi+\Phi(\theta)\psi}|\cdot\varepsilon_{\theta\phi+\Phi(\theta)\psi,z}>\varepsilon_{\phi+\Phi'(\theta)\psi,z}, \end{cases}$$

  *in which $|\varepsilon_{g_2',\theta\phi+\Phi(\theta)\psi}|$ represents the absolute value of the elasticity of $g_2'$ with respect to the amount of period-2 public goods consumption $\theta\phi+\Phi(\theta)\psi$, $\varepsilon_{\theta\phi+\Phi(\theta)\psi,z}>0$ represents the elasticity of the amount of period-2 public goods consumption with respect to the federal transfers $z$, and $\varepsilon_{\phi+\Phi'(\theta)\psi,z}>0$ represents the elasticity of $\phi+\Phi'(\theta)\psi$ with respect to $z$.*

**Proof.** First, we let $\dot{b}(\theta)\equiv\beta(\theta)$ as before, and write the generalized Hamiltonian as:

$$\begin{aligned}\mathcal{H} \ &= v(\theta)f(\theta)+\mu_1(\theta)[V(b(\theta),z(\theta),\theta)-v(\theta)]f(\theta)+\mu_2(\theta)\beta(\theta)-\gamma z(\theta)f(\theta)\\ &\quad+\eta_1(\theta)g_2'(\theta\phi(b(\theta),z(\theta),\theta)+\Phi(\theta)\psi(b(\theta),z(\theta),\theta))\\ &\quad\times[\phi(b(\theta),z(\theta),\theta)+\Phi'(\theta)\psi(b(\theta),z(\theta),\theta)]+\eta_2(\theta)\beta(\theta),\end{aligned}$$

where $\mu_1(\theta)$, $\mu_2(\theta)$ and $\gamma$ are non-negative Lagrangian multipliers, and $\eta_1(\theta)$ and $\eta_2(\theta)$ are co-state variables. The first-order necessary conditions are

$$\begin{aligned}\mathcal{H}_z \ &= \mu_1(\theta)V_z(b(\theta),z(\theta),\theta)f(\theta)-\gamma f(\theta)\\ &\quad+\eta_1(\theta)\{g_2''\cdot[\theta\phi_z+\Phi(\theta)\psi_z][\phi+\Phi'(\theta)\psi]+g_2'\cdot[\phi_z+\Phi'(\theta)\psi_z]\}=0,\end{aligned}\tag{59}$$

$$\mathcal{H}_\beta \ = \ \mu_2(\theta)+\eta_2(\theta)=0,\tag{60}$$

$$\dot{\eta}_1(\theta) \ = \ -\mathcal{H}_v \ = \ [\mu_1(\theta)-1]f(\theta),\tag{61}$$

and

$$\begin{aligned}\dot{\eta}_2(\theta) \ &= \ -\mathcal{H}_b\\ &= \ -\mu_1(\theta)\{g_1'-[\Phi(\theta)(1+r)-\theta]g_2'\}f(\theta)\\ &\quad -\eta_1(\theta)\{g_2''\cdot[\theta\phi_b+\Phi(\theta)\psi_b][\phi+\Phi'(\theta)\psi]+g_2'\cdot[\phi_b+\Phi'(\theta)\psi_b]\}.\end{aligned}\tag{62}$$

In addition, we have the following transversality conditions:

$$\eta_1(\theta) \;=\; \eta_2(\theta) \;=\; 0 \;\text{ for } \forall\theta \in \{\underline{\theta}, \bar{\theta}\}. \tag{63}$$

Following the first-order approach, we must have $\mu_2(\theta) = 0$ for all $\theta \in \Theta$. By (60) we thus have $\eta_2(\theta) = 0$ for all $\theta \in \Theta$, and hence we must have $\dot{\eta}_2 \equiv 0$. Applying $\dot{\eta}_2 \equiv 0$, $\mu_1(\theta)f(\theta) > 0$ and (63) to (62) reveals that $g_1' = [\Phi(\theta)(1 + r) - \theta]g_2'$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in part (i). Also, applying (63) to (59) reveals that $\mu_1(\theta)V_z(b(\theta), z(\theta), \theta) = \gamma$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in part (ii).

By applying Implicit Function Theorem to the FOCs given by (56), we can still have those partial derivatives given by equations (42) and (43). In consequence, we get by (42), (43), (46) and Assumption 5.2 that

$$\underbrace{g_2''}_{<0} \cdot \underbrace{[\theta\phi_b + \Phi(\theta)\psi_b]}_{<0} \cdot \underbrace{[\phi + \Phi'(\theta)\psi]}_{\geq 0} + \underbrace{g_2'}_{>0} \cdot \underbrace{[\phi_b + \Phi'(\theta)\psi_b]}_{>0} > 0, \tag{64}$$

which combined with $\dot{\eta}_2 \equiv 0$, $\mu_1(\theta)f(\theta) > 0$ and (62) concludes the proof of part (i).

Finally, using (42), (43), (45) and Assumption 5.2 shows that

$$\underbrace{g_2'' \cdot [\theta\phi_z + \Phi(\theta)\psi_z]}_{<0} \cdot \underbrace{[\phi + \Phi'(\theta)\psi]}_{\geq 0} + \underbrace{g_2' \cdot [\phi_z + \Phi'(\theta)\psi_z]}_{>0}.$$

Thus, the sign of this formula is positive whenever $\phi + \Phi'(\theta)\psi$ is sufficiently close to zero from above, applying which to equation (59) gives the second result in part (ii). If, however, $\phi + \Phi'(\theta)\psi$ is sufficiently larger than zero, then we have by using (42), (43), (45) and Assumption 5.2 again that:

$$\begin{aligned} &\underbrace{g_2'' \cdot [\theta\phi_z + \Phi(\theta)\psi_z]}_{-} \cdot \underbrace{[\phi + \Phi'(\theta)\psi]}_{+} + \underbrace{g_2' \cdot [\phi_z + \Phi'(\theta)\psi_z]}_{+} \; < 0 \\ \Leftrightarrow \;\; &\underbrace{-\frac{[\theta\phi + \Phi(\theta)\psi]g_2''}{g_2'}}_{|\varepsilon_{g_2', \theta\phi+\Phi(\theta)\psi}|} \cdot \underbrace{\frac{z[\theta\phi_z + \Phi(\theta)\psi_z]}{\theta\phi + \Phi(\theta)\psi}}_{\varepsilon_{\theta\phi+\Phi(\theta)\psi, z}} > \underbrace{\frac{z[\phi_z + \Phi'(\theta)\psi_z]}{\phi + \Phi'(\theta)\psi}}_{\varepsilon_{\phi+\Phi'(\theta)\psi, z}} \end{aligned}$$

applying which to equation (59), therefore, concludes the proof of part (ii). ■

The key message conveyed by Proposition 5.1 can be summarized as follows. First, in the asymmetric-information optimum only the intertemporal allocation at the endpoints of type distribution is not distorted with respect to the first-best. Second, the presence of the asymmetric information between center and regions prevents full insurance from happening.

We now proceed to the implementation of the asymmetric-information optimum established in Proposition 5.1 through decentralized regional debt decisions, which lead to the intertemporal allocation features that intertemporal rate of substitution between current and future public goods consumption equals intertemporal rate of transformation:

$$\frac{g_1'\left(\phi(b(\theta), z(\theta), \theta)\right)}{g_2'\left(\theta\phi(b(\theta), z(\theta), \theta) + \Phi(\theta)\psi(b(\theta), z(\theta), \theta)\right)} = \Phi(\theta)(1 + r) - \theta,$$

which is desired by each region for any given amount of federal transfers. As shown in the text, the task facing the center is to design intergovernmental grants scheme that guarantees incentive compatibility for all regions. Indeed, we can obtain the following result:

**Proposition 5.2** *The grant scheme $z^*(b)$ that decentralizes the asymmetric-information optimum $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ is a nonlinear nondecreasing function of b, almost everywhere differentiable, with the slope*

$$\frac{dz^*}{db}\begin{cases} = \; 0 & \text{for } b \in \{b^*(\underline{\theta}), b^*(\bar{\theta})\}; \\ > \; 0 & \text{for } b \in (b^*(\underline{\theta}), b^*(\bar{\theta})). \end{cases}$$

**Proof.** Making use of the first-order necessary condition for incentive compatibility, we get

$$\frac{dz}{db} = \frac{dz}{d\theta}\frac{d\theta}{db} = -\frac{V_b}{V_z},$$

in which we have shown above that $V_z > 0$ always holds true. As we focus on the case without bunching, we get from (62) and (64) that

$$\begin{aligned} & - V_b\left(b(\theta), z(\theta), \theta\right) \\ & = \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}\{g_2'' \cdot [\theta\phi_b + \Phi(\theta)\psi_b][\phi + \Phi'(\theta)\psi] + g_2' \cdot [\phi_b + \Phi'(\theta)\psi_b]\} > 0 \end{aligned}$$

for all $\theta \in (\underline{\theta}, \bar{\theta})$. We thus have for all $b \in (b^*(\underline{\theta}), b^*(\bar{\theta}))$ that:

$$\frac{dz}{db} = \frac{\eta_1(\theta(b))\{g_2'' \cdot [\theta(b)\phi_b + \Phi(\theta(b))\psi_b][\phi + \Phi'(\theta(b))\psi] + g_2' \cdot [\phi_b + \Phi'(\theta(b))\psi_b]\}}{\mu_1(\theta(b))f(\theta(b))V_z} > 0$$

in which by a little abuse of notation $\theta(b)$ denotes the inverse of $b(\theta)$, which does exist by Lemma 5.1. Finally, applying (63) immediately completes the proof. ■

The main message of Proposition 5.2 is that federal transfers and local debt in general paly a complementary role for regional insurance provision. Except for the bottom and top types whose intertemporal allocations are not distorted in the asymmetric-information optimum, the optimal funding structure in the case with observable expenditure on IPGs exhibits the following feature: regions of a higher degree of intergenerational externality should issue more debt and receive more federal transfers than regions of a lower degree of intergenerational externality.

## II: The Case with Observable Physical Output of Public Goods

Using Assumption 5.1, the value function given by (12) can be rewritten as:

$$\begin{aligned} V(b, z, \theta) \; \equiv \; \max_{G_1, G_2} \;\; & u_1(y_1 + b + z - G_1) + g_1(G_1) \\ & + u_2(y_2 - b(1+r) - \Psi(\theta)G_2) + g_2(\theta G_1 + G_2). \end{aligned} \tag{65}$$

The first-order conditions are thus given by:

$$\begin{aligned} u_1'(y_1 + b + z - G_1) \; &= \; g_1'(G_1) + \theta g_2'(\theta G_1 + G_2) \;\; \text{and} \\ \Psi(\theta)u_2'(y_2 - b(1+r) - \Psi(\theta)G_2) \; &= \; g_2'(\theta G_1 + G_2). \end{aligned} \tag{66}$$

Let $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$. Applying Envelope Theorem to (65) produces the following first-order necessary condition for incentive compatibility:

$$\begin{aligned} \dot{v}(\theta) \; = \; & - u_2'\left(y_2 - b(\theta)(1+r) - \Psi(\theta)\psi(b(\theta), z(\theta), \theta)\right) \Psi'(\theta)\psi(b(\theta), z(\theta), \theta) \\ & + g_2'\left(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta)\right) \phi(b(\theta), z(\theta), \theta), \end{aligned} \tag{67}$$

in which $G_1(\theta) \equiv \phi(b(\theta), z(\theta), \theta)$ and $G_2(\theta) \equiv \psi(b(\theta), z(\theta), \theta)$.

Similar to Lemma 5.1, we now arrive at:

**Lemma 5.2** *Under Assumption 5.1, then the global optimality of truth-telling strategy is guaranteed by the second-order condition $\dot{b}(\theta) \leq 0$ for all $\theta \in \Theta$.*

**Proof.** Applying Envelope Theorem to the value function (65) again gives us $V_b(b, z, \theta) = u_1' - (1+r)u_2'$ and $V_z(b, z, \theta) = u_1' > 0$ for all $\theta \in \Theta$. Under Assumption 5.1, we get

$$\frac{\partial}{\partial \theta}\left[\frac{V_b(b, z, \theta)}{V_z(b, z, \theta)}\right] = (1+r)\frac{u_2'' \Psi'(\theta) G_2}{u_1'} < 0, \tag{68}$$

which thus guarantees the Spence-Mirrlees property. The second-order condition for incentive compatibility can be expressed as:

$$\dot{b}(\theta) \cdot V_z(b(\theta), z(\theta), \theta) \cdot \frac{\partial}{\partial \tilde{\theta}}\left(\frac{V_b(b(\theta), z(\theta), \tilde{\theta})}{V_z(b(\theta), z(\theta), \tilde{\theta})}\right)\Bigg|_{\tilde{\theta}=\theta} \geq 0,$$

which combined with (68) reveals that $\dot{b}(\theta) \leq 0$ must hold. ■

Now, applying (67) and Lemma 5.2, the optimization problem facing the center is formalized as:

$$\max \int_{\underline{\theta}}^{\bar{\theta}} v(\theta) f(\theta) d\theta$$

$$\begin{aligned}
s.t. \quad v(\theta) &= V(b(\theta), z(\theta), \theta); \\
\int_{\underline{\theta}}^{\bar{\theta}} z(\theta) f(\theta) d\theta &\leq 0; \\
\dot{v}(\theta) &= -u_2'\left(y_2 - b(\theta)(1+r) - \Psi(\theta)\psi(b(\theta), z(\theta), \theta)\right)\Psi'(\theta)\psi(b(\theta), z(\theta), \theta) \\
&\quad + g_2'\left(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta)\right)\phi(b(\theta), z(\theta), \theta); \\
\dot{b}(\theta) &\leq 0.
\end{aligned}$$

By solving this problem we arrive at the following proposition:

**Proposition 5.3** *Suppose Assumption 5.1 holds. In the asymmetric-information case without bunching, the welfare optimum $\{b^*(\theta), z^*(\theta)\}_{\theta \in \Theta}$ satisfies:*

*(i) Suppose Assumption 4.2 holds. Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we have:*

$$\frac{u_1'(c_1^*(\theta))}{u_2'(c_2^*(\theta))} \begin{cases} = 1+r & \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < 1+r & \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

*(ii) Let $\mu_1(\theta) > 0$ be the Lagrangian multiplier on the value constraint $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$ of any type-$\theta$ region who is reporting truthfully, we have:*

- *$V_z\left(b^*(\theta), z^*(\theta), \theta\right) = \gamma/\mu_1(\theta)$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$;*

- *If* $|\varepsilon_{g_2',\theta G_1}| \leq 1$*, in which the elasticity is given by* $\varepsilon_{g_2',\theta G_1} \equiv g_2'' \cdot \theta\phi/g_2'$*, then*

$$V_z\left(b^*(\theta), z^*(\theta), \theta\right) < \gamma/\mu_1(\theta)$$

*for* $\theta \in (\underline{\theta}, \bar{\theta})$*.*

**Proof.** First, we let $\dot{b}(\theta) \equiv \beta(\theta)$ as before, and write the generalized Hamiltonian as:

$$\begin{aligned}\mathcal{H} &= v(\theta)f(\theta) + \mu_1(\theta)[V(b(\theta), z(\theta), \theta) - v(\theta)]f(\theta) - \mu_2(\theta)\beta(\theta) - \gamma z(\theta)f(\theta) \\ &\quad - \eta_1(\theta)u_2'\left(y_2 - b(\theta)(1+r) - \Psi(\theta)\psi(b(\theta), z(\theta), \theta)\right)\Psi'(\theta)\psi(b(\theta), z(\theta), \theta) \\ &\quad + \eta_1(\theta)g_2'\left(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta)\right)\phi(b(\theta), z(\theta), \theta) + \eta_2(\theta)\beta(\theta),\end{aligned}$$

where $\mu_1(\theta)$, $\mu_2(\theta)$ and $\gamma$ are non-negative Lagrangian multipliers, and $\eta_1(\theta)$ and $\eta_2(\theta)$ are co-state variables. The first-order necessary conditions are

$$\begin{aligned}\mathcal{H}_z &= \mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta) \\ &\quad + \eta_1(\theta)\left[u_2'' \cdot \Psi(\theta)\Psi'(\theta)\psi\psi_z - u_2' \cdot \Psi'(\theta)\psi_z\right] \\ &\quad + \eta_1(\theta)\left[g_2'' \cdot (\theta\phi_z + \psi_z)\phi + g_2' \cdot \phi_z\right] = 0,\end{aligned} \tag{69}$$

$$\mathcal{H}_\beta = -\mu_2(\theta) + \eta_2(\theta) = 0, \tag{70}$$

$$\dot{\eta}_1(\theta) = -\mathcal{H}_v = [\mu_1(\theta) - 1]f(\theta), \tag{71}$$

and

$$\begin{aligned}\dot{\eta}_2(\theta) &= -\mathcal{H}_b \\ &= -\mu_1(\theta)[u_1' - (1+r)u_2']f(\theta) \\ &\quad + \eta_1(\theta)\{-u_2'' \cdot [1 + r + \Psi(\theta)\psi_b]\Psi'(\theta)\psi + u_2' \cdot \Psi'(\theta)\psi_b\} \\ &\quad - \eta_1(\theta)[g_2'' \cdot (\theta\phi_b + \psi_b)\phi + g_2' \cdot \phi_b].\end{aligned} \tag{72}$$

In addition, we have the following transversality conditions:

$$\eta_1(\theta) = \eta_2(\theta) = 0 \text{ for } \forall\theta \in \{\underline{\theta}, \bar{\theta}\}. \tag{73}$$

Following the first-order approach, we must have $\mu_2(\theta) = 0$ for all $\theta \in \Theta$. By (70) we thus have $\eta_2(\theta) = 0$ for all $\theta \in \Theta$, and hence we must have $\dot{\eta}_2 \equiv 0$. Applying $\dot{\eta}_2 \equiv 0$, $\mu_1(\theta)f(\theta) > 0$ and (73) to (72) reveals that $u_1' = (1+r)u_2'$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in part (i). Also, applying (73) to (69) reveals that $\mu_1(\theta)V_z(b(\theta), z(\theta), \theta) = \gamma$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in part (ii).

By applying Implicit Function Theorem to the FOCs given by (66), we can still have those partial derivatives given by equations (53) and (54). In consequence, we get by (53) and (54) that

$$\theta\phi_b + \psi_b = \frac{\{[\theta\Psi(\theta) - (1+r)]u_1'' - (1+r)g_1''\}\Psi(\theta)u_2''}{Q} < 0 \tag{74}$$

and

$$1 + r + \Psi(\theta)\psi_b = \frac{[1 + r - \theta\Psi(\theta)]u_1''g_1'' + (1+r)g_1''g_2''}{Q} > 0. \tag{75}$$

Applying $\dot{\eta}_2 \equiv 0$, Assumption 5.1, (53), (54), (74) and (75) to (72) yields that

$$\begin{aligned} & \mu_1(\theta)[u_1' - (1+r)u_2']f(\theta) \\ = \; & \eta_1(\theta)\{\underbrace{-u_2'' \cdot [1+r+\Psi(\theta)\psi_b]\Psi'(\theta)\psi}_{+} + \underbrace{u_2' \cdot \Psi'(\theta)\psi_b}_{-}\} \\ & - \eta_1(\theta)\underbrace{[g_2'' \cdot (\theta\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]}_{+}. \end{aligned} \tag{76}$$

As such, applying $\mu_1(\theta)f(\theta) > 0$, (55) (which uses Assumption 4.2), and Assumption 5.1 to (76) concludes the proof of part (i).

In addition, applying (53), (54) and Assumption 5.1 to (69) gives rise to

$$\begin{aligned} & [\mu_1(\theta)V_z(b(\theta), z(\theta), \theta) - \gamma]\, f(\theta) \\ = \; & -\eta_1(\theta)\underbrace{[u_2'' \cdot \Psi(\theta)\Psi'(\theta)\psi\psi_z - u_2' \cdot \Psi'(\theta)\psi_z]}_{+} \\ & -\eta_1(\theta)\left\{[g_2' + g_2'' \cdot \theta\phi]\underbrace{\phi_z}_{+} + \underbrace{g_2'' \cdot \psi_z\phi}_{+}\right\}, \end{aligned}$$

which combined with

$$g_2' + g_2'' \cdot \theta\phi \geq 0 \;\Leftrightarrow\; \underbrace{-\frac{\theta\phi g_2''}{g_2'}}_{\varepsilon_{g_2',\theta G_1}} \leq 1$$

completes the proof of part (ii). ■

The key message conveyed by Proposition 5.3 can be summarized as follows. First, in the asymmetric-information optimum only the intertemporal allocation at the endpoints of type distribution is not distorted with respect to the first-best. Second, the presence of the asymmetric information between center and regions prevents full insurance from happening.

Next we characterize the scheme of federal transfers than decentralizes the asymmetric-information optimum established in Proposition 5.3.

**Proposition 5.4** *Suppose Assumptions 4.2 and 5.1 hold. The grant scheme $z^*(b)$ that decentralizes the asymmetric-information optimum $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ is a nonlinear non-decreasing function of b, almost everywhere differentiable, with the slope*

$$\frac{dz^*}{db} \begin{cases} = \; 0 & \textit{for } b \in \{b^*(\underline{\theta}), b^*(\bar{\theta})\}; \\ > \; 0 & \textit{for } b \in (b^*(\bar{\theta}), b^*(\underline{\theta})). \end{cases}$$

**Proof.** Using the first-order necessary condition for incentive compatibility gives

$$\frac{dz}{db} = \frac{dz}{d\theta}\frac{d\theta}{db} = -\frac{V_b}{V_z},$$

in which we have shown above that $V_z > 0$ always holds true. As we focus on the case without bunching, we get from (76) (which uses Assumption 5.1) and (55) (which uses

Assumption 4.2) that

$$\begin{aligned}
& -V_b\left(b(\theta), z(\theta), \theta\right) \\
= & -\frac{\eta_1(\theta)}{\mu_1(\theta) f(\theta)} \underbrace{\{-u_2'' \cdot [1 + r + \Psi(\theta)\psi_b]\Psi'(\theta)\psi + u_2' \cdot \Psi'(\theta)\psi_b\}}_{-} \\
& + \frac{\eta_1(\theta)}{\mu_1(\theta) f(\theta)} \underbrace{[g_2'' \cdot (\theta\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]}_{+}
\end{aligned}$$

for all $\theta \in (\underline{\theta}, \bar{\theta})$. We thus have for all $b \in (b^*(\bar{\theta}), b^*(\underline{\theta}))$ that:

$$\begin{aligned}
\frac{dz}{db} = & -\frac{\eta_1(\theta(b))\{-u_2'' \cdot [1 + r + \Psi(\theta(b))\psi_b]\Psi'(\theta(b))\psi + u_2' \cdot \Psi'(\theta(b))\psi_b\}}{\mu_1(\theta(b)) f(\theta(b)) V_z} \\
& + \frac{\eta_1(\theta(b))[g_2'' \cdot (\theta(b)\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]}{\mu_1(\theta(b)) f(\theta(b)) V_z} > 0,
\end{aligned}$$

in which by a little abuse of notation $\theta(b)$ denotes the inverse of $b(\theta)$, which does exist by Lemma 5.2. Finally, applying (73) immediately completes the proof. ■

The main message of Proposition 5.4 is that federal transfers and local debt in general paly a complementary role for regional insurance provision. Except for the bottom and top types whose intertemporal allocations are not distorted in the asymmetric-information optimum, the optimal funding structure in the case with observable physical output of IPGs exhibits the following feature: regions of a higher degree of intergenerational externality should issue less debt and receive less federal transfers than regions of a lower degree of intergenerational externality.