

Household Debt Overhang and Transmission of Monetary Policy*

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Abstract

We investigate how the level of household indebtedness affects the monetary transmission mechanism in the U.S. economy. Using state-dependent local projection methods, we find that the effects of monetary policy are less powerful during periods of high household debt. In particular, the impact of monetary policy shocks is smaller on GDP, consumption, residential investment, house prices and household debt during a high-debt state. We then build a partial equilibrium model of borrower households with financial constraints to rationalize these facts. The model points to the weakening of the home-equity loan channel as a possible reason for the decline in monetary policy effectiveness when initial debt levels are high.

Keywords: Household debt, monetary policy, home-equity loans

JEL Classification: E21, E32, E52

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1 Introduction

In this paper, we investigate whether the prevailing level of household debt can have a moderating effect on the transmission of monetary policy, an issue that has gained more importance in light of the tepid recovery following the recent financial crisis. In the first part of the paper, we employ *state*-dependent local projection methods as in Jorda (2005) and Ramey and Zubairy (2018) to generate empirical impulse responses of various macroeconomic variables to a monetary policy shock in the U.S., conditional on the existing level of household debt when the monetary stimulus occurs. Our results indicate that the effectiveness of monetary stimulus can be curtailed during periods of high household debt. In particular, the effects of a monetary shock on GDP, consumption, residential investment, house prices and household debt are significantly smaller when the initial level of household debt-to-GDP ratio is high relative to its long-run trend. These results are robust to employing alternative identification schemes for monetary shocks using timing restrictions and the narrative identification approach of Romer and Romer (2004). Our baseline sample uses pre-crisis data, but the results are by and large robust to using a longer time-series that includes the post-crisis period and constructing the high and low debt states employing alternative filtering techniques.

In the second part of the paper, we rationalize our empirical findings using a small-scale theoretical model that features some of the key channels of the monetary policy transmission mechanism operating through borrower households, and examine how the effectiveness of these channels can change based on the households' initial debt stock. The model is a simplified version of the model in Alpanda and Zubairy (2017), and features long-term fixed- or adjustable-rate debt contracts as in Kydland et al. (2012) and Garriga et al. (2017), distinguishing between the stock and the flow of debt. The model is purposefully kept simple and partial equilibrium to focus on the effects of a policy rate cut on the interest burden of borrowers (i.e., interest rate channel), and their new borrowing through home-equity loans (i.e., home-equity loan channel).¹ Using our model, we show that both channels of monetary transmission can be curtailed when existing debt levels are high, with the latter channel playing a more important role. In particular, agents' borrowing capacity becomes severely constrained under high levels of initial debt rendering the home-equity channel mute, despite the monetary stimulus' favorable effects on house prices and therefore home equity levels.² We show that the interest rate channel is effective in reducing a borrowers' interest burden on pre-existing debt if the loans are adjustable rate, regardless of debt levels. However, under fixed-rate contracts, the interest-rate channel of monetary policy can also be impaired, since agents are unable to lower the interest burden on their existing debt through refinancing when their existing

¹Note that the home-equity loan channel directly affects new borrowing or the flow of debt, unlike the interest rate channel that primarily affects the stock of pre-existing debt (at least when loans are adjustable rate or refinancing is allowed for fixed-rate loans). Garriga et al. (2017) also features the interest rate channel operating on the stock of debt, while their channel operating only through new loans are related to purchases of new houses (i.e., residential investment) rather than home-equity loans featured in our framework.

²The home-equity channel is similar to the "financial accelerator" mechanism in Kiyotaki and Moore (1997) and Bernanke et al. (1999). In our model, the agents build home equity as house prices increase regardless of their existing debt level, but they can tap into this newly increased home equity only when their initial debt levels are low and they satisfy the credit constraint on home equity extraction.

debt levels are high and they do not possess adequate equity in their homes as a result. Thus, the overall expansionary impact of monetary stimulus is curtailed under high initial debt levels through the weakening of both channels, especially the home-equity channel, and the agents are forced to delever, reminiscent of a debt overhang, before they can benefit from an interest rate cut (Dynam, 2012).

In the earlier literature, the state dependence of the effectiveness of monetary policy has been studied mostly in the context of recessionary versus expansionary periods. See, for instance, Weise (1999), Lo and Piger (2005) and Tenreyro and Thwaites (2016).³ In contrast, we focus on the state of household debt to capture the potential non-linearity in the responses to monetary policy shocks. Note that high debt episodes in our sample do not necessarily coincide with recessionary episodes and in fact, recessions are distributed equally between our high debt and low debt episodes; hence, we are capturing a different non-linearity in the responses to monetary policy.⁴

There is also a vast literature looking at the effectiveness of monetary policy particularly in the Great Recession, and some of the channels discussed are through household debt levels. Mian and Sufi (2014) argue that after house prices collapsed during the recent recession, households significantly reduced their spending levels in order to rebuild their lost retirement savings, and because they no longer had sufficient home equity to use as collateral for borrowing or to be able to refinance into a lower mortgage rate. Similarly, Sufi (2015) argues that monetary policy since the crisis has been ineffective “because it has channeled interest savings and additional credit to exactly the households that are least likely to change their spending in response. The households that would normally spend most aggressively out of monetary policy shocks are heavily indebted or have seen their credit scores plummet, rendering them either unwilling or unable to boost spending”. Others also find that during this period some households that wanted to borrow further found it more difficult to do so, since their home equity had dwindled, which made it harder (or impossible) for them to tap into their home equity lines of credit (Bhutta and Keys, 2016) or to refinance into lower mortgage rates (Chen et al., 2013; Beraja et al., 2015).⁵ The aforementioned studies mostly focus on the Great Recession period and use cross-sectional techniques. For instance, Beraja et al. (2015) use MSA loan-level data from the U.S. to show that the expansionary effects of monetary policy following the financial crisis were weaker in U.S. states where house prices were more depressed,

³Others, such as Angrist et al. (2017) and Barnichon and Matthes (2016) explore non-linearities in the responses to monetary policy by considering whether contractionary policy has different effects from expansionary policy.

⁴Tenreyro and Thwaites (2016) show that the effects of monetary policy are less powerful in recessions as opposed to expansions and in Section 3.4, we provide consistent evidence of how the effects of monetary policy shocks are weakened further when a high debt episode coincides with a recession.

⁵There are other reasons why high household debt may curtail the effectiveness of monetary policy. To the extent that high household debt is accompanied by an excess supply of housing during a boom, a monetary policy expansion during the bust may become less effective due to the inability to increase construction activity and housing starts. The possibility of future leverage-related financial crises may also moderate the effects of current monetary policy. For example, if the probability of a future crisis depends positively on the aggregate debt level (or on the debt gap) as in Jorda et al. (2015) and Alpanda and Ueberfeldt (2016), then agents may become more cautious about borrowing and consuming further in periods with high aggregate debt relative to periods with low aggregate debt, even in normal times.

attributing this to the weakening of the refinancing channel when home equity levels are low.⁶ Other examples of studies employing micro-data and focusing primarily on the Great Recession include Bhutta and Keys (2016), Di Maggio, Kermani, and Palmer (2016) and Di Maggio et. al (2017). In this paper, we analyze the role of household indebtedness in the transmission of monetary policy in the time series dimension, by considering high- and low-debt episodes in the U.S. over the last few decades. Furthermore, we end our baseline sample in 2007 excluding the Great Recession period, but nevertheless arrive at rather similar findings.

While our theoretical focus is on the limited ability of highly-indebted households to borrow in response to a rate cut, they may also be less willing to borrow when they are already highly indebted. Banerjee (2011) argues that buying a house is a commitment for future cash outflows, and agents may increase precautionary savings following a house purchase financed by a mortgage. Mody et al. (2012) find that at least two-fifths of the increase in household saving between 2007 and 2009 can be attributed to the precautionary savings motive, and highlight the effects of income uncertainty on precautionary savings in a small-scale partial equilibrium model of *saver* households. Our partial equilibrium model is similar to theirs, except that our model features *borrower* households and focuses on the effects of initial debt on borrowing and expenditure.⁷

We should also note that there is not full consensus in the literature regarding the relationship between the levels of household debt and the strength of monetary policy transmission. In particular, Calza et al. (2016) find that countries with high mortgage debt-to-GDP ratios exhibit larger responses to monetary policy. Note however that they are focusing on the level of debt, which can be thought of as a proxy for financial development. In contrast, we focus on the *debt gap*, which is the deviation of household debt-to-GDP from its trend, where financial development might be a factor driving the trend. As our theoretical model in Section 4 illustrates, monetary policy can have a larger impact on the economy when the *trend level* of debt is higher, since the interest rate channel is stronger in this case, consistent with the findings of Calza et al. (2016). Nevertheless, monetary policy can become less effective overall during periods of high *debt gaps*, since home equity extractions may become curtailed. Also, Agarwal and Qian (2014) and Baker (2017) find that positive income shocks have a larger impact on households that are highly indebted and are credit or liquidity constrained, suggesting that monetary stimulus could potentially also have stronger effects as the overall level of debt increases.⁸

⁶Charles Goodhart (2014), on the other hand, argues that the decline in monetary policy effectiveness during the Great Recession is mainly due to the weakening in the bank lending and bank capital channels of monetary policy transmission, due to an increase in excess reserves and regulatory capital requirements, respectively.

⁷There is also a related literature that focuses on debt overhang in businesses following Myers (1977). In particular, debt increases the probability of a bankruptcy and therefore loss of collateral, which has a disincentivizing effect on acquiring new investment goods through debt. In the housing realm, this type of debt overhang can be manifested as households' unwillingness to spend on maintenance, an important component of residential investment and durable consumption goods (Melzer, 2016). There may also be a related adverse impact on household labor supply due to household debt overhang (Bernstein, 2016).

⁸Also see Albuquerque and Krustev (2015) who use data on U.S. states to find that excessive indebtedness exerted a meaningful drag on consumption over and beyond income and wealth effects during the post-crisis recovery, but that indebtedness begins to bite only at high levels, indicating a non-linear relationship between the level of initial household debt and consumption.

The next section introduces the econometric model we use to test whether the effectiveness of monetary policy depends on the level of household debt, and Section 3 presents the results from this empirical framework. Section 4 introduces a small-scale partial equilibrium model that illustrates some of the key channels of the monetary policy transmission mechanism that operate through borrower households, and examines how the strength of these channels changes based on the initial debt stock of borrowers. Section 5 concludes.

2 Econometric methodology

We follow the methodology of Ramey and Zubairy (2018) and apply the local projection technique proposed in Jordà (2005) to estimate state-dependent models and calculate impulse responses. The Jordà method simply requires estimation of a series of regressions for each horizon, h , and for each variable. The linear model looks as follows:

$$z_{t+h} = \alpha_h + \psi_h(L)y_t + \beta_h shock_t + \varepsilon_{t+h}, \text{ for } h = 0, 1, 2, \dots \quad (1)$$

where z is the variable of interest, y is a vector of control variables, $\psi_h(L)$ is a polynomial in the lag operator, and $shock$ is the identified monetary shock. The coefficient β_h gives the response of z at time $t+h$ to the shock at time t . Thus, one constructs the impulse responses as a sequence of the β_h 's estimated in a series of separate regressions for each horizon.

This method is easily adapted to estimating a state-dependent model. In particular, we estimate a set of regressions for each horizon h as follows:

$$z_{t+h} = I_{t-1} [\alpha_{A,h} + \psi_{A,h}(L)y_t + \beta_{A,h} shock_t] + (1 - I_{t-1}) [\alpha_{B,h} + \psi_{B,h}(L)y_t + \beta_{B,h} shock_t] + \varepsilon_{t+h}, \quad (2)$$

where $I_{t-1} \in \{0, 1\}$ is a dummy variable that indicates the *state* of the economy in terms of household indebtedness before the monetary policy shock hits. In particular, I_{t-1} takes a value of 1 in the high-debt state and 0 otherwise. We discuss the construction of this dummy variable in more detail in the next subsection. We allow all the coefficients of the model (other than deterministic trends) to vary according to this state of the economy. One particular complication associated with the Jordà method is the serial correlation in the error terms induced by the successive leading of the dependent variable. Thus, we use the Newey-West correction for our standard errors (Newey and West, 1987).

We could potentially conduct the analysis employing a threshold VAR, but the local projection methodology is more robust to model misspecification as it does not impose any underlying restrictions on the dynamics of the variables in the system like VARs. Another merit of using the Jordà's local projection methodology is that, in contrast to regime-switching VARs, we do not need to take a stance on how long a given state lasts, and how the economy transitions between high and low

debt states, when constructing impulse responses. The coefficients β_h 's, in the equations above, capture the average effect of the monetary policy shock based on the initial state, given by high debt or low debt, and thus also embed the average effects of the shock for a possible change in the state. Additionally, the natural transitions in the data between the two states due to other factors in the economy are captured by the lagged control variables on the right hand side.⁹ Ramey and Zubairy (2018) show that these implicit assumptions regarding the transition of the state in the local projection methodology versus threshold VARs can potentially lead to very different impulse response functions. Given that the high and low debt states, as we define in the next section, are particularly persistent, both methodologies result in similar findings in our case. In Appendix E, we show that our results are robust to employing a threshold vector autoregression (TVAR).

2.1 Defining the high-debt state

In order to test whether the transmission of monetary shocks depends on the initial level of household debt, we first need to define which periods constitute a high-debt state. We base our state variable on the household debt-to-GDP ratio, since this accounts for the effects of population growth and changes in economic conditions. In order to define the high- and low-debt states, we construct a *debt gap* measure by considering the deviation of the debt-to-GDP ratio from a smooth trend. We construct this trend by running a Hodrick and Prescott (1997) (HP) filter with a very high smoothing parameter, $\lambda = 10^4$. This approach is useful in capturing the longer duration of credit cycles, and has been previously used in the literature (e.g. Drehmann and Tsatsaronis, 2014; Bernadini and Peersman, 2016).¹⁰ In particular, our choice of λ assumes that credit cycles are twice as long as business cycles.¹¹ We show a comparison of the implied debt gap under this definition with alternative measures of debt overhang and alternative values of the smoothing parameter, λ , in Appendix B. Overall, since our approach employs a discrete or dummy variable approach in characterizing high- and low-debt states, these differences in definitions do not yield very different turning points.

Given the well-known endpoint problem of the two-sided HP-filter, we filter the debt-to-GDP series from 1951 to 2015, but exclude both the start and end of this sample in our baseline case and use debt gap data only from 1955 to 2007. Figure 1 shows the level of household debt-to-GDP ratio for the full sample and the vertical lines indicate the subsample that we employ in our estimation. For the sample period 1952q1-2015q3, we have a positive debt gap (i.e., a high-debt

⁹There are some disadvantages as well, since the local projection methodology is non-parametric, there is a loss in efficiency and precision, particularly at longer horizons. In our application of monetary policy shocks, we thus focus on impulse response functions for short-run horizons up to 16 quarters.

¹⁰In particular, the BIS assigns credit-to-GDP gap measure a lot of importance as a guide to policymakers, given that it has been shown to be a useful early warning indicator for banking crises.

¹¹Typically, λ is set at 1600 when extracting business cycle frequencies up to 8 years. Ravn and Uhlig (2005) show that the filter parameter should be adjusted by multiplying it with the fourth power of the observation frequency ratio. Our choice of 10^4 , is close to $1600 \times 2^4 \approx 2.5 \times 10^4$, implying twice the business cycle. BIS tends to use a higher smoothing parameter of 5×10^5 , when they construct credit-to-GDP gap, since they use it as an indicator for banking crises, which on average occur every 20 to 25 years in the samples they consider.

state) in nearly 50 percent of the sample. As shown in Figure 1, the high-debt state corresponds with four distinct periods: 1956q2-1968q4, 1979q1-1980q4, 1985q4-1992q3 and 2003q2-2011q1.¹² The high debt episodes identified coincide with the rise and fall of household debt around the credit crunch of 1966, the savings and loan crisis in the 1980s and the large change in household debt accompanied with the house price boom in the run-up to the Great Recession.

In order to further understand the reason behind considering the gap instead of the level of household debt-to-GDP to define debt states, it is useful to see the evolution of the variable over time. Figure 1 plots the level of household debt-to-GDP ratio from 1951 to 2015, along with its smooth HP-trend. It is apparent that the debt-to-GDP has had an upward trend throughout the sample, except for the large deleveraging episode following the recent financial crisis. The trend level of debt-to-GDP has likely been increasing over time due to financial innovations, such as advances in credit scoring, and technological progress in related fields that have significantly eased the flow of information, reducing frictions between lenders and borrowers. In addition, since the 1980s, there has also been a strengthening in potential fundamentals driving household debt, such as house prices and home-ownership rates, which determine collateral for borrowing, accompanied by a decline in mortgage rates, which determines borrowing costs. Although these developments on the trend level of debt may have also affected monetary policy transmission, our focus here is whether state-dependence of monetary policy is related to the deviations of debt from its fundamental (or trend) level. This choice is also partly informed by our theoretical model in Section 4, which is also consistent with considering the deviation of debt from a steady state level.

2.2 Identifying monetary shocks

In our baseline specification, the main identifying assumption is that current GDP and inflation are in the information set of the monetary authority. This is a standard identification approach employed in structural VARs that identify monetary shocks using residuals from a 3-variable VAR, where the federal funds rate is ordered last after GDP and inflation and employs a Cholesky decomposition. In our case, this is equivalent to using the contemporaneous federal funds rate as the shock in Equations (1) and (2), and ensuring that the contemporaneous and the lagged values of GDP and inflation, along with the lagged values of federal funds rate, are part of y_t in Equations (1) and (2). This method of identification allows us to use a sample period spanning 1955q1-2007q4. The start of the sample is characterized by the availability of quarterly data for federal funds rate, and the sample ends in 2007q4 to avoid the ZLB period on the federal funds rate. As noted before, close to 50 percent of the sample is classified as high debt period for this sample period. In Appendix B, we also show that the distribution of these monetary shocks in high- and low-debt states look very similar in terms of sign and size.

We also conduct a robustness check on our results by identifying the monetary policy shocks as in Romer and Romer (2004). These shock series are obtained as residuals from a regression of the

¹²In Section 3.3, we also consider alternative methods of filtering.

first difference of the intended federal funds rate target, identified using a narrative approach, on lagged values of output and inflation and Greenbook forecasts, which are all in the Federal Reserve’s information set. As argued by Romer and Romer (2004), the obtained residuals are exogenous with respect to the evolution of economic activity. However, this shock is available for a shorter sample spanning 1969q1-2007q4.¹³ For this sample period, close to 36 percent of the sample is classified as a high-debt period.

3 Empirical Results

In this section, we report results from the state-dependent model on the effects of monetary policy shocks, first for the case where the monetary shock is identified under timing restrictions (i.e., Cholesky decomposition), and then for a shorter sample and with the alternative identification of monetary shocks following Romer and Romer (2004). Note that in both cases we consider our baseline definition of household debt gap, constructed using a two-sided HP-trend. We then conduct several robustness checks based on the definition of the state and threshold, as well as sample choice. Finally, we explore how the results are affected if the high-debt states and recessions interact.

3.1 Identification of monetary shock under timing restrictions

The source and the exact definition of the data series used in the analysis are given in Appendix A. In our baseline model, we use two lags of GDP, inflation and federal funds rate as control variables, as suggested by both the Akaike and Schwartz information criteria.¹⁴ In addition, for each variable of interest, besides these three, we also add two lags of the variable itself to the set of controls.

Figure 2 shows the impulse response functions to a 100 basis points (bps) expansionary shock to the federal funds rate for our baseline specification. We first consider results from the linear model, as shown in the second column. In response to an expansionary monetary shock, we observe a rise in GDP, investment and consumption, where the responses peak between 8 to 10 quarters after impact. The response of inflation, on the other hand, is negative on impact and increases above the initial position with a delay of few quarters, and is thus subject to the well documented *price puzzle*. In response to an expansionary monetary shock, we also see a rise in the debt-to-GDP ratio and house prices.

The third column of Figure 2 shows the impulse response functions to a monetary shock for both the high debt (blue dashed) and low debt (red dot-dashed) states. Note that the responses of GDP, investment, consumption, debt-to-GDP and house prices are significantly positive in the low-debt state. They are also shaped similar to the linear case, and are typically of larger magnitude relative to the linear model. In particular, the GDP response peaks at 0.7 percent in response to a 100 bps shock to the federal funds rate in the low-debt state, relative to 0.5 percent in the linear

¹³We use the updated Romer-Romer shock series up to 2007q4 from Wieland and Yang (2017).

¹⁴We choose lag length based on the zero horizon ($h = 0$) equations for GDP and the federal funds rate as z_t in Equation (1).

model. The price puzzle is also worse in the low-debt state relative to the linear case. In contrast, in the high-debt state, the responses of all the variables, including GDP, consumption, investment and debt-to-GDP are muted, and not significantly different from zero, at most horizons.¹⁵

We then consider the responses of subcomponents of consumption and investment to the monetary policy shock in Figure 3. In the linear case, we see that in response to an expansionary shock, there are positive and hump-shaped responses for durable and services consumption as well as for residential investment, whereas non-durable consumption and non-residential investment increase with a delay. The state-dependent responses show that, in general, all components of consumption and investment have more robust responses in the low-debt state. Durable consumption has a significantly higher response in the low-debt state across almost all horizons after impact, and thus seems to be a major driver of the difference between the total consumption responses in the two states. Residential investment also has a significantly larger response in the low-debt state for 6 to 8 quarters after impact, and is driving the difference across states in terms of gross investment.

The state dependent responses reveal differences in the propagation of monetary policy shocks under the high- and low-debt states at different horizons. In order to further assess the total effectiveness of monetary policy in each state, we also compute the *cumulative* impulse responses. Figure 4 shows the cumulative effects of each variable in the high- and low-debt states, computed using the integral of the corresponding impulse response function. The cumulative effects clearly illustrate that, for all real variables, there is a much larger positive impact during the low-debt state relative to the high-debt state. The only exception is for inflation, where the effects are reversed, and this is due to the worsening of the prize puzzle in the low-debt state as mentioned above. The first panel, however, shows that the federal funds rate also responds differently after impact, and overall falls much less in the low-debt state. In order to verify whether the weaker response of the federal funds rate might be responsible for the more robust response of other variables in the low-debt state, we show the normalized cumulative responses of all the variables by the cumulative response of the federal funds rate. As the second column of Figure 4 shows, even after controlling for the differential responses of the federal funds rate, the cumulative effects of a monetary shock are much larger in the low-debt state than in the high-debt state. The only exception is the effect of monetary policy shocks on house prices, which seem to be of comparable size in the two states following this normalization.

Overall, these results lead us to conclude that the effectiveness of monetary policy in stimulating the economy is reduced when households are highly indebted. These effects can be explained by a relatively muted response of both consumption and investment. Overall, consumption and residential investment seem to be playing a big role in driving the differences in the GDP responses between high and low debt states. In addition, we also document that the responses of household debt-to-GDP, which comprises mostly of mortgage debt, and house prices also have smaller responses in the

¹⁵In Appendix C, we formally test whether the impulse responses in the high debt state are statistically different from responses in the low debt states. The GDP response is statistically different across states for various horizons between 6 and 12 at the 10 percent significance level.

high-debt state.

3.2 Identification of monetary shock as in Romer and Romer (2004)

Next, we conduct the same analysis as in the previous subsection, but use the extended series for the Romer and Romer (2004) monetary shock instead. As mentioned earlier, under this specification, due to the availability of Greenbook forecasts, we start the sample at a later date in 1969. Hence, in this specification, we are dropping the 1956q2-1968q4 high debt episode used in the previous section, when our data was starting in 1955.

Figure 5 shows the impulse response functions to a Romer and Romer monetary shock. In the linear model, shown in the middle column, GDP, consumption and investment again have hump shaped responses, peaking at around 8-10 quarters after the shock hits the economy. The inflation response is muted on impact, and starts rising with a delay. With this identification specification and the shorter sample, the price puzzle is no longer a pronounced issue. Similar to the alternative identification scheme, both debt-to-GDP and house prices respond positively to an expansionary monetary shock.

The last column of Figure 5 shows the state-dependent responses to the monetary shock. Once again the responses in the low-debt state (red dot-dashed) tend to be more robust than the responses to a shock hitting the economy in a high-debt state (blue dashed). This is especially evident in the case of consumption, debt-to-GDP and between horizons 6 to 12 quarters for GDP and investment. Figure 6 shows the impulse response functions of the sub-components of consumption and investment. All components of consumption and residential investment have larger responses in the low-debt state.

Figure 7 shows the *cumulative* effects of the monetary shocks in the two states. The left panel shows that the cumulative effects on GDP, consumption, investment and debt-to-GDP are much larger in the low-debt state. Once we normalize for the effects of the federal funds rate, the differences across the two states become much smaller, particularly relative to the differences we obtained using the baseline identification and sample in the previous subsection. However, there is still evidence to suggest a significant deviation in the cumulative effects on consumption and debt-to-GDP based on the state.

To sum up, under this alternative identification scheme for the monetary shock following Romer and Romer (2004) and a shorter sub-sample, we find slightly weaker, but still positive, evidence of state-dependence in the responses of variables to a monetary shock based on the level of household debt. Namely, in this case too, GDP, its components and debt-to-GDP have a muted response and are stimulated to a much smaller degree in response to a monetary shock when the economy is in a high-debt state.

3.3 Robustness checks

In this section, we consider various robustness checks on our baseline specification. We consider alternative definitions for our state variable, the threshold used to construct the state variable, and also consider a different sample. For these robustness checks, we use our baseline (i.e., VAR-based) identification scheme for the monetary shock, unless otherwise indicated.

3.3.1 Debt gaps under alternative filtering assumptions

Since the two-sided HP filter incorporates future information in constructing the trend, we conduct a robustness check of our baseline results using a debt gap measure constructed using a one-sided HP filter with the same smoothing parameter. The implied debt gap for this case is shown in the top panel of Figure 8 along with our baseline two-sided HP filter gap.

Figure 9 shows the impulse response functions to a monetary policy shock identified using timing restrictions with this alternative debt gap measure to construct the high and low debt states. Overall, the differences in the GDP responses across states are not so apparent, but the consumption response is again significantly higher in the low debt state than the high debt state. The results are much more robust, and even a bit stronger, when we employ the Romer and Romer identification scheme, shown in second column of the same figure. Namely, the response of output, investment and consumption to a monetary shock are significantly larger in the low debt state than a high debt state for many horizons up to 12 quarters.

We then construct another alternative debt gap series using the Baxter and King (1999) bandpass filter, where we isolate frequencies between 4 and 64 quarters. Thus it is twice as long as the standard business cycle frequencies used in the literature, and is largely consistent with our HP-filter smoothing parameter. The implied debt gap is also shown in the top panel of Figure 8. Note under this alternative filtering methodology we have a few additional very short duration high debt periods in the mid 1970s and 1990s.

The third column of Figure 9 shows the impulse response functions using this alternative debt gap measure to construct the high and low debt states. Overall, the results look very similar to those in Figure 2, where we had used the HP filter to construct the debt gap. In particular, we again find evidence of stronger and more robust responses of GDP, investment, consumption and deb-to-GDP in the low-debt state, and relatively muted responses in the high-debt state. The last column of Figure 9 shows the impulse response functions when we employ the Romer and Romer identification for the monetary policy shock. Again the results are very similar to our baseline case shown in Figure 5. The responses of most real variables are larger with an expansionary monetary policy shock in the low-debt state relative to the high-debt state.

3.3.2 Using only mortgage debt to construct the state variable

In our baseline specification, we had considered the gap in the *total* household debt-to-GDP as our state variable. We check the robustness of our results to the use of only mortgage debt (as a ratio

to GDP) instead, using the same HP-filter smoothing parameter of 10^4 . On average, mortgage debt accounts for 66 percent of the total household debt. Figure 8 shows the resulting mortgage debt gap in this case, which lines up fairly closely with our baseline definition using the total household debt gap measure. Figure 10, top panel, shows the impulse response when we use this alternative debt gap as the state variable. Note that, we again find that output has a much more robust response to expansionary monetary policy in the low-debt gap state.

3.3.3 Alternative debt threshold

In the baseline case, our debt gap threshold to divide the sample into two was effectively 0. If, however, we want to focus only on very high debt periods, we could instead define periods of high debt as when the debt gap is larger than the median (i.e., when the debt-to-GDP ratio is at least 2 percent above trend). This results in 36 percent of the sample be characterized as high debt, shown in Figure 8.

Figure 10, second panel, summarizes the impulse responses when we use this alternative debt threshold. The response of GDP in the low debt (red dot-dashed line) state is much larger in response to a monetary shock. In particular, we again find that GDP has a positive and robust response in the low-debt state.

3.3.4 Controlling for lagged debt

When we identified the monetary policy shocks in our baseline case with timing restrictions, we assumed that GDP and inflation were in the information set of the monetary authority. We now allow for the possibility that the monetary authority also takes into account information regarding household debt. In particular, we now include lagged household debt as an additional control variable in our baseline specification, when we construct the responses of the fed funds rate, GDP and inflation. The results are shown in Figure 10, third panel. Note that the results in this case look similar to our baseline case, with the response of GDP being much larger in the low-debt state than the high-debt state.

3.3.5 Accounting for the price puzzle

As noted earlier, our baseline specification in Figure 2 gives rise to the price puzzle, an issue encountered in the related literature as well. In particular, in our linear model, the response of inflation to an expansionary monetary policy shock is negative in the first few quarters, and is statistically significantly negative during the quarter after the shock hits the economy. We also note that in our state-dependent model, the price puzzle is worsened during the low-debt state, while the response of inflation is positive in the high-debt state.

In order to overcome the price puzzle, we follow the literature and include the logarithm of commodity prices as a control variable in our baseline specification, and allow both lags and the contemporaneous values of this variable in the information set of the monetary authority (Christiano

et. al, 1999). The last panel of Figure 10 shows the responses of federal funds rate, GDP and inflation when we include this variable in our analysis. There are three things to note. First, the addition of commodity prices certainly helps mitigate the price puzzle in the linear model with the response of inflation being statistically insignificant for the first few quarters and then turning positive (not shown). Second, the price puzzle now weakens in the low-debt state, but the response of inflation is still negative for a few quarters (while remaining positive in the high-debt state). Finally, note that the addition of commodity prices as a control variable does not affect the main results for GDP in the state-dependent model. In particular, the response of GDP to an expansionary monetary shock is much more positive and robust during the low-debt state, relative to the high-debt state.¹⁶

3.3.6 Importance of each high-debt episode

We want to assess whether a particular high-debt episode is the major driver of our baseline results regarding the state dependence of monetary policy shocks' effects on the economy, based on household indebtedness. In order to examine this systematically, we reclassify each of our four high debt episodes in our baseline case as low debt periods, one by one and conduct the analysis.

Figure 11 shows the response of GDP in both the high-debt and the low-debt state to a monetary policy shock for each of the four cases. First, note that the response of GDP in the low-debt state (red dot-dashed line) remains larger than the response in the high-debt state (blue dashed line) at most horizons in all cases. Second, note that the responses of GDP across the two states look very similar to our baseline results. The only exception is the case when we remove 1979q1-1980q4 from the high-debt state. In that case the response of GDP is in fact negative in the high-debt state, but the 90 percent confidence bands are also larger. Overall, this suggests that our results are not driven by one particular outlier episode.

3.3.7 Extending the sample to include the Great Recession

While we have data available for all real variables, we are forced to end our sample in 2007q4 in the baseline case, since the federal funds rate is subject to the ZLB in the subsequent period. In order to additionally consider the sample period 2008q1-2015q3, we employ the *shadow* federal funds rate constructed by Wu and Xia (2016) for this sub-period. Note that in this case, our sample includes the Great Recession. In addition, as is apparent from Figure 1, our last high debt episode is now longer and lasts until 2011. Figure 12 shows the results from running the linear and the state-dependent models using our baseline monetary shock identification and specification over this longer sample period. The second column shows that for this longer sample, GDP, consumption and investment respond to the monetary policy shock with a persistent hump-shape. Also, both the debt-to-GDP ratio and house prices rise in response to the expansionary monetary shock. The last column shows the state-dependent responses. For horizons between 5- 10 quarters, the responses of

¹⁶The literature has noted that excluding the pre-1984 sample helps to mitigate the prize puzzle. When we conduct the analysis excluding the pre-1984 sample, we still find evidence of larger effects of monetary policy in the low debt state on GDP.

GDP and its components are statistically significantly larger in the low-debt state than the high-debt state. Debt-to-GDP also rises more robustly in the low-debt state, and has a muted response in the high-debt state. The response of house prices however is virtually the same across the two different states.

3.4 Interaction of high debt periods with recessions

In this section, we dig deeper into our baseline findings about the state-dependent effects of monetary policy shocks, and consider whether the state of the economy plays an additional moderating role. Figure 1 shows the household debt-to-GDP ratio (solid line) and its trend (dashed line), and the grey shaded areas indicate the NBER recessions. It is clear that the recessionary episodes are distributed almost equally through both the high- and low-debt periods. Out of the nine recessionary episodes in the sample running until 2007q4, only three recessions occur completely in the high-debt state. If we extend the sample to the end of 2015, then the last episode, the Great Recession, occurs during the high-debt state. Tenreyro and Thwaites (2016) find empirical evidence that the effects of monetary policy on macroeconomic variables are more powerful in expansions than in recessions.¹⁷ However, given the distribution of recessions across the two debt states, we can conclude that the results about the effectiveness of monetary policy being dependent on household indebtedness is not driven solely by the state of the economy.¹⁸

One might conjecture, though, that if the high-debt state occurs during a recession, then the effects could be exacerbated. This would mean that the relative effectiveness of monetary policy might worsen further during periods that are characterized as both a high-debt state and a recession. In order to test this conjecture, we run our state-dependent model, where we assume that the dummy variable I_{t-1} takes a value of 1 in Equation (2) when we are in the high-debt state *and* in an NBER recession, and 0 otherwise. For this exercise, we consider the longer sample, running until 2015q3, as described above among the robustness checks, in order to include additional high debt observations occurring during a recession. Note that only about 8 percent of observations constitute a high-debt and recession state.¹⁹

Figure 13 shows the impulse response functions using this new definition of the state, where we interact high debt *and* recession. The first column shows the state-dependent results, where the response to monetary shock in a high-debt/recession state are given by the blue dashed lines, and otherwise are given by the red dashed-dotted lines. In response to an expansionary shock, we now observe that GDP *falls* in the high debt/recession state, while it rises otherwise. This is also particularly striking in light of the federal funds rate response, which returns to steady state within a couple quarters in the high-debt/recession state. On the other hand, in the other state, federal

¹⁷They consider a sample period of 1969-2007, and identify shocks based on Romer and Romer (2004).

¹⁸Tenreyro and Thwaites (2016) also find that the effects of contractionary monetary policy shocks are larger than the effects of expansionary shocks. Figure 17 in Appendix B shows the distribution of both types of monetary shocks used in our analysis, and shows that we have rather similar looking distribution of shocks across the high and low debt states. This suggests that the specific distribution of shocks is also not driving our main results on state-dependence.

¹⁹For the sample period 1955-2007, this constitutes only 6 percent of the sample.

funds rate has a much more persistent response and stays negative for longer. Consumption and investment also fall after a few quarters in the high debt/recession state. Household debt and house prices have relatively muted responses in the high debt/recession state versus otherwise. Overall, this suggests that when we consider high debt states which are also characterized by a recession, the effectiveness of monetary policy is further reduced significantly.

In order to quantify the role of recessions, we compare the *cumulative* effects of monetary policy in the high-debt state, both for the case where we distinguish between recessions and where we do not. More precisely, the second and third column of Figure 13 show the cumulative effects of monetary policy in the high-debt state (blue dashed line) where we do not draw any distinction between recessions and expansions,²⁰ and the cumulative effects in the high-debt/recession state (black dot-dashed line). The second column shows that, for all real variables the effects are smaller in the high debt/recession state. In particular, for GDP, consumption and investment, the cumulative effects are negative at longer horizons following an expansionary monetary shock.²¹ Since the cumulative effects on the federal funds rate are also different in the two cases, in the last column we normalize for the effects on the federal funds rate. In that case too, we see that the cumulative effects on real variables are much smaller, particularly at longer horizons, in the high-debt/recession state versus the high-debt state.

We established earlier that the effectiveness of monetary policy is limited during a high-debt state. The results in this section show that this ineffectiveness is further exacerbated when the high-debt state coincides with a recession. In fact, if the state of the economy characterized by both high level of indebtedness and recession, expansionary monetary policy can have negative effects on GDP.

4 Small-scale model with debt overhang

In this section, we build a small-scale, partial-equilibrium, endowment-economy model to illustrate some of the key channels of the monetary policy transmission mechanism that operate through *borrower* households, and examine how the effectiveness of these channels change based on the initial debt stock of borrowers. The model features long-term, fixed or adjustable-rate mortgage contracts as in Kydland et al. (2012) and Garriga et al. (2017), and allows for home equity extraction and refinancing within this set-up similar to Alpanda and Zubairy (2017). The model is purposefully kept simple and partial equilibrium to focus on the effects of a policy rate cut on the interest burden of borrowers (i.e., interest rate channel), and the relaxation of their borrowing constraint through increased house prices and home equity (i.e., home equity channel).

²⁰Note, these are the same cumulative effects as in the high debt state shown in Figure 4, except that we are now considering a longer sample that runs until the end of 2015.

²¹Note that in this figure we are showing the response for the level of real household debt (in logs) rather than the response of debt-to-GDP. The latter is also increasing in the high debt/recession state, but only because the drop in GDP is larger than the drop in household debt, resulting in the debt-to-GDP ratio to increase.

4.1 A partial equilibrium model

Consider a borrower household, whose preferences are represented by a period utility function $u(c_t, h_t)$, where c_t and h_t denote real consumption and housing, respectively. For simplicity, we abstract from residential investment in our model, and assume that the household owns a constant \bar{h} units of housing in real terms and housing does not depreciate (i.e., $h_t = \bar{h}$ for all t). The household is assumed to be infinitely-lived, and discounts future utility with a factor $\beta < 1$.

The borrower's period budget constraint is given by

$$c_t + (R_{t-1}^m + \kappa) \frac{D_{t-1}}{P_t} \leq y + \frac{L_t}{P_t}, \quad (3)$$

where P_t denotes the aggregate price level, D_{t-1} denotes the nominal stock of debt carried from the previous period, L_t is the amount of new borrowing in period t , and y denotes real income received each period. The average (net) interest on the borrower's existing mortgage debt is denoted by R_{t-1}^m , while the borrower also pays κ percent of the debt as a principal payment each period. For simplicity, we assume that the aggregate price level increases by a constant inflation factor of π each period (i.e., $\pi = P_t/P_{t-1}$ for all t), and the real endowment income level, y , is a constant each period.

The borrower's stock of debt evolves according to the following law of motion

$$D_t = (1 - \kappa) D_{t-1} + L_t, \quad (4)$$

where new borrowing, L_t , is taken in the form of a home equity loan, and is thus subject to a borrowing constraint of the form

$$L_t \leq \max \{0, \phi P_t q_t \bar{h} - (1 - \kappa) D_{t-1}\}, \quad (5)$$

where q_t denotes the relative price of housing and ϕ is the regulatory loan-to-value ratio on borrowing.²² The max operator above captures the notion that households become fully borrowing constrained when they do not have adequate equity in their houses. Note that $1 - \phi$ percent of the house value has already been pledged as collateral for the original mortgage, and thus cannot be pledged against home equity loans taken on top of the first lien.²³

In terms of the duration of the interest rate on the loan, we consider three cases: (i) fixed-rate loans with no refinancing, (ii) fixed-rate loans with a refinancing option, and (iii) adjustable-rate loans. New loans in period t are taken at the ongoing interest rate of R_t^f . Given that R_{t-1}^m denotes the average interest burden on the existing stock of debt, we can characterize how the borrower's average interest burden evolves over time as they take out new loans or refinance old ones by the

²²Note that new loans in Kydland et al. (2012) and Garriga et al. (2017) are loans for the purchase of new houses (i.e., residential investment). In contrast, our setup abstracts from residential investment, and new loans reflect home-equity loans, which are constrained by the borrower's net equity on existing homes.

²³Similarly, if agents build home equity through a house price increase, they cannot pledge more than ϕ percent of this increase as collateral when extracting equity.

following expression:

$$R_t^m D_t = R_{t-1}^m (1 - \Phi_t) (1 - \kappa) D_{t-1} + R_t^f [L_t + \Phi_t (1 - \kappa) D_{t-1}], \quad (6)$$

where $\Phi_t \in \{0, 1\}$ is an indicator function denoting whether the stock of debt brought from the previous period, $(1 - \kappa) D_{t-1}$, is refinanced in period t . In the case of fixed-rate loans with no refinancing (case i), we set $\Phi_t = 0$ for all t , which renders the above condition to $R_t^m D_t = R_{t-1}^m (1 - \kappa) D_{t-1} + R_t^f L_t$, where the current fixed rate affects the overall average interest burden only through new loans. In the case of fixed-rate loans with a refinancing option (case ii), the borrower chooses $\Phi_t \in \{0, 1\}$ each period, but exercising this option is subject to the regulatory LTV constraint denoted in (5), similar to home equity extractions. Thus, when the borrower does not have enough equity in the house and therefore cannot acquire any new loans, the debt stock would evolve as $D_t = (1 - \kappa) D_{t-1}$, and the average interest on mortgage debt would stay constant (i.e., $R_t^m = R_{t-1}^m$). Finally, for adjustable-rate loans (case iii), we set $\Phi_t = 1$ for all t , and the law of motion for the average interest burden reduces to $R_t^m = R_t^f$, since the current rate applies to all loans, including existing debt brought from the previous period.

4.1.1 Determination of interest rates and house prices

We assume that the policy rate set by the central bank, R_t , follows an AR(1) process with a persistence parameter ρ_R :

$$R_t = (1 - \rho_R) R + \rho_R R_{t-1} + \varepsilon_{R,t}, \quad (7)$$

where R denotes the steady-state level of the policy rate, ρ_R is the persistence parameter, and $\varepsilon_{R,t}$ is an *i.i.d.* monetary policy shock. The current interest rate on fixed-rate long-term loans, R_t^f , is then determined by a weighted sum of current and future policy rates based on the expectations hypothesis as

$$R_t^f = \kappa \sum_{s=0}^{\infty} (1 - \kappa)^s R_t, \quad (8)$$

given that the duration of the loans is (approximately) $1/\kappa$. When loans are adjustable-rate, we set $R_t^f = R_t$ for all t , and thus, changes in the policy rate pass through to the current borrowing rate fully.²⁴

Finally, we close the model by specifying a stochastic process on house prices as

$$\log q_t = \rho_q \log q_{t-1} + \varepsilon_{q,t} - \rho_{qR} (R_{t-1} - R), \quad (9)$$

where ρ_q is the persistence parameter, and the last term allows for a (lagged) feedback effect from interest rates to house prices with a pass-through parameter of ρ_{qR} .²⁵ This feedback effect is

²⁴In Alpanda and Zubairy (2017), this follows from the arbitrage condition between the saver households' incentives to give adjustable-rate long-term loans to borrowers versus purchasing 1-period government bonds.

²⁵Note that in the above formulation, we have also assumed, without loss of generality, that the steady-state value

important to generate a credit channel for monetary policy transmission, whereby a policy rate cut increases house prices, relaxing the borrowing constraint of households and inducing them to borrow further. When this channel is shut off (i.e., $\rho_{qR} = 0$), monetary policy transmission works exclusively through the interest rate channel, whereby a rate cut reduces the average interest burden of borrowers, R_t^m , leaving them with a higher level of disposable income to purchase consumption goods.

4.2 Solution of the model

Assuming the utility function obeys the usual conditions (e.g., strictly increasing and concave), the borrowing constraint binds every period as long as the households discount the future sufficiently (i.e., $\beta \ll 1/(1+R)$). We follow Iacoviello (2005), and assume this holds. The law of motion of debt in (4) and the borrowing constraint in (5) can now be combined to solve for the policy functions for the debt stock in equilibrium:

$$d_t = \begin{cases} \frac{1-\kappa}{\pi} d_{t-1} & \text{if } d_{t-1} > \frac{\pi\phi}{1-\kappa} q_t \bar{h} \\ \phi q_t \bar{h} & \text{if } d_{t-1} \leq \frac{\pi\phi}{1-\kappa} q_t \bar{h} \end{cases}, \quad (10)$$

where $d_t = D_t/P_t$ denotes the real stock of debt. In particular, when debt levels are sufficiently high, agents cannot borrow more, so they start to slowly deleverage by paying off a portion of their principal each period, reminiscent of a debt overhang. When debt levels are low however, they borrow against housing equity up to the allowed loan-to-value ratio, ϕ . This implies that the response of debt to a change in house prices in our model will be asymmetric, conditional on the existing level of debt and home equity. This asymmetry is similar to that assumed in Justiniano et al. (2015), but here depends on the debt level (instead of the change in house prices) and is derived explicitly using long-term mortgage contracts.

The consumption level of agents can now be solved using their budget constraint in (3). In particular, the asymmetry in the evolution of debt will affect the agents' consumption profile as well:

$$c_t = \begin{cases} y - \frac{R_{t-1}^m + \kappa}{\pi} d_{t-1} & \text{if } d_{t-1} > \frac{\pi\phi}{1-\kappa} q_t \bar{h} \\ y + \phi q_t \bar{h} - \frac{1+R_{t-1}^m}{\pi} d_{t-1} & \text{if } d_{t-1} \leq \frac{\pi\phi}{1-\kappa} q_t \bar{h} \end{cases}. \quad (11)$$

Note that how the average interest burden of borrowers, R_{t-1}^m , evolves over time depends on the loan type and whether the refinancing option is available and exercised. Considering fixed-rate loans with no refinancing (case i), the average interest burden of borrowers evolves in equilibrium as

$$R_t^m = \begin{cases} R_{t-1}^m & \text{if } d_{t-1} > \frac{\pi\phi}{1-\kappa} q_t \bar{h} \\ R_{t-1}^m \left(\frac{1-\kappa}{\pi} \right) \frac{d_{t-1}}{d_t} + R_t^f \frac{l_t}{d_t} & \text{if } d_{t-1} \leq \frac{\pi\phi}{1-\kappa} q_t \bar{h} \end{cases}, \quad (12)$$

where the interest rate cut reduces the interest burden of the borrower only through newly acquired

for the relative price of housing, q , is equal to 1.

loans, $l_t = L_t/P_t$. In the case of high initial debt, the agents cannot borrow any further, and therefore the average interest on loans stays the same.

Under fixed-rate loans with a refinancing option (case ii), the interest burden can also be reduced through refinancing of existing loans following a rate cut; thus, the equilibrium average interest burden evolves as

$$R_t^m = \begin{cases} R_{t-1}^m & \text{if } d_{t-1} > \frac{\pi\phi}{1-\kappa}q_t\bar{h} \\ R_{t-1}^m \left(\frac{1-\kappa}{\pi}\right) \frac{d_{t-1}}{d_t} + R_t^f \frac{l_t}{d_t} & \text{if } d_{t-1} \leq \frac{\pi\phi}{1-\kappa}q_t\bar{h} \text{ and } R_t^f \geq R_{t-1}^m \\ R_t^f & \text{if } d_{t-1} \leq \frac{\pi\phi}{1-\kappa}q_t\bar{h} \text{ and } R_t^f < R_{t-1}^m \end{cases} . \quad (13)$$

Note that, conditional on satisfying the LTV constraint, agents exercise their refinancing option only when the current fixed rate is below the average interest burden on existing loans (i.e., $R_t^f < R_{t-1}^m$). When this is the case, the borrowers' overall interest burden is reduced to the current borrowing rate of R_t^f , while only their new borrowing reflects R_t^f when they do not refinance (but still take out home equity loans on the margin nevertheless).²⁶ Finally, when loans are adjustable-rate (case iii), we have $R_t^m = R_t^f = R_t$, irrespective of the existing level of debt or interest rates.

We can now investigate how a monetary policy shock would affect borrowers' consumption in our simple set-up. At the impact period $t = 0$, there is no change in consumption, since the interest paid on existing debt, R_{t-1}^m , is pre-determined and the interest rate is assumed to affect house prices only with a lag (see Equation 9). For periods following the impact period, the derivative of consumption with respect to the policy rate is given by

$$\frac{\partial c_t}{\partial R_{t-1}} = \begin{cases} 0 & \text{if } d_{t-1} > \frac{\pi\phi}{1-\kappa}q_t\bar{h} \\ -\frac{1}{\pi} \frac{\partial R_{t-1}^f}{\partial R_{t-1}} l_{t-1} + \phi\bar{h} \frac{\partial q_t}{\partial R_{t-1}} & \text{if } d_{t-1} \leq \frac{\pi\phi}{1-\kappa}q_t\bar{h} \end{cases} \text{ for } t > 0, \quad (14)$$

when loans are fixed-rate with no refinancing (case i). With a refinancing option (case ii), the corresponding expression becomes

$$\frac{\partial c_t}{\partial R_{t-1}} = \begin{cases} 0 & \text{if } d_{t-1} > \frac{\pi\phi}{1-\kappa}q_t\bar{h} \\ -\frac{1}{\pi} \frac{\partial R_{t-1}^f}{\partial R_{t-1}} l_{t-1} + \phi\bar{h} \frac{\partial q_t}{\partial R_{t-1}} & \text{if } d_{t-1} \leq \frac{\pi\phi}{1-\kappa}q_t\bar{h} \text{ and } R_t^f \geq R_{t-1}^m \\ -\frac{1}{\pi} \frac{\partial R_{t-1}^f}{\partial R_{t-1}} d_{t-1} + \phi\bar{h} \frac{\partial q_t}{\partial R_{t-1}} & \text{if } d_{t-1} \leq \frac{\pi\phi}{1-\kappa}q_t\bar{h} \text{ and } R_t^f < R_{t-1}^m \end{cases} \text{ for } t > 0. \quad (15)$$

Finally, when loans are adjustable rate (case iii), the response of consumption to the policy rate

²⁶Our formulation with fixed-rate loans with refinancing above also suggests that the strength of monetary policy can be different in a monetary tightening versus an easing cycle, because of the asymmetry in the refinancing decision with respect to interest rates. In particular, in an easing cycle, the current fixed interest-rate, R_t^f , would likely go below the agents' average interest rate burden on existing debt, R_{t-1}^m , incentivising them to refinance existing loans. In a tightening cycle however, this channel would likely be shut off, suggesting that monetary policy is likely to be more effective during an easing cycle relative to a tightening cycle. We do not explore this issue in our current paper and leave it for future research.

would be

$$\frac{\partial c_t}{\partial R_{t-1}} = \begin{cases} -\frac{1}{\pi} \frac{\partial R_{t-1}^f}{\partial R_{t-1}} d_{t-1} & \text{if } d_{t-1} > \frac{\pi\phi}{1-\kappa} q_t \bar{h} \\ -\frac{1}{\pi} \frac{\partial R_{t-1}^f}{\partial R_{t-1}} d_{t-1} + \phi \bar{h} \frac{\partial q_t}{\partial R_{t-1}} & \text{if } d_{t-1} \leq \frac{\pi\phi}{1-\kappa} q_t \bar{h} \end{cases} \text{ for } t > 0. \quad (16)$$

The above expressions highlight that an interest rate cut affects next period's consumption through two channels in our model. The first is the standard interest-rate channel, working through both new and old loans, whereby lower policy rates reduce the overall interest burden of borrowers, R_t^m , allowing them to consume more as a result. For new loans acquired in period t (i.e., l_t), the decline in interest rates has a direct, positive income effect on borrowers' consumption in future periods by reducing the interest they will pay on these loans acquired at the margin. This effect occurs regardless of the type of loan acquired by the borrower, but is stronger with adjustable-rate loans, since the pass-through from the policy rate, R_t , to the borrowing rate, R_t^f , is larger in this case, since $\partial R_t^f / \partial R_t = 1$ for adjustable-rate loans, while $\partial R_t^f / \partial R_t \ll 1$ for fixed-rate loans.

The interest-rate channel can also reduce the borrowers' interest burden on existing debt brought from the previous period (i.e., $(1 - \kappa) d_{t-1} / \pi$). This effect is operational only if the loans are adjustable rate, or if the agent can refinance existing fixed-rate loans at the reduced rate. With adjustable-rate loans, this effect gets stronger as the existing debt level increases, since the same rate cut leads to a larger decline in the borrowers' interest burden. When loans are fixed rate however, the strength of this effect becomes non-linear in the existing level of debt. In particular, after a certain threshold debt level, the LTV constraint is no longer satisfied and households cannot refinance their existing loans. In this latter case, the interest rate channel is rendered completely mute, since the agent is now unable to borrow at the cheaper rates or reduce its interest burden on existing loans.

The second channel through which a rate cut affects consumption in our model is the home equity channel, which works through the relaxation of the borrowing constraint as house prices increase and borrowers' home equity positions improve, which allows agents to borrow further.²⁷ The magnitude of this effect is captured by the term $\phi \bar{h} (\partial q_t / \partial R_{t-1})$ in the above expressions. The agents build home equity as house prices increase regardless of their existing debt level, but they can tap into this newly increased home equity only when their initial debt levels are low and they satisfy the LTV constraint on home equity extraction.²⁸ Thus, this credit channel also becomes non-operational and the effectiveness of monetary policy is again curtailed under high levels of existing debt, and agents are thus forced to deleverage despite the interest rate cut, reminiscent of a debt overhang.²⁹

²⁷This is essentially the "financial accelerator" effect introduced in Kiyotaki and Moore (1997) and Bernanke et al. (1999).

²⁸Note, also that the model could generate the dampened responses to monetary policy shocks during recessions shown in Section 3.4, as long as recessions are associated with lower levels of house prices. This would suggest a smaller proportion of borrowers are able to extract the increase in home equity that results from the policy rate cut. In our formulation, we did not explicitly link the state of the economy (as can be proxied by income levels) to the level of house prices and the distribution of home equity, but mechanically building this link would generate the desired result. In particular, we could consider negative shocks to the aggregate income level, y , which would also lead to a decrease in house prices, q , and lower home equity levels, which would then render monetary policy less effective through the same channels as discussed here.

²⁹Our borrowing constraint formulation above assumes a constant cost of borrowing up to the limit, after which no

In the next section, we present impulse responses of key model variables to monetary policy shocks to illustrate the aforementioned channels quantitatively. Our results indicate that, while both the interest rate and the home equity channels are rendered less effective when agents hold high levels of existing debt, the latter is quantitatively more important in generating this result.

4.3 Theoretical impulse responses to a monetary policy shock

In this section, we first parameterize the model and then present impulse responses of model variables to a 25 bps (i.e., 100 bps in annualized terms) monetary policy shock. The parameters of the model are set to fairly standard values. The policy rate at the steady state, R , is set to 0.01, reflecting a 4 percent nominal interest rate in annualized terms, while the constant inflation factor, π , is set to 1.005, for an annual inflation rate of 2 percent. Without loss of generality, the level of output, y , is normalized to 1. The share of debt principal paid out every period, κ , is set to 0.0125, reflecting an average loan duration of 20 years.³⁰ The LTV ratio for housing loans, ϕ , is set to 90 percent, while the housing quantity parameter, \bar{h} , is set equal to 4. These parameters imply that the borrower’s steady-state housing-to-income and debt-to-income ratios in annualized terms are 1 and 0.9, respectively. Finally, for the exogenous processes on interest rates and house prices, we set the persistence parameters ρ_R and ρ_q to 0.85 and 0.5, respectively, while the response coefficient of house prices to interest rates, ρ_{qR} , is set equal to 4 in the baseline simulations to ensure that an annualized 100 bps decline in policy rates leads to a peak response in house prices of 1.4 percent, similar to our estimates in the empirical analysis of Section 3.

Figure 14 presents the borrower’s impulse responses for key variables to an annualized 100 bps innovation in the monetary policy shock under three different types of loans; namely, (i) fixed-rate loans with no refinancing, (ii) fixed-rate loans with a refinancing option, and (iii) adjustable-rate loans. In each case, we compare the impulse responses of model variables starting from two initial levels of debt; the first is the steady-state level of debt corresponding to an initial debt-to-annual income ratio of 0.9, and the second is a case where the initial debt level is 20 percent above the steady state. The impulse responses are computed by simulating the model separately with and without the monetary shock, and then taking the difference of these paths (i.e., “shock minus control”).³¹

The results are qualitatively, and to some extent quantitatively, similar for the three types of

further borrowing is allowed (i.e., an infinite cost of borrowing). In fact, all that is required for the above results to go through is an increasing and convex cost function, which would generate an increase in the marginal cost of borrowing as initial debt levels rise. This may be motivated, for example, by an increase in the default risk premium charged by lenders as the borrower gets more leveraged, similar to the agency cost models of Carlstrom and Fuerst (1997) and Bernanke et al. (1999).

³⁰This figure reflects a weighted average of the durations of the original mortgage loan, which tends to be longer than 20 years, and home equity loans taken on top of the first lien, which are typically of shorter duration. Census Bureau’s American Housing Survey in 2011 indicates that the average remaining term of all outstanding housing loans is around 19.6 years (Alpanda and Zubairy, 2017).

³¹In the “high initial debt” case, we start the simulations off the steady state only for the initial stock of debt, while setting the initial policy rate before the shock and the price of housing to their steady-state values. Note that the control simulations in the absence of the monetary shock still imply transitional dynamics, but our “shock minus control” approach here isolates these dynamics from the impulse responses.

loans we consider. In particular, consumption demand increases more in the low initial debt case, based on the relative strength of both the interest rate and the home equity channels, with the latter playing a more significant role. Note that the interest rate channel refers to the increase in consumption that occurs as a result of the decline in the average interest burden of agents, R_t^m , following the policy rate cut. In particular, the decrease in the policy rate, R_t , reduces the rate at which the agents can borrow at the margin and refinance existing loans (i.e., R_t^f), which then reduces their average interest burden (i.e., R_t^m) in future periods. Note that this pass-through from the policy rate to the average interest burden is strongest under adjustable-rate loans (case iii), while the resulting decline in average interest under fixed-rate loans are relatively more modest, especially in the absence of a refinancing option (case i). The differences in the strength of the interest rate channel above also translates into differences in the borrower’s consumption response under each type of loan, with the strongest response being associated with adjustable-rate loans. Nevertheless, the pattern, and to some degree the magnitude, of the consumption responses are similar under the three types of loans, indicating that the main driver of consumption dynamics in the model is not the interest rate channel, but rather the home equity channel, which results from the relaxation of the borrowing constraint as the rate cut raises house prices and home equity. When initial debt levels are low, the borrower is able to tap into this newly increased amount of home equity, and raise her consumption level accordingly. With a high level of initial debt however, the agent does not have enough home equity to take out any new loans initially, and is forced to deleverage during the first 9 periods.³² The stronger response of consumption under low initial debt lasts for several periods, but consumption slowly declines below the high initial debt case in the following periods, as the new debt accumulated in the low debt case now needs to be paid back with interest.³³

Under fixed-rate loans (cases i and ii), the high debt borrower is also unable to take advantage of the lowered interest rates through refinancing during the first 9 periods, and therefore, not only the home equity channel, but also the interest rate channel, is fully shut off.³⁴ Thus, consumption does not respond to the monetary policy shock until period 10, when the agent has now built enough home equity to be able to borrow again and refinance existing debt. By that time however, the impact of the monetary policy shock is already diminished, and therefore, the overall effect on consumption is smaller (and lagged), relative to the case with lower initial debt levels. Under adjustable-rate loans (case iii), the home equity channel is also delayed until period 10, while the agent builds up equity to satisfy the LTV constraint on new borrowing. The interest-rate channel however is now

³²Note that the impulse response for the debt stock is shown as 0 during these periods, since the same deleveraging occurs in both the “shock” and the “control” scenarios, with no additional impact of the monetary policy shock on the debt stock. Note also that the equilibrium level of home equity increases further in the high initial debt case relative to low initial debt, since, in the latter case, part of the increase in home equity is extracted through home equity loans.

³³Note that our model allows agents to extract 100% of the eligible home equity for consumption purposes, which results in a large spike in borrowing and consumption right after the monetary policy shock. One can slow this process by assuming that agents are able to extract only a fraction of their available home equity each period, as illustrated later in Figure 15.

³⁴This is similar to Beraja et al. (2015) who study how housing equity differences, driven by house price variation affect the aggregate consequences of monetary policy through its effects on mortgage refinancing, and also find that large house price declines result in muted consumption responses.

operational starting from the impact period, as the decline in the policy rate applies to all loans, including existing debt brought from the previous period. The resulting reduction in the borrower’s interest burden thus leads to a slight increase in the borrower’s consumption levels during the initial 9 periods as well. Under adjustable rate mortgages (case iii), this interest rate channel is actually stronger for a higher level of initial debt, but since the home equity channel is not operational in this case, consumption demand responds less than the case with low initial debt.

If we isolate the interest rate channel by shutting off the feedback effect from the policy rate to house prices, by setting $\rho_{qR} = 0$ in Equation 9 (shown in Appendix F), the increase in consumption here is simply due to the increase in disposable income, given the decline in the interest burden of debt. Since there is no change in house prices, the “home equity channel” is shut off and the expansionary monetary policy shock does not lead to any *additional* borrowing.³⁵ Under adjustable-rate mortgages (case iii), monetary policy in fact becomes *more* effective with high initial debt, since the decline in the interest rate now leads to a larger income effect, and therefore, to a bigger boost in consumption. With fixed-rate mortgages (cases i and ii) however, the interest rate channel depends primarily on the refinancing of existing debt, which in turn depends on whether the agent satisfies the LTV constraint. With a high initial debt level, the agent is constrained during the initial periods, and cannot refinance to reduce the interest burden on existing debt. After period 10, the agent has built enough equity and can borrow again at a lower rate and refinance old loans, which reduces the agent’s overall interest burden slightly, but this effect is small relative to the case with low initial debt. Note that, for each type of loan, the magnitude of the consumption response is significantly smaller in Appendix F relative to the baseline case in Figure 14, indicating that the contribution of the interest rate channel in generating the baseline results in Figure 14 is rather limited, while the home equity channel plays a far more important role.

So far, we have considered only two initial debt levels for each loan type, but similar results are obtained if we instead consider a distribution of initial debt across borrowers, and examine how monetary policy effectiveness would change as the distribution is shifted to the right and the share of agents that are unable to extract equity from their houses is increased. Figure 15 summarizes these results under fixed-rate loans with a refinancing option, as the initial debt distribution is shifted to the right by 20%.³⁶ For this experiment, we also assume that agents are able to extract only a fraction (5%) of their available home equity each period. This spreads out the impact of the home equity channel across more periods, and, although not crucial for the key message here, helps generate more persistent impulse responses for consumption, more in line with the empirical estimates presented in Section 3 of the paper.³⁷ The responses of borrowing and consumption are

³⁵Note that new borrowing, l_t , is indeed positive in the low initial debt case (and after 9 periods, in the high debt case), but there is no additional new borrowing as a result of the monetary shock; i.e., the “shock” and “control” scenarios generate the same time paths for l_t .

³⁶For the control simulation, we consider a uniformly-distributed initial level of debt, centered around the steady state of level of debt as before (i.e., a debt level of 3.6, implying a debt-to- annual income ratio of 0.9), and with endpoints at 20% below and above this steady state value. This is for illustrative purposes, but the results are robust to changes in the endpoints considered or the type of the distribution for initial debt.

³⁷Partial extraction of available home equity is also consistent with the empirical evidence on home equity extraction

significantly weaker under the high initial-debt distribution, since a higher share of agents cannot extract the increased home equity to finance consumption and cannot refinance existing loans to reduce their interest burden. Over time, agents build equity, and the share of agents extracting equity rises to 1, as both economies converge to the same steady state. During the transition path however, some agents are fully constrained and cannot extract equity under both distributions, but much more so under the high initial-debt case.

4.3.1 Discussion

Our partial equilibrium model described above is illustrative, and abstracts from other channels that may also be potentially important. First, in our partial equilibrium set-up, we have assumed an exogenous relationship between interest rates and house prices, but this link could crucially depend on the amount of housing demand. If existing levels of high debt restricts this demand coming from borrower households, the impact on house prices would likely be muted as well. This, in turn, would lead to a further weakening in the home equity channel, and thus lower monetary policy effectiveness, as initial debt levels grow. This suggests that endogenizing house price formation in our set-up would likely not overturn the main conclusion, and may even strengthen it.

Second, a rate cut would also lead to a small increase in the inflation rate, thereby reducing the *real* debt burden of borrowers through the debt-deflation effect (Fisher, 1933). Other things equal, this favorable effect would be stronger when the initial debt stock is larger, thereby increasing the effectiveness of a rate cut. We abstract from this channel by setting the inflation rate to a constant. Note, however that the evidence for monetary policy’s impact on inflation is rather mixed. In particular, many VAR studies, as well as our empirical analysis in Section 3, point to an initial decrease in inflation following monetary easing, a notion that has been dubbed as the “price puzzle”. Incorporating this initial decrease in inflation would slightly strengthen our results for the short term by further weakening the effectiveness of monetary policy under high initial debt levels, but this would also weaken our results in the medium term. Instead, we assume a constant inflation rate in our model, and abstract from the debt-deflation effect altogether.

Third, our model abstracts from “saver” households which would provide the financing to the borrowers in our model. The decline in interest rates would also impact the consumption smoothing and investment patterns of these households. In particular, the resulting decline in the interest income of savers and the increase in house prices would likely offset some of the increase in demand coming from borrower households, and this adverse effect would likely be stronger as the existing stock of debt is higher.

In Appendix G, we consider a general equilibrium version of our model, which incorporates the three aforementioned channels. In particular, the general equilibrium model features saver households who provide the financing to borrowers, and features endogenously-determined inflation and house prices. The results from this extension indicate that our main conclusions are robust

rates in the data (Alpanda and Zubairy, 2017).

to these changes. Unlike Garriga et al. (2017), our model abstracts from residential investment, and does not allow for new mortgage lending except for home equity loans taken on top of the first lien. In a world where home equity loans can also be used to finance residential investment expenditures (e.g., to cover maintenance costs or to finance the down-payment on a secondary home or an investment property), the effects of monetary policy on residential investment will also become more muted as initial debt levels increase.³⁸

4.4 Additional evidence on the home-equity channel

Our theoretical model presented above suggests that the weakening of the home-equity loan channel may be a possible reason for the decline in monetary policy effectiveness when initial debt levels are high. In this subsection, we empirically test this idea using the state-dependent model of Section 2, and explore whether the impulse response of home-equity extractions to a monetary shock features state-dependence with respect to household debt. Since a considerable portion of home-equity extractions are associated with refinancing activity, we also explore whether refinancing response is state dependent.

Note that the available data we have on home-equity loans and refinancing activity are limited, with a shorter time series spanning 1990-2015. In what follows, we conduct our baseline analysis over this shorter sample period. Figure 16 shows the impulse responses of federal funds rate, GDP and inflation in the first three rows, and also shows the responses of home-equity loans (HE loans), which is defined as home-equity loans as a ratio of total household debt, and refinancing, which is data on all refinancing mortgage originations from the Mortgage Bankers Association.

The middle column shows impulse responses from the linear model. Note that in this shorter sample of 1990-2015, in response to a monetary policy shock, we see a muted response of GDP, which is in fact negative at many horizons.³⁹ The response of home-equity loans is negative on impact and rises slowly, while refinancing goes up on impact. The last column shows the responses from the state-dependent model. In the high-debt state (blue dashed response), the response of GDP is once again smaller than the response in the low-debt state (red dot-dashed), and is in fact negative, while it is positive in the low-debt state. The state-dependent responses also suggest that in response to expansionary monetary policy, both home equity loans and refinancing activity increase much more in the low-debt state relative to the high-debt state, where the difference in the home-equity loan

³⁸There are also possible interactions with housing-related fiscal policy that may attenuate the effects of monetary policy. For instance, as interest rates fall, agents would be able to deduct less mortgage interest, which would increase their overall tax burden on their income. This adverse effect will be larger when existing debt levels are high. Similarly, property tax payments increase along with the increase in house prices. If the existing levels of debt and housing stocks are correlated, this effect by itself could also attenuate the effectiveness of monetary policy under high levels of debt.

³⁹This is consistent with findings in other papers that focus on this period. For example, Ramey (2016) shows that for this and shorter sample periods starting in 1990s, expansionary monetary policy shocks under some identifications can have negative effects on GDP. In order to deal with this identification issue, which possibly arises due to expectations regarding monetary policy changes, some papers use high frequency identification methods (Gertler and Karadi, 2015).

response is in the longer horizons, and the difference in the refinancing response is on impact and at shorter horizons.

Given the short sample and the problems associated with identification of monetary policy shocks in the more recent samples discussed above, we view this exercise as suggestive evidence that supports the basic transmission mechanism we highlight, but deserves further attention that is beyond the scope of this paper.

5 Conclusion

In this paper, we use a state-dependent time-series model to find that the effectiveness of monetary policy in the U.S. is curtailed during periods of high household debt. These results are robust to alternative definitions of high- and low-debt periods, and two leading methodologies of identification of monetary policy shocks. Our small-scale theoretical set-up highlights one possible channel as to why this may occur; namely that higher initial debt levels may slow down the increase in home equity extractions when policy rates are cut.

Our results indicate that the effectiveness of monetary policy was likely hindered during the recent recession, given the significant amount of debt households had accumulated previously. Our results also suggest that, in general, when high levels of leverage accompany recessions, alternative tools to monetary policy should also be considered in order to stimulate the economy.

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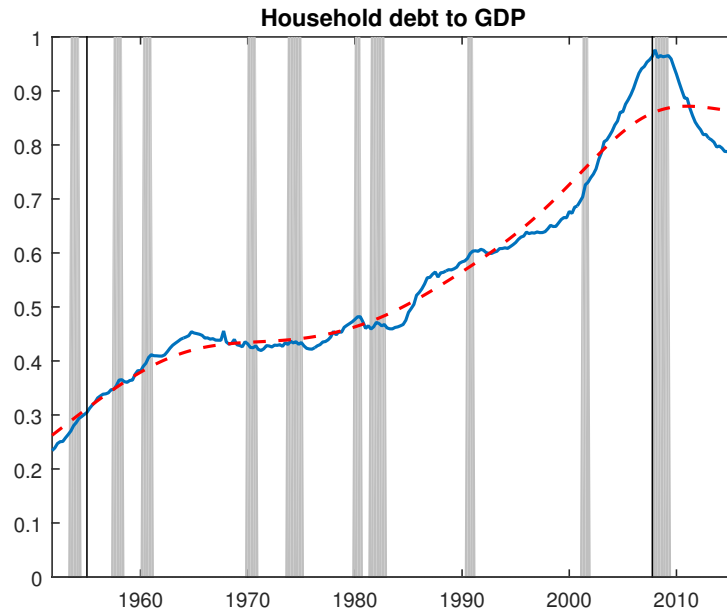


Figure 1: Household debt to GDP ratio, for the full sample, 1951q2-2015q3. The trend (red dashed) is constructed by running a HP filter with a very high smoothing parameter, 10^4 . The vertical lines mark 1955q1, start of the sample and 2007q4, the end of the sample used in the baseline estimation. The grey shaded regions indicate NBER recession dates.

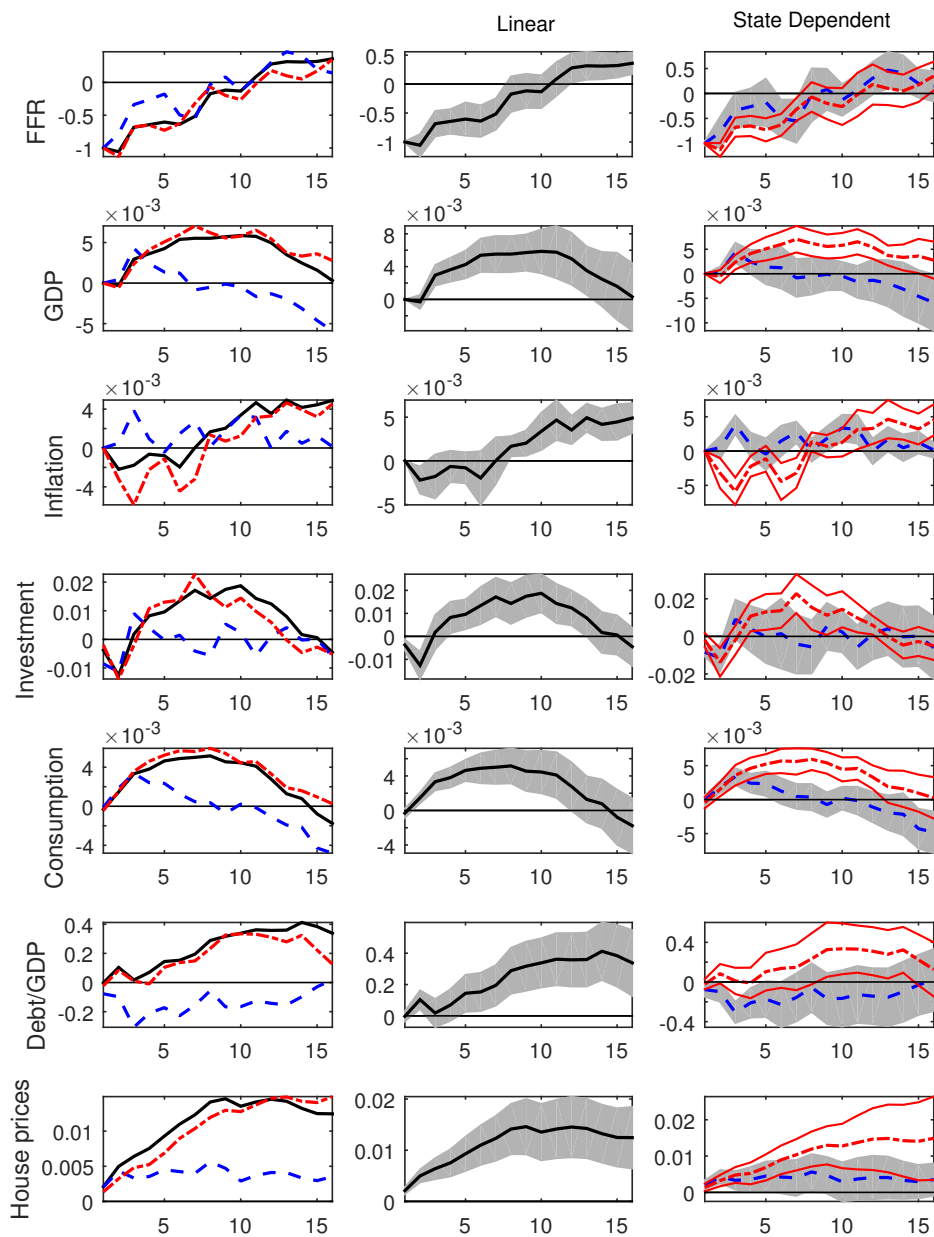


Figure 2: IRFs to a monetary shock identified with timing restrictions. The first column shows the point estimate for IRFs in linear (black solid), high debt (blue dashed) and low debt (red dot-dashed) state. The second column shows the IRFs in the linear case with 90% confidence band. The third column shows state-dependent IRFS for the high debt state (blue dashed line) and low debt state (red dot-dashed line), with their respective 90% confidence band.

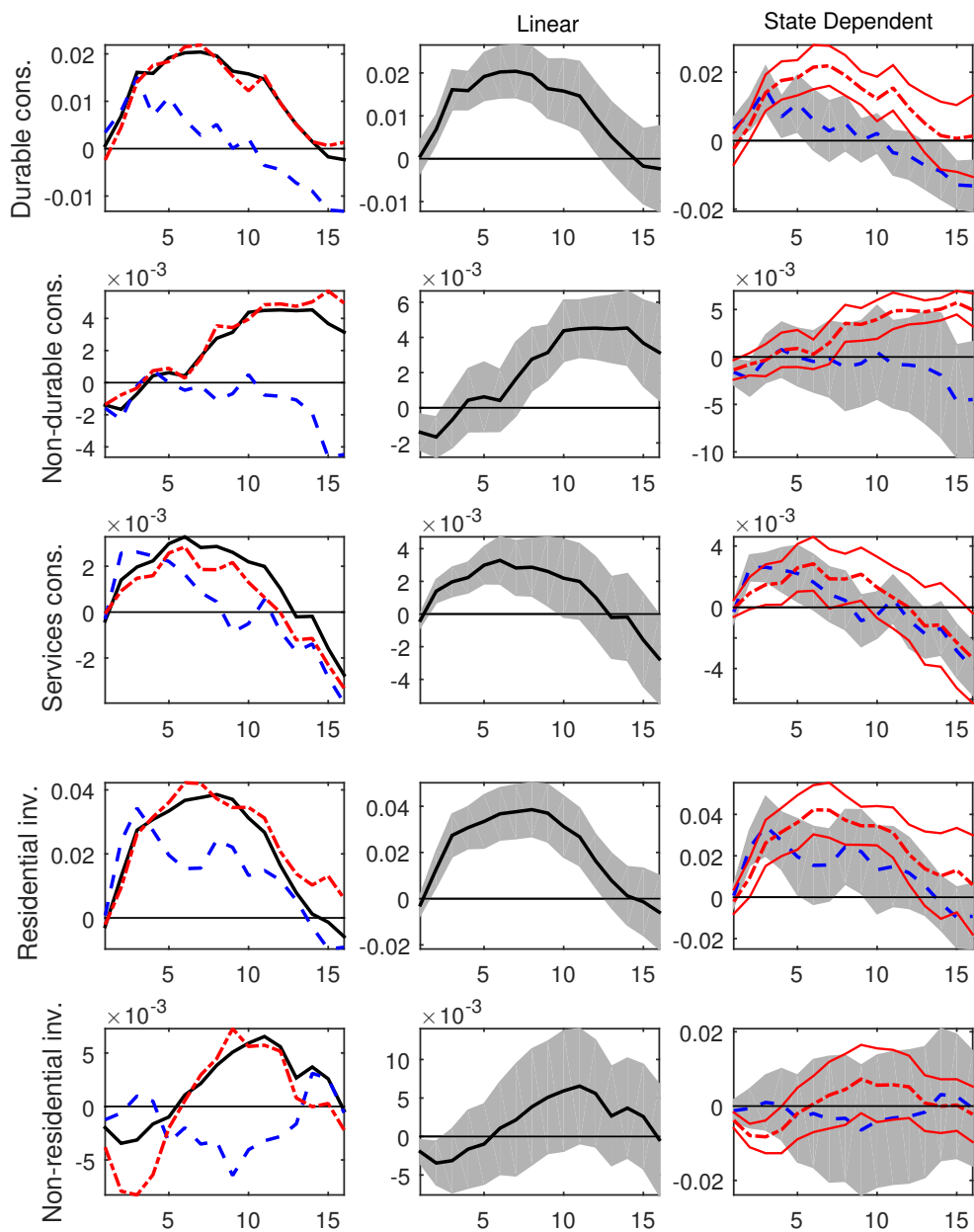


Figure 3: IRFs to a monetary shock identified with timing restrictions. The first column shows the point estimate for IRFs in linear (black solid), high debt (blue dashed) and low debt (red dot-dashed) state. The second column shows the IRFs in the linear case with 90% confidence band. The third column shows state-dependent IRFS for the high debt state (blue dashed line) and low debt state (red dot-dashed line), with their respective 90% confidence band.

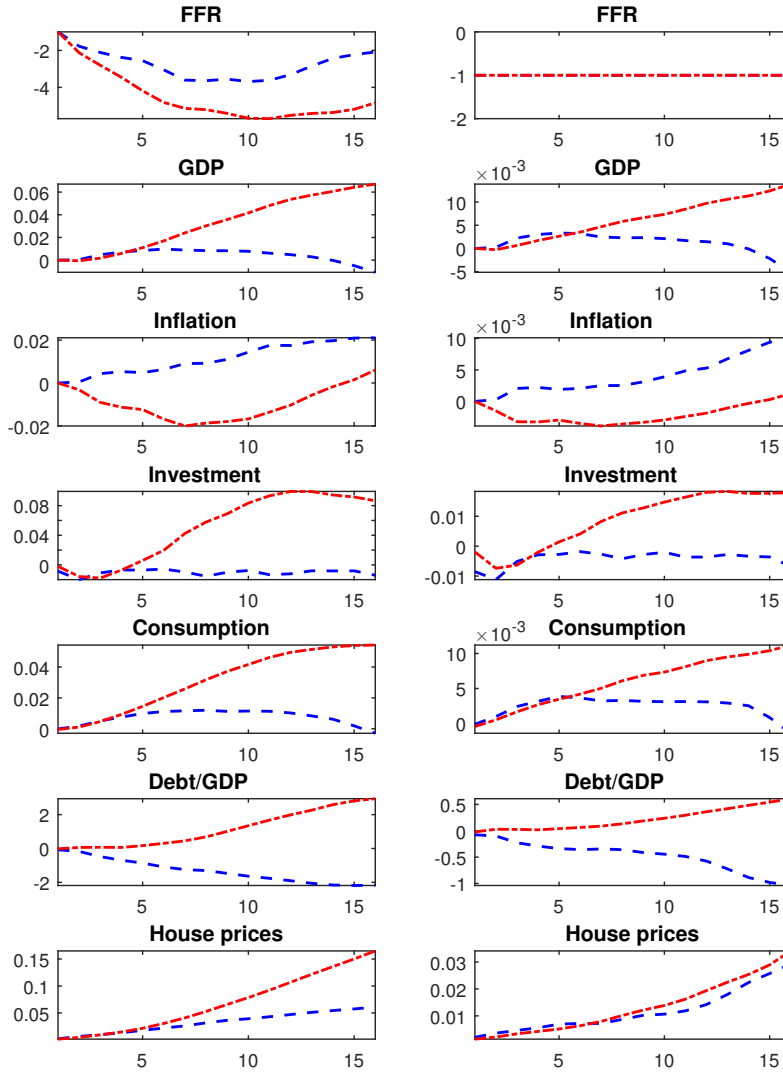


Figure 4: Cumulative effects of a monetary shock identified with timing restrictions. The first column shows the cumulative effects of a monetary shock on a variable in high debt (blue dashed) and low debt (red dot-dashed) state. The second column shows the cumulative effect of a monetary shock on a variable normalized by the cumulative response of federal funds rate in that particular state, for high debt (blue dashed) and low debt (red dot-dashed) state.

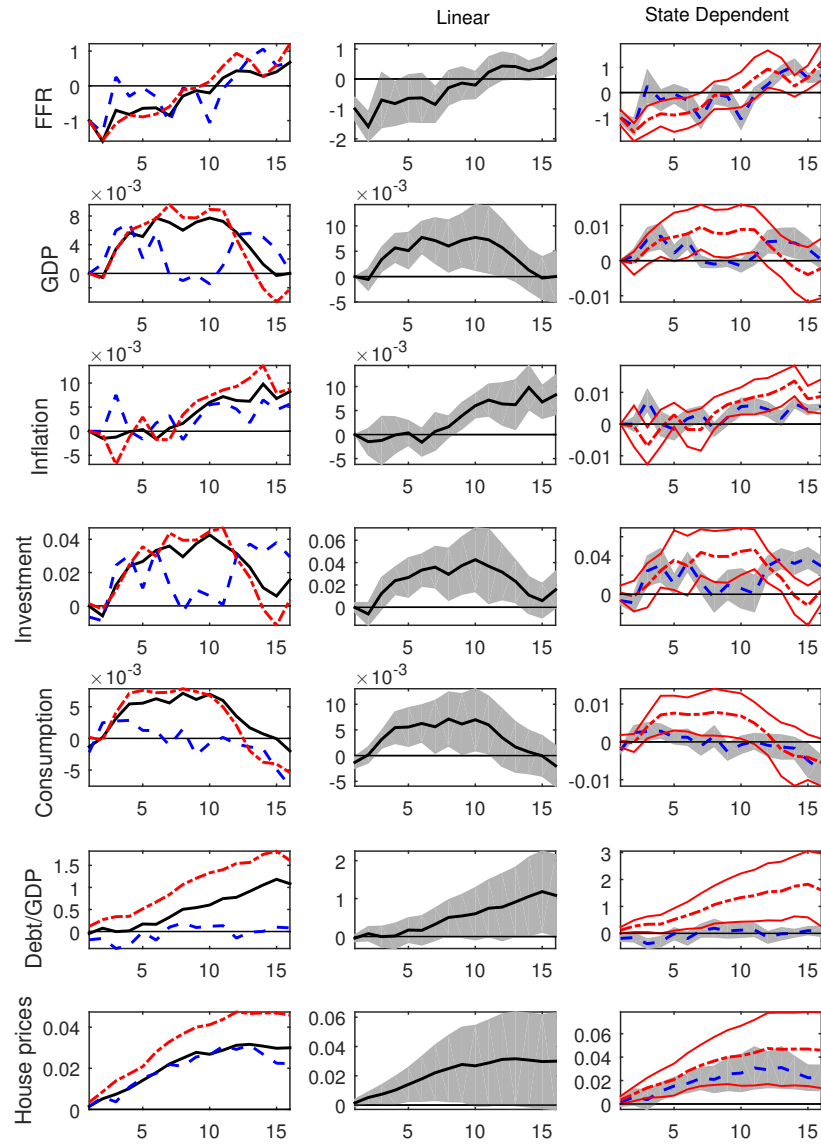


Figure 5: IRFs to a monetary shock identified as in Romer and Romer (2004), for the sample 1969q1-2007q4. The first column shows the point estimate for IRFs in linear (black solid), high debt (blue dashed) and low debt (red dot-dashed) state. The second column shows the IRFs in the linear case with 90% confidence band. The third column shows state-dependent IRFs for the high debt state (blue dashed line) and low debt state (red dot-dashed line), with their respective 90% confidence band.

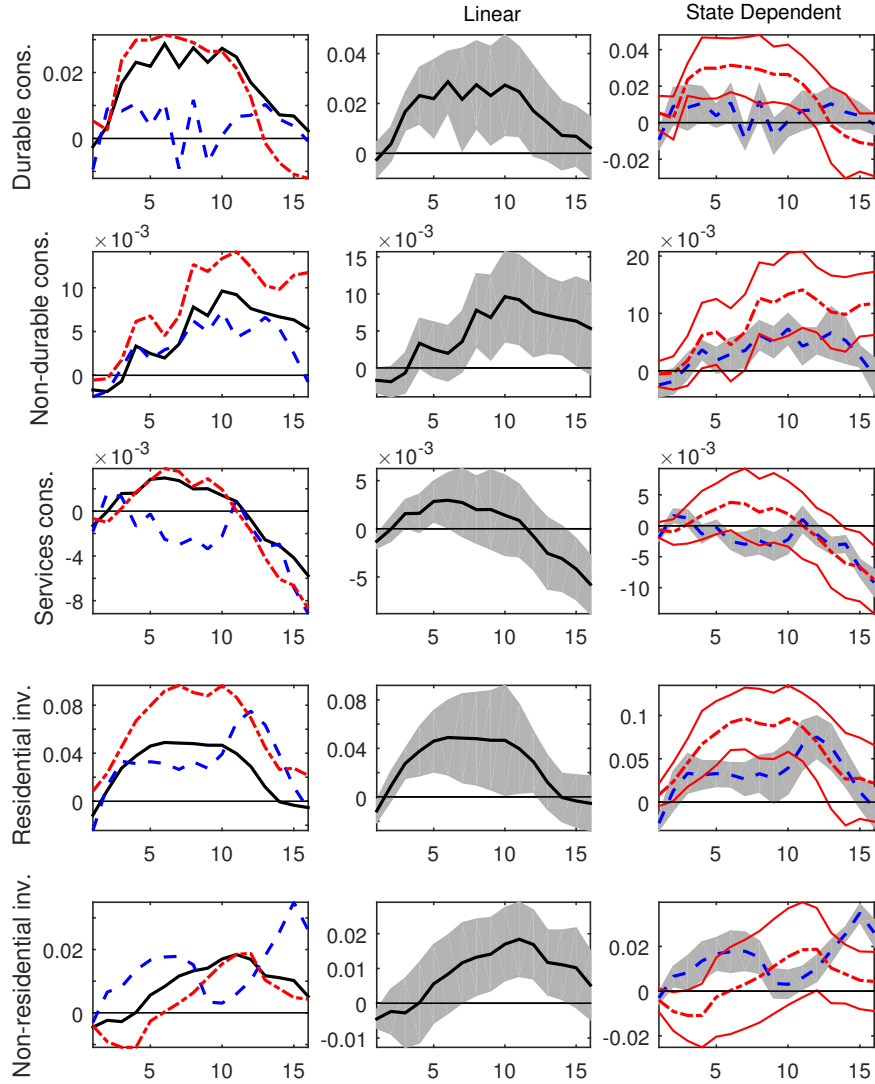


Figure 6: IRFs to a monetary shock identified as in Romer and Romer (2004), for the sample 1969q1-2007q4. The first column shows the point estimate for IRFs in linear (black solid), high debt (blue dashed) and low debt (red dot-dashed) state. The second column shows the IRFs in the linear case with 90% confidence band. The third column shows state-dependent IRFs for the high debt state (blue dashed line) and low debt state (red dot-dashed line), with their respective 90% confidence band.

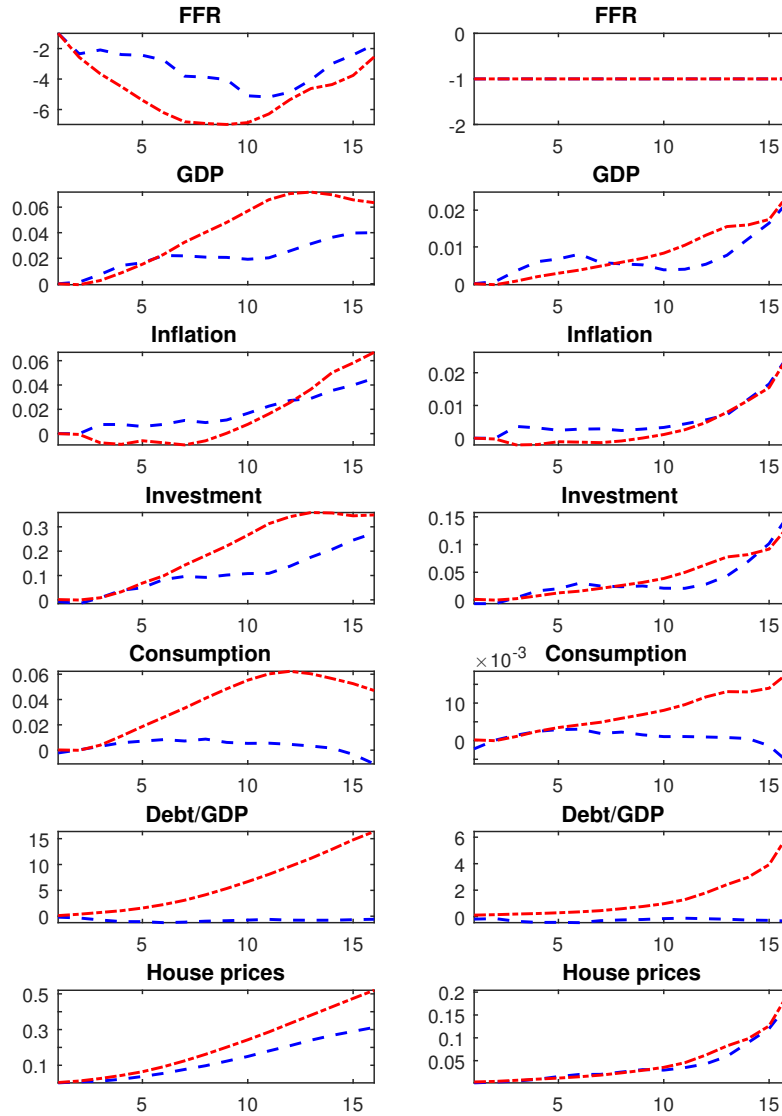


Figure 7: Cumulative effects of a monetary shock identified as in Romer and Romer (2004), for the sample 1969q1-2007q4. The first column shows the cumulative effects of a monetary shock on a variable in high debt (blue dashed) and low debt (red dot-dashed) state. The second column shows the cumulative effect of a monetary shock on a variable normalized by the cumulative response of federal funds rate in that particular state, for high debt (blue dashed) and low debt (red dot-dashed) state.

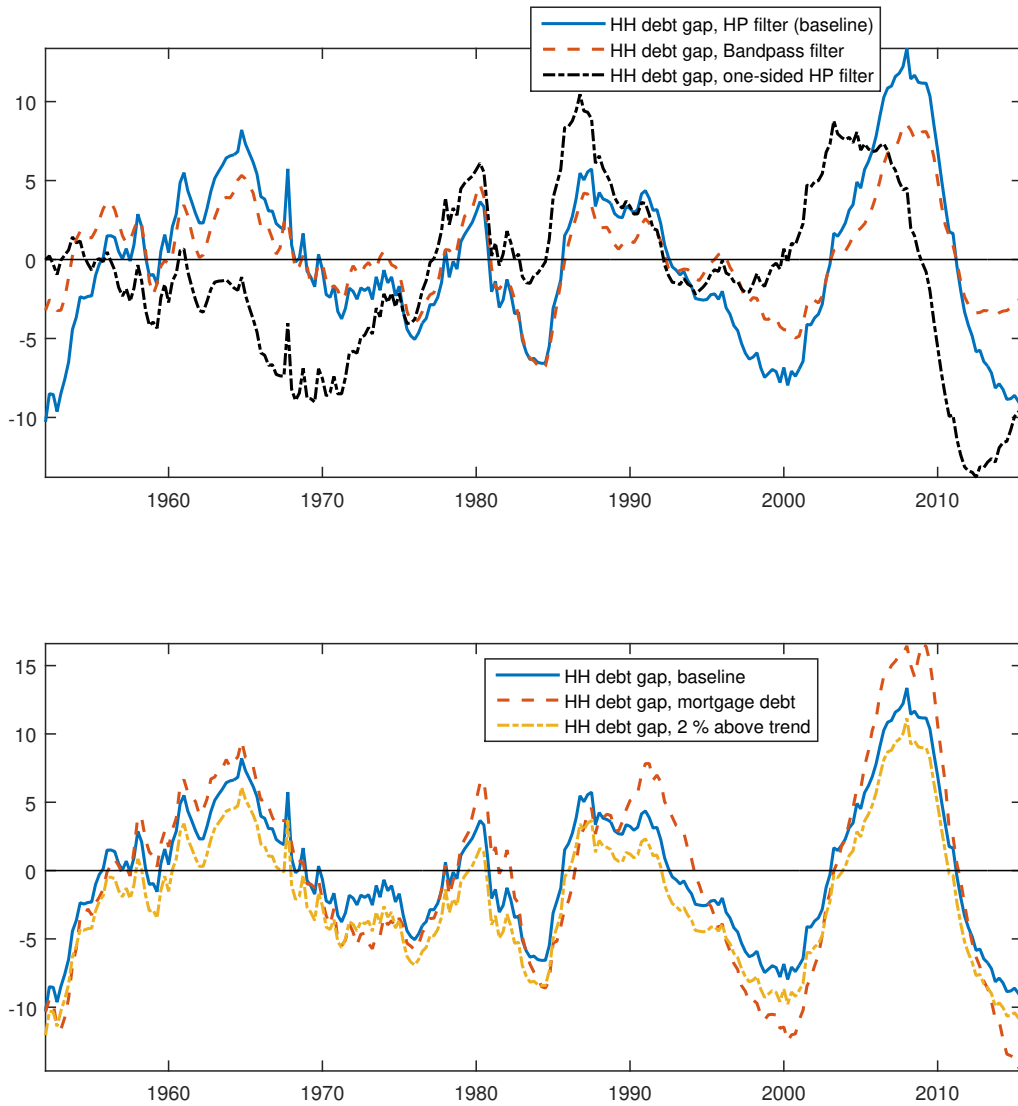


Figure 8: Robustness check: Comparing our baseline debt gap with alternative definitions of the debt gap. The top panel shows the baseline debt gap (solid line) constructed using the HP filter with $\lambda = 10^4$ and alternative measures of debt gap using a bandpass filter (dashed line), with frequencies between 4 and 64 quarters, and with the one-sided HP filter (dash-dotted line) with $\lambda = 10^4$. The bottom panel shows the baseline debt gap (solid line), debt gap with mortgage debt as state variable instead of total household debt (dashed line) and debt gap with high debt state defined as being 2 percent above our baseline trend (dot-dashed line).

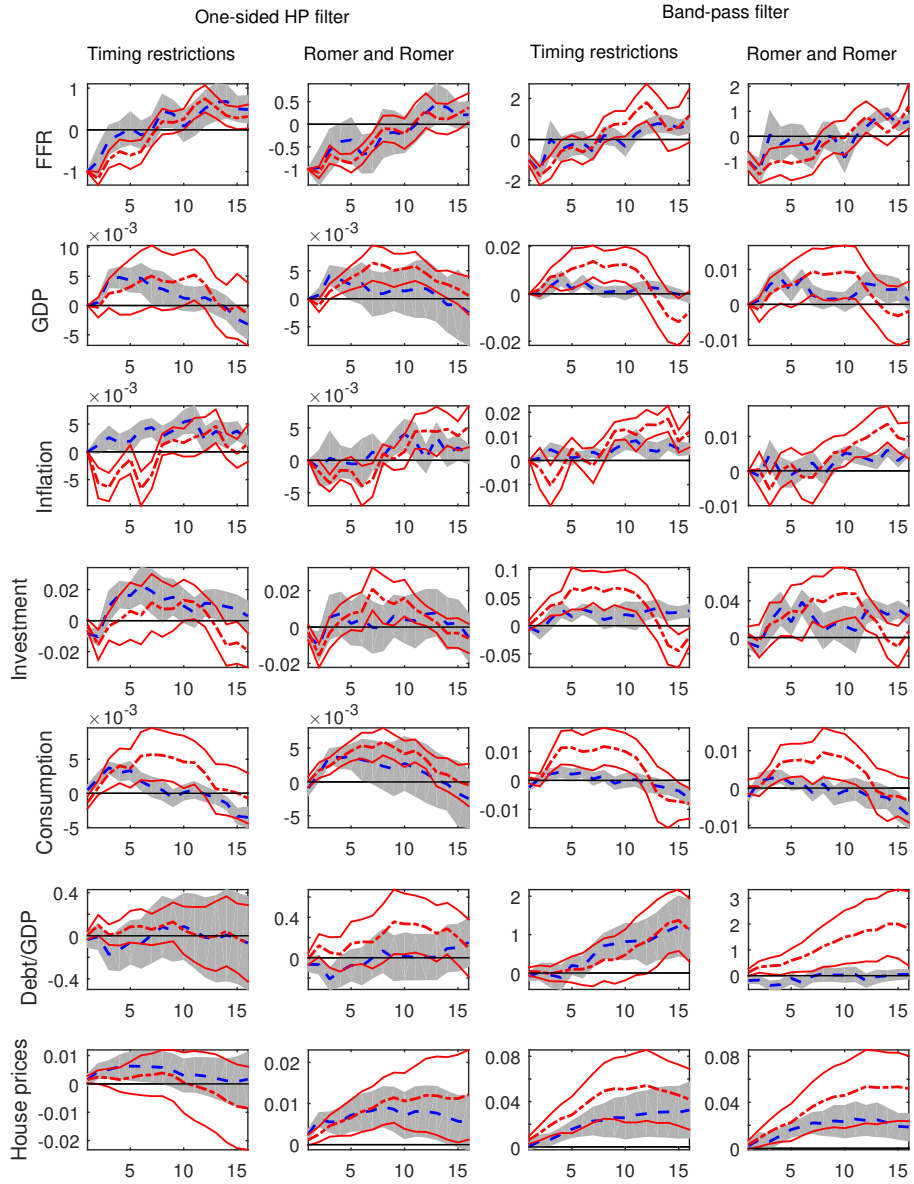
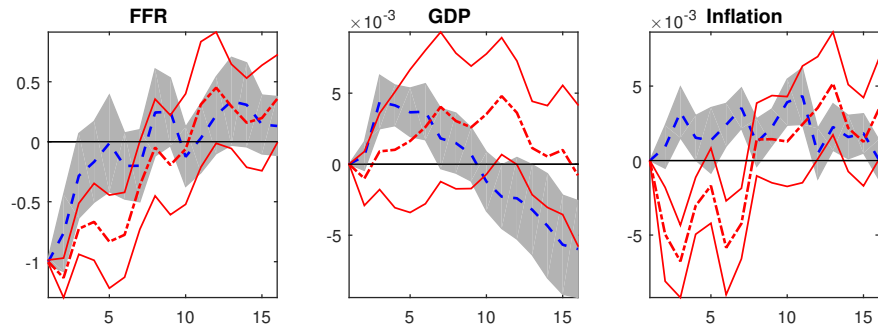
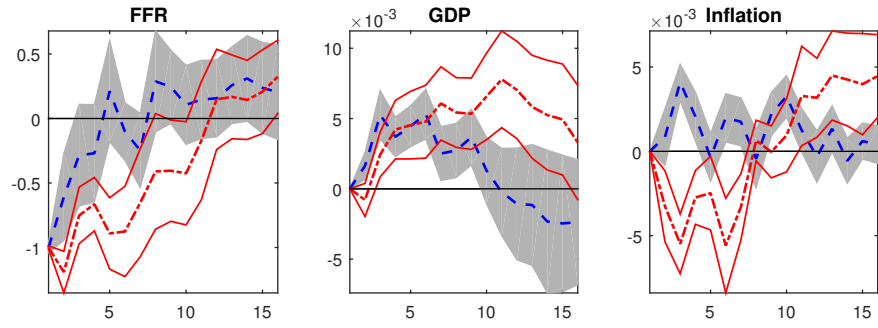


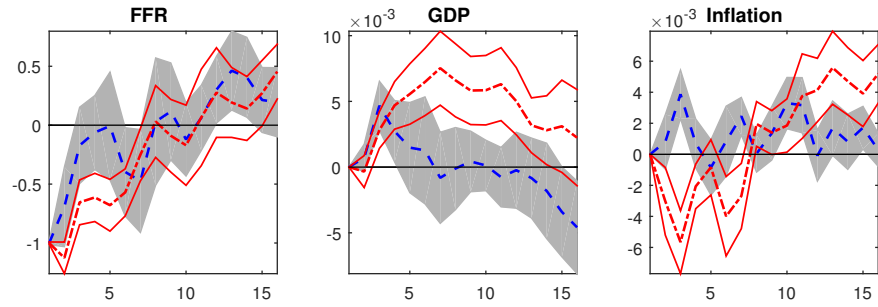
Figure 9: Robustness check: IRFs to a monetary shock under alternative filters to define high and low debt states. The first column shows the impulse response functions using one-sided HP filter with timing restriction identification, and the second column shows it with Romer and Romer shocks. The third column shows the impulse response functions using BP filter with timing restriction identification, and the fourth column shows it with Romer and Romer shocks. All columns shows state-dependent IRFS for the high debt state (blue dashed line) and low debt state (red dot-dashed line), with their respective 90% confidence band.



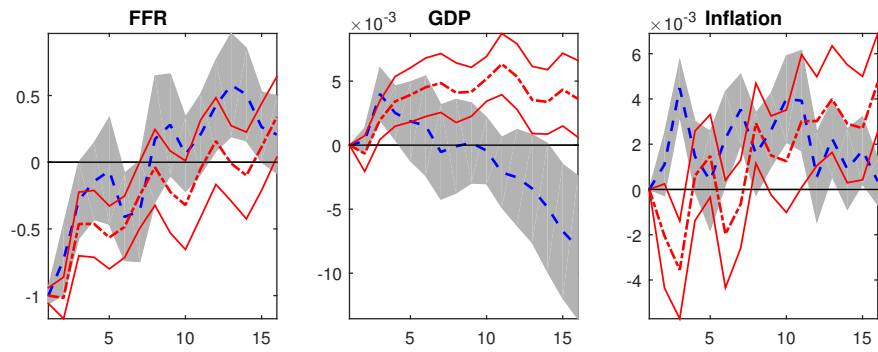
(a) Mortgage debt as a state variable



(b) High debt state defined as being 2 percent above trend



(c) Debt as an additional control variable in identification



(d) Commodity prices as an additional control variable in identification

Figure 10: Various robustness checks as described above. The figures show state-dependent IRFs for the high debt state (blue dashed line) and low debt state (red dot-dashed line), with their respective 90% confidence band.

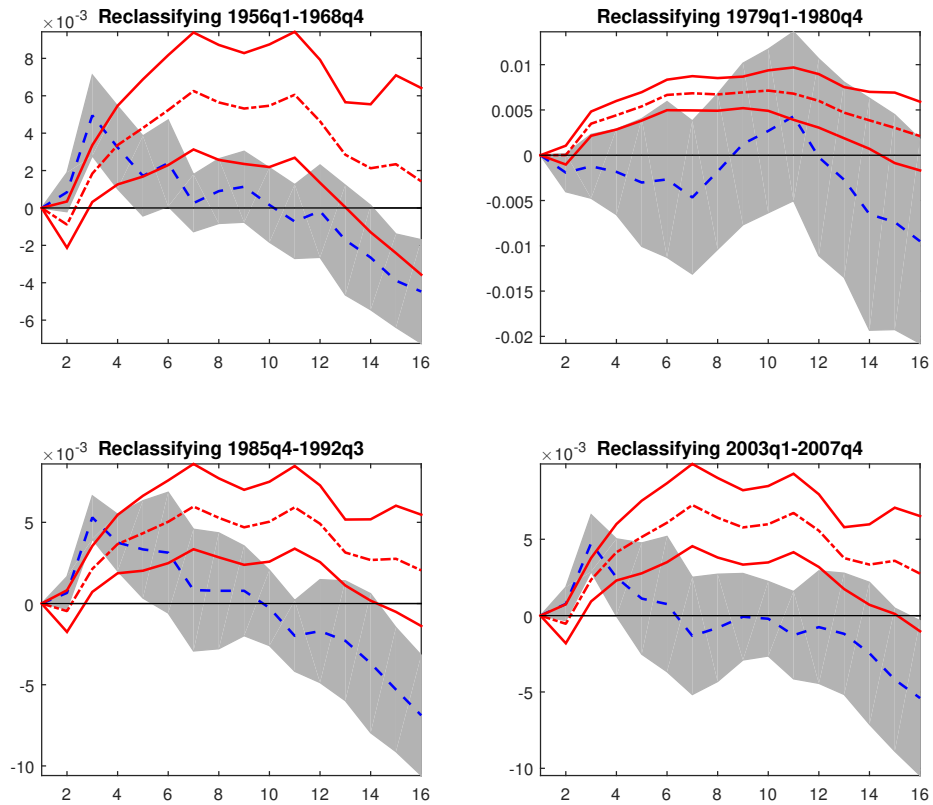


Figure 11: Robustness check: IRFs of GDP to a monetary shock in the high debt (blue dashed) and low debt (red dot-dashed) state for the sample where we re-classify the specified period from a high debt state to a low debt state. The state dependent IRFs are shown with their respective 90% confidence band.

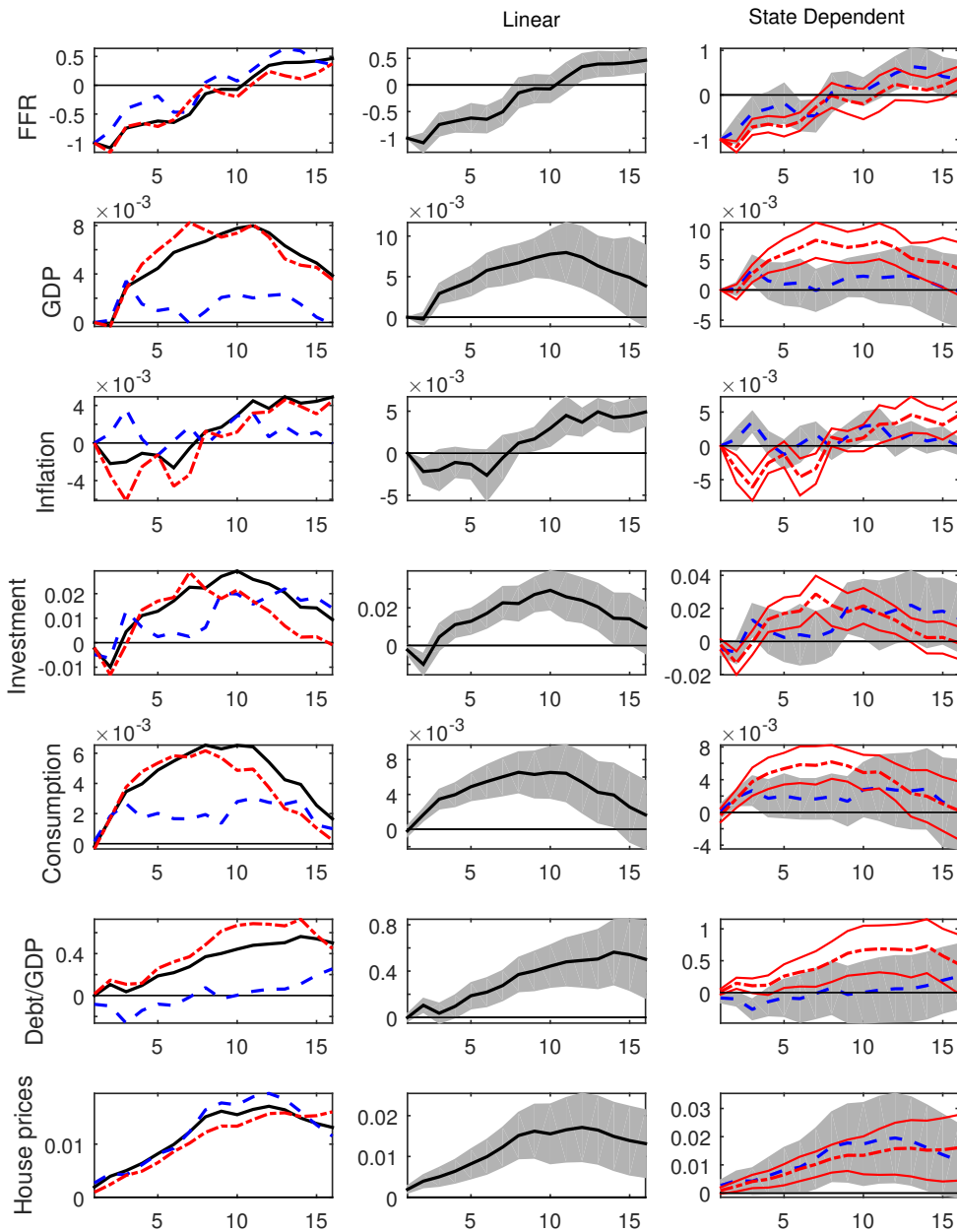


Figure 12: Robustness check: IRFs to a monetary shock with our baseline specification, for a longer sample, 1955q1-2015q3. The first column shows the point estimate for IRFs in linear (black solid), high debt (blue dashed) and low debt (red dot-dashed) state. The second column shows the IRFs in the linear case with 90% confidence band. The third column shows state-dependent IRFs for the high debt state (blue dashed line) and low debt state (red dot-dashed line), with their respective 90% confidence band.

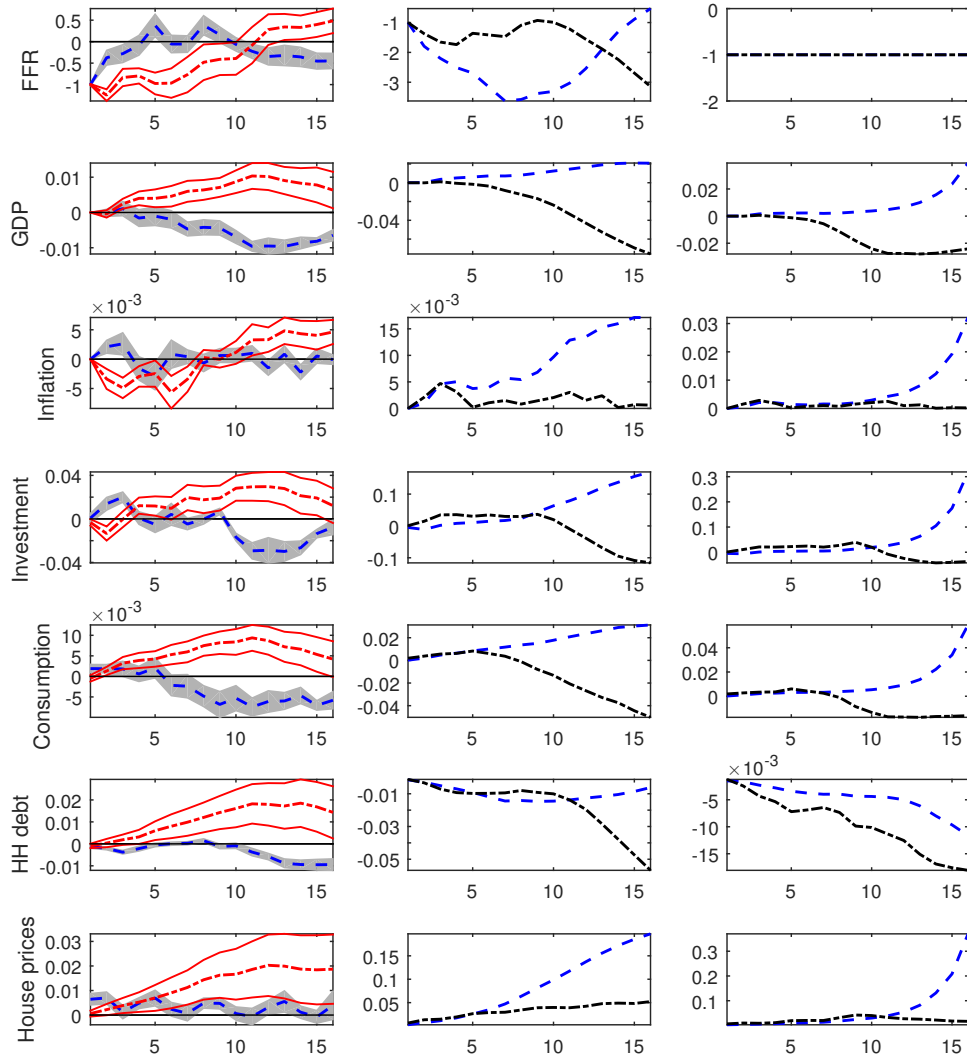
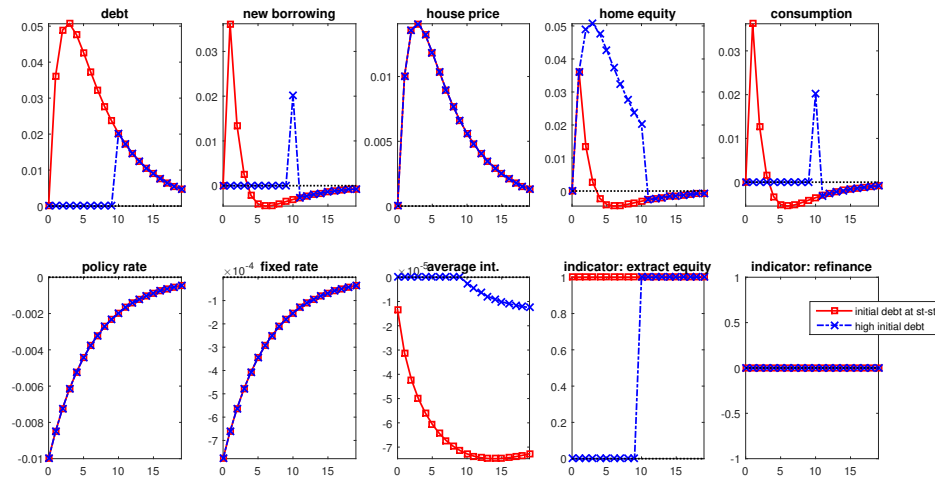
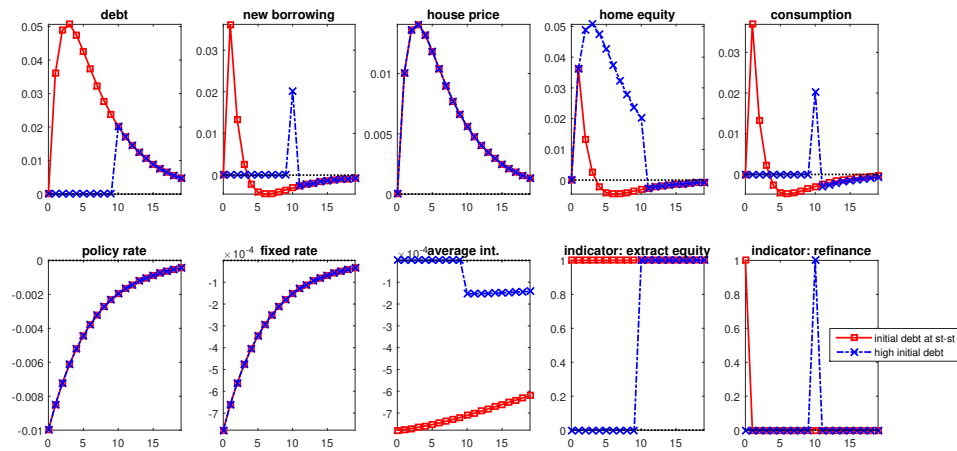


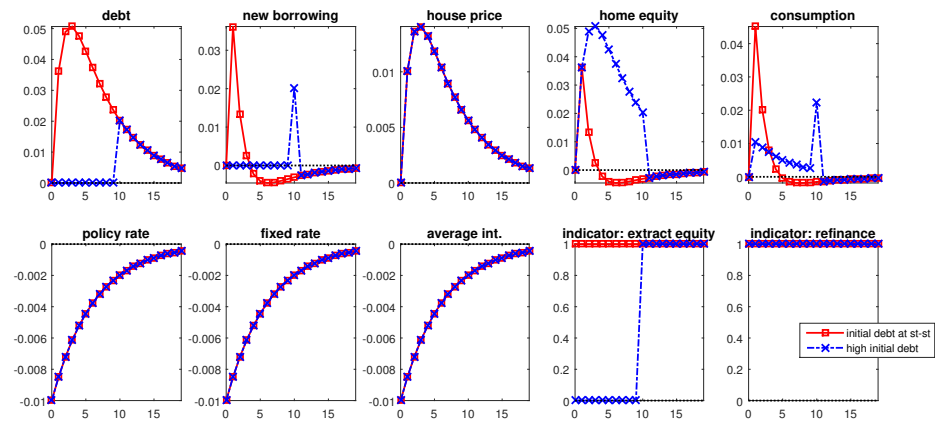
Figure 13: Robustness check: IRFs to a monetary shock for high debt states coinciding with recession for a longer sample, 1955q1-2015q3. The first column shows state-dependent IRFs for the high debt state *and* recession (blue dashed line) and otherwise (red dot-dashed line), with their respective 90% confidence band. The second column shows the cumulative effects of a monetary shock on a variable in high debt *only* (blue dashed) and high debt state coinciding with recession (black dot-dashed) state. The third column shows the cumulative effect of a monetary shock on a variable normalized by the cumulative response of federal funds rate in that particular state, for high debt *only* (blue dashed) and high debt coinciding with recession (black dot-dashed) state.



(a) Fixed-rate loans with no refinancing



(b) Fixed-rate loans with refinancing option



(c) Adjustable-rate loans

Figure 14: Impulse responses from our theoretical model of model variables to an annualized 100 bps monetary policy shock for steady state debt level (red line with circles) and the high debt state (blue line with crosses).

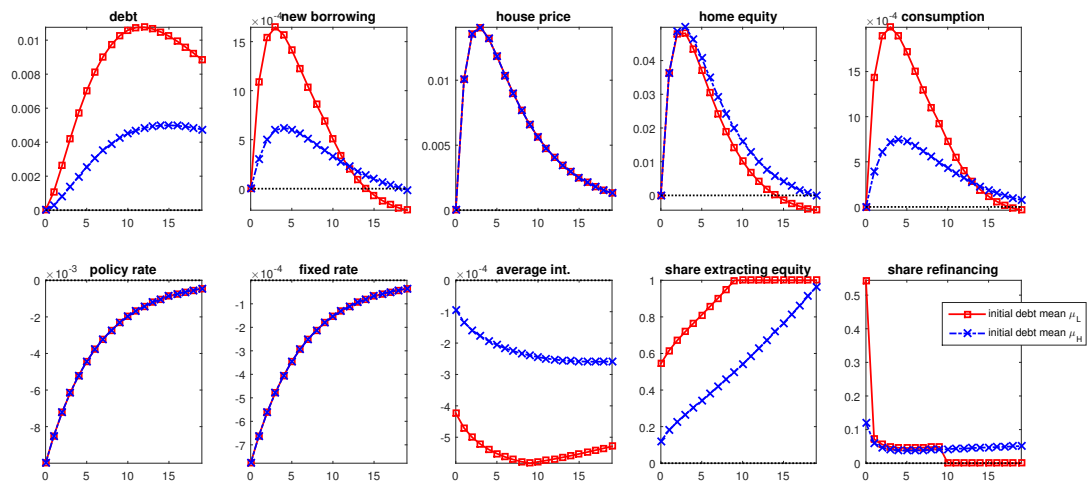


Figure 15: Impulse responses from our theoretical model of model variables to an annualized 100 bps monetary policy shock, when we aggregate over a distribution of households, when $\mu = \mu_L$ versus $\mu = \mu_H$. The figure shows the response for the low debt households, $\mu = \mu_L$, (red line with circles) and the high debt households, with $\mu = \mu_H$ (blue line with crosses).

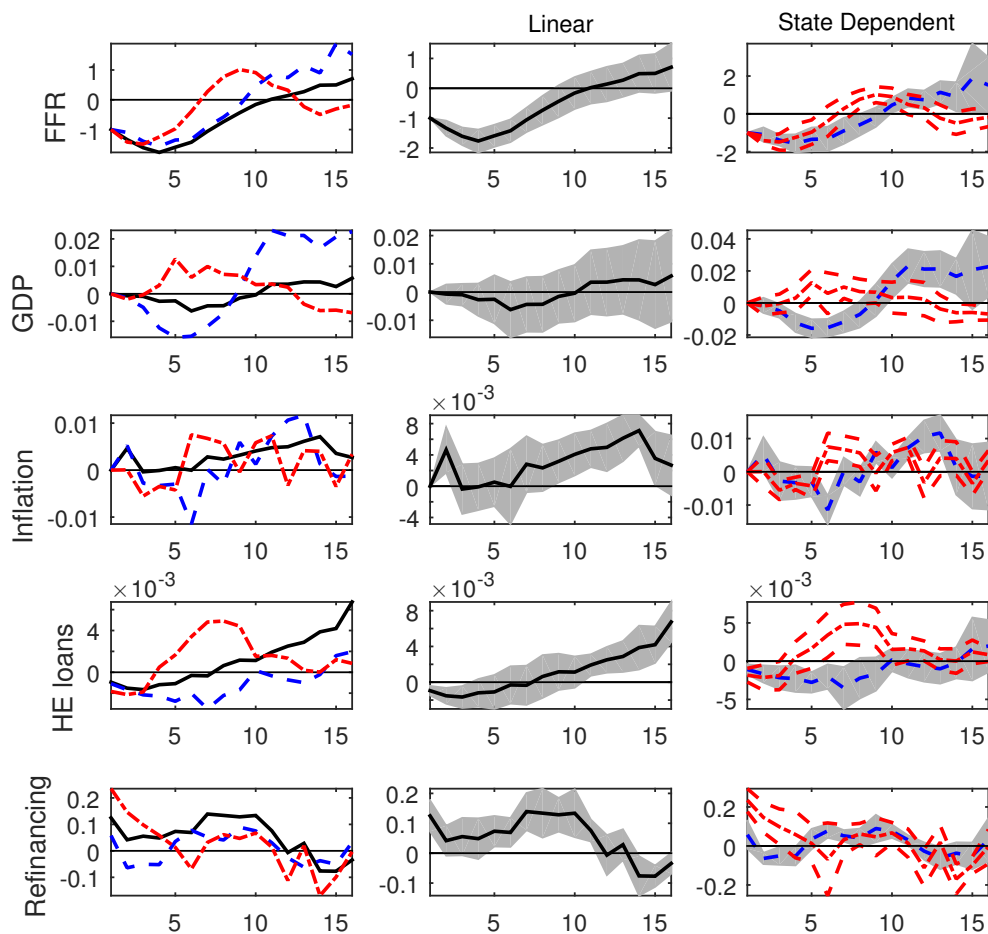


Figure 16: IRFs of home equity loans (HE loans), refinancing and other variables to a monetary shock, for the sample period: 1990q4-2015q3. The first column shows the point estimate for IRFs in linear (black solid), high debt (blue dashed) and low debt (red dot-dashed) state. The second column shows the IRFs in the linear case with 90% confidence band. The third column shows state-dependent IRFs for the high debt state (blue dashed line) and low debt state (red dot-dashed line), with their respective 90% confidence band. Note: We use home equity loans as a share of total household debt.

A Appendix on data sources

Table 1: Data Sources

Data	Description	Source
GDP	Nominal GDP	BEA
PGDP	GDP deflator	BEA
Consumption	Nominal Personal consumption expenditures	BEA
Investment	Nominal Gross private investment	BEA
Residential Investment	Nominal Residential investment	BEA
Population	Civilian Noninstitutional Population, 16 and over (CNP16OV)	FRED
Federal funds rate	FFR	FRED
Mortgage debt	Households; Home Mortgages; Liability (HHMSDODNS)	FRED
Household debt	Households; Liability (CMDEBT)	FRED
House price	Real house price index	Robert Shiller's data webpage
Wu and Xia shadow rate		Atlanta Fed website
Home equity loans	Z1/FL893065125.Q	FRB Financial Accounts
Refinancing	Refinancing applications index	Mortgage Bankers Association

Note: Real values of GDP and its expenditure components were all deflated using the GDP deflator.

B Additional Details about Debt Gap and Monetary Shocks

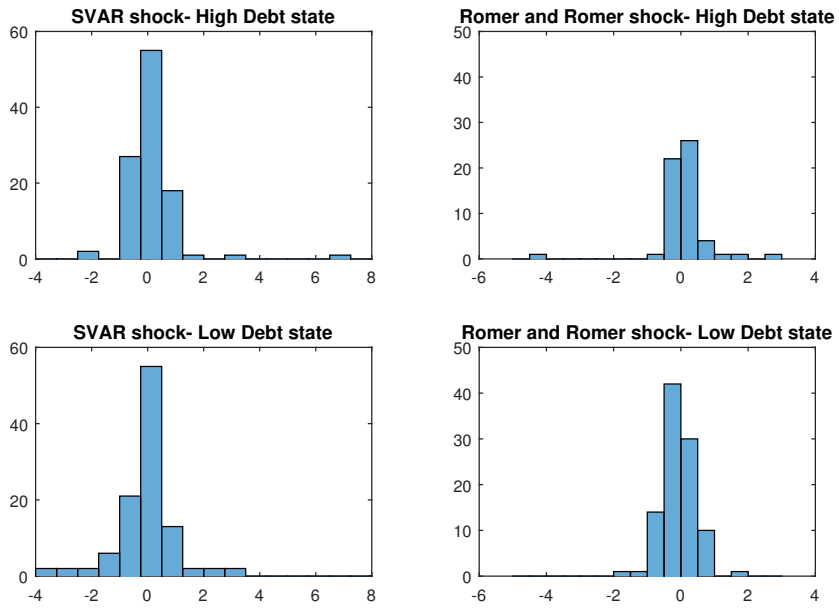


Figure 17: Histograms of SVAR and Romer-Romer monetary policy shocks by household debt state. The top panel shows the distribution of the monetary policy shocks in the high debt state, and the bottom panel shows the low debt state.

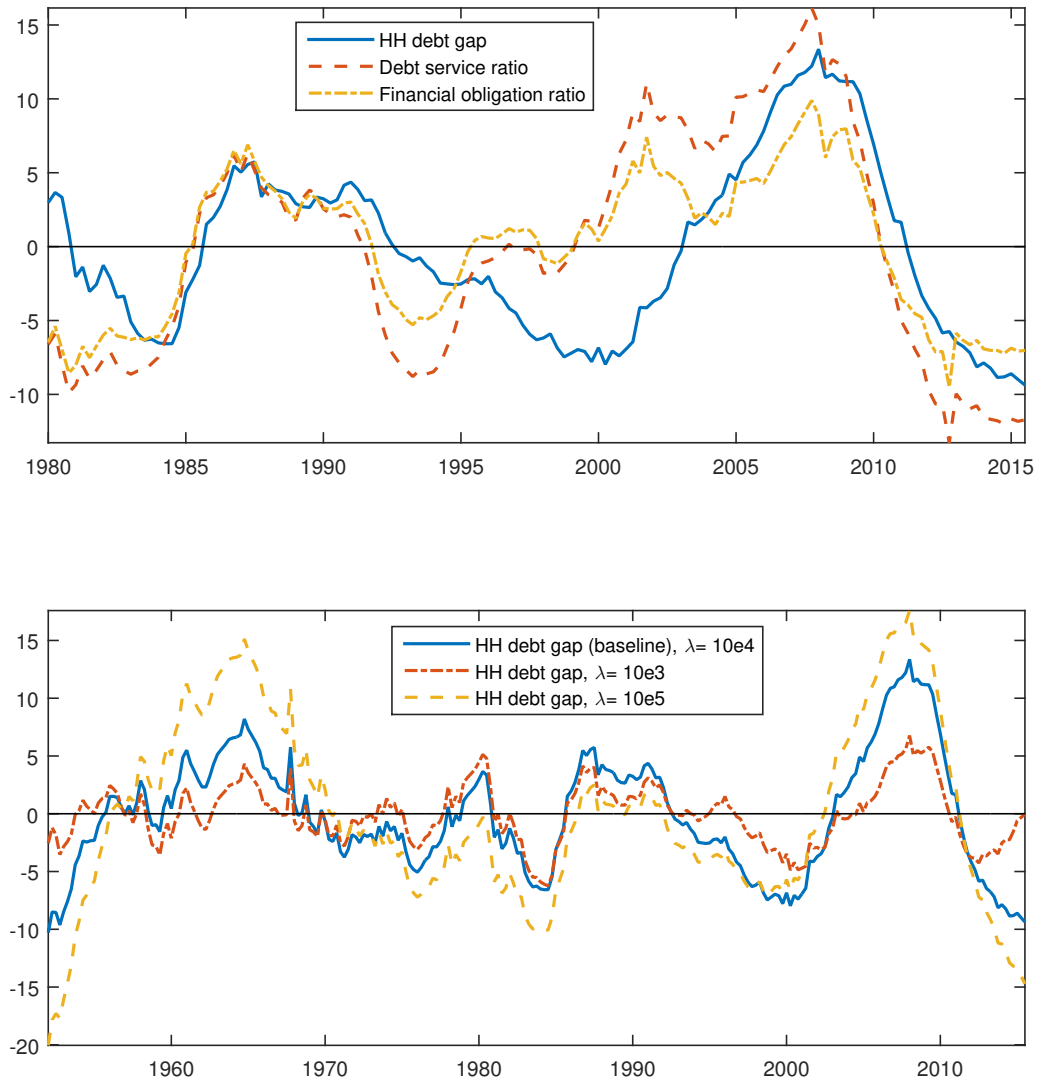


Figure 18: Comparison of the debt gap with alternative measures of debt overhang for the overlapping sample. The first panel of the figure compares our baseline debt gap with the measures of household debt services and financial obligation ratio percent deviations from their respective means. Source: Federal Reserve Board. Note: Household debt service ratio (DSR) is the ratio of total required household debt payments to total disposable income, including required mortgage and scheduled consumer debt payments. The Financial Obligations Ratio (FOR) is a broader measure than the debt service ratio. It includes rent payments on tenant-occupied property, auto lease payments, homeowners' insurance, and property tax payments. The second panel shows the implied debt gap under alternative values of the smoothing parameter, λ in the HP filter.

C Statistical significance of baseline results

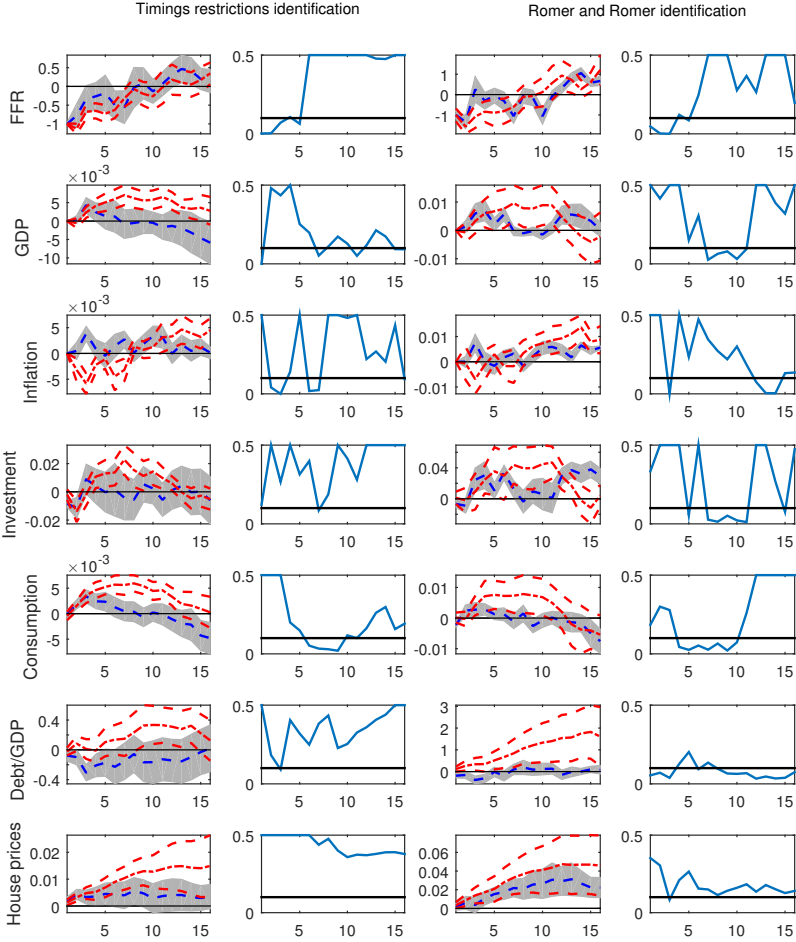


Figure 19: The first and third column shows the impulse response function to a monetary shock in high debt (blue dashed) and low debt (red dot-dashed) state, under the respective identification. The second and last column show the p-value for the null hypothesis that the response in high debt is equal to the response in the low debt state at a given horizon. The p-value are capped at 0.5. The solid black line is at 0.1, at the 10% significance level.

D Robustness Check: One-sided HP and Band-Pass filters to define debt gap

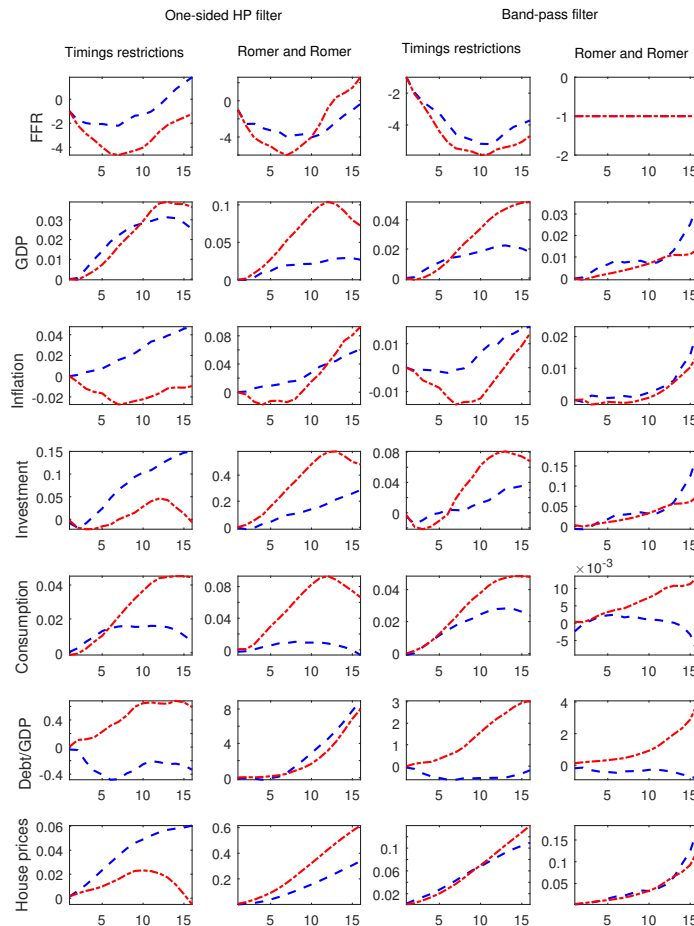


Figure 20: Cumulative effects of a monetary shock under alternative filters to define high and low debt states. The first column shows results for one-sided HP filter with timing restriction identification, and the second column shows it with Romer and Romer shocks. The third column shows the impulse response functions using BP filter with timing restriction identification, and the fourth column shows it with Romer and Romer shocks. All figures show the effects in high debt (blue dashed line) and low debt state (red dot-dashed line).

E Robustness Check: Threshold VAR

We also consider a threshold VAR, as a robustness check of our baseline empirical results. More specifically, we consider the following threshold VAR to look at the state dependent effects of monetary policy based on household debt:

$$Y_t = I_{t-1}A(L)Y_{t-1} + (1 - I_{t-1})B(L)Y_{t-1} + u_t, \quad (17)$$

where $u_t \sim N(0, \Omega_t)$, and $\Omega = I_{t-1}\Omega_A + (1 - I_{t-1})\Omega_B$. Here, as before, I is the dummy variable indicating high-debt state, and $A(L)$ and $B(L)$ are polynomials of order 2. In order to identify a monetary shock we order federal funds rate after macroeconomic aggregates such as GDP, consumption, investment and inflation, but before house prices and household debt, before doing a Cholesky decomposition.

While our baseline Jorda method allows for natural transition across states, the VAR methodology assumes that we stay in a given state for a long time. Given that the average duration of both high and low debt states in our sample are around 13 quarters, the short-run impulse response function using the threshold-VAR methodology are consistent with the data.

Figure 21 shows the resulting IRFs in the linear and state dependent case. Note the state dependence results are robust to this different methodology and almost all variables are less responsive to monetary policy in the high-debt state. The state-dependence in investment is weaker than our baseline case, whereas the only exception is the case of house prices, where the state dependence is reversed.

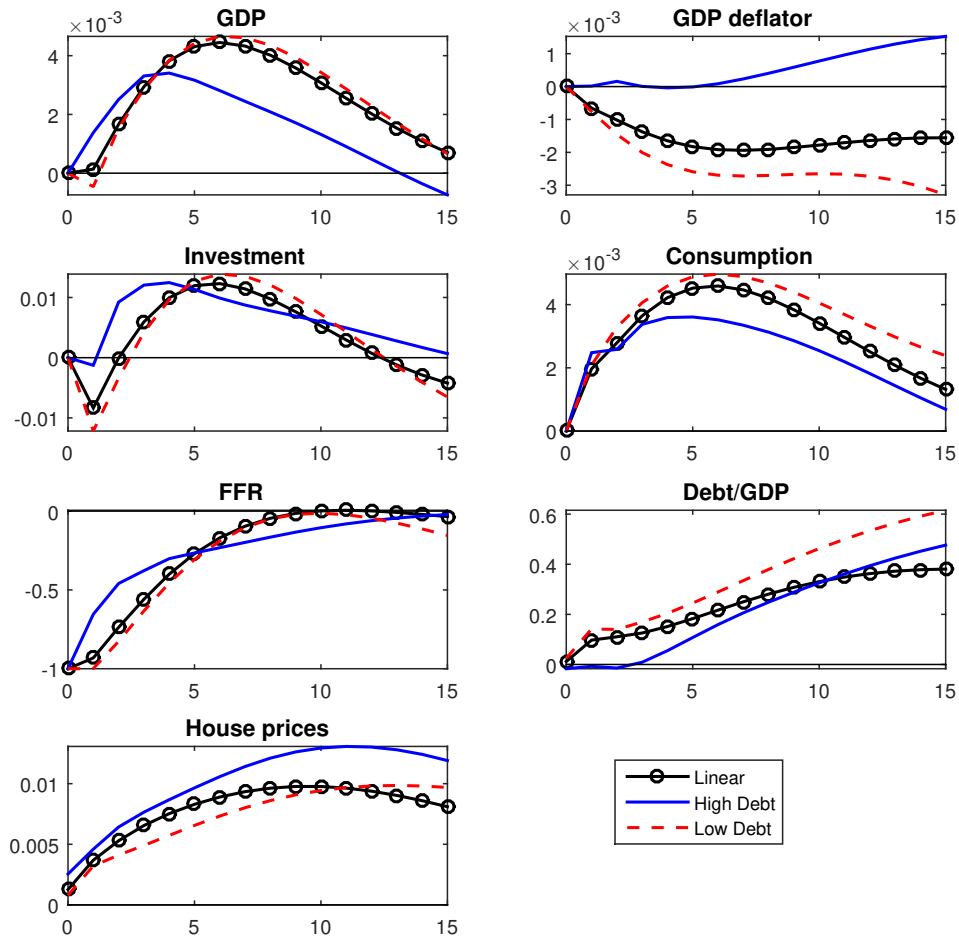
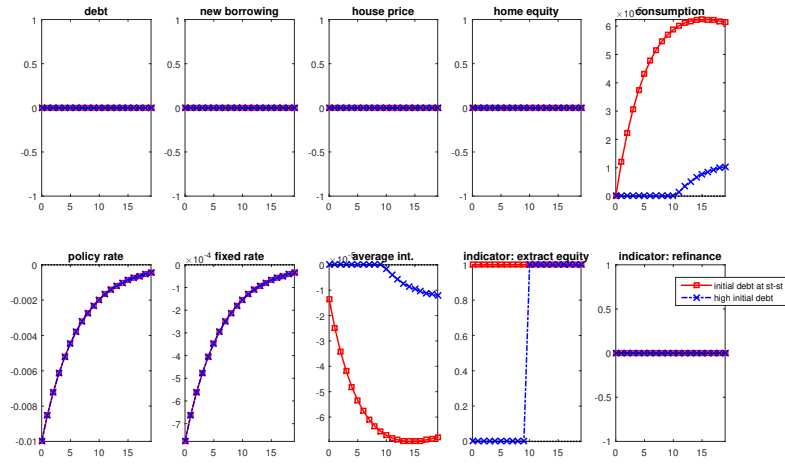


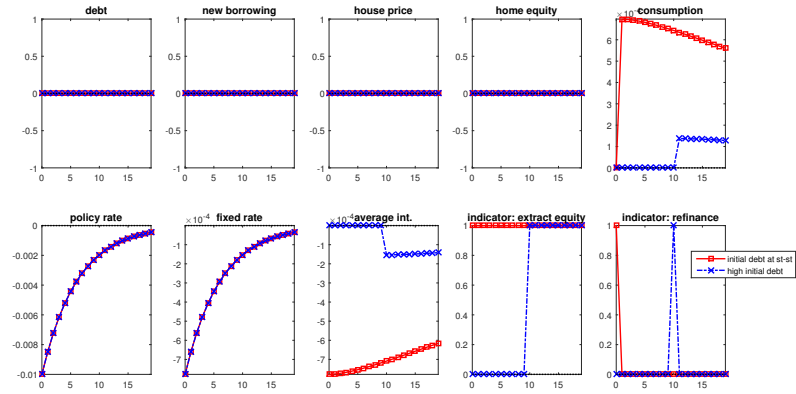
Figure 21: Robustness check: IRFs to a monetary shock using the threshold VAR approach. The figure shows the point estimate for IRFs in linear (black solid), high debt (blue dashed) and low debt (red dot-dashed) state.

F Theoretical responses to a monetary shock: Interest rate channel only

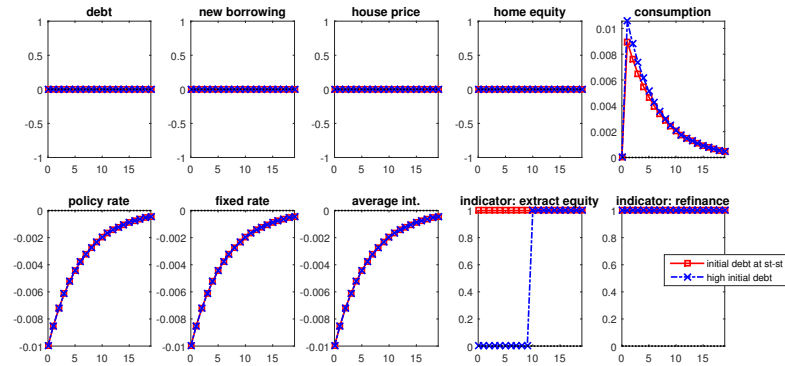
In order to isolate the interest rate channel from the home equity channel, we show the theoretical impulse response functions to a monetary policy in the models considered in Section 4, where we set $\rho_{qR} = 0$ in Equation (9). The following figures, thus show the analogous responses to a monetary policy shock for the three cases considered, as Figure 14, where we only show the interest rate channel.



(a) Fixed-rate loans with no refinancing



(b) Fixed-rate loans with refinancing option



(c) Adjustable-rate loans

Figure 22: Impulse responses from our theoretical model of model variables to an annualized 100 bps monetary policy shock when we shut down feedback effects to house prices. The figure shows the response for the steady state debt level (red line with circles) and the high debt state (blue line with crosses).

G General Equilibrium Extension of Theoretical Model

In this Appendix, we extend the partial equilibrium model in Section 4 to include saver households, and consider a general equilibrium version of the model with endogenous labor supply (and income), endogenous house price formation and variable inflation rates. Here, we consider the case of adjustable-rate mortgages. Similar to Iacoviello (2005), the model features two types of agents which differ in terms of their time discount factors. In particular, the impatient households (identified with subscript I) discount the future more heavily than the patient households (identified by subscript P); hence, $\beta_P > \beta_I$. Their period utility functions are identical, and are given by

$$u(c_{i,t}, h_{i,t}, n_{i,t}) = \log c_{i,t} + \xi \log h_{i,t} - \frac{n_{i,t}^{1+\vartheta}}{1+\vartheta}, \text{ for } i \in \{P, I\} \quad (18)$$

where ξ determines the relative importance of housing in utility, n_i denotes labor supply, and ϑ is the inverse of the Frisch-elasticity of labor supply.

We retain the assumption that there is no residential investment in the model and the aggregate housing level is a constant, but allow housing to be traded across the two types of households; hence, $h_{P,t} + h_{I,t} = \bar{h}$. The budget constraint of patient households is given by

$$c_{P,t} + q_t (h_{P,t} - h_{P,t-1}) + \frac{B_t}{P_t} + \frac{L_t}{P_t} \leq w_{P,t} n_{P,t} + (1 + R_{t-1}) \frac{B_{t-1}}{P_t} + (R_{t-1} + \kappa) \frac{D_{t-1}}{P_t} + \frac{\Pi_t}{P_t}, \quad (19)$$

where B_t denoted nominal holdings of 1-period government bonds (assumed to be in zero supply), $w_{P,t}$ is the wage rate of patient households, and Π_t denotes the pure profits of monopolistically competitive firms, which is transferred to patient households in lump-sum fashion. The budget constraint of impatient households is given by

$$c_{I,t} + q_t (h_{I,t} - h_{I,t-1}) + (R_{t-1} + \kappa) \frac{D_{t-1}}{P_t} \leq w_{I,t} n_{I,t} + \frac{L_t}{P_t}, \quad (20)$$

where $w_{I,t}$ is the wage rate of impatient households. Their borrowing constraint is now modified as

$$\frac{L_t}{P_t} = \phi q_t (h_{I,t} - h_{I,t-1}) + \max \left\{ 0, \phi q_t h_{I,t-1} - (1 - \kappa) \frac{D_{t-1}}{P_t} \right\}. \quad (21)$$

Thus, as opposed to the partial equilibrium model, we now allow agents to borrow up to ϕ percent of the housing value at purchase (i.e., first lien), but allow home equity loans (i.e., second lien) only when their home equity level surpasses the threshold level, similar to the partial equilibrium model we analyzed before.

The production part of the model is standard. In particular, we consider a unit of measure of monopolistically competitive intermediate goods producers indexed by j , that face quadratic price adjustment costs (with a level parameter κ_p), and produce differentiated output, $y_t(j)$, using the

following production function

$$y_t(j) = zn_{P,t}(j)^\psi n_{I,t}(j)^{1-\psi} - f, \quad (22)$$

where z is the level of total factor productivity (TFP), ψ is the share of patient household labor, and f denotes the fixed cost in production. The differentiated goods of intermediate goods producers are aggregated by perfectly competitive producers, as is standard in New Keynesian set-ups. In equilibrium, the resource constraint of the economy is given by

$$c_{P,t} + c_{I,t} = y_t - \frac{\kappa_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 y_t, \quad (23)$$

where y_t denotes aggregate output, and the inflation rate is determined via a New Keynesian Phillips curve, which can be derived from the first-order conditions of the monopolistically competitive intermediate goods producers as

$$\left(\frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} = E_t \left[\left(\beta_P \frac{\lambda_{P,t+1}}{\lambda_{P,t}} \right) \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1} y_{t+1}}{\pi y_t} \right] - \frac{\eta - 1}{\kappa_p} (1 - \theta \Omega_t), \quad (24)$$

where λ_P denotes the Lagrange multiplier on the patient household budget constraint, η is the elasticity of substitution among the differentiated intermediate goods, $\theta = \eta / (\eta - 1)$ is the average mark-up that the monopolistically competitive firms charge, and Ω_t denotes their marginal cost of production.

Monetary policy is conducted via a Taylor rule that is given by

$$R_t = \rho R_{t-1} + (1 - \rho) \left(R + a_\pi \log \frac{\pi_t}{\pi} \right) + \varepsilon_{R,t}, \quad (25)$$

where a_π denotes the long-run response coefficient with respect to inflation.

G.1 Parameterization and impulse responses

We set the patient households' discount factor, β_P , to 0.995, which along with the steady-state inflation factor, π , of 1.005, implies a 4 percent nominal interest rate in annualized terms at the steady state, similar to our partial equilibrium model. Similarly, we set the share of debt principal paid out every period, κ , to 0.0125, and the LTV ratio for new housing purchases, ϕ , 0.9 as before.

The discount factor for impatient households, β_I , is set to 0.97, the level parameter for housing in the utility function, ξ , is set to 0.12, and the share parameter in the production function, ψ , is set to 0.65, following Iacoviello and Neri (2010). We set ϑ to 1, implying a unit Frisch-elasticity of labor supply, and η to 11, implying that firms set a 10 percent average markup when setting prices over their marginal cost. The price stickiness parameter, κ_p , is set to 100, implying that the slope of the New Keynesian Phillips curve is 0.1, in line with estimates in the literature. Finally, for the smoothness parameter on the Taylor rule, ρ , is set to 0.85, similar to its corresponding value in the partial equilibrium model, and the long-run response coefficient for inflation, a_π , is set to 1.5.

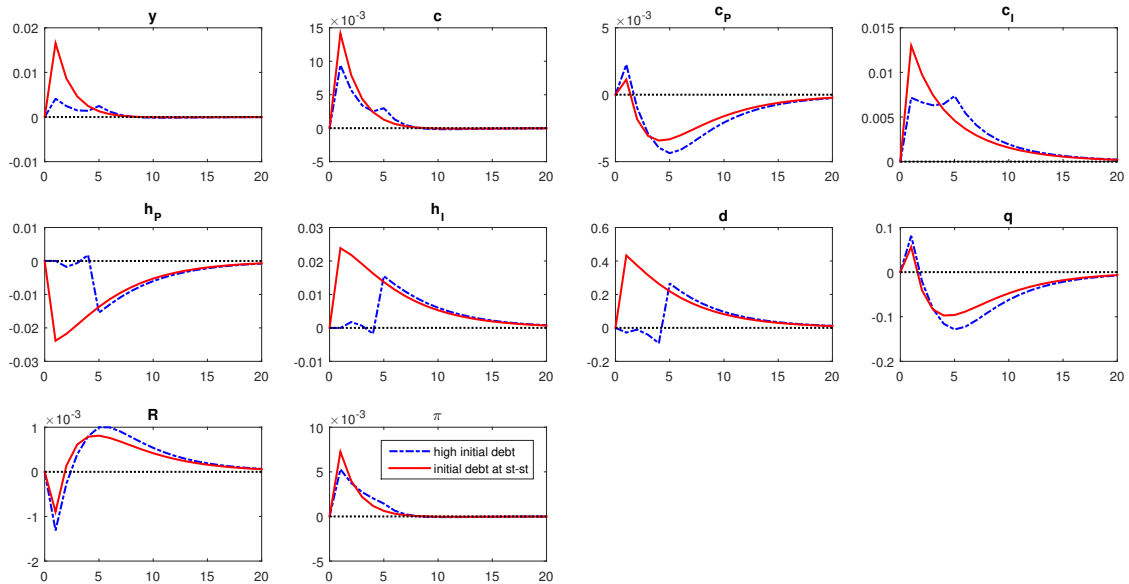


Figure 23: Impulse responses from our theoretical model of model variables to an annualized 100 bps monetary policy shock, in the general equilibrium model. The figure shows the response for the steady state debt level (red solid line) and the high debt state, where initial debt level is assumed to be 10% above the steady state (blue dashed line).

We compute impulse responses using the exact non-linear version of the model and a perfect foresight solution following an unexpected monetary policy shock.⁴⁰ In the high-debt case, we start the model at the steady state for all variables, except for the initial debt level which is assumed to be 10% above the steady state. As can be observed from Figure 23, in this case, the impact of the monetary policy shock is muted for impatient household’s real debt stock, d , in the initial periods following the shock due to the debt overhang effect. Note that inflation increases less in the high debt case, but this effect is not strong enough to reverse the impact of the monetary shock on the real debt profile of borrowers. The smaller increase in borrowing weakens the stimulatory impact of the monetary shock on overall consumption and output. Thus, the results in the general equilibrium model regarding the efficacy of a monetary shock under high debt are by and large similar to those we obtained in the partial equilibrium model.

⁴⁰To compute the transition path from the initial to the terminal steady state, we use the Matlab routines available in *Dynare*. The model converges to the terminal steady state after 100 periods, corresponding to 25 years. The transition path is computed by imposing the initial and terminal values and simultaneously solving a system of nonlinear equations that characterize equilibrium in all periods using a Newton method.