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Fear the Machine?

by

Michael D. Bradley
The George Washington University

and

Dennis W. Jansen
Texas A&M University

Abstract: We examine the impact of technological change in a world with dual production technologies of differing capital intensities and growing productivity of the more capital intensive technology. The model is one of overlapping generations with savings of the young providing productive capital in the following period. As society moves toward adoption of the newer more capital intensive technology, the equilibrium impact can result in lower wages of the young, lower savings, and hence a lower per-capita capital stock and a lower level of utility. These effects persist until the less capital-intensive technology falls into complete disuse. After that happens, the increase in productivity of the more capital-intensive technology generates the expected effects, raising wages, capital, and utility. During the transition a policy of subsidizing labor so as to maintain the income of the young can mitigate the decline in the capital stock and the resultant impact on the economy.

*Michael D. Bradley is Professor of Economics at The George Washington University (Washington, DC). Dennis W. Jansen is Professor of Economics and Director of the Private Enterprise Research Center at Texas A&M University (College Station, TX).

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1. Introduction.

The increased use of advanced automation and artificial intelligence has led to concerns about the impact of such technological advances on workers' wages and jobs. This conceptual concern has been heightened by recent labor market movements such as a decreasing labor share of GDP and the stagnation of median wages over time. Real median income in the U.S. economy grew fairly rapidly from 1953 to about 1973, then more slowly in the 1980s and 1990s. Real median family income reached a relative peak of $70,732 in 2000. Real median family income then stagnated, not exceeding the previous peak until 2016, when it achieved a level of $72,707. It is fair to say that median family income has been stagnate since the turn of the century, achieving a compound growth rate barely above zero, 0.18%. Given the predominance of labor income in median family income, this is a reflection of declining relative payments to labor. In addition, there has been a decrease in the labor force participation rate, even among those of prime working age, although the cause of this is highly debated and no doubt partially due to demographics and to the response of the work force to the Great Recession. Nevertheless, the traditionally sanguine view of the impact of technology on wages and employment is starting to be questioned.

Concerns about advanced automation and artificial intelligence on wages and employment are often simplified to concerns about "robots" and the replacement of jobs with robot "labor." Moreover concerns about robot technology has moved from popular press to academic literature as these issues are starting to be rigorously investigated in that literature. For example, Sachs, Benzell and LaGarda (2015) investigate these issues by defining "robot" production as a technology that requires zero labor. They embody this technology in an overlapping generations model with two technologies, one that uses both capital and labor and one (the robot sector) that uses just capital. They find that increases in productivity in the robot sector can lead to declines in output, wages, and employment across the economy. Acemoglu and Restrepo (2016) employ a representative agent model with CES production technology in which output is determined by the sum of individual tasks. Some tasks can be done either by capital or by labor, but other tasks can only be completed by labor. Because firms can substitute capital for labor, the impact of automation on employment and wages depends upon whether labor has a comparative advantage in the new tasks created by technological change. If it does, wages and employment can increase, if it does not, they can fall.

More recently, Abeliansky and Prettner (2017) and Prettner (2017) employ a discrete time Solow model with Cobb-Douglas technology in which automation capital is a perfect substitute for labor. In this model, as automation capital increases, wages fall as robots substitute for labor, leading to lower labor income. However, the resulting lower marginal product of labor does not cause output per capita (as opposed to per worker) to fall.

Given that this literature is in its infancy, it is not surprising that there is little commonality in terms of the framework chosen to analyze the impact of robots. In fact, there is no agreement on how robot technology should be embodied in a macroeconomic model. In some instances there is a single technology in which automation or robots are captured either by as a separate input or by technology that requires little or no labor. In other instances, automation or robots are captured by two competing technologies, one that embodies a typical role for capital and labor and one (the robot technology) relying solely, or almost solely, on capital.

In this paper we begin the process of bridging that gap by examining the potential impact of automation or robots both ways. Within the same model, we first investigate automation in a single production technology in which the share of output shifts from favoring labor to favoring capital. This approach is consistent with the observed declines in labor's share of output. We then allow for two different technologies, one traditional and one robotic, but allow enough generality so that the robotic sector can employ a very small amount of labor.[1]

We incorporate these two approaches to robots in a straightforward model that allows for elements that feature in the debate over robots and their impact on workers. The model is an overlapping generations model with cohorts living two periods. This model allows for differential impacts of robots within a cohort (differential impacts on young and old of a given cohort) and across cohorts (differential impacts on young of the current cohort and old of the previous cohort).

This overlapping generations model also captures the general equilibrium feature that the labor income of the young is what is available to fund both consumption and saving by the young, and saving by the young funds the capital that is available for production and hence consumption by the old next period. A reduction in earnings of the young can translate not just into lower consumption but also into lower capital in future periods. Finally, we use the model to investigate the degree that tax policies can be used to offset any negative impacts of increased use of robotics.

2. The Model

Our model has overlapping generations of two-period-lived cohorts. Each generation consumes while young and old, and gets utility from consumption in each period. In each cohort, the young generation is born without an endowment of capital, and supplies labor inelastically. Labor earnings of the young generation fund consumption and savings. Savings

[1] In our view it is unlikely that in any foreseeable future there will a productive technology that does not require any human oversight, monitoring, or coordination. In any event, a pure robotic technology is just a special case of our model.

take the form of capital (traditional capital and/or robots depending upon the version of the model being analyzed) that generates income to fund consumption when old. Each generation has perfect foresight. Shocks are of the so-called "MIT shock" variety.[2]

We look at two versions of the model, each featuring a different way of analyzing an increasing use of "robots" or automation. The first version of the model includes only a traditional technology for production, which uses capital and labor to generate output, with a Cobb Douglas form and a relatively high labor share of output (for example, 0.7). As production in this economy becomes more automated, the labor share of output declines. In the second version of the model, there is a second technology available for production that uses robots and labor to generate output. It is a robot technology in the sense that production is much less labor intensive.[3] This technology also has a Cobb Douglas form but it has a relatively low labor share of output (for example, 0.1 or 0.3). In this version of the model, increased use of robots is captured by a shift in production from the traditional technology to the robot technology Labor and capital are allocated across these two alternative production technologies to generate output. Labor is homogeneous in use across the two technologies, so the marginal product of labor is equalized across the two technologies. Similarly, investment in capital is fungible and can be allocated to either the traditional technology or the robot technology. The marginal product of capital is also equalized across the two technologies.

Capital, both traditional and robot, depreciates completely in use. Thus, capital purchased at time t is used for production at time t+1, after which it is completely depreciated.

[2] That is, we solve the model as if there will be no shocks, and then a shock occurs to that steady state, and then we solve for the new steady state as if there will be no further shocks.

[3] In the extreme, a robot technology could conceivably use no labor in the production of output. To date, however, robot technologies have required at least a small amount of labor for monitoring and maintenance, if nothing else.

The household side of the model is the same in both versions. Households (individuals) get utility according to the utility function:

(1) $$U_t = \ln(c_t^y) + \beta \ln(c_{t+1}^o)$$

Here c_t^y is consumption of cohort t when young, c_{t+1}^o is consumption of cohort t when old, and β is the subjective rate of discount.

The budget constraint of the young and old cohorts are:

(2) $$c_t^y + s_t^y = w_t l_t (1-\tau_L)$$

(3) $$c_{t+1}^o = s_t^y [1 + rr_{t+1}(1-\tau_{KR})]$$

Here s_t^y is saving of cohort t (when young), w_t is the real wage, l_t is household labor supply, τ_L is the tax on wage income, $(1+rr_{t+1})$ is the return in *t+1* of saving at time *t* and τ_{KR} is the tax on capital income. Note that the tax rates can be positive or negative, with a negative tax rate indicating a subsidy. Taxes are solely used to redistribute income across generations. A positive labor tax rate and a negative capital tax rate (subsidy) is a transfer from the young generation to the old generation, like Social Security. This is a one-way transfer because only the old generation receives capital income and those earnings constrain the old generation's consumption. In contrast, if $\tau_L < 0$ and $\tau_{KR} > 0$, then a subsidy flows from the old generation to the young generation as the old generation does not supply labor.

Constraints (2) and (3) imply an intertemporal budget constraint for cohort t:

(4) $$c_{t+1}^o = [w_t l_t (1-\tau_L) - c_t^y][1 + rr_{t+1}(1-\tau_{KR})]$$

Household optimization implies the following intertemporal relationship:

(5) $$c_{t+1}^{o} = \beta c_t^{y}(1 + rr_{t+1}(1 - \tau_{KR}))$$

Equations (4) and (5) and lead to the following conditions:

(6) $$\begin{aligned} c_t^{y} &= \left(\frac{1}{1+\beta}\right) w_t l_t (1-\tau_L) \\ s_t^{y} &= \left(\frac{\beta}{1+\beta}\right) w_t l_t (1-\tau_L) \\ c_{t+1}^{o} &= \left(\frac{\beta}{1+\beta}\right) w_t l_t (1-\tau_L)[1 + rr_{t+1}(1-\tau_{KR})] \end{aligned}$$

The resource constraint for this economy indicates that output is split among the government good, consumption and saving (i.e. investment) of the current young, and consumption of the current old. There is no government net borrowing or lending in the model, and for simplicity there is no government spending, so the sum of labor and capital taxes are zero.

(7a) $$\begin{aligned} Y_t &= N_t(c_t^{y} + s_t^{y}) + N_{t-1} c_t^{o} \\ &= N_t w_t l_t (1-\tau_L) + N_{t-1}\left(\frac{\beta}{(1+\beta)}\right) w_{t-1} l_{t-1} (1-\tau_L)[1 + rr_t(1-\tau_{KR})] \end{aligned}$$

(7b) $$0 = N_t w_t l_t \tau_L + N_{t-1} s_{t-1}^{y} rr_t \tau_{KR}$$

Here Y_t is aggregate output and N_s is the generation young at time s. For convenience, the total labor supply per household, l_t, will be set to unity.

For future reference it is useful to note that, using (7b), (7a) can be rewritten as:

$$(8)\quad \begin{aligned} Y_t &= N_t w_t l_t (1-\tau_L) + N_{t-1}\left(\frac{\beta}{1+\beta}\right) w_{t-1} l_{t-1}(1-\tau_L)[1+rr_t(1-\tau_{KR})] \\ &= N_t w_t l_t + N_{t-1}\left(\frac{\beta}{1+\beta}\right) w_{t-1} l_{t-1}(1-\tau_L)(1+rr_t) \end{aligned}$$

Equation (8) makes clear that, ceteris paribus, an increase in taxes on labor will lower saving and hence lower the future capital stock. A subsidy to labor will do the opposite.

Production occurs using one or both of the following production functions:

$$(9)\quad Y_t^K = A_t (K_{t-1})^{\alpha_K} (L_t^K)^{1-\alpha_K};\ L_t^K = N_t l_t^K$$

$$(10)\quad Y_t^R = \theta_t (R_{t-1})^{\alpha_R} (L_t^R)^{1-\alpha_R};\ L_t^R = N_t l_t^R$$

Equation (9) provides the traditional production function using traditional capital and labor to produce output, and equation (10) provides the production function using robot capital and labor to produce output. The output from either production process is identical. Firms employ a capital stock (traditional or robot) determined in the previous period, and labor from the current young. The terms A_t and θ_t index productivity of the two production technologies. We will consider a situation in which there is growing productivity in the robot sector and we will assume that labor intensity of the robot technology is lower than the labor intensity of the traditional technology, or that $\alpha^R > \alpha^K$. Population grows at a constant rate, n, so $N_t = N_{t-1}(1+n)$.

At time t, the young generation provides labor to firms in exchange for the economy-wide market wage per unit of effective labor, which firms take as given. Firms have a capital stock, determined by previous savings, which they also take as given. The predetermined capital stock is owned by the old generation, but firms decide how to divide this capital stock between traditional capital and robot capital. Capital earns its economy-side marginal product, which funds consumption by the old generation. Finally, capital depreciates fully in the time span of the old generation.

Profit maximization ensures that wages equal to the marginal product of labor and are equalized over the two alternative production technologies.[4] The predetermined capital stock, the level of total labor supply, and the levels of productivity in each of the production technologies then determine the economy-wide wage.

3. A Single Production Technology with Changing Capital Share

We first illustrate the impact of technological change in a model where there is only one technology available to firms. This allows us to look at the impact on the economy of a change in the share of output going to capital. One way to view adoption of "robot" technology in an actual economy is that it reduces labor's share of output.[5] Less labor and more capital is needed to product output using the robot technology, so in an economy characterized by increasing use of robot technology, labor's share of output will fall and capital's share will rise. Recent evidence suggests that this may be happening for the U.S. economy. For example as Figure 1 shows, the Bureau of Labor Statistics has measured a persistent downward movement in labor's share of output.

The model with a single production technology provides an excellent framework for examining the impact of a change in the capital intensity of production, and can provide insight into the potential effects of widespread robot use. That is, the model can be used to compare two

[4] Profit maximization by the firms leads to equality of both marginal products across the two technologies Firms allocate effective labor and capital between the traditional and robot sectors accordingly, as they face a common wage and common cost of capital across the two technologies.

[5] See, for example, Autor, Dorn, Katz, Patterson, and Reenen (2017) and Giandrea and Sprague (2017).

states of the world, one with a relatively low capital share and one with a relatively high capital share.

With only one production function, equation (9), the overlapping generations equilibrium is characterized by that production function, the household optimization conditions, equation (6), particularly $s_t^y = \left(\frac{\beta}{1+\beta}\right) w_t l_t (1-\tau_L)$, the firm's optimization conditions:

(11)
$$\begin{aligned} MPL_t^K &\equiv (1-\alpha_K) A_t \left(\frac{K_{t-1}}{L_t^K}\right)^{\alpha_K} = w_t \\ MPK_{t-1} &\equiv \alpha_K A_t \left(\frac{K_{t-1}}{L_t^K}\right)^{\alpha_K - 1} = 1 + rr_t \end{aligned}.$$

and the market clearing conditions for capital and labor:

(12)
$$\begin{aligned} K_{t-1} &= s_{t-1}^y N_{t-1} \\ L_t^K &= N_t. \end{aligned}$$

Combining these conditions yields the associated steady state conditions (dropping the time subscript):

(13) $$\frac{K}{L^K} = \frac{1}{1+n} \frac{\beta}{1+\beta} w(1-\tau_L)$$

Equations (11), (12) and (13) imply steady-state equilibrium values for the wage, capital return, the capital-labor ratio, and output per worker:

(14a) $$w = \left[A(1-\alpha_K)\right]^{\frac{1}{1-\alpha_K}} \left[\frac{\beta(1-\tau_L)}{(1+n)(1+\beta)}\right]^{\frac{\alpha_K}{1-\alpha_K}}.$$

(14b) $$1 + rr = \frac{\alpha_K (1+n)(1+\beta)}{\beta(1-\alpha_K)(1-\tau_L)}.$$

(14c) $$\frac{K}{L^K} = \left[\frac{(1-\alpha_K)\beta A(1-\tau_L)}{(1+n)(1+\beta)}\right]^{\frac{1}{1-\alpha_K}}$$

(14d) $$\frac{Y}{L^K} = \left[\frac{(1-\alpha_K)\beta(1-\tau_L)}{(1+n)(1+\beta)} \right]^{\frac{\alpha_K}{1-\alpha_K}} A^{\frac{1}{1-\alpha_K}}$$

As expected, an increase in productivity raises the steady state wage both through increasing productivity and augmenting the capital-labor ratio. The tax on labor has the opposite effect, lowering the capital-labor ratio, output per worker, and the wage. It leads to lower saving by the young generation, so the labor tax reduces the capital stock and thus increases the return to capital. Because it would lead to higher savings and a larger capital stock, a labor subsidy would work in the other direction, raising the capital-labor ratio, output per worker and the wage. Higher population (labor force) growth lowers the capital-labor ratio, the wage and output per worker. Note too, that the growth of productivity results in an increase in the capital-labor ratio that reduces the marginal product of capital by exactly as much as the increase in productivity is increasing the marginal product of capital, so that in steady state equilibrium the return to capital is not impacted by increases in total factor productivity. Finally, a tax on capital does not affect the steady state values of these variables, because saving does not depend upon the return to capital with a log utility function.

While the steady state value for the marginal product of capital in equation (11) is not impacted by changes in the productivity term, A, this is not so for the marginal product of labor – an increase in A will increase the marginal product of labor and the wage rate.

The overlapping generations structure means that the current young generation, born with no endowment, must fund both consumption and saving from their labor earnings. Thus saving, and hence the capital stock in the next period, are tied to labor earnings. Changes in the economy that alter labor earnings of the young will also alter the capital stock in the future period, when the current young are then old. In any given period t, the capital stock is predetermined from the past saving decisions of the current old. The capital-labor ratio and output per unit of labor, depend directly on the productivity term, and are influenced by both the capital share and the subjective rate of time preference. One way that an increased use of automation or robots would manifest itself in the economy would be through increases in the share of output going to capital. In the extreme, in a purely robotic industry, capital's share would be one-hundred percent and labor's share would be zero. Even in a more realistic case, labor's share would be well below current levels. As the economy employs more automation, more sectors will experience rising capital shares and the overall all economy-wide capital share will rise.

We will thus explore the impact of changes in the capital share on output and capital in this version of model, as the structure of this model leads to interesting general equilibrium impacts of changes in the capital share, even in the case of a single technology. In other words, we capture the impact of an increased use of robots by simulating the overlapping generations model for different values of the capital share. We assume standard values for the rate of population growth (initially we set n=0), the discount rate (we set our per-generation discount rate equal to β^{30}, where the annual discount rate is 0.96), and we normalize the size of the total factor productivity variable (setting A=10).

Suppose that increased use of robot technology causes an economy to experience an exogenous increase in the capital share, or, equivalently, a decrease in the labor share. This change leads to lower labor income and hence a lower capital stock and the effects can be large, to put it mildly. As shown in Table 1, if the capital share changes from 0.3, a value in the range of those typically assigned in calibration exercises, to a value of 0.7, the general equilibrium impact is substantial. With no other changes in the model, the steady state capital stock falls to

14.4 percent of its previous steady state level, and steady state output declines to 33.5 percent of its previous steady state level. Steady state consumption of the young and the old also decline, to 14.4 percent and 78.1 percent of their previous values. Finally, steady state utility falls to 11.2 percent of its previous value. These are just simulation values and their quantitative interpretation must be made with caution. On one hand, they could suggest that the posed change in capital share is so large that it could only happen over a long period of time. On the other hand, the large impacts could suggest the potential importance of such changes in the production technology for wages, output, consumption, and welfare.

Table 1 also presents results for a more modest increase in the capital share, to 0.5. This increase causes a fall in the steady state capital stock to 66.5 percent of its initial value, and a fall in output to 93.1 percent of its initial level. Utility declines to 87.7 percent of its prior level. These results are for a constant level of productivity, $A=10$.

Changes in productivity can mitigate some of the effects of a rising capital share, and we also calculate steady state values when the productivity term rises simultaneously with the increase in the capital share. The increase in the productivity term was chosen to generate the same level of steady state output as in the baseline economy. Nevertheless, the model produces a decline in the capital stock, in consumption of the young, and in utility, but the impact is somewhat muted.

A similar exercise is illustrated for an increase in alpha to 0.7, and again to 0.9. These large increases in the capital share generate increasingly large changes in steady state capital, output, and utility. The increase in alpha to 0.7 results in output that is only 33.5 percent of the previous steady state level.

In Table 2 we illustrate the adjustment process from one steady state to another. In this case, we show the movement from the baseline steady state (A, α_K) = (10, 0.3), to the steady state with (A, α_K) = (10, 0.7). The full adjustment process takes quite a few generations, although most of the adjustment occurs in the first few periods. The adjustment results in lower welfare for all generations, except that the old generation in the period of the shock experiences an unexpected increase in their wellbeing. That is because at the time of the shock the old generation has locked in the size of the capital stock. The shock raises the marginal product of capital thereby giving the old generation an unexpected increase in their capital income. Meanwhile the young generation earns a lower wage and, because they save less, generates a lower capital stock – but a higher return on capital – for when they are old. Here the net impact is to reduce utility for those young at the time of the shock, and for all future generations as well. This is because in all subsequent generations the lower labor income results in lower saving and lower capital, and the increase in the marginal product of capital is not sufficient to compensate for the decline in labor income and saving. Over time the economy converges to the new steady state with much lower capital, lower output, consumption, and much lower welfare.

The adjustment in the levels of output, capital, and the wage rate as the economy adjusts from one steady state to another are shown graphically in Figure 2. The reduction in all three of these variables occurs in a few periods, so after five or six periods, all three variables are near their eventual steady state levels, even though the full adjustments continues for many more periods. The precipitous drop in output and wages is apparent in the graph.

We can also use this version of the model to investigate the ability of a tax and subsidy policy to mitigate the negative effects on wages and output caused by the decline in labor's share. We impose a tax on capital income (τ_{KR} > 0) and specify that the proceeds are used to provide a subsidy for labor income (τ_L < 0). We require the tax rate to satisfy two constraints. First, the proceeds of the tax on capital return must just equal the amount of subsidy to labor income, so the government's budget remains in balance. Second, τ_{KR} must be set so that the subsidy to labor income must equal the decline in the wage caused by the increase in capital's share of output. The first constraint is given by equation 7b above and the second constraint is given by $\tau_{KR} = \frac{\Delta w}{s^y rr}$.

Table 3 presents the results of imposing the capital income tax and labor income subsidy. The first result of note is that, unlike the previous analysis without the tax and subsidy, output rises. The labor income subsidy allows the young households to continue their saving, so the capital stock, and thus the capital-labor ratio does not fall. Consequently the productivity increase associated with a higher capital share translates into an output increase. In other words, with the subsidy, there is no capital stock decline to offset the increased productivity. In addition, with the tax and subsidy in place, utility unambiguously increases with an increase in the capital share. The subsidy keeps the young generation's income and consumption at the pre-shift level and the higher level of output increases income and consumption for the old generation.[6]

The above exercises are meant to show the steady state impact of a change in the capital share. In the next section we turn to our main interest, the evolution of this economy when it faces the choice between a traditional production process with a 'traditional' capital share of 0.3,

[6] Gasteiger and Prettner (2017) also examine a "robot tax" in an overlapping generations model\, but their model features a very different production function in which robots and labor are perfect substitutes. While they find that a tax and transfer scheme mitigates some of the negative impacts increasing use of robots, they find that it is insufficient to sustain the pre-robot growth rate.

and a ‘robot’ production process with an assumed capital share of 0.7. For equal values of the productivity term, we find that the economy will only use the traditional process, but as productivity in the robot sector increases there are increasing resources devoted to the robot production process, and the economy moves toward production that has a lower labor share and exhibits some of the features we saw in this section.

Finally, for comparison with a more traditional technological change, Table 4 illustrates the adjustment process as the economy transits from one steady state to another after an increase in capital productivity or total factor productivity. In this case we show the movement from the baseline steady state = (10, 0.3), to the steady state with = (20, 0.3). This results in the expected increase in consumption of both young and old, and in utility.

4. Multiple Production Technologies

In this section, we look at the version of the model in which there are two production technologies available to firms, one with a relatively low capital share (the traditional technology) and one with a relatively high capital share (the robot technology). These production functions are given by equations (9) and (10), respectively. Multiple production technologies can coexist as long as, ceteris paribus, the productivity ratio θ / A is not too small or too large, where the meaning of too small or too large depends on the values of α_K and α_R. When the ratio gets too large or small, then only one of the two technologies will be used.

In the multiple technologies version of the model, the equilibrium conditions include the household saving equation (equation 6) and the firm’s optimization conditions with respect to the traditional technology, equation (11), as in the single production technology case. However, when there are multiple production technologies, profit maximization drives firms to hire capital and labor so as to equalize the marginal products of each factor across sectors. This leads to a second set of firm optimization conditions for the robot technology.

(15)
$$MPR_{t-1} \equiv \alpha_R \theta_t \left(\frac{R_{t-1}}{L_t^R} \right)^{\alpha_R - 1} = 1 + rr_t$$
$$MPL_t^R \equiv (1 - \alpha_R) \theta_t \left(\frac{R_{t-1}}{L_t^R} \right)^{\alpha_R} = w_t$$

The model is closed with the multiple technology versions of the market clearing conditions for capital and labor:

(16)
$$K_{t-1} + R_{t-1} = N_{t-1} s_{t-1}^y$$
$$L_t^K + L_t^R = N_t \quad .$$

With two technologies available to produce the same product, we can have both interior solutions where both technologies are used simultaneously, while satisfying equations (11) and (15), and corner solutions where only one of the two technologies will be adopted. We first examine the nature of interior solutions and then describe conditions under which corner solutions arise. When both technologies are employed simultaneously, the following conditions govern the steady state capital – labor ratios in the traditional sector and the robot sector, respectively:

(17)
$$k^* \equiv \left(\frac{K}{L^K} \right)^* = \left(\frac{\alpha_K}{\alpha_R} \right)^{\frac{\alpha_R}{\alpha_R - \alpha_K}} \left(\frac{1 - \alpha_K}{1 - \alpha_R} \right)^{\frac{1 - \alpha_R}{\alpha_R - \alpha_K}} \left(\frac{A}{\theta} \right)^{\frac{1}{\alpha_R - \alpha_K}}$$

(18)
$$r^* \equiv \left(\frac{R}{L^R} \right)^* = \left(\frac{\alpha_K}{\alpha_R} \right)^{\frac{\alpha_K}{\alpha_R - \alpha_K}} \left(\frac{1 - \alpha_K}{1 - \alpha_R} \right)^{\frac{1 - \alpha_K}{\alpha_R - \alpha_K}} \left(\frac{A}{\theta} \right)^{\frac{1}{\alpha_R - \alpha_K}}$$

Equations (17) and (18), along with (16), can be used to develop the relationship between steady state total saving per capita and the labor allocation to traditional capital and robot capital:

(19) $$S_{t-1} / N_t = [k^*(L_t^K / N_t) + r^*(1 - L_t^K / N_t)]$$

We also have the following equation for steady state saving, reflecting the fact that when we have an interior solution both production processes are utilized:

(20)
$$\begin{aligned} S_{t-1} / N_t &= \frac{\beta}{1+\beta}\Big((w_{t-1}^K L_{t-1}^K + w_{t-1}^R (N_{t-1} - L_{t-1}^K))\Big) / N_t)(1-\tau_L) \\ &= \frac{\beta}{1+\beta}\Big((1-\alpha_K)A(k^*)^{\alpha_K}(L_{t-1}^K / N_{t-1})^* + (1-\alpha_R)\theta(r^*)^{\alpha_R}(1-(L_{t-1}^K / N_{t-1})^*)\Big)\left(\frac{1}{(1+n)}\right)(1-\tau_L) \\ &= \frac{\beta}{1+\beta}\left(\frac{1}{(1+n)}\right)(1-\tau_L)w^* \end{aligned}$$

Equations (19) and (20) give us the following solutions for the portion of the labor force in the traditional sector:

(21)
$$\begin{aligned} \left(\frac{L^K}{N}\right)^* &= \left(\frac{\frac{\beta}{(1+\beta)}(1-\alpha_R)\theta(r^*)^{\alpha_R}(1-\tau_L) - r^*}{\frac{\beta}{(1+\beta)}(1-\alpha_R)\theta(r^*)^{\alpha_R}(1-\tau_L) - r^* + k^* - \frac{\beta}{(1+\beta)}(1-\alpha_K)A(k^*)^{\alpha_K}(1-\tau_L)}\right) \\ &= \frac{r^* - \frac{\beta}{(1+\beta)}(1-\alpha_K)A(k^*)^{\alpha_K}(1-\tau_L)}{r^* - k^*} = \frac{r^* - \frac{\beta}{(1+\beta)}w^*(1-\tau_L)}{r^* - k^*} \end{aligned}$$

This equation together with (19) can determine steady state savings and hence the steady state aggregate capital stock, traditional capital plus robots. The overall capital-labor ratio for the economy when both production processes are utilized is given by:

(22) $$\frac{K+R}{L^K + L^R} = \frac{1}{1+n}\frac{\beta}{1+\beta}(1-\alpha_K)A(k^*)^{\alpha_K}(1-\tau_L) = \frac{1}{1+n}\frac{\beta}{1+\beta}w(1-\tau_L)$$

Note that in equation (22), the tax rate on labor matters for the aggregate capital-labor ratio. This occurs even though the ratio ($K/L^{K)}$ and ($R/L^{R),}$ given in equations (17) and (18), do not depend on the tax rate. The aggregate capital-labor ratio depends on the share of labor allocated to the two sectors, and this allocation of labor depends on the labor tax rate. Although changes in the labor tax rate will reallocate labor across the two sectors, it will not change the optimal capital-labor ratio within the sectors. Moreover, a comparison of the aggregate capital-labor ratio in equation (22) and the capital-labor ratio in the case of a single production technology in equation (13) shows that the formulas written in terms of the wage rate are identical. In both cases the economy's capital-labor ratio is a simple linear function of the after-tax wage.

Equation (21) holds for the interior solution, in which both technologies coexist, and this happens only for a certain range of parameter values, particularly productivity terms A and θ. If the right hand side of equation (21) is greater than unity, then the economy is at a corner solution, and only the traditional technology is used. No resources would be allocated to the robot technology. If the right hand side of equation (21) is less than or equal to zero, then again the economy is at a corner solution, one where only the robot technology is used. Behavior in the corner solutions is described by the results presented above for the single production technology.

We now consider changes in the productivity of the robot technology and how it leads to reallocation of resources to the traditional and robot technologies. We proceed in two steps. First we look at the economy with a given stock of capital, to focus on the implications of improvement of increases in the productivity of the robot as the productivity of the traditional technology is held constant. We then relax this assumption and allow the capital stock to respond to the technological change. We find that as the robot technology becomes more productive, both capital and labor shift away from the traditional technology and toward the robot technology. An economy initially using the traditional technology will, as robot productivity grows, eventually shift to only using the robot technology.

5. How the Steady State Changes as Robot Productivity Increases: Results for a Constant Aggregate Capital Stock

Figures 3 through 5 reflect a set of calibrations for the parameters that facilitate illustration of the effects of an increase in robot productivity. To achieve that goal, the aggregate capital stock, the sum of the traditional capital stock K and the robot capital stock R, are all set to unity. We hold the capital stock constant in this section to show the direct effects of the increase in robot productivity. Later we will look at the indirect impact of this change on the size of the capital stock itself.

Here the capital share in the traditional technology is set to 0.3, and the capital share in the robot technology is set to 0.9. To highlight the changes in robot productivity, level of productivity in the traditional sector, A, is constant and equal to 1, while the level of productivity in the robot sector, θ varies.

Figure 3 graphs the allocation of labor as θ increases. As θ rises sufficiently, (above 0.48 in our simulation), labor is allocated the traditional sector to robot sector, going from being primarily in the traditional sector to being split between the two sectors. When robot productivity rises enough (2.8 in our simulation), almost all labor is employed in the robot sector. The economy has switched to the robot production function.

Although both labor and capital switch between the traditional sector and the robot sector, the rate at which they switch is not the same. The robot sector starts out with nearly no labor and almost no capital, but the ratio of robot capital to labor is high. As both labor and capital move into the robot sector, the robot capital to labor ratio falls. Moreover, both capital and labor are moving from a sector in which the marginal product of labor is relatively high and the marginal product of capital is relatively low. Thus, the movement in resources causes the economy's marginal product of labor to fall and the economy-wide marginal product of capital to rise. Figure 4 illustrates the impact on the marginal product of labor.

With the shift in resources from the traditional sector to the robot sector, the output of traditional sector falls while the output of the robot sector rises. But because the robot sector becomes more productive as θ rises, total output rises. Figure 5 shows the change in output in the traditional sector and the robot sector. Traditional output sector declines, but output in the robot sector rises to more than make up for the decline in the traditional sector, so economy-wide output increases.

Finally, the marginal product of labor and capital are the payments to the two factors. Because they are both the same across sectors and because total labor and total capital both equal one, the marginal products also represent the total earnings for the two factors. Figure 6 graphs the response of the economy's labor share and capital share as θ increases. When robot productivity reaches the point labor starts switching (at 0.48 in our simulation), most output is from the traditional sector, and, in the steady state, the capital share must is near 0.3 and the labor share consequently is near 0.7. As θ increases the shift in production to the robot sector causes the capital share to increase. Eventually, as θ gets high enough, almost all production is in the robot sector, and consequently the capital share nears 0.9.

6. How the Steady State Changes with Robot Productivity Increases: General Equilibrium with Dual Production Processes

In the previous section, we investigated the impact of a change in robot productivity in an environment in which the capital stock did not change. We now extend that analysis to allow capital changes to take place. In addition to the above effects, the overlapping generations structure of our model ties capital investment to labor earnings of the young. As robot technology productivity increases and resources shift to that technology, the labor share declines, and both saving and the capital stock decline. This effect is large and means that there is a substantial period where the growing productivity in the robot sector does not yield proportionate increases in output or consumption, and where welfare may decline. Eventually if the robot technology productivity increases enough, the traditional technology will be abandoned and at that point increases in productivity of the robot technology have the expected impact on output, consumption, and utility.

The model is recalibrated to accommodate capital growth and the conditions from the previous section are relaxed. As robot productivity improves, so robot productivity approaches the traditional sector productivity, labor starts shift into the robot technology, although the movement is gradual. In fact, half of labor is still used in the traditional sector when robot productivity is well above traditional sector productivity. Figure 7 graphs the labor allocation in the traditional sector and the robot sector as robot productivity increases. Part of what happens here compared to our previous results (Figure 3) is that the capital stock is declining with the fall in wages, so there is a pressure increasing the marginal product of capital in both production technologies.

The movements in the capital stock are shown in Figure 8. This figure graphs capital allocated to the robot technology and to the traditional technology, but also graphs the aggregate capital stock, as this is generally declining due to the effect of the declining marginal product of labor on savings of the young. This decline in savings and decline in the overall capital stock results in the more gradual movement of resources to the robot technology. In fact, capital continues to decline in the aggregate until the traditional technology is abandoned. At that point further increase in robot productivity lead to increases in the wage rate, increased saving, and increased capital. That is also the point at which further increases in robot productivity no longer change the marginal product of capital.

The movements in labor and capital have differential effects on output in the individual sectors, as well as on total economy-wide output. This is shown in Figure 9. The net effect on output is rather small for a large range of values of θ. In fact, under a different parameterization, it is possible that output could fall, as the increased productivity of robots and increased resource allocation to the robot sector can be insufficient to make up for the decline in aggregate capital. Eventually as the productivity of the robot technology increases, and the increased resource use in the robot production function, leads to growth in output. However, the large increases in output must wait until the traditional technology is abandoned.

To further understand what is happening in this model, Figure 10 shows wage changes and marginal product of capital changes as a function of robot productivity. Once θ is sufficiently large (above 2 in our simulation), the robot technology starts to be used in production, which leads to a decline in the marginal product of labor and the decline in the aggregate capital stock as shown in Figure 8. This generally causes the effects shown above, with a lower capital stock leading to a long period of near-constant output with lower capital, lower earnings, and lower utility. Indeed, wages decline monotonically with θ up to the point where the traditional technology is abandoned, at which point increases in robot productivity lead to strong increases in the marginal product of labor and hence in the wage.

Finally, Figure 10 also graphs the marginal product of capital, which rises once firms have an opportunity to employ robots in production, and continues to rise throughout the range of θ graphed here, until the traditional technology is abandoned, at which point the marginal product of capital levels out at a high level.

7. The Path to New Steady States when Robot Productivity Increases: Dual Production Processes

Our final analysis investigates the movement between steady states when the economy is employing both technologies. We have already shown the adjustment process between steady states when technology parameters change and the economy is using a single production function. We have also shown the changes in steady states when the productivity of the robot production technology increases relative to the traditional production technology. Now we look at the adjustment process between steady states when technology changes, and both technologies are being used.

Once an economy is in the parameter space where it is using both technologies, the adjustment process between steady states is through an adjustment in the fraction of workers devoted to each production process, and this takes place with no change in the capital to labor ratios in either the traditional or the robot production processes. Consequently, there is no change in the wage rate after the initial period. After an increase in the productivity of the robot technology, there is a period in which the inherited capital stock is higher than the steady state due to the higher wage in the period before the technological change. This higher wage means there is higher savings and thus a larger capital stock and means that the share of labor devoted to each technology is different than the new steady state values. But in the adjustment period the wage falls to the new steady state level, bequeathing the next generation with the new, lower steady state level of capital, and this leads the economy, in the second period after the change, to be in the new steady state.

In the period in which the robot technology become more productive, the equation calculating the share of labor dedicated to the traditional technology, equation (21), does not hold. Equation (21) calculates the steady state labor share. Instead, the transitional share of labor is calculated from equation (19) as:

$$(23) \qquad \frac{L_t^K}{N_t} = \left(\frac{S_{t-1}}{N_t} - r^* \right) / \left(k^* - r^* \right)$$

where S_{t-1} is the capital stock held by the current old in period t. This will be the equilibrium capital stock from the (old) steady state that held prior to the change in robot productivity. In this case, the fraction of labor devoted to the old technology is unchanged in period t, despite the change in technology. Then in the second period after the change in robot productivity the fraction of labor devoted to the old technology will change according to equation (21).

Table 5 illustrates the change from the initial steady state to a new steady state when robot productivity increases sharply and both technologies are used. In the first period with the new higher robot productivity, the aggregate per capita capital stock does not change but the wage rate declines and the marginal product of capital increases. This also leads to a large reduction in the fraction of labor devoted to the traditional technology, a reduction that overshoots the new steady state. There is a consequent fall in consumption by the young, an increase in consumption by the old. The old benefit from a windfall. The utility of the period-1 young is lower, and is at the new steady state level. In the second period, the decline in the wage rate leads to a lower level of saving and a lower capital stock. These will now be at the new steady state levels. The fraction of the labor force devoted to the traditional technology rises compared to the previous period, but is still below the value at the old steady state. Consumption by the young is unchanged, while consumption of the old declines. These are now the new steady state levels. The new steady state has lower consumption by the young, higher consumption by the old, and lower utility.

Lastly, Table 6 presents the results of introducing a labor tax, and the tax rate is set to be a subsidy to labor that exactly offsets the impact of the decline in the wage rate. The tax rate thus keeps the after-tax wage constant. The labor subsidy is paid for with a tax on capital, equally applied to both forms of capital, robots and traditional capital. The subsidy keeps saving by the young constant, and maintains the value of the capital stock at the original steady state level. There is no change in the after-tax wage, no change in consumption by the young, and an increase in consumption by the old. Utility increases. The steady state is reached immediately in the period of the change in robot productivity.

8. Conclusion

The advent of sophisticated, artificial-intelligence drive production technology has caused economists to begin contemplating the implications of technological change in which labor is a little used factor. Rigorous analysis of this issue has just begun and, as one would expect, there are a diversity of approaches to analyzing it. For example, there are differences in how sophisticated automation (or robot technology) should be incorporated into a macro model. In this paper we embody the two main approaches into different versions of the same overlapping generations model. In the first version, the effects robot technology are captured by analyzing a single production technology in which the share of capital is rising substantially, reflecting the reduced need for labor in roboticized industries. In the second version, we have two separate technologies and allow the robotic sector to compete for capital and labor with the traditional technology sector.

Our results in both versions raise concerns about the possible implications of automated production technologies for both employment and labor income. A shift to technologies with relatively small labor shares holds the potential to, at least for a period of time, lower wages and labor income. This is true even in a relatively modest definition of robots -- a rising capital share in a traditional technology -- as well as a stronger version in the form of a competing technology that requires little or no labor. It is also true even if the shift to the robotic sector is caused by an ongoing increase in robot productivity. However, in the competing technologies version of the model, once an economy shifts entirely to a robotic technology, then further increases in robot productivity lead to higher wages and output. We find this result, moreover, whether or not the capital stock is allowed to vary, but the impact is stronger when it does. Finally, because the negative impact of automation technology affects wages, not capital returns, and because only the young generation supplies labor, we find that a tax and subsidy scheme holds the potential for mitigating the negative impacts.

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Table 1. Steady State Results: Increase in Capital Share (alpha) with and without Compensating Change in Total Factor Productivity (A)

A	alpha	K	L	Y	MPL	MPK	C(y)	S(y)	C(o)	U
10	0.3	1.939	1	12.198	8.54	1.89	6.60	1.94	3.66	2.268
10	0.5	1.290	1	11.356	5.68	4.40	4.39	1.29	5.68	1.989
10.3642	0.5	1.385	1	12.198	6.10	4.40	4.71	1.39	6.10	2.082
10	0.7	.278	1	4.085	1.23	10.27	0.95	0.28	2.86	0.254
13.8844	0.7	.831	1	12.198	3.66	10.27	2.83	0.83	8.84	1.670
10	0.9	3.65E-07	1	1.61E-05	1.61E-06	39.63	1.24E-06	3.65E-07	1.45E-05	-16.873
38.7261	0.9	0.277	1	12.1981	1.22	39.63	0.94	0.28	10.98	0.645

Table 2. Adjustment to New Steady State After a Change in the Capital Share

alpha	Time	K(t-1)	Y(t)	w(t)	MPK(t)	$C_Y(t)$	$S_Y(t)$ =K(t+1)	$C_O(t)$	$U_Y(t)$
0.3	ss	1.94	12.20	8.54	1.89	6.60	1.94	3.66	2.27
0.7	1	1.94	15.90	4.77	5.74	3.69	1.08	11.13	1.89
0.7	2	1.08	10.58	3.17	6.83	2.45	0.72	7.40	1.40
0.7	3	0.72	7.95	2.39	7.72	1.84	0.54	5.57	1.06
0.7	4	0.54	6.51	1.95	8.41	1.51	0.44	4.56	0.82
0.7	5	0.44	5.66	1.70	8.93	1.31	0.39	3.96	0.65
0.7	6	0.39	5.13	1.54	9.32	1.19	0.35	3.59	0.53
0.7	7	0.35	4.79	1.44	9.59	1.11	0.33	3.36	0.45
0.7	8	0.33	4.57	1.37	9.79	1.06	0.31	3.20	0.39
0.7	9	0.31	4.42	1.33	9.93	1.02	0.30	3.09	0.35
0.7	10	0.30	4.32	1.29	10.04	1.00	0.29	3.02	0.32
0.7	ss	0.28	4.09	1.23	10.27	0.95	0.28	2.86	0.25

Table 3. Adjustment to New Steady State After a Change in the Capital Share, with Changes in Subsidy on Labor and Tax on Capital											
Alpha	Time	K(t-1)	Y(t)	w(t)	MPK(t)	C_Y(t)	S_Y(t) =K(t+1)	C_O(t)	U(t)	τ(L)	τ(K)
0.3	Ss	1.94	12.20	8.54	1.89	6.60	1.94	3.66	2.27	0	0
0.7	1	1.94	15.90	4.77	5.74	6.60	1.94	7.36	2.47	-.79	.41
0.7	2	1.94	15.90	4.77	5.74	6.60	1.94	7.36	2.47	-.79	.41
0.7	ss	1.94	15.90	4.77	5.74	6.60	1.94	7.36	2.47	-.79	.41

Note: the tax/subsidy rates are set so that the subsidy on labor is paid by the tax on capital, leaving budget balance, and the subsidy on labor is chosen to keep after-subsidy labor income constant after the increase in the capital share.

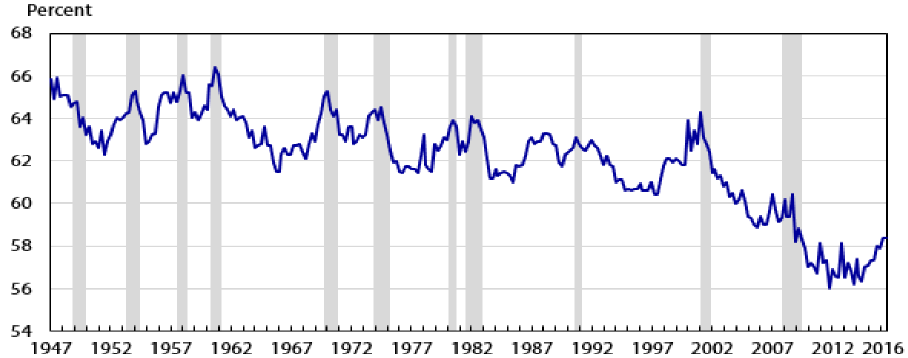
Table 4. Adjustment to New Steady State After a Change in TFP									
A	Time	K(t-1)	Y(t)	w(t)	MPK(t)	C_Y(t)	S_Y(t) =K(t+1)	C_O(t)	U_Y(t)
10	Ss	1.94	12.20	8.54	1.89	6.60	1.94	3.66	2.2682
20	1	1.94	24.40	17.08	3.77	13.20	3.88	7.32	3.2261
20	2	3.88	30.04	21.02	2.32	16.25	4.78	9.01	3.4524
20	3	4.78	31.97	22.38	2.01	17.30	5.08	9.59	3.5203
20	4	5.08	32.57	22.80	1.92	17.62	5.18	9.77	3.5407
20	5	5.18	32.76	22.93	1.90	17.72	5.21	9.83	3.5468
20	6	5.21	32.81	22.97	1.89	17.75	5.22	9.84	3.5486
20	7	5.22	32.83	22.98	1.89	17.76	5.22	9.85	3.5492
20	8	5.22	32.83	22.98	1.89	17.76	5.22	9.85	3.5493
20	ss	5.22	32.83	22.98	1.89	17.76	5.22	9.85	3.5494

Table 5. Adjustment to New Steady State in the Dual Technology Case, After a Change in the Productivity of Robot Capital, θ. (Traditional Capital Productivity A = 2)												
θ	t	$\frac{K_{t-1}}{L_t^K}$	$\frac{R_{t-1}}{L_t^R}$	$\frac{L_t^K}{N_t}$	$\frac{L_t^R}{N_t}$	$\frac{Y_t}{N_t}$	w_t	MPK_t	c_t^y	$\frac{S_{t-1}}{N_t}$	c_t^o	U_t^y
2	ss	0.266	5.590	0.9999	0.0001	1.35	0.941	1.515	0.672	0.267	0.404	0.834
6	1	0.043	0.896	0.7373	0.2627	2.00	0.543	5.460	0.388	0.267	1.456	0.725
6	2	0.043	0.896	0.8695	0.1305	1.38	0.543	5.460	0.388	0.154	0.841	0.725
6	ss	0.043	0.896	0.8695	0.1305	1.38	0.543	5.460	0.388	0.154	0.841	0.725

Table 6. Adjustment to New Steady State in the Dual Technology Case, After a Change in the Productivity of Robot Capital, θ. (Traditional Capital Productivity A = 2) with Changes in Subsidy on Labor and Tax on Capital												
θ	t	$\frac{K_{t-1}}{L_t^K}$	$\frac{R_{t-1}}{L_t^R}$	$\frac{L_t^K}{N_t}$	$\frac{L_t^R}{N_t}$	$\frac{Y_t}{N_t}$	w_t	MPK_t	c_t^y	$\frac{S_{t-1}}{N_t}$	c_t^o	U_t^y
2	ss	0.266	5.590	0.9999	0.0001	1.35	0.941	1.515	0.672	0.267	0.404	0.834
6	1	0.043	0.896	0.7373	0.2627	2.00	0.543	5.460	0.672	0.267	1.055	1.095
6	2	0.043	0.896	0.7373	0.2627	2.00	0.543	5.460	0.672	0.267	1.055	1.095
6	ss	0.043	0.896	0.7373	0.2627	2.00	0.543	5.460	0.672	0.267	1.055	1.095

Note: the tax/subsidy rates are set so that the subsidy on labor is paid by the tax on capital, leaving budget balance, and the subsidy on labor is sufficient to keep after-subsidy labor income constant after the increase in alpha, so that consumption and saving of the young are not impacted by the change in θ.

Figure 1. Labor's share of output in the nonfarm business sector, first quarter 1947 through third quarter 2016



Note: Shaded areas indicate recessions, as determined by the National Bureau of Economic Research.
Source: U.S. Bureau of Labor Statistics.

Figure 2. Dynamic Adjustment of Output, Capital and Wages to a Change in the Capital Share

(Change is from alpha = .3 to alpha = .7)

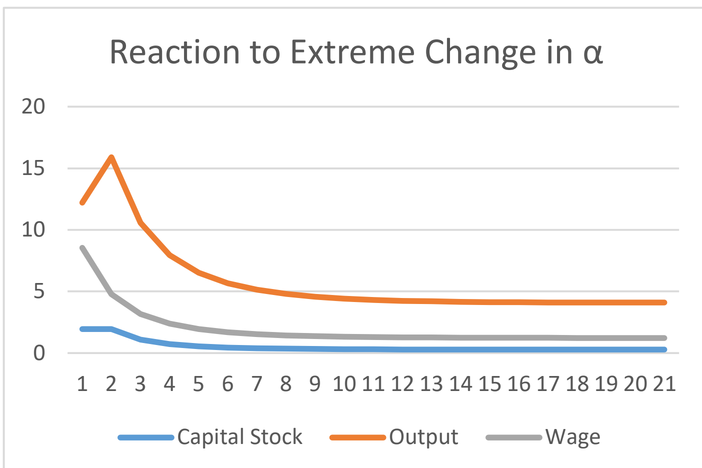


Figure 3. Change in Labor Allocation as θ Increases.

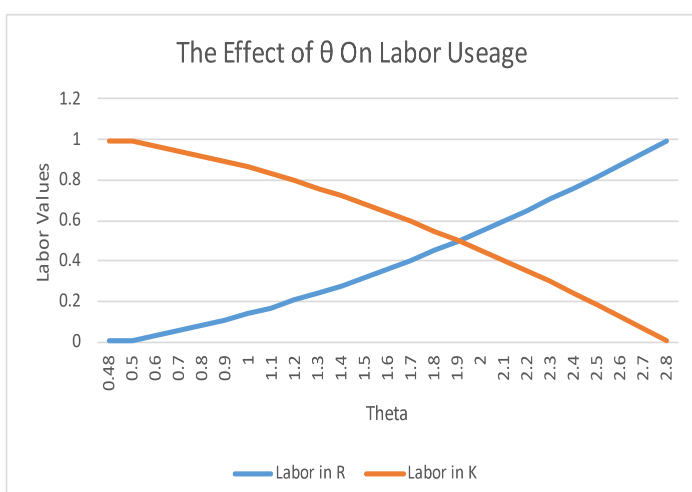


Figure 4. Change in MP of Labor as θ Increases.

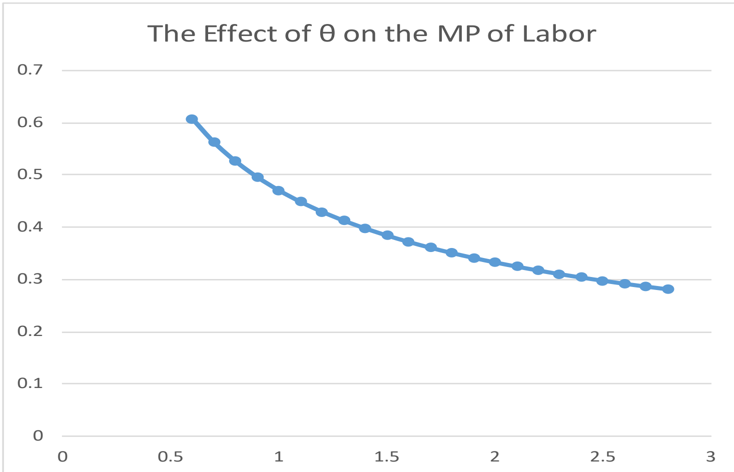


Figure 5. The Output Response to Increases in θ

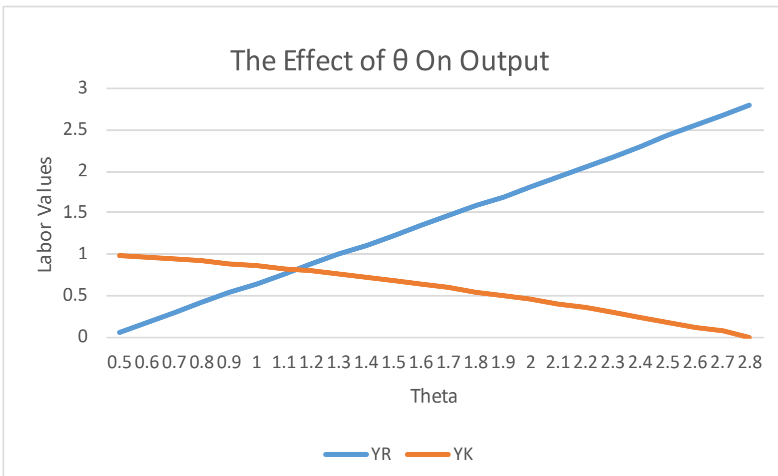


Figure 6. Labor and Capital Shares as θ Increases.

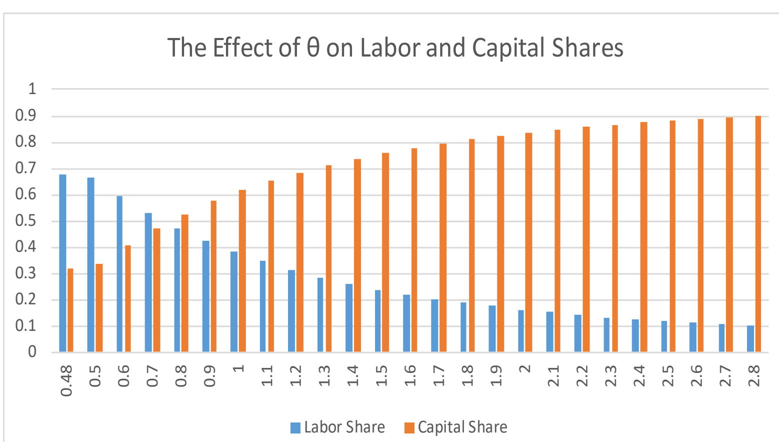


Figure 7. Steady State Labor Allocation as Robot Technology Productivity Increases

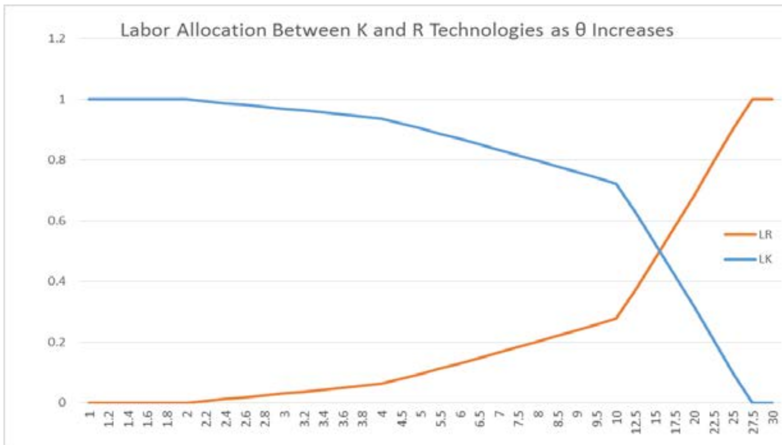


Figure 8. Steady State Capital Allocation as Robot Technology Productivity Increases

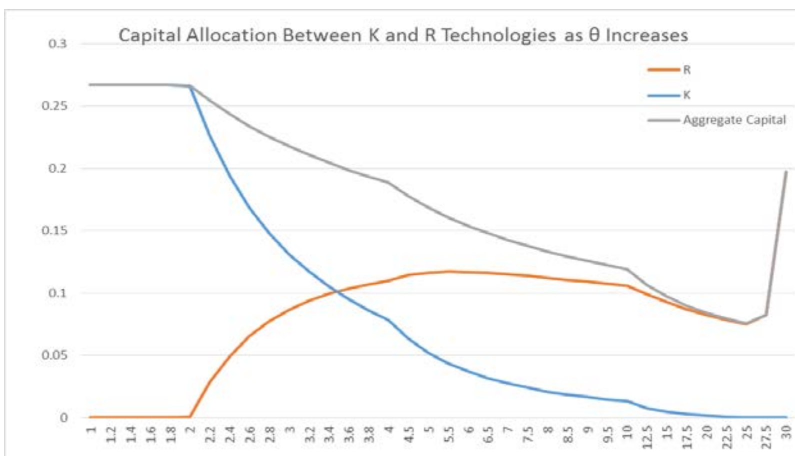


Figure 9. Steady State Output across Sectors as Robot Technology Productivity Increases

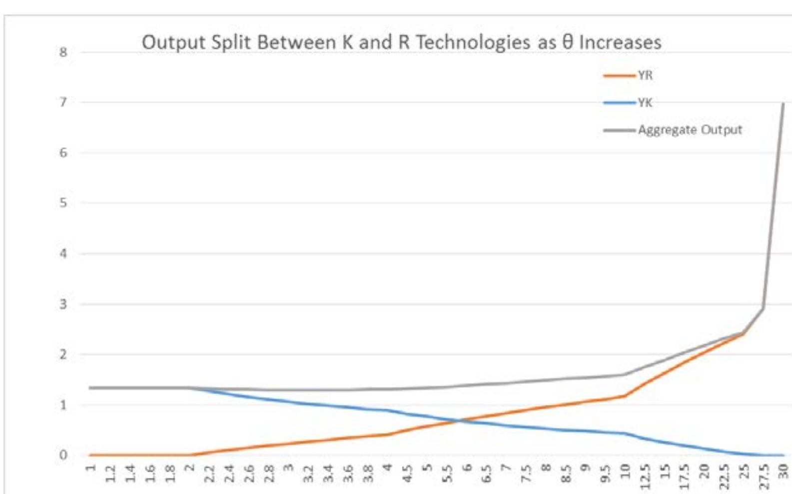


Figure 10. Steady State Wage and MPK Changes as Robot Technology Productivity Increases