The Problem of Abdul, the Newsboy

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1 Introduction

Consider the problem of a newsboy, Abdul[1]. Abdul needs to buy copies of tomorrow's newspaper from Houston to sell at his stand in College Station, and has to pay for it today. Abdul does not know how many people will come to his stand to buy newspapers tomorrow: some days more people come, other days only a few ! However, Abdul has been in the business long enough. So, while he does not know the exact number of people who will come to buy, he can have a distribution of demand. That is, he can say things like "Well, I think I can confidently say, that there is a 50% chance that there will be at least 15 people who will come to buy my newspaper tomorrow" and so on.

Each newspaper that Abdul buys, costs him \$4. He sells each of them for \$5 to his customers. Abdul can return the unsold papers back, but only at a price of \$0.20 - Late newspapers have very little value !

Abdul's problem is : How many newspapers should he buy?

Note that if Abdul buys too many newspapers, he will not be able to sell some and will have to return them, losing \$3.80 on every unsold paper. However, if he buys too few newspapers, he will have to turn some customers away. For every such unsatisfied customer, Abdul looses his profit (\$1).

Note the general nature of Abdul's sourcing/procurement problem. Abdul has to buy/source the product before knowing exact demand. This is precisely the problem of fashion industry. Firms like Zara and Benetton (we will read about these!) and many others decide on how much they should buy and produce, based on forecasts (probability distributions) of demand.

[1] This case is intended as a basis for class discussion and is prepared specifically for SCMT 335 class of Spring 2021 for the Undergrad participants at the Texas A&M University. email: anupam@tamu.edu.

2 Solving Abdul's problem

First, we need to know the distribution of the demand (how much he has been selling, on an average). One way to do this is to use Abdul's wisdom. Put alternatively, we can use historical data to construct an empirical distribution function. Other alternative ways are to poll experts to obtain a forecast (Wisdom of the Crowds), or to ask customers (maybe reward them to reveal their information), or even assume distribution, and then estimate parameters of that distribution.

Second, we also need to know the financial consequences of being wrong. If Abdul buys too little, how much does it hurt him? Typically this is the lost profit on a sale (Abdul loses \$1 on each lost sale).

Also, if Abdul buys too much, how much does it hurt him? Typically, this is the production cost, less any value that can be salvaged - in Abdul's case it is \$3.80.

2.1 Building Intuition: Abdul's Demand

When Abdul thinks about how much he wants to order for tomorrow, his main target is the actual sales that he can make. There may be known factors - Abdul might be selling more on weekends, or there may be unknown factors - some customers may be traveling on weekends and may not be there, or new customers might be vacationing in Abdul's city, or there maybe a big news item, which can spike the demand. But, Abdul can - via his experience and judgement - estimate his next day's demand. Say this is as below:

# of Newspapers demanded	10	11	12	13	14	15	16	17	18	19	20
Probability	.04	.06	.09	.11	.13	.14	.13	.10	.11	.05	.04

Given such a data, we know how to calculate the mean and the standard deviation of demand for newspapers. The mean, also called the expected demand, is usually denoted by the Greek letter μ or the mathematical symbol E[D], where E stands for expectation operator and D denotes the demand. The mean is calculated by multiplying the demand outcomes with their respective probabilities, as under

$$\begin{aligned}\mu &= E[D] = 10*0.04 + 11*0.06 + 12*0.09 + 13*0.11 + 14*0.13 + 15*.14 + \\ &\quad 16*0.13 + 17*0.10 + 18*0.11 + 19*0.05 + 20*0.04 \\ &= 15\end{aligned}$$

So, on an average, Abdul sells 15 newspapers.

It is very common to suggest that Abdul should buy 15 newspapers daily. As business scholars, we are interested in the deeper question - How many newspapers should Abdul buy, so that his expected profit is the largest? How can he make the most money? Let us investigate.

We will assume that Abdul is a risk neutral person - he neither wants to take a lot of risk (so he does not order 20 newspapers) nor does he want to play absolutely safe (so he does not order just 10). Rather, he would maximize his *average or expected profit.*

2.2 Building Intuition: Abdul's Profits

How do we calculate Abdul's expected profits? Let us take the case when Abdul orders 15 newspapers. He pays \$60 to buy these 15 newspapers. Note that the actual demand can be anything - from 10 to 20 newspapers. However, if the demand is more than or equal to 15, Abdul *can sell ONLY 15 newspapers*, and has nothing left over for remaining customers. If demand is less than 15, Abdul sells *equal to demand* - so if demand is 11, Abdul can sell ONLY 11, and has 4 newspapers left unsold.

We can show this case (Abdul buys 15 newpapers) as follows

Demand	10	11	12	13	14	15	16	17	18	19	20
Probability	.04	.06	.09	.11	.13	.14	.13	.10	.11	.05	.04
Abdul can sell	10	11	12	13	14	15	15	15	15	15	15

So how much money will Abdul make if he decides to buy 15 newspapers? We can calculate this by adding Abdul's profit for each demand scenario. So when demand turns out to be 10, Abdul makes \$50 from selling 10 newspapers and has to return the remaining 5 papers for getting \$1. In all, he makes \$51, however he has paid \$60 - so he incurs a loss. This scenario (of demand being 10) has a probability of 0.04. We add all these scenarios, noting that Abdul's cost will always be \$60, but his sales revenue will depend on the actual demand. Note that Abdul makes a profit of \$15 whenever demand is ≥ 15.

$$\begin{aligned} E[profit] &= E[(Sales + Salvage) - Cost] \\ &= 0.04 * \$(51-60) + 0.06 * \$(55.8-60) + 0.09 * \$(60.6-60) \\ &\quad +0.11 * \$(65.4-60) + 0.13 * \$(70.2-60) \\ &\quad +(0.14+0.13+0.10+0.11+0.05+0.04) * \$(75-60) \\ &= \$9.91 \end{aligned}$$

So we can make the statement: **Abdul can expect to make** \$9.91, ***on an average*, if he orders 15 newspapers**. What if he orders something different? We can calculate like the above. The resultant profits look like the table below (Perform calculations and satisfy yourself!)

Abdul orders	Abdul's Expected Profit
10	$10
11	$10.81
12	$11.33
13	$11.42
14	$10.98
15	$9.91
16	$8.18
17	$5.82
18	$2.98
19	-$0.39
20	-$4

Table 1: Abdul's expected profits for various levels of orders

Do you see something interesting? If you were a consultant advising Abdul, you would tell him "Dear Abdul, you must order only 13 newspapers every day. You would make, on an average, $11.42 daily by doing so".

The quantity of 13 newspapers is the optimal quantity - *Abdul cannot make more money, on an average, by ordering any other quantity – given his newspaper demand and his economics (buying and selling costs).*

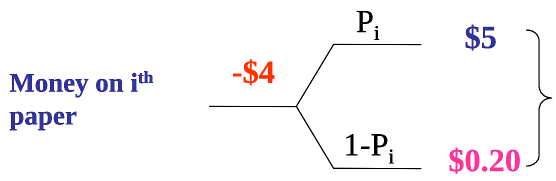
2.3 Building intuition further - Solving on the Margin

We can analyze Abdul's problem in a different and intuitive way. Let the optimal quantity that Abdul should order be i. What will happen if Abdul orders $i-1$ newspapers? He will make less money (on an average)! What if he orders $i+1$ newspapers? Again, he will make less money on an average. This observation is what we now use, by looking at Abdul's problem as that of ordering the i^{th} unit of newspaper. Should Abdul order the i^{th} newspaper (as in should he order the 12th newspaper? Should he order the 13th ? .... and so on)

Let us define the probability that demand will be equal to or higher than i newspapers as P_i. In simple (!) language of mathematical symbols, we can denote this statement as

$$P(D \geq i) = P_i$$

Now look at the following figure, which is a probability tree. It gives us Abdul's problem in a succinct way. The left hand branch tells us that Abdul spends $4 for every newspaper that he needs to buy. This money needs to be paid beforehand, is a sunk cost, and therefore we show it as negative. The upper right hand branch tell us how much money Abdul makes in case the demand actually occurs for the i^{th} paper. The lower branch tells us that in case demand is less than i, Abdul needs to return the i^{th} newspaper, and gets only 20 cents.



Abdul's problem is then simply put as

- Should I order the i^{th} paper?

or

- Should I NOT order the i^{th} paper?

It is easy to calculate Abdul's benefit from ordering the i^{th} newspaper.

$$Benefit = P_i * \$5 + (1 - P_i) * \$0.20$$

$$Abdul's\ Cost = \$4$$

Abdul has to judge if benefit $\geq$ cost by ordering. If so, then he must order the i^{th} unit. This judgement gives us the optimal quantity i, since at the optimal quantity, the benefit from an extra unit, is just equal to the cost that is incurred.

If we do equate the cost and benefit, we get

$$P_i * \$5 + (1 - P_i) * \$0.20 = \$4,$$

which gives us

$$P_i = 0.79$$

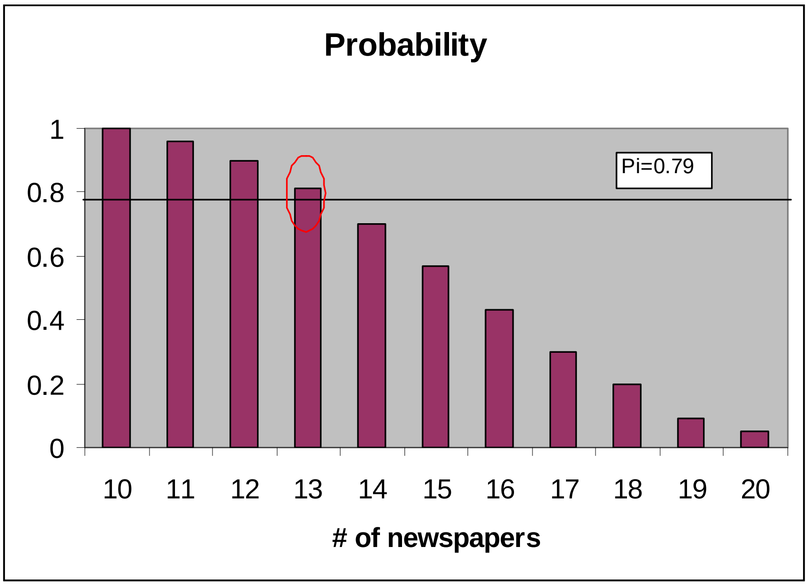
This calculation tells us that the optimal quantity i which Abdul should order must be such that it satisfies the relationship $P_i = 0.79$. In other words, we must order a quantity such that the probability of demand occurring equal to or greater than i equals 0.79.

To actually find the optimal quantity i, we look at Abdul's demand distribution.

# of Newspapers demanded (i)	10	11	12	13	14	15	16	17	18	19	20
Probability ($D = i$)	.04	.06	.09	.11	.13	.14	.13	.10	.11	.05	.04

Note that the first row gives us the various values that i can take, whereas the second row gives the probability of that exact demand occurring. We can now add one more row to this table, calculating the probability that demand will be equal to or more than i for every unit i.

# of Newspapers demanded, i	10	11	12	13	14	15	16	17	18	19	20
Probability ($D = i$)	.04	.06	.09	.11	.13	.14	.13	.10	.11	.05	.04
Probability ($D \geq i$) $= P_i$	1	0.96	0.9	0.81	0.70	0.57	0.43	.30	.2	.09	.04



Do you see the solution in the above table? If we had a continuous distribution, we could have chosen an exact solution, but here, we must choose the solution of 13 newspapers (since the optimal quantity is very close to our calculated solution). Abdul would be better off than ordering either 12 or 14 (and we can calculate how much better. How?)

Graphically, we can show the solution as follows

It is customary to denote the optimal quantity i with an asterisk - that is , we usually write $i^* = 13$.

2.4 Quantifying Abdul's Business

We have information now to calculate Abdul's quality of service - how often will he run out of newspapers, how much will he sell on an average, and so on.

2.4.1 Expected Lost Sales

Abdul will lose sales whenever his demand exceeds his inventory. We can calculate this as under, taking into account the cases whenever Abdul's demand will be greater than the optimal quantity $i^* (= 13)$. In other words, we are calculating the expected number of customers Abdul turns away, on an average - this will happen whenever Abdul encounters the 14th customer, the 15th customer,... and so on.

$$
\begin{aligned}
Expected\ Lost\ sales &= E[D - i^*], D > i \\
&= (14 - 13) * P(D = 14) + (15 - 13) * P(D = 15) + ... \\
&= 1 * 0.13 + 2 * 0.14 + 3 * 0.13 + 4 * 0.10 + 5 * 0.11 + 6 * 0.05 + 7 * 0.04 \\
&= 2.33
\end{aligned}
$$

So, on an average, Abdul will have 2.33 unsatisfied customers.

2.4.2 Expected Sales

Abdul will do best by ordering the optimal quantity $i^*(= 13)$. But what do we expect his average sales to look like? This figure can be got from deducting the expected lost sales figure above from the mean demand.

$$
\begin{aligned}
Expected\ Sales &= E[D] - (E[D - i^*], D > i) \\
&= \mu - (E[D - i^*], D > i) \\
&= 15 - 2.33 \\
&= 12.67
\end{aligned}
$$

Expected sales are less than expected demand (μ) because Abdul's inventory of 13 newspapers is insufficient to cover all possible demand levels. Thus, there will be days when demand exceeds 13 newspapers and Abdul will lose sales. Of course, higher inventory levels will allow higher expected sales, but such a decision would not be economically justified, given Abdul's economics.

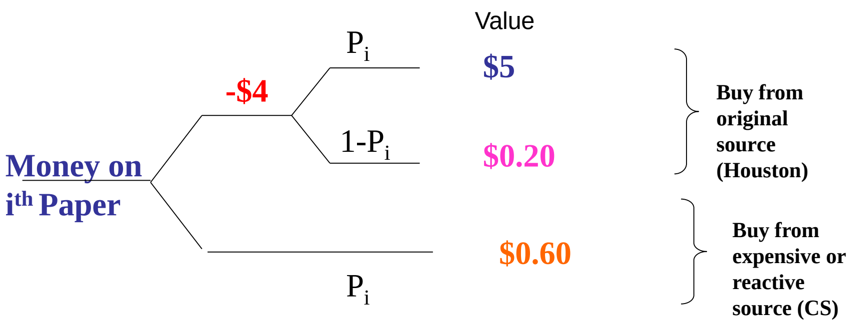
2.4.3 Expected Salvage (or Unsold Inventory)

Abdul will have unsold stock, which he will need to return whenever his demand is less than his inventory $i^*(= 13)$. We can calculate this by focusing on cases whenever Abdul's demand will be less than the optimal quantity.

$$
\begin{aligned}
Expected\ Salvage &= i^* - Expected\ Sales \\
&= 13 - 12.67 = 0.33
\end{aligned}
$$

2.4.4 Expected Profits

We can calculate the expected profits by using the numbers calculated above. This can be done by deducting the cost of ordering the i^* newspapers, from the revenue received from sales and salvage.



$$\begin{aligned} Expected\ profits &= \$5 * (Expected\ Sales) + \$0.20 * (Expected\ Salvage) - \$4 * i^* \\ &= \$11.42 \end{aligned}$$

Note that this figure matches with the expected profit figure calculated in table 1. At the order quantity i^* expected profit is optimized.

3 Seeking Abdul's options - Fast Response/ Reactive Capacity

Now let us consider an interesting option that Abdul might have. Suppose there is a local news-shop in College Station, which also sells newspapers. Instead of ordering from Houston, Abdul can, if he wishes, order newspapers from this local shop. The local shop does not require any advance money, but sells newspapers to Abdul at the rate of \$4.40. This option is costly, yet flexible – if Abdul runs out of newspapers, he can always go to the local shop and order some more, instantly, to service his demand.

How does this option (we can call it Abdul's reactive capacity) change Abdul's option tree? Let us investigate.

Abdul has an option of buying every unit of newspaper from either the Houston source or the College Station source. He makes $\$5 - \$4.40 = \$0.60$ from each newspaper bought locally. The Houston buying economics remains unchanged. Note that Abdul would satisfy his complete demand in this case - he does not need to let any customer go without a newspaper !

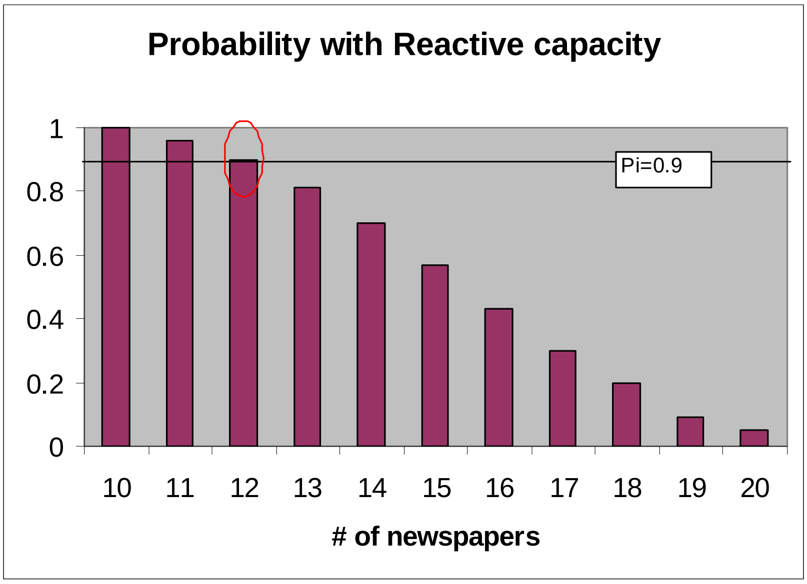
Abdul's problem with a reactive capacity is then simply put as

- Should I order the i^{th} paper from Houston?

or

- Should I order the i^{th} paper from College Station?

Abdul has to judge if benefit of buying from Houston $\geq$ benefit of buying from College Station. If so, then he must order the i^{th} unit from Houston. This judgement gives us the optimal quantity



i, since at the optimal quantity, the benefit from Houston is just equal to the benefit from College Station.

$$Houston\ Benefit = P_i * \$5 + (1 - P_i) * \$0.20 - \$4$$

$$College\_Station\ Benefit = P_i * \$0.60$$

equating, we get

$$P_i * \$5 + (1 - P_i) * \$0.20 - \$4 = P_i * \$0.60$$

which gives us

$$P_i = 0.9$$

This calculation tells us that the optimal quantity i which Abdul should order must be such that it satisfies the relationship $P_i = 0.9$ now. In other words, *Abdul would order less from Houston* - He will order only 12 newspapers !(which makes sense - Abdul now has a safety net)

The advantage of the College Station option is that it allows Abdul to react to demand levels in excess of his Houston order. Since College Station cost ($4.4) is less than Abdul's selling price ($5), sales will always be met. The reactive replenishment option leads Abdul to moderate his initial Houston order. Indeed, College Station does present an alternate supply source that can be profitably used if demand levels exceed inventory levels. Therefore, less can be ordered initially from Houston.

Note that although the introduction of the College Station option fundamentally changes the nature of the problem, it changes the marginal analysis of the Houston option only in one respect:

it eliminates for Abdul the risk of lost sales. There is still the potential of ordering too much from Houston and incurring salvage losses.

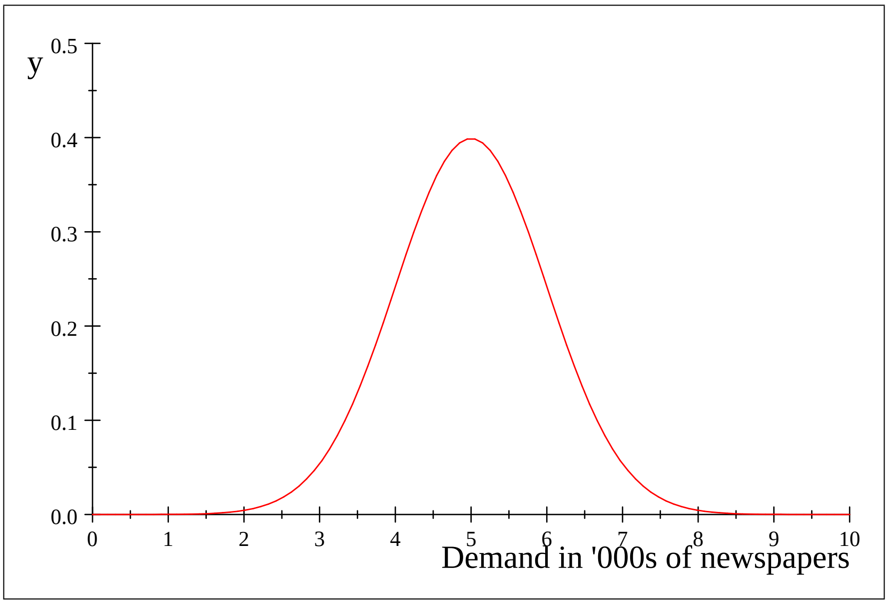
We can again calculate various parameters of Abdul's business, and this exercise is left for you!

4 Continuous Distribution case

Continuous distributions are often used for convenience, even if the actual distribution is discrete (like Abdul's newspaper demand). The plus point is that we can calculate the inventory levels precisely, and the optimal quantities can be determined exactly. Note that in case of a continuous distribution, we have

$$P(D \geq i) = P(D > i)$$

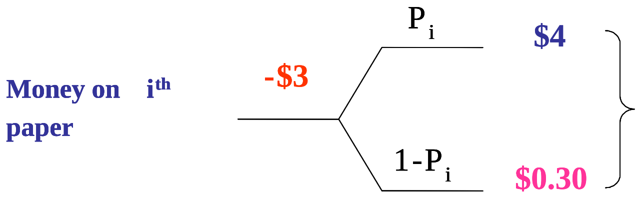
Let us now consider that Abdul, owing to his superb forecasting ability, has been hired by the Houston newsagent. The newsagent deals with the publisher directly, and Abdul can put in a single order daily for all the newspapers to be bought. Abdul analyzes the past demand and estimates the demand to be normally distributed with a mean of $\mu = 5000$ and a standard deviation of $\sigma = 1000$ newspapers.



The newsagent sells the newspapers to newsboys at \$4 per newspaper, and gets newspapers at \$3 from the publisher. Whatever does not get sold to the newsboys, and during the day, can be sold in the evening for \$0.30.

The option tree for the newsagent can now be drawn.

We can now calculate the probability for ordering the optimal quantity P_i just as in the discrete case. Evaluating, we get



$$P_i * \$4 + (1 - P_i) * \$0.30 = \$3,$$

giving us $P_i = 0.73$.

4.1 Finding optimal ordering quantity

We can find out the optimal inventory level i^* from the above information. We utilize the *standardized normal probability table* to determine our answers (Appendix A, Table A). This table gives us for each possible value z, the probability that a standardized Normal random variable (distributed with mean 0 and standard deviation 1) is greater than or equal to z. Alternatively, we can say that this table gives us the right tail of normal distribution - Recall that z is in terms of the number of standard deviations (σ) to the right of the mean (μ).

There are 3 steps in determining the optimal inventory level i^* from table A.

1. Find the closest probability value to $P_i = 0.73$ in Table A: This is 0.72907 (Column 2)

2. Determine the value of z associated with this probability. : This is -0.61

3. Translate z into the value of i^*. Since z is the number of standard deviations to the right of the mean, i^* is easily determined via

$$\begin{aligned} i^* &= \mu + z\sigma \\ &= 5000 - 0.61 * 1000 = 4390 \end{aligned}$$

4.2 Finding expected sales

Abdul knows from his College Station experience that expected sales equal expected demand minus expected lost sales. To calculate expected lost sales, we must use the standard normal loss table (Table B in appendix), which lists the expected sales losses for a standard normal demand when inventory is z standard deviations away from mean demand.

There are 3 steps in determining the expected sales losses from table B for inventory level i^*.

1. Find the value of z associated with inventory level i^*: We determined it above, it is -0.61

2. Find the loss value for $z = -0.61$ in the standard normal loss table (Table B)

$$L_n(-0.61) = 0.77594$$

3. Translate $L_n(-0.61)$ into expected loss sales. Because $L_n(-0.61)$ equals the lost sales for a standard normal distribution, the value of expected loss sales for Abdul is easily determined as under

$$\begin{aligned} Expected\ Lost\ sales &= E[D - i^*], D > i \\ &= L_n(z) * \sigma \\ &= L_n(-0.61) * \sigma \\ &= 0.77594 * 1000 \simeq 776 \text{ newspapers.} \end{aligned}$$

Now we can calculate Expected sales.

$$\begin{aligned} Expected\ Sales &= \mu - Expected\ Lost\ sales \\ &= 5000 - 776 \\ &= 4224 \text{ newspapers} \end{aligned}$$

Also, the expected number of newspapers sold at a discount in the evening (salvage) can be calculated as

$$\begin{aligned} Expected\ Salvage &= i^* - Expected\ Sales \\ &= 4390 - 4224 = 166 \text{ newspapers.} \end{aligned}$$

Therefore the expected profit of the newsagent is

$$\begin{aligned} Expected\ profit &= \$4 * Expected\ Sales + \$0.3 * Expected\ Salvage - \$3 * i^* \\ &= \$3775.8 \end{aligned}$$

5 Conclusion

Abdul's problem, as we noted earlier also, is quite generic. The solution tool, also called the newsboy model, applies where the product is perishable, or there is no inventory carried over to the next selling period (like newspapers, fashion garments, T-shirts saying "Aggies 2015"....).

The newsboy model is a technique that tells us what is the optimal quantity to produce when we don't know the exact demand, but only have a distribution of demand. The technique trades off the consequences of producing too much with the consequences of producing too little.

Note that even if we produce optimally, we may still have losses due to unsold product or lost sales; but the total financial consequences of demand-supply mismatches would have been minimized. The tool can be used both for discrete and continuous demand cases.