Higher Education Center at McAllen, Texas

MMET 275
MECHANICS FOR TECHNOLOGISTS

PRACTICE PROBLEMS WITH SOLUTIONS

By:

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PREFACE

This content is designed to help students and professionals alike to improve their problem-solving skills, in mechanics. Whether you are preparing for a test or simply want to sharpen your abilities, the practice problems included here, offer a diverse range of challenges that will help you develop a deeper understanding of key concepts of fundamentals on mechanics.

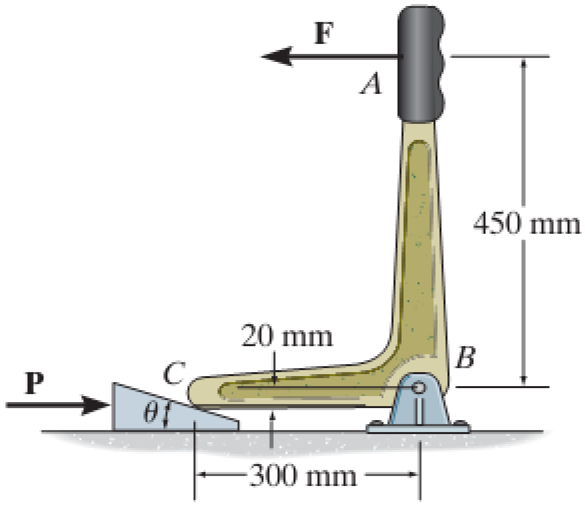
The problems are focused on the specific topics, such as free body diagram, equations of equilibrium, force vectors, resultant force, trusses, moment of inertia, friction, internal forces, center of gravity, and includes a collection of practice problems with step-by-step solutions. These problems are carefully crafted to illustrate important concepts and techniques and are designed to build on one another as you progress through the concepts.

In addition to the practice problems, several tips and tricks for solving problems efficiently and effectively, have been highlighted. Whether you are new to a subject or looking to deepen your understanding, these problems will provide you with the tools you need to succeed.

I hope that this problem set will serve as a valuable resource for students and professionals alike, and that the practice problems and solutions contained within will help you achieve your academic and professional goals.

PROBLEM 1

Determine the smallest horizontal force P required to move the wedge to the right. The coefficient of static friction at all contacting surfaces is $\mu_s = 0.3$. Set $\theta = 15^0$ and F = 400 N. Neglect the weight of the wedge.



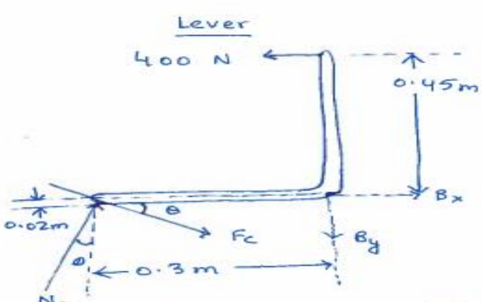
Procedure

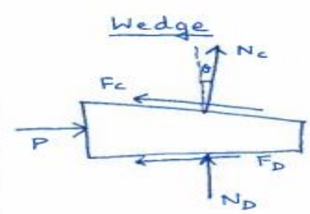
1: Draw FBD of wedge and lever

2 : Apply EoE to wedge and lever

3 : Write the friction equations

4 : Common - sense check.





Applying EoE to the above FBD of lever ; that is:

$\Sigma M_B = 0$

or, $400(0.45) + F_c \cos\theta\,(0.02) + F_c \sin\theta\,(0.3) - N_c \cos\theta\,(0.3) + N_c \sin\theta\,(0.02) = 0$

or, $400(0.45) + 0.3 N_c \cos 15^\circ (0.02) + 0.3 N_c \sin 15^\circ (0.3) - N_c \cos 15^\circ (0.3) + N_c \sin 15^\circ (0.02) = 0$ [← Substituting ① & ②]

$\Rightarrow N_c = 704.47$ N → ③

The wedge is on the verge of sliding to the right. Hence the frictional forces F_c and F_D acting to the left, can be written as:

$F_c = \mu_s N_c = 0.3 N_c$ → ①

$F_D = \mu_s N_D = 0.3 N_D$ → ②

Applying $\Sigma F_x = 0$ and $\Sigma F_y = 0$ to the FBD of wedge, we obtain the following:

$\Sigma F_x = 0$

or, $P - 0.3 N_c \cos 15^\circ - 0.3 N_D - N_c \sin 15^\circ = 0$

or, $P - 0.3(704.47)\cos 15^\circ - 0.3 N_D - (704.47)\sin 15^\circ = 0$ → (4)

$\Sigma F_y = 0$

or, $N_D + 0.3 N_c \sin 15^\circ - N_c \cos 15^\circ = 0$

or, $N_D + 0.3(704.47)\sin 15^\circ - (704.47)\cos 15^\circ = 0$

$\therefore \quad N_D = 625.76 \text{ N}$ → (5)

Substituting (5) in (4),

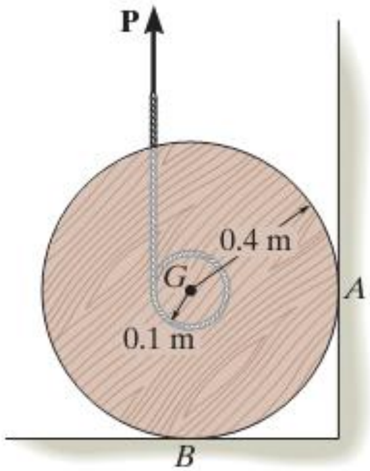
$0 = P - 0.3(704.47)\cos 15^\circ - 0.3(625.76) - (704.47)\sin 15^\circ$

$\Rightarrow \boxed{P = 574 \text{ N}}$

$\therefore$ Smallest horizontal force $P = 574$ N (Answer)

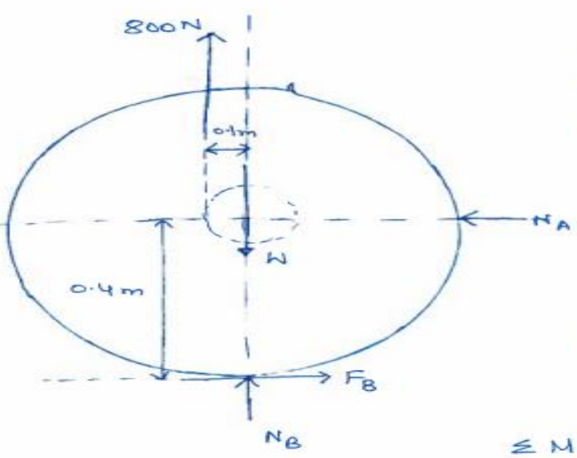
PROBLEM 2

The spool has a mass of 200 kg and rests against the wall and on the floor. If the coefficient of static friction at B is $(\mu_s)_B = 0.3$, the coefficient of kinetic friction is $(\mu_k)_B = 0.2$, , and the wall is smooth, determine the friction force developed at B when the vertical force applied to the cable is P = 800 N.



Procedure

1: Draw the FBD
2: Apply EoE
3: Write friction equation



No friction of Wall A, as it is mentioned as smooth.

Applying EoE,

$\Sigma F_x = 0$

or, $F_B - N_A = 0 \rightarrow (i)$

$\Sigma F_y = 0$

or, $N_B + 800 - W = 0$

or, $N_B + 800 - 200(9.81) =$

$\therefore N_B = 1162$ N

$\Sigma M_o = 0$

or, $-800(0.1) + F_B(0.4) = 0$

$\Rightarrow F_B = 200$ N

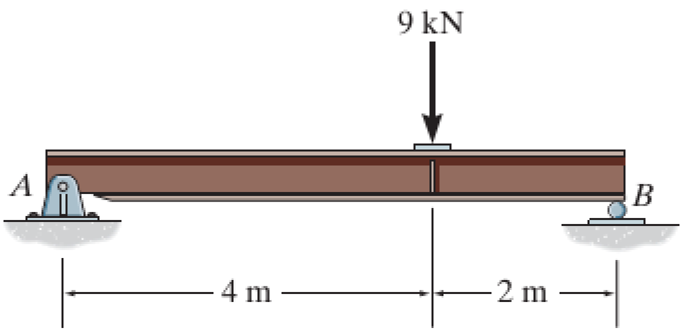
$(F_B)_{max} = \mu_s N_B = 0.3(1162)$

or, $(F_B)_{max} = 348.6$ N > 200 N

$\therefore \boxed{F_B = 200 \text{ N}}$ (Answer)

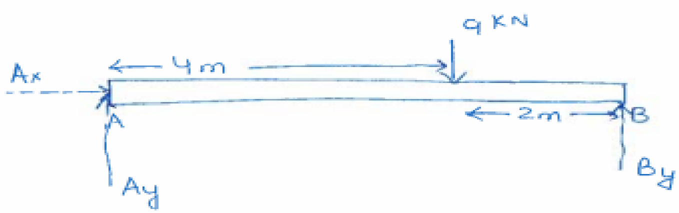
PROBLEM 3

Draw the shear and moment diagrams for the simply supported beam.



Procedure

1) Draw FBD of the beam
2) Find the support reactions
3) Define the sections as needed
4) Apply sum of forces / moments for V/M
5) Plot the SFD & BMD.



$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0 \Rightarrow A_y + B_y = 9$

$\Sigma M_A = 0 \Rightarrow 9(4) - B_y(6) = 0 \Rightarrow B_y = 6$ kN

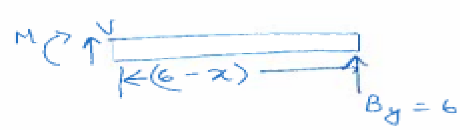
$\therefore A_y = 3$ kN & $B_y = 6$ kN

For $0 \leq x < 4$m

$V - 3$ kN $= 0 \Rightarrow V = 3 \rightarrow$ ①

$\Sigma M_A = 0 \Rightarrow M - 3x = 0$

or, $M = 3x \rightarrow$ ②



For, 4m $< x \leq 6$m

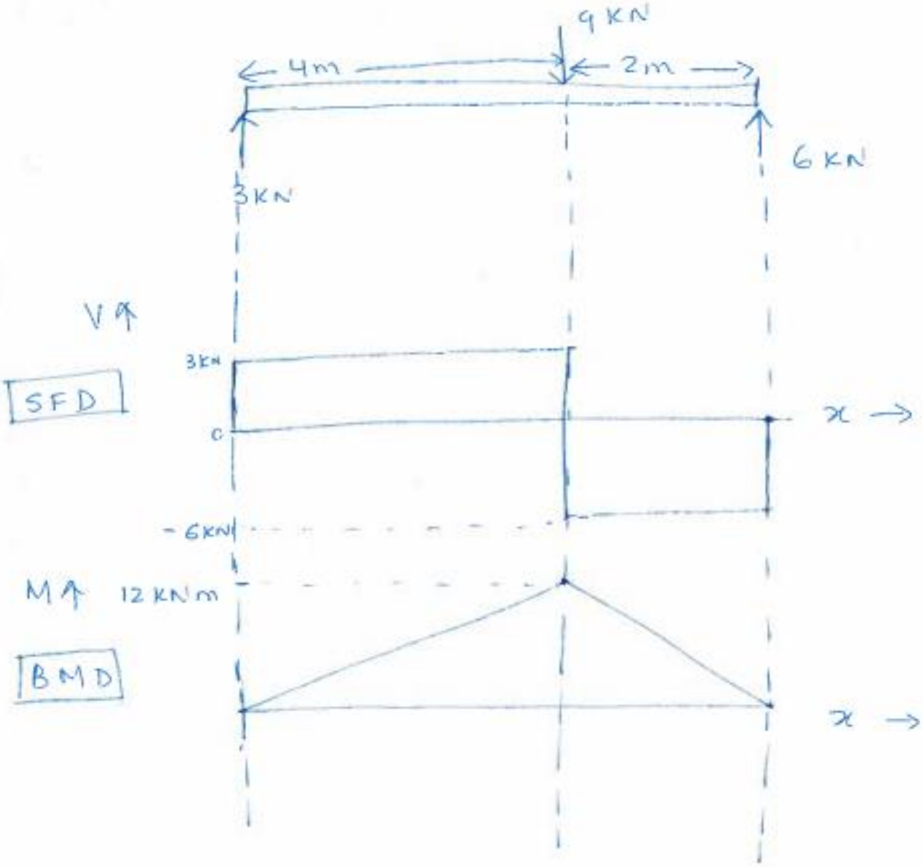
$V = -6 \rightarrow$ ③

$\Sigma M = 0$

$\Rightarrow 6(6-x) - M = 0$

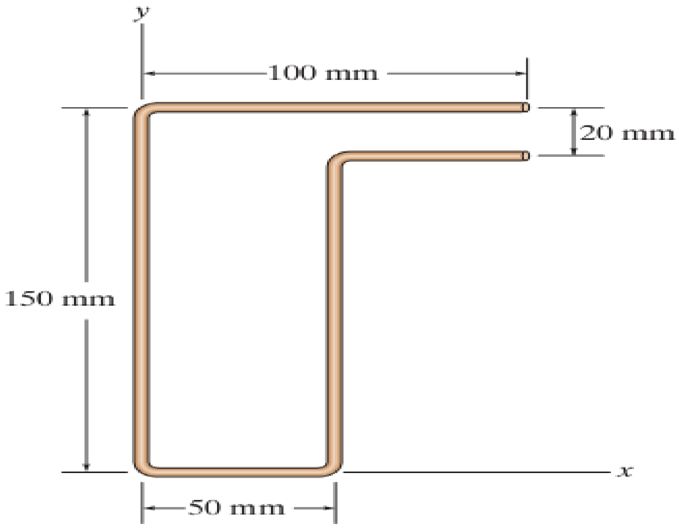
$\Rightarrow M = (36 - 6x) \rightarrow$ (

Now based on ①, ②, ③, ④ for $0 \le x < 4m$ and $4m < x \le 6m$ respectively, we have.



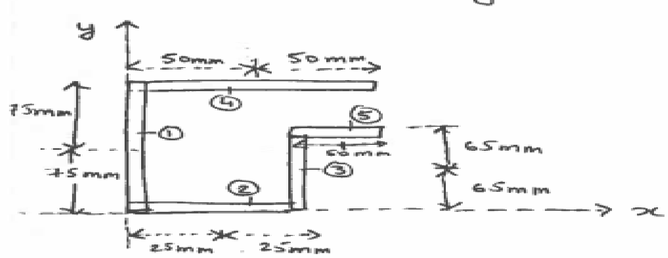
PROBLEM 4

Locate the centroid $(\bar{x}, \bar{y})$ of the uniform wire bent in the shape shown:



Procedure

1) Divide the composite body into simpler shapes
2) Make a table to tabulate the size and weight of the simpler shapes.
3) Find the centroid of the simpler shapes by fixing the x-y axes.
4) Sum columns of the table & apply formulae.



Segment	L (mm)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}L$ (mm^2)	$\bar{y}L$ (mm^2)
1	150	0	75	0	11250
2	50	25	0	1250	0
3	130	50	65	6500	8450
4	100	50	150	5000	15000
5	50	75	130	3750	6500
	480			16500	41200

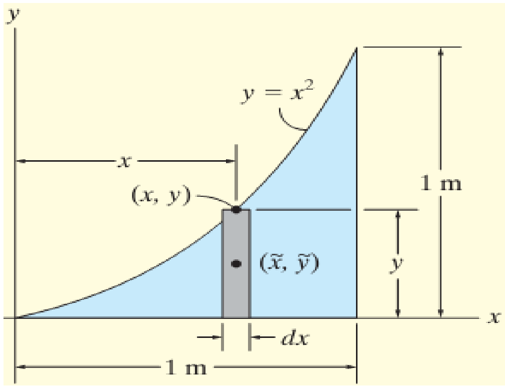
$$\therefore \quad \bar{x} = \frac{\sum \bar{x}L}{\sum L} = \frac{16500\ mm^2}{480\ mm} = 34.4\ mm$$

$$\bar{y} = \frac{\sum \bar{y}L}{\sum L} = \frac{41200\ mm^2}{480\ mm} = 85.8\ mm$$

[Answer]

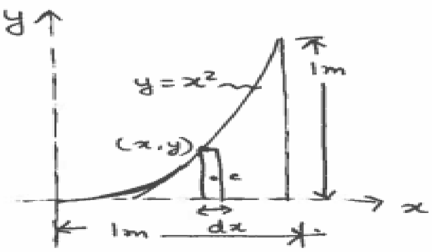
PROBLEM 5

Locate the centroid of the area.



Procedure

1) $y = f(x)$, so we need to choose a vertical element
2) Express dA of length y and breadth dx.
3) Find the centroid co-ordinates
4) Integrate.



The vertical element of height ~~dA~~ y and width dx, has an area $dA = y\,dx$.

If the element intersects the curve $y = x^2$ at (x, y), then the centroid of the element is $(x, y/2) = c$

$$\therefore\ \bar{x} = \frac{\int_A \hat{x}\,dA}{\int_A dA} = \frac{\int_0^{1m} x y\,dx}{\int_0^{1m} y\,dx} = \frac{\int_0^{1m} x(x^2)\,dx}{\int_0^{1m} x^2\,dx}$$

$$\text{or,}\quad \bar{x} = \frac{\left.\frac{x^4}{4}\right|_0^1}{\left.\frac{x^3}{3}\right|_0^1} = 0.75\,m.$$

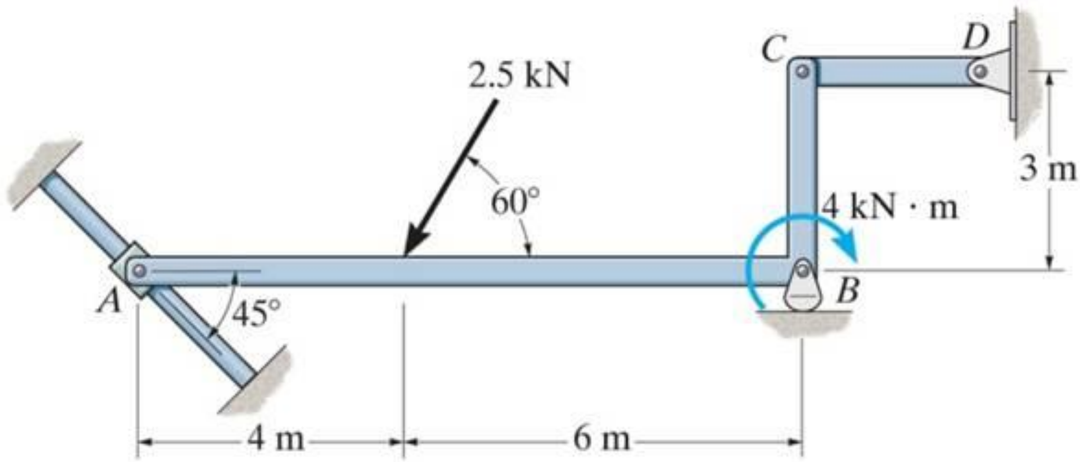
$$\therefore\ \bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1m} (y/2)\,y\,dx}{\int_0^{1m} y\,dx} = \frac{\int_0^{1m} (x^2/2)\,x^2\,dx}{\int_0^{1m} x^2\,dx}$$

$$\text{or,}\quad \bar{y} = \frac{0.1}{0.33} = 0.3\,m$$

$$\therefore\ \boxed{(\bar{x}, \bar{y}) = (0.75\,m,\ 0.3\,m)}$$ [Answer]

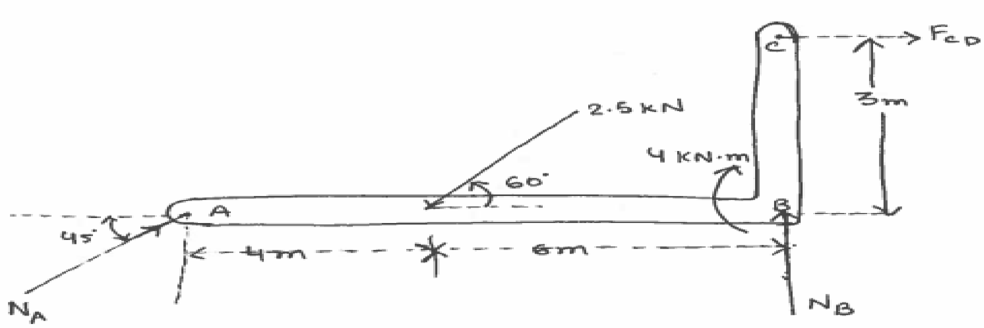
PROBLEM 6

Draw the FBD of member ABC, which is supported by a smooth collar at A, roller at B, and link CD.



Procedure

1) Draw the outline shape A – B – C
2) Indicate all external Forces and Moments
3) Label all loads and dimensions.



N_A : Smooth collar reaction on member ABC

N_B : Reaction force on ABC due to roller support at B

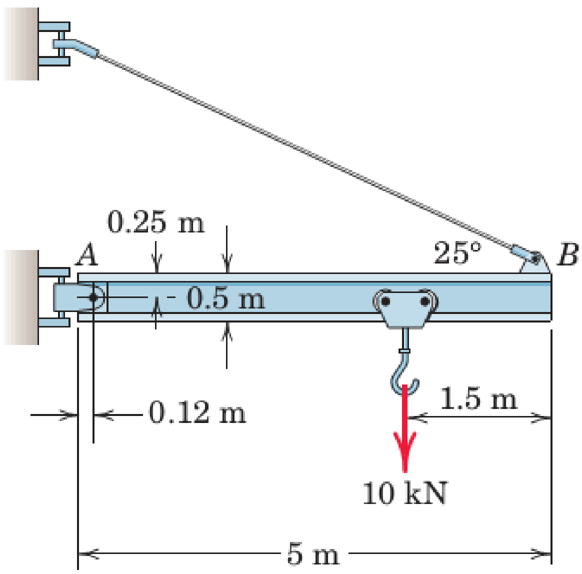
F_{CD} : Short link reaction on member ABC.

2.5 kN : Externally applied force on member ABC.

4 kN·m : Externally applied couple moment on ABC.

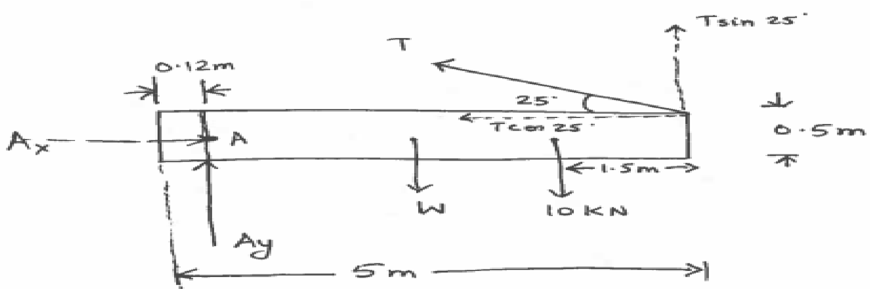
PROBLEM 7

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.



Procedure

1) Draw the FBD
2) Apply the EoE



Mass of beam = 95 kg /m of length

∴ Weight of beam = 95 kg × 5m × 9.81 m s^{-2} = 4.66 kN = W

W acts at a distance $\left(\frac{5}{2} - 0.12\right)$ m from A.

10 kN acts at a distance (5 − 1.5 − 0.12) m from A

∴ Applying $\Sigma M_A = 0$

or, $W\left(\frac{5}{2} - 0.12\right) + 10(5 - 1.5 - 0.12) - T\sin 25°(5 - 0.12) - T\cos 25°(0.5/2) = 0$

or, $4.66(2.5 - 0.12) + 10(5 - 1.5 - 0.12) - T\sin 25°(5 - 0.12) - T\cos 25°(0.25) = 0 \Rightarrow$ **T = 19.61 kN** (Ans.)

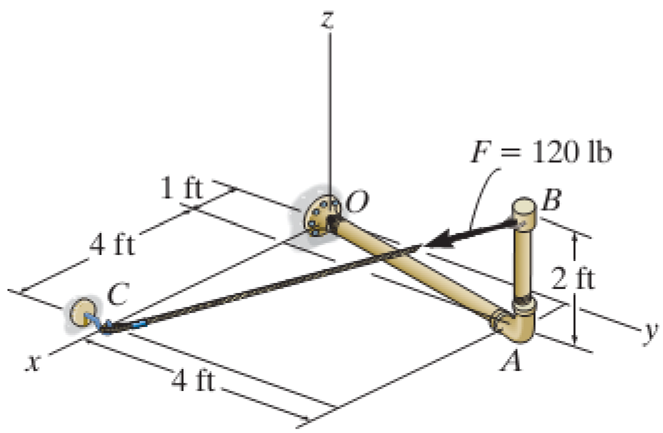
Now Applying $\Sigma F_x = 0$, $A_x - 19.61\cos 25° = 0 \Rightarrow A_x = 17.77$ kN

$\Sigma F_y = 0 \Rightarrow A_y + 19.61\sin 25° - 4.66 = 10 \Rightarrow A_y = 6.37$ kN

∴ $A = \sqrt{A_x^2 + A_y^2} = \sqrt{17.77^2 + 6.37^2}$ = **18.88 kN = A** (Ans)

PROBLEM 8

Determine the moment of force F about point O. Express the result as a Cartesian vector.



Procedure

1) Moment is the cross-product of displacement vector and Force vector.

2) We need to use vector formulation.

We need to figure out the moment of F about pt. O.

From the given figure,

$OB = r_B = 1\hat{i} + 4\hat{j} + 2\hat{k}$

Magnitude of F = 120 lb.

Unit vector of F = $BC = (OC - OB)/|BC|$

$= \frac{1}{|BC|}\{(5\hat{i} + 0\hat{j} + 0\hat{k}) - (1\hat{i} + 4\hat{j} + 2\hat{k})\}$

$= \frac{4\hat{i} - 4\hat{j} - 2\hat{k}}{|BC|}$

$= \frac{4\hat{i} - 4\hat{j} - 2\hat{k}}{\sqrt{(4)^2 + (-4)^2 + (-2)^2}}$

$\therefore F_{BC} = 120 \times \frac{4\hat{i} - 4\hat{j} - 2\hat{k}}{\sqrt{(4)^2 + (-4)^2 + (-2)^2}}$ lb.

$= [80\hat{i} - 80\hat{j} - 40\hat{k}]$ lb.

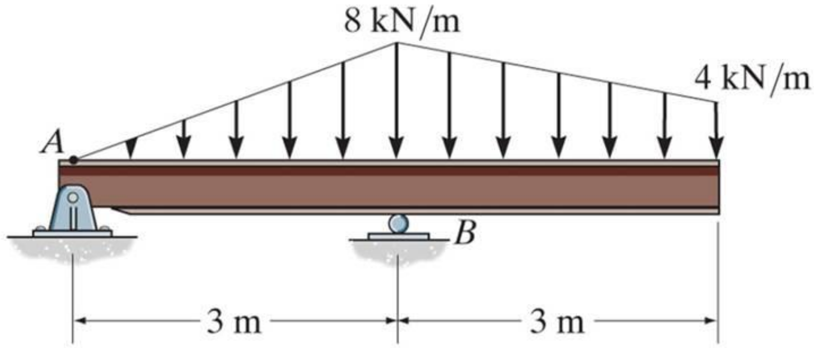
$\therefore M_O = r_B \times F_{BC} = \begin{vmatrix} i & j & k \\ 1 & 4 & 2 \\ 80 & -80 & -40 \end{vmatrix}$

(Answer)

$= i\begin{vmatrix} 4 & 2 \\ -80 & -40 \end{vmatrix} - j\begin{vmatrix} 1 & 2 \\ 80 & -40 \end{vmatrix} + k\begin{vmatrix} 1 & 4 \\ 80 & -80 \end{vmatrix} = \boxed{[200\hat{j} - 400\hat{k}] \text{ lb ft}}$

PROBLEM 9

Given the loading on the beam, find the equivalent force and its location from point A.

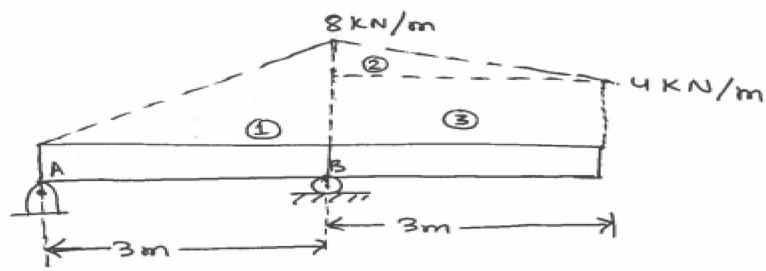


Procedure

1) Divide the distributed loading into simpler geometry.

2) Find resultant point load as a ΣF for each of the simpler geometries.

3) Find the location of the F_R by $(M_R)_A = \Sigma M_A$



For ①, $F_{①} = \frac{1}{2}(8\ \text{kN/m})(3\text{m})$

②, $F_{②} = \frac{1}{2}(8-4)\ \text{kN/m} \cdot (3)\text{m}$

③, $F_{③} = (4\ \text{kN/m})(3\text{m})$

$\therefore\ F_R = \Sigma F_y = \{\frac{1}{2}(8)(3) + \frac{1}{2}(4)(3) + (4)(3)\}\ \text{kN}$

or, $F_R = 30\ \text{kN} \downarrow$

Centroid for ③ is at $(1.5\text{m} + 3\text{m})$ from A

Centroid for ② is at $\{\frac{1}{3}(3\text{m}) + 3\text{m}\}$ from A

Centroid for ① is at $\{-\frac{1}{3}(3\text{m}) + 3\text{m}\}$ from A

Now taking the moment of F_R ~~to~~ and equating it to the sum of moments of the forces $F_{①}$, $F_{②}$ and $F_{③}$.

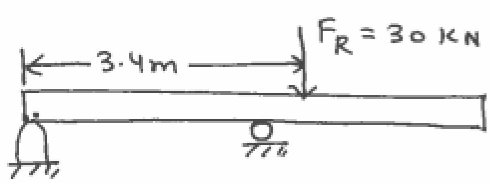
$$(M_R)_A = \Sigma M_A$$

or, $$F_R \times (x) = F_{③}(4.5)m + F_{②}(4m) + F_{①}(2m)$$

or, $$30\ KN\ (x) = \frac{1}{2}(8)(3)(2) + \frac{1}{2}(4)(3)(4) + (4)(3)(4.5)$$

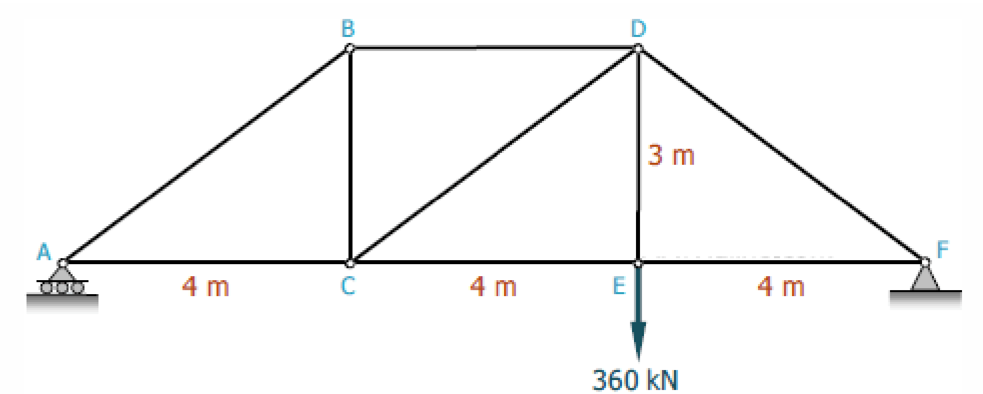
or, $$x = 3.4\ m.$$

∴ The resultant point load F_R is located at a distance of 3.4 m from point A.



PROBLEM 10

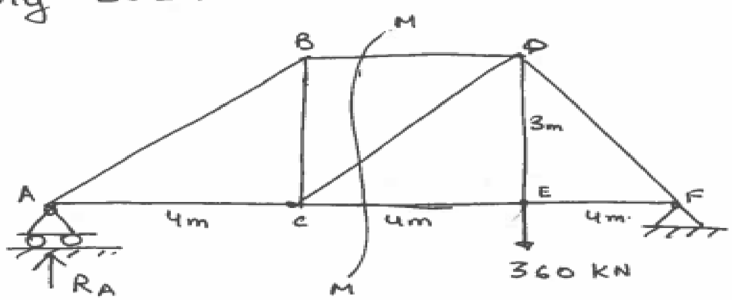
Using the method of sections, determine the force in members BD, CD, and CE of the roof truss shown in Figure



Procedure

1) Cut an appropriate section

2) Apply EoE.



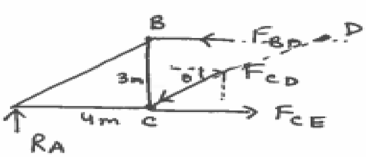
M–M is the section.

Applying $\Sigma M_F = 0 \Rightarrow R_A(4+4+4) - 360(4) = 0$

$\Rightarrow \boxed{R_A = 120 \text{ KN}}$

Applying $\Sigma M_C = 0 \Rightarrow F_{BD}(3) - 120(4) = 0$

$\Rightarrow \boxed{F_{BD} = 160 \text{ KN} \quad C}$ Compression [Answer]



$\Sigma F_y = 0$

$R_A = F_{CD} \sin\theta = F_{CD}\left(\frac{3}{5}\right)$

$F_{CD} = \frac{5 \times 120}{3}$ KN $= 200$ KN

[Answer] $\boxed{F_{CD} = 200 \text{ KN}}$ Compression

Applying $\Sigma M_D = 0$

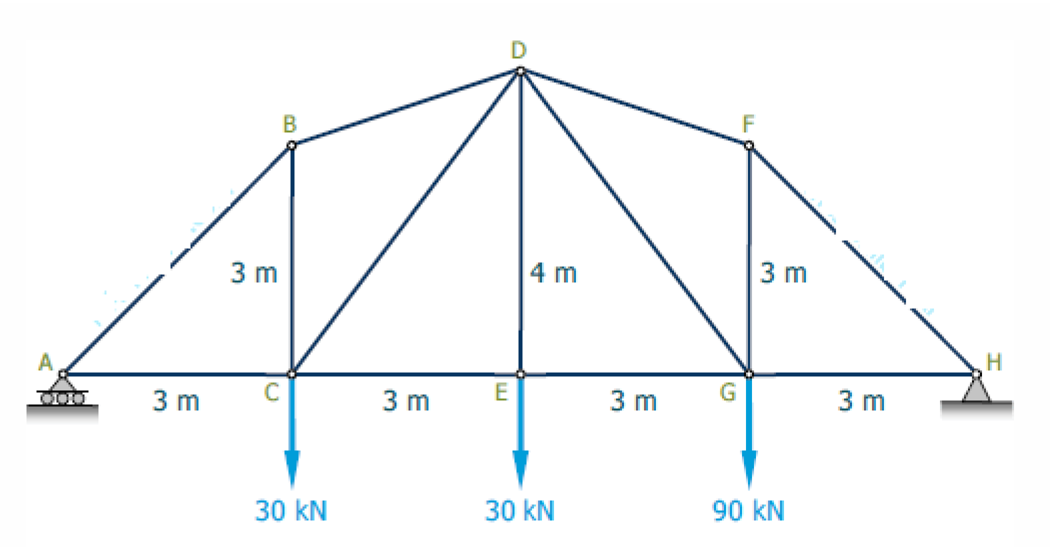
$F_{CE}(3) = R_A(4+4)$

or, $F_{CE} = (120 \times 8)/3$ KN

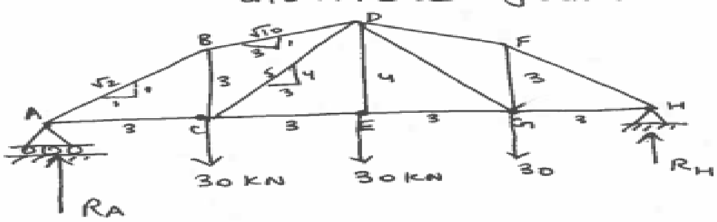
$\therefore \boxed{F_{CE} = 320 \text{ KN} \quad \text{Tension}}$ [Answer]

PROBLEM 11

Determine the force in members AB, BD, and CD of the truss shown in Figure.



Procedure : Draw FBD of entire structure and the individual joints respectively.

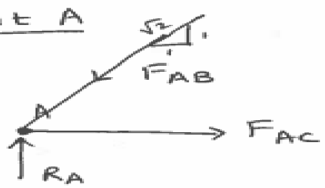


Applying $\sum M_H = 0$

$\Rightarrow R_A(3+3+3+3) - 30(3+3+3) - 30(3+3) - 30(3) = 0$

$\Rightarrow \boxed{R_A = 60 \text{ KN}}$

Joint A



$\sum F_y = 0$

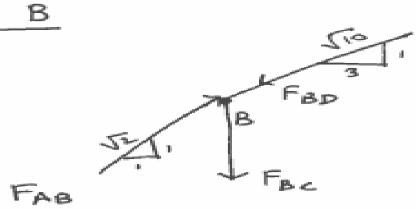
$\therefore R_A - F_{AB}\frac{1}{\sqrt{2}} = 0$

or, $F_{AB} = R_A\sqrt{2}$

or, $F_{AB} = 60\sqrt{2}$

$\therefore \boxed{F_{AB} = 84.85 \text{ KN Compression}}$

Joint B



$\sum F_x = 0$

or, $F_{BD}\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{2}}F_{AB} = 0$

or, $F_{BD} = \frac{1}{\sqrt{2}}F_{AB}\frac{\sqrt{10}}{3}$

or, $F_{BD} = \frac{1}{\sqrt{2}}(84.85)\frac{\sqrt{10}}{3}$

$\therefore \boxed{F_{BD} = 63.24 \text{ KN compression}}$

At Joint B

$$\sum F_y = 0$$

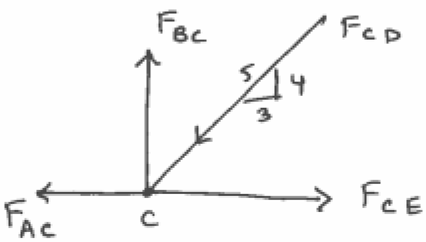
$$-F_{BC} - \frac{1}{\sqrt{10}} F_{BD} = -\frac{1}{\sqrt{2}} F_{AB}$$

or, $$-F_{BC} - \frac{1}{\sqrt{10}} 63.24 = -\frac{1}{\sqrt{2}}(84.85)$$

or, $$F_{BC} = \frac{1}{\sqrt{2}}(84.85) - \frac{1}{\sqrt{10}}(63.24)$$

$\therefore$ $\boxed{F_{BC} = 40 \text{ KN Tension}}$

Joint C



$$\sum F_y = 0$$

or, $$F_{CD} \frac{4}{5} + 30 = 40$$

or, $$F_{CD} = (40-30)\frac{5}{4}$$

$\therefore$ $\boxed{F_{CD} = 12.5 \text{ KN}}$ Compression

$\therefore$

F_{AB} = 84.85 KN Compression

F_{BD} = 63.24 KN Compression

F_{CD} = 12.5 KN Compression

[Answer]