ESSAYS ON RESOURCE PLANNING AND UTILIZATION TO ACHIEVING OPERATIONAL EFFICIENCY IN SERVICE ORGANIZATIONS

A Dissertation

by

MAYUKH MAJUMDAR

Submitted to the Graduate and Professional School of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Chair of Committee,	Chelliah Sriskandarajah
Co-Chair of Committee,	Anupam Agrawal
Committee Members,	Bala Shetty
	Steve Boivie
Head of Department,	Richard Metters

May 2022

Major Subject: Business Administration

ABSTRACT

The broadening of scope in Operations Management (OM) has facilitated new opportunities to study resource utilization and planning in service organizations for higher operational efficiency. In this dissertation, I investigate operational issues in online retailing logistics, healthcare, and social media. In my first essay, I focus on supply logistics in online retailing by analyzing safety stock allocation and transshipments to minimize their operational cost. Using a stochastic optimization framework, my proposed approach solves the problem optimally for smaller networks and offers a decomposition method for larger networks. I demonstrate its potential to save significant outbound shipping costs for an online retailer.

In my second essay, I examine Accountable Care Organizations (ACO), a network of healthcare providers who collaborate to offer high-quality care at a reduced cost. To address their performance issues, I analyze the role of provider composition in delivering primary care and ACO experience on performance. Using a dataset of ACOs under Medicare, I find that (i) ACOs achieve better performance by utilizing more nurse practitioners, (ii) having more primary care services delivered by specialists does not result in better quality and (iii) high performing ACOs focus more on savings in the initial phases under any risk model and consider quality initiatives in the higher risk model. Collectively, the results provide guidelines on improving ACO performance.

In my third essay, I examine how firms can deliver better content to their target audience via social media posts and maximize user engagement. Developing social media posts with compelling features that capture users' attention is a vital task, albeit a challenging one. I propose a data-driven optimization framework for analyzing and publishing social media posts across multiple platforms. This framework captures users' preferences via analytics to develop better content for planning horizon under a firm's limited budget. In that regard, I identify the number of features for each platform to be included in social media posts and schedule them throughout the planning horizon in the context of maximizing user engagement. The models and results in the third chapter can help firms improve their social media campaign and achieve higher user engagement.

DEDICATION

To my mother Rama Majumdar and my father Prabir Kumar Majumdar.

Thank you for your constant support and guidance along the way.

ACKNOWLEDGMENTS

I would like to express my gratitude to my committee members and co-authors for their constant guidance, my parents for their never-ending support, and my friends for their encouragements.

I would like to thank my PhD advisors, Dr. Chelliah Sriskandarajah and Dr. Anupam Agrawal, for their outstanding mentoring, patience, and feedback throughout my PhD tenure. I have been fortunate to receive knowledge and wisdom from them not only in conducting research and teaching students and but also in values and ethics in life in general. I aspire to become an outstanding academician like them and mentor students in the future.

I am extremely fortunate to have Dr. Bala Shetty and Dr. Steve Boivie as my PhD committee members. I want to thank them for their guidance through my research and their service in my academic journey. I also would like to thank my co-authors Dr. Subodha Kumar and Dr. Arun Sen, for their engagement and guidance over the years to enhance my research projects.

I also would like to convey my gratitude to all the faculty members at the Department of Information and Operations Management, especially Dr. Richard Metters, for his leadership and constant support to the PhD students. I am thankful to have guidance, especially from Dr. Michael Ketzenberg, Dr. James Abbey, Dr. Andres Jola. Sanchez, Dr. Rajiv Mukherjee, Dr. Rogelio Oliva, Dr. Gregory Heim, Dr. Neil Geismar, and Dr. Esmaeil Keyvanshokooh for their help through PhD seminars and professional development. I am extremely thankful to Ms. Donna Shumaker, Ms. Theresa Ralston, Ms. Veronica Stilley, and Ms. Tammy Louther for their help and assistance throughout my journey.

I am extremely grateful to share my PhD journey with my incredibly talented peers - Seulchan Lee, Huseyn Abdulla, and Han K. Oh. I want to thank all my friends for their support throughout the time, especially Mr. Rupam Dutta, Mr. Apratim Mukherjee, Ms. Esha Dutta, Mr. Shauktik Kumar, Ms. Aarushi Verma, Mr. Niladri Das, and Ms. Swathi Murali, for being supportive of my career decision and constantly motivating me. At this time, the world is devastated by a pandemic, and I want to thank all the Covid warriors for keeping our families safe.

Finally, I am thankful to my parents who are my first teachers, best friends, and my constant pillars of support. They taught me values, kindness, respectfulness, and honesty. They have always inspired me to be a better person and contribute to the society. I can not come this far without their love and sacrifice.

CONTRIBUTORS AND FUNDING SOURCES

Contributors

The committee member co-chaired by Professor Chelliah Sriskandarajah and Professor Anupam Agrawal from the Department of Information and Operations Management have supported this work. Also, Professor Bala Shetty from the Department of Information and Operations Management and Professor Steve Boivie the Department of Management have been an active part in advising and reviewing this work. Essay 2 was reviewed by Professor Arun Sen from the Department of Information and Operations Management and Essay 3 was reviewed by Professor Subodha Kumar from Temple University.

All other works for this dissertation were performed by the student independently.

Funding Sources

No other outside source of funding was provided.

NOMENCLATURE

OM	Operations Management
FC	Fulfillment Centers
PAAS	Procedure Allocate Safety Stock
MIP	Mixed Integer Program
MNLIP	Mixed Nonlinear Integer Program
SAA	Sample Average Approximation
HOM	Healthcare Operations Management
ACO	Accountable Care Organizations
CMS	Center for Medicare and Medicaid Services
PCP	Primary Care Providers
NP	Nurse Practitioners
FQHC	Federally Qualified Health Center
CHC	Community Health Center
RHC	Rural Health Clinic
CDC	Centers for Disease Control and Prevention
SSD	Single Shot Detector
R-CNN	Region-based Convolutional Neural Network
ZINB	Zero-Inflated Negative Binomial

TABLE OF CONTENTS

LIST OF FIGURES

FIGURE Page

LIST OF TABLES

TABLE Page

1. INTRODUCTION AND LITERATURE REVIEW

Operations management (OM), as a community, has seen significant changes in the last few decades and has improved substantially in its effort to find effective solutions to complex problems. Currently, scholars in the OM discipline are looking beyond the traditional manufacturing domain and applying operation research in service organizations. From networks of e-commerce fulfillment centers to patient-centric healthcare units to user-focused social media platform advertisements, organizations constantly deal with complex processes to make their decisions. Though there are substantial differences in different industries' operations, all are faced with one common challenge: operational efficiency in utilizing and planning resources.

In this dissertation, I analyze operational issues in online retailing, healthcare, and social media advertising. The first essay focuses on safety stock management and transshipments in the online retailing industry. In the second essay, I study the issues associated with accountable care organizations (ACO). More specifically, in the second essay, I provide an empirical analysis of ACO characteristics based on a sample of ACOs in the US. Finally, in the third essay, I provide an optimization framework to help firms develop better advertising content on social media platforms by utilizing social media analytics.

1.1 Safety Stock Allocation in an Online Retailing Network: A Stochastic Optimization Approach

Ecommerce, or online retailing, is the buying or selling of goods or services on the Internet. However, amidst the growth in sales figures, e-commerce firms are falling short on profit margins compared to other industries. In 2020, Amazon spent approximately \$61.1 billion on shipping, up from around \$21.7 billion in 2017 (Statista 2021). This high cost can be attributed to the expensive outbound shipments to customer locations. Online retailers often utilize lateral transshipments to meet the immediate shortfall at a location by moving stocks from another location with a surplus. Such transshipments are expensive and depend on the inventory level of the location having surplus stocks. One way to reduce such expensive transshipments is to hold additional safety stocks across the network.

In this chapter, I study the allocation of safety stocks in a large network of multiple fulfillment centers to minimize the total cost composed of transportation, inventory holding, and lost sales. This chapter finds an optimal and robust solution for a network consisting of one to six fulfillment centers using a stochastic optimization method. For larger networks, I propose a decomposition method that (i) groups nearby fulfillment centers (FC) into a number of clusters, (ii) connects those clusters by using a Minimum Spanning Tree algorithm, and (iii) forms a hub-and-spoke network where safety stocks are allocated first in the clusters and then in the hub and spoke FCs. This chapter contributes to the existing literature on transshipments by providing a safety stock allocation strategy. I offer an implementable solution procedure to manage an extensive network of FCs to the practice. Additionally, by solving a network of 6 FCs, this chapter significantly advance the current research paradigm that limits solutions to networks with 3 FCs.

1.1.1 Policy Implications

I address the following questions for the management of an online retailer: (i) what is an efficient mechanism to allocate safety stock in a network that reduces expensive transshipment, (ii) how does the performance of the allocation mechanism vary substantially across networks, and iii) how robust is the allocation mechanism for changes in the demand. The safety stock allocation is an important problem for an online retailer since an insufficient quantity of safety stock, combined with expensive transshipments, can significantly increase the cost of meeting customer demand.

Wei et al. (2021) is the most recent work that examines the order fulfillment process of an omnichannel retailer but considers only two nearby stores to deliver customer orders. Glazebrook et al. (2015) is another study on hybrid transshipments in a multi-node network that considers three locations and extends to 10 and 50 locations via simulations. I employ a stochastic optimization approach and solve a network of up to six FCs optimally by incorporating a large number of sample paths representing uncertain demands. This chapter is the first to solve a network of 6 FCs optimally to the best of my knowledge. I employ the Sample Average Approximation approach

(SAA) to show the robustness of the approach for a network with 166 FCs. The incorporation of such a huge number of FCs is a first in the online retailing inventory literature.

1.2 ACO Service Delivery and Experience on Financial and Quality Performance - An Empirical Examination

In this chapter, I examine Accountable Care Organizations (ACOs) and their performance implications on reduced healthcare cost and increased care quality. ACOs consist of primary care physicians (PCPs), specialists, nurse practitioners (NPs), hospitals, and other healthcare providers and facilities, who provide coordinated healthcare to their patients for reducing healthcare costs and improving care coordination. ACOs that meet both financial and quality standards are incentivized by their payers through financial rewards. Currently, there are nearly 1000 ACOs across the US, operating with commercial, Medicare, and Medicaid contracts (Solutions 2021). In the rising cost of healthcare in the United States which has reached $3.8 trillion in 2019, or 17.7 percent of the gross domestic product (GDP) (Martin et al. 2021), ACOs are touted as attractive options to reduce healthcare costs. Despite ACOs' efforts to curb spending and improve quality of care, the Centers for Medicare and Medicaid Services (CMS) do not consider that the financial and quality performance have been enhanced by the ACOs (CMS 2021).

I examine (i) provider composition in delivering primary care services, namely specialists and nurse practitioners (NPs) as a proportion to primary care providers (PCP) and (ii) ACO experience on ACO performance. Using a dataset of ACOs, comprising 1908 observations between 2016 and 2019, I perform an empirical analysis and present important findings. The results suggest that ACOs achieve better performance by utilizing more NPs. Additionally, having more primary care services delivered by specialists is financially unsustainable in the long term and does not result in better quality. High performing ACOs focus more on savings in the initial phases under any risk model and consider quality initiatives in the higher risk model. Additionally, I explore the geographical and behavioral aspects of care by including the Center of Disease Control (CDC) annual data in a post-hoc analysis. First, the findings suggest that providing access to care at non-urban and underserved areas may not negatively impact the cost and quality performance of

ACOs. The addition and analysis of behavioral variables suggest that ACOs in states with poor routine check-up times and cost of care are negatively associated with quality performance.

1.2.1 Policy Implications

This chapter makes practical contributions to the healthcare operations management (HOM) literature by exploring the ACO characteristics associated with the financial and quality performance of ACOs. Service providers (or providers) play a critical role in population health management in service operations, especially in HOM. Their actions determine the quality of care and the cost of healthcare. Payers such as CMS push providers to take more accountability in reducing healthcare costs while increasing access to care and quality. The ACO model is a vital program started by the CMS and later adopted by other payers to achieve the triple aim of healthcare and motivate providers to transition from a volume-based to a value-based approach. ACOs are PCP-focused organizations that manage the care episodes of patient populations. However, they often deliver services through non-PCP and non-physicians. The findings in this chapter offer guidelines on how ACOs service delivery via provider composition and experience affect their performance.

1.3 Know Your Users Before You Spend: A Data-Driven Optimization to Enhance User Engagement using Visual Analytics

With the increasing presence of social media platforms, delivering attractive posts to advertise products on these platforms has become crucial for firms to engage their target audience. Firms are expected to spend almost $225 billion on social media advertising in 2024, surpassing paid search and television advertising (IndiaPartner 2021). However, creating content with compelling features in posts that match what their users prefer to engage with is challenging. Content development for posts has implications for firms' objectives; a recent industry report (Gary 2021) finds that posts failing to attract the users attention have negative implications on a brand's reputation. Even though content development via social media analytics is essential for firms, it has not been examined rigorously in the literature. To bridge this important gap, in essay 3, I propose an optimization framework for analyzing and publishing social media posts across multiple platforms. I incorporate social media analytics to obtain user information and guide content development efforts using those insights for a planning horizon under a firm's limited budget.

I consider a planning horizon where the firm wants to develop and publish posts on single or multiple social media platforms. The firm requires information on users' preferences and analyzes their own and competitors' data to understand relevant image features. Social media marketing campaigns require firms to operate across multiple platforms, resulting in more resources and higher costs. However, users across platforms differ in their attributes. Thus, understanding the relationship between features included in a post and the corresponding user engagement becomes crucial at each platform. This chapter explores the relationship between engagement and features using empirical analysis and data on social media posts on Facebook and Instagram and model it in the optimization framework. I combine and run advanced analytical and econometric methods on the dataset to explore the relationship between features and engagement.

This chapter provides structural results for the two platforms - Facebook and Instagram and offer insights on content development activities. In this regard, I identify the number of features that contribute to the maximum and minimum engagement levels on Instagram and Facebook, respectively. I establish the relationship between user engagement and the number of features included in the content. Further, I also illustrate the higher performance of the approach that combines both user-base and engagement intensity against the user-base budget allocation approach; it shows a difference of 11-12% in engagement. Additionally, I offer managerially relevant guidelines on other aspects of media posts.

1.3.1 Policy Implications

In this chapter, I analyze how social media analytics and content development activities can be jointly utilized to deliver better posts for users to achieve higher engagement. This chapter fills an important gap in the literature by analyzing social media content development strategies using a data-driven optimization framework and offering valuable insights. I develop an optimization model that maximizes user engagement under a limited budget. The model parameters and the functional form of user engagement are estimated via an empirical analysis based on the data from Facebook and Instagram. This chapter develops insights into the following aspects: (i) the relationship between user engagement and image features across platforms, (ii) the budget allocation for the competing costs of social media analytics and content development activities in the model, and (iii) the budget allocation and content development strategies on single and multiple platforms. This chapter has substantial managerial implications for designing social media content development strategies.

In summary, my dissertation develops managerially relevant guidelines to address the resource planning and utilization issues in service organizations in e-commerce, healthcare, and social media advertising. Utilizing state-of-the-art empirical and analytical tools and existing theories in management science, I develop a research portfolio that extends the current stock of knowledge in these domains and offer policy implications to both academia and practitioners.

The reminder of the dissertation is organized as follows. Chapter 2 examines the allocation safety stock in an e-commerce fulfillment center network. Chapter 3 analyzes the performance implications of ACOs. Chapter 4 proposes an optimization framework that guides a firm's social media content development by utilizing analytics. Chapter 5 concludes the dissertation by summarizing the chapters.

2. Safety Stock Allocation in an Online Retailing Network: A Stochastic Optimization Approach

2.1 Introduction

Online retailing or E-tailing refers to the sale of goods and services through the Internet and the shipment of goods directly to customer locations. Product pricing, website management, marketing, and order fulfillment are the key decisions for an online retailer, with the latter being one of the most expensive and critical operations (Maltz et al. 2004). Order fulfillment encompasses the entire process of receiving, processing, packaging, and shipping orders, with shipping cost dominating the total order cost. Online shoppers expect free or cheaper shipping with fast delivery, forcing e-commerce retailers to absorb a large part of the shipping cost. Fast delivery requires expensive shipments and product availability at the right place and at the right time. When a shortage occurs at one location, an online retailer may fulfill that order from a different fulfillment center (FC) at a higher shipping cost, a strategy known as lateral transshipment. This chapter addresses the problem of allocating safety stock to multiple locations in an online retailing network while allowing lateral transshipment and demand uncertainty. The problem is computationally quite challenging. We provide several strategies to bring the problem to a manageable level.

Having an efficient distribution network is paramount in online retailing to ensure shorter wait times for customers to receive their orders. A fulfillment center represents a node in a distribution network. On-time delivery is a crucial driver of consumer repurchase, and any failure or delay in delivery may lead to diminished repurchases (Reichheld and Schefter 2000). Thirty-eight percent of shoppers with poor delivery experience will never shop with the retailer again (Lopienski 2018). To fulfill customer expectations (e.g., Amazon's recent introduction of a one-day prime shipping policy), online retailers keep the shipping charges as low as possible. Lopienski (2018) also reports that 61% of shoppers will abandon their carts if shipping charges are too high. In 2020, Amazon spent approximately \$61.1 billion on shipping, up from around \$21.7 billion in 2017 (Statista

2021). This enormous expenditure can be attributed to their almost free or low-cost shipping strategies. For example, Amazon, the largest online retailer, ships products to its Prime customers for a minimal charge (currently $119 annually or $12.99 monthly), which totaled $25.21 billion in 2020 (Digital360 2021). Walmart charges a flat shipping fee of $5.99 per order under $35 to most locations in the US [1]. While customers prefer free shipping, they are less reluctant to pay for expedited delivery (ProShip 2017).

Shipping cost is a variable cost in that it increases with the quantities sold. One component of the rising shipping cost is the cost resulting from expensive transshipments. While traditional retailers transship items within the network of stores and warehouses, online retailers use lateral transshipment (hereafter transshipment for simplicity) to fulfill customer orders directly from FCs. They may transship directly to a customer location or indirectly via a customer's nearest FC. The safety stock allocation in this chapter covers both of these options for transshipment with the primary goal of ensuring the customer demand is met while keeping the total cost down.

Ideally, an online retailer would satisfy the demand from an FC nearest to the customer. However, in the event of a stockout at the local FC, the retailer may use a distant FC with surplus inventory to fulfill customer demand. Acimovic and Graves (2017) explain this as a demand spillover phenomenon. Consider a situation with three FCs. If a stockout occurs at FC 1, FC 2 will serve Customer Region 1, whose closest facility is FC 1. Following Acimovic and Graves (2017), a local stockout leads to higher outbound shipping costs. With higher demand uncertainty, outbound shipping costs tend to increase as online retailers transship more from distant FCs. The expensive transshipments may be avoided by storing a sufficient amount of safety stock at each FC to prevent local stockouts.

Safety stock represent a level of insurance to help mitigate the risk of stockouts due to uncertainty in supply and demand. We focus only on demand uncertainty in this chapter. An optimal safety stock level may protect against demand uncertainty, prevent supply chain disruptions, improve customer satisfaction, and compensate for forecast inaccuracies. Despite the cost savings

[1]The information on Walmart and Amazon are based on the data available at the time of this chapter.

achieved by minimizing expensive transshipments, a large volume of safety stock at one location may also lead to higher lateral transshipment, resulting in financial and operational inefficiencies. Thus, an efficient allocation of safety stock over a large network of FCs is critical for achieving an appropriate balance between a satisfactory service level and rising costs. Each customer region may exhibit different demand patterns, and each FC should ideally meet those demands with the least amount of safety stock for the assortment it carries.

Since customers are not concerned with the source location of their orders, online retailers pool or aggregate inventory from multiple stocking locations. Pooling reduces overall system inventory and provides additional flexibility in replenishing the customer demand for an FC (Lee 1987). We pool safety stock across an online retailer's network to ensure that the required customer service level is met. We also incorporate lost sales in this model because maintaining a 100% service level can be expensive and inefficient. While a certain level of lost sales is economically acceptable, retailers do not desire a high amount of unmet demand. These factors motivate online retailers to operate in a way that minimizes the total expenditures consisting of inventory, transportation, and lost sales. Note that transportation costs in this chapter refer to the costs associated with outbound shipping from FC to customers that include shipment and transshipment costs only. Minimizing other transportation costs is not the focus of this chapter.

Previous studies mostly consider transshipments in the traditional retail setting: within warehouses, retail stores, or between warehouses and retail stores. In the traditional retailing model, products are sold and shipped to customers through physical stores. In contrast, online retailers ship products directly from their FCs to customer locations. Therefore, the difference between traditional retailing and online retailing lies in the order fulfillment process. Additionally, online shopping requires customers to wait for a period after they place their orders, whereas traditional retailers deliver items immediately. Online retailers may also ship items directly from their suppliers. While we recognize that fulfillment by external suppliers to customers is possible, it is not the focus of this chapter. The reader may refer to the work by Xiao and Xu (2018) for coordination and capacity issues with the online platform and third-party seller coordination.

2.1.1 Goals

We address a difficult problem of allocating safety stock over a network of fulfillment centers in the face of demand uncertainty. The goal here is to develop an easily implementable safety stock allocation mechanism that may benefit online retailers. An online retailer must deliver the product upon receiving an order if the ordered item was displayed on the website. We consider the allocation process as a discrete stochastic optimization model with uncertain demands represented by sample paths from a normal distribution. Such representation of demand is consistent with the literature (Campbell et al. 1998). For the ease of development, we use single item orders that include such items as computers, laptops, and backpacks that are delivered to customers one at a time. We assume that the retailer follows a periodic review order-up-to policy where it periodically places system-wide orders to its suppliers. Our single period model can be repeated throughout the year, as in Acimovic and Graves (2017). At the end of each period, the retailer calculates the safety stock at the FCs and may order the amount needed at these FCs, along with the required inventory.

For our analysis, we use the online retailing networks of Amazon and Flipkart as canonical examples. Because of the difficulty of solving the safety stock allocation problem in a large network, we use a divide and conquer strategy where the network is divided into multiple clusters of FCs, connected via a Minimum Spanning Tree (MST) graph. By doing so, we model the cheapest transportation link between any two clusters, avoiding long lead time delays and minimizing expensive distant transshipments. Within each cluster, we model the FCs in a hub-and-spoke system. The distance between two FCs within a cluster tends to be small, and therefore, the hub-and-spoke system allows us to use equal transshipment costs between any two nodes within that cluster. The hub-and-spoke structure is consistent with the fulfillment strategy of many large online retailers. We employ our solution strategy first to a network with two FCs serving two customer regions and examine the relationship between the standard deviation of the demand and safety stock allocation. We then extend our analysis to three FCs with three customer regions. Finally, we use the insights from the solution for two and three FC networks, along with the clustering and hub-and-spoke approach, to solve allocation in much larger networks and provide online retailers an easily implementable tool for managing their inventory.

2.1.2 Contributions

This chapter proposes a safety stock allocation approach in a multi-location online retailing network. The safety stock allocation is an important problem for an online retailer since an insufficient quantity of safety stock, combined with expensive transshipments, can significantly increase the cost of meeting customer demand. As discussed in the next section, there has been some work in the literature on safety stock allocation in online retailing. However, we are not aware of any attempt in the literature that explicitly addresses safety stock allocation for medium or large online retailers under demand uncertainty and lateral transshipments. Our approach to handling a larger retailing network begins with the solution for the simpler case of two FCs and two regions, where we present a stochastic optimization model with uncertain demands represented by sample paths from the demand distribution. We determine the proportion of safety stock allocated to two FCs for both equal and unequal variances of the demand at two regions.

The optimal allocation of safety stock quickly becomes intractable as the retailing network grows in the size of the number of nodes (FCs). We develop a novel approach to solve larger problems by sequentially employing the following steps: 1) the global network is broken into several clusters using an integer program, 2) within each cluster we identify a hub-and-spoke system using a p-center algorithm 3) all hub nodes in the network are then linked through a MST to keep the transshipment cost between clusters as small as possible, 4) system-wide safety stock is then allocated to the hub of each cluster via a stochastic optimization algorithm, 5) spoke nodes (also refereed to as *leaf nodes*) in each cluster are pooled to form one FC, and the share of that cluster's safety stock is allocated to the hub and pooled FC, and consequently to individual leaf FCs.

Wei et al. (2021) is the most recent work that examines the order fulfillment process of an omnichannel retailer but considers only two nearby stores to deliver customer orders. Glazebrook et al. (2015) is another study on hybrid transshipments in a multi-node network that considers three locations and extends to 10 and 50 locations via simulations. We employ a stochastic optimization approach and solve a network of up to six FCs optimally by incorporating a large number of sample paths representing uncertain demands. This is the first study to solve a network of 6 FCs optimally to the best of our knowledge. For networks with more than 6 FCs, we use the decomposition described above to obtain a near-optimal solution. We employ the Sample Average Approximation approach (SAA) to show the robustness of our approach for a network with 166 FCs. The incorporation of such a huge number of FCs is a first in the online retailing inventory literature.

We address the following questions for the management of an online retailer: (i) what is an efficient mechanism to allocate safety stock in a network that reduces expensive transshipment, (ii) how does the performance of the allocation mechanism vary substantially across networks, and iii) how robust is the allocation mechanism for changes in the demand. Considering the demand volatility observed in practice, we offer a cost-efficient inventory allocation and transportation approach while maintaining a required service level. We demonstrate that our decomposition approach offers a near-optimal solution by assessing its performance on a sizable testbed of relevant instances of mid-sized and large networks. We report extensive computational results to validate the robustness of our approach under different parametric conditions.

This chapter is organized as follows. We proceed with a brief literature review in Section 2.2 and study two and three FC networks in Section 2.3. We also prove some important results in Section 2.3 that are used for analyzing larger networks. Section 2.4 is the key part of this chapter, where we model a large network of FCs under a MST graph and explore its properties. Key contributions in this section include the decomposition of the network into clusters and each cluster into a hub-and-spoke system. Section 2.5 presents an implementation of the decomposition method and evaluation of its performance. In Section 2.6, we describe the SAA approach and validate the robustness of our heuristic for a large network. In Section 2.7, we analyze three practical safety stock allocation approaches used in practice and investigate the cost implications for those approaches. We present the managerial insights in Section 2.8 and conclude in Section 2.9 with implications for further research.

2.2 Literature Review

The general problem of minimizing the total costs of transportation, inventory holding, and lost sales has been extensively studied in the operations management literature. This chapter contributes to the literature by specifically addressing safety stock allocation while balancing the costs of transshipment, inventory, and lost sales for an online retailer. Variants of this problem have been addressed by a few researchers, prominent among those are the contributions of Glazebrook et al. (2015) and Acimovic and Graves (2017). While these and other important studies inspire our research, as we discuss below, what we address in this chapter differs from the current literature in several ways. To the best of our knowledge, this is the first attempt to simultaneously consider transshipment and safety stock management for a large online retailer in the presence of demand uncertainty.

There is a rich literature in the operations management field on various transshipment strategies. Krishnan and Rao (1965), Karmarkar and Patel (1977), Robinson (1990), and Archibald et al. (1997) are some of the leading pieces that explore transshipment within the same firm. Krishnan and Rao (1965) develop a single-period two location model. Robinson (1990) extends the literature by considering multiple identical locations and two non-identical location models. Research on transshipment has primarily focused on transshipment that can be broadly classified as: (i) proactive transshipment when transshipment is allowed before the demand is realized and (ii) reactive transshipment when transshipment is allowed after the demand is realized. For a brief review of the transshipment types, the reader may refer to the work by Paterson et al. (2011). We model reactive transshipment where an online retailer transships orders after the selling period begins. Archibald et al. (2010) propose a novel heuristic for reactive transshipments. Hu et al. (2008) and Chen et al. (2015) consider reactive transshipment with uncertain capacity. In the first stage, they characterize the optimal ordering, and in the second stage, they model reactive transshipment. The work in this chapter differs from the transshipment literature in that we use an online retailing context with a focus on managing a balance between inventory and transshipment costs for a network with a large number of FCs. We also address demand uncertainty in a stochastic optimization framework. In Table 1, we compare this chapter to the problems analyzed in the prior literature. Some of these papers, as rightly pointed out by Herer et al. (2006), incorporate high levels of complexity into their models, rendering them analytically intractable. We adopt a framework that may provide useful insights to practitioners, while making sure the intractability is handled through a creative solution approach.

This chapter is in part motivated by Glazebrook et al. (2015), who address transshipment in car parts industry. For a central depot periodically replenishing the stores, they propose a hybrid transshipment approach for meeting immediate shortfall and proactive transshipment for inventory balancing. Their decisions include: (i) choosing the sending location for transshipment, and (ii) determining the number of items to be transshipped. They provide an optimal solution for the case of three locations and propose a heuristic for the multi-location (10 and 50 locations) and multi-item orders. There are important differences between Glazebrook et al. (2015) and our research. We formulate a cost minimization problem for an online retailer, replenished by a supplier. As opposed to Glazebrook et al. (2015), we focus on the efficient allocation of safety stock in an online retailing environment under reactive lateral transshipment. Unlike the car parts industry, where proactive transshipment is more common since replenishments from the central depot requires more time and money, reactive transshipment allows online retailers to take advantage of the time gap between order placement and order fulfillment to pursue the best fulfillment strategy. Our modeling approach also differs from Glazebrook et al. (2015). We explicitly model safety stock allocation and transshipment via stochastic optimization, where a large number of scenarios effectively represent demands. We prove a number of results that allow us to tackle very large online networks. We solve the model optimally for 6 FCs, versus the three-FC networks considered in Glazebrook et al. (2015), and provide an easily implementable heuristic for larger networks.

The second stream of literature related to our research is inventory management in online retailing. Several studies focus on order fulfillment at FCs to minimize shipping costs (Acimovic and Graves 2015, Jasin and Sinha 2015, Lei et al. 2018, Xu et al. 2009). This chapter builds on this stream of literature and focuses on the simultaneous consideration of transshipment policies and safety stock allocation in online retailing. One of the key studies that motivate this work is that of Acimovic and Graves (2017). They point out some of the difficulties inherent in using traditional inventory management to an online environment. For their industrial partner, one of the largest online retailers, the traditional status quo policy increases inventory imbalance and local stockouts, resulting in higher outbound shipping costs through “additional spillovers" or transshipments. They suggest a heuristic based on a stochastic linear program for inventory allocation policy. In the proposed policy, they determine the order amount for each FC by estimating the safety stock and demand realized during the lead time and review period. With 1080 scenarios, each having 63 demand sample paths along with varying lead time, cycle service level, and inventory shift magnitude, they show that their heuristic reduces costs associated with transshipments or the additional cost of meeting the demand spilled over to other nodes.

This chapter extends the work of Acimovic and Graves (2017) in several aspects. To reduce the overall inventory imbalance, they propose the allocation of inventory and the safety stock in the same proportion across all FCs. In contrast, we determine the allocation of safety stock with the objective of minimizing the total costs associated with transportation, safety stock holding, and lost sales. More specifically, our approach involves the decomposition of a large network that reduces the complexity and helps in efficiently allocating inventory. Through our theoretical insights and numerical analysis, we show that our decomposition approach produces results reasonably close to optimality. Additionally, our decomposition strategy allows us to solve much larger problems with significant uncertainty in demands across the entire network.

2.3 Problem Formulation

Because the focus of our research is its applicability to online retailers, we closely studied the fulfillment processes of two major online retailers - Amazon and Flipkart. We visited Flipkart's office in Bengaluru, India, and had an extensive conversation with a senior executive regarding their network and fulfillment process. We also had conversations with employees of Amazon who are involved in order fulfillment. Flipkart, the largest e-commerce player in India, connects thousands of sellers to millions of customers across the country. Since the government policy in India does not

Table 2.1: A review of closely related works

Work	Objective	Size	Methodology	Replenish-ment policy	Transshipment policy	Lost demand	Safety stock
Archibald et al. (1997)	Optimal reorder policy with minimized inventory cost	Small	Stochastic dynamic programming (DP)	Periodic	Reactive	Emergency order	No
Axsater (2003)	Decision rule to cover whole or a part of demand with transshipments	Small	Stochastic dynamic programming	Continuous	Reactive	Back-ordering	No
Herer et al. (2006)	Optimal order up-to policy and optimal transshipment policy	Small	Integrated Infinitesimal Perturbation Analysis IPA/LP algorithm	Periodic	Reactive	Lost sales	No
Archibald (2007)	Optimal replenishment and transshipment decisions	Small	Markov decision process and heuristics	Periodic	Reactive	Emergency order	No
Hu et al. (2008)	Effect of capacity on replenishment and transshipment policies	Small	Two-stage backward induction	Periodic	Reactive	Lost sales	No
Zhao et al. (2008)	Transshipment policy triggered by both demand arrivals and production completions	Small	Dynamic Programming	Continuous	Hybrid	Lost sales	Yes
Archibald et al. (2010)	Efficient transshipment policies for reactive transshipments	Small	Stochastic DP and heuristics	Continuous	Reactive	Emergency order	No
Glazebrook et al. (2015)	Selection of transshipping location and item-types under a centralized system	Medium	Stochastic DP and heuristic	Periodic	Hybrid	Lost sales and backordering	Yes
Acimovic and Graves (2017)	Inventory allocation to reduce demand spillovers (transshipments)	Medium	Stochastic linear program (SLP) and heuristics	Periodic	Reactive	Lost sales	Yes
This chapter	Efficient allocation of safety stock with a network decomposition approach to reduce expensive reactive transshipments	Large	Stochastic optimization approach and heuristic	Periodic	Reactive	Lost sales	Yes

allow online retailers to hold their own inventory, Flipkart acts as a platform that connects buyers and sellers. Through their robust distribution network, Flipkart ships the items from sellers to their FCs and based on customer orders, fulfill them from their FCs, mostly from customers' nearby centers. As long as they have an item available at any FC, the website does not show out-of-stock status. Based on their historical data and a better understanding of customer preferences, Flipkart inventory allocation is robust and efficient. However, the website's product availability status means that Flipkart has to fulfill the order upon request from any FC in their network, resulting in transshipments. They use both proactive and reactive transshipments. Reactive transshipments mostly meet immediate shortages. Flipkart has large FCs near the metropolitan cities and smaller FCs across the tier 2 and tier 3 cities. The larger FCs offer more product variety along with higher inventory levels. Also, during shortages, these large fulfillment centers fulfill orders in distant regions since immediate transportation from sellers on short notice is expensive and may not always be possible. Flipkart's five most popular orders are for single item products.

Amazon, another major retailer, has a significant market share in the Indian online retail landscape; however, their FCs are mostly located in tier 1 cities. On the other hand, in the US, Amazon is the biggest e-commerce player having its presence across the country. In the US, e-commerce firms can have their own inventory. Amazon sells its own products along with vendor-supplied products. The inventory practice of Amazon closely resembles our work in two areas: (i)40 percent of the products sold via Amazon are their own brands, and therefore, can be categorized as single supplier products, and (ii) Amazon also follows a hub-and-spoke system with a combination of large and small FCs. For example, the Moreno Valley FC in California is the largest among the 14 FCs in the California region for Amazon. It is one of the largest and crucial FCs for Amazon in terms of both shipment and employment, supplying (shipping and transshipping) a large population of California. Any supply shortage in nearby centers is likely redirected to the larger warehouse.

The modeling assumptions stated below are based on our understanding of the operations of the two largest e-commerce retailers and the existing literature. In Section 2.3.1, we analyze a network of two FCs serving two customer regions, which include the cases of equal and unequal demand variances. Next, in Section 2.3.2, we extend our analysis to three fulfillment centers configured as a hub-spoke system, serving three customer regions. We construct a generalized model in Section 2.4 and apply the analytical insights from this section.

Assumptions

- We consider customers that place a single-item order and do not return the item. Return management is not the focus of this chapter. However, the single item orders are quite common for online retailers. Many of the popular products available on Flipkart are single item orders.

- The fulfillment centers periodically replenish the single item by a single supplier at the same per unit cost. The single supplier assumption exists in the literature (Archibald et al. 2010). The number and location of FCs are predetermined. Many papers consider simultaneous replenishments for retail networks under periodic review policy (e.g. Cao and Silver 2005, Herer et al. 2006, Archibald et al. 2009).

- The online retailer follows a periodic review joint replenishment policy with identical review period T for all FCs. Several papers measure base-stock levels in a periodic review environment (e.g. Robinson 1990, Herer et al. 2006). The supply lead time from the supplier to any FC, L is a fixed constant and is known. FCs across the network are assumed to have the same review period for convenience.

- The model includes lost sales; we penalize the firm for not meeting the realized demand by a per unit missed demand cost. However, as noted by Nobel and van der Heeden. (2000), “The lost sales model is intrinsically more difficult..., and no exact results have been reported yet." Lee and Hong (2003) demonstrate the expression for the total cost per unit of time but comment that it is an intractable task. One approach is to associate lost sales with a service level requirement, with the aim of achieving a satisfactory compromise between holding cost and customer service (Faaland et al. 2019). Additionally, our discussions with the practitioners reveal that the estimated penalty is the loss of revenue (product price) the sale would have provided. One common notion with lost sales is the loss of customer goodwill or their repurchase intention. However, the existing literature shows mixed evidence; while Kim and Lennon (2011) use experiments to show that stock-outs cause consumers to experience negative emotions, which adversely affect the retailer's image and repurchase decisions. Dadzie et al. (2005) do not find any significant impact of stock-outs on customer decisions for books, clothes, and shoes. Firms often use price promotions to lower consumer dissatisfaction due to stock-outs. Additionally, most online retailers do not display out-of-stock items. In order to develop tractable analysis we initially fix the service level in our model. However, later in Section 2.7, we analyze the impact of service level on the lost sales.

For Flipkart, the penalty cost is not direct since it does not own any inventory. Flipkart shares a profit percentage for each sale. However, every unmet demand causes a seller to lose a selling opportunity, and Flipkart loses its profit percentage. Note Flipkart works with sellers to create demand forecasts and forecast errors hurt sellers and Flipkart. In that sense, our lost sales assumption seems reasonable for both markets.

- We do not consider backordering. Glazebrook et al. (2015) show that the lost sales and backordered sales models in their work with multiple stocking locations produce comparable results. Online retailers do not use backordering since online customers do not wait to switch to another platform for their desired products. Thus, we include only the lost sales cost component in our model.

- Any FC can ship an item to any customer location. However, the cost may vary for shipment or transshipment. Flipkart and Amazon both show items on their website as long as they are available at warehouses in their markets.

- We assume that shipment and transshipment costs are exogenously given.

- We model consumer demand using a normal distribution and assume independent demand at all FCs. We discuss it later in this section.

- System-wide demand over the total on-hand inventory across all FCs is unmet. We do not consider supplier fulfillment. This assumption is true, especially for Flipkart. The sellers are small to medium enterprises that do not possess efficient logistics capabilities. Our focus is not supplier fulfillment.

- Transshipments occur from one FC having a surplus to another FC facing a stock out.

2.3.1 Two Fulfillment Centers and Two Regions

Let us consider two FCs facing independent and stochastic demand for a specific item. Two facilities FC_1 and FC_2 primarily serve two customer regions, R_1 and R_2, respectively. If FC_1 (FC_2) is unable to meet the demand of R_1 (R_2), FC_2 (FC_1) fulfills the unmet demand of R_1 (R_2) if it has a surplus after serving the demand of its local region. We assume that the per unit outbound shipping cost is higher if a region's local FC fails to meet the demand. Due to demand uncertainty, it is a common practice for FCs to keep safety stock to maintain the system wide service level requirement. While a high level of safety stock is expensive, a low level of safety stock may lead to higher transportation costs, resulting in a trade-off between transportation costs and holding costs.

Due to the random nature of demand, there may exist several possible demand scenarios (or sample paths). Each combination of demand for these two regions can affect the shipments and transshipments. Let ρ^s denote the probability of occurrence of each scenario s. The two FCs meet all demand unless there is a system-wide stock-out. As long as a product is displayed on its website, the online retailer will deliver orders for that product from any region. Any excess demand, more than the system-wide on-hand inventory, is lost. The safety stock allocation choice must be made optimally in anticipation of all possible demand scenarios. It is a one-time decision made at the beginning of a period.

Region R_1 has a mean daily demand d_1 and daily variance σ_1^2. Region R_2 has a mean daily demand d_2 and daily variance σ_2^2. Let the lead time at each facility be L and review period T. Also, let the system wide demand variance be $\sigma_0^2 = \sigma_1^2 + \sigma_2^2$ (Independence assumption). The system wide safety stock requirements is $z\sigma_0\sqrt{T+L}$. We develop our model to allocate the system wide safety stock $z\sigma_0\sqrt{T+L}$, optimally among the two FCs to minimize the total cost. Note that the system level target inventory is U_p, where $U_p = (L+T)(d_1+d_2)+z\sigma_0\sqrt{T+L}$. The system-wide order is placed in every period is $(U_p - I_1 - I_2)$, where $(I_1 + I_2)$ is the on-hand inventory at both FCs at the review period. When the order is received, it is split among the FCs such that the target inventory at FC_i is U_i, where $U_i = (L+T)d_i + \lambda_i z\sigma_0\sqrt{T+L}$, $i = 1, 2$, $\lambda_1 + \lambda_2 = 1$, and λ_i is the proportion of safety stock allocated at FC_i.

An FC transships to a stocked-out FC only if the former has a surplus. As the selling period approaches, the online retailer's accuracy in predicting the demand improves. In other words, the online retailer can better predict the imminent stock-out and excess inventory across the network. Therefore, during $(T + L)$, the online retailer's forecast may approximately mimic the actual demand and it can facilitate transshipments after accounting for the local demand. Next, we define our model. The parameters and variables are listed in Table 2.2.

Table 2.2: Parameters and variables

Parameters:	
c_p	Shipping cost per unit from the FC closest to the customer
c_s	Cost of transshipping one unit, which is the shipping cost from the distant (i.e., secondary) FC to customer location ($c_s > c_p$)
d_1	Mean daily demand of region R_1
d_2	Mean daily demand of region R_2
σ_1^2	Daily variance of demand at region R_1
σ_2^2	Daily variance of demand at region R_2
σ_0^2	System wide demand variance σ_0^2, where $\sigma_0^2 = \sigma_1^2 + \sigma_2^2$
T	Review time at each facility
L	Lead time at each facility
h	Holding cost at each facility
g	Penalty cost for each lost demand
Variables:	
X_1^s	Units shipped from FC_1 to R_1 for scenario s
X_2^s	Units shipped from FC_2 to R_2 for scenario s
X_{12}^s	Units shipped from FC_1 to R_2 for scenario s
X_{21}^s	Units shipped from FC_2 to R_1 for scenario s
Y_1^s	Lost sales for region 1 for scenario s
Y_2^s	Lost sales for region 2 for scenario s
e_1^s	Realized demand at FC_1 under scenario s
e_2^s	Realized demand at FC_2 under scenario s
λ_1	Proportion of safety stock allocated at FC_1
λ_2	Proportion of safety stock allocated at FC_2

The firm incurs a cost of c_p to ship each product to a customer from the nearest facility. If it ships from a distant FC, the firm incurs a per unit transshipment cost c_s, where $c_p < c_s$. Holding cost for each unit at either FC is h. We assume that any demand not satisfied is lost and a penalty cost of g is incurred for each unit of unmet demand. We assume identical holding and penalty costs across the FCs, similar to Glazebrook et al. (2015).

We use a normal distribution for customer demand with 99.72% of the demands within three standard deviations from the mean. We explore 7 different demand realizations at each FC: $d-3\sigma$, $d-2\sigma$, $d-\sigma$, d, $d+\sigma$, $d+2\sigma$, and $d+3\sigma$, for a total of 49 scenarios (i.e., sample paths, e_i^s). The objective is to find the optimal allocation of safety stock that minimizes the expected cost over all scenarios. e_1^s and e_2^s are the realized demands at the two FCs for scenario s.

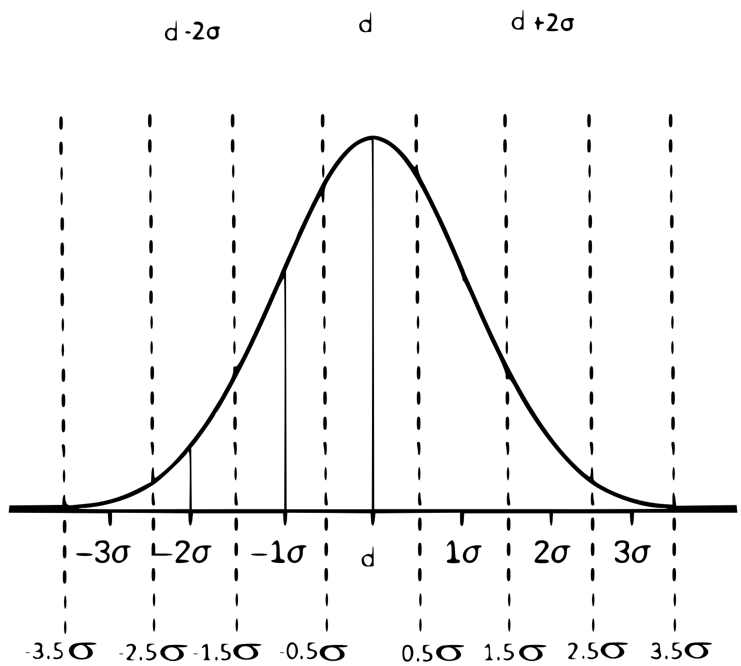


Figure 2.1: Normal distribution up to $\pm 3\sigma$

The seven points noted above provide a reasonable approximation of normal distribution covering its entire region to a large extent. Normal distribution is continuous; however, we discretize the distribution at seven points as mentioned above. We assume that the occurrence of demand between d+0.5σ and d-0.5σ is represented by the value of d (the mean). The area between d+0.5σ

and d-0.5σ in the normal table is the probability of occurrence of demand d. The probability of occurrence of other six demand values ($d-3\sigma$, $d-2\sigma$, $d-\sigma$, $d+\sigma$, $d+2\sigma$, and $d+3\sigma$), are shown in Figure 2.1.

Now, let us consider that two FCs, FC_1 and FC_2, ship X_1^s and X_2^s to regions R_1 and R_2, respectively for each scenario s. For any excess demand, FC_1 and FC_2 ship X_{12}^s and X_{21}^s to regions R_1 and R_2, respectively, at a unit cost of c_s. Unmet demands, Y_1^s and Y_2^s are penalized at a unit cost of g for each scenario s. Decision variables are λ_1 and λ_2, which are the proportions of safety stock deployed at FC_1 and FC_2, respectively. The objective function is the sum of transportation cost, inventory holding cost and lost sales cost for all s sample paths. In the formulation below, K_1 = 49.

Problem MIP_{2FC}:

$$\textbf{Min}\ \Psi_1 = \lambda_1 z\sqrt{T+L}\sigma_0 h + \lambda_2 z\sqrt{T+L}\sigma_o h + \sum_{s=1}^{K_1}\rho^s\pi^s$$

Subject to:

$$\pi^s = (X_1^s+X_2^s)c_p + (X_{12}^s+X_{21}^s)c_s + (Y_1^s+Y_2^s)g, \quad s=1,\ldots,K_1 \tag{2.1}$$

$$X_1^s + X_{12}^s \leq (L+T)d_1 + \lambda_1 z\sqrt{T+L}\sigma_0, \quad s=1,\ldots,K_1 \tag{2.2}$$

$$X_2^s + X_{21}^s \leq (L+T)d_2 + \lambda_2 z\sqrt{T+L}\sigma_0, \quad s=1,\ldots,K_1 \tag{2.3}$$

$$Y_1^s \geq e_1^s - X_1^s - X_{21}^s, \quad s=1,\ldots,K_1 \tag{2.4}$$

$$Y_2^s \geq e_2^s - X_2^s - X_{12}^s, \quad s=1,\ldots,K_1 \tag{2.5}$$

$$\lambda_1 + \lambda_2 = 1 \tag{2.6}$$

$$X_1^s, X_{21}^s, X_2^s, X_{12}^s, Y_1^s, Y_2^s: \ \text{Integer variables}, \quad s=1,2,\ldots,K_1 \tag{2.7}$$

$$\lambda_1, \lambda_2 \geq 0 \tag{2.8}$$

The objective function minimizes the total costs, consisting of transportation, holding, and penalty costs for all sample paths. Constraint set (2.1) represents the total cost of transportation and lost sale for each sample path. Constraint sets (2.2) and (2.3) limit the shipments from FC_1 and FC_2 to the on-hand inventories in their respective locations. That is, none of the two FCs can ship or transship more than their inventories. The lost sales are estimated in Constraint

Table 2.3: Different demand situations (two FC case)

Sample path(s)	e_1^s	Probability of e_1^s	e_2^s	Probability of e_2^s	ρ^s
1	$\mu^2 d_1$-$\mu\sigma_1$	0.242	$\mu^2 d_2$	0.383	0.092686
2	$\mu^2 d_1$-2$\mu\sigma_1$	0.061	$\mu^2 d_2$	0.383	0.023363
3	$\mu^2 d_1$-3$\mu\sigma_1$	0.006	$\mu^2 d_2$	0.383	0.002298
4	$\mu^2 d_1$	0.383	$\mu^2 d_2$	0.383	0.146689
5	$\mu^2 d_1$+$\mu\sigma_1$	0.242	$\mu^2 d_2$	0.383	0.092686
6	$\mu^2 d_1$+2$\mu\sigma_1$	0.061	$\mu^2 d_2$	0.383	0.023363
7	$\mu^2 d_1$+3$\mu\sigma_1$	0.006	$\mu^2 d_2$	0.383	0.002298
8	$\mu^2 d_1$-$\mu\sigma_1$	0.242	$\mu^2 d_2$-$\mu\sigma_2$	0.242	0.058564
9	$\mu^2 d_1$-2$\mu\sigma_1$	0.061	$\mu^2 d_2$-$\mu\sigma_2$	0.242	0.014762
10	$\mu^2 d_1$-3$\mu\sigma_1$	0.006	$\mu^2 d_2$-$\mu\sigma_2$	0.242	0.001452
11	$\mu^2 d_1$	0.383	$\mu^2 d_2$-$\mu\sigma_1$	0.242	0.092686
12	$\mu^2 d_1$+$\mu\sigma_1$	0.242	$\mu^2 d_2$-$\mu\sigma_2$	0.242	0.058564
13	$\mu^2 d_1$+2$\mu\sigma_1$	0.061	$\mu^2 d_2$-$\mu\sigma_2$	0.242	0.014762
14	$\mu^2 d_1$+3$\mu\sigma_1$	0.006	$\mu^2 d_2$-$\mu\sigma_2$	0.242	0.001452
15	$\mu^2 d_1$-$\mu\sigma_1$	0.242	$\mu^2 d_2$-2$\mu\sigma_2$	0.061	0.014762
16	$\mu^2 d_1$-2$\mu\sigma_1$	0.061	$\mu^2 d_2$-2$\mu\sigma_2$	0.061	0.003721
17	$\mu^2 d_1$-3$\mu\sigma_1$	0.006	$\mu^2 d_2$-2$\mu\sigma_2$	0.061	0.000366
18	$\mu^2 d_1$	0.383	$\mu^2 d_2$-2$\mu\sigma_2$	0.061	0.023363
19	$\mu^2 d_1$+$\mu\sigma_1$	0.242	$\mu^2 d_2$-2$\mu\sigma_2$	0.061	0.014762
20	$\mu^2 d_1$+2$\mu\sigma_1$	0.061	$\mu^2 d_2$-2$\mu\sigma_2$	0.061	0.003721
21	$\mu^2 d_1$+3$\mu\sigma_1$	0.006	$\mu^2 d_2$-2$\mu\sigma_2$	0.061	0.000366
22	$\mu^2 d_1$-$\mu\sigma_1$	0.242	$\mu^2 d_2$-3$\mu\sigma_2$	0.006	0.001452
23	$\mu^2 d_1$-2$\mu\sigma_1$	0.061	$\mu^2 d_2$-3$\mu\sigma_2$	0.006	0.000366
24	$\mu^2 d_1$-3$\mu\sigma_1$	0.006	$\mu^2 d_2$-3$\mu\sigma_2$	0.006	0.000036
25	$\mu^2 d_1$	0.383	$\mu^2 d_2$-3$\mu\sigma_2$	0.006	0.002298
26	$\mu^2 d_1$+$\mu\sigma_1$	0.242	$\mu^2 d_2$-3$\mu\sigma_2$	0.006	0.001452
27	$\mu^2 d_1$+2$\mu\sigma_1$	0.061	$\mu^2 d_2$-3$\mu\sigma_2$	0.006	0.000366
28	$\mu^2 d_1$+3$\mu\sigma_1$	0.006	$\mu^2 d_2$-3$\mu\sigma_2$	0.006	0.000036
29	$\mu^2 d_1$-$\mu\sigma_1$	0.242	$\mu^2 d_2$+$\mu\sigma_2$	0.242	0.058564
30	$\mu^2 d_1$-2$\mu\sigma_1$	0.061	$\mu^2 d_2$+$\mu\sigma_2$	0.242	0.014762
31	$\mu^2 d_1$-3$\mu\sigma_1$	0.006	$\mu^2 d_2$+$\mu\sigma_2$	0.242	0.001452
32	$\mu^2 d_1$	0.383	$\mu^2 d_2$+$\mu\sigma_2$	0.242	0.092686
33	$\mu^2 d_1$+$\mu\sigma_1$	0.242	$\mu^2 d_2$+$\mu\sigma_2$	0.242	0.058564
34	$\mu^2 d_1$+2$\mu\sigma_1$	0.061	$\mu^2 d_2$+$\mu\sigma_2$	0.242	0.014762
35	$\mu^2 d_1$+3$\mu\sigma_1$	0.006	$\mu^2 d_2$+$\mu\sigma_2$	0.242	0.001452
36	$\mu^2 d_1$-$\mu\sigma_1$	0.242	$\mu^2 d_2$+2$\mu\sigma_2$	0.061	0.014762
37	$\mu^2 d_1$-2$\mu\sigma_1$	0.061	$\mu^2 d_2$+2$\mu\sigma_2$	0.061	0.003721
38	$\mu^2 d_1$-3$\mu\sigma_1$	0.006	$\mu^2 d_2$+2$\mu\sigma_2$	0.061	0.000366
39	$\mu^2 d_1$	0.383	$\mu^2 d_2$+2$\mu\sigma_2$	0.061	0.023363
40	$\mu^2 d_1$+$\mu\sigma_1$	0.242	$\mu^2 d_2$+2$\mu\sigma_2$	0.061	0.014762
41	$\mu^2 d_1$+2$\mu\sigma_1$	0.061	$\mu^2 d_2$+2$\mu\sigma_2$	0.061	0.003721
42	$\mu^2 d_1$+3$\mu\sigma_1$	0.006	$\mu^2 d_2$+2$\mu\sigma_2$	0.061	0.000366
43	$\mu^2 d_1$-$\mu\sigma_1$	0.242	$\mu^2 d_2$+3$\mu\sigma_2$	0.006	0.001452
44	$\mu^2 d_1$-2$\mu\sigma_1$	0.061	$\mu^2 d_2$+3$\mu\sigma_2$	0.006	0.000366
45	$\mu^2 d_1$-3$\mu\sigma_1$	0.006	$\mu^2 d_2$+3$\mu\sigma_2$	0.006	0.000036
46	$\mu^2 d_1$	0.383	$\mu^2 d_2$+3$\mu\sigma_2$	0.006	0.002298
47	$\mu^2 d_1$+$\mu\sigma_1$	0.242	$\mu^2 d_2$+3$\mu\sigma_2$	0.006	0.001452
48	$\mu^2 d_1$+2$\mu\sigma_1$	0.061	$\mu^2 d_2$+3$\mu\sigma_2$	0.006	0.000366
49	$\mu^2 d_1$+3$\mu\sigma_1$	0.006	$\mu^2 d_2$+3$\mu\sigma_2$	0.006	0.000036

Table 2.4: Decision variables for Problem MIP_{2FC} when $z = \sqrt{2}$ (two FC case)

s	e_1^s	X_{12}^s	e_2^s	X_{21}^s	$Y_1^s + Y_2^s$
1	$\mu^2 d_1$-$\mu\sigma$	0	$\mu^2 d_2$	0	0
2	$\mu^2 d_1$-2$\mu\sigma$	0	$\mu^2 d_2$	0	0
3	$\mu^2 d_1$-3$\mu\sigma$	0	$\mu^2 d_2$	0	0
4	$\mu^2 d_1$	0	$\mu^2 d_2$	0	0
5	$\mu^2 d_1$+$\mu\sigma$	0	$\mu^2 d_2$	0	0
6	$\mu^2 d_1$+2$\mu\sigma$	0	$\mu^2 d_2$	$(1-2\delta)\mu\sigma$	0
7	$\mu^2 d_1$+3$\mu\sigma$	0	$\mu^2 d_2$	$(1-2\delta)\mu\sigma$	$\mu\sigma + 0$
8	$\mu^2 d_1$-$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
9	$\mu^2 d_1$-2$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
10	$\mu^2 d_1$-3$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
11	$\mu^2 d_1$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
12	$\mu^2 d_1$+$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
13	$\mu^2 d_1$+2$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$(1-2\delta)\mu\sigma$	0
14	$\mu^2 d_1$+3$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$(2-2\delta)\mu\sigma$	0
15	$\mu^2 d_1$-$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
16	$\mu^2 d_1$-2$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
17	$\mu^2 d_1$-3$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
18	$\mu^2 d_1$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
19	$\mu^2 d_1$+$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
20	$\mu^2 d_1$+2$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	$(1-2\delta)\mu\sigma$	0
21	$\mu^2 d_1$+3$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	$(2-2\delta)\mu\sigma$	0
22	$\mu^2 d_1$-$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
23	$\mu^2 d_1$-2$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
24	$\mu^2 d_1$-3$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
25	$\mu^2 d_1$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
26	$\mu^2 d_1$+$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
27	$\mu^2 d_1$+2$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	$(1-2\delta)\mu\sigma$	0
28	$\mu^2 d_1$+3$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	$(2-2\delta)\mu\sigma$	0
29	$\mu^2 d_1$-$\mu\sigma$	$2\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
30	$\mu^2 d_1$-2$\mu\sigma$	$2\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
31	$\mu^2 d_1$-3$\mu\sigma$	$2\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
32	$\mu^2 d_1$	$2\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
33	$\mu^2 d_1$+$\mu\sigma$	$2\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
34	$\mu^2 d_1$+2$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	$(1-2\delta)\mu\sigma + 2\delta\mu\sigma$
35	$\mu^2 d_1$+3$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	$(2-2\delta)\mu\sigma + 2\delta\mu\sigma$
36	$\mu^2 d_1$-$\mu\sigma$	$(1+2\delta)\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
37	$\mu^2 d_1$-2$\mu\sigma$	$(1+2\delta)\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
38	$\mu^2 d_1$-3$\mu\sigma$	$(1+2\delta)\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
39	$\mu^2 d_1$	$(1+2\delta)\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
40	$\mu^2 d_1$+$\mu\sigma$	$(2\delta)\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	$0 + \mu\sigma$
41	$\mu^2 d_1$+2$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	$(1-2\delta)\mu\sigma + (1+2\delta)\mu\sigma$
42	$\mu^2 d_1$+3$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	$(2-2\delta)\mu\sigma + (1+2\delta)\mu\sigma$
43	$\mu^2 d_1$-$\mu\sigma$	$(2+2\delta)\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
44	$\mu^2 d_1$-2$\mu\sigma$	$(2+2\delta)\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
45	$\mu^2 d_1$-3$\mu\sigma$	$(2+2\delta)\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
46	$\mu^2 d_1$	$(1+2\delta)\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	$0 + \mu\sigma$
47	$\mu^2 d_1$+$\mu\sigma$	$(2\delta)\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	$0 + 2\mu\sigma$
48	$\mu^2 d_1$+2$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	$(1-2\delta)\mu\sigma + (2+2\delta)\mu\sigma$
49	$\mu^2 d_1$+3$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	$(2-2\delta)\mu\sigma + (2+2\delta)\mu\sigma$

sets (2.4) and (2.5). The sum of the two allocation parameters, λ_1 and λ_2 is normalized to 1 in Constraint (2.6). Constraint sets (2.7) and (2.8) provide the nonnegativity constraints. Note that λ_1 and λ_2 allocate the safety stock at FC_1 and FC_2, respectively. Problem MIP_{2FC} can be solved optimally using CPLEX.

2.3.1.1 Equal Variance

First, we consider a special case where both regions have the same variance σ^2. The probabilities of the occurrence of each scenario (ρ^s) are described in Table 2.3. In total, we have 49 sample paths with different probabilities. Each sample path contains the costs associated with shipments, transshipments, safety stock holding, and penalty costs. Note that Constraint set (2.2) quantifies the total cost for each sample path in the previous objective function and also in this special case. We derive the optimal solution of safety stock allocation in the following Lemma.

Lemma 1. *For any fixed service level no less than 50% and $c_s > c_p$, the optimal safety stock allocation is $\lambda_1 = \lambda_2 = 0.5$ for the two FCs case with equal standard deviation, σ.*

Proof: Since $\lambda_1 + \lambda_2 = 1$, we may assume that λ_1 = 0.5 + δ and λ_2 = 0.5 - δ, where δ is in [0, 0.5]. Note that $\sigma_1 = \sigma_2 = \sigma$. The total inventory at FC_1 and FC_2 are $(L+T)d_1$+$z\sqrt{L+T}(0.5+\delta)\sigma_0$ and $(L+T)d_2$+$z\sqrt{L+T}(0.5-\delta)\sigma_0$, respectively, where $\sigma_0 = \sqrt{2}\sigma$. Let us also denote $\sqrt{L+T}$ = μ and $L+T$ = μ^2. Note that

Total system inventory is $\mu^2(d_1+d_2)$ + $z\sqrt{2}\mu\sigma$.

Inventory at FC_1 is $\mu^2 d_1$ + $z\sqrt{2}\mu\sigma(0.5+\delta)$.

Inventory at FC_2 is $\mu^2 d_2$ + $z\sqrt{2}\mu\sigma(0.5-\delta)$.

For ease of exposition, we fix the service level to 92% such that $z = \sqrt{2}$. However, the proof can easily be generalized to any service level no less than 50%. With the updated z parameter, we can rewrite the inventory levels for the system and FCs. The inventory levels as follows:

Total system inventory is $\mu^2(d_1+d_2)$ + $2\mu\sigma$.

Inventory at FC_1 is $\mu^2 d_1$ + $(1+2\delta)\mu\sigma$.

Inventory at FC_2 is $\mu^2 d_2$ + $(1-2\delta)\mu\sigma$.

We can rewrite the MIP_{2FC} based on these inventory levels Thus, we present the problem in the following manner.

Problem MIP_{2FC}:

$$\textbf{Min } \Psi_1 = (1+2\delta)\mu\sigma h + (1-2\delta)\mu\sigma h + \sum_{s=1}^{K_1} \rho^s \pi^s$$

Subject to:

$$\begin{aligned}
\pi^s &= (X_1^s + X_2^s)c_p + (X_{12}^s + X_{21}^s)c_s + (Y_1^s + Y_2^s)g, \quad s = 1, \ldots, K_1 \\
X_1^s + X_{12}^s &\leq \mu^2 d_1 + (1+2\delta)\mu\sigma, \quad s = 1, \ldots, K_1 \\
X_2^s + X_{21}^s &\leq \mu^2 d_2 + (1-2\delta)\mu\sigma, \quad s = 1, \ldots, K_1 \\
Y_1^s &\geq e_1^s - X_1^s - X_{21}^s, \quad s = 1, \ldots, K_1 \\
Y_2^s &\geq e_2^s - X_2^s - X_{12}^s, \quad s = 1, \ldots, K_1 \\
\lambda_1 + \lambda_2 &= 1
\end{aligned}$$

Table 2.4 provides the transshipments and lost sales decisions for each sample path s along with the realized demands. From Table 2.4, we infer the following claim.

Claim 1: The total lost sale for any sample path s, $(Y_1^s + Y_2^s)$ does not depend on the value of δ.

From the objective function Ψ_1 of MIP_{2FC}, the following claim is also obvious as holding cost is the same at two FCs.

Claim 2: The total inventory cost does not depend on the value of δ for a fixed service level $z > 0$.

Claim 3: The objective function Ψ_1 is minimized when $\delta = 0$.

This claim is proven as follows: Ψ_1 can be written as $\Psi_1 = Z_1 + Z_2$, where Z_1 is the cost component does not depend on δ and Z_2 is the cost component depends on δ.

Note that Z_1 consists of costs of lost sale, inventory holding, and other transportation costs that do not depend on δ. Z_2 consists of all transshipment costs (columns corresponding to X_{12}^s and X_{21}^s in Table 2.4) that depend on δ. Thus, Z_2 can be expressed as follows:

$Z_2 = [2\delta\mu(\rho^{29} + \rho^{30} + \rho^{31} + \rho^{32} + \rho^{33} + \rho^{36} + \rho^{37} + \rho^{38} + \rho^{39} + \rho^{40} + \rho^{43} + \rho^{44} + \rho^{45} + \rho^{46} + \rho^{47}) - 2\delta\mu(\rho^6 + \rho^7 + \rho^{13} + \rho^{14} + \rho^{20} + \rho^{21} + \rho^{27} + \rho^{28})](c_s - c_p)$,

That is,

$Z_2 = [2\delta\mu(\rho^{29} + \rho^{32} + \rho^{33} + \rho^6 + \rho^7 + 3\rho^{13} + 3\rho^{14} + \rho^{20} + 2\rho^{21} + \rho^{28}) - 2\delta\mu(\rho^6 + \rho^7 + \rho^{13} +$

$\rho^{14} + \rho^{20} + 2\rho^{21} + \rho^{28})](c_s - c_p)$.

This implies that

$Z_2 = [2\delta\mu(\rho^{29} + \rho^{32} + \rho^{33} + 2\rho^{13} + 2\rho^{14})](c_s - c_p)$.

Since $c_s > c_p$, Z_2 is minimized when $\delta = 0$.

Thus we conclude that the optimal allocation is $\lambda_1 = \lambda_2 = 0.5$ so that the expected cost is the minimum.

2.3.1.2 Unequal variances

Now, we extend Section 2.3.1.1 and consider two FCs with unequal demand variances. Let us assume that the standard deviation of demand at FC_1, σ_1=$k\sigma_2$, where k is any number. The probabilities of the occurrence of each scenario (ρ^s) are described in Table 2.3.

Theorem 1. *For any fixed service level no less than 50% and $c_s > c_p$, the optimal safety stock allocation for the two FCs where σ_1=$k\sigma_2$ follows: $\lambda_1 = \frac{k}{k+1}$ and $\lambda_2 = \frac{1}{k+1}$.*

Proof: Without loss of generality we set $\sigma_1 \geq \sigma_2 = \sigma$ and $k \geq 1$. Note that the result is true for $k = 1$ per Lemma 1. Now we first show the result for $k = 2$. Then we show the validity of the result for any $k > 2$. We start the analysis by fixing k at 2. Since $\lambda_1 + \lambda_2 = 1$, we may assume for $k = 2$ that $\lambda_1 = \frac{2}{3} + \delta$ and $\lambda_2 = \frac{1}{3} - \delta$, where δ is in [0, 1/3]. The total inventory at FC_1 and FC_2 are $(L+T)d_1$+$z\sqrt{L+T}(\frac{2}{3} + \delta)\sigma_0$ and $(L+T)d_2$+$z\sqrt{L+T}(\frac{1}{3} - \delta)\sigma_0$, respectively, where $\sigma_0 = \sqrt{5}\sigma$. Let us also denote $\sqrt{L+T}$ = μ and $L+T$ = μ^2.

For convenience we fix the service level such that $z = \sqrt{5}$. However, the proof can easily be generalized to any service level no less than 50%. Now we present the inventory levels:

Total system inventory is $\mu^2(d_1 + d_2)$ + $5\mu\sigma$.

Inventory at FC_1 is $\mu^2 d_1$ + $\mu\sigma(\frac{10}{3} + 5\delta)$.

Inventory at FC_2 is $\mu^2 d_2$ + $\mu\sigma(\frac{5}{3} - 5\delta)$.

Note that, δ is in [0, 1/3]. However, there are two cases based on the value of δ: Case 1: $1/3 \geq \delta \geq 2/15$; Case 2: $0 \leq \delta < 2/15$. In Case 1, FC_1 can satisfy demand upto 2 one standard deviations and FC_2 can satisfy mean demand. In contrast, under Case 2, FC_1 and FC_2 both can satisfy demand upto 1 one standard deviation. We show the resulting transshipments and lost sales in Table A.1 and Table A.2 in the Appendix.

Similar to the approach taken in Section 2.3.1.1, we extend the constraint of Problem MIP_{2FC} as follows:

$$\textbf{Min } \Psi_1 = (\frac{10}{3} + 5\delta)\mu\sigma h + (\frac{5}{3} - 5\delta)\mu\sigma h + \sum_{s=1}^{K_1} \rho^s \pi^s$$

Subject to:

$$\begin{aligned}
\pi^s &= (X_1^s + X_2^s)c_p + (X_{12}^s + X_{21}^s)c_s + (Y_1^s + Y_2^s)g, \quad s = 1, \ldots, K_1 \\
X_1^s + X_{12}^s &\leq \mu^2 d_1 + (\frac{10}{3} + 5\delta)\mu\sigma, \quad s = 1, \ldots, K_1 \\
X_2^s + X_{21}^s &\leq \mu^2 d_2 + (\frac{5}{3} - 5\delta)\mu\sigma, \quad s = 1, \ldots, K_1 \\
Y_1^s &\geq e_1^s - X_1^s - X_{21}^s, \quad s = 1, \ldots, K_1 \\
Y_2^s &\geq e_2^s - X_2^s - X_{12}^s, \quad s = 1, \ldots, K_1 \\
\lambda_1 + \lambda_2 &= 1
\end{aligned}$$

We exhibit the realized demands, transshipments, and lost sales in Table A.1 and Table A.2. From these two tables, the following claim is obvious.

Claim 1: The total lost sales for any sample path s, $(Y_1^s + Y_2^s)$ does not depend on the value of δ. It is because the total system-wide lost sales do not change with safety stock allocation. This is true for both cases 1 and 2.

From the objective function Ψ_1 of MIP_{2FC}, the following claim is also obvious as holding cost is the same at two FCs.

Claim 2: The total inventory cost does not depend on the value of δ.

Claim 3: The objective function Ψ_1 is minimized when $\delta = 0$.

This claim is proven as follows: Ψ_1 can be written as $\Psi_1 = Z_1 + Z_2$, where Z_1 is the cost component that does not depend on δ and Z_2 is the cost component that depends on δ.

Case 1: As in Lemma 1, we show from Table A.1 that Z_2 can be expressed as follows:

$Z_2 = [5\delta\mu(\rho^{29} + \rho^{30} + \rho^{31} + \rho^{32} + \rho^{33} + \rho^{34} + \rho^{36} + \rho^{37} + \rho^{38} + \rho^{39} + \rho^{40} + \rho^{41} + \rho^{43} + \rho^{44} + \rho^{45} + \rho^{46} + \rho^{47} + \rho^{48}) - 5\delta\mu(\rho^7 + \rho^{14} + \rho^{21} + \rho^{28})](c_s - c_p)$.

That is, $Z_2 = [5\delta\mu(\rho^{29} + \rho^{30} + \rho^{31} + \rho^{32} + \rho^{33} + \rho^{34} + \rho^{36} + \rho^{37} + \rho^{39} + \rho^{40} + \rho^{41} + \rho^{43} + \rho^{44} + \rho^{48})](c_s - c_p)$.

Since $c_s > c_p$, Z_2 is minimized when $\delta = 0$.

Case 2: As in Lemma 1, we show from Table A.2 that Z_2 can be expressed as follows:

$Z_2 = [5\delta\mu(\rho^{36} + \rho^{37} + \rho^{38} + \rho^{39} + \rho^{40} + \rho^{43} + \rho^{44} + \rho^{45} + \rho^{46} + \rho^{47}) - 5\delta\mu(\rho^{6} + \rho^{7} + \rho^{13} + \rho^{14} + \rho^{20} + \rho^{21} + \rho^{27} + \rho^{28} + \rho^{34} + \rho^{35})](c_s - c_p)$.

That is, $Z_2 = [5\delta\mu(\rho^{36} + \rho^{37} + \rho^{38} + \rho^{39} + \rho^{40} + \rho^{43} + \rho^{44} + \rho^{45} + \rho^{46} + \rho^{47}) - 5\delta\mu(\rho^{6} + \rho^{7} + 2\rho^{13} + 2\rho^{14} + \rho^{20} + 2\rho^{21} + \rho^{28})](c_s - c_p)$.

As $(\rho^{36} + \rho^{37} + \rho^{38} + \rho^{39} + \rho^{40} + \rho^{43} + \rho^{44} + \rho^{45} + \rho^{46} + \rho^{47}) = (\rho^{6} + \rho^{7} + 2\rho^{13} + 2\rho^{14} + \rho^{20} + 2\rho^{21} + \rho^{28})$, we have $Z_2 = 0$ under Case 2, $0 \leq \delta < 2/15$. Thus we may set $\delta = 0$.

The proof is similar for $k = 2$ with $\lambda_1 = \frac{2}{3} - \delta$ and $\lambda_2 = \frac{1}{3} + \delta$, where δ is in [0, 2/3].

Thus we conclude that the optimal safety stock allocation is λ_1 =kλ_2 where σ_1=$k\sigma_2$ under the condition that k=2.

General k: So far, we show that the result hold for $k = 1$ and $k = 2$. We now show that it will be true for general k.

For σ_1=$k\sigma_2$, where $\sigma_2 = \sigma$:

Total system inventory is $\mu^2(d_1 + d_2)$ + $z\mu\sigma\sqrt{k^2+1}$.

For convenient, we assume the service level, z = (k+1)/$\sqrt{k^2+1}$.

Case 1: $0 \leq \delta \leq \frac{1}{k+1}$. We assume that $\lambda_1 = \frac{k}{k+1} + \delta$ and $\lambda_2 = \frac{1}{k+1} - \delta$, where δ is in $[0, \frac{1}{k+1}]$.

Inventory at FC_1 is $\mu^2 d_1$ + $k\mu\sigma$ + $(k+1)\delta\mu\sigma$.

Inventory at FC_2 is $\mu^2 d_2$ + $\mu\sigma$ - $(k+1)\delta\mu\sigma$.

Similar to the proof for $k = 1$ and $k = 2$, we show below the optimal allocation where δ = 0.

For this allocation, we can show that FC_1 can always serve a demand upto one standard deviation. However, as δ increases from 0 to $\frac{1}{k+1}$, FC_1 stores more inventory. For, δ = 0, it has no safety stock after one serving one standard deviation of demand. On the other hand, δ = $\frac{1}{k+1}$ yields an extra $\mu\sigma$ safety stock for FC_1. Under this condition, FC_2 has no safety stock; it can only serve the mean demand. However, with δ = 0, it can serve a demand up to one standard deviation. Therefore, we clearly have two cases: (i) δ = 0 and (ii) $\delta > 0$. Note that, in either case, the total system inventory and lost sales remain the same; only transshipments between the FCs change due to the change in their respective inventories.

Case 2: $0 \leq \delta \leq \frac{k}{k+1}$. We assume that $\lambda_1 = \frac{k}{k+1} - \delta$ and $\lambda_2 = \frac{1}{k+1} + \delta$, where δ is in $[0, \frac{k}{k+1}]$.

Total system inventory remains the same. We allocate (kλ - δ) proportion to FC_1 and (λ + δ) proportion to FC_2.

Inventory at FC_1 is $\mu^2 d_1$ + $k\mu\sigma$ - $(k+1)\delta\mu\sigma$.

Inventory at FC_2 is $\mu^2 d_2$ + $\mu\sigma$ + $(k+1)\delta\mu\sigma$.

Similar to above procedure, we can show the optimal allocation where δ = 0.

Again, we have two cases: (i) δ = 0 where FC_1 and FC_2 both can serve demand upto one standard deviation with results are exactly similar to Table A.3 and (ii) $0 < \delta \leq \frac{k}{k+1}$. Under (ii), FC_2 can serve demand more than one standard deviation and FC_1 can serve only the mean demand. In Table A.5, we list the results. Now, FC_2 can serve demands upto 2 and 3 standard deviations under the conditions $\delta \geq$ 1/(k+1) and $\delta \geq$ 2/(k+1). Both conditions are favorable following our assumption that k $\geq$ 2. With $\delta \geq$ 2/(k+1), FC_2 never faces a stock-out. Note that the changes in δ can be explained by the sample paths 36-49. Thus, we list only the sample paths between 36-49 corresponding to the deltas in Table A.6, Table A.7, and Table A.8. Note that Table A.5 does not include the sample paths between 36 and 49.

We can simplify the expressions from Table A.6, Table A.7, and Table A.8 and show that the total transshipments under cases 2a, 2b, and 2c are:

Transshipments under case 2a: $21\mu\sigma + 6k\mu\sigma - 8(k+1)\delta\mu\sigma$.

Transshipments under case 2b: $14\mu\sigma + 6k\mu\sigma - (k+1)\delta\mu\sigma$.

Transshipments under case 2c: $2\mu\sigma + 6k\mu\sigma + 5(k+1)\delta\mu\sigma$.

With the sample path probabilities for each sample paths between 36-49 being the same and unit transshipment costs are equal, we can claim that the total costs are increasing with δ. In other words, case 2a exhibits the least cost among the three cases under the boundary conditions of δ. Now, we can just combine Table A.6 with the sample paths in Table A.5 and compare them with the results for δ = 0.

Table A.3 yields the expression $0.122\mu\sigma[c_s - c_p]$ for $\delta = 0$. By combining the results for the sample paths across Tables 16 and 17, we get $(0.122\mu\sigma + 0.21(k+1)\delta\mu\sigma\ )[c_s - c_p]$. Clearly, we get the lowest cost under the condition that $\delta = 0$. Our proof is complete for case 2. When we look at case 1 (Table A.4), we find the expression for transshipments as $(0.122\mu\sigma + 0.242(k+1)\delta\mu\sigma\ )[c_s - c_p]$, which is more than $0.122\mu\sigma[c_s - c_p]$.

Therefore, we prove Theorem 1 and conclude that the optimal allocation of safety stock for the two FCs where σ_1=$k\sigma_2$ follows: $\lambda_1 = \frac{k}{k+1}$ and $\lambda_2 = \frac{1}{k+1}$.

2.3.2 Pooling losses

In the previous subsection, we observe a network of two fulfillment centers. However, networks generally consist of more than two fulfillment centers and thus, the allocation problem may quickly become complex. To reduce the complexity of the problem, we propose a hub-and-spoke system as described in the next subsection. Figure 2.2 exhibits a hub-and-spoke model for a three FC network.

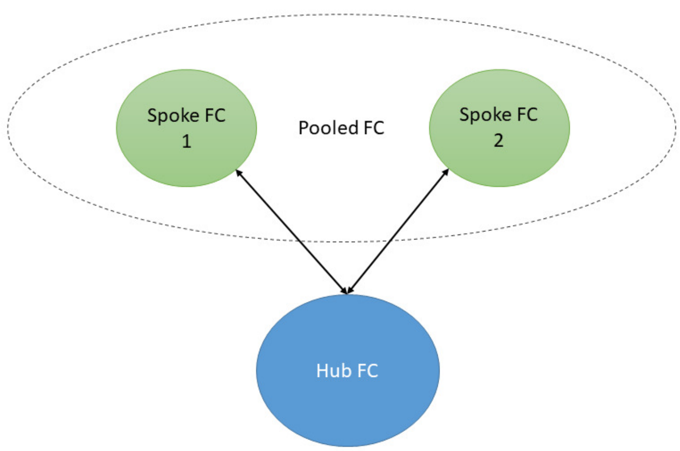


Figure 2.2: A network of three FCs

In terms of utility, a hub-and-spoke system offers several advantages. A hub-and-spoke system requires fewer main routes and allows flows in both directions. With the number of transshipments being a vital component of this chapter, the hub-and-spoke is appropriate to model consistent flows in both directions. Moreover, it reduces the number links for the same number nodes. The reader may refer to Lumsden et al. (1999) for a detailed discussion of the benefits of a hub-and-spoke system for cost reduction and flexibility. The primary motivation behind the analysis of a hub-and-spoke system is twofold: (i) common practice in the industry where few nearby FCs fulfill the demand of a customer region, and (ii) possibility for decomposition of any large network of fulfillment centers into several smaller and manageable hub-and-spoke systems.

In the next Section, we decompose a network by forming clusters to group nearby FCs and join these clusters to maintain the system connectivity. Our hub-and-spoke configuration in this subsection paves the way for the overall network decomposition followed by the assignment of safety stock. We make a few basic assumptions: (1) the distance between any two FCs within a cluster is smaller than the distance between two FCs in two different clusters, and (2) within each cluster, the leaf (or spoke) nodes are equidistant from the hub node. In other words, we assume that the transportation cost between any leaf node and the hub node is the same within each cluster. Therefore, we pool the leaf nodes within each cluster and form a single node as shown in Figure 2.2 where we pool spoke nodes 1 and 2 to form a pooled node. We reduce n leaf nodes (in a cluster) each with σ_i standard deviation to a single node with a pooled standard deviation. This approximation is possible because of our equidistant nodes assumption and the independence of demand among the FCs. In the next section, we model a large network under a MST graph by dividing the entire network into a number of linked clusters. Within each cluster, the FCs constitute a hub-and-spoke system. Therefore, the insights generated from this section guide the work in the subsequent sections.

In a hub-and-spoke system, we model one FC as the hub and pool the leaf FCs to one FC, which we call FC_f. While this simpler model is easy to study and explain, there is a trade-off associated with the simpler model. The pooling is virtual; in reality, these FCs exist as separate nodes. In the pooled network, there are two stock-out situations - either the hub or the pooled FC face stock-outs. However, in reality, each of the leaf FCs, now parts of FC_f may face stock-outs randomly. Let us illustrate this phenomenon with an example. We consider a network with three FCs - FC_1,

FC_2, and FC_3 with FC_1 being the hub FC. We pool FC_2 and FC_3 to form a pooled FC_f. As explained above, this simpler model has two stock-out situations - either the hub or the pooled FC face stock-outs. However, when we consider each FC_2 or FC_3, we can decide whether our simpler model is appropriate or not. We show that, the stock-outs at either FC_2 or FC_3, causing the transshipment costs of δ is small compared to the total costs of shipments and transshipments. We provide an MIP formulation for the three FC network. Note that the three FC model can be reduced to a hub-and-spoke model by combining FC_2 and FC_3 to a single pooled FC. Let the reduced structure with the total cost of Ψ_2 has 49 sample paths. Here, the transshipments occurs between FC_1 and the pooled FC. In the formulation below, K_2 = 343.

Problem MIP_{3FC}:

$$\textbf{Min } \Psi_3 = \lambda_1 z\sqrt{T+L}\sigma_o h + \lambda_2 z\sqrt{T+L}\sigma_o h + \lambda_3 z\sqrt{T+L}\sigma_o h + \sum_{s=1}^{K_2} \rho^s \pi^s$$

Subject to:

$$\pi^s = (X_1^s + X_2^s + X_3^s)c_p + (X_{12}^s + X_{21}^s + X_{13}^s + X_{31}^s + X_{23}^s + X_{32}^s)c_s + (Y_1^s + Y_2^s + Y_3^s)g, \quad s = 1, \ldots, K_2 \tag{2.9}$$

$$X_1^s + X_{12}^s + X_{13}^s \le (L+T)d_1 + \lambda_1 z\sqrt{T+L}\sigma_0, \quad s = 1, \ldots, K_2 \tag{2.10}$$

$$X_2^s + X_{21}^s + X_{23}^s \le (L+T)d_2 + \lambda_2 z\sqrt{T+L}\sigma_0, \quad s = 1, \ldots, K_2 \tag{2.11}$$

$$X_3^s + X_{31}^s + X_{32}^s \le (L+T)d_3 + \lambda_3 z\sqrt{T+L}\sigma_0, \quad s = 1, \ldots, K_2 \tag{2.12}$$

$$Y_1^s \ge e_1^s - X_1^s - X_{21}^s - X_{31}^s, \quad s = 1, \ldots, K_2 \tag{2.13}$$

$$Y_2^s \ge e_2^s - X_2^s - X_{12}^s - X_{32}^s, \quad s = 1, \ldots, K_2 \tag{2.14}$$

$$Y_3^s \ge e_3^s - X_3^s - X_{13}^s - X_{23}^s, \quad s = 1, \ldots, K_2 \tag{2.15}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 \tag{2.16}$$

$$X_1^s, X_2^s, X_3^s, X_{12}^s, X_{21}^s, X_{13}^s, X_{31}^s, X_{23}^s, X_{32}^s, Y_1^s, Y_2^s, Y_3^s : \text{ Integer variables}, \quad s = 1, 2, \ldots, K_2 \tag{2.17}$$

$$\lambda_1, \lambda_2, \lambda_3 \ge 0 \tag{2.18}$$

Note that the above formulation is similar to MIP_{2FC} except for one additional FC. MIP_{3FC} has three inventory equations (Constraint sets (2.10), (2.11), and (2.12)), three equations associated with lost sales (Constraint sets (2.13), (2.14), and (2.15)), and three allocation parameters, λ (Constraint sets (2.18)). This three FC configuration allows transshipments between all nodes.

Thus $\Psi_3 = \Psi_2 + \Delta$, where Δ captures the costs associated with missed transshipments cost due to pooling FC_2 and FC_3 to a single pooled node FC. Next, the following theorem provides an upper bound for Δ with respect to the total cost.

Theorem 2. *In a three fulfillment center approximate model for the hub-and-spoke system with leaf nodes having $\sigma_2 = \sigma_3 = \sigma$ and the hub node with $\sigma_1 = k\sigma$, $k \geq 2$ and with the service level, $z = \frac{2+k}{\sqrt{2+k^2}}$, and $c_s > c_p$, we have $\frac{\Delta}{\Psi_2+\Delta} \leq \frac{0.0246 c_s \mu\sigma}{\mu^2(d_1+d_2+d_3)c_p - 0.2189\mu\sigma c_p + 0.3144\mu\sigma c_s}$, where Δ is the amount of cost that the hub-and-spoke system does not capture due to approximation.*

Proof: Note that we consider a network of three FCs - FC_1 (hub FC), FC_2, and FC_3 with standard deviation of demands σ_1, σ_2, and σ_3. For the ease of calculation, let us assume that $\sigma_2 = \sigma_3 = \sigma$ and $\sigma_1 = \mathrm{k}\sigma$ where $k \geq 2$. Note that, without pooling, FC_2 and FC_3 can transship items between each other when (i) FC_1 is out-of-stock and (iii) one of FC_2 and FC_3 is out-of-stock and the other has inventory. We have a total of 343 sample paths. We need to show that the total probability of these occurrences is negligible.

Total system inventory is $\mu^2(d_1 + d_2 + d_3)$ + (k+2)$\mu\sigma$, where $z = \frac{2+k}{\sqrt{2+k^2}}$.

Inventory at FC_1 is $\mu^2 d_1$ + k$\mu\sigma$.

Inventory at FC_2 is $\mu^2 d_2$ + $\mu\sigma$.

Inventory at FC_3 is $\mu^2 d_3$ + $\mu\sigma$.

Note that, in many cases, either FC_1 or one of FC_2 and FC_3 can transship to the out-of-stock FC between FC_2 and FC_3 if two FCs have enough surplus to meet the extra demand. However, based on our problem construction, we utilize the hub FC, FC_1, to serve the extra demand in those cases. Similarly, when the hub and one of the leaf FCs are out-of-stock and the overall system observes lost sales, the remaining leaf FC with surplus inventory transships items for the out-of-stock hub FC.

Out of 343 sample paths, we find 17 sample paths where transshipments occur between FC_2 and FC_3, which are ignored due to pooling. Let us define the probabilities associated with the sample paths as ρ_j^{ik} where *i* refers to the FCs (1, 2, or 3), *j* refers to the occurrence of demand (1, 2,.. 7), and *k* refers to the sample path numbers. Thus, the probabilities associated with missed transshipments can be expressed as: $\rho_5^{1,40}\rho_6^{2,40}\rho_4^{3,40}+\rho_5^{1,47}\rho_7^{2,47}\rho_4^{3,47}+\rho_5^{1,89}\rho_6^{2,89}\rho_3^{3,89}+\rho_5^{1,96}\rho_7^{2,96}\rho_4^{3,96}$ $+\rho_5^{1,138}\rho_6^{2,138}\rho_2^{3,138}+\rho_5^{1,145}\rho_7^{2,145}\rho_2^{3,145}+\rho_5^{1,187}\rho_6^{2,187}\rho_1^{3,187}+\rho_5^{1,194}\rho_7^{2,194}\rho_1^{3,194}+\rho_5^{1,250}\rho_4^{2,150}\rho_6^{3,150}$

$+\rho_5^{1,257}\rho_3^{2,257}\rho_6^{3,257}+\rho_5^{1,264}\rho_2^{2,264}\rho_6^{3,264}+\rho_5^{1,271}\rho_1^{2,271}\rho_6^{3,271}+\rho_5^{1,299}\rho_4^{2,299}\rho_7^{3,299}+\rho_5^{1,306}\rho_3^{2,306}\rho_7^{3,306}$
$+\rho_5^{1,313}\rho_2^{2,313}\rho_7^{3,313}+\rho_5^{1,320}\rho_1^{2,320}\rho_7^{3,320}+\rho_6^{1,321}\rho_1^{2,321}\rho_7^{3,321}$.

The reader may refer to Table A.9 for the sample paths and their probabilities. In Table A.9, we exhibit the 17 sample paths where we omit the transshipments. The next step is to include the cost parameters. Let Ψ_2' that reflects all the shipment and transshipment costs under pooling model without loss sales. Thus $\Psi_2' \leq \Psi_2$.

That is, $\frac{\Delta}{\Psi_2+\Delta} \leq \frac{\Delta}{\Psi_2'+\Delta} \leq \frac{\sum_{i,j,k(k\in A)} \rho_j^{ik} M^k c_s}{\sum_{i,j,k} \rho_j^{ik}[Sp^k c_p+Tp^k c_s]+\sum_{i,j,k(k\in A)} \rho_j^{ik} M^k c_s}$

In the above inequality, *M* represents the missed transshipment volume where *Sp* and *Tp* denote the shipment and transshipment volumes under pooling. Δ is expressed in the inequality as $\sum_{i,j,k(k\in A)} \rho_j^{ik} M^k c_s$. Corresponding to all sample paths, the costs associated with these missed transshipments can be quantified to be $0.0246 C_s \mu\sigma$. Thus $\Delta = 0.0246 C_s \mu\sigma$. We show the sample paths Table A.9.

For all 343 sample paths, the total costs consisting of all shipments and transshipments in Problem MIP_{3FC} can be expressed as $\mu^2(d_1+d_2+d_3)c_p$ - 0.2189$\mu\sigma c_p$ + 0.3144$\mu\sigma C_s$. Note that $\mu^2(d_1+d_2+d_3)c_p$ - 0.2189$\mu\sigma c_p$ + 0.3144$\mu\sigma c_s = \Psi_2' + \Delta \leq \Delta + \Psi_2 = \Psi_3$ as the left hand expression does not consist of lost sales cost.

Thus, $\frac{\Delta}{\Psi_2'+\Delta} \leq \frac{0.0246 c_s \mu\sigma}{\mu^2(d_1+d_2+d_3)c_p - 0.2189\mu\sigma c_p + 0.3144\mu\sigma c_s}$. Hence the results follows as $\frac{\Delta}{\Psi_2+\Delta} \leq \frac{\Delta}{\Psi_2'+\Delta}$.

Next, we run a numerical analysis to show that the bound in Theorem 2 is very small for practically relevant scenarios. Assuming c_s = 1.5c_p, we obtain $\frac{\Delta}{\Psi_2'+\Delta} \leq \frac{0.0369 c_p \mu\sigma}{\mu^2(d_1+d_2+d_3)c_p + 0.2527\mu\sigma c_p}$. Note that the costs of missed transshipments due to pooling can be quantified as $0.0369 c_p \mu\sigma$.

Observation 1. *In Theorem 2, we prove the bound for the ratio of missed transshipment cost under pooling, Δ to the total cost Ψ_3. Our numerical analysis in Table 2.5 shows that for the test data, the ratio, $\frac{\Delta}{\Psi_2'+\Delta}$, is much smaller compared to the total costs of shipments and transshipments. This justifies the use of the approximated hub-and-spoke system.*

Table 2.5: Missed transshipments between FC_2 and FC_3 for three FC case

Sl. no	μ	d_1	d_2	d_3	σ	$\frac{\Delta}{\Psi_2'+\Delta}$ (in %)
1.	1	500	500	500	10	0.024
2.	1	500	500	500	20	0.048
3.	1	500	500	500	50	0.122
4.	1	500	500	500	100	0.242
5.	1	500	500	500	200	0.484
6.	2	500	500	500	10	0.012
7.	2	500	500	500	20	0.024
8.	2	500	500	500	50	0.122
9.	2	500	500	500	100	0.242
10.	2	500	500	500	200	0.484

2.4 Generalization of the Proposed Method

In this section, we extend the simple models discussed in the previous section to develop approaches to allocate safety stock in large networks such as those used by Amazon and Flipkart.

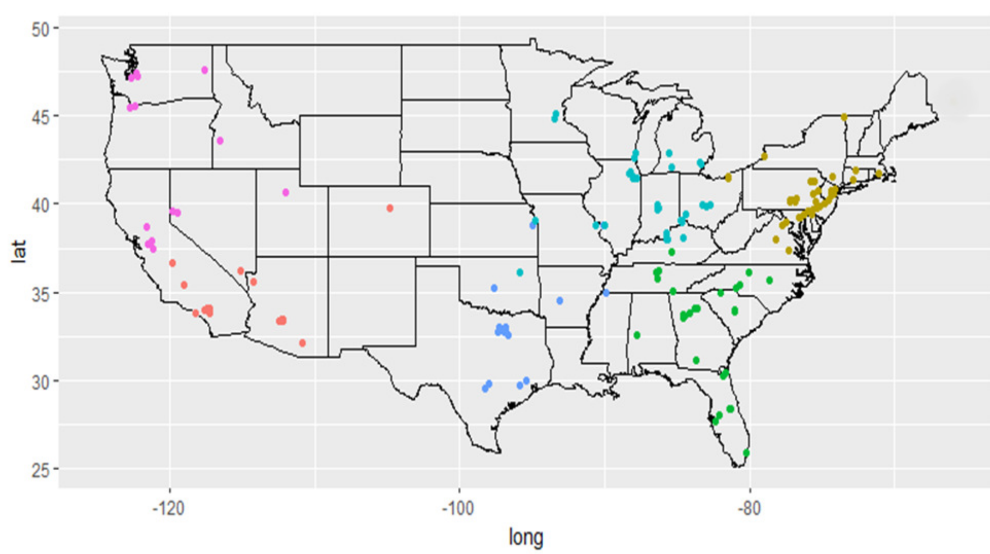


Figure 2.3: FC network of a large retailer, $n = 166$

Here, we illustrate the application of our two previous models towards the decomposition of a large online retailing network and the allocation of safety stock across the network. We use a canonical network of a large online retailer with 166 FCs as shown in Figure 2.3. However, first, we describe a general model MIP_{MP} for a network having n fulfillment centers and K sample paths. A network of such size (Figure 2.3) is hard to analyze and solve and thus, we propose a general decomposition-allocation strategy, **Procedure Allocate-Safety Stock** (PASS). Before that, we describe the objective function and the constraints of MIP_{MP}.

Problem MIP_{MP}:

$$\textbf{Min } \Psi_g = \sum_{i=1}^{n} \lambda_i z \sqrt{(T+L)\sum_{i=1}^{n} \sigma_i^2 h} + \sum_{s=1}^{K} \rho^s \pi^s$$

Subject to:

$$\pi^s = \sum_{i=1}^{n} X_i^s c_p + \sum_{i=1}^{n}\sum_{j=1}^{n} X_{i,j}^s c_{s_i,j} + \sum_{i=1}^{n} Y_i^s g, \quad s = 1,\dots,K \tag{2.19}$$

$$X_i^s + \sum_{j=1}^{n} X_{ij}^s \leq (T+L)d_i + \lambda_i z \sqrt{(T+L)\sum_{i=1}^{n} \sigma_i^2}, \quad s = 1,\dots,K, \forall i \tag{2.20}$$

$$Y_i^s \geq e_i^s - X_i^s - \sum_{i=1}^{n} X_{ij}^s, \quad s = 1,\dots,K, \forall j \tag{2.21}$$

$$\sum_{ß=1}^{n} \lambda_i = 1 \tag{2.22}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} X_{ij}^s = 0, \quad s = 1,\dots,K, i = j \tag{2.23}$$

$$\lambda_i \geq 0 \tag{2.24}$$

$$X_i^s, X_{ij}^s, Y_i^s \; : \; \text{Integer variables}, \quad s = 1,2,\dots,K \tag{2.25}$$

The objective function minimizes the total holding, transportation, and penalty costs. Constraint (2.20) sets an inventory upper bound to the number of shipments and transshipments from each FC. Constraint (2.21) equates the resultant demand and lost sales at each fulfillment center. Constraint (2.22) normalizes the total safety stock allocation to 1. Constraint (2.23) negates any shipments within the same FC. Finally, constraints (2.25) and (2.24) set the non-negativity criteria. We have a total of K sample paths, which is so large that it is virtually impossible to handle with the currently available tools. In our canonical example, $K = 7^{166}$. We reduce the complexity of our problem by proposing the PASS approach. First, we break down the network into a number of clusters to manage the complexity. The clustering of nearby FCs is supported in the literature (Wei et al. 2018). An online retailer can choose the best fulfillment strategy by selecting a fulfillment source from multiple nearby FCs. Next, following the hub-and-spoke approach, we connect the clusters to allow lateral transshipments. Then, we use MIP_{MP} to find the cluster level allocation and use Theorem 1 to distribute the safety stock from the cluster level to the FC level.

Algorithm 1 Procedure Allocate-Safety Stock (PASS)

Input: Network data of n FCs, Demand data, Cost parameters.
Begin
Step 1: Divide the network of n FCs into a number of clusters using $MIP_{Cluster}$, where each cluster groups nearby FCs to form a hub-spoke system.
Step 2: Locate the hub FC for each cluster using the *p-center* algorithm.
Step 3: Link the clusters' hubs with a Minimum Spanning Tree (MST) graph to create the cost-effective transshipment links between clusters.
Step 4: Following the cluster formations at step 3, we consider each cluster as a node and run MIP_{MP} to estimate the cluster-wise safety stock allocation.
Step 5: Model each cluster as a hub-and-spoke network to allocate safety stock at each FC using Theorem 1.
End
Output: Allocation of safety stock at each FC at each cluster.

2.4.1 Clustering

Each online order is fulfilled from the nearest FC based on the available inventory. Beyond that, the online retailer would transship the item(s) from another nearby FC or transfer the order to that FC. Given that online customers are unconcerned about the supply source of their order assuming no change in the price, online retailers can choose the best fulfillment option. Yet, this flexibility may become expensive for distant FCs. For example, it may be cheaper to transship an item to any FC in Texas from an FC located in Arkansas or Tennessee than from an FC in Washington. In other words, a reduction in the distance a package would travel reduces the cost of shipping and time in transit. Wei et al. (2018), in the context of omnichannel retailing, remark that the current practice is to ship online orders from at most two nearby stores (fulfillment options in the context of omnichannel retailing). We do not restrict our model to any specific number of nearby locations; we allow multiple transshipping points to fulfill customer orders in a particular region while allowing transshipment cost to increase as the distance increases. This helps us to group FCs in clusters. The distance between two FCs within a cluster is smaller than that of two FCs across two clusters. In other words, the cost of shipment within the cluster is lower than that across the clusters.

We divide the network into several clusters and allocate the safety stock to each cluster. First, we develop a Mixed Integer Programming (MIP) formulation, $MIP_{Cluster}$, to assign FCs to clusters, based on the pair-wise distances. To determine the upper bound of the distances between two points within a cluster, we use the practice followed by the United States Postal Service (USPS). They estimate the shipping cost based on the weight of an item and the distance between the sender and receiver. To decide on the distance, USPS divides the country into 8 zones. For a package of certain weight, it is cheaper to ship an item from zone 1 to zone 2 compared to a shipment from zone 1 to zone 8. We observe that a facility can ship items within zone 1, 2, 3, 4, and 5 at the same cost. For further distant zones, the facility transships items at a higher cost. USPS categorizes up to 600 miles as zone 4 and up to 1000 miles as zone 5, from zone 1. We use the same metric to constrain the upper bound of the distances between two points within a cluster. As we deal with single items, the weight of each item is assumed to be equal. We define the variables and parameters below and formulate model $MIP_{Cluster}$.

Table 2.6: Parameters and variables for clustering

Parameters:	
l_{ij}	Distance between point i and point j
l	Maximum distance between two points in a cluster
n	Number of fulfillment centers
m	Maximum number of clusters
Variables:	
N_c	= 1 indicates that cluster c is selected
	= 0 indicates that cluster c is not selected
p_{ic}	= 1 indicates that point i is in cluster c
	= 0 indicates otherwise
y_{ijc}	= 1 indicates if locations i and j are assigned to the same cluster c
	= 0 indicates otherwise

The primary purpose of $MIP_{Cluster}$ is to form clusters of nearby FCs. The pair-wise distances of all fulfillment centers are used in the model to form the clusters. The objective is to minimize the number of clusters given that the distance between any two centers in a cluster is bounded.

Here, we discuss the constraints of $MIP_{Cluster}$. Constraint (2.26) provides an upper bound for the number of clusters. For computational efficiency, we limit the number of possible clusters. The second constraint (2.27) ensures that each location is assigned to only one cluster. Constraint (2.28) limits the number of locations in a cluster, where M is a large number. Constraint (2.29) ensures that if and only if locations i and j are assigned to the same cluster, the arc between locations i and j resides within a cluster. Constraint (2.30) limits the distance between two points in a cluster. Finally, we have non-negativity conditions through constraint (2.31). We use CPLEX to solve the problem. We set the upper bound of the number of clusters to be 20 which seems reasonable. We use three different upper bounds for the distances between two points within a cluster: 800 miles, 900 miles, and 1000 miles. The number of clusters found with these three upper limits is 6 which is what we use in the next subsection.

Problem $MIP_{Cluster}$:

$$\textbf{Min} \sum_{c=1}^{m} N_c$$

Subject to:

$$\sum_{c=1}^{m} N_c \leq m \tag{2.26}$$

$$\sum_{i=1}^{m} p_{ic} = 1 \,, \forall i \tag{2.27}$$

$$\sum_{c=1}^{m} p_{ic} \leq MN_c \,, \forall i \tag{2.28}$$

$$y_{ijc} \geq p_{ic} + p_{jc} - 1 \,, \forall i, j, c \tag{2.29}$$

$$y_{ijc} l_{ij} \leq l \,, \forall i, j, c \tag{2.30}$$

$$y_{ijc}, p_{ic}, N_c \in 0, 1 \tag{2.31}$$

2.4.2 Linking Clusters under a MST graph

A straightforward solution would be to restrict transshipments across clusters, wherein each cluster acts as an independent node. Then the allocation of safety stock at the FCs becomes easy. Existing literature also supports transshipments within groups (Herer et al. 2002). Independent clusters constrain transshipments and, therefore, may result in high lost sales, which is undesirable. Such an independent structure is not realistic in the online retailing context as orders may be fulfilled from FCs of distant clusters when cluster-wide stock-outs occur. Thus, we focus on how to transship items across clusters economically. An interconnected less-expensive network can be established under a MST graph, which is an undirected graph where all vertices are connected without any cycles and exhibits the lowest possible total edge weight. In other words, in MST, the pairs of unconnected nodes represent long distances requiring expensive transshipments. The shortest path in the MST fulfills our objective of the cheapest transshipment between any two nodes.

We compute the distances between the clusters by first identifying the central point (hub FC) in a cluster. We develop the within cluster network as a hub-and-spoke network as described in the previous section. To locate the hub FC at each cluster, we utilize the *p-center* approach. For each cluster, we find a point (or the centroid) and measure the distances to other points in that cluster. The algorithm for the *p-center* problem is as follows:

Algorithm 2 P-center algorithm (To locate the hub FCs, adapted from Shier (1977))

Input: Network data of FCs with coordinates and their cluster assignments.
Begin
    Initialize **N** as the number of clusters (where $N = 6$ in our example).
    It can also be treated as the index of clusters.
    While (**N**) do
    Step 1: for each point i in cluster **N**
    Step 2: set i = 1
    Step 3: Compute distance vector d, consists of the distances between
        i and all other points (j) within cluster N (($j \neq i$))
    Step 4: Set maximum distance Maxdist := maximum(d)
    Step 5: Add MaxDist to **maxD** where maxD is the list of the maximum pair-wise distances
        of each iteration and set i = i+1
    Minimum distance in each cluster **N** := minimum(Maxdist)
    N =**N**+1 and go to Step 1
    End(while)
    Output: A list of N hub or central FCs .
End

With the *p-center* algorithm, we generate the MST (Figure 2.4). The distance between the center points of cluster 1 and 6 (or the point minimizing the maximum distance in each cluster) is 564 miles. Similarly, the distance between two center points of clusters 1 and 5 is 1198 miles. The distances between clusters 5 and 4, 5 and 3, and 3 and 2 are 328, 594, and 958 miles, respectively.

The *p-center* approach provides us with: (i) the distance between clusters and (ii) the hub FCs for each cluster.

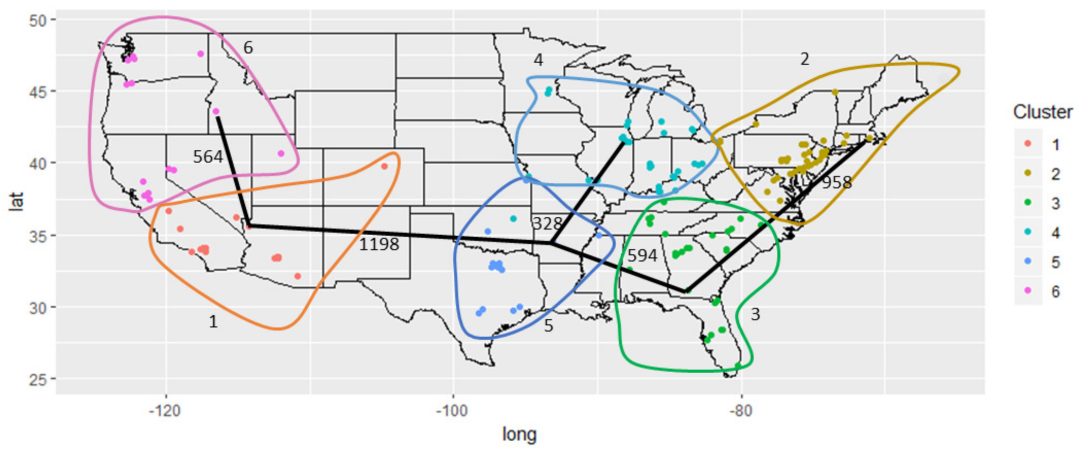


Figure 2.4: A large size problem: MST of 6 clusters, $n = 166$

2.4.3 Hub-and-Spoke Network

The clusters provide us the distribution of FCs in an aggregate form. We allocate safety stock to each cluster, based on their pooled standard deviations of demand (σ_o). For our illustrative example, the σ_o parameters are not readily available. As the actual demand data is not available, we collected and scaled the square footage information of each FC from online sources to estimate a proxy for σ_o. In their empirical work in the medical supply chain, Shapiro and Byrnes (1992) find that lower demand variance is associated with less storage space. Since facility planning requires a substantial investment, online retailers build and expand their facilities with a demand forecast; else, the excess space does not add any value. Therefore, we believe that the FC size is a reasonable proxy for its standard deviation of demand since. However, our results are robust to any demand variance parametric values, including the actual numbers observed in practice.

We now focus on the safety stock assignment within the clusters. Clusters can be made up of one or more FCs. However, we observe that clusters are rarely formed with one FC. To reduce the complexity of the problem, we employ the theory of stocks centralization by using the "square root law of locations." Remember, our primary objective for clustering is to group FCs that are closer to one another. The distance between any two FCs within a cluster is smaller than the distance between two FCs in two different clusters. Based on the cluster formation, the transportation cost between any leaf node and the hub node is the same within each cluster. Therefore, we pool the leaf nodes within each cluster and form a single node; we move from a system with σ_{total} standard deviation of demand to a pooled system with $\sqrt{m}\,\sigma_{total}$ standard deviation of where m denotes the number of leaf nodes within a cluster. This approximation is possible because the transportation cost between any leaf node and the hub node is the same within each cluster.

2.4.4 Safety stock allocation to clusters

We extend our MIP model, MIP_{MP}, to assign safety stock with n=6. We model all the FCs within each cluster as a single node, and thus, the entire system is aggregated into six nodes. The distribution of demand is also aggregated at the cluster level. The system-wide safety stock level remains unchanged. The per unit transshipment cost across the clusters is used as the per unit transshipment cost in this aggregate network. Note that this system does not generate the actual cost of transportation, safety stock holding, and lost sales; it provides us the safety stock allocation parameter (λ_c) at the cluster level. For six nodes, we have a total of 117,649 sample paths. We include the transshipment links, as described in Figure 2.4.

Table 2.7 displays the total safety stock assignment at each cluster.

Table 2.7: Safety stock assignment within six clusters following Theorem 1

$Cluster_i$	Adjacent clusters	Pooled STDDEV of $Cluster_i$	Safety stock at $Cluster_i$
1	5, 6	402	272
2	3	518	518
3	2, 5	416	273
4	5	452	412
5	1, 3, 4	358	242
6	1	330	327

Notes: The standard deviations are rounded up to integer values.

2.5 Heuristic PASS: Implementation

In the previous section, we describe the procedure (PASS) that decomposes a large network of FCs into a cluster level hub-and-spoke system to facilitate the allocation of safety stock. We determine the system-wide safety stock level and allocate them to each cluster in Section 2.4.4. The clusters are linked under a MST graph that enables transshipments across the regions. In this section, we have two objectives: (i) allocation of safety stock from cluster-level to FC-level, and (ii) compare the performance of the optimal results from MIP_{MP} and the results obtained from our proposed PASS.

2.5.1 Distribution of Stock at the FC level

We extend our allocation approach from Section 2.4.4 and distribute the safety stock at every fulfillment center in two stages: (i) allocate the safety stock at the hub FC and pooled FC and (ii) dispense the shared safety stock from the pooled FC to the individual leaf FCs that make up the pooled FC.

Table 2.8 exhibits the allocation of safety stock at the hub and pooled FCs. Column σ_h/σ_0 denotes the ratio of the standard deviation at the hub FC and pooled standard deviation of the leaf FCs. Next, we again allocate the safety stock at each leaf FC from the pooled safety stock following Theorem 1.

Table 2.8: Safety stock assignment at hub and leaf FCs

$Cluster_i$	σ_h/σ_0 of $Cluster_i$	Safety stock at hub FC of $Cluster_i$	Pooled safety stock at leaf FCs of $Cluster_i$
1	114/385	62	210
2	94/509	81	437
3	95/405	52	221
4	80/444	63	349
5	80/348	45	197
6	80/3201	65	262

Notes:
1. The safety stock are rounded to integer values.
2. σ_h and σ_o are the standard deviation of demands of the hub FC and the pooled FC

Note that the general problem, MIP_{MP}, can produce an optimal solution. However, when the number of FCs is large (i.e., $n \geq 7$), even the latest version of CPLEX (version 12.9) may not guarantee a feasible solution for an instance of the MIP_{MP} within a reasonable amount of time.

For such large networks, PASS can be deployed to allocate safety stock. In what follows, we test the efficiency of the proposed approach PASS.

2.5.2 Performance Analysis of PASS

We employ our algorithm PASS to provide an efficient allocation strategy that can be used in any size of the network, especially the large ones. In this section, we analyze the performance of PASS. The gap between the performance of our proposed approach and the optimal results obtained from MIP_{MP} is a good measure of the efficiency of the allocation strategy. To illustrate this, we follow a simplistic setting by choosing a network of four FCs in two clusters and extending the process to a network of six FCs. Note that, we divide the six FCs into clusters to allow pooling in our analysis. We follow a two-step process: (i) we check the performance deviation from the optimal results in these two settings and show that, with a higher n, the performance deviation does not increase substantially, and (ii) we test PASS for a variety of cost settings under a six-FC network and show that the performance is consistent. The former validates PASS's usefulness in any network, especially the larger ones, while the latter ensures the robustness under different cost parameters.

We determine a sample size *k* of networks that establishes a statistically small gap between the optimality of MIP_{MP} and PASS. We choose *k* and repetitively collect *v* samples (replications). We choose *k* = 2401 and *v* = 35 for n=4. We illustrate the algorithm, **Sample selection**, for generating *v* four-FC networks. First, we randomly select two different clusters and the corresponding hub FCs from our sample of 166 FCs. Next, we randomly select one leaf FC from each cluster as the hub FC is already predetermined. Our algorithm selects a different network each time. However, the cluster-hub configuration remains the same. For six-FC networks, we follow the same process, with the only exception of randomly selecting two leaf FCs from each cluster.

We fix the values of the cost parameters (shipment transshipment, holding, and penalty costs at \$7, \$15, \$20, and \$28, respectively), keep the service level at 97.7%, and set $(T + L)$ at 1 in our computational experiments. Note that we do not report the safety stock holding cost in the results;

Algorithm 3 Sample selection

Input: Network data of FCs with cluster assignments, hub and leaf FCs, demand, and cost parameters.
Begin
Choose sample size (N) = *v*.
While (**N**) do
Step 1: Randomly select two different clusters and two hub FCs. If the clusters are not connected as shown in Figure 2.4, repeat this step. Otherwise, go to step 2.
Step 2: Choose a leaf FC from each cluster.
Step 3: Check whether the entire network has two hub FCs and two leaf FCs; if the same configuration already selected before, go to step 2. Otherwise, go to step 1.
End(while)
Output: A list of *v* network configurations.
End

the system-wide safety stock and the costs associated with it remain the same. Table 2.9 displays the performance comparison. The detailed results for all sample paths are provided in Tables A.10 and A.11. The average difference, in terms of overall cost, is 0.012 % with a maximum deviation of 0.05%. When we extend it to a network of 6 FCs, our method again performs reasonably well with an average of 0.015% deviation from the optimal results (Table 2.9). The maximum deviation is 0.03%. The results do not show any substantial deviation between the two types. These findings indicate that our decomposition approach performs reasonably well in comparison with the optimal setup, even for large networks.

Table 2.9: Performance analysis (Differences in overall cost) with two clusters

Type	Min diff (%)	Max diff (%)	Avg diff (%)
4 FC	0	0.05	0.012
6 FC	0.01	0.059	0.041

The above numbers provide reasonable support to the fact that our approximation method may work well when we include more nodes in a network. For further analysis, we test it under three different cost settings for a subsample of 17 randomly selected samples for each setting. We do not see any noticeable differences between the three settings. c_{s1} and c_{s2} ($c_{s2} > c_{s1}$) refer to the per unit transshipment costs within and between clusters, respectively. In Table 2.10, approximation refers to the results obtained from PASS.

Table 2.10: Different cost settings for 6FC networks

c_p \$7, c_{s1} \$12, c_{s2} \$18, h \$20, and g \$28			c_p \$10, c_{s1} \$15, c_{s2} \$22, h \$25, and g \$35			c_p \$10, c_{s1} \$20, c_{s2} \$25, h \$28, and g \$35		
Cost(Optimal)	Cost(Approximation)	Difference (%)	Cost(Optimal)	Cost(Approximation	Difference (%)	Cost(Optimal)	Cost(Approximation	Difference (%)
229334	229464	0.057	302273	302392	0.039	302555.6657	302669.6657	0.037679017
230471	230567	0.042	302270	302385	0.038	302418.1045	302531.1045	0.037365488
229459	229510	0.022	302251	302366	0.038	302501.6077	302676.6077	0.057850932
227417	227521	0.046	302306	302423	0.039	302583.9861	302694.9861	0.03668403
213081	213191	0.052	302233	302351	0.039	302622.4025	302781.4025	0.052540724
230469	230535	0.028	302316	302439	0.041	302532.336	302645.336	0.037351379
228868	228969	0.044	302295	302416	0.040	302444.015	302548.015	0.034386529
211739	211827	0.042	302422	302535	0.037	302396.1099	302500.1099	0.034391977
211827	211923	0.045	302286	302400	0.038	302616.7486	302730.7486	0.037671411
211739	211839	0.047	302156	302311	0.051	302486.266	302598.266	0.037026474
211716	211819	0.049	302347	302459	0.037	302423.1907	302524.1907	0.03339691
211753	211874	0.057	301818	301987	0.056	302442.1904	302555.1904	0.037362512
211618	211713	0.045	302762	302887	0.041	302573.8737	302683.8737	0.036354758
211481	211591	0.052	302361	302469	0.036	302610.5179	302782.5179	0.056838738
211665	211761	0.045	302229	302391	0.054	302400.4962	302507.4962	0.03538354
211699	211802	0.049	302189	302299	0.036	301027.3802	301211.3802	0.061124008
230509	230575	0.028	302193	302304	0.037	301031.1861	301136.1861	0.034880107

Here we discuss one important aspect of a multi-nodal structure. The MST establishes the connecting links between the clusters, and this translates to the transshipment links in our model. For example, in the U.S. network, if cluster 1 is out-of-stock, cluster 6 may tranship if it has sufficient inventory. Interestingly, we need to address the situation if both clusters 1 and 6 are out-of-stock. In this case, cluster 1 may be supplied by cluster 5 if it has a surplus whereas cluster 6 is allowed to face lost sales. Intuitively, it is more costly to facilitate transshipments between two disjoint clusters than two connected clusters. Note that we do not model transshipments between two disjoint clusters that are not connected under the MST. However, the omission of such links between disjoint clusters does not impede the order fulfillment strategy as our stochastic optimization approach efficiently allocates the system-wide safety stock to each cluster aimed at maintaining a balance between safety stock holding and transshipments. Instead of expensive distant transshipments, we compare it with the cost of holding safety stock at the closest FC and choose the best fulfillment strategy. Our stochastic optimization approach estimates all the probabilities of stock-outs at all clusters and provides a robust solution.

2.6 Large Scale Problems: Performance Evaluation of PASS via Robust Optimization

In the previous section, we implement PASS and show the optimality gap. Due to computational limitation of solving MIP_{MP} exactly, we evaluate the performance of PASS using a small version of the problem with four and six FCs. We show that the gap between the results obtained from MIP_{MP} and PASS is between 0.03% and 0.05%. In this section, using a large scale problem encountered in practice (as shown in Figure 2.4), we demonstrate the efficiency of PASS. We first perform a stochastic optimization approach to find a robust solution for Problem MIP_{MP} that minimizes the expected total cost. We then compare this solution with that of the solution obtain by PASS.

2.6.1 Sample Average Approximation

The stochastic version of the problem MIP_{MP} requires an exponential number of sample paths ($K = 7^n$) for large n, and thus cannot be solved in a reasonable time. In order to deal with this issue, researchers use Sample Average Approximation Method (SAA) (Kleywegt and de Mello. 2001) which repetitively uses a smaller number of sample paths to obtain approximate solutions. In this sampling approach, we determine an appropriate sample size, k, that confirms a statistically small gap between the optimum of the general problem and the solution by SAA. We start with a sampling process with a sample size (k) and υ (the number of replications of samples tested).

We use a large scale problem encountered in practice (as shown in Figure 2.4) to illustrate our approach. Using the probability distribution for the demand data at each FC, we start with a sampling process with the sample size (k) and the number of replications of samples tested (υ). The value of υ is increased for a given value of k until a specified precision is reached. If the precision is not reached then we need to increase sample size, k, and repeat the process until the precision is reached. Note that the SAA problem corresponding the MIP_{MP} can be formulated as the following integer program, where the solution is $z = (X_i^s, X_{ij}^s, Y_i^s)$:

Problem MIP_{SAA}:

$$\textbf{Min } \pi_k(z) = \frac{1}{k}\sum_{s=1}^{k} \pi^s(\lambda_i, X_i^s, X_{ij}^s, Y_i^s)$$

Subject to:

$$\pi^s(\lambda_i, X_i^s, X_{ij}^s, Y_i^s) = \sum_{i=1}^{n} X_i^s c_p + \sum_{i=1}^{n}\sum_{j=1}^{n} X_{i,j}^s c_{s_i,j} + \sum_{i=1}^{n} \lambda_i z \sqrt{(T+L)\sum_{i=1}^{n}\sigma_i^2 h} + \sum_{i=1}^{n} Y_i^s g, \quad s = 1,\ldots,k \tag{2.32}$$

$$X_i^s + \sum_{j=1}^{n} X_{ij}^s \leq (T+L)d_i + \lambda_i z \sqrt{(T+L)\sum_{i=1}^{n}\sigma_i^2}, \quad s = 1,\ldots,k, \forall i \tag{2.33}$$

$$Y_i^s \geq e_i^s - X_i^s - \sum_{i=1}^{n} X_{ij}^s, \quad s = 1,\ldots,k, \forall j \tag{2.34}$$

$$\sum_{ß=1}^{n} \lambda_i = 1 \tag{2.35}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} X_{ij}^s = 0, \quad s = 1,\ldots,k, i = j \tag{2.36}$$

$$X_i^s, X_{ij}^s, Y_i^s : \text{Integer variables}, \quad s = 1,2,\ldots,k \tag{2.37}$$

$$\lambda_i \geq 0 \tag{2.38}$$

We start by randomly generating k sample paths. Then, we run the SAA problem and collect the optimal objective value for the first sample, $\pi_k^1(z^1)$. Similarly, we obtain $\pi_k^2(z^2), \pi_k^3(z^3), \ldots, \pi_k^{\upsilon}(z^{\upsilon})$ for υ replications. Note that $\pi_k^r(z^r)$ is an unbiased estimator of Ψ_g^*. An estimator of Ψ_g^*, the optimal objective value of Problem MIP_{MP}, is given by $\bar{\pi}_k^{\upsilon} = \frac{\pi_k^1(z^1)+\pi_k^2(z^2)+\ldots+\pi_k^{\upsilon}(z^{\upsilon})}{\upsilon}$, where $\pi_k^r(z^r)$ denotes the optimal objective value of the r^{th} SAA replication, $r = 1,\ldots,\upsilon$.

We can also calculate the sample variance $V_k^{\upsilon} = \frac{\sum_{r=1}^{\upsilon}[\pi_k^r(z^r)-\bar{\pi}_k^{\upsilon}]^2}{\upsilon-1}$. After υ replications, an approximate $100(1-p)$ percent confidence interval for the expected objective value (i.e., $E[\Psi_g^*]$) is given by $\bar{\pi}_k^{\upsilon} \pm t_{\upsilon-1,1-p/2}\sqrt{\frac{V_k^{\upsilon}}{\upsilon}}$. The half-length of this confidence interval given is denoted $\zeta(\upsilon,p) = t_{\upsilon-1,1-p/2}\sqrt{\frac{V_k^{\upsilon}}{\upsilon}}$.

If the estimate $\bar{\pi}_k^{\upsilon}$ satisfies $\frac{\bar{\pi}_k^{\upsilon}-E[\Psi_g^*]}{E[\Psi_g^*]} = \xi$, then we can claim that $\bar{\pi}_k^{\upsilon}$ has a relative error of ξ. If we keep increasing replications until $\frac{\zeta(\upsilon,p)}{\bar{\pi}_k^{\upsilon}}$ is less than or equal to ξ, then $\bar{\pi}_k^{\upsilon}$ has a relative error at most $\frac{\xi}{1-\xi}$ with a probability of approximately $1-p$ (Law 2015). Therefore, to obtain an estimate of $E[\Psi_g^*]$ with a relative error of ξ and a confidence level of $100(1-p)$ percent, we follow the following procedure (Law 2015) to determine the size of υ for the given sample size k:

Below we list the steps of the procedure defined in Law (2015).

- **Step 1:** Set $\upsilon_0 = 10$ random replications (each with $k = 100$ sample paths), and set $\upsilon = \upsilon_0$.

- **Step 2:** Calculate $\bar{\pi}_k^{\upsilon}$ and $\zeta(\upsilon, p)$ from $\pi_k^1, \pi_k^2, \ldots, \pi_k^{\upsilon}$. Note that $\pi_k^r, r = 1, \ldots, \upsilon$ are obtained by solving Problem SAA.

- **Step 3:** If $\frac{\zeta(\upsilon,p)}{\bar{\pi}_k^{\upsilon}} < \frac{\xi}{1+\xi}$, then use $\bar{\pi}_k^{\upsilon}$ as the point estimate for Ψ_g^* and stop. Otherwise, replace υ by $\upsilon + 1$, proceed with another iteration, and go to Step 2.

2.6.2 Numerical Analysis

In order to evaluate performance of PASS on larger problems, we first devise a stochastic optimization approach (using MIP_{SAA}) to find robust solution for MIP_{MP} that minimizes the expected cost. We then compare this solution with that of the solution given by PASS.

Finding the total expected cost for the solution given by heuristic, PASS for a large size problem in Figure 2.4 is not straightforward. We estimate the total expected cost two different ways:

- (a) We solve the problem for six FCs using CPLEX by treating each cluster node as one consolidated FC (found by PASS). We denote the six-cluster model $PASS_{across}$. The model inputs include the six clusters found previously with their cluster standard deviations and the outputs are the total expected cost and allocation of safety stock at each cluster. Note that $PASS_{across}$ omits all the transshipments that occur within a cluster (both between hub and leaf node, and among nodes represented by the leaf node). However, we include the transportation cost of each cluster's demand which is an underestimate of the actual logistics cost as transshipments within each cluster are ignored. Note that $PASS_{across}$ includes the transshipment cost across the clusters. The clusters are connected under the MST graph as shown in Figure 5.

- (b) In the second approach, $PASS_{within}$, we model each cluster as a hub-and-spoke system where the hub is determined by our *p-center algorithm* and leaf nodes within each cluster are pooled into a single node, resulting in two FCs in each cluster. For the entire network, we obtain 12 FCs with each cluster having two nodes (a hub node and a leaf node). However, it is computationally impossible to solve a network of 12 FCs together. This leads us to solve each cluster individually and sum all the costs obtained from each of six clusters. This approach also has limitations: (i) omission of the transshipments across clusters, and (ii) the missed transshipments (between nodes within a consolidated leaf node in each cluster) due to pooling. In Section 2.3.2, we demonstrate that the pooling loss in (ii) is negligible as compared to the overall cost.

In (a) and (b), we discuss two possible ways to measure the performance of PASS. Both approaches have their limitations due to underestimation of the total cost. However, we argue that the cost obtained from $PASS_{within}$ is likely to be lower than that from $PASS_{across}$ because transshipments across clusters are more expensive. We now present the results for the three models: $PASS_{across}$, $PASS_{within}$, and MIP_{SAA}. In our computational experiment, we set the mean demand at each node at 400. Other parameters remain unchanged as in Section 2.5.2. After 15 replications of MIP_{SAA}, we obtain a solution that satisfies the condition in Step 3 of SAA.

Table 2.11: Costs comparison: $PASS_{across}$, $PASS_{within}$, and MIP_{SAA}

$PASS_{across}$	$PASS_{within}$	MIP_{SAA}
\$469,406	\$460,137	\$532,733

As we expected, the cost estimated by $PASS_{across}$ is more than that given by $PASS_{within}$ (Table 2.11). On the other hand, MIP_{SAA} produces the highest costs among the three models mainly because it captures all costs specified in MIP_{MP}. In spite the computational complexities associated with solving such large networks, we offer a new approach to systematically evaluate the performance of PASS. It has to be noted that we use the theoretical insights gained in this chapter to devise Heuristic PASS where we first determine the cluster safety stock allocation and then, determine the allocation at all FCs within each cluster. The complexity arises when we include all 166 FCs to compute the total cost for the network. Thus, we propose a combined two stage method, $PASS_{SAA}$: (i) using PASS to estimate the safety stock allocation at each FC (namely, $PASS_{across}$) and (ii) then employing SAA by including the λs estimated in (i) to calculate the total expected cost (namely, $PASS_{SAA}$).

Table 2.12: Comparison: MIP_{SAA} and $PASS_{SAA}$

MIP_{SAA}	$PASS_{SAA}$	$\frac{\zeta(\upsilon,p)}{\bar{\pi}_k^{\upsilon}}$	$\frac{\xi}{1+\xi}$
\$532733.93	\$540584.38	0.009	0.0145

Observe that $PASS_{across}$ provides the cluster-wise safety stock allocation. Following Theorem 1, we then allocate safety stock at the hub FCs and leaf FCs in all clusters. With fixed safety stock allocation (i.e., fixed λs for all 166 FCs) above, we execute $PASS_{SAA}$, which is equivalent of running MIP_{SAA} with fixed λs. For $PASS_{SAA}$, we start with $\upsilon_0 = 10$ random replications $k = 100$ for each replication. After 15 replications, $PASS_{SAA}$ generates a total cost of \$540,584, which is expectedly higher (1.47% higher) than MIP_{SAA}. From Table 2.12, we can observe that $PASS_{SAA}$ appears to be reasonably closer to MIP_{SAA}. $PASS_{SAA}$ provides an upper bound estimator to Ψ_g^*, the objective function of MIP_{MP}. Thus, based on our computational experiments, we conclude that the total expected cost obtained from $PASS_{SAA}$ would be close to the optimal. Overall, our heuristic PASS offers an efficient allocation of safety stock across a large network.

2.7 Extensions Used in Practice

In managing online retailing operational costs, an efficient allocation of safety stock is an important task for an operations manager. In the previous subsection, we consider a real-world online retailing network safety stock allocation problem, propose an allocation approach for medium and larger networks, and discuss the managerial implications in the context of operational efficiency. We also develop a methodology offering solutions close to optimal. In this section, we explore the cost implications of various allocation methods that have been used in practice. For that, we analyze the four cost components: shipment, transshipment, lost sales, and safety stock costs. We study three practical safety stock allocation approaches that may be used in practice due to operational simplifications. We investigate the cost implications for those approaches using networks having between two and six FCs. Since we illustrate this in small to medium size problems, we use MIP_{MP}. First, we compare the performance of our approach (called as "decentralized system" investigated in the previous sections) in contrast to the "centralized system," where the entire safety stock is assigned to only one hub FC among a cluster of FCs. Second, we study the effect of varying service levels on the total cost in the decentralized system. Finally, we compare our decentralized system (i.e., pooling of safety inventory) with that of having no-pooling of safety stock (i.e., no transshipments between FCs). For simplicity, we assume the demand variance of all FCs to be equal. Other cost parameters used are as in Section 2.6.2.

2.7.1 Comparison between Decentralized and Centralized Systems

In previous sections, we consider an online retailing network where safety stock is allocated across all FCs in a decentralized manner. This allocation allows transshipments between FCs. i.e., if an FC faces stock-out, an adjacent FC having surplus can fulfill the shortage. Online retailers often implement a policy to store safety stocks at one location (say hub FC) among a cluster of FCs for operational convenient. Thus, this safety stock allocation approach is of practical interest, i.e., *allocate safety stock at one FC only* (called as "centralized system"). However, stocking at one location may result in increased transshipments if other locations face frequent stock-outs. Therefore, we investigate the cost implication associated with this policy. That is, we compare our decentralized system with that of the centralized case. To describe the centralized case more formally, we let $\lambda_1 = 1$ and $\lambda_i = 0$ for $i \neq 1$. Note that the total system-wide safety stock costs remain unchanged for the two scenarios.

We denote the centralized and decentralized cases as *cen* and *decen*, respectively. From Table 2.13 we observe that the decentralized cases achieve a substantial amount of cost savings over the centralized case allocation. For example, when there are five FCs in the network, one may save 2.48$\%$ of their outbound shipping cost which amounts to a sizable saving for a company with a large outbound shipping expenditure. The decentralized allocation policy appears to result in similar savings for small, medium, and large networks.

2.7.2 Impact of Service Level on Cost Components: Decentralized System

For planning purposes, it is also important for managers to know the impact of service level on the total cost. With this information, managers can evaluate the change in the total cost due to the

Table 2.13: Comparison between decentralized and centralized systems

	2FC		3FC		4FC		5FC		6FC	
Cost components (in $)	Decen	Cen	Decen	Cen	Decen	Cen	Decen	Cen	Decen	Cen
Shipment	6907.37	6741.2	10254.4	9993.26	13550.2	13228.8	16917.5	16512.7	20233	19771.3
Transshipment	0	356.09	479.04	1038.74	1056.2	1758.68	1290.56	2160.77	1715.84	2711.39
Lost sales	426.6	426.6	214.3	214.3	51.78	51.78	271.2	265.5	360.1	348.6
Difference (in %) in total costs ($\frac{cen-decen}{decen}$)	2.58		2.72		2.24		2.48		2.34	

level of safety stock allocation in the decentralized system. Higher service may be effective in reducing lost sales and transshipments. However, it can increase the safety stock costs substantially. We compute and compare the shipment, transshipment, lost sales, and safety stock costs for service levels (in %) $\in$ {90, 92, 95, 97, and 99}, respectively. Table 2.14 summarizes the percentage cost differences associated with shipments, transshipments, safety stock, lost sales, and the total cost for different service levels with respect to 90% base level. The total costs that include safety stock costs sharply increase with the increasing service level. As expected, the shipments increase while transshipments decrease as the service level goes up.

In Figure 2.5, we observe the costs for networks with four, five, and six FCs. The safety stock holding costs increase across all networks as we increase the service level from 90% to 99%, resulting in lower lost sales. On the other hand, as observed from Figure 2.5, the higher the availability of safety stock (i.e., higher service level) the lower the transshipment cost. Furthermore, the shipment costs increase with the service level. The relationship between shipments and transshipments with varying service level warrants further discussion. Higher service level increases system safety stock resulting in higher safety stock at the individual FC level. With demand parameters remain unchanged, cheaper shipments replace expensive transshipments as service level increases. However, the total cost also increases with a higher service level since safety stock holding is costly. The increase in the total cost is significant, so the decision-makers should determine the appropriate balance between the system service level and the corresponding total cost of serving customer demand.

Table 2.14: Changing costs (in%) from 90% service level: decentralized system

Network size	Cost parameters	92%	95%	97%	99%
2FC	Shipment	0.5	0.79	1.03	1.45
	Transshipment	0	0	0	0
	Lost sales	-24.52	-38.88	-50.18	-70.78
	Safety stock	8.52	27.9	46.51	80.62
	Total cost	1.87	7.72	13.43	23.9
3FC	Shipment	0.44	1.33	1.77	2.13
	Transshipment	-13.11	-39.34	-51.85	-61.17
	Lost sales	-18	-54.02	-73.93	-93.3
	Safety stock	8.52	27.9	46.51	80.62
	Total cost	1.99	6.64	11.54	20.99
4FC	Shipment	0.44	1.29	2.15	2.84
	Transshipment	-10.25	-30.23	-50.72	-67.06
	Lost sales	-22.12	-53.58	-73.07	-91.21
	Safety stock	8.52	27.9	46.51	80.62
	Total cost	1.83	6.14	10.28	18.56
5FC	Shipment	0.32	1.03	1.73	2.82
	Transshipment	-5.5	-20.32	-36.01	-62.31
	Lost sales	-22.09	-50.76	-73.1	-90.87
	Safety stock	8.52	27.9	46.51	80.62
	Total cost	1.62	5.46	9.17	16.29
6FC	Shipment	0.18	0.96	1.59	2.73
	Transshipment	-4.13	-17.26	-30.5	-56.06
	Lost sales	-2.13	-55.88	-72.53	-91.14
	Safety stock	8.08	27.9	46.51	80.62
	Total cost	1.62	5.03	8.53	15.02

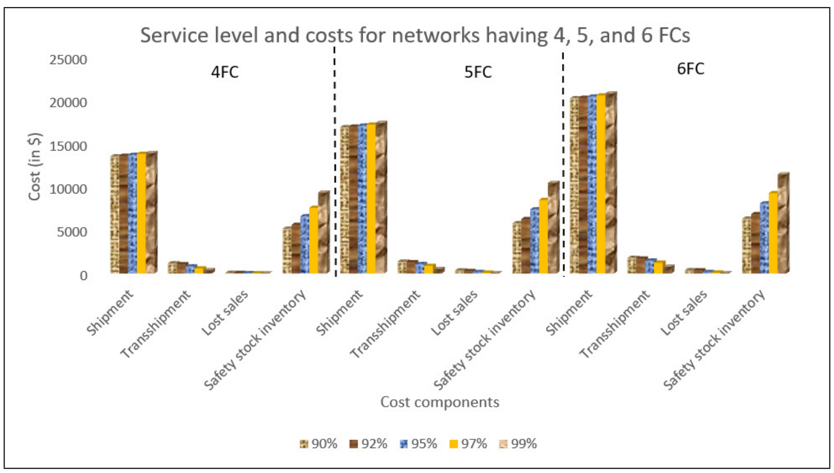


Figure 2.5: Impact of service level on cost components

2.7.3 Comparison between Pooling and No-Pooling Systems

It has to be noted that the pooling of safety stock and optimal allocation to FCs in the decentralized system is the main focus of this chapter. We may also refer to this decentralized system as a "pooling system." In the "no-pooling system", FCs are assigned safety stock individually, thereby increasing the amount of safety stock deployed. This situation may be encountered when a network consists of unconnected FCs that belong to the same company, but each FC operates as an independent business unit. We assume transshipments do not occur in a no-pooling system as FCs operate independently. However, in the pooling system, lateral transshipments across the FCs in a network reduce the system-wide safety stock level due to pooling.

We compare the cost components in pooling and no-pooling systems. Note that the total cost includes safety stock cost. Table 2.15 provides a representative subset of the outcomes associated with pooling and no-pooling systems. The total cost for the pooling system includes the transshipment cost. Figure 2.6 illustrates that as the network grows in the number of FCs, the cost difference between the pooling and no-pooling systems increases sharply. For example, the difference is 13.57% for a network of two FCs and almost 30% for a network of five FCs. As expected, the use of a pooling system performs much better in terms of the overall cost than a system with no-pooling. The purpose of this section is to quantify the amount of saving that is significant due to pooling systems.

Table 2.15: Comparison between pooling and no-pooling systems (92% service level)

	2FC		3FC		4FC		5FC		6FC	
Cost components (in $)	Pooling	No-pooling	Pooling	No-pooling	Pooling	No-pooling	Pooling	No-pooling	Pooling	No-pooling
Shipment	6907.37	6950.2	10254.4	10435.7	13550.2	13928.2	16917.5	17427.6	20233	20933.1
Transshipment	0	0	479.04	0	1056.31	0	1290.56	0	1715.84	0
Lost sales	426.6	255.22	214.3	383.21	51.78	511.46	271.2	639.91	360.11	762.45
Safety stock	3976	5640	4872	8460	5600	11280	6272	14100	6832	16920
Total cost	11309.9	12845.42	15819.8	19278.91	20258.29	25719.66	24751.32	32167.51	29140.95	38615.55

This section explores the variants of the safety stock allocation approaches that may be encountered in practice. Specifically, we address three variants: (i) cost saving associated with our decentralized system over the centralized case, (ii) impact of service level on the total cost in the decentralized system, and (iii) comparison between pooling and no-pooling systems. In summary, numerical results from the three computational studies suggest that

• for a given service level, the decentralized safety stock allocation results in significantly lower total cost than that of the centralized scenario,

• higher service levels in the decentralized system increase the safety stock holding cost and reduce lost sales and transshipments. However, the total cost increase significantly with the service level, and

• pooling system performs better than no-pooling system. The cost saving is significant in the pooling system.

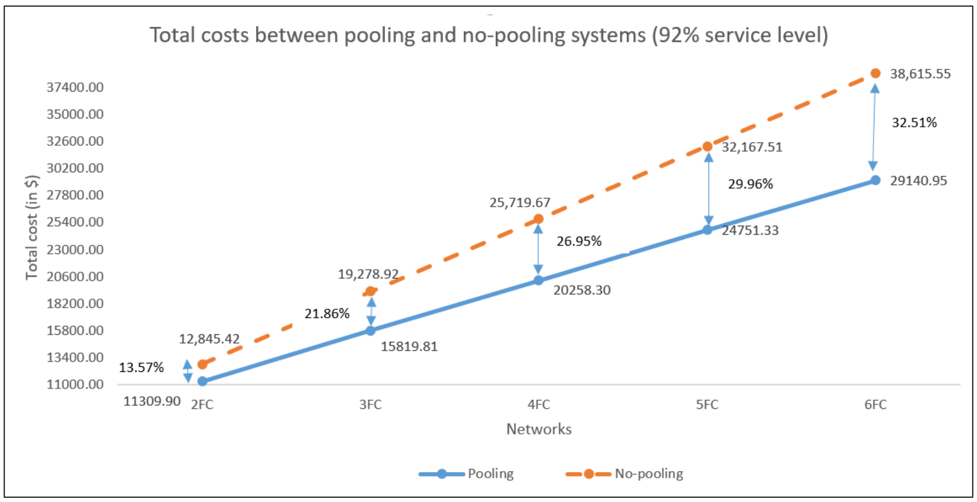


Figure 2.6: Total costs of pooling and no-pooling systems

2.8 Managerial Insights

In this chapter, we address the order fulfillment challenges faced by online retailing managers. We specifically investigate the approaches to allocate safety stock in online distribution networks. The need and desire to continually reduce operating costs while balancing inventory and transportation costs is a common issue online retailers face around the world. Additionally, demand uncertainty, coupled with the size of their networks, makes it more difficult for online retailers to systemically allocate safety stock at the right place and at the right time. The problem of safety stock allocation, therefore, must be studied under these conditions. In this section, we present managerial insights and discuss the advantages and possible extensions of this chapter.

Considering demand uncertainty and network size, our MIP model, MIP_{MP}, can efficiently allocate safety stock by reducing expensive transshipments and excess inventory. However, the current state-of-the-art MIP software cannot solve instances with more than 6 fulfillment centers within a reasonable amount of time. Our approach, therefore, emphasizes on reducing complexity associated with safety stock planning. For example, by creating clusters of fulfillment centers, managers can holistically administer a large network of FCs with ease. This approach is consistent with the industry practice of grouping nearby FCs while maintaining the control over every individual FC as closely as possible. The cluster-based approach helps in storing inventory at the right place that may result in cost-effective and faster delivery. We also validate the efficiency of our approach in Section 2.5.2 and Section 2.6.2; our procedure (PASS) provides solutions reasonably close to the optimum for small, medium, and larger networks.

In addition to its computational benefits, our proposed approach offers useful practical extensions relevant to online retail operations. First, we consider stocking locations in a network. Until now, we decompose a large network into several clusters and allocate the safety stock across all fulfillment centers. However, online retailers may not allocate safety inventory at all locations for ease of operations and expensive warehousing cost. Efficiency may arise by allocating safety inventory only at hub FCs of the clusters. Our research may guide in identifying those locations.

Online retailers often provide customers with multiple delivery options or delivery heterogeneity; they may charge customers for expedited deliveries. The amount charged may compensate for the extra transportation costs fully or partially. The transshipment costs for expedited deliveries are more than normal mode of shipping. Delivery options may also vary across geographic regions. For example, urban areas may exhibit more expedited delivery requests than suburban areas. Our model can apply to the multiple delivery setting if the distribution of delivery options can be estimated in terms of fraction of customers demand for various delivery options at each FC. A rigorous mathematical analysis of safety stock allocation with delivery options is possible by modifying model MIP_{MP} with those options. However, with the availability of historical data of different delivery options of customer demand, PASS can be adopted for safety stock allocation.

There are additional relevant managerial extensions of this chapter. We assume that cost parameters do not vary within a cluster, given that fulfillment centers within a region may not exhibit different cost characteristics. However, retailers may encounter varying holding costs, and therefore, the model can be extended to observe the relationship between safety stock allocation and the ratio of demand uncertainty and holding costs (σ_i/h_i). Another interesting extension is the inclusion of supplier lead time uncertainty. For analytical tractability, we use a single supplier with identical lead time across the network. Based on their geographical operations, a supplier may have different lead times in different regions. Additionally, the supplier lead times may change with multiple suppliers. Our model can be utilized to understand the relationship between supplier lead time (L_i) and demand uncertainty σ_i. Several important questions can be similarly addressed by running and re-running our model after adjusting certain variables. For example, shipment or transshipment costs can be adjusted at the network level or across the clusters. Managers can modify demand parameters frequently to account for rapid changes in demand.

2.9 Conclusion

We study the safety stock allocation mechanism in an online retailing fulfillment centers network with an objective of minimizing the total cost consisting of the costs of transportation, inventory holding, and penalty for lost sales. Currently, online retailers distribute safety stock myopically (Acimovic and Graves 2017) across its network and therefore, under uncertain demand, they may encounter expensive transshipments or higher inventory holding costs. In this chapter, we describe a simple and easy-to-implement approach for allocating safety stock in any network. With the current state-of-the-art computational capability, we can optimally allocate safety stocks in a network having no more than 6 FCs. For larger networks, we begin by clustering the FCs. Next, with *p-center* algorithm, we select the hub and spoke FCs and link the clusters under the Minimum Spanning Tree (MST). This process lets us select the cheapest transportation route between any two nodes. Finally, we allocate the safety stock across the network, starting at the cluster-level to FC-level. Our theoretical results assist us to allocate safety stock at each FC near-optimally. To the best of our knowledge, this chapter is the first research to optimally solve a network of 6 FCs and suggest inventory allocation for larger networks using the stochastic optimization approach. None of the published papers in inventory allocation stream has computationally solved an inventory allocation problem for more than 3 FCs.

We assess our decomposition method in the context of both the average-case and worst-case performance. The computational study suggests that our proposed near-optimal method performs fairly well compared to the optimal results. From the practice point-of-view, this chapter can contribute significantly to the managerial decision. Despite substantial differences among online retailers in terms of inventory allocation and shipment strategies, our proposed method offers an easy-to-implement safety stock allocation process intending to reduce excess operational costs. Amazon reported approximately \$21.7 billion expenditure in outbound shipping in 2017. Flipkart's annual costs in transportation and distribution in 2017 were approximately \$324 million[2]. Therefore, even 1-2% of savings in transportation may result in significant dollars in saving. With an increasing trend in online shopping across the world, the net savings can be significant, and the benefits can be passed on to the consumers.

We further observe many interesting avenues for future research. We model customer demand using normal distribution and discretized it for analytical tractability. It has to be noted that the discretization approximation is reasonable and can be used for any other distribution. Future research may model demand using other appropriate distributions and validate our findings. Second, our approach is valid for items ordered and delivered as a single-item. In doing so, we devise a procedure on the allocation of safety inventory in any size of networks. Future studies may extend the research in this chapter by incorporating multi-item orders. Third, supply-side uncertainty can be considered as an interesting extension. However, the simultaneous consideration of supply and demand uncertainty may increase the complexity.

We believe that this chapter has significant implications for practice. Online retailers struggle with high transportation costs, facing a trade-off between overstocking and expensive transshipments. While these are difficult challenges, our research addresses these problems tactically. We recommend that practitioners need to move from a myopic approach of allocating safety stock to

[2] https://www.livemint.com/Money/TcqdDf30s06hvtSuyM5I1L/If-Flipkart-losses-havent-alarmed-Walmart-Amazon-Indias-s.html

a more planned and cost-efficient process to derive significant cost benefits.

3. ACO Service Delivery and Experience on Financial and Quality Performance - An Empirical Examination

3.1 Introduction

Healthcare spending in the United States (U.S.) reached $3.8 trillion in 2019, or 17.7 percent of the gross domestic product (GDP), exhibiting a consistent risk in healthcare spending over time (Martin et al. 2021). U.S. healthcare faces significant obstacles from the fragmented nature of its payment and delivery systems due to little or no accountability from a single group of physicians to guide a patient's health. The care episode of a patient may be distributed across many sites, resulting in duplication of tests, unnecessarily high costs, and inadequate care quality. Also, fragmented care renders unnecessary hospitalizations, causing poor care quality and inefficient resource utilization (Freeman et al. 2021). The Affordable Care Act 2010 (ACA) created Accountable Care Organizations (ACOs) to improve population health and care quality and reduce healthcare costs. ACOs consist of primary care physicians (PCPs), specialists, nurse practitioners (NPs), hospitals, and other healthcare providers and facilities, who provide coordinated healthcare to their patients for reducing healthcare costs and improving care coordination. ACOs that meet both financial and quality standards are incentivized by their payers through financial rewards.

ACOs have increased both in terms of numbers and population coverage over time. The ACO model requires the providers across care settings to take responsibility for the spending and quality of care for a defined population. Currently, there are nearly 1000 ACOs across the US, operating with commercial, Medicare, and Medicaid contracts (Solutions 2021). ACOs have exhibited higher average performance rates than other medical groups (Federal 2018). By making providers more accountable for the total costs of care (care episode), the ACO model aims to provide better care coordination. For example, Akron Children's Health Collaborative, a new ACO, will take responsibility for 100,000 children insured under Medicaid (Medcity 2021). Despite efforts to curb spending and improve quality, the Centers for Medicare and Medicaid Services (CMS) estimates that financial and quality performance have not been substantially enhanced by the ACOs (CMS 2021). Several ACOs do not demonstrate substantial cost reductions and quality improvements (McKinsey. 2020), and over 30 organizations quit their contracts because they struggled to reach their respective benchmarks (LaPointe 2018). Consequently, the number of ACO participation becomes slower compared to previous periods due to the risk and uncertainty associated with performance (McWilliams and Chen 2020).

Given the number of ACOs and the rise in covered population, clearly, it is important from an industry perspective to ensure ACOs become more efficient in delivering care. ACO model shows promises amidst the rising cost of healthcare worldwide. This new and innovative patient-care model has garnered interest in other countries, including Canada, especially for patients with complex needs (Peckham et al. 2018). Within the broader umbrella of efficiency in operations, service delivery via provider composition and experience play important roles in ACO performance (Wilson et al. 2020). However, in spite of such importance, there is limited work in the literature on analyzing efficiency issues in ACOs, especially in terms of utilizing these resources. To bridge this important gap in the literature, we empirically examine the impact of primare care service delivery (or broadly service delivery) via provider composition and experience on ACO financial and quality performance. Our work sheds light on this operationally relevant problem for ACOs that is relevant for researchers and policymakers, and we present valuable and actionable insights.

3.1.1 Background of ACOs

In the 1970s, the Health Maintenance Organization Act (HMO) became popular with its value-based payment methods. In the nineties, integrated delivery networks (IDN) evolved but were unsuccessful due to the misalignment of incentives between hospitals and providers (Burns and Pauly 2012). The first set of ACOs was established in 2012 with the triple aims of access, care, and quality. ACOs rely heavily on Health Information Technologies (HIT) and decision support systems for better coordination among the participants, centered around PCPs as the gatekeepers of care. The introduction of ACOs was also a departure from the traditional fee-for-service (FFS) structure to a more value-based model through the incorporation of shared savings. Shared savings is a payment strategy that incentivizes providers to decrease health care spending for a population by sharing a percentage of savings with the payers realized from their efforts.

The Medicare Shared Savings Program (MSSP), the largest of the ACO programs, incentivizes ACOs to reduce spending and improve the quality of care. ACOs can be classified based on two risk contract models - upside and downside. New ACOs commonly participate under the upside risk only contract or one-side model through Track 1, where they are not penalized for exceeding the benchmark spending, are eligible to share savings for three years and may extend the contract for an additional three years. On the other hand, CMS encourages larger and experienced ACOs to participate under the downside risk or two-sided models (Tracks 2 and 3). Under these models, ACOs are penalized for exceeding the spending threshold. While these ACO share the losses, the shared savings percentage is higher than that of Track 1. In 2018, with the support of the National Association of ACOs (NAACOS), CMS introduced an additional contract - Track 1+, a two-sided model with lower risk levels and was introduced to help ACOs transition to higher risk models (Tracks 2 and 3). The three downside tracks put the ACOs under more risk-sharing.

Over the past decade, research on ACOs has gained a lot of momentum as the industry focus has been gradually shifting from the popular volume-based practices towards value-based approaches with innovative payment models. Fisher et al. (2006) did a seminal work that explains the fundamentals of ACOs and Burwell (2019) explains the ACO model through the lens of payer-provider partnerships. The initial works classify ACOs across several dimensions - (i) public and private programs (Fisher et al. 2012), healthcare IT infrastructure (Fisher and Shortell 2010), depth of the network (Shortell et al. 2015), and ACO size (Shortell et al. 2010). These studies, among others, clearly distinguish the characteristics of different ACOs and how heterogeneity in size or structure may influence ACO performance. While these studies provide a synthesized evidence on how ACOs differ, performance improvements can be achieved by many factors, which needed more in-depth investigations. Later studies provided more insights on the organizational characteristics and performance, using more granular data and utilizing the outcomes from these early works. As Ganguli et al. (2020) remark, physician composition may play a larger role in ACO success.

3.1.2 Research Questions and Contributions

Prior research and industry studies identified service delivery and ACO experience as key issues affecting ACO performance (Wilson et al. 2020, McKinsey. 2020). Given these findings, we assert that the role of service delivery through provider composition and experience on ACO performance merits additional investigation. Understanding how these factors influence performance could inform ACOs in deploying initiatives at the organizational level. We have chosen three factors to study their influences on ACO performance as follows: (a) provider composition or the proportion of primary care services provided by specialists and NPs to the primary care services provided by PCPs, (b) ACO experience, and (c) the interaction between experience and the service delivery on ACO performance.

The first factor involves an examination of *provider composition* with regard to primary care delivery. It is important because while the ACO attempts to shift healthcare to a more PCP-patient-oriented delivery, there is a lack of available PCPs (Heath 2018). In this scenario, it is pertinent to examine how ACOs can use specialists and NPs to improve their performance and how geographical boundaries will affect their usage. Given the above facts and trends, we argue that policymakers and ACO practitioners must understand the performance implications of provider composition. This examination of provider composition relates to the healthcare objective of access to care and its financial and quality implications.

The second factor pertains to *ACO experience*. Since the formation of ACOs in 2012, some ACOs have shown consistent performance over time, and some have even expanded their operations in other states to treat more patients as revealed both in our data and CMS reports (CMS 2021). However, many ACOs drop out of the program each year, exhibiting weak financial or quality performance (LaPointe 2018). The CMS allows ACOs to join the program with a flexible setting but wants them to take more accountability by posing stricter benchmarks over time. ACOs must perform satisfactorily in terms of savings and quality to achieve shared savings and survive under downside-risk contracts. Under each risk model, the ACOs perform the same operations and improve their experience in delivering better care. On the other hand, as they transition from a low risk to a high risk model, they take more accountability and initiatives. For this reason, we examine the impact of experience, both in terms of years of operation and risk model, on ACO performance.

We also explore the *moderating roles of experience and risk model on the relationship between provider composition and ACO performance* as the third factor. As they gain experience within a risk model, it is pertinent to observe how that experience affects the impact of service delivery via provider composition on ACO performance. It is also interesting to explore the moderating role of risk model on the relationship as higher risk models require stringer benchmarks, both in terms of quality and financial performance. Therefore, we individually analyze the moderating role of experience within a risk model and risk model on the relationship between provider composition and performance.

We perform an empirical analysis using longitudinal performance data of ACOs between 2016 and 2019 with more than 600 ACOs with a Medicare contract and rigorous methods to obtain a fine-grained picture of how ACOs are performing and how they can improve under industry constraints. We find that provider composition has two different effects on ACO performance. On the one hand, ACOs generate more short-term savings by providing more primary care services via specialists and NPs. On the other hand, using specialists for primary care reduces the quality of care, even in the short-term. This is because specialists are not trained to provide holistic care. With regard to experience, we also find that depending on their contracts, ACOs can be strategic through their operational experience and risk model. They should focus on cost reduction in their initial years and slowly invest in quality and process improvement initiatives over time. Finally, ACOs having more primary care services through NPs gain more savings both in the short-term (experience) and long-term (risk model). However, ACOs need to improve the quality of care as having more specialists and NPs providing primary care services hurt them in the long-term.

In a post-hoc analysis, we find that ACOs with more primary care services through federally qualified health centers (FQHC), a prominent safety-net center, are associated with higher savings rates. This finding should encourage ACOs to form associations with these centers in rural areas to deliver primary care and increase access to care to prevent expensive hospitalizations or delay in proper care. In another post-hoc analysis, we use the socio-economic data from the Center for Disease Control (CDC) and find that ACOs operating in states with more population having infrequent check-ups and failed visit doctors because of cost are not associated with higher quality score. Collectively, these results demonstrate the policy implications for ACO models in areas of less access to care.

This chapter makes practical contributions to the healthcare operations management (HOM) literature by exploring the ACO characteristics associated with the financial and quality performance of ACOs. Service providers (or providers) play a critical role in population health management in service operations, especially in HOM. Their actions determine the quality of care and the cost of healthcare. Payers such as CMS push providers to take more accountability in reducing healthcare costs while increasing access to care and quality. The ACO model is a vital program started by the CMS and later adopted by other payers to achieve the triple aim of healthcare and motivate providers to transition from a volume-based to a value-based approach. ACOs are PCP-focused organizations that manage the care episodes of patient populations. However, they often deliver services through non-PCP and non-physicians. Our findings offer guidelines on how ACOs service delivery via provider composition and experience affect their performance. We present managerially relevant insights to improve the ACO model further. In the next subsection, we briefly describe the related works in the literature and highlight our contributions.

3.1.3 Literature Review

This work aims to analyze the impact of service delivery and experience in the context of ACOs. There is substantial research in operations management that associates service delivery and experience with organizational performance (Chandrasekaran et al. 2012, Anand et al. 2021). Research on ACOs focusing on organizational aspects shows that the ACO model needs continuous examination on their performance evaluations, especially in a highly competitive environment with multiple providers, requiring effective coordination between ACOs and their care delivery system. McClellan et al. (2010) highlight the importance of primary care services and clinical transformation on ACO performance and suggest that the proper utilization of primary care services should be a priority for the ACOs. Multiple studies point out significant variations in primary care practices (Finkelstein et al. 2017, Weigel et al. 2016, Weiss et al. 2013). Kaufman et al. (2019) observe highly variable financial performance across ACOs. Other studies identify several factors that impact the success of ACOs, such as the type of ACOs (McWilliams et al. 2018), the responsibility of PCPs in coordinating care (Rittenhouse et al. 2009), specialist office visits (Shetty et al. 2019), utilization of NPs (Perloff et al. 2016), ACO experience, sizes, and prior experience with more stringent contractual measures (Muhlestein et al. 2016, McWilliams et al. 2016).

Prior research, thus, reveals that ACO competence is a combination of their resources, such as the care providers and the system, which has grown over time through experience. However, only a few studies have explored the impact of service delivery through provider composition and experience on performance. Ouayogode et al. (2017) analyze association of ACOs with hospitals and ACO beneficiaries across different geographical regions, mostly observing the performance difference between rural and other regions. Zhu et al. (2019) study the proportion of primary care providers and physician leadership on ACO performance. Unlike these studies, our focus on service delivery via provider composition and experience provides more insights on the operational characteristics of ACOs that can be utilized to improve their performance. We specifically examine the proportion of primary care services provided by specialists and NPs over PCPs instead of the number of providers and how ACOs utilize their experience across risk models. Additionally, we also consider two important extensions related to ACO geographical operations and socio-economic factors.

We present the theoretical development and hypotheses in Section 2. Section 3 describes our data, variables, empirical results, robustness checks, and post-hoc analysis. In Section 4, we discuss the policy implications. Section 5 describes the contributions and future opportunities of this chapter.

3.2 Theoretical Development and Hypotheses

This work seeks to advance our understanding of the impact of service delivery via provider composition and experience on ACO performance through the lens of service delivery and organizational learning literature. Existing literature estimates that physicians' decisions account for approximately 80% of all spending by the ACOs (Mendelson et al. 2017). Increasing the number of PCPs is considered important in the ACO model, and ACOs' success on parameters of improved care, better population health, and reduced costs may largely depend on its ability to attract PCPs. We visited one of the biggest ACOs in Texas several times during this research and received valuable information. They have 335 PCPs working under the ACO model and they plan to increase PCPs and NPs to deliver more primary care.

However, the national shortage of PCPs is a challenge for the ACOs (Siddiqui and Berkowitz 2014). The shortage of PCPs and the high importance of primary care services prompt healthcare groups, including ACOs, to look beyond PCPs for primary care services and deploy healthcare through specialists and NPs. Delivering primary care services through specialists is not uncommon, as the shortage of generalist physicians is not a new phenomenon (Aiken et al. 1979, Janson and Weiss 2004, Olfson et al. 2020). Similarly, NPs are also attractive options for primary care services given their training and reduced expenses. Healthcare delivery is a social setting that follows the rules and policies to engage patients, guided by the policymakers, organizations, and the providers' own training.

While healthcare research has received much attention in the literature, the literature highlights a need to understand further the impact of provider composition within service delivery and the impact of ACO experience, which adds to the prominence of this chapter. We explain our theoretical development using a simple conceptual diagram in Figure 3.1 that offers a high-level overview of the relationship between ACO service delivery, experience, and performance. We denote direct relationships by solid arrows and moderating behavior of experience on the relationship between ACO service delivery and performance using dotted arrows. It is important to note that the service delivery variables, primary care by specialists and primary care by NPs, are constructed by taking the proportion of primary care services provided by specialists and NPs to the total number of primary care services provided by PCPs. In this way, we focus on service utilization rather than the number of providers in this chapter.

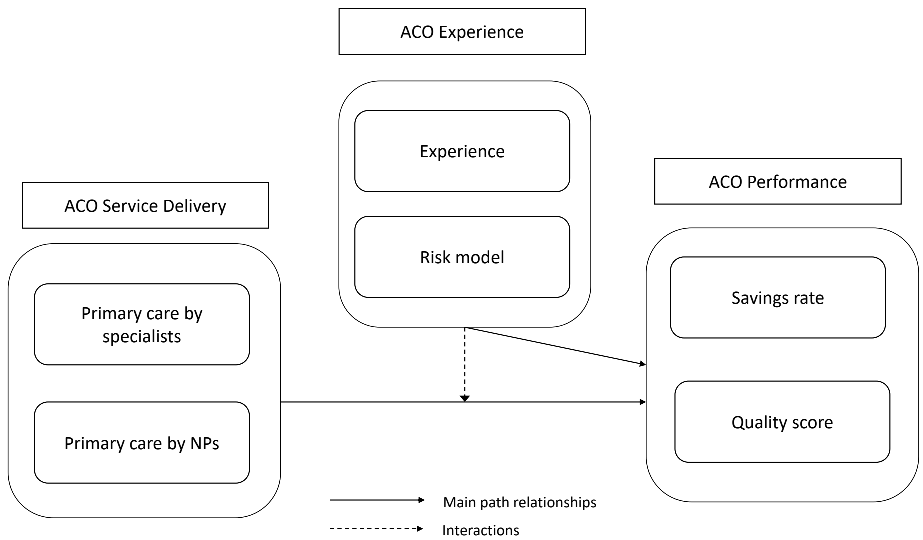


Figure 3.1: Conceptual model: ACO service delivery and experience on performance

3.2.1 ACO Service Delivery

The ACO model is rooted in the abilities of PCPs as designated providers for every patient to ensure integrated care (Starfield 1998). A PCP is defined as a physician specializing in Family Medicine, Internal Medicine, or Pediatrics who provides care to patients at the point of first contact PCPs observe the patients and may refer them to specialists, nursing homes, or hospitals based on care requirements. PCPs are the gatekeepers of care coordination, experts in communication with patients and other providers, and mostly take a generalist approach to care. However, ACOs are also impacted by the shortage of PCPs and may utilize specialists and NPs to offer primary care services. We, therefore, examine healthcare service delivery by looking at primary care services provided by specialists and NPs. We ground our hypotheses from the existing literature on primary care access and healthcare quality. Senot et al. (2016) discuss two broader quality categories

- conformance quality and experiential quality. Conformance quality indicates the degree to which a product meets the required specifications (Garvin 1987). For the ACOs, the conformance quality refers to the set of conditions set by CMS and the ACO board. On the other hand, experiential quality refers to the interactions between the caregivers and the patients and measures the perceived quality (Chandrasekaran et al. 2012). We discuss the quality implications of ACO service delivery using specialists and NPs through the lens of primary care access, ACO setup, and quality dimensions.

3.2.1.1 Primary Care by Specialists

Access to primary care can aid in positive health outcomes (Starfield and Macinko 2005). ACOs focus on disease management through preventive actions by offering timely care and coordinated practice to reduce costs of care while maintaining the quality of patient care. Primary care providers are an essential platform for general care and, more importantly, early diagnosis and treatment of disease. Without timely intervention, a patient's health may deteriorate, which would require hospitalization, resulting in higher spending and poor quality of care. The proportion of primary care physicians has not changed substantially between 1996 and 2015 and is unlikely to improve in the next few years (Lancet 2019). Primary care is also less expensive than specialty care and patients having easy access to PCPs tend to spend less money on health services and obtain better care. A recent study on 620 ACOs shows that a balance of PCPs and specialists results in a cost reduction of $1129 per beneficiary along with lower ED visits, lower hospital discharges, and reduced skilled nursing facility discharges (Shetty et al. 2019). Therefore, it is pertinent to analyze the relationship between specialists offering primary care services compared to that by PCPs and ACOs. Having specialists perform primary care services may offer benefits in terms of access to care and ACO supervision by ensuring conformance quality. While specialty charges are higher than PCPs on average, having primary care to patients may prevent expensive future procedures and improve the quality of care. Thus, higher rates of primary care services offered through specialists can help ACOs achieve their financial outcomes. We posit that:

Hypothesis 1A: ACOs providing more primary care services through specialists are likely to have higher savings than other ACOs, on average.

Dagger et al. (2007) define technical quality as the "expertise, professionalism, and competency of a service provider." The authors relate the definition to a physician's knowledge and skills in diagnosis and analysis. Patients seek higher levels of expertise from their providers in terms of diagnosis of disease and care coordination. Primary care contributes to access to care and appropriateness, including avoiding redundant treatments. As Baicker and Chandra (2004) remark, care appropriateness may suffer at the hands of specialists who are trained to work at the level of specific diseases. In other words, specialists may "overdo" primary care services that require more generalist approaches. Specialists lack the generalist approach to primary care, resulting in a lower experiential quality when they communicate with their patients and provide services. Through continuous treatments and follow-ups, PCPs are more familiar with their patients. The long-term relationship facilitates a generalist approach that prioritizes and personalizes care best suited for different patients, resulting in higher experiential quality.

Primary care services provided by specialists may result in lowering the desired quality performance. Studies show that seeking primary care services through a specialist may not achieve better quality; specialists can overestimate the likelihood of illness in patients, leading to adverse effects and medical errors (Franks et al. 1992, Hashem et al. 2003). While the collaborative environment and norms set by the ACOs may reduce the overall cost of care of primary care services through specialists, the quality of care can deteriorate. Thus, specialists may offer substantial conformance quality in providing primary care services, but their experiential quality compared to PCPs may worsen, leading to lower patient satisfaction. Another view offers a perspective of the experience gained by specialists who offer their services at multiple places. While that may help expand specialist knowledge, a substantial portion of that knowledge is not portable (Huckman and Pisano 2006), especially for generic primary care services. For specialists, we hypothesize the following:

Hypothesis 1B: ACOs providing more primary care services through specialists are likely to have lower quality than other ACOs, on average.

3.2.1.2 Primary Care by NPs

Physician shortages and an aging population have increased the importance of NPs. There are approximately 234,000 licensed NPs in the United States, and they form an important element in urban and rural primary care practices. Primarily, offering more primary care through NPs includes two advantages : (i) NPs can perform many services that PCPs provide and at a lower cost, and (ii) NPs can be trained quicker than PCPs. NPs can work both autonomously and collaborate with other physicians while providing primary care services. An expansion in the number of NPs offering primary care services may keep increasing the access to basic primary care, especially in remote areas. American Academy of Nurse Practitioners estimates that approximately 90% of NPs receive primary care training, and 75% practice primary care (AANP 2014). DesRoches et al. (2013) comment that NPs are more likely to give primary care services to disadvantaged Medicare patients than physicians. Perloff et al. (2016) find that primary care through NPs is more cost-efficient than that through physicians for Medicare beneficiaries. The authors note that "Medicare program could obtain significant cost savings if more NPs were providing primary care services to beneficiaries." With NPs, ACOs can also maintain conformance quality as NPs are trained to follow guidelines and coordinate with physicians. We hypothesize:

Hypothesis 2A: ACOs providing more primary care services through NPs are likely to be associated with higher savings than other ACOs, on average.

Quality of care encompasses both clinical and experiential aspects. NPs take a more generalist approach to patient care given their holistic education in clinical training. Their care delivery constitutes patient's overall wellness and a "sense of caring for others (Callister and Hobbins-Garbett 2000)." Wilson et al. (2005) find that NPs can provide almost the same quality of care as an expert and even better than non-expert physicians. Swan et al. (2015) also confirm the quality of NPs. In a primary care setting, we can consider PCPs as the expert. Horrocks et al. (2002) find that NPs, in some cases, may even provide better care than doctors. The same study shows that NPs did longer consultations and more investigations than physicians. Stanik-Hutt et al. (2013), in a systemic review of research on NPs, find that quality of care provided by NPs is mostly at the same level as that provided by physicians and better in some cases. Additionally, NPs work with multiple physicians, observing a variety of processes and and learning from them, This can be beneficial since expertise in healthcare industry is often obtained through implicit learning (KC and Tushe 2021). The holistic education of NPs, coupled with their experience and more time with the patients may help better patient care and satisfaction, indicating a higher experiential quality. Therefore, we hypothesize:

Hypothesis 2B: ACOs providing more primary care services through NPs are likely to be associated with higher care quality than other ACOs, on average.

3.2.2 Experience

Experience plays a vital role in the sustenance of the ACO model. The CMS wants ACOs to make fast transitions to two-sided contracts. Two-sided contracts have two major benefits: (i) letting ACOs take more responsibility in reducing cost and quality and (ii) inducing more physicians to join ACOs for higher financial rewards (Song and Fisher 2016). Recent studies show that ACOs participating for a longer time generate greater savings per beneficiary (Kocot 2016, Introcaso and Berger 2017). Organizational learning curve theory asserts that cumulative production experience is associated with better performance (Dutton and Thomas 1984, Argote et al. 2000). Firms can learn by (i) performing the same activity over time (autonomous learning) and (ii) undertaking improvement activities (induced learning) (Levy 1965). March (1991) argues that organizations try to trade-off between these two via exploitation or refining their systems and exploration or investing in new opportunities.

In our context, ACOs focus on both types of learning to ensure better financial and quality performance. However, ACOs in different stages may show heterogeneity in their learning process. ACOs in their initial stage or under the one-sided contracts may adhere more to the norms set by CMS and their own leadership boards and focus on autonomous learning because they have higher motivation to perform the repeated tasks and reduce the costs of operations. However, more focus on autonomous learning may hurt their inductive learning, reducing quality improvement initiatives. Such a strategy may hurt their quality score since CMS assigns a substantial weightage on quality innovation measures to construct quality score variable. On the other hand, advanced ACOs, under the two-sided contracts, are under pressure to perform better due to higher penalties. These ACOs are more likely to have more process improvement initiatives than the ACOs under no penalty. Due to this, they can incur more cost of operations and are likely to generate less shared savings than the ACOs under one-side risk contracts under the current ACO model. Therefore, we hypothesize a contrasting relationship between cost and quality through the lens of autonomous learning and induced learning and posit:

Hypothesis 3A: ACOs with more experience are likely to be associated with greater savings than other ACOs, on average.

Hypothesis 3B: ACOs with more experience are likely to be associated with lower quality score than other ACOs, on average.

Hypothesis 4A: ACOs in a higher risk model are likely to be associated with lower savings than other ACOs, on average.

Hypothesis 4B: ACOs in a higher risk model are likely to be associated with higher quality score than other ACOs, on average.

3.2.3 Experience and Risk Model on Service Delivery Through Specialists and NPs

Experience and risk are two different ACO characteristics. While the former represents the exploitation part of learning where ACOs mostly learn by doing, the latter is related to ACOs improvement caused by their transition to a higher risk model. The literature on organizational learning theory posits that organizations may learn differently based on their capacities (e.g. Cohen and Levinthal 1990, Miner and Mezias 1996) . Experience can facilitate learning, but opportunities need to be exploited to achieve substantial gains (Dutton and Thomas 1984). Organizational learning can be broadly represented as a set of three consecutive tasks in a cyclical fashion - (i) enactment or information receival and action, (ii) selection of the information, and (iii) retention of the information. Through these cycles, new information becomes available to the organization's members and organizations may or may not incorporate them in their structure. Organizations may reexamine their approaches and utilize the new information to improve their services. The new information can come from both exploitation and exploration. However, exploration offers a higher degree of change to the existing set of norms. Thus, within the same risk models, ACOs mostly learn by focusing on their savings rate. They utilize their physicians and non-physicians to deliver services and achieve to stay lower than the expenditure benchmark. Higher risk models come with different action items and may force the ACOs to change the way they perform their operations. Two-sided risk contracts result in more shared savings but require ACOs to share the losses. Thus, the ACO members also need to be familiar with the new practices and act on them for better performance.

ACOs facilitate communication among their members. Since PCPs are the primary provider of primary care services and accountable for their patients' care episodes, the ACO model may get the most out of utilizing more PCPs as the gatekeepers. Due to the shortage of PCPs or unavailability at locations, ACOs utilize specialists to provide primary care. Yet, the relationship between the service delivery through specialists and performance may vary across experience and risk models. Specialists, while adhering to ACO policies and routines, may not be efficient in terms of both financial and quality performance in the long run. While access to care through specialists gives ACOs the benefit of preventing expensive treatments, such benefits may not outweigh the cost in the long run. Most specialists are still hired on fee-for-service contracts, and thereby, shared savings generated by the ACO may be offset by a loss of FFS revenue (Barnett and McWilliams 2018). Financial incentives to specialists is also an ineffective lever for cost reduction and quaity improvement measures in the long-term (Khullar et al. 2018). Also, while primary care is mostly viewed as an "approach to providing healthcare," specialists are usually trained in sets of specific services. Thus, as an ACO grows over time, both in years and risk contracts, having a high proportion of primary care through specialists may not be efficient both in terms of financial and quality performance.

Hypothesis 5A: Experience negatively moderates the relationship between primary care services by specialists and ACO savings rate.

Hypothesis 5B: Experience negatively moderates the relationship between primary care services by specialists and ACO quality score.

Hypothesis 5C: Risk model negatively moderates the relationship between primary care services by specialists and ACO savings rate.

Hypothesis 5D: Risk model negatively moderates the relationship between primary care services by specialists and ACO quality score.

NPs, on the other hand, work mostly with PCPs, are well-trained in primary care services, have a holistic patient care approach, and are likely to be more flexible with organizational changes. ACOs can change their provider composition from year to year and shift how they deliver care. Between 2013 and 2018, NPs, on average, grew from 17.6 percent to 25 percent as a proportion of total caregivers, while the proportion of PCPs declined from 60 percent to 42 percent during the same period (Nyweide et al. 2020). While NPs work more with physicians, Horrocks et al. (2002) and Stanik-Hutt et al. (2013) find that their care quality is as good as that of PCPs and billing rates are lower. Medicare patients receive more annual check-ups and chronic care management visits in which NPs can play a better role (Ganguli et al. 2017). Thus, as ACOs switch to a higher-risk model, utilizing NPs can become more cost-effective, as highlighted in the literature (Salsberg 2015). Yet, a higher risk model may not necessarily see a better quality of score of an ACO utilizing more NPs.

Aledade, one of the most successful ACOs in the MSSP program, remarks that "Primary care physicians are ideally positioned to be leaders in this (downside risk) process. Many PCPs have deep roots in their communities and are already familiar with their colleagues' strengths, weaknesses, and opinions on quality improvement initiatives (Aledade 2020)." Thus, as other ACOs also note, PCPs develop strong networks over time across other physicians and patients. Additionally, care delivery needs to be centered around a value-based system rather than a single PCP (Nyweide et al. 2020). This becomes even more challenging because of the limit to NPs due to state and federal health policies. This may hurt the ACOs in a higher risk model in improving access to care and quality of care when diffusing more responsibilities to NPs from physicians turns difficult. Therefore, we post the following hypotheses.

Hypothesis 6A: Experience positively moderates the relationship between primary care services by NPs and ACO savings rate.

Hypothesis 6B: Experience positively moderates the relationship between primary care services by NPs and ACO quality score.

Hypothesis 6C: Risk model positively moderates the relationship between primary care services by NPs and ACO savings rate.

Hypothesis 6D: Risk model negatively moderates the relationship between primary care services by NPs and ACO quality score.

3.3 Data, Variables, and Methodology

This section describes our data collection process, variables, and the econometric model to test our hypotheses developed in the previous section.

3.3.1 Data

Our unit of analysis is the ACO. We analyze our research questions on a panel dataset of Medicare ACOs primarily for two reasons: (i) Medicare is still the largest single payer of ACO contracts and (ii) data availability. CMS records multiple quality measures under four broad categories - patient/caregiver experience, care coordination/patient safety, preventive health, and at-risk population. The last broad measure includes diabetes, hypertension, vascular disease, heart failure, and artery disease. In addition to these measures, our data also include the performance against benchmark and improvement to the quality score variable. We investigate our hypotheses using the dataset on ACOs between 2016 and 2019. From July 2019, CMS introduced the new "pathways to success" program that introduced substantial changes to the ACO program. Thus, the 2019 data include the data for the first six months in 2019 standardized by the CMS. Every year, several new ACOs engage with CMS with Medicare ACO contracts while some ACOs drop out of the program. Therefore, it is difficult to obtain a consistent and identical set of ACOs across years. Our cleaned-up sample consists of 675 ACOs in an unabalanced panel, comprising 1908 observations across 4 years. We use STATA 14 to perform our statistical analysis.

Although data for ACOs were available from 2013, we decided to use the 2016-2019 time period due to the following reasons. First, a period starting from 2016 provides a good window to capture the ACO operations mostly for two reasons. First, 2016 was the first year in which the early ACOs, who joined in 2012 or 2013, had their benchmarks reset. Second, CMS also started comparing ACO location to the estimate ACO efficiency[1]. Additionally, our data include both experienced and new experienced ACOs since the early ACOs in the program complete 3 years by 2016. Quality measures changed from 2016, thus the comparison of quality scores is difficult (Saunders et al. 2017). Additionally, starting 2016, CMS started a new initiative called Meaningful Measures for further refinement of performance metrics, consistent with the ACO model objectives. Finally, the number of ACOs have significantly increased over time. Of the 103 MSSP ACOs that started in 2013, 74 ACOs remained in 2016, which is a reasonable number according to the experts (Broome 2017). Therefore, we do not lose much information by excluding the first 3-year ACOs and focusing on the newer performance data.

3.3.2 Variables

We measure the performance of ACOs using savings rate and quality score for each ACO *i* ACO in year *t*. Below, we describe variables along with the independent variables and additional controls. We provide the descriptive statistics and correlation analysis of the variables in Tables A1 and A2 in Online Appendix.

3.3.2.1 Dependent Variables

ACOs receive shared savings if they do not exceed their benchmark spendings while maintaining quality of care. This motivates us to include savings rate and quality as the dependent variables[2].

Savings rate (sav_{it}) - It captures the savings rate for an ACO *i* in year *t* and is estimated by CMS by subtracting the assigned beneficiary expenditures from the total benchmark expenditures

[1] https://www.healthindustrywashingtonwatch.com/2016/06/articles/regulatory-developments/medicare-medicaid-services-regulations/cms-finalizes-changes-to-medicare-shared-savings-programaco-benchmark-rebasing-rules/

[2] https://www.cms.gov/Research-Statistics-Data-and-Systems/Downloadable-Public-Use-Files/SSPACO

as a percent of total benchmark expenditures. A positive number represents a positive savings rate and a negative number denotes a negative savings or loss. The benchmark expenditures are decided by the CMS earlier, which are standardized based on the size and type of ACOs along with several other factors. There are 680 total observations with a negative savings rate, almost 36% of total observations.

Quality is a multi-dimensional construct and, therefore, is difficult to measure and test. Studies in OM measure quality at the product or the organizational level. In this chapter, we examine the effects of our variables of interest on quality; this measure of quality is at the ACO or the firm-level. Firm-level quality measure has been used across industries such as airlines, food and drug industries, dairy and meat industries, and hospitals (Theokary and Justin Ren 2011).

Quality score ($qual_{it}$) - CMS assigns a quality of score of 1 (or 100%) in the first performance year of ACO if all measures were completely reported; otherwise, it assigns a score of less than 1 if one or more measures were not completely reported. From the second performance year onwards, CMS considers both the reporting of all measures and the ACO's performance against established benchmarks to calculate the quality score. The choice of a uni-dimensional quality variable such as the quality score not only captures a wide variety of quality scores but also significantly reduces the complexity of variable selection. For the quality score analysis, we drop the first year ACOs since they all received a quality score of 1.

3.3.2.2 Independent Variables

Our primary variables of interest are:

PC by specialists ($pcsp_{it}$) - Since our focus on healthcare service delivery through non PCPs is a comparable measure, we measure PC by specialists by dividing the total primary care services provided by specialists to primary care services provided by PCPs. This operation allows us to measure the proportion of primary care services offered by non PCPs and provides a better stand-point to analyze the service delivery. Thus, we do not focus on the number of physicians but the number of primary care services in all cases.

PC by NPs ($pcnp_{it}$) - Similar to the PC by specialists, we operationalize PC by NPs by dividing the total primary care services provided by NPs to primary care services provided by PCPs.

Experience (exp_{it}) - This variable denotes the number of years an ACO is associated with a payment contract with Medicare under a particular risk model.

Risk model ($risk_{it}$) - We assign a Risk model score of 0 to ACOs under one-sided or upside risk only contract. Since a majority of ACOs are in one-sided contract, we combined Tracks 2, 3, and 1+ and assign the Risk model score 1 to those ACOs.

3.3.2.3 Control Variables

We describe a list of control variables relevant to this work, which are represented as a vector X in our regression equation.

Beneficiaries (ben_{it}) - This variable controls for ACO size. Note that an ACO must have a minimum 5,000 attributed patients or beneficiaries. However, CMS calculates the attributed patients based on the full 12 months eligibility. This variable is adjusted downwards and may show a few ACOs with less than 5,000 attributed patients in the data summary[3]. We perform a log-transformation on this variable to ensure normality assumptions are maintained.

States ($states_{it}$) - This variable denotes the number of states an ACO operates.

PCPs (pcp_{it}) - We control the effect of the number of PCPs. Following Shortell et al. (2015), who use the percentage of primary care providers in their cluster analysis, we use the total number of primary care physicians in our analysis. This variable denotes the total number of PCPs associated with ACO *i* in year *t*.

Specialists (sp_{it}) - It refers to the number of specialists associated with ACO *i* in year *t*. Specialists have an essential role in the ACO framework.

NPs (np_{it}) - It denotes the number of NPs associated with an ACO.

Female (fem_{it}) - It controls the number of female beneficiaries.

Inpatient expenditures (inp_{it}) and Outpatient expenditures (out_{it})- control for the outpatient and inpatient processes, by using the weighted expenditures.

[3]The reader may refer to https://www.naacos.com/assets/docs/pdf/2019/Final-NAACOS-AsTreatedDID-SavingsEstimateReport2017.pdf for the attribution method used by the CMS.

CMS estimate the output variables by already accounting for the population risk and therefore, we do not include the combined measure of four risk categories - average ESRD (End Stage Renal Disease) HCC risk score, average Disabled HCC risk score, average Aged/Dual HCC risk score, and average Aged/Non-Dual HCC risk score. These combined measures are highly correlated with the number of beneficiaries. All the Tables in Appendix B. In Table B.1, we see that ACOs have more specialists than PCPs and NPs, on average. This suggests the ongoing shortage of PCPs in the healthcare market and more focus on specialty providers.

3.3.3 Econometric Model

We examine how service delivery through primary care services through specialists and NPs and experience in terms of number of years and risk model affect ACO savings rate and quality score. We thus formally present our main effects models as follows

$$sav_{it} = \beta_1 pcsp_{it} + \beta_2 pcnp_{it} + \beta_3 exp_{it} + \beta_4 risk_{it} + \gamma \mathbf{X} + \epsilon^1_{it} \tag{3.1}$$

$$qual_{it} = \beta_5 pcsp_{it} + \beta_6 pcnp_{it} + \beta_7 exp_{it} + \beta_8 risk_{it} + \eta \mathbf{X} + \epsilon^2_{it} \tag{3.2}$$

where β_1 and β_1 denote the effect of PC by specialists and PC by NPs on savings rate. If they are both positive, then we can infer that both contribute to positive savings rate. On the other hand, β_5 being negative and β_6 being positive indicate that PC by specialists have negative impact on quality and PC by NPs have positive impact on quality. The coefficients for exp_{it} and $risk_{it}$ have opposite effects on both savings rate and quality score. For example, a positive β_3 and a negative β_7 support that exp_{it} positively affects savings rate and negatively affects quality score. In contrast, a negative β_4 and a positive β_8 support that $risk_{it}$ negatively affects savings rate and positively affects quality score. These results are shown in Models 2 and 5 in Appendices A4 and A5, respectively.

To observe how each exp_{it} and $risk_{it}$ moderates the relationship between service delivery and ACO performance, we observe the interactions in Equations 3.1 and 3.2. For each model, we add the interaction terms one at a time. First, we observe how the two experience variables influence the relationship between pc by specialists and ACO performance, following our hypotheses that β_{13}, β_{15}, β_{21}, and β_{23} are all negative. On the other hand, the moderating effects of experience variables on the relationship between pc by NPs and ACO performance are mixed. While we expect β_{14}, β_{16}, and β_{22} to be positive, β_{24} negatively moderates the relationship between quality score and PC by NPs. These results are displayed in Models 3 and 6 in Tables B.3 and B.4 in Appendix B.

$$sav_{it} = \beta_9 pcsp_{it} + \beta_{10} pcnp_{it} + \beta_{11} exp_{it} + \beta_{12} risk_{it} + \beta_{13} exp_{it} X pcsp_{it} + \beta_{14} exp_{it} X pcnp_{it} + \beta_{15} risk_{it} X pcsp_{it} + \beta_{16} risk_{it} X pcnp_{it} + \zeta \mathbf{X} + \epsilon_{it}^3 \tag{3.3}$$

$$qual_{it} = \beta_{17} pcsp_{it} + \beta_{18} pcnp_{it} + \beta_{19} exp_{it} + \beta_{20} risk_{it} + \beta_{21} exp_{it} X pcsp_{it} + \beta_{22} exp_{it} X pcnp_{it} + \beta_{23} risk_{it} X pcsp_{it} + \beta_{24} risk_{it} X pcnp_{it} + \lambda \mathbf{X} + \epsilon_{it}^4 \tag{3.4}$$

Our data include multiple ACOs over a period of four time periods. With panel data, we can estimate the effects of our independent variables on outcome variables over time. Panel data helps us with time-invariant omitted variables. The assumption of the random effects model is that the unobserved effect between panels is uncorrelated with the explanatory variables. In a broader sense, in a random effects specification, ACO effects are characterized by a time-invariant component, τ_i. The component τ_i is the random disturbance characterizing ACO i and it is constant through time. From a practical point of view, random effects model is more appealing since ACOs are mostly operated according to the CMS guidelines. However, there may exist unobserved effects between panels. For this, we perform Hausman specification test, which rules out the random effects model ($\tilde{\chi}^2$ = 88.37, df=12) and we choose to proceed with the fixed effects assumption. For the quality score analysis, we use Tobit regression.

3.3.3.1 Accounting for Possible Sources of Bias

In our data, there may remain potential biases associated with endogeneity and causality that require a brief discussion. The nature of our dataset alleviates many concerns about the possible biases. The performance variables, savings rate and quality score are reported by CMS at the end of each performance year. The independent variables that we use in our analysis are reported by the ACOs before the dependent variables are measured by the CMS. Additionally, the dependent variables are generated by the CMS and the independent variables are reported by each ACO. The time-lag between our dependent and independent variables help remove the concerns related to reverse causality.

Our dependent variables are not direct measures and are used in terms of savings and quality ratings. Senot (2019) remarks that savings and improvement in quality may come from both positive (effective care) and negative factors (low-quality care). Since our focus is on service delivery through provider composition and ACO experience and their impact on ACO performance, we control for the standardized outpatient and inpatient expenditures to understand how they affect savings rate and quality score. This can be attributed to the ACO operations; ACOs attempt to lower expensive inpatient admissions and keep outpatient admissions for the needed patients so that they can reduce cost and increase quality.

3.4 Results

We present our results and discuss the robustness tests we did to validate our findings. The estimation results for savings rate and quality score are presented in Tables B.3 and B.4, respectively.

3.4.1 Impact of Service Delivery through Provider Composition

H1A and H1B posit that ACOs providing more primary care services through specialists are likely to be associated with higher savings and lower quality scores. Model 2 in Table B.3 and Model 5 in Table B.4 test these two hypotheses and show that the PC by specialists coefficient is positive and significant for savings rate (0.0089, $p < 0.1$) and negative and significant for quality score (-0.0044, $p < 0.05$). Thus, ACOs having more specialists offering primary care services as a percentage of primary care services provided by PCPs are associated with higher savings and lower quality, supporting both H1A and H1B. On the other hand, H2A and H2B argue that ACOs providing more primary care services through NPs are likely to be associated with higher savings and higher quality scores. The coefficient for PC by NPs in Model 2 in Table B.3 and Model 5 in Table B.4 is positive and significant for savings rate (0.0132, $p < 0.05$) and negative and insignificant for quality score (-0.004, $p > 0.1$). The results highlight that primary care by more NPs is associated with higher savings for an ACO. Thus, we find support for H2A. On the other hand, H2B is not supported.

3.4.2 Impact of Experience

We further examine the two experience related variables - experience (years) and risk model. Savings rate is positively associated with experience (0.0122, $p < 0.01$) in Table B.3. This indicates that one additional year in ACO operations is associated with 0.0122 higher savings rate, on average, supporting Hypothesis 3A. On the other hand, risk model is negatively associated with savings rate (-0.00974, $p < 0.1$) in Table B.3. This result suggests that ACOs in a two-sided risk contract generate 0.00974 lower savings rate than ACOs in a one-side contract, on average, showing support for Hypothesis 4A. As hypothesized, experience (years) and risk model are negatively and positively associated with quality score, respectively supporting Hypotheses 3B and 4B (Table B.4. The coefficient of experience is -0.0017 ($p < 0.1$), suggesting that an additional year in experience is associated with a decrease of 0.0017 unit in quality score. The coefficient for the risk model variable (0.013, $p < 0.01$) suggests that an ACO in a two-sided contract is associated with 0.013 higher quality score than an ACO in a one-sided contract, on average.

3.4.3 Interactions

To better understand the interaction effects, we created the margin plots for the statistically significant coefficients in Figures 3.2 and 3.3. H5A - H5D hypothesize the interactions between PC by specialists and experience and risk model on both savings rate and quality score. Column 3 of Table B.3 shows that the interactions between PC by specialists and risk model on quality score (-0.0230, $p < 0.05$) is negative and significant. Thus, ACOs in a higher risk model having more primary care services through specialists are associated with lower quality score, on average. This supports H5D. We visually describe the interaction in Figure 3.2 and it shows reduction in quality score as PC by specialists increases.

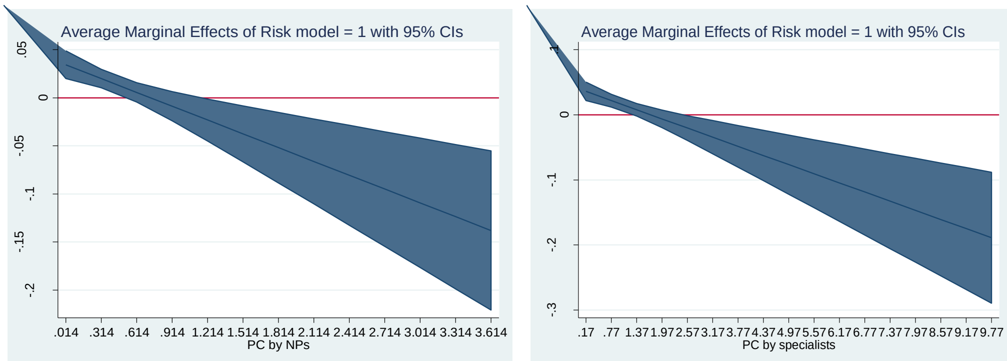


Figure 3.2: Interactions between PC by NPs and risk model and PC by specialists and risk model on quality score

Columns 3 and 6 from Tables B.3 and B.4 show no support form Hypotheses 5A, 5B, and 5C. We hypothesize the interactions between PC by NPs and experience and risk in H6A - H6D. For savings rate, the interaction coefficients between PC by NPs and experience (0.0063, $p < 0.05$) and PC by NPs and risk model (0.05368, $p < 0.05$) are both statistically significant, supporting Hypotheses 6A and 6C (Figure 3.3). More experienced ACOs with higher primary care services by NPs always generate higher savings. Also, NPs are effective in delivering in primary care services when ACOs are in the higher risk model. PC by NPs and risk model (-0.047, $p < 0.01$) on quality score from Model 6 in Table B.4 is negative and significant, supporting H6D. The findings suggest that more primary care services through other physicians and NPs do not help in getting better quality scores to ACOs. H6B is not supported.

3.4.4 Robustness Checks

We perform additional analysis to test the robustness of our findings. The results are available in Tables B.5 and B.6. First, as a validation of the main effects of our regression model of savings rate, we perform quantile regression at 5, 101 25, 50, 75, and 90th percentiles and display the results

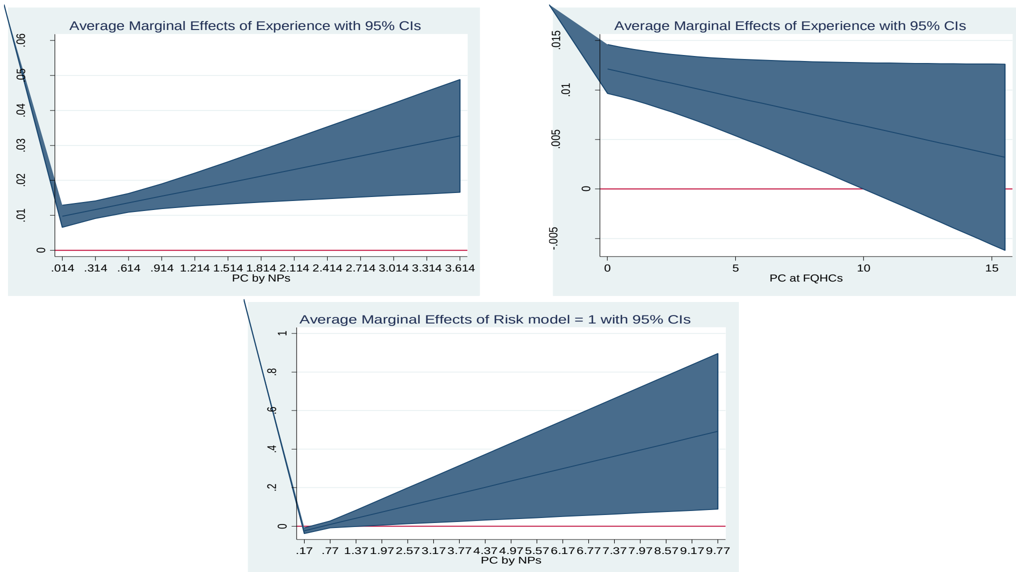


Figure 3.3: Interactions between PC by NPs and experience (years) and risk model on savings rate

in Table B.5. The impact of PC by specialists is consistent across all the percentiles. Experience has a slightly negative impact on savings rate at 5th percentile and risk model has positive effect below 50th percentiles. These findings are not surprising since these ACOs are at lower quantiles. We expect the risk model to be effective at a higher quantile since only 12.68% ACOs in our sample are in two-sided risk models. We observe a similar pattern for the primary care by NPs at lower quantiles. Thus, these effects are mostly driven by ACOs with a higher number of primary care services by NPs. This finding is also reasonable given the construction of these variables and the sample characteristics. Most ACOs usually have a lower number of this and therefore, at lower fractiles, the estimation becomes challenging. Thus, these results need to be considered carefully; bigger ACOs are better in utilizing NPs and FQHCs. Our interaction plots also suggest how NP primary care services varies across experience and risk model. Altogether, this suggests that smaller ACOs need to look at utilizing NPs more efficiently.

Next, we utilize a bootstrapped tobit regression to check the robustness of our main effects on quality score. The results are presented in Table B.6. Since most ACOs have a higher quality score, the bootstrapping may help finesse our results. We find that the the regression results do not differ from our main results, thereby supporting the robustness of our main effects model for quality score.

3.4.5 Post-Hoc Analysis

In this subsection, we present two additional analyses related to the use of FQHCs to provide primary care services and the impact of socio-demographic factors on ACO outcomes. Additionally, we also present our findings from our main results.

3.4.5.1 The Mechanism of Federally Qualified Health Centers

A specific feature of ACO service delivery that is worth examining involves care coordination with multiple providers, especially outside urban areas where access to care is limited. In such areas, care is provided through local centers such as federally qualified health centers (FQHC), community health centers (CHC), and rural health centers (RHC), which Sandberg et al. (2014) refer to as safety-nets. Across the country, approximately 22 million Americans are served by over 1100 CHCs and these health centers provide services to people below the poverty line as well as racial and ethnic minorities (Shin et al. 2013). Takach and Buxbaum (2013), in their study of these centers across eight states for Medicaid beneficiaries, find evidence that local community health teams reduce high-cost utilization and improve the quality of care. Socioeconomic factors play a crucial role in the lack of access to primary, causing higher rates of hospitalizations that can be prevented (e.g. Hansell 1991, Stevens 2002). Parchman and Culler (1999) observe that primary care shortage areas have more preventable hospitalizations. On the other hand, some studies indicate that more primary care physicians do not necessarily lead to better access (Schreiber and Zielinski 1997). Hence, we perform an analysis to relate to the set of variables associated with safety-net to ACO performance and conjecture safety-net has a positive association with both savings rate and quality score.

We operationalize safety-net using three variables: (i) primary care services through FQHCs (PC at FQHCs), (ii) association with CHC (Assoc CHC), and (iii) association with RHCs (Assoc RHC), where (ii) and (iii) are binary variables indicating whether an ACO is associated with CHCs or RHCs or not. We create the PC at FQHCs variable by dividing the number of primary care services at FQHCs by the total primary care services. We expect better access to primary care through these centers, possibly reducing preventable hospitalizations and other serious outcomes. This may lead to less spending for the ACOs and better quality of care to the patients. Primary care services have been consistently associated with fewer specialists and emergency rooms (Martin et al. 1989), not abundant in these remote areas. Thus, we expect these variables are positively associated both with savings rate and quality score. We present our results in Table B.7.

ACOs offering more primary care services at FQHCs as a percentage of total primary care services are also associated with a positive savings rate (0.0021, $p < 0.01$). In other words, between two ACOs with a difference of one unit in PC at FQHCs, the ACO with the higher PC at FQHC is associated with 0.0021 percent more savings. The intuition is that more primary care services at FQHCs result in higher access to care in areas that are often underserved and observe a lack of physicians. While research indicates that FQHCs are associated with lower spending compared to traditional primary care settings (Sokol 2020), our result suggests that ACOs can take the financial risk in providing care to patients at FQHCs to lower healthcare spending. Indeed, a positive association between savings rate and higher primary care services at FQHCs indicates that ACOs should increase their access beyond urban areas. PC at FQHC does not have a statistically significant impact on quality score. On the other hand, the two binary variables, Assoc CHC and Assoc RHC, do not exhibit any significant relationship with savings rate or quality score. This result can happen for several reasons, the sample being the primary one. Among 1908 observations, only 526 ACOs are associated with RHCs, while 402 are associated with CHCs. On the other hand, this result indicates that association with these clinics may not necessarily increase cost and reduce care quality.

3.4.5.2 Socio-demographic Factors on ACO performance

ACO model was implemented to increase access to care. Urban areas, in general, have more providers, and therefore, the competition for patients may motivate them to form ACOs (Lewis et al. 2013). However, their performance may also depend on several socio-demographic factors related to access to care and how patients interact in the system. With the increase in healthcare costs, families also face increased cost-sharing in their health plans (Claxton et al. 2009). Our discussions with the ACO in Texas reveal that they engage more patients in their system through continuous follow-ups and routine checks to prevent delays in care. However, infrequent check-ups may hurt the ACO performance. We operationalize the socio-demographic construct using two state-level variables from the CDC data - Medcost and Checkup. The former indicates infrequent visits to providers due to cost, and the latter refers to the length of time during two routine check-ups. We control for the population using two additional variables from the CDC dataset - general health (Genhealth) and income level (Income). Note that CDC constructs these variables using aggregated surveys at the state level.

We report the results in Table B.8. Medcost and Checkup have no significant impact on savings rate. On the other hand, they are both negative and significant on quality score. The coefficients of Medcost and Checkup are -0.3912 ($p < 0.01$) and -0.0809 ($p < 0.01$). Thus, ACOs quality performance may hurt in states where people do not visit their healthcare providers for higher cost and duration between routine visits are longer. These observations, though at aggregate level, necessitates some policy interventions that may bolster the ACO model in the future.

3.4.5.3 Other Results

We discuss the hypothesized main and interaction effects, which are insignificant, followed by some interesting results from our analysis. Note that the insignificant outcomes may arise due to (i) insufficient data or (ii) other phenomenon or a combination of both. On average, primary care services through NPs do not show any significant impact on quality score. Physicians enjoy full authority to diagnose and treat all conditions in every state. In contrast, NPs in some states practice independently from physicians (full authority), while in some other states operate under the supervision of physicians (partial authority) (Cassidy 2012). This may explain that a higher number of NPs not fully utilized in providing primary care services that are being reflected in the insignificant impact on quality score. However, Model 5 in Table B.4 suggests that more primary care through NPs is associated with higher quality scores, on average, hinting that NPs may have direct and indirect effects on quality performance, which can be an exciting future research avenue.

Column 3 of Table A3 suggests that the interaction terms associated with PC by specialists on savings rate are not significant. Additional research questions may arise from this: What type of specialists are being engaged in delivering primary care services? There are various types of specialists, ranging from working in specific fields to being closer to primary care practices. Therefore, as ACOs become more experienced and move to higher-risk contracts, they may judiciously use specialists. Similar reasoning may follow for the insignificant relationships for the interaction terms involving experience and specialists on quality score.

3.5 Discussions and Implications

This chapter makes several contributions to the HOM literature by analyzing the antecedents of ACO performance and their implications. To the best of our knowledge, this work is the first to focus on the understudied relationship between ACO service delivery and experience on ACO performance and how ACO experience moderates the relationship between service delivery and performance. We operationalize ACO service delivery via provider composition, which consists of the primary care services provided through specialists and NPs. This operationalization of service delivery focuses on how ACOs utilize their non-PCP workforce to deliver care and maintain access to care for their patients. We focus on these two characteristics in their short-term and long-term implications. While the data used in this chapter predate the new "Pathways to Success" program launched by the CMS on July 2019, the implications are still applicable. This new program still maintains the core of the ACO model. However, it requires ACOs to take up more resposibility (or downside risk contract) sooner than before. Our questions are pertinent in the general ACO network model and the findings can help in the "Pathways to Success" program.

3.5.1 ACO Service Delivery

Our results suggest that having primary care services by specialists and NPs may contribute to more savings, on average. Service delivery is an important aspect of the ACO model since most cost reduction of ACOs comes from reducing expensive hospitalizations and treatment delays through better access to care. ACOs differ by their size in terms of physicians, patient population, and geographical presence, among others. We find that the smallest ACO in our sample has 2 PCPs, and the largest one consists of 5697 PCPs. Similarly, the ACO with the smallest patient size has 152 beneficiaries, while the largest one has more than 200,000 patients. Such diversity requires the efficient composition of providers to deliver better care at a lower cost. In spite of being a PCP-centered model, ACOs, on average, have a higher number of specialists and provide more primary care services than PCPs. This suggests that ACOs often utilize other providers to offer primary care services. However, in the long term, under a higher risk model, using specialists for primary care services may not be beneficial for ACOs.

On the other hand, NPs provide both short-term and long-term gains in terms of higher savings. Yet, ACOs need to take proactive actions to utilize more NPs to offer primary care under higher risk models to avoid a reduction in quality of care. Existing research suggests that NPs may not always enjoy full autonomy during their work, which may affect patient care. Existing research reveals that nurse satisfaction and working conditions can improve patient care quality (Poghosyan et al. 2010), which, in turn, may affect the ACO quality. The assignment or attribution of a patient to an ACO requires that the patient receives at least one primary care service provided by a PCP each year; this may hurt NPs' autonomy and satisfaction in general. ACOs may consider these findings and may take necessary under the directives of the respective healthcare guidelines across the states they operate.

Our post-hoc analysis reveals that ACOs with higher primary care services through FQHCs generate more shared savings, on average. This finding should encourage ACOs to extend their operations beyond urban areas and collaborate more with FQHCs to provide healthcare for vulnerable populations. This chapter suggests ACOS, especially Medicare ACOs, should not exclude the patients in those areas or the patients who do not have proper access to quality healthcare. Contrary to popular beliefs, this chapter shows that FQHCs do not necessarily increase operations cost, showing higher savings without reduced quality scores for the ACOs. This indicates that basic primary care services alone may improve the healthcare landscape in underserved areas, which may motivate ACOs to operate across urban and distant areas. CMS should encourage ACOs to reach out to FQHCs and also RHCs, especially in areas where access to quality healthcare is poor. The goal is to increase access to care. Finally, ACOs can also do more in having more PCPs engaged in their organizations. While there are multiple reports on the shortage of PCPs, ACOs still should expand their network by reaching out to more PCPs. In their new plan, CMS introduced telemedicine services to risk-taking ACOs that may reduce the workload of PCPs in the future, which, in turn, may allow more PCP hours to the patients. Furthermore, CMS can use these tools to better access the population for regular check-ups at a lower cost.

3.5.2 ACO Experience

We find that experience is positively associated with savings rate and negatively associated with quality score. In contrast, the risk model has a negative impact on savings and a positive impact on quality. Organizational learning theory guides us to analyze ACOs learning over time and how that experience directly affects performance and also indirectly influence the relationship between service delivery and performance. In the beginning, ACOs are more focused on sustainability and continue to learn by doing or utilizing exploitation. Under a higher risk model, they invest more in quality improvement initiatives, and therefore, try to trade-off exploration with exploitation. Our findings suggest that ACOs under downside contracts may generate low savings but display higher quality scores than ACOs under upside-only contracts, on average. The result is not surprising. ACOs often refrain from taking downside risk contracts or drop out of the program due to fear of losses. Several studies show that the number of ACOs willing to take the downside risk is small. This chapter also show the same concern; on average, ACOs generate lower savings under downside risk. On the other hand, the new "Pathways to Success" program pushes ACOs to take downside risk as soon as possible. However, CMS, along with other payers, needs to consider the burden on the ACOs and they must come up with innovative solutions, which should not be one-size-fits-all. Some policymakers also suggested that CMS change the risk adjustment methodologies to allow risk scores to increase over time, giving ACOs more time to adjust.

To thrive in an extremely competitive healthcare environment with increasing population and health expenditure, ACOs must position themselves to enjoy financial sustainability as well as expanded coverage of the patient population, where experience may play a key role. Our findings indicate how ACOs usually learn. Under any risk model, ACOs focus more on learning through policy adherence and organizational routines and focus more on cost reduction to generate more savings. As they move to a higher-risk model, ACOs need to change their strategies to prepare for the shared loss policy, thereby forcing them to implement innovative processes to increase quality. Two-sided risk contracts ensure that ACOs share the losses with CMS while enjoying a higher shared savings percentage than that in a one-sided contract.

3.6 Conclusion

Rising health expenditure is one of the major concerns for policymakers in the US. The healthcare system suffers from a fragmented nature of care under fee-for-service agreements, which results in excessive waste and duplicate tests due to the low level of coordination among the health providers. Under the ACA 2010, ACOs were created to tackle the issue of increasing healthcare costs while providing a high quality of care through a coordinated system across healthcare providers. The ACO model has been widely implemented across the country during the past decade with participation from both public and private payers. Over time, the ACOs have brought more patients under their umbrella, and since 2017, the net savings have increased. However, healthcare spending, specifically Medicare spending, which accounts for the health expenditure of older people and young people with long-term disabilities, is likely to increase over time due to several factors, including population aging. Additionally, there are significant performance variations across the ACOs since the program's implementation. With these concerns and CMS' ambitious plan of saving approximately $2.9 billion over ten years, ACOs need to provide high quality care while being financially sustainable. From both the healthcare and OM perspectives, it is imperative to examine the ACOs through these two critical elements - savings and quality. Using data on ACO characteristics and performance between 2016 and June 2019, we analyze the performance implications of ACOs and suggest best practices.

PCPs are the gatekeepers in healthcare delivery and ACOs are modeled as a PCP-centered healthcare unit forming a network of specialists, hospitals, and nursing facilities. However, there is an acute shortage of PCPs. In the words of Dr. Adrian Billings, chief medical officer for Preventative Care Health Services, Texas - “That means sicker patients, that means more costly or care. That means less productivity. That means more death.” In this work, we focus on how ACOs can utilize non-PCP workforce and the implications on performance. While specialists are not inexperienced in primary care services, they may not offer services similar to an expert PCP. This dissimilarity may reflect on financial and quality performance as ACOs gain more experience as well as move to a higher level of risk sharing. Our results confirm that delivering primary care through specialists is financially unsustainable and does not result in better quality. In the short-term, ACOs can utilize them to deliver primary care services to extend their care to more people and prevent costlier healthcare services. On the other hand, NPs can become beneficial in achieving long-term and short-term ACO objectives.

We also analyze on how experience plays a role in ACO performance. Our data and analysis reveal that ACOs may focus more on savings in the initial phases under a risk model and gradually consider improving quality as they gain more experience. The results suggest that ACO performance varies across different stages. Additionally, the relationship between ACO service delivery through specialists and NPs and performance may vary with ACO experience. Overall, these findings provide valuable insights for managing the ACO model better, both by ACOs themselves and payers such as Medicare.

There are some limitations associated with the work in this chapter. We observe ACO characteristics and performance using aggregate panel data, and a large number of ACOs are not in a risk-sharing model. Our sample is also limited to Medicare ACOs as they form the largest of the existing payer contracts. The inclusion of Medicaid and private contracts may provide better estimates. Our discussions with ACOs reveal that all contracts are nearly the same, and we expect similar findings with ACOs associated with private payers. Finally, a lot of ACOs use isolated, decentralized health IT systems. An examination of the health IT implementation and financial performance can provide an interesting research avenue.

4. Know Your Users Before You Spend: A Data-Driven Optimization to Enhance User Engagement using Visual Analytics

4.1 Introduction

Social media platforms are attractive channels for firms since they can reach out to a large audience ubiquitously to promote their products. A recent report shows that approximately 4.14 billion people across the planet used social media in October 2020, which is around half of the global population (DataReportal 2020). Furthermore, most of these users spend a substantial amount of time online. In the US, users spend approximately 2 hours every day on social media sites (Henderson 2020). Given that social media platforms offer substantial flexibility and benefits to firms in addition to the high user activities, it is not surprising that social media advertising is expected to grow at 10.93% annually between 2021 and 2025 (Statista 2021). Firms are expected to spend almost $225 billion on social media advertising in 2024, surpassing paid search and television advertising (IndiaPartner 2021).

As a result of increasing social media advertising and an increase in the number of users, firms produce and deliver a massive number of attractive posts every day for the users in the social media. Consequently, it is an uphill task for a firm to get its content noticed by the users.[1] Thus, producing a post for social media platforms imposes many challenges since firms compete for users' attention, and only a fraction of users may like and engage with a post (Gitlin 2021). Content development for posts has implications for firms' objectives; a recent industry report (Gary 2021) finds that posts failing to attract the users attention have negative implications on a brand's reputation. Consequently, content development requires special attention, which in turn needs an understanding of features in a post that are attractive to the users. Social media analytics plays a key role in getting insights on user preferences, and its cost is expected to reach $9.3 billion in 2026, consistent with the increasing number of social media users and firms' focus on obtaining market intelligence (ResearchAndMarket 2021).

[1] We interchangeably use post and content

Given the increase in social media marketing, clearly, it is an important problem from an industry perspective on how to develop relevant posts with images. Businesses rely on social media platforms to market their products while consumers want to learn about the products for purchasing decisions (Forbes 2021). However, in spite of the importance of jointly analyzing user preferences via analytics and content development activities, there is little work reported in the literature in this regards. To bridge the gap in the literature, we propose a data-driven optimization framework for deploying social media posts across multiple platforms by efficiently utilizing social media analytics to understand users' preferences and develop content for posts accordingly during a planning horizon under a firm's limited budget. We also present valuable insights to managerially relevant questions.

4.1.1 Motivation

Social media analytics provides important input in creating better content that is more attractive and engaging to an audience (Marta 2021). The deluge of enormous social media information, including unstructured textual data and visual data such as images, has opened up new opportunities for firms to understand users' choices and content attributes that engage them. In this regard, measuring user engagement or the value of social media benefits has become crucial (Hallock et al. 2019). Users show their engagement via various actions, such as liking a post or commenting on it. With the growing prevalence of social media analytics tools, firms can rapidly collect, combine, and analyze data to gather information on users' preferences for features included in posts. The interactive nature of social media platforms allows firms to engage their audience with firm-generated content (FGC) and helps them analyze the effectiveness of their content quantitatively. In such light, we seek to establish a connection between image features of social media posts and user engagement.[2]

Social media posts with images are a vital element of a firm's advertising effort; a study by MDG (2018) reveals that online posts with an image received 94% more engagement on average than posts without images. Images in posts are information artifacts that can influence users' deci-

[2]Image features or features represent the features of image.

sions to engage with them on online platforms. We focus on understanding features of the images that affect user attention. Setting in the narrative framing concept by Baumgartner (2002), we argue that users go through a sense-making process as they observe images having many features. These features are associated with user engagement (Zhao et al. 2019, Zhang et al. 2021). As can be seen from empirical analysis performed later in the chapter, the features in this chapter primarily refer to the objects used in the post. We operationalize two types of features - focal features that display the product or group of products advertised and additional features that help create a dynamic environment and instill a sense-making process, stimulating user engagement and increasing brand popularity (Wulf et al. 2019).

We operationalize these two types of features following the findings of two deep learning algorithms - the simpler Single Shot Detector (SSD) and the advanced Faster Region-based Convolutional Neural Network (R-CNN). The Faster R-CNN extracts additional features (or advanced features) that may help users form a better narrative, albeit at a higher effort cost to the firm. Focal features are the primary advertising objects and thus can be easily extracted using simpler methods such as SSD. This process also allows us to quantitatively examine the costs associated with advanced social media analytics on user engagement.

In essence, users engage with a firm's post if the firm identifies, creates, and publishes relevant image features in it that attract users' attention. In this regard, social media analytics become an important input to content development for posts. A recent survey of 250 business executives reveals that 85% of them believe social media data are a primary source of business intelligence, and 60% of them agree to invest more resources into social media analytics (SproutSocial 2021). Even if a firm decides to outsource these operations, they need a team in-house to coordinate with the partners.[3] For example, Unmetric, a popular analytics firm, offers services to firms wanting to use social media insights. Their price for the basic analytic starts from $1000 and can increase depending on the depth of service.[4] Within social media analytics, the extraction and analysis

[3]https://www.forbes.com/sites/forbescommunicationscouncil/2021/08/17/the-real-cost-of-content-it-could-be-greater-than-you-think/?sh=3efb66d6169a

[4]https://demo.unmetric.com/analyze

of image features on consumer sentiments towards the post is important (Shin et al. 2020). On the other hand, content development requires planning and investment. According to the numbers published by the Business Development Bank of Canada, business-to-consumer or B2C companies should allocate a budget of 5-10% of their total revenue to marketing, out of which social media budget can be around 24% (BDC 2020).

In spite of their significance for firms, social media content development efforts incorporating insights form data analytics have not been examined rigorously. Some key issues in this context include: (i) how does a firm use social media analytics to develop content and determine the desirable features to be included in posts under limited budget and (ii) how does a firm manage a portfolio of social media platforms for their content publication. Hence, we focus on the utilizing the insights obtained via social media analytics on the user preferences in developing the content of posts to be published across multiple platforms and offer managerially relevant guidelines.

4.1.2 Goals and Contributions

Firms spend significant amount of resources on social media posts for advertising and seek higher user engagement on their posts (Marta 2021). Consequently, generating more engagement on their posts is imperative for brand awareness and potential future sales. To increase user engagement on posts under a limited budget, we propose a data-driven optimization framework that guides a firm's content development activities and scheduling strategy for posts to be published across multiple platforms during a planning horizon. The focus of this work is on organic or free FGC of firms in two social media platforms: Instagram and Facebook.

FGC are more popular over paid advertisements primarily for two reasons. First, paid advertisements (or ads) failed to attract substantial engagement or interaction from users. A recent study reports that sponsored or paid post on Instagram received less user engagement, on average, than organic or free content (Fractl 2017). Second, paid or targeted ads are created to reach at least a specific number of users, introducing bias in the relationship between content and engagement (Lee et al. 2018).

Several engagement metrics are considered in this chapter. Likes and comments were used to measure user engagement for each post on Instagram. For Facebook, we extract the number of comments and shares. These online engagement metrics are public information, offer timely information to firms, and act as reliable proxies to measure user engagement and potential future sales (Jaakonmäki et al. 2017, Ma et al. 2018). Based on the observations in practice and our empirical analysis of data from Facebook and Instagram, we analyze a problem where a firm wants to develop their social media posts for single or multiple platforms under a budget constraint. The main features of our optimization framework are presented below:

- *Determining the features for social media posts for maximum user engagement for single or multiple platforms.*

  We consider a planning horizon where the firm wants to develop and publish posts on single or multiple social media platforms. The firm requires information on users' preferences and analyzes their own and competitors' data to understand relevant image features. Social media marketing campaigns require firms to operate across multiple platforms, resulting in more resources and higher costs. However, users across platforms differ in their attributes. Thus, understanding the relationship between features included in a post and the corresponding user engagement becomes crucial at each platform. We explore this relationship between engagement and features using empirical analysis and data on social media posts on Facebook and Instagram and model it in our optimization framework. We combine and run advanced analytic methods and econometrics method on our dataset to explore the relationship between features and engagement.

  We find a clear difference in user behavior across platforms regarding what they like to see on social media post. Instagram users prefer fewer features on posts. For example, in the furniture industry, we observe that engagement starts to reduce if more than eight features are included in a post in addition to the focal features. On the other hand, Facebook users prefer more features on posts. This shows that firms need to treat each platform differently for developing their advertising posts with relevant features.

- *Obtaining the maximum overall engagement under a limited budget.*

  Both content development and social media analytics activities require substantial investment. Our model deals with the trade-off between these completing nativities and finding answers to the following question: “How much analytics can achieve the desired level of features to be included in their content under a budget constraint while maximizing the user engagement?"

  We show that social media analytics efforts reduce as the firm faces tighter budget limitations. Engagement also differs across platforms; therefore, firms must select features by prioritizing platform(s). As the budget tightens, we show that the firm can obtain the highest overall engagement following our framework. In the context of the study specific to a certain category of firms, we show that Facebook receives more priority, and thus, more budget are allocated to Facebook as it generates higher overall engagement.

- *Developing an easy-to-implement solution.*

  We model social media user engagement as a Mixed Non-Linear Integer Programming (MNILP) formulation incorporates the functional form of the relationship between the user engagement and the features included in the post. This function relationship is estimated from the empirical analysis. However, MNILP does not result in analytically tractable solutions. We then transform the models into equivalent linear formulations for both single and multiple platforms that are solvable efficiently. We show that the linearized model is equivalent to solving the MNILP since the feasible integer solution is also feasible to the MNILP and vice-versa. The objective of our model is to maximize user engagement subject to the budget constraint that includes the costs of social media analytics and content development activities.

  We provide structural results for the two platforms - Facebook and Instagram and offer insights on content development activities. In this regard, we identify the number of features that contribute to the maximum and minimum engagement levels on Instagram and Facebook, respectively. We establish the relationship between user engagement and the number of features included in the content. Further, we also illustrate the better performance of our approach that combines both user-base and engagement intensity on a platform compared to a scenario where budget is allocated only based on a platform's user-base; it shows a difference of 11-12% in engagement. In addition to that, we also offer additional managerially relevant guidelines on other aspects of social media posts.

The remainder of this chapter is organized as follows. In Section 4.2, we provide a review of related works and emphasize the contributions of this chapter to the literature. In Section 4.3, we describe the model setting, discuss the main empirical findings, and their applications in the optimization model. In Section 4.4, we present our optimization model, structural properties, computational experiments, and provide managerial insights. In Section 4.5, we discuss the extensions of this chapter. We conclude the chapter in Section 4.6 with managerial and industrial implications and future research directions.

4.2 Literature Review

This chapter draws from two major streams of research: (i) social media user engagement and (ii) operations literature related to resource allocation. Following the discussions of relevant studies of these streams, we underline our contributions.

4.2.1 Social Media User Engagement

First, this chapter builds and contributes to the literature on social media user engagement on advertising posts. User engagement on social media advertising posts increases a firm's exposure and their products' appeal to online users (Coursaris et al. 2016, Dessart et al. 2015). The social media user engagement literature is vast and touches many aspects of the antecedents of user engagement. The difficulties associated with user engagement are multifaceted as it involves analyzing unstructured social media data. For instance, numerous studies analyze textual data to reveal the antecedents of user engagement (Kumar et al. 2016, Naylor et al. 2012). Researchers also examine visual data such as images and their impact on user engagement. While it is essential from the industry point of view, research on social media posts with images in user engagement is relatively nascent. We briefly summarize some of these works here.

Li and Xie (2020) find that the presence of images and their quality enhance user engagement across different industries and different social media platforms. Wulf et al. (2019) find that images affect the popularity of cars in both economy and premium segments. Additionally, the popularity and applications of deep learning algorithms have accelerated the research in image analytics and social media. Liu et al. (2020) develop a convolutional neural network model and examine the impact of images on brand popularity. Ma et al. (2018) observe that images boost online hotel reviews. Zhang et al. (2021) empirically show that high-quality verified images stimulate the demand of Airbnb properties.

Much of the study in the domain is related to how images enhance user engagement across different industries. However, none of these studies consider the development of social media posts with images having relevant features and how to extract those features using social media analytics. In this chapter, we analyze not only the antecedents of user engagement, but also how to develop content using those insights accordingly. In a related study, Shin et al. (2020) utilize deep learning methods to explore user engagement using textual and visual data. They focus on measuring user engagement on social media platforms. This is a key difference with our work; we explore the functional form of image features-user engagement relationship to guide social media post design and development under a budget-constrained environment. In Table 4.1, we highlight the critical differences between this work and that by Shin et al. (2020).

4.2.2 Resource Allocation

The work in this chapter also shares some similarities with resource allocation research that have been extensively studied in the operations literature (e.g., Loch and Kavadias 2002, Bish and Wang 2004, Klingebiel and Rammer 2014). More specifically, this chapter partly relates to the emerging field of new product development (NPD) under budget constraints. The problem considered in this chapter can be considered as a NPD framework: the social media analytics process for designing content acts as a design phase, while the content development phase is the manufacturing phase. Scholars in NPD have looked at domains such as funding (Santiago and Vakili

Table 4.1: Comparison between this work and Shin et al. (2020)

	This chapter	Shin et al. (2020)
Goals	A data-driven optimization framework to guide social media content development using social media analytics to maximize user engagement	A predictive analytics approach to understand the antecedents of higher user engagement
Method	Analytical modeling using empirical results	Empirical
Focus	utilizing budget to gain maximum engagement by using the relevant number of features	classifying specific features related to user engagement
Deep learning algorithms	SSD and Faster R-CNN	Yahoo CNN
Image analysis	Yes	Yes
Text analysis	Yes	Yes
Platform	Multiple	Single (Tumblr)
Social media advertising costs	Yes	No

2005), resource allocation decision-making (Hutchison-Krupat and Kavadias 2015), development of pipeline (Ding and Eliashberg 2002), and most importantly, the phases of NPD (Bajaj et al. 2004) such as design and manufacturing.

Although there have been several studies on NPD, none of the past studies are applicable in our setting because of the novelty of our context, requiring different methods and analyses. For example, firms need to continuously monitor user engagement and develop social media posts in accordance with frequently changing user needs. Yet, most works consider one phase of the problem. Kumar et al. (2020) demonstrate a theoretical model of user engagement on digital platforms for content providers' advertisement sequence. Mallipeddi et al. (2021a) analyze social media posts on Twitter to explore the relationship between tone of posts and engagement. Unlike the aforementioned studies in the social media context, our model is unique in that we jointly consider two phases in social media advertising and extend the resource allocation domain to social media marketing. We explicitly show the use of social media analytics to develop better social media posts, which is an important contribution of this chapter.

4.3 Problem Setting

The firm desires to maximize user engagement on their social media platforms by efficiently deploying social media analytics and content creation efforts using relevant features to be included in the posts. In order to achieve this, we propose an optimization framework for analyzing and publishing social media posts across multiple platforms for a planning horizon under a firm's limited budget. Our framework incorporates the trade-off between the completing costs associated with social media analytics and content development efforts to achieve the maximum engagement.

We introduce the basic model setting for maximizing user engagement for firms, and then perform an empirical analysis to estimate parameters for the model. We provide a description of the social media marketing ecosystem where firms publish their posts (i.e., advertisements) on social media platforms where users engage with them. Information on their own and competitors' posts act as an external source of knowledge for the firms to know about their users (Choudhury and Harrigan 2014). Knowledge is a key element of competitive advantage and firms use it into their decision making process to offer better products and services to their customers (Jansen et al. 2005). We illustrate the approach on two social media platforms, Instagram and Facebook.

4.3.1 Social Media Marketing Ecosystem

In a typical social media marketing system, a firm develops and publishes posts that contain image with features on one or multiple platforms. Upon seeing a post on their social media accounts, an Instagram user may engage in the following manner: like or comment or do both; and a Facebook user may share or comment or engage in both. These user reactions help firms extract valuable insights from users' engagements. For example, a study by Palmer et al. (2013) reveals that Burberry, a leading fashion brand, utilizes social media data to identify shopper trends and performs timely customizations to their products accordingly. The same study also reports that Walmart uses social media data that include user reactions such as comments and check-ins to predict user demand for a product.

As discussed earlier, user engagement or the extent to which a user interacts with a service and frequently wants to use that service is an important outcome metric for the firm. In analyzing user engagement data, we take engagement types such as like, comment, and share, into consideration. In the following, we discuss three important aspects of social media ecosystem: (i) social media user engagement (ii) features of post and user engagement on social media platforms, and (iii) costs associated with social media marketing.

4.3.1.1 Social Media User Engagement.

To capture users' engagement level toward a post, we let x_{lmk} be the volume of engagement type m on post k at platform l. The firm derives benefits by maximizing user engagement: $\sum_{l=1}^{L} \sum_{m=1}^{M} \sum_{k=1}^{K} x_{lmk}$. Firms usually publish one post each day as revealed in our data and other studies (Cui et al. 2018). Thus, in this chapter, each k could represent one day and therefore, the planning horizon consists of K number of days. Our data also reveal that engagement on posts occur within a day from the post publication date. For example, we analyze a sample of 32 posts on Instagram across multiple firms over a three week period of time and observe that 94% likes and 84% comments occur within a day.

Firms usually post once every day to provide fresh content to their users (Innovations 2021). Therefore, we implement a single post publication per period policy in developing our optimization model. Another aspect of our model is the inclusion of both single and multiple platforms. The context of multiple platforms requires distinctions since each platform has its own characteristics and impact. Following Aichner and Jacob (2015), we utilize social media impact factor for a platform which is defined in the context of a firm using multiple platforms. The impact factor for a platform which is a fraction user basis as compared to total user basis for multiple platform, can be expressed by dividing the number of active users per month for the platform by total active users per month across multiple platforms. We also assign weights z_{lm} to each engagement type m for each platform l.

4.3.1.2 *Features of Post and User Engagement on Social Media Platforms.*

Creating posts with images is challenging, especially when the firm operates across multiple social media platforms. Practitioners suggest that firms should not post identical content on multiple platforms since each platform is different (Amibbola 2021). Also, a firm should not post same content regularly as it reduces user engagement (Mallipeddi et al. 2021b). Thus, a firm must tailor its social media marketing strategy to create and customize image features in a post that attracts and engages users by competing among millions of other posts published every minute.

Recall that a firm may emphasize its focal features or simple features in an image and introduce additional features that create an impression of the overall narrative on users. Since focal features portray the main information or primary products, firms may enhance them to a certain extent. On the other hand, they can add the number of advanced features to significantly enhance the post that in turn improves user engagement. In our optimization model, we represent simple and advanced features as s_{lk} and f_{lk}, respectively. Therefore, user engagement x_{lmk} can be expressed as x_{lmk} = $f(s_{lk}, f_{lk}, \tau)$, where the vector τ represent additional parameters derived from the empirical analysis. We estimate the engagement-feature association, x_{lmk} = $f(s_{lk}, f_{lk}, \tau)$ from the empirical analysis.

4.3.1.3 *Costs Associated with Social Media Marketing.*

In the context of social media marketing, Wu et al. (2020) find that firms allocate resources to perform in-house market research and create their advertisements. This entails two major costs: social media analytics and content development. We denote U_l as the analytics effort in social media for every platform l. Social media analytics typically involves the analysis and insights gained from structured and unstructured social media data (SproutSocial 2021). As mentioned in an industry report (Hootsuite 2021), more insights from social media platforms require higher analytics effort, increasing the costs. Our modeling framework incorporates this cost and guides as to how a firm should invest its resources in getting more information.

Social media marketers should be cautious about similar or outdated posts on their pages.

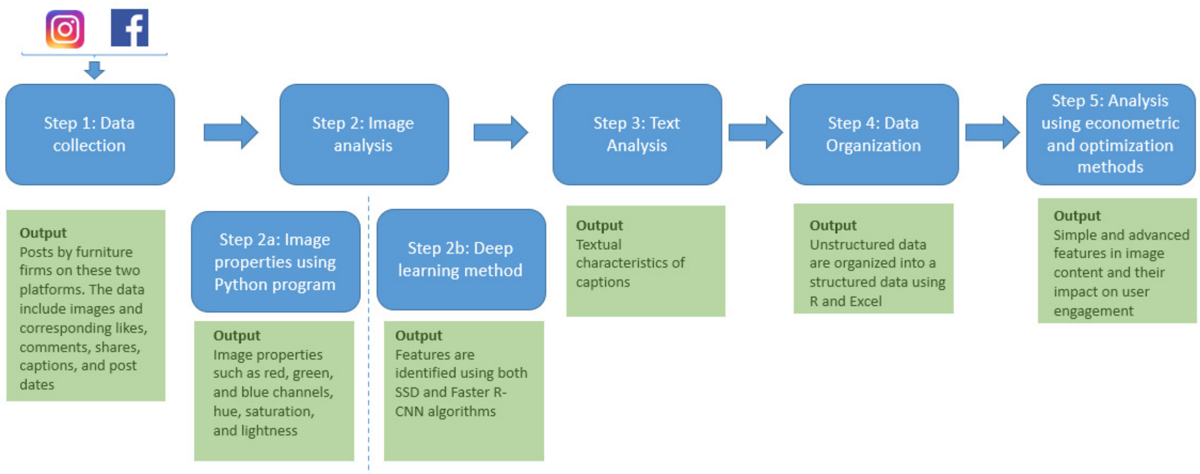
Content development using new and relevant features are essential in the social media advertising landscape. It requires costs associated with making images, which may consist of photographers, human actors, objects, renting space, etc. We capture a substantial part of this cost by assuming a unit development cost per feature for each s_{lk} and f_{lk}. For example, Socialistics (2021) mentions that organizations usually spend on social media activities anywhere between \$6,000 and \$10,500 per month, primarily depending on the frequency of posting, number of platforms, and graphics. The cost increases as more sophisticated social media analytics are used. For example, The Content Factory, a leading social media outsourcing vendor, charges \$4000 for three social media platforms only for content customizations and hashtags (TCF 2021). Another study by Reports (2021) finds that it may cost \$50 per each graphic or object on social media content. For ease of exposition, our optimization model includes a constant development cost, “a" per unit for every feature to be included in a post. The model also includes a fixed cost C_f of a technical team performing a social media analytics. C_f does not include the content development cost. As discussed earlier, a firm may need either an in-house or a third-party outsourced social media management team. We represent the budget constraint as $a\sum_{l=1}^{L}\sum_{k=1}^{K}(s_{lk}+f_{lk}) + e\sum_{l=1}^{L}U_l + LC_f \leq B$, where B denotes the limited budget and e is a per unit effort cost of including features in a post.

4.3.2 Data Collection and Empirical Analysis

To investigate the impact of image features and select the relevant advanced features for achieving the maximum engagement on single or multiple platforms, we conduct an empirical analysis using data on posts from Instagram and Facebook. To establish the relationship between engagement and features, we follow these two stages: (i) extracting simple and advanced features by running machine learning algorithms (SSD and Faster R-CNN) and (ii) performing an econometric analysis to understand the relationship between user engagement and the features extracted in (i) while controlling for other factors. In this subsection, we briefly describe the process and use of empirical outcomes in our optimization model. Figure 4.1 graphically illustrates the empirical analysis step-by-step. Appendix C.1.1 provides the details of the data collection.

We ground our empirical work in the process model developed by O’Brien and Toms (2008)

Figure 4.1: Social media content analysis process



and the narrative framing theory by Baumgartner (2002) to postulate the relationship between user engagement and advanced features. User engagement occurs through a process where users start an engagement with a system, which is a social media post in our research context, and maintain their engagement, followed by disengagement and re-engagement. Images convey information to users, which can influence their decision-making (Duarte et al. 2012). These pieces of information help in forming a mental image of the product(s) advertised on social media. An image not only consists of focal products (or simple features) but also comprises other additional features (or advanced features) that offer a visual experience as a whole. This narrative framing of images via the advanced features helps form users' cognitive thinking and subjective interpretation when they see a post, resulting in excitement from the users. In line with the marketing literature on user reactions (Yin et al. 2017), we argue that increasing additional features (or additional information) in an image may impact non-linearly on users' reactions. In other words, we conjecture that the association between advanced features and user engagement depends on how much information is helpful for users.

An understanding of the relationship between user engagement and features is important for firms as they need to decide how many advanced features they should use to help users in their decision making process without overdoing it or exceeding their budget. Users express their excitement via liking a post, commenting on it, and sharing with others, leading to user engagement on a post. Deep learning methods have facilitated opportunities to identify image features that influence user engagement at a substantially low cost and without human biases associated with data-intensive projects (Wu et al. 2015).

4.3.2.1 Deep Learning Methods.

We apply deep learning algorithms to extract features of the images included in social media posts. This is a major part of social media analytics, which encompasses a range of activities that can reveal insights on user engagement and desired features by exploring the relationships in unstructured datasets (IBM 2021). These algorithms facilitate an information-gathering process from images and how much of that information effectively generates higher user engagement, similar to the textual analytics processes through automated software tools. Consistent with the literature, we use pre-trained robust deep learning algorithms on our social media data to extract the features from images (Shin et al. 2020). The use of algorithms pre-trained on large existing datasets and applied on a new and smaller dataset is known as transfer learning. This approach is widely used for higher accuracy, especially when the collected data is not large enough for model training. We also manually analyze the features extracted after applying the pre-trained algorithms and ensure the accuracy and detection performance.

More details of the deep learning approach is provided in Appendix C.1.2. We illustrate a sample output of the deep learning methods in Table 4.2, where the features extracted by algorithms SSD and Faster R-CNN are shown. We denote the number of features extracted by SSD as simple features. Advanced features are the additional features extracted by the Faster R-CNN, representing more information in images (Column 5 in Table 4.2). Note that simple and advanced features together represent the total number of features in an image. SSD is a simple algorithm with one layer and therefore, is designed to extract the most prominent feature(s) of an image. This is in line with the content marketing strategies of firms where they focus on clarity or the absence of ambiguity in an advertisement where the primary objects are prominently highlighted (Erdem and Swait 1998). Social media advertising strategies utilize the same philosophy, especially given the space limitations and users' attention span. Faster R-CNN is an advanced algorithm, and thus, it can extract the additional features in an image.

Table 4.2: Sample output from deep learning algorithms

Post	SSD	Faster R-CNN	No. of Simple features	No. of Advanced features
1	['couch']	['vase', 'vase', 'chair', 'couch']	1	3
2	['chair']	['vase', 'chair', 'chair', 'surfboard', 'couch', 'potted plant', 'potted plant']	1	6

4.3.2.2 Empirical Findings.

As hypothesized in our empirical analysis, the features in a post influence the user engagement. In order to formulate and solve our optimization model that maximizes the user engagement (to be developed in the next section), we need to understand the relationship between user engagement for a post and the corresponding features in it. Therefore, we collect data from social media platforms and perform the empirical analysis to discover this relationship. The features in a post are extracted by the machine learning algorithms. Our empirical analysis reveals that the user engagement in a platform is strongly related to the number of simple and advanced features included in a post. Following the theoretical support, we introduce a quadratic term for advanced features in the regression model. Table C.1 (in Online Appendix) lists the variables used in the empirical analysis on Instagram and Facebook data. The summary statistics for the variables used in Instagram and Facebook analyses are displayed in Tables C.2 and C.3, respectively. More details of the empirical analysis are provided in Appendix C.1.3.

Given the form of our engagement variables, which exhibit the properties of count variables, we deploy negative binomial regression. We specify the model specifications in Appendix C.1.3.1 and discuss the results in Appendix C.1.3.2 and report the findings for Instagram in Table C.4, followed by the Facebook results in Table C.5. We also perform additional robustness tests by using quantile regression and zero-inflated negative binomial regression and illustrate them in Appendix C.1.3.4. We briefly summarize the primary findings below.

As postulated, the number of advanced features follow a non-linear relationship with likes on Instagram, and shares and comments on Facebook, although exhibiting different patterns. The patterns for the two platforms are displayed in Figures 4.2 and 4.3. We do not find any signifi-

Figure 4.2: Instagram analysis

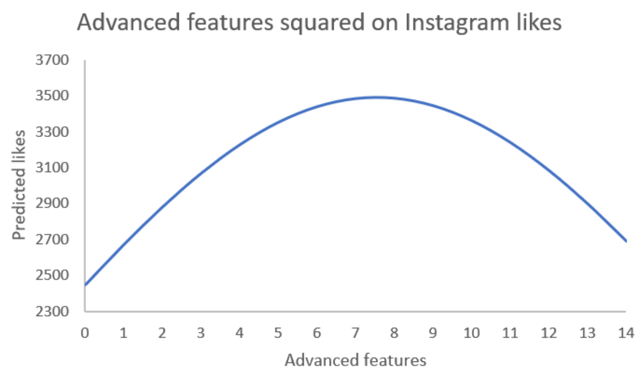
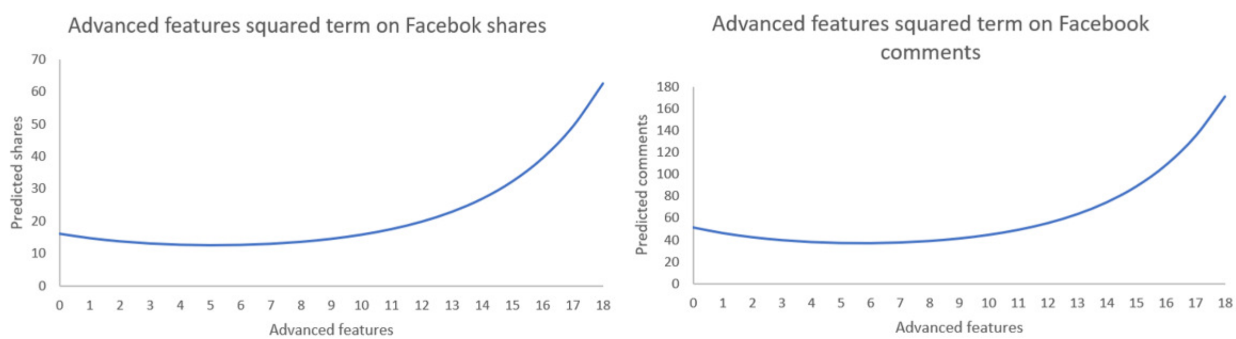


Figure 4.3: Facebook analysis



cant relationship between advanced features and comments on Instagram. Following the approach suggested by Hayes (2017), we estimate the user engagement values (e.g., the number of likes on Instagram) from the regression equation by averaging all other variables and multiplying them with their corresponding coefficients as shown in Table 4.4. Thus, we estimate the engagement levels due to the number of advanced features and other parameters in a post and include this non-linear relationship in our optimization model to be developed in the next section. We present the key findings of our empirical analysis below. We incorporate these insights and other empirically derived parameters to build our optimization model.

Empirical Insight 1: *The relationship between user engagement for a post and the corresponding number of advanced features in it is nonlinear and differs across engagement types and platforms. In particular, the empirical analysis reveals that the relationships between the number of advanced* *features and likes on Instagram, and comments and shares on Facebook are quadratic.*

Empirical Insight 2: *Instagram users do not prefer too many features on images in a post as excessive features hinder the aesthetics of the post. On the other hand, Facebook users prefer more features in a post as they interact well with those users.*

In the next section, an optimization model is developed with the aim of creating a social media marketing strategy for a firm. The model provides a plan for designing social media posts and scheduling them during a planning horizon in order to maximize user engagement, x_{lmk} (e.g., the number of likes on Instagram) under a firm's limited budget B, where l, m, and k denote the platform type, the user engagement type, and the identity of the post, respectively. We provide the model parameters and variables for the single platform single engagement type in Table 4.3, where indices l and m are dropped due to the model specificity to single platform and single engagement type. Our empirical analysis suggests the following functional from for the relationship between the user engagement, x_{lmk}, and the number of simple features, s_{lmk} and the number of advanced features, f_{lmk} in post k.

$$x_{lmk} = b_{lm} + \beta_{lm} s_{lmk} + \zeta_{lm} f_{lmk} + \eta_{lm} f_{lmk}^2,$$

where b_{lm} is a constant, β_{lm} is coefficient for simple features, and ζ_{lm} and η_{lm} are coefficients for advanced features (linear and quadratic, respectively). These coefficients are estimated via empirical analysis for a planning horizon and are provided in Table 4.4. This functional relationship is used in our optimization model to be developed in the next section.

4.4 Problem Formulation

As mentioned in the previous section, we use the findings from our empirical analysis to develop an optimization model with the aim of creating a social media marketing strategy for a firm. More specifically, we use the function relationship, $x_{lmk} = b_{lm} + \beta_{lm} s_{lmk} + \zeta_{lm} f_{lmk} + \eta_{lm} f_{lmk}^2$ that describes the user engagement in our model.

Our model devises a plan for designing social media posts and scheduling them during a plan-

Table 4.3: Parameters and variables for one platform and one engagement type

Parameters	
a	Content creation Cost of adding one feature to post k, $k = 1, 2, \ldots, K$
B	Total allocated budget for a planning horizon
u_1, u_2	Upper bound for simple and advanced features extracted in the previous planning horizon, respectively.
b	Impact factor of post measures on user engagement of post k, obtained from empirical analysis, $k = 1, 2, \ldots, K$
β	Impact factor of simple features on user engagement of post k, derived from empirical analysis
ζ	Linear impact factor of advanced features on user engagement of post k, estimated in empirical analysis
η	Quadratic impact factor of advanced features on user engagement of post k, obtained from empirical analysis
w_k	Weight of post k on user engagement, where $\sum_{k=1}^{7} w_k = 1$.
C_f	Fixed cost incurred for social media analytics activities for a planning horizon
e	Unit cost of putting an effort for extracting/analyzing advanced features using the advanced deep learning method Faster R-CNN.
q_k	Number of features of focal product(s) in content k ($q_k \leq s_k$).
Variables	
U	Maximum number of advanced features to be included during the planning horizon
x_k	User engagement of post k, $k = 1, 2, \ldots, K$
s_k	Number of simple features to be included in the image of post k , $k = 1, 2, \ldots, K$
f_k	Number of advanced features to be included in image of post k, $k = 1, 2, \ldots, K$

Table 4.4: Parameters obtained from empirical analysis

Coefficients	Instagram ($l = 1$)	Facebook ($l = 2$)	
	Likes ($m = 1$)	Shares ($m = 2$)	Comments ($m = 3$)
b_{lm}	7.2900	2.8400	2.6300
β_{lm}	-0.0340	-0.0500	0.0036
ζ_{lm}	0.0940	-0.0988	-0.1140
η_{lm}	-0.0062	0.0090	0.0100
u_{1l}	10	19	19
u_{2l}	14	18	18

ning horizon in order to maximize user engagement under a firm's limited budget. In Section 4.4.1, we develop a plan for a single platform with both single and multiple engagement types. We present the generalized case for multiple platforms and engagement types in Section 4.4.3.

4.4.1 Single Platform Single Engagement Type

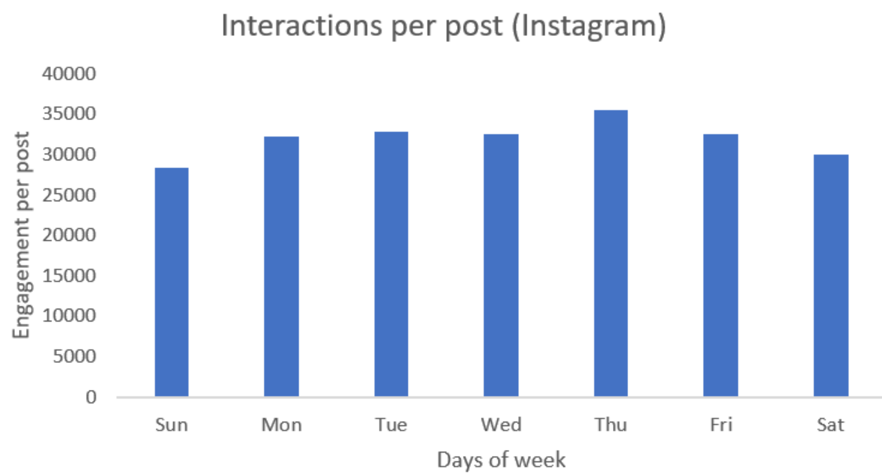
We present the single platform and single engagement type version of the generalized problem. The firm's objective is to maximize user engagement during a planning horizon comprising of K periods, where $k = 1, 2, \ldots, K$ and each period represents a day. Similar to the analysis by Mallipeddi et al. (2021b), we consider a planning horizon of $K = 14$ days or two weeks. The firm operates under limited budget to be allocated for the costs associated with the firm's content development activities and the social media analytics tasks during the planning horizon. Recall that our data reveal that a firm creates and publishes approximately one post per period. Thus, we could consider K posts (usually one per day) are posted during the planning horizon.

In the context of social media analytics, the firm gathers social media data concerning firms' posts, performs feature extraction from posts using deep learning algorithms, and establishes the feature-engagement functional relationship and estimates relevant parameters associated with the relationship using an econometric analysis. The firm then decides the number of features to be included in each post and schedules the posts during a planning horizon. Given the focal features (referred to as simple features in this chapter) for a product to be advertised, the firm also needs to decide the number of advanced features to be included in a post on different platforms. As observed in our empirical analysis, the Faster R-CNN identifies the advanced features in a post, including the simple features extracted by SSD.

We represent the features by defining two decision variables: s_k (respectively, f_k) denote the number of simple (respectively, advanced) features to be included in the post k. Also, our empirical analysis reveals that the number of features is finite; s_k and f_k have upper bounds u_1 and u_2, respectively, which are estimated from our data, specific to each platform. Within simple features, we denote q_k ($q_k \leq s_k$) as the focal features specific to a product(s) to be used in a post. We maintain this relationship throughout our analysis. Finally, we also mention here that for multiple platforms l and engagement types m, x_{lmk} denote the user engagement (e.g., the number of likes on Instagram) for post (i.e., content) k. For a single platform with single engagement type, x_k denote the user engagement where indices l and m are dropped.

We allocate budget B that include the cost of performing the following activities for a planning horizon: (i) content creation and (ii) social media analytics. The content creation cost for a post depends on the number of features to be included in it. Recall that the firm analyzes posts in the previous planning horizon to decide features in post k in the current planning horizon. Our objective is to assist firms in developing a social media marketing strategy by deciding the number of features to be included in the posts that have to be scheduled during the planning horizon. This involves analyzing the data, understanding features that influence the user engagement, and designing better post by specifying features to be included in the post. These insights are derived

Figure 4.4: Instagram: engagement intensity per post during days of week



from social media analytics.

Firms perform social media analytics by continuously monitoring and analyzing user engagement, and observing user preferences for its own posts and those of other similar organizations (IBM 2021). They use these insights to deliver better and relevant posts. Therefore, we update the parameters in Table C.10 for each planning horizon and run our optimization model with updated parameter for each period. Thus, the goal is to analyze social media data across the platforms periodically, derive empirical insights on the relationship between user engagement types and image features in the posts, and design the image content to be posted for the next planning horizon. We follow that no two posts are the same during a planning horizon in the same platform, consistent with the suggestions for not using identical and repetitive posts by practitioners (Amibbola 2021).

Next, we discuss three important attributes to be included in our model: (i) engagement intensity during a week, (ii) cost of social media analytics and (iii) content creation cost.

4.4.1.1 Engagement Intensity.

The existing literature on social media user engagement reports mixed evidence on engagement intensity over a week, often different across industries. Some studies find that posts on weekdays generate a relatively higher intensity of engagement than posts on weekends, on average (Arens 2021). While the data did not reveal any statistically significant difference of user engagement across the days, it is not surprising that users access social media platforms from their phones or computers depending on their convenient time. For example, Kumar et al. (2019) find that users do not engage on social media shopping sites during the weekends. To be general, we introduce an engagement intensity via weight w_k, for user engagement for post k which depends on day the post is scheduled. We set $\sum_{k=1}^{7} w_k = 1$ for one week during the planning horizon where w_k assumes lower values during the weekend. Note that weights w_k may differ with the industry. For example, Mariani et al. (2018) find that user engagement of tourism organizations on Facebook is positively associated with weekend postings. This highlights the importance of continuous monitoring of user engagement during each planning horizon and finding weights based on observed data. In our model, we use the weights derived from a survey done by Unmetric on 100 firms' Instagram posts and user engagement as depicted in Figure 4.4, which clearly shows that engagement is relatively lower on Saturdays and Sundays.

4.4.1.2 Social Media Analytics Cost.

Firms may perform social media analytics in-house or outsource them. Even if a firm outsources their analytics to a third-party, they still require a team to coordinate with the vendor regularly. Studies suggest that many firms perform the analytics in house for better control and ease of decision making (Wu et al. 2020). The in-house team monitors social media activities and aids the content designers to develop effective posts for the next planning horizon. The team (i) collects data from multiple platforms on multiple firms that include images, text information, and user reactions, (ii) extracts features from the images, and (iii) analyzes their impact on the user engagement.

We refer the cost of maintaining a team, C_f, as fixed cost, amortized for the planning horizon. There is a variable cost eU per planning horizon, where U is the upper bound on f_k (or U_l for f_{lk}) quantifying the team's effort (include computational power) required to perform social media analytics and e represents the per unit analytics cost in extracting and analyzing the number of advance features, f_k bounded by variable U for a planning horizon. Thus, the costs of feature extraction and analysis consist of a fixed cost C_f and a variable cost eU per planning horizon.

Estimating e is challenging; the firm needs resources on data collection, data structuring since most data are unstructured, and running computational algorithms on the data. These operations are costly and time-consuming. The firm may devote more effort U to extract more (advanced or additional) features and get better understanding of user engagement, which may reduce budget allocation for content creation activities, C_c. We perform computational experiments on a testbed of various parametric values of e to estimate user engagement outcomes and the corresponding cost components. Without loss of generality, our framework applies to both in-house or outsourced analytics operations as higher analytics effort incurs more cost in both cases.

4.4.1.3 Content Development Cost.

Similar to analytics operations, the firm may possess in house design team(s) having required expertise in developing posts or it may subcontract this activity to another firm. The content creation involves one or more teams designing and developing images with desirable features for a post. The team may consists of designers and photographers requiring office space. Thus, each additional feature in the image requires more effort and resources, besides renting or buying the objects associated with the features. As discussed earlier, we denote a linear cost per feature to be included in a post is a. Thus, for a planning horizon, the total content creation cost is $a \sum_{k=1}^{K}(s_k + f_k)$, each post k having $(s_k + f_k)$ features. The same approach extends to the generalized model where each content has $(s_{lk} + f_{lk})$ features. We now present below our model, SMM_s for the single platform and single engagement type. The objective of SMM_s is to maximize the weighted user engagement, Π_s, where weight w_k is introduced to capture the intensity of user engagement in period k. As indicated before, we omit the indices l and m in SMM_s.

We now describe the constraints in SMM_s. In Constraint (4.1), we impose the budget constraint with three cost components. The first component $(a \sum_{k=1}^{K}(s_k + f_k))$ is the content creation cost for K posts, each post k having $(s_k + f_k)$ features. The second component (eU) refers to the social media analytics cost which is variable cost. The third component (C_f) represents the fixed cost of the social media analytics cost. Constraint set (4.2) which is obtained by our empirical analysis, illustrates the functional relationship between user engagement and image features to be included in the post. Note that the parameters, b, β, ζ, and η are estimated from our empirical analysis (refer, Table C.10). However, this constraint introduces non-linearity to our model through a quadratic term. Constraint sets (4.3) and (4.4) restrict the total number of simple and advanced features below their upper bound values, respectively, where U is the decision variable. We obtain the upper bound values from our empirical data. In Constraint set (4.5), the decision variable U is bounded by u_2 which is obtained from our empirical analysis. Constraint sets (4.6) provide lower bounds set by the firm on the number of simple features for post k. Note that q_k represent the number of features of a focal product(s) the firm wants to advertise in period k, such as a bed or chairs for a furniture firm, which are easier to extract and therefore, belong to the simple features category. In Constraint sets (4.7). we define the nonnegativity constraints.

Problem SMM_s:

$$\textbf{Max } \Pi_s = \sum_{k=1}^{K} w_k x_k$$

Subject to:

$$a \sum_{k=1}^{K}(s_k + f_k) + eU + C_f \leq B \tag{4.1}$$

$$x_k = b + \beta s_k + \zeta f_k + \eta f_k^2, \ \ \forall k \tag{4.2}$$

$$s_k \leq u_1, \ \ \forall k \tag{4.3}$$

$$f_k \leq U, \ \ \forall k \tag{4.4}$$

$$U \leq u_2, \ \ \forall k \tag{4.5}$$

$$s_k \geq q_k, \ \ \forall k \tag{4.6}$$

$$x_k, s_k, f_k, U : \ \text{Integer variable } (\geq 0), \ \ \forall k \tag{4.7}$$

4.4.2 Solving Single Platform Problem SMM_s for a Platform

We illustrate our solution approach and algorithms for two platforms, Instagram and Facebook, as each platform has certain attributes characterizing its user engagement. We illustrate those attributes in the subsections below. The proofs of Lemmas and Theorems are provided in

Appendix C.2.1.

4.4.2.1 Solving Single Platform Problem SMM_s for Instagram.

Attributes of Instagram on user engagement are given under Case 1, where $\beta_{lm} < 0$, $\zeta_{lm} > 0$, $\eta_{lm} < 0$. We now provide the solution approach by linearizing Problem SMM_s and a polynomial time algorithm to solve it. Any platform belonging to this case may use the approach/algorithm presented in this subsection.

Case 1: $\beta_{lm} < 0$, $\zeta_{lm} > 0$, $\eta_{lm} < 0$. Example of such platform is Instagram in our case study demonstrated in Table 4.4, where $l = 1$ and $m = 1$. We have following results for the platform satisfying condition in Case 1.

We let $q_0 = \sum_{k=1}^{K} q_k$ and $f_0 = \sum_{k=1}^{K} f_k$.

Since $\beta_{lm} < 0$, in Problem SMM_s, in order to have maximum user engagement, f_k, we must have $s_k = q_k, \forall k$. Thus, the following results follow.

Lemma 2. *In Problem SMM_s, the maximum user engagement is obtained by $s_k = q_k$, $\forall k$ and $\sum_{k=1}^{K} s_k = q_o$.*

The next result provides the optimal number of advanced features, f_k^* that maximizes the user engagement for period k.

Lemma 3. *In Problem SMM_s under condition Case 1 for platform l and engagement type m for a given value of s_k, the maximum user engagement for period k is x_k^*, when either $f_k^* = \lfloor \frac{\zeta_{lm}}{2|\eta_{lm}|} \rfloor$ or $f_k^* = \lfloor \frac{\zeta_{lm}}{2|\eta_{lm}|} \rfloor + 1$ whichever provides maximum x_k^*.*

Lemma 4. *As $\zeta_{lm} > 0$ and $\eta_{lm} < 0$, in Problem SMM_s for platform l and engagement type m, $U_0 = \min\{u_2, f_k^*\}$ and $U \leq U_0$.*

The next lemma quantifies f_0 for given budget B, where $f_0 = \sum_k f_k$.

Lemma 5. *In Problem SMM_s under condition Case 1, the maximum, $f_0 = \lfloor \frac{(B - C_f - eU - aq_0)}{a} \rfloor$.*

Theorem 3. *Problem $LSMM_s(Case1)$ under condition Case 1 is equivalent to linear version of Problem SMM_s.*

Problem $LSMM_s(Case1)$**:**

$$\textbf{Max } \Pi_s = \sum_{k=1}^{K} w_k x_k$$

Subject to:

$$a\sum_{k=1}^{K} s_k + a\sum_{k=1}^{K}\sum_{j=0}^{f_k^*} j y_{k,j} + eU + C_f \leq B \tag{4.8}$$

$$x_k = \beta s_k + \sum_{j=0}^{f_k^*} g_j y_{k,j}, \ \ \forall k \tag{4.9}$$

$$\sum_{j=0}^{f_k^*} y_{k,j} = 1, \ \ \forall k \tag{4.10}$$

$$s_k \leq u_1, \quad \forall k \tag{4.11}$$

$$\sum_{j=0}^{f_k^*} j y_{k,j} \leq U, \quad \forall k \tag{4.12}$$

$$U \leq f_k^*, \tag{4.13}$$

$$s_k \geq q_k, \quad \forall k \tag{4.14}$$

$$x_k, s_k, U: \ \text{Integer variable } (\geq 0), \ \ \forall k \tag{4.15}$$

$$y_{k,j} \in \{0,1\} \ \ \forall k; \ \ \forall j \tag{4.16}$$

In Table C.11, we enumerate the Instagram engagement (likes) values for each f_k, which we derive from our empirical results. The highest engagement x_k^* occurs at $f_k^* = 8$. We find these values from our empirical analysis for each Instagram and Facebook (later).

4.4.2.2 Solving Single Platform Problem SMM_s for Facebook (Shares).

Attributes of Facebook on user engagement (Shares) are given under Case 2, where $\beta_{lm} < 0$, $\zeta_{lm} < 0$, $\eta_{lm} > 0$. We now provide the solution approach by linearizing Problem SMM_s. Any platform belonging to this case may use the appraoch presented in this subsection.

Case 2: $\beta_{lm} < 0$, $\zeta_{lm} < 0$, $\eta_{lm} > 0$. Example of such platform is Facebook (Shares) in our case study, see Table 4.4, where $l = 2$ and $m = 2$. We have following results for the Platform satisfying condition in Case 2.

As in Case 1, we must have $s_k = q_k, \forall k$ and $\sum_{k=1}^{K} s_k = q_o$ (Lemma 2). The next result provides the optimal number of advanced features, f_k^* that minimizes the user engagement for period k.

Lemma 6. *In Problem SMM_s under condition Case 2 for platform l and engagement type m for a given value of s_k, the minimum user engagement x_k^* occurs at period k, when either $f_k^* = \lfloor \frac{|\zeta_{lm}|}{2\eta_{lm}} \rfloor$ or $f_k^* = \lfloor \frac{|\zeta_{lm}|}{2\eta_{lm}} \rfloor + 1$ whichever provides minimum x_k^*.*

4.4.2.3 Solving Single Platform Problem SMM_s for Facebook (Comments).

Attributes of Facebook on user engagement (Comments) are given under Case 3, where $\beta_{lm} > 0$, $\zeta_{lm} < 0$, and $\eta_{lm} > 0$. We now provide the solution approach by linearizing Problem SMM_s. Any platform belonging to this case may use the approach presented in this subsection.

Case 3: $\beta_{lm} > 0$, $\zeta_{lm} < 0$, $\eta_{lm} > 0$. Example of such platform is Facebook (Shares) in our case study, see Table 4.4, where $l = 2$ and $m = 3$. We have following results for the Platform satisfying condition in Case 2.

The next result provides the optimal number of advanced features, f_k^* that minimizes the user engagement for period k.

Lemma 7. *In Problem SMM_s under condition Case 3 for platform l and engagement type m for a given value of s_k, the minimum user engagement x_k^* occurs at period k, when either $f_k^* = \lfloor \frac{|\zeta_{lm}|}{2\eta_{lm}} \rfloor$ or $f_k^* = \lfloor \frac{|\zeta_{lm}|}{2\eta_{lm}} \rfloor + 1$ whichever provides minimum x_k^*.*

Unlike Case 1, Case 2 and 3 have convex patterns and therefore, demonstrate the minimum engagements that occur at f_k = 5 and 6 for Facebook shares and comments, respectively. Since all other structural properties remain the same, we can modify Problem $LSSM_s(Case1)$ by replacing f_k^* with u_2, the upper bound of advanced features on Facebook. u_2 remains the same for both shares and comments as these two engagement types belong to the same platform. The specifics of the Problem $LSMM_s(Cases2-3)$ are formally described in Appendix C.2.

Table 4.5: Weights w_k (in multiples of 10)

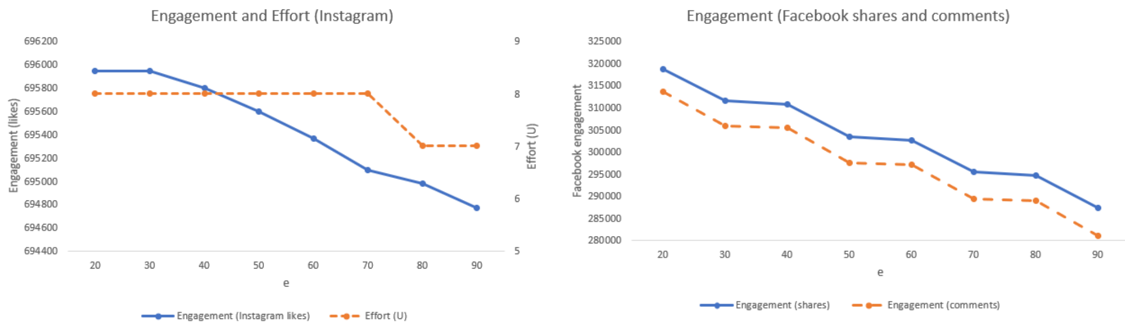
w_1	w_2	w_3	w_4	w_5	w_6	w_7
10	12	16	14	16	14	10

4.4.2.4 Numerical Analysis.

In summary, the analysis of two platforms and corresponding three engagement types lead to several interesting and distinct insights, as illustrated in the Lemmas. Prior to analyzing the generalized framework, we present the numerical analysis on single platform and single engagement type, first by running models, $LSMM_s(Case1)$ and $LSMM_s(Cases2-3)$. Next, we combine shares and comments for Facebook and offer insights for single platform and multiple engagement types problem ($LSSM_f$). We fix a at 20 and vary e between 20 and 90, with increments of 10. q_k refers to the number of features in the focal products in the post k, and q_k is generated randomly from the set of integer number between 1 and 5, fixed for all platforms. The values for w_k are selected based on the numbers from Figure 4.4, where the planning horizon starts on a Sunday and ends on a Saturday during a two week period. Table 4.5 shows the values of w_k, where $k = 1, 2, \ldots, 7$. We repeat the same values for $k = 8, 9, \ldots, 14$ and fix them across platforms and engagement types for comparison.

The results of the linearized formulation for Instagram are presented in Table C.14. We graphically illustrate the Instagram engagement results on the left panel of Figure 4.5. As can be seen, the total engagement (likes) on Instagram reduces as e increases because the budget allocation tightens for content development activities. Consequently, all posts use q_k focal features and the optimal number of advanced features, f_k^* when e is low (below 40). Thus, the highest engagement of 695946 (x_k^*) occurs when e is set at lowest ($e = 20$). However, as e increases, fewer than optimal number of advanced features (f_k^*) are included on posts to be published on the days with low engagement intensity. Also U starts to reduce overall as well as the budget allocation becomes more stringent for both activities (Social media analytics and content development) when e exceeds 80. The numerical experiment on Instagram supports our theoretical results for Case 1. On average, firms exploit their social media analytics fully on Instagram under lower e (i.e., lower budget al-

Figure 4.5: Single platform analysis



location for content development activities) and achieves the maximum user engagement, x_k^* by including the optimal number of advanced features (f_k^* = 8) on posts as specified by Lemma 3.

Managerial Insight 1: *The relationship between the number of likes on Instagram and the advanced features exhibits a concave function, implying that likes on Instagram initially increase with the number of advanced features and then reduce after a threshold. In our context, likes on posts attain the maximum value when the number of advanced features is eight and decreases afterward.*

Managerial Insight 2: *Budget restriction affects the overall engagement on likes as the optimal number of advanced features are not included across all posts. The firm obtains the maximum possible engagement corresponding to their budget by reducing the number of advanced features on posts that are published on the days with lower engagement intensity.*

The firm must focus on adding more advanced features for posts (i.e., more content development activities) to be published on days having relatively high engagement intensity. More budget allocation for social media analytics affects the content development activities which in turn, forces the firm to use fewer advanced features on posts to be published during the days with less engagement intensity. In our case study, Tuesdays and Thursdays receive the highest engagement intensity, the firm may utilize more advanced features on posts to be published during those days.

Similar to the above setting, we run extensive computation experiments for Facebook. Tables C.15 and C.16 display the results for Facebook shares and comments, respectively. We visually depict the results for both Facebook shares and comments on the right panel of Figure 4.5.

Since U for Facebook remained constant at 18, we do not display it in the figure. Unlike the relationship between likes and advanced features on Instagram, both engagement types on Facebook follow a convex relationship with advanced features. Table C.12 shows that f_k^* = 5 minimizes the shares. Additionally, the engagement at f_k = 12 is substantially higher than that at f_k = 0. Table C.15 shows the results for Facebook shares. The firm utilizes f_k = u_2 (or, 18 for Facebook) advanced features on posts to be published other than the weekends. Among week days, posts scheduled for Tuesdays and Thursdays continue to include u_2 advanced features even at a very high value of e. This is not surprising for Facebook shares since the firm receives more benefits by not using any advanced features than $f_k \leq 11$. This is evident from the fact that at least one post during each planning horizon is assigned 12 advanced features. We observe a similar pattern for Facebook comments in Table C.16, where the minimum engagement occurs at f_k^* = 6. User engagement at f_k = 13 substantially exceeds the engagement at f_k = 0. The results conform our theoretical insights for the convex relationship between advanced features and Facebook shares (respectively, comments).

Managerial Insight 3: *The relationship between the user engagement measured by the number of shares (respectively, comments) on Facebook and advanced features reveals a convex pattern. The engagement decreases initially with the number of advanced features and rises after exceeding certain thresholds. For example, in the furniture industry context, the number of shares (respectively, comments) increases as advanced features exceed 5 (respectively, 6).*

Managerial Insight 4: *When the budget tightens, the firm may use more advanced features on posts for days with higher engagement intensity and less or zero advanced features on posts for days with lower engagement intensity.*

Facebook results offer interesting insights. Under a sufficient budget, the firm uses the maximum number of advanced features on its posts. When the budget tightens, the maximum number of advanced features are placed on the posts for days with higher engagement intensity days, and other posts have less or zero number of advanced features. In cases with no advanced features on a post, the firm may place only the focal features and add further textual information.

Next, we run the numerical analysis for the case with multiple engagement types combined for a single platform. The model can be run by assigning weights to the engagement types. We assign Facebook shares and comments two different weights of 0.7 (μ_2) and 0.3 (μ_3), respectively and analyze the solutions. We present a combined formulation for Facebook in Problem $LSSM_f$ in Appendix C.2. There are two major differences between Problems $LSSM_f$ and $LSSM_s(Cases2{-}3)$: (i) the addition of index m to denote the engagement type, (ii) the engagement weight variables μ_2 and μ_3, and (iii) g_{mj}, the enumerated engagement values for each f_k. The number of simple and advanced features are platform dependent but engagement type independent.

The results are presented in Table C.17. Since the engagement types are from the same platform, the number of features, both simple and advanced, the effort U, and the upper limits for both features remain common. x_{1k} and x_{2k} refer to the engagement from shares and comments, respectively. Our model, $LSSM_f$ efficiently selects the number of advanced features for the posts to be published on a platform to achieve the maximum total weighted user engagement across all engagement types, depending on their weights.

For a platform with multiple engagement types, the firm must focus on the engagement metrics they prefer. For example, sharing helps in reaching out to more target audiences. On the other hand, the firm can gather more information on user sentiment through comments. The firm can set the appropriate weights depending on the impact of each engagement type on their business context. Since our model, $LSSM_f$ incorporates both engagement types, the firm can derive maximum benefits by following our approach. In the next subsection, we present the generalized problem, including all platforms and engagement types in a single formulation.

4.4.3 Multiple Platforms and Multiple Engagement Types - Generalized Problem

Firms desire their presence across multiple social media platforms in order to reach varieties of users. Moreover, users also retrieve information from multiple platforms (Zhou et al. 2015). Thus, firms need comprehensive social media marketing management strategies by considering multiple platforms for getting the maximum user engagement through their organic posts. This raises the following key challenging aspects that must be incorporated into the generalized model.

- Modalities of engagement may differ within and across the platforms. The engagement types signal the degree of interest of a user. For example, a user's degree of interest on a post through comments are highly valued than likes within a platform.

- Our single platform analysis shows that inclusion of the number of advanced features in a post have opposite influences on user engagement in different platforms. For example, Facebook and Instagram users have differing engagement levels in viewing the advanced features in a post. We include this aspect into the generalized problem.

- A firm's content publication across platforms is a significant lever in reaching out to a larger audience and impacting user engagement. Thus, the firm goal is to achieve the highest possible overall engagement from all platforms. This requires a standardized approach of incorporating above mentioned aspects of different platforms within a multi-platform framework to achieve the highest benefits.

In the previous approach, we show that the firm solves its problem by running the model individually for each platform and combining engagement types within it. However, such an approach may be suboptimal as the budget allocation across the platforms must be determined optimally. Thus, we develop here the generalized problem with the multiple platforms and multiple engagement types. The firm's objective is to maximize the overall user engagement across multiple platforms during a planning horizon.

Firms create content for a post on social media platforms to advertise their products and often customize the content of a post with modifications in multiple platforms, saving them content creation costs substantially and letting them reach out to their audience faster. While reusing the same content of a post, a firm may boost its brand messaging. However, such cross-posting efforts may not yield benefits in terms of user engagement since each platform has its own characteristics. Our model also captures the customization approach by tailoring the number of features on posts to be published on different platforms.

In the multiple platform problem, we distinguish different user engagement types via the parameter z_{lm}, which represents the importance and impact of the platform and its engagement types on the firm. Besides assigning weights for days of a week, this parameter differentiates platforms and their engagement types. z_{lm} depends on the user-base for each platform l, v_l and the weight of an engagement type, μ_{lm}. We follow the model developed by Aichner and Jacob (2015), where they describe two metrics - social media impact factor (SMIF) and social media use (SMU). The authors calculate SMIF of every platform l by dividing the active users of platform l to the total active users across platforms, $V = \sum_{l=1}^{2} v_l$. We use v_l/V to represent SMIF and estimate it from the firm's social media accounts. Next, SMU of each platform is measured by computing the user reactions through engagement types. For each platform, they compare different engagement types and assign weights accordingly. For example, one share is ten times more valuable than one like and twice the value of a comment on Facebook. We capture the engagement weights within each platform. Thus, we construct z_{lm} using v_l and μ_m and present them in Table 4.6. It is obvious that we set $z_{1,2} = z_{1,3} = 0$ and $z_{2,1} = 0$.

Table 4.6: Platform and engagement weights

Coefficients	Instagram ($l = 1$)	Facebook ($l = 2$)	
	Likes ($m = 1$)	Shares ($m = 2$)	Comments ($m = 3$)
Platform engagement weight Engagement weight	$\frac{v1}{V}$ $\mu_{11} = 1.0$	$\frac{v2}{V}$ $\mu_{22} = 0.7$	$\frac{v2}{V}$ $\mu_{23} = 0.3$
(Engagement+ Platform) weight, z_{lm}	$z_{1,1} = \mu_{11}(\frac{v1}{V})$	$z_{2,2} = \mu_{22}(\frac{v2}{V})$	$z_{2,3} = \mu_{23}(\frac{v2}{V})$

In the generalized model, we optimize the user engagement across platforms with the objective of creating a template for content designers to develop different posts for a planning horizon. With the design parameters across the platform in hand (e.g., the number of focal and advanced features to be included in the post for each platform), designers can choose to customize their content while creating posts for different platforms. We continue with our assumption that the firm publishes different posts for different days/platforms. However, note that the firm publishes one post per day for each platform. Thus, the variables representing features, s_{lk} and f_{lk} are engagement type independent.

Problem SMM_G**:**

$$\textbf{Max } \Pi_g \quad = \quad \sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k=1}^{K} z_{lm} w_k x_{lmk}$$

Subject to:

$$a\sum_{l=1}^{L}\sum_{k=1}^{K}(s_{lk}+f_{lk}) + e\sum_{l=1}^{L} U_l + LC_f \leq B \tag{4.17}$$

$$x_{lmk} = b_{lm} + \beta_{lm} s_{lk} + \zeta_{lm} f_{lk} + \eta_{lm} f_{lk}^2, \forall k; \quad \forall l; \quad \forall m \tag{4.18}$$

$$s_{lk} \leq u_{1l}, \quad \forall k; \quad \forall l \tag{4.19}$$

$$f_{lk} \leq U_l, \quad \forall k; \quad \forall l \tag{4.20}$$

$$U \leq u_{2l}, \quad \forall k, \quad \forall l \tag{4.21}$$

$$s_{lk} \geq q_{lk}, \quad \forall k; \quad \forall l \tag{4.22}$$

$$x_{lmk}, s_{lk}, f_{lk}, U_l: \text{ Integer variable } (\geq 0), \ \forall l; \ \forall m; \ \forall k \tag{4.23}$$

Constraint set (4.17) limits the three cost parameters under the total budget, similar to the single platform and single engagement type case although with platform (l) notations. In Constraint set (4.18), we associate user engagement with our empirical results for each platform l and user engagement m. However, the features are independent of m. Constraint sets (4.19) and (4.20) restrict the simple and advanced features within their upper bound and the decision variable U_l, respectively. In Constraint (4.21), we restrict the decision variable U_l under the upper bound of advanced features u_{2l} for each platform l. We set the focal products(s) criterion through Constraint set (4.22). In Constraint set (4.23), we define the nonnegativity constraints.

We follow a similar approach we used for the single platform formulations and linearize Constraint (4.18) and present the linear version $LSMM_G$ of SMM_G in Appendix C.2. We linearize the nonlinear Constraint (4.18) by using Constraints (C.21) and (C.22). There are substantial differences between the linearized general model $LSMM_G$ and the single platform combined formulation $LSMM_f$. The binary variable, $y_{l,k,j}$ now depends on the platform l since the firm can vary the number features on content across platforms. Depending on the convex or concave relationships between advanced features and engagement, the firm can either use the upper bound on the advanced features or the optimal advanced features, represented by J^l. For example, J^l take the values of $f^*_{l=1,k}$ for Instagram and $u_{2,l=2}$ for Facebook. The enumerated values, g_{lmj} depend on both platform and engagement types.

4.4.3.1 Numerical Analysis - Generalized Framework.

Figure 4.6: Multiple platform analysis

Next, we run $LSSM_G$ and present the numerical results in Table C.18. In our example, the user-base on Facebook is relatively higher than that of Instagram. We assign 100000 and 80000 users to Facebook and Instagram, respectively, which are scaled numbers based on our data. For ease of exposition, we kept q_k similar across two platforms and set a budget of \$12000, which is within the range of firms' social media budgets as discussed earlier. Furthermore, consistent with our single platform results, we assume the same engagement intensities across the days within the planning horizon. The generalized problem is unique for two more reasons - (i) the functional relationship between engagement-feature is different across multiple platforms (ii) the number of advanced features to be included in a post is different across multiple platforms. We find that as the costs of analytics increase, the number of advanced features on Instagram posts reduces steadily while Facebook posts include the maximum number of advanced features. In the case study at furniture company, limited budget affects Instagram posts, primarily because Instagram has a lower user-base as compared to Facebook.

We illustrate the results in Figure 4.6. The left and right graphics on the top panel illustrate the engagement levels and U on Instagram and Facebook, respectively. Facebook comments remain the same, and therefore we omit them in Figure 4.6. As e increases, likes and U for Instagram show a downward curve. On the other hand, shares on Facebook increase moderately. Consequently, overall weighted engagement across two platforms declines as e increases.

Managerial Insight 6: *The firm must focus on the overall user engagement across the platforms and allocate large portion of the budget on the platform that offers high engagement. The number of advanced features for posts at low engagement platform is sensitive to the per unit variable cost of analytics (e). As this cost increases, the number of advanced features on posts for low engagement platform reduces. Thus, the firm must prioritize the platforms according to the user engagement and allocate accordingly more advanced features for posts to be placed at platforms with high engagement. In the case study example, the furniture company obtains more overall engagement by reducing advanced features in Instagram posts and maintaining high number of advanced features on Facebook posts.*

4.5 Extensions

In this section, we discuss relevant extensions of our model that we observe by running the computational experiments. First, we compare two cases: our approach and a policy where the firm allocates budget solely based on the user-base of a platform. The latter policy is not uncommon (HootSuite 2021). Second, we also discuss the engagement levels on Facebook with the number of advanced features.

4.5.1 Comparison Against User-Base Budget Allocation

We consider a case where the allocation of budget are done according to a platform's user-base to maximize user engagement. The number of users are different across platforms. Therefore, a firm may want to distribute their budget accordingly. This approach entails the separation of social media analytics and content development by platforms. For presentation purpose, we report the overall user engagement and the difference from the generalized linear model in Appendix C.19. As can be seen, for every value of e, the average engagement is lower when budget is pre-allocated. The difference is consistently within the range of 11-12%. Here, U_1 and U_2 represent the effort levels on Instagram and Facebook, respectively (refer Appendix C.18). The firm fully exploits social media analytics on both platforms and ignores the level of engagement. This produces sub-optimal performances since the firm misses out the opportunity to prioritize content development for Facebook and fails to gain higher overall engagement.

Managerial Insight 7: *User-base focused budget allocation may produce sub-optimal performance compared to our approach.*

However, the results require a cautious explanation. First, increasing overall engagement across platforms may not be ideal for a firm that solely wants to advertise on one platform. Additionally, the firm may want to attract more users on their less popular platform, and a user-based-focused budget allocation may negatively impact that strategy.

4.5.2 Facebook Engagement

The structural results show that Facebook users prefer either fewer or more features. Both comments and shares show similar patterns. As observed from Tables C.15 and C.16, social media analytics effort (U) remain the same as budget tightens. However, in most cases, advanced features on posts are either the maximum (18) or the minimum (0). Thus, the firm utilizes user insights and develops posts accordingly to obtain the highest possible engagement. Under more budget restrictions, the firm obtains more benefits than not having any advanced features than having fewer of them. The firm must exclusively focus on focal features and generate substantial engagement in such a scenario.

4.6 Discussion and Conclusion

The popularity of social media platforms has opened up new opportunities for firms to advertise their products and reach their target audience. These platforms facilitate real-time interactions between social media posts and users, allowing firms to monitor and collect user feedback information which has enormous operational value for the firms (Lee et al. 2018). Firms can use the information to deliver better social media posts to users. However, developing posts is challenging. A survey by Sebastian (2021) finds that only 40.6% of online users find the advertisements on social media platforms relevant. Social media marketing depends on relevant information and firms are increasingly using advanced analytics tools such as deep learning algorithms to mine images and uncover trending patterns (Linkfluence 2021). Thus, social media analytics has become an essential tool to discover users' preference that are valuable information for firms to design and develop posts attractive to their users (ResearchAndMarket 2021).

Most companies today, irrespective of their sizes, are spending substantial resources on social media platforms to gain competitive advantage. This includes spending more money to advertise their products. Since the proliferation of multimedia social platforms, image-based content has become instrumental in advertising products. For example, IKEA, a leading global furniture retailer, constantly uses pictures with home decor to provide an integrated experience to its users. This include having many features within an image, not just the focal product(s). Victor Bayata, head of mobile solutions at IKEA, said "So it's very important for us to understand these [people] behaviors because that is how we start supporting them and bringing relevant communication. Our ambition as a company is all about, again, bringing the customer a unique, relevant and integrated experience that can allow us to engage."[5] In this context, by utilizing social media analytics to de-

[5] https://www.retaildive.com/ex/mobilecommercedaily/ikea-exec-selling-through-instagram-creates-personalized-

velop posts using appropriate features on the images, firms can improve user engagement on social media platforms, which in turn, increases the potential future sales.

4.6.1 Managerial and Industry Implications

In this chapter, we analyze how social media analytics and content development activities can be jointly utilized to deliver better posts for users to achieve higher engagement. We fill an important gap in the literature by analyzing social media content development strategies using a data-driven optimization framework and offering valuable insights. We develop an optimization model that maximizes user engagement under a limited budget. Our model parameters and the functional form of user engagement are estimated via an empirical analysis based on the data from Facebook and Instagram. We develop insights into the following aspects: (i) the relationship between user engagement and image features across platforms, (ii) the budget allocation for the competing costs of social media analytics and content development activities in our model, and (iii) the budget allocation and content development strategies on single and multiple platforms. Our work has substantial managerial implications for designing social media content development strategies.

We derive parameters and the functional form of the relationship between user engagement and image features from our empirical analysis. We use two different deep learning algorithms and identify simple and advanced features in posts. Simple features define the image of the focal product(s) to be advertised. Advanced features represent the additional features in a post that enhances the context of an advertised product to the users. Our empirical analysis show that these additional features across platforms improve user engagement, although nonlinearly. We develop technique to linearize this relationship in our optimization model. The relationship between user engagement and advanced features are different across platforms, which is also captured in the optimization model. We find a clear difference in user behavior across platforms where Instagram and Facebook users prefer relatively low and high number of features in posts, respectively. These findings show that a “one-size-fits-all" strategy does not work; firms must tailor their posts according to the platforms’ user preferences.

---

user-experience

We offer guidelines on developing social media posts by exploring the interaction between social media analytics and social media content development and model their costs in the optimization framework. We guide how a firm can achieve the highest overall engagement as their budget changes. Our models derive insights for both single and multiple social media platform strategies. In the context of analyzing two platforms, we find that as budget tightens, the firm allocates fewer advanced features on posts to be published (i) on days that exhibit relatively low engagement intensity and (ii) on the platform which has cumulatively less user-base and engagement. This work can be extended to three or more social media platforms without the loss of generality. Our numerical analysis shows that a budget allocation policy solely based on a platform's user-base produces suboptimal performance compared to our model solution, which considers user-base and engagement intensity together.

We now discuss some additional results obtained in our empirical analysis and their implication to practice. Even though the following discussions do not claim causality, they are substantially related to social media marketing. First, the post interval or the duration between two successive posts is negatively associated with user engagement across all platforms. Given the dynamic nature of social media advertising and competition among marketers to seek user attention, infrequent social media activity provides negative outcomes. Users may disengage because of untimely information and no communication. Thus, firms should frequently publish posts on their social media accounts. Second, different platforms have different user preferences. For example, Instagram users do not like more features in an image but prefer more textual information, as evidenced from the positive relationship between word count of a post on Instagram and the number of likes. In contrast, Facebook users are attracted to high number of features and less textual details, revealed by the negative association between word count and shares and comments on Facebook. Finally, we also observe how image color properties can influence engagement. Saturation refers to the strength of color present within a picture and it negatively impacts engagement on Facebook. Since Facebook users prefer more features, images with high saturation levels may lead to negative outcomes.

4.6.2 Limitations and Future Research

We present some limitations of this chapter which can open up exciting future research avenues. First, we collected data from Instagram and Facebook from the accounts of leading furniture firms and used the pre-trained deep learning models to analyze them. The parameters and the functional form of engagement are both derived from the empirical results based on that data. While our optimization framework is generalizable for other industries, future research may examine multiple datasets from different industries and extend the application domain. Second, we use the number of features of images along with other relevant post information such as textual descriptions, image color properties, and post time and analyze user engagement. Further research can classify different features or use clustering properties to determine a particular feature's role in engagement. However, such a study may face challenges in specifying the class of features, which is a complex and subjective task and requires substantial time and effort. Third, our optimization model does not directly incorporate the relationship between textual information and image features. We show that they act differently on different platforms. Future studies may include this relationship in social media user engagement research. Finally, for this work, we do not focus on algorithmic development, and therefore, use pre-trained models to illustrate the social media analytics utility. Studies may consider developing and tuning pre-trained models and also create comparative research works.

5. CONCLUSION

In my dissertation, I analyze challenging and practically relevant operational efficiency issues in service organizations. I specifically explore the resource utilization and planning problems in e-commerce or online retailing, healthcare, and social media advertising domains. Motivated by the order fulfillment challenges faced by online retailing managers, my first essay specifically investigates the approaches to allocate safety stock in online distribution networks. The need and desire to continually reduce operating costs while balancing inventory and transportation costs is a common challenge online retailers face worldwide. Additionally, demand uncertainty, coupled with the size of their networks, makes it more difficult for online retailers to systemically allocate safety stock at the right place and at the right time.

Considering demand uncertainty and network size, the generalized formulation of safety stock and transshipment model can efficiently allocate safety stock by reducing expensive transshipments. However, the current state-of-the-art MIP software cannot solve instances with a large number of fulfillment centers within a reasonable amount of time. Therefore, my approach emphasizes reducing the complexity associated with safety stock planning. For example, by creating clusters of fulfillment centers, managers can holistically administer a large network of FCs with ease. This approach is consistent with the industry practice of grouping nearby FCs while maintaining control over every individual FC as closely as possible. The cluster-based approach helps in storing inventory at the right place and can result in cost-effective and faster delivery. I also validate the efficiency of my approach, and the proposed PASS provides solutions reasonably close to the optimum for small, medium, and larger networks.

This study fills a gap in the online retailing and inventory management literature by proposing a new and practical model-driven solution approach for safety stock allocation. Despite substantial differences among online retailers in terms of inventory allocation and shipment strategies, the proposed method offers an easy-to-implement safety stock allocation process intending to reduce excess operational costs. Even 1-2% of savings in transportation may result in significant savings.

With an increasing trend in online shopping worldwide, the net savings can be significant, and the benefits can be passed on to the consumers.

My second essay explains the characteristics of ACOs in terms of their care delivery through provider composition and experience that affect their performance. It is particularly significant since the rising health expenditure is one of the major concerns for policymakers in the US. The healthcare system suffers from a fragmented nature of care under fee-for-service agreements, which results in excessive waste and duplicate tests due to the low level of coordination among the health providers. Under the ACA 2010, ACOs were created to tackle the issue of increasing healthcare costs while providing a high quality of care through a coordinated system across healthcare providers. The ACO model has been widely implemented across the country during the past decade with participation from both public and private payers. Over time, the ACOs have brought more patients under their umbrella, and since 2017, the net savings have increased. However, healthcare spending, specifically Medicare spending, which accounts for the health expenditure of older people and young people with long-term disabilities, is likely to increase over time due to several factors, including population aging.

PCPs are the gatekeepers in healthcare delivery and ACOs are modeled as a PCP-centered healthcare unit forming a network of specialists, hospitals, and nursing facilities. However, there is an acute shortage of PCPs. In the words of Dr. Adrian Billings, chief medical officer for Preventative Care Health Services, Texas - "That means sicker patients, that means more costly or care. That means less productivity. That means more death." The second essay first analyzes how ACOs can utilize non-PCP workforce and the implications on their performance. While specialists are not inexperienced in primary care services, they may not offer services similar to an expert PCP. This dissimilarity may reflect on financial and quality performance as ACOs gain more experience as well as move to a higher level of risk-sharing. My results confirm that delivering primary care through specialists is financially unsustainable and does not result in better quality. In the short-term, ACOs can utilize them to deliver primary care services to extend their care to more people and prevent costlier healthcare services. On the other hand, NPs can become beneficial in achieving long-term and short-term ACO objectives.

I also analyze how experience plays a role in the ACO performance. The data and analysis reveal that ACOs may focus more on savings in the initial phases under a risk model and gradually consider improving quality as they gain more experience. The results suggest that ACO performance varies across different stages. Additionally, the relationship between ACO service delivery through specialists and NPs and performance may vary with ACO experience. Overall, these findings provide valuable insights for managing the ACO model better, both by ACOs themselves and payers such as Medicare.

This study on ACOs makes several contributions to the HOM literature by analyzing the antecedents of ACO performance and their implications. To the best of my knowledge, this work is the first to focus on the understudied relationship between ACO service delivery and experience on ACO performance and how ACO experience moderates the relationship between service delivery and performance. I operationalize ACO service delivery via provider composition, which consists of the primary care services provided through specialists and NPs. This operationalization of service delivery focuses on how ACOs utilize their non-PCP workforce to deliver care and maintain access to care for their patients. I focus on these two characteristics in their short-term and long-term implications. While the data used in this study predate the new "Pathways to Success" program launched by the CMS on July 2019, the implications are still applicable. This new program still maintains the core of the ACO model. However, it requires ACOs to take up more resposibility (or downside risk contract) sooner than before. My questions are pertinent in the general ACO network model and the findings can help in the "Pathways to Success" program.

My third essay explores how social media analytics and content development can be jointly utilized to deliver better image content to users to achieve higher engagement. This chapter fills an important gap in the literature by analyzing social media content development strategies using a data-driven optimization framework and offering insights into a social media content ecosystem. I develop an optimization model that maximizes user engagement under budget constraints. The model parameters and the functional form of user engagement are estimated via an empirical analysis based on the data from Facebook and Instagram. I show insights on (i) user engagement and image feature relationships across platforms, (ii) social media analytics and content development costs in the model, and (iii) insights on single and multiple platforms. This work has substantial managerial implications for designing social media content development strategies. I also discuss some additional results from the empirical analysis.

I derive parameters and the functional form of engagement from the empirical analysis. This chapter utilizes two different deep learning algorithms and operationalize simple and advanced features. Advanced features represent the additional features in an image that explains the context of an advertised product to the users. The empirical analysis show that these additional features across platforms improve user engagement, although nonlinearly. I linearize the relationship from the empirical results and employ in the optimization model. The relationship between engagement and advanced features are different across platforms, which is also captured in the optimization model. I find a clear difference in user behavior across platforms where Instagram and Facebook users prefer relatively low and high number of features in image posts, respectively. These findings show that a “one-size-fits-all" strategy does not work; firms must tailor their content according to the platforms’ user preferences.

I also find that as budget tightens, the firm allocates fewer advanced features on posts to be published (i) on days that exhibit relatively low engagement intensity and (ii) on platform which has cumulatively less user-base and engagement. Without the loss of generality, this work can be extended to three or more social media platforms. I numerically show that a budget allocation policy solely based on a platform’s user-base produces sub-optimal performance compared to the framework, which considers user-base and engagement intensity together.

Social media marketing depends on relevant information and firms are increasingly using advanced analytics tools such as deep learning algorithms to mine images and uncover trending patterns. My work sheds light on the social media marketing landscape and identifies factors relevant to user engagement under a firm’s budget allocation to develop the appropriate content to be posted on social media platforms. I offer guidelines on developing image content by exploring the interaction between social media analytics and social media content development and model their costs in the optimization framework. The numerical analysis exhibits the changing levels user engagement according to the changing costs of social media analytics.

In conclusion, my dissertation focuses on multiple industries and utilizes multiple methods - econometric method, stochastic optimization, and data-driven optimization to analyze the efficiency problems in online retailing, healthcare, and social media advertising. The research works contribute to the growing areas in OM, explore managerially relevant questions, and offer actionable guidelines. I believe that the findings discussed in the dissertation will enhance the service organizations by making them more efficient.

REFERENCES

AANP (2014) *Prioritising primary care in the USA.* Available at NursePractitionersinPrimaryCare (last accessed: August 16, 2021).

Acimovic J, Graves SC (2015) Making better fulfillment decisions on the fly in an online retail environment. *Manufacturing & Service Operations Management* 17(1):34–51.

Acimovic J, Graves SC (2017) Mitigating spillover in online retailing via replenishment. *Manufacturing & Service Operations Management* 19(3):419–436.

Aichner T, Jacob F (2015) Measuring the degree of corporate social media use. *International Journal of market research* 57(2):257–276.

Aiken LH, Lewis CE, Craig J, Mendenhall RC, Blendon RJ, Rogers DE (1979) The contribution of specialists to the delivery of primary care: a new perspective. *New England Journal of Medicine* 300(24):1363–1370.

Aldous KK, An J, Jansen BJ (2019) View, like, comment, post: Analyzing user engagement by topic at 4 levels across 5 social media platforms for 53 news organizations. *In Proceedings of the International AAAI Conference on Web and Social Media, July 2019* 13:47–57.

Aledade (2020) *For Physician-Led ACOs, Building Trust Brings Success with Downside Risk*. Available at https://doi.org/10.1016/S0140-6736(19)31678-2 (last accessed: June 16, 2021).

Amibbola A (2021) *Should You Post the Same Content to All Social Networks?* Available at https://mauconline.net/should-you-post-the-same-content-to-all-social-networks/ (last accessed: November 20, 2021).

Anand G, Chandrasekaran A, Sharma L (2021) Sustainable process improvements: Evidence from intervention-based research. *Journal of Operations Management* 67(2):212–236.

Archibald T, Sassen S, Thomas L (1997) An optimal policy for a two depot inventory problem with stock transfer. *Management Science* 43(2):173–183.

Archibald TW (2007) Modelling replenishment and transshipment decisions in periodic review multilocation inventory systems. *Journal of the Operational Research Society* 58(7):948–956.

Archibald TW, Black D, Glazebrook KD (2009) An index heuristic for transshipment decisions in multilocation inventory systems based on a pairwise decomposition. *European Journal of Operational Research* 192(1):69–78.

Archibald TW, Black DP, Glazebrook KD (2010) The use of simple calibrations of individual locations in making transshipment decisions in a multi-location inventory network. *Journal of the Operational Research Society* 61(2):294–305.

Arens E (2021) *The best times to post on social media in 2021.* Available at https://sproutsocial.com/insights/best-times-to-post-on-social-media/ (last accessed: October 23, 2021).

Argote L, Ingram P, Levine JM, Moreland RL (2000) Knowledge transfer in organizations: Learning from the experience of others. *Organizational Behavior and Human Decision Processes* 82(1):1–8.

Audrezet A, de Kerviler G, Moulard JG (2086) Authenticity under threat: When social media influencers need to go beyond self-presentation. *Journal of Business Research* .

Axsater S (2003) A new decision rule for lateral transshipments in inventory systems. *Management Science* 49(9):1168–1179.

Baicker K, Chandra A (2004) Medicare spending, the physician workforce, and beneficiaries' quality of care: Areas with a high concentration of specialists also show higher spending and less use of high-quality, effective care. *Health Affairs* 23(Suppl1):W4–184.

Bajaj A, Kekre S, Srinivasan K (2004) Managing npd: Cost and schedule performance in design and manufacturing. *Management Science* 50(4):527–536.

Barnett ML, McWilliams JM (2018) Changes in specialty care use and leakage in medicare accountable care organizations. *The American Journal of Managed Care* 24(5):E141.

Baumgartner H (2002) Toward a personology of the consumer. *Journal of Consumer Research* 29(2):286–292.

BDC (2020) *What is the average marketing budget for a small business?* Available at https://www.bdc.ca/en/articles-tools/marketing-sales-export/marketing/what-average-marketing-budget-for-small-business (last accessed: October 10, 2021).

Bish EK, Wang Q (2004) Optimal investment strategies for flexible resources, considering pricing and correlated demands. *Operations Research* 52(6):954–964.

Broome T (2017) *Shared Savings Program 2016 Results*. Available at https://www.ajmc.com/view/cms-releases/-medicare-shared/-savings-program/-2016-results (last accessed: August 11, 2019).

Burns LR, Pauly MV (2012) Accountable care organizations may have difficulty avoiding the failures of integrated delivery networks of the 1990s. *Health Affairs* 31(11):2407–2416.

Burwell SM (2019) *Building a system that works: the future of health care.* Available at https://www.healthaffairs.org/do/10.1377/hblog20161212 (last accessed: January 15, 2022).

Callister LC, Hobbins-Garbett D (2000) "enter to learn, go forth to serve": Service learning in nursing education. *Journal of Professional Nursing* 16(3):177–183.

Cameron AC, Trivedi PK (2013) Regression analysis of count data. *Cambridge university press* 53.

Campbell A, Clarke L, Kleywegt A, Savelsbergh M (1998) The inventory routing problem. *Fleet Management and Logistics* 95–113.

Cao DB, Silver EA (2005) A dynamic allocation heuristic for centralized safety stock. *Naval Research Logistics* 52(6):513–526.

Cassidy A (2012) Nurse practitioners and primary care. *Health Affairs* 33(3).

Chandrasekaran A, Senot C, Boyer KK (2012) Process management impact on clinical and experiential quality: Managing tensions between safe and patient-centered healthcare. *Manufacturing and Service Operations Management* 14(4):548–566.

Chen X, Gao X, Hu Z (2015) A new approach to two-location joint inventory and transshipment control via l-convexity. *Operations Research Letters* 43(1):65–68.

Cheng G, Han J, Lu X (2017) Remote sensing image scene classification: Benchmark and state of the art. *Proceedings of the IEEE* 105(10):1865–1883.

Choudhury MM, Harrigan P (2014) Crm to social crm: the integration of new technologies into customer relationship management. *Journal of Strategic Marketing* 22(2):149–176.

Ciregan D, Meier U, Schmidhuber J (2012) Multi-column deep neural networks for image classification. *IEEE Conference on Computer Vision and Pattern recognition* 3642–3649.

Claxton G, DiJulio B, Whitmore H, Pickreign J, McHugh M, Finder B, Osei-Anto A (2009) Job-based health insurance: Costs climb at a moderate pace: Premiums grew about 5 percent from 2008 to 2009, as average family coverage reached 13,375. *Health Affairs* 28(Suppl1):1002–1012.

CMS (2021) *Performance Year Financial and Quality Results.* Available at `https://data.cms.gov/medicare-shared-savings-program/performance-year-financial-and-quality-results` (last accessed: December 26, 2021).

Cohen WM, Levinthal DA (1990) Absorptive capacity: A new perspective on learning and innovation. *Administrative Science Quarterly* 128–152.

Coursaris CK, Van Osch W, Balogh BA (2016) Do facebook likes lead to shares or sales? exploring the empirical links between social media content, brand equity, purchase intention, and engagement. *In 2016 49th Hawaii international conference on system sciences (HICSS) IEEE* 2016, January:3546–3555.

Cui R, Gallino S, Moreno A, Zhang DJ (2018) The operational value of social media information. *Production and Operations Management* 27(10):1749–1769.

Dadzie KQ, Chelariu C, Winston E (2005) Customer service in the internet-enabled logistics supply chain: website design antecedents and loyalty effects. *Journal of Business Logistics* 26(1):53–78.

Dagger TS, Sweeney JC, Johnson LW (2007) A hierarchical model of health service quality: scale development and investigation of an integrated model. *Journal of service research* 10(2):123–142.

DataReportal (2020) *Global Social Media Overview.* Available at `https://datareportal.com/social-media-users` (last accessed: October 26, 2020).

DesRoches CM, Gaudet J, Perloff J, Donelan K, Iezzoni LI, Buerhaus P (2013) Using medicare data to assess nurse practitioner–provided care. *Nursing Outlook* 61(6):400–407.

Dessart L, Veloutsou C, Morgan-Thomas A (2015) Consumer engagement in online brand communities: a social media perspective. *Journal of Product and Brand Management* 24(1):28–42.

Digital360 (2021) *Amazon Prime reaches 200 million members worldwide.* Available at `https://www.digitalcommerce360.com/article/amazon-prime-membership/` (last accessed: August 2, 2021).

Ding M, Eliashberg J (2002) Structuring the new product development pipeline. *Management Science* 48(3):343–363.

Duarte J, Siegel S, Young L (2012) Trust and credit: The role of appearance in peer-to-peer lending. *Review of Financial Studies* 25(8):2455–2484.

Dutton JM, Thomas A (1984) Treating progress functions as a managerial opportunity. *Academy of Man-*

agement Review 9(2):235–247.

Erdem T, Swait J (1998) Brand equity as a signaling phenomenon. *Journal of consumer Psychology* 7(2):131–157.

Faaland B, McKay M, Schmitt T (2019) A fixed rate production problem with poisson demand and lost sales penalties. *Production and Operations Management* 28(3):516–534.

Federal (2018) *Medicare Program; Medicare Shared Savings Program; Accountable Care Organizations-Pathways to Success and Extreme and Uncontrollable Circumstances Policies for Performance Year 2017.* Available at `https://www.federalregister.gov/documents/2018/12/31/2018-27981/medicare-program-medicare-shared-savings-program-accountable-care-organizations-pathways-to-success` (last accessed: December 20, 2020).

Finkelstein A, Gentzkow M, Hull P, Williams H (2017) Adjusting risk adjustment–accounting for variation in diagnostic intensity. *The New England journal of medicine* 376(7):608.

Fisher ES, Shortell SM (2010) Accountable care organizations: accountable for what, to whom, and how. *JAMA* 304(15):1715–1716.

Fisher ES, Shortell SM, Kreindler SA, Van Citters AD, Larson BK (2012) A framework for evaluating the formation, implementation, and performance of accountable care organizations. *Health Affairs* 31(1):2368–2378.

Fisher ES, Staiger DO, Bynum JP, Gottlieb DJ (2006) Creating accountable care organizations: The extended hospital medical staff: A new approach to organizing care and ensuring accountability. *Health Affairs* 25(Suppl1):W44–W57.

Forbes (2021) *Social Media Has Over-Promised, But Under-Delivered For Retailers. Here's The Fix.* Available at `https://www.forbes.com/sites/pamdanziger/2021/05/23/social-media-has-over-promised-but-under-delivered-for-retailers-heres-the-fix/?sh=2340e1711686` (last accessed: December 15, 2021).

Fractl (2017) *Organic vs Sponsored Instagram Posts.* Available at `https://www.frac.tl/work/marketing-research/organic-vs-sponsored-instagram-posts/` (last accessed: November 2, 2020).

Franks P, Clancy CM, Nutting PA (1992) Gatekeeping revisited—protecting patients from overtreatment.

New England Journal of Medicine 327(6):424–429.

Freeman M, Robinson S, Scholtes S (2021) Gatekeeping, fast and slow: An empirical study of referral errors in the emergency department. *Management Science* 67:4209–4232.

Ganguli I, Lupo C, Mainor AJ, Orav EJ, Blanchfield BB, Lewis VA, Colla CH (2020) Association between specialist compensation and accountable care organization performance. *A Health Services Research* 55(5):722–728.

Ganguli I, Souza J, McWilliams JM, Mehrotra A (2017) Trends in use of the us medicare annual wellness visit, 2011-2014. *JAMA* 317(21):2233–2235.

Gary W (2021) *8 Reasons Social Media Campaigns Fail.* Available at `https://everconvert.com/8-reasons-social-media-campaigns-fail/` (last accessed: December 15, 2021).

Gitlin J (2021) *74 percent of people are tired of social media ads—but they're effective.* Available at `https://www.surveymonkey.com/curiosity/74-of-people-are-tired-of-social-media-ads-but-theyre-effective/` (last accessed: October 2, 2021).

Glazebrook K, Paterson C, Rauscher S, Archibald T (2015) Benefits of hybrid lateral transshipments in multi-item inventory systems under periodic replenishment. *Production and Operations Management* 24:311–324.

Hallock W, Roggeveen AL, Crittenden V (2019) Firm-level perspectives on social media engagement: an exploratory study. *Qualitative Market Research: An International Journal* .

Hansell MJ (1991) Sociodemographic factors and the quality of prenatal care. *American Journal of Public Health* 81(8):1023–1028.

Hashem A, Chi MT, Friedman CP (2003) Medical errors as a result of specialization. *Journal of Biomedical Informatics* 36(1-2):61–69.

Hayes AF (2017) Introduction to mediation, moderation, and conditional process analysis: A regression-based approach. *Guilford publications* .

Heath S (2018) *NPs, PAs Could Reduce Primary Care Physician Shortage Nearly 70 Percent.* Available at `https://patientengagementhit.com/news/nps-pas-could-reduce-primary-care-physician-shortage-nearly-70` (last accessed: June 16, 2021).

Henderson G (2020) *How Much Time Does The Average Person Spend On Social Media?* Available at `https://www.digitalmarketing.org/blog/how-much-time-does-the-`

average-person-spend-on-social-media (last accessed: October 10, 2021).

Herer YT, Tzur M, Yucesan E (2002) Transshipments: An emerging inventory recourse to achieve supply chain leagility. *International Journal of Production Economics* 80(3):201–212.

Herer YT, Tzur M, Yucesan E (2006) The multilocation transshipment problem. *IIE transactions* 38(3):185–200.

Hootsuite (2021) *10 Social Media Analytics Tools that Will Do the Math For You.* Available at https://blog.hootsuite.com/social-media-analytics/ (last accessed: November 23, 2021).

HootSuite (2021) *How to Create a Social Media Budget for Every Size of Business*. Available at https://blog.hootsuite.com/the-7-components-of-every-social-media-budget/ (last accessed: December 19, 2021).

Horrocks S, Anderson E, Salisbury C (2002) Systematic review of whether nurse practitioners working in primary care can provide equivalent care to doctors. *BMJ* 324(7344):819–823.

Hu X, Duenyas I, Kapuscinski R (2008) Optimal joint inventory and transshipment control under uncertain capacity. *Operations Research* 56(4):881–897.

Huckman RS, Pisano GP (2006) The firm specificity of individual performance: Evidence from cardiac surgery. *Management Science* 52(4):473–488.

Hutchison-Krupat J, Kavadias S (2015) Strategic resource allocation: Top-down, bottom-up, and the value of strategic buckets. *Management Science* 61(2):391–412.

IBM (2021) *What is social media analytics?* Available at https://www.ibm.com/topics/social-media-analytics (last accessed: November 23, 2021).

IndiaPartner (2021) *Social media ad spends to reach $177 billion in 2022, overtaking television at $174 billion: Zenith report.* Available at https://www.businessinsider.in/advertising/media/article/social-media-ad-spends-to-reach-177bn-in-2022-overtaking-television-at-174-billion-zenith-report/articleshow/88116003.cms (last accessed: November 23, 2021).

Innovations S (2021) *How Often Should Businesses Post on Social Media?* Available at https://www.sharpinnovations.com/blog/2021/07/how-often-should-a-businesses-post-on-social-media (last accessed: December 15, 2021).

Introcaso D, Berger G (2017) Mssp year two medicare acos show muted success. *Health Affairs* .

Jaakonmäki R, Müller O, Vom Brocke J (2017) The impact of content, context, and creator on user engagement in social media marketing. *In Proceedings of the 50th Hawaii international conference on system sciences, January 2017* .

Jansen JJP, Van den Bosch FAJ, Volberda HW (2005) Managing potential and realized absorptive capacity: how do organizational antecedents matter? *Academy of Management Journal* 48(6):999–1015.

Janson S, Weiss K (2004) A national survey of asthma knowledge and practices among specialists and primary care physicians. *Journal of Asthma* 41(3):343–348.

Jasin S, Sinha A (2015) An lp-based correlated rounding scheme for multi-item ecommerce order fulfillment. *Operations Research* 63(6):1336–1351.

Karmarkar U, Patel N (1977) The one-period, n-location distribution problem. *Naval Research Logistics* 4:559–575.

Kaufman BG, Spivack BS, Stearns SC, Song PH, O'Brien EC (2019) Impact of accountable care organizations on utilization, care, and outcomes: a systematic review. *Medical Care Research and Review* 76(3):255–290.

KC D, Tushe S (2021) The effects of multisiting on productivity and quality. *Manufacturing and Service Operations Management* 23(4):803–818.

Khullar D, Wolfson D, Casalino LP (2018) Professionalism, performance, and the future of physician incentives. *JAMA* 320(23):2419–2420.

Kleywegt AS A J, de Mello TH (2001) The sample average approximation method for stochastic discrete optimization. *SIAM Journal of Optimization* 12(2):479–502.

Klingebiel R, Rammer C (2014) Resource allocation strategy for innovation portfolio management. *Strategic Management Journal* 35(2):246–268.

Kocot WR SL (2016) Medicare acos: Incremental progress, but performance varies. *Health Affairs* .

Krishnan K, Rao V (1965) Inventory control in n warehouses. *Journal of Industrial Engineering* XVI(3):212–215.

Kumar A, Bezawada R, Rishika R, Janakiraman R, Kannan PK (2016) From social to sale: The effects of firm-generated content in social media on customer behavior. *Journal of Marketing* 80(1):8–25.

Kumar A, Salo J, Li H (2019) Stages of user engagement on social commerce platforms: Analysis with the navigational clickstream data. *International journal of electronic commerce* 23(2):179–211.

Kumar S, Tan Y, Wei L (2020) When to play your advertisement? optimal insertion policy of behavioral advertisement. *Information Systems Research.* .

Lancet (2019) *Prioritising primary care in the USA*. Available at `https://resources.aledade.com/blogs/for-physician-led-acos-building-trust-brings-success-with-downside-risk` (last accessed: July 6, 2021).

LaPointe J (2018) *After a Slow 2017, ACOs Grow and Expand Their Contracts in 2018.* Available at `https://revcycleintelligence.com/news/after-a-slow-2017-acos-grow-and-expand-their-contracts-in-2018` (last accessed: July 25, 2019).

Law AM (2015) Simulation modeling and analysis. fifth edition. *McGraw Hill, New York, NY* .

Lee D, Hosanagar K, Nair HS (2018) Advertising content and consumer engagement on social media: evidence from facebook. *Management Science* 64(11):5105–5131.

Lee HL (1987) A multi-echelon inventory model for repairable items with emergency lateral transshipments. *Management science* 33(10):1302–1316.

Lee JE, Hong Y (2003) A stock rationing policy in an (s, s)-controlled stochastic production system with 2-phase coxian processing times and lost sales. *International Journal of Production Economics* 83:299–307.

Lei Y, Jasin S, Sinha A (2018) Joint dynamic pricing and order fulfillment for e-commerce retailers. *Manufacturing and Service Operations Management* 20(2):269–284.

Levy FK (1965) Adaptation in the production process. *Management Science* 11(6):B–136.

Lewis VA, Colla CH, Carluzzo KL, Kler SE, Fisher ES (2013) Accountable care organizations in the united states: market and demographic factors associated with formation. *Health Services Research* 48(6pt1):1840–1858.

Li Y, Xie Y (2020) Is a picture worth a thousand words? an empirical study of image content and social media engagement. *Journal of marketing research* 57(1):1–19.

Linkfluence (2021) *AI Basics: How AI and Machine Learning Supercharge Your Social Media Marketing.* Available at `https://www.linkfluence.com/blog/ai-basics-how-ai-machine-learning-supercharge-social-media-marketing` (last accessed: November 23, 2021).

Liu L, Dzyabura D, Mizik N (2020) Visual listening in: Extracting brand image portrayed on social media.

Marketing Science 39(4):669–686.

Loch CH, Kavadias S (2002) Dynamic portfolio selection of npd programs using marginal returns. *Management Science* 48(10):1227–1241.

Lopienski K (2018) *Ecommerce Fulfillment: The Unappreciated Yet Vital Strategy Brands Use to Win Loyal Customers.* Available at https://www.bigcommerce.com/blog/ecommerce-fulfillment/ (last accessed: December 15, 2018).

Lumsden K, Dallari F, Ruggeri R (1999) Improving the efficiency of the hub and spoke system for the skf european distribution network. *International Journal of Physical Distribution and Logistics Management* 29(1):50–66.

Ma Y, Xiang Z, Du Q, Fan W (2018) Effects of user-provided photos on hotel review helpfulness: An analytical approach with deep leaning. *International Journal of Hospitality Management* 71:120–131.

Mallipeddi R, Janakiraman R, Kumar S, Gupta S (2021a) The effects of social media tone on engagement: Evidence from indian general election 2014. *Information Systems Research* 32(1):212–237.

Mallipeddi RR, Kumar S, Sriskandarajah C, Zhu Y (2021b) A framework for analyzing influencer marketing in social networks: selection and scheduling of influencers. *Management Science* .

Maltz A, Rabinovich E, Sinha R (2004) Logistics: the key to e-retail success. *Supply Chain Management Review* 8:48–54.

March JG (1991) Exploration and exploitation in organizational learning. *Organization Science* 2(1):71–87.

Mariani MM, Mura M, Di Felice M (2018) The determinants of facebook social engagement for national tourism organizations' facebook pages: A quantitative approach. *Journal of Destination Marketing and Management* 8:312–325.

Marta (2021) *A Complete Guide to Social Media Analysis.* Available at https://brand24.com/blog/guide-to-social-media-analysis/ (last accessed: December 15, 2021).

Martin AB, Hartman M, Lassman D, Catlin A, Team NHEA (2021) National health care spending in 2019: Steady growth for the fourth consecutive year. *Health Affairs* 40:14–24.

Martin DP, Diehr P, Price KF, Richardson WC (1989) Effect of a gatekeeper plan on health services use and charges: a randomized trial. *American Journal of Public Health* 79(12):1628–1632.

McClellan M, McKethan AN, Lewis JL, Roski J, Fisher ES (2010) A national strategy to put accountable care into practice. *Health Affairs* 29(5):982–990.

McKinsey (2020) *The math of ACOs.* Available at https://www.mckinsey.com/industries/healthcare-systems-and-services/our-insights/the-math-of-acos (last accessed: December 28, 2021).

McWilliams JM, Chen AJ (2020) *Understanding The Latest ACO "Savings": Curb Your Enthusiasm And Sharpen Your Pencils—Part 1.* Available at https://www.healthaffairs.org/do/10.1377/forefront.20201106.719550/full/. (last accessed: January 15, 2022).

McWilliams JM, Hatfield LA, Chernew ME, Landon BE, Schwartz AL (2016) Early performance of accountable care organizations in medicare. *New England Journal of Medicine* 374(24):2357–2366.

McWilliams JM, Hatfield LA, Landon BE, Hamed P, Chernew ME (2018) Medicare spending after 3 years of the medicare shared savings program. *New England Journal of Medicine* 379(12):1139–1149.

MDG (2018) *It's All About the Images [Infographic].* Available at https://www.mdgadvertising.com/marketing-insights/infographics/its-all-about-the-images-infographic/ (last accessed: November 20, 2021).

Medcity (2021) *Akron Children's, CareSource to launch ACO.* Available at https://medcitynews.com/2021/09/akron-childrens-caresource-to-launch-aco/ (last accessed: January 11, 2022).

Mendelson A, Kondo K, Damberg C, Low A, Motúapuaka M, Freeman M, Kansagara D (2017) The effects of pay-for-performance programs on health, health care use, and processes of care: a systematic review. *Annals of internal medicine* 166(5):341–353.

Miner AS, Mezias SJ (1996) Ugly duckling no more: Pasts and futures of organizational learning research. *Organization Science* 7(1):88–99.

Muhlestein D, Saunders R, McClellan M (2016) Medicare accountable care organization results for 2015: the journey to better quality and lower costs continues. *Health Affairs* 1407–1423.

Naylor RW, Lamberton CP, West PM (2012) Beyond the "like" button: The impact of mere virtual presence on brand evaluations and purchase intentions in social media settings. *Journal of Marketing* 76(6):105–120.

Nobel RD, van der Heeden M (2000) A lost-sales production/ inventory model with two discrete production modes. *Commun. Stat.-Stochastic Models* 16(5):453–478.

Nyweide DJ, Lee W, Colla CH (2020) Accountable care organizations' increase in nonphysician practitioners may signal shift for health care workforce: The growth of non-physician practitioners participating in medicare shared savings program acos. *Health Affairs* 39(6):1080–1086.

O'Brien HL, Toms EG (2008) What is user engagement? a conceptual framework for defining user engagement with technology. *Journal of the American society for Information Science and Technology* 59(6):938–955.

Olfson M, Zhang V, Schoenbaum M, King M (2020) Buprenorphine treatment by primary care providers, psychiatrists, addiction specialists, and others: Trends in buprenorphine treatment by prescriber specialty-primary care providers, psychiatrists, and addiction medicine specialists. *Health Affairs* 39(6):984–992.

Ouayogode MH, Colla CH, Lewis VA (2017) Determinants of success in shared savings programs: An analysis of aco and market characteristics. *In Healthcare* 5(1-2):53–61.

Palmer D, Mahldhar V, Gallzia T, Sharma V (2013) Reengineering business intelligence. *Westlake: Deloitte University Press.* .

Parchman ML, Culler SD (1999) Preventable hospitalizations in primary care shortage areas: an analysis of vulnerable medicare beneficiaries. *Archives of Family Medicine* 8(6):487.

Paterson C, Kiesmüller G, Teunter R, Glazebrook K (2011) Inventory models with lateral transshipments: A review. *European Journal of Operational Research* 210(2):125–136.

Peckham A, Rudoler D, Bhatia D, Fakim S, Allin S, Marchildon G (2018) Accountable care organizations and the canadian context. *Rapid Review* 9.

Perloff J, DesRoches CM, Buerhaus P (2016) Comparing the cost of care provided to medicare beneficiaries assigned to primary care nurse practitioners and physicians. *Health services research* 51(4):1407–1423.

Poghosyan L, Clarke SP, Finlayson M, Aiken LH (2010) Nurse burnout and quality of care: Cross-national investigation in six countries. *Research in Nursing and Health* 33(4):288–298.

ProShip (2017) *Top Four Reasons Customers Pay for Premium Shipping.* Available at `https://www.proshipinc.com/about/news/infographic-top-four-reasons-customers-pay-premium-shipping` (last accessed: August 5, 2018).

Reichheld FF, Schefter P (2000) E-loyalty: your secret weapon on the web. *Harvard Business Review* 78(4):105–113.

Ren S, He K, Girshick R, Sun J (2015) Faster r-cnn: Towards real-time object detection with region proposal networks. *In Advances in neural information processing systems* 91–99.

Reports U (2021) *How Much Content Creation Costs in 2021?* Available at https://www.upreports.com/blog/how-much-content-creation-cost-2020/ (last accessed: December 19, 2021).

ResearchAndMarket (2021) *Global Social Media Analytics Market to Nearly Triple in Size by 2026, Reaching $9.3 Billion | A Complete Industry Assessment - ResearchAndMarkets.com.* Available at https://www.businesswire.com/news/home/20211119005497/en/Global-Social-Media-Analytics-Market-to-Nearly-Triple-in-Size-by-2026-Reaching-9.3-Billion-A-Complete-Industry-Assessment---ResearchAndMarkets.com (last accessed: November 29, 2021).

Rittenhouse DR, Shortell SM, Fisher ES (2009) Primary care and accountable care—two essential elements of delivery-system reform. *New England Journal of Medicine* 361(24):2301–2303.

Robinson L (1990) Optimal an approximate policies in multiperiod, multilocation inventory models with transshipments. *Operations Research* 38(2):278–295.

Salsberg ES (2015) Is the physician shortage real? implications for the recommendations of the institute of medicine committee on the governance and financing of graduate medical education. *Academic Medicine* 90(9):1210–1214.

Sandberg SF, Erikson C, Owen R, Vickery KD, Shimotsu ST, Linzer M, DeCubellis J (2014) Hennepin health: a safety-net accountable care organization for the expanded medicaid population. *Health Affairs* 33(11):1975–1984.

Santiago LP, Vakili P (2005) On the value of flexibility in r&d projects. *Management Science* 51(8):1206–1218.

Saunders R, Muhlestein D, McClellan M (2017) *Medicare accountable care organization results for 2016: seeing improvement, transformation takes time.* Available at https://www.healthaffairs.org/do/10.1377/forefront.20171120.211043/full/ (last accessed: August 11, 2019).

Schreiber S, Zielinski T (1997) The meaning of ambulatory care sensitive admissions: urban and rural perspectives. *The Journal of Rural Health* 13(4):276.

Sebastian N (2021) *Social Media Advertising: Must Know User Preferences.* Available at https://www.goodfirms.co/resources/social-media-advertising-user-preferences (last accessed: October 2, 2021).

Senot C (2019) Continuity of care and risk of readmission: An investigation into the healthcare journey of heart failure patients. *Production and Operations Management* 28(8):2008–2030.

Senot C, Chandrasekaran A, Ward PT (2016) Role of bottom-up decision processes in improving the quality of health care delivery: A contingency perspective. *Production and Operations Management* 25(3):458–476.

Shetty VA, Balzer LB, Geissler KH, Chin DL (2019) Association between specialist office visits and health expenditures in accountable care organizations. *JAMA network open* 2(7):e196796–e196796.

Shier DR (1977) A min-max theorem for p-center problems on a tree. *Transportation Science* 11(3):243–252.

Shin D, He S, Lee GM, Whinston AB, Cetintas S, Lee KC (2020) Enhancing social media analysis with visual data analytics: A deep learning approach. *MIS Quarterly* 44(4):1459–1492.

Shin P, Sharac J, Alvarez C, Rosenbaum S, Paradise J (2013) Community health centers in an era of health reform. *Kaiser Family Foundation* Washington DC.

Shortell S, Sehgal N, Bibi S (2015) An early assessment of accountable care organizations' efforts to engage patients and their families. *Med Care Res Rev 2015* 72:580–604.

Shortell SM, Casalino LP, Fisher ES (2010) How the center for medicare and medicaid innovation should test accountable care organizations. *Health Affairs* 29(7):1293–1298.

Siddiqui M, Berkowitz SA (2014) Shared savings models for acos-incentivizing primary care physicians. *Journal of general internal medicine* 29(6):832–834.

Socialistics (2021) *Cost Breakdown: How Much Should Social Media Marketing Cost?* Available at https://socialistics.com/social-media-marketing-cost/ (last accessed: December 23, 2021).

Sokol E (2020) *How an ACO, FQHC Model Leveraged Social Determinants of Health.* Available at https://healthitanalytics.com/news/how-an-aco-fqhc-model-leveraged-social-determinants-of-health (last accessed: July 1, 2021).

Solutions CH (2021) *QA: Proven ACO Strategies for Enhancing Care Quality and Generating*

Savings. Available at https://www.coniferhealth.com/knowledge-center/proven-aco-strategies/ (last accessed: December 26, 2021).

Song Z, Fisher ES (2016) The aco experiment in infancy—looking back and looking forward. *JAMA* 316(1):705–706.

SproutSocial (2021) *The Future of Social Media: New Data for 2021 & Beyond.* Available at https://sproutsocial.com/insights/data/harris-insights-report/ (last accessed: October 2, 2021).

Stanik-Hutt J, Newhouse RP, White KM, Johantgen M, Bass EB, Zangaro G, Weiner JP (2013) The quality and effectiveness of care provided by nurse practitioners. *The Journal for Nurse Practitioners* 9(8):492–500.

Starfield B (1998) Primary care: balancing health needs, services, and technology. *Religion in America* .

Starfield B, Macinko J (2005) Contribution of primary care to health systems and health. *Milbank Q* 83:457–502.

Statista (2021) *Social Media Advertising.* Available at https://www.statista.com/outlook/dmo/digital-advertising/social-media-advertising/worldwide (last accessed: October 10, 2021).

Stevens GD (2002) Racial and ethnic disparities in the quality of primary care for children. *The Johns Hopkins University* .

Swan M, Ferguson S, Chang A, Larson E, Smaldone A (2015) Quality of primary care by advanced practice nurses: a systematic review. *International Journal for Quality in Health Care* 27(5):396–404.

Takach M, Buxbaum J (2013) Care management for medicaid enrollees through community health teams. *The Commonwealth Fund* May 2013.

TCF (2021) *How Much Does Social Media Marketing Cost?* Available at https://contentfac.com/how-much-does-social-media-marketing-cost/ (last accessed: December 19, 2021).

Theokary C, Justin Ren Z (2011) An empirical study of the relations between hospital volume, teaching status, and service quality. *Production and Operations Management* 20(3):303–318.

Wei L, Kapuscinski R, Jasin S (2021) Shipping consolidation across two warehouses with delivery deadline and expedited options for e-commerce and omni-channel retailers. *Manufacturing and Service Operations Management* 23:1634–1650.

Weigel PA, Ullrich F, Shane DM, Mueller KJ (2016) Variation in primary care service patterns by rural-urban location. *The Journal of Rural Health* 32(2):196–203.

Weiss JM, Smith MA, Pickhardt PJ, Kraft SA, Flood GE, Kim DH, Pfau PR (2013) Predictors of colorectal cancer screening variation among primary care providers and clinics. *The American journal of gastroenterology* 108(7):1159.

Wilson IB, Landon BE, Hirschhorn LR, McInnes K, Ding L, Marsden PV, Cleary PD (2005) Quality of hiv care provided by nurse practitioners, physician assistants, and physicians. *Annals of internal medicine* 143:729.

Wilson M, Guta A, Waddell K, Lavis J, Reid R, Evans C (2020) The impacts of accountable care organizations on patient experience, health outcomes and costs: a rapid review. *Journal of Health Services Research and Policy* 25:130–138.

Wu CW, Guaita Martínez JM, Martín Martín JM (2020) An analysis of social media marketing strategy and performance in the context of fashion brands: The case of taiwan. *Psychology and Marketing* 37(9):1185–1193.

Wu Z, Wang X, Jiang YG, Ye H, Xue X (2015) Modeling spatial-temporal clues in a hybrid deep learning framework for video classification. *In Proceedings of the 23rd ACM international conference on Multimedia* October:461–470.

Wulf J, Mettler T, Ludwig S, Herhausen D (2019) A computational visual analysis of image design in social media car model communities. *Journal of the Academy of Marketing Science* .

Xiao W, Xu Y (2018) Should an online retailer penalize its independent sellers for stockout? *Production and Operations Management* 27(6):1124–1132.

Xu P, Allgor R, Graves S (2009) Benefits of reevaluating real-time order fulfillment decisions. *Manufacturing and Service Operations Management* 11(2):340–355.

Yadav N, Binay U (2017) Comparative study of object detection algorithms. *International Research Journal of Engineering and Technology (IRJET)* 4(11):586–591.

Yim D, Malefyt T, Khuntia J (2021) Is a picture worth a thousand views? measuring the effects of travel photos on user engagement using deep learning algorithms. *Electronic Markets* 1–19.

Yin D, Bond SD, Zhang H (2017) Keep your cool or let it out: Nonlinear effects of expressed arousal on perceptions of consumer reviews. *Journal of Marketing Research* 54(3):447–463.

Zhang S, Lee D, Singh PV, Srinivasan K (2021) What makes a good image? airbnb demand analytics leveraging interpretable image features. *Management Science* .

Zhao H, Ryan JK, Deshpande V (2008) Optimal dynamic production and inventory transshipment policies for a two-location make-to-stock system. *Operations Research* 56(2):400–410.

Zhao Z, Zhu H, Xue Z, Liu Z, Tian J, Chua MCH, Liu M (2019) An image-text consistency driven multimodal sentiment analysis approach for social media. *Information Processing and Management* 56(6):102097.

Zhou X, Liang X, Zhang H, Ma Y (2015) Cross-platform identification of anonymous identical users in multiple social media networks. *IEEE transactions on knowledge and data engineering* 28(2):411–424.

Zhu X, Mueller K, Huang H, Ullrich F, Vaughn T, MacKinney AC (2019) Organizational attributes associated with medicare aco quality performance. *The Journal of Rural Health* 35(1):68–77.

APPENDIX A

SAFETY STOCK ALLOCATION IN AN ONLINE RETAILING NETWORK: A STOCHASTIC OPTIMIZATION APPROACH

Table A.1: Decision variables for Problem MIP_{2FC} when $z = \sqrt{5}$ (two FC unequal variance, case 1: $1/3 \geq \delta \geq 2/15$)

s	e_1^s	X_{12}^s	e_2^s	X_{21}^s	$Y_1^s + Y_2^s$
1	$\mu^2 d_1$-$2\mu\sigma$	0	$\mu^2 d_2$	0	0
2	$\mu^2 d_1$-$4\mu\sigma$	0	$\mu^2 d_2$	0	0
3	$\mu^2 d_1$-$6\mu\sigma$	0	$\mu^2 d_2$	0	0
4	$\mu^2 d_1$	0	$\mu^2 d_2$	0	0
5	$\mu^2 d_1$+$2\mu\sigma$	0	$\mu^2 d_2$	0	0
6	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$	0	0
7	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$	$\mu\sigma(\frac{5}{3} - 5\delta)$	$\mu\sigma + 0$
8	$\mu^2 d_1$-$2\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
9	$\mu^2 d_1$-$4\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
10	$\mu^2 d_1$-$6\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
11	$\mu^2 d_1$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
12	$\mu^2 d_1$+$2\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
13	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
14	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$\mu\sigma(\frac{8}{3} - 5\delta)$	0
15	$\mu^2 d_1$-$2\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
16	$\mu^2 d_1$-$4\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
17	$\mu^2 d_1$-$6\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
18	$\mu^2 d_1$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
19	$\mu^2 d_1$+$2\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
20	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
21	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	$\mu\sigma(\frac{8}{3} - 5\delta)$	0
22	$\mu^2 d_1$-$2\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
23	$\mu^2 d_1$-$4\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
24	$\mu^2 d_1$-$6\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
25	$\mu^2 d_1$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
26	$\mu^2 d_1$+$2\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
27	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
28	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	$\mu\sigma(\frac{8}{3} - 5\delta)$	0
29	$\mu^2 d_1$-$2\mu\sigma$	$\mu\sigma(5\delta - \frac{2}{3})$	$\mu^2 d_2$+$\mu\sigma$	0	0
30	$\mu^2 d_1$-$4\mu\sigma$	$\mu\sigma(5\delta - \frac{2}{3})$	$\mu^2 d_2$+$\mu\sigma$	0	0
31	$\mu^2 d_1$-$6\mu\sigma$	$\mu\sigma(5\delta - \frac{2}{3})$	$\mu^2 d_2$+$\mu\sigma$	0	0
32	$\mu^2 d_1$	$\mu\sigma(5\delta - \frac{2}{3})$	$\mu^2 d_2$+$\mu\sigma$	0	0
33	$\mu^2 d_1$+$2\mu\sigma$	$\mu\sigma(5\delta - \frac{2}{3})$	$\mu^2 d_2$+$\mu\sigma$	0	0
34	$\mu^2 d_1$+$4\mu\sigma$	$\mu\sigma(5\delta - \frac{2}{3})$	$\mu^2 d_2$+$\mu\sigma$	0	0
35	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	$\mu\sigma(\frac{8}{3} - 5\delta) + \mu\sigma(5\delta - \frac{2}{3})$
36	$\mu^2 d_1$-$2\mu\sigma$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
37	$\mu^2 d_1$-$4\mu\sigma$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
38	$\mu^2 d_1$-$6\mu\sigma$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
39	$\mu^2 d_1$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
40	$\mu^2 d_1$+$2\mu\sigma$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
41	$\mu^2 d_1$+$4\mu\sigma$	$\mu\sigma(5\delta - \frac{2}{3})$	$\mu^2 d_2$+$2\mu\sigma$	0	$0 + \mu\sigma$
42	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$+$2\mu\sigma$	0	$\mu\sigma(\frac{8}{3} - 5\delta) + \mu\sigma(\frac{1}{3} + 5\delta)$
43	$\mu^2 d_1$-$2\mu\sigma$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
44	$\mu^2 d_1$-$4\mu\sigma$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
45	$\mu^2 d_1$-$6\mu\sigma$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
46	$\mu^2 d_1$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
47	$\mu^2 d_1$+$2\mu\sigma$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
48	$\mu^2 d_1$+$4\mu\sigma$	$\mu\sigma(5\delta - \frac{4}{3})$	$\mu^2 d_2$+$3\mu\sigma$	0	$0 + 2\mu\sigma$
49	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$+$3\mu\sigma$	0	$\mu\sigma(\frac{8}{3} - 5\delta) + \mu\sigma(\frac{4}{3} + 5\delta)$

Table A.2: Decision variables for Problem MIP_{2FC} when $z = \sqrt{5}$ (two FC unequal variance, case 2: $0 \leq \delta < 2/15$)

s	e_1^s	X_{12}^s	e_2^s	X_{21}^s	$Y_1^s + Y_2^s$
1	$\mu^2 d_1$-$2\mu\sigma$	0	$\mu^2 d_2$	0	0
2	$\mu^2 d_1$-$4\mu\sigma$	0	$\mu^2 d_2$	0	0
3	$\mu^2 d_1$-$6\mu\sigma$	0	$\mu^2 d_2$	0	0
4	$\mu^2 d_1$	0	$\mu^2 d_2$	0	0
5	$\mu^2 d_1$+$2\mu\sigma$	0	$\mu^2 d_2$	0	0
6	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$	$\mu\sigma(\frac{2}{3} - 5\delta)$	0
7	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$	$\mu\sigma(\frac{5}{3} - 5\delta)$	$\mu\sigma + 0$
8	$\mu^2 d_1$-$2\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
9	$\mu^2 d_1$-$4\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
10	$\mu^2 d_1$-$6\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
11	$\mu^2 d_1$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
12	$\mu^2 d_1$+$2\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
13	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$\mu\sigma(\frac{2}{3} - 5\delta)$	0
14	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$\mu\sigma(\frac{8}{3} - 5\delta)$	0
15	$\mu^2 d_1$-$2\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
16	$\mu^2 d_1$-$4\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
17	$\mu^2 d_1$-$6\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
18	$\mu^2 d_1$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
19	$\mu^2 d_1$+$2\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	0	0
20	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	$\mu\sigma(\frac{2}{3} - 5\delta)$	0
21	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$-$2\mu\sigma$	$\mu\sigma(\frac{8}{3} - 5\delta)$	0
22	$\mu^2 d_1$-$2\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
23	$\mu^2 d_1$-$4\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
24	$\mu^2 d_1$-$6\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
25	$\mu^2 d_1$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
26	$\mu^2 d_1$+$2\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	0	0
27	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	$\mu\sigma(\frac{2}{3} - 5\delta)$	0
28	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$-$3\mu\sigma$	$\mu\sigma(\frac{8}{3} - 5\delta)$	0
29	$\mu^2 d_1$-$2\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
30	$\mu^2 d_1$-$4\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
31	$\mu^2 d_1$-$6\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
32	$\mu^2 d_1$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
33	$\mu^2 d_1$+$2\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
34	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	$\mu\sigma(\frac{2}{3} - 5\delta)$	0
35	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	$\mu\sigma(\frac{2}{3} - 5\delta)$	$2\mu\sigma + 0$
36	$\mu^2 d_1$-$2\mu\sigma$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
37	$\mu^2 d_1$-$4\mu\sigma$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
38	$\mu^2 d_1$-$6\mu\sigma$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
39	$\mu^2 d_1$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
40	$\mu^2 d_1$+$2\mu\sigma$	$\mu\sigma(\frac{1}{3} + 5\delta)$	$\mu^2 d_2$+$2\mu\sigma$	0	0
41	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$+$2\mu\sigma$	0	$\mu\sigma(\frac{2}{3} - 5\delta) + \mu\sigma(\frac{1}{3} + 5\delta)$
42	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$+$2\mu\sigma$	0	$\mu\sigma(\frac{8}{3} - 5\delta) + \mu\sigma(\frac{1}{3} + 5\delta)$
43	$\mu^2 d_1$-$2\mu\sigma$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
44	$\mu^2 d_1$-$4\mu\sigma$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
45	$\mu^2 d_1$-$6\mu\sigma$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
46	$\mu^2 d_1$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
47	$\mu^2 d_1$+$2\mu\sigma$	$\mu\sigma(\frac{4}{3} + 5\delta)$	$\mu^2 d_2$+$3\mu\sigma$	0	0
48	$\mu^2 d_1$+$4\mu\sigma$	0	$\mu^2 d_2$+$3\mu\sigma$	0	$\mu\sigma(\frac{2}{3} - 5\delta) + \mu\sigma(\frac{4}{3} + 5\delta)$
49	$\mu^2 d_1$+$6\mu\sigma$	0	$\mu^2 d_2$+$3\mu\sigma$	0	$\mu\sigma(\frac{8}{3} - 5\delta) + \mu\sigma(\frac{4}{3} + 5\delta)$

Table A.3: Decision variables for Problem MIP_{2FC} for k (two FC unequal variance, case 1: δ = 0)

s	e_1^s	X_{12}^s	e_2^s	X_{21}^s	$Y_1^s + Y_2^s$
1	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$	0	0
2	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$	0	0
3	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$	0	0
4	$\mu^2 d_1$	0	$\mu^2 d_2$	0	0
5	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$	0	0
6	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$	$\mu\sigma$	$\mu\sigma(k-1)$ + 0
7	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$	$\mu\sigma$	$\mu\sigma(2k-1)$ + 0
8	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
9	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
10	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
11	$\mu^2 d_1$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
12	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
13	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	2$\mu\sigma$	$\mu\sigma(k-2)$ + 0 [k>2; no lost sales for k$\leq$2]
14	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	2$\mu\sigma$	$\mu\sigma(2k-2)$ + 0
15	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
16	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
17	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
18	$\mu^2 d_1$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
19	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
20	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	3$\mu\sigma$	$\mu\sigma(k-3)$ + 0 [k>3; no lost sales for k$\leq$3]
21	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	3$\mu\sigma$	$\mu\sigma(2k-3)$ + 0
22	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
23	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
24	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
25	$\mu^2 d_1$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
26	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
27	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	4$\mu\sigma$	$\mu\sigma(k-4)$ + 0 [k>4; no lost sales for k$\leq$4]
28	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	4$\mu\sigma$	$\mu\sigma(2k-4)$ + 0
29	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
30	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
31	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
32	$\mu^2 d_1$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
33	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
34	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	k$\mu\sigma$ + 0
35	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	2k$\mu\sigma + 0$
36	$\mu^2 d_1$-k$\mu\sigma$	$\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
37	$\mu^2 d_1$-2k$\mu\sigma$	$\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
38	$\mu^2 d_1$-3k$\mu\sigma$	$\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
39	$\mu^2 d_1$	$\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
40	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	0 + $\mu\sigma$
41	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	$k\mu\sigma$ + $\mu\sigma$
42	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	$2k\mu\sigma$ + $\mu\sigma$
43	$\mu^2 d_1$-k$\mu\sigma$	2$\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
44	$\mu^2 d_1$-2k$\mu\sigma$	2$\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
45	$\mu^2 d_1$-3k$\mu\sigma$	2$\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
46	$\mu^2 d_1$	2$\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0 [$k > 2$]
47	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	0+2$\mu\sigma$
48	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	k$\mu\sigma$ + 2$\mu\sigma$
49	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	2k$\mu\sigma$ + 2$\mu\sigma$

Table A.4: Decision variables for Problem MIP_{2FC} for k (two FC unequal variance, case 1: $0 < \delta \leq \frac{1}{k+1}$)

s	e_1^s	X_{12}^s	e_2^s	X_{21}^s	$Y_1^s + Y_2^s$
1	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$	0	0
2	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$	0	0
3	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$	0	0
4	$\mu^2 d_1$	0	$\mu^2 d_2$	0	0
5	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$	0	0
6	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$	$\mu\sigma - (k+1)\delta\mu\sigma$	$\mu\sigma(k-1)$ + 0
7	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$	$\mu\sigma - (k+1)\delta\mu\sigma$	$\mu\sigma(2k-1)$ + 0
8	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
9	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
10	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
11	$\mu^2 d_1$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
12	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
13	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu\sigma(k-2)$ + 0 [k>2; no lost sales for k≤2]
14	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu\sigma(2k-2)$ + 0
15	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
16	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
17	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
18	$\mu^2 d_1$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
19	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
20	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	$3\mu\sigma - (k+1)\delta\mu\sigma$	$\mu\sigma(k-3)$ + 0 [k>3; no lost sales for k≤3]
21	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	$3\mu\sigma - (k+1)\delta\mu\sigma$	$\mu\sigma(2k-3)$ + 0
22	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
23	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
24	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
25	$\mu^2 d_1$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
26	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
27	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	$4\mu\sigma - (k+1)\delta\mu\sigma$	$\mu\sigma(k-4)$ + 0 [k>4; no lost sales for k≤4]
28	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	$4\mu\sigma - (k+1)\delta\mu\sigma$	$\mu\sigma(2k-4)$ + 0
29	$\mu^2 d_1$-k$\mu\sigma$	$(k+1)\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
30	$\mu^2 d_1$-2k$\mu\sigma$	$(k+1)\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
31	$\mu^2 d_1$-3k$\mu\sigma$	$(k+1)\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
32	$\mu^2 d_1$	$(k+1)\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
33	$\mu^2 d_1$+k$\mu\sigma$	$(k+1)\delta\mu\sigma$	$\mu^2 d_2$+$\mu\sigma$	0	0
34	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	k$\mu\sigma$+ $(k+1)\delta\mu\sigma$ + $(k+1)\delta\mu\sigma$
35	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	2k$\mu\sigma$+ $(k+1)\delta\mu\sigma$ + $(k+1)\delta\mu\sigma$
36	$\mu^2 d_1$-k$\mu\sigma$	$\mu\sigma$ + $(k+1)\delta\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
37	$\mu^2 d_1$-2k$\mu\sigma$	$\mu\sigma$ + $(k+1)\delta\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
38	$\mu^2 d_1$-3k$\mu\sigma$	$\mu\sigma$ + $(k+1)\delta\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
39	$\mu^2 d_1$	$\mu\sigma$ + $(k+1)\delta\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
40	$\mu^2 d_1$+k$\mu\sigma$	$(k+1)\delta\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0 + $\mu\sigma$
41	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	($k\mu\sigma$ - $(k+1)\delta\mu\sigma$) + ($\mu\sigma$ + $(k+1)\delta\mu\sigma$)
42	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	($2k\mu\sigma$ - $(k+1)\delta\mu\sigma$) + ($\mu\sigma$ + $(k+1)\delta\mu\sigma$)
43	$\mu^2 d_1$-k$\mu\sigma$	$2\mu\sigma$ + $(k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
44	$\mu^2 d_1$-2k$\mu\sigma$	$2\mu\sigma$ + $(k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
45	$\mu^2 d_1$-3k$\mu\sigma$	$2\mu\sigma$ + $(k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
46	$\mu^2 d_1$	$2\mu\sigma$ + $(k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0 [$k > 2$]
47	$\mu^2 d_1$+k$\mu\sigma$	$(k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0+($2\mu\sigma$ + $(k+1)\delta\mu\sigma$)
48	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	($k\mu\sigma$ - $(k+1)\delta\mu\sigma$) + ($2\mu\sigma$ + $(k+1)\delta\mu\sigma$)
49	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	($2k\mu\sigma$ - $(k+1)\delta\mu\sigma$) + ($2\mu\sigma$ + $(k+1)\delta\mu\sigma$)

Table A.5: Decision variables for Problem MIP_{2FC} for k (two FC unequal variance, case 2: $0 < \delta \leq \frac{k}{k+1}$)

s	e_1^s	X_{12}^s	e_2^s	X_{21}^s	$Y_1^s + Y_2^s$
1	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$	0	0
2	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$	0	0
3	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$	0	0
4	$\mu^2 d_1$	0	$\mu^2 d_2$	0	0
5	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$	$(k+1)\delta\mu\sigma$	0
6	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$	$\mu\sigma + (k+1)\delta\mu\sigma$	$\mu\sigma(k-1)$ + 0
7	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$	$\mu\sigma + (k+1)\delta\mu\sigma$	$\mu\sigma(2k-1)$ + 0
8	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
9	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
10	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
11	$\mu^2 d_1$	0	$\mu^2 d_2$-$\mu\sigma$	0	0
12	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$(k+1)\delta\mu\sigma$	0
13	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$2\mu\sigma + (k+1)\delta\mu\sigma$	$\mu\sigma(k-2)$ + 0 [k>2; no lost sales for k≤2]
14	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$-$\mu\sigma$	$2\mu\sigma + (k+1)\delta\mu\sigma$	$\mu\sigma(2k-2)$ + 0
15	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
16	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
17	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
18	$\mu^2 d_1$	0	$\mu^2 d_2$-2$\mu\sigma$	0	0
19	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	$(k+1)\delta\mu\sigma$	0
20	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	$3\mu\sigma + (k+1)\delta\mu\sigma$	$\mu\sigma(k-3)$ + 0 [k>3; no lost sales for k≤3]
21	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$-2$\mu\sigma$	$3\mu\sigma + (k+1)\delta\mu\sigma$	$\mu\sigma(2k-3)$ + 0
22	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
23	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
24	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
25	$\mu^2 d_1$	0	$\mu^2 d_2$-3$\mu\sigma$	0	0
26	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	$(k+1)\delta\mu\sigma$	0
27	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	$4\mu\sigma + (k+1)\delta\mu\sigma$	$\mu\sigma(k-4)$ + 0 [k>4; no lost sales for k≤4]
28	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$-3$\mu\sigma$	$4\mu\sigma + (k+1)\delta\mu\sigma$	$\mu\sigma(2k-4)$ + 0
29	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
30	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
31	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
32	$\mu^2 d_1$	0	$\mu^2 d_2$+$\mu\sigma$	0	0
33	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	$(k+1)\delta\mu\sigma$	0
34	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	$(k+1)\delta\mu\sigma$	k$\mu\sigma$ + 0
35	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+$\mu\sigma$	$(k+1)\delta\mu\sigma$	2k$\mu\sigma$ + 0

Table A.6: Decision variables for Problem MIP_{2FC} for k (two FC unequal variance, case 2a: $0 < \delta < \frac{1}{k+1}$)

s	e_1^s	X_{12}^s	e_2^s	X_{21}^s	$Y_1^s + Y_2^s$
36	$\mu^2 d_1$-k$\mu\sigma$	$\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
37	$\mu^2 d_1$-2k$\mu\sigma$	$\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
38	$\mu^2 d_1$-3k$\mu\sigma$	$\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
39	$\mu^2 d_1$	$\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	0	0
40	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	$(k+1)\delta\mu\sigma + (\mu\sigma - (k+1)\delta\mu\sigma)$
41	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	$(k\mu\sigma + (k+1)\delta\mu\sigma) + (\mu\sigma - (k+1)\delta\mu\sigma)$
42	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	$(2k\mu\sigma + (k+1)\delta\mu\sigma) + (\mu\sigma - (k+1)\delta\mu\sigma)$
43	$\mu^2 d_1$-k$\mu\sigma$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
44	$\mu^2 d_1$-2k$\mu\sigma$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
45	$\mu^2 d_1$-3k$\mu\sigma$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
46	$\mu^2 d_1$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0 [$k > 2$]
47	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	$(k+1)\delta\mu\sigma + (2\mu\sigma - (k+1)\delta\mu\sigma)$
48	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	$(k\mu\sigma + (k+1)\delta\mu\sigma) + (2\mu\sigma - (k+1)\delta\mu\sigma)$
49	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	$(2k\mu\sigma + (k+1)\delta\mu\sigma) + (2\mu\sigma - (k+1)\delta\mu\sigma)$

Table A.7: Decision variables for Problem MIP_{2FC} for k (two FC unequal variance, case 2b: $\frac{1}{k+1} \leq \delta < \frac{2}{k+1}$)

s	e_1^s	X_{12}^s	e_2^s	X_{21}^s	$Y_1^s + Y_2^s$
36	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	0
37	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	0
38	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	0
39	$\mu^2 d_1$	0	$\mu^2 d_2$+2$\mu\sigma$	0	0
40	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	$(k+1)\delta\mu\sigma - \mu\sigma$	$\mu\sigma$ + 0
41	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	$(k+1)\delta\mu\sigma - \mu\sigma$	(k$\mu\sigma$ + $\mu\sigma$) + 0
42	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	$(k+1)\delta\mu\sigma - \mu\sigma$	(2k$\mu\sigma$ + $\mu\sigma$) + 0
43	$\mu^2 d_1$-k$\mu\sigma$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
44	$\mu^2 d_1$-2k$\mu\sigma$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
45	$\mu^2 d_1$-3k$\mu\sigma$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
46	$\mu^2 d_1$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0 [$k > 2$]
47	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	$(k+1)\delta\mu\sigma + (2\mu\sigma - (k+1)\delta\mu\sigma)$
48	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	$(k\mu\sigma + (k+1)\delta\mu\sigma) + (2\mu\sigma - (k+1)\delta\mu\sigma)$
49	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	$(2k\mu\sigma + (k+1)\delta\mu\sigma) + (2\mu\sigma - (k+1)\delta\mu\sigma)$

Table A.8: Decision variables for Problem MIP_{2FC} for k (two FC unequal variance, case 2c: $\frac{2}{k+1} \leq \delta < \frac{k}{k+1}$)

s	e_1^s	X_{12}^s	e_2^s	X_{21}^s	$Y_1^s + Y_2^s$
36	$\mu^2 d_1$-k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	0
37	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	0
38	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	0	0
39	$\mu^2 d_1$	0	$\mu^2 d_2$+2$\mu\sigma$	0	0
40	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	$(k+1)\delta\mu\sigma - \mu\sigma$	$\mu\sigma$ + 0
41	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	$(k+1)\delta\mu\sigma - \mu\sigma$	(k$\mu\sigma$ + $\mu\sigma$) + 0
42	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+2$\mu\sigma$	$(k+1)\delta\mu\sigma - \mu\sigma$	(2k$\mu\sigma$ + $\mu\sigma$) + 0
43	$\mu^2 d_1$-k$\mu\sigma$	$2\mu\sigma - (k+1)\delta\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	0	0
44	$\mu^2 d_1$-2k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	0
45	$\mu^2 d_1$-3k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	0	0
46	$\mu^2 d_1$	0	$\mu^2 d_2$+3$\mu\sigma$	0	0 [$k > 2$]
47	$\mu^2 d_1$+k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	$(k+1)\delta\mu\sigma - 2\mu\sigma$	2$\mu\sigma$ + 0
48	$\mu^2 d_1$+2k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	$(k+1)\delta\mu\sigma - 2\mu\sigma$	(k$\mu\sigma$ + 2$\mu\sigma$) + 0
49	$\mu^2 d_1$+3k$\mu\sigma$	0	$\mu^2 d_2$+3$\mu\sigma$	$(k+1)\delta\mu\sigma - 2\mu\sigma$	(2k$\mu\sigma$ + 2$\mu\sigma$) + 0

Table A.9: Missed transshipments between FC_2 and FC_3 for three FC case

s	e_1^s	e_2^s	e_3^s	Probability	Transshipments
40	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	$\mu^2 d_3$	0.0056000	$\mu\sigma$
47	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	$\mu^2 d_3$	0.0005000	$\mu\sigma$
89	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	$\mu^2 d_3$-$\mu\sigma$	0.0035000	$\mu\sigma$
96	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	$\mu^2 d_3$-$\mu\sigma$	0.0003500	2$\mu\sigma$
138	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	$\mu^2 d_3$-2$\mu\sigma$	0.0009000	$\mu\sigma$
145	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	$\mu^2 d_3$-2$\mu\sigma$	0.0000880	2$\mu\sigma$
187	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$+2$\mu\sigma$	$\mu^2 d_3$-3$\mu\sigma$	0.0000880	$\mu\sigma$
194	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$+3$\mu\sigma$	$\mu^2 d_3$-3$\mu\sigma$	0.0000087	2$\mu\sigma$
250	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$	$\mu^2 d_3$+2$\mu\sigma$	0.0056000	$\mu\sigma$
257	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$-$\mu\sigma$	$\mu^2 d_3$+2$\mu\sigma$	0.0035000	$\mu\sigma$
264	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$-2$\mu\sigma$	$\mu^2 d_3$+2$\mu\sigma$	0.0009000	$\mu\sigma$
271	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$-3$\mu\sigma$	$\mu^2 d_3$+2$\mu\sigma$	0.0000880	$\mu\sigma$
299	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$	$\mu^2 d_3$+3$\mu\sigma$	0.0005500	$\mu\sigma$
306	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$-$\mu\sigma$	$\mu^2 d_3$+3$\mu\sigma$	0.0003500	2$\mu\sigma$
313	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$-2$\mu\sigma$	$\mu^2 d_3$+3$\mu\sigma$	0.0008800	2$\mu\sigma$
320	$\mu^2 d_1$+k$\mu\sigma$	$\mu^2 d_2$-3$\mu\sigma$	$\mu^2 d_3$+3$\mu\sigma$	0.0000087	2$\mu\sigma$
321	$\mu^2 d_1$+2k$\mu\sigma$	$\mu^2 d_2$-3$\mu\sigma$	$\mu^2 d_3$+3$\mu\sigma$	0.0000021	2$\mu\sigma$

Table A.10: Performance analysis with 4 FCs across 2 clusters ($\sum_{i=1}^{n} \lambda_i = 100$)

Optimal Method					PASS					Difference	
Cost	λ_1	λ_2	λ_3	λ_4	Cost	λ_1	λ_2	λ_3	λ_4	Diff	Diff in (%)
147825	32	22	30	16	147845	36	19	32	13	20	0.013
147259	34	24	18	24	147340	37	23	16	24	81	0.055
147823	32	22	22	24	147847	35	19	25	21	24	0.016
147929	31	22	25	22	147958	36	18	28	18	29	0.019
148085	31	22	34	13	148089	34	20	35	11	5	0.003
147958	31	22	21	16	147965	33	21	22	24	7	0.005
147283	34	24	16	26	147293	39	23	15	23	9	0.006
147317	34	24	24	18	147344	38	22	24	16	27	0.018
148095	30	21	27	22	148134	32	20	28	20	39	0.026
146622	38	27	12	23	146638	49	23	11	17	16	0.011
147825	30	21	20	29	147905	34	19	23	24	79	0.053
148520	28	19	26	27	148572	30	20	27	23	52	0.035
147623	32	23	16	19	147674	34	23	16	27	52	0.035
147805	32	23	28	17	147825	37	19	31	13	21	0.014
147203	35	25	11	29	147205	40	24	10	26	1	0.001
146985	36	25	33	6	146987	37	32	25	6	2	0.001
148129	31	22	20	27	148135	32	21	20	27	6	0.004
147055	25	24	21	30	147057	25	25	21	29	2	0.001
147151	24	24	24	28	147151	24	25	24	27	0	0.000
146738	26	26	20	28	146750	26	26	20	28	12	0.008
146008	30	30	15	25	146010	31	30	15	24	2	0.001
148030	21	22	26	31	148033	22	22	26	30	3	0.002
147356	24	24	34	18	147356	24	24	34	18	0	0.000
147081	25	25	30	20	147083	25	25	30	20	2	0.001
146843	26	31	15	28	146843	27	31	16	26	0	0.000
148389	20	25	31	24	148390	21	24	30	25	0	0.000
147636	23	27	23	27	147638	23	27	23	27	2	0.001
146607	27	32	33	8	146609	27	39	27	7	1	0.001
147636	23	27	27	23	147636	23	27	27	23	0	0.000
147573	28	28	28	16	147575	27	29	28	16	2	0.001
148184	25	25	20	30	148186	25	25	21	29	1	0.001
147630	27	27	23	23	147632	28	26	23	23	2	0.001
147178	29	29	12	30	147220	33	28	10	29	42	0.029
147864	27	26	14	33	147867	28	26	15	31	3	0.002
146771	32	31	16	21	146772	32	32	17	19	1	0.001

Table A.11: Performance analysis with 6 FCs across 2 clusters ($\sum_{i=1}^{n} \lambda_i = 600$)

Optimal							PASS							Difference	
Cost	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	Cost	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	Diff	Diff in (%)
228899	155	110	90	85	105	55	228979	139	133	82	93	93	60	80	0.034
226783	176	45	128	100	59	92	226881	154	60	132	104	72	78	98	0.042
228835	155	49	105	73	144	74	228909	156	65	79	148	57	95	73	0.031
229334	151	71	91	90	121	77	229419	151	76	89	109	91	84	85	0.036
230471	105	75	130	81	124	85	230562	165	70	81	103	106	75	91	0.038
229459	150	104	88	104	104	50	229547	152	108	85	94	108	53	88	0.037
227417	168	74	74	108	71	105	227492	171	80	55	120	54	120	75	0.032
227229	171	62	80	86	90	111	227299	171	50	89	120	66	104	70	0.030
230486	142	101	81	93	83	100	230552	144	112	64	103	64	113	66	0.027
227894	164	44	119	114	81	78	227996	164	48	106	118	81	83	102	0.043
227389	169	72	100	88	88	83	227499	170	50	110	120	64	86	110	0.047
227424	169	81	112	119	51	68	227449	172	86	102	120	43	77	25	0.011
229646	146	80	113	70	123	68	229738	150	83	97	107	89	74	92	0.039
211629	139	130	59	58	141	73	211760	145	108	77	138	77	55	131	0.059
213212	144	106	66	118	118	48	213321	178	82	56	118	118	48	109	0.049
213108	150	57	123	103	45	122	213218	178	48	90	118	54	112	110	0.050
211637	122	143	39	173	66	57	211733	125	106	75	178	49	67	96	0.044
211687	106	127	44	162	74	87	211790	118	96	62	164	55	105	103	0.047
211710	110	119	33	155	109	74	211805	112	86	66	160	74	102	95	0.043
211693	106	88	79	152	122	53	211796	108	62	98	155	87	90	103	0.047
211756	93	135	50	137	138	47	211853	98	98	78	137	100	89	97	0.044
211672	118	82	67	165	108	60	211777	116	60	90	169	77	88	105	0.048
211600	115	94	115	168	30	78	211699	122	66	134	174	48	56	99	0.045
211730	129	103	65	155	83	65	211818	130	78	92	154	68	78	88	0.040
211849	126	124	19	148	125	58	211945	127	91	90	150	98	44	96	0.043
211717	126	61	103	111	127	72	211817	130	92	78	138	104	58	100	0.045
211600	140	3	92	152	111	102	211701	144	20	79	171	100	86	101	0.046
211780	113	53	138	104	100	92	211875	117	71	106	138	78	90	95	0.043
211737	88	53	118	124	68	149	211858	139	58	103	132	70	98	121	0.055
212091	75	201	80	137	37	70	212196	134	114	62	162	46	82	105	0.048
211786	125	70	68	152	90	95	211879	143	90	51	175	91	50	93	0.042
211675	112	130	135	74	74	75	211770	134	114	62	162	46	82	95	0.043
211637	133	133	79	138	78	39	211724	150	134	62	125	62	67	87	0.040
211627	137	73	86	118	56	130	211725	159	68	65	136	73	99	98	0.045
211804	57	0	338	119	2	84	211910	151	67	130	128	44	80	106	0.048

APPENDIX B

ACO SERVICE DELIVERY AND EXPERIENCE ON FINANCIAL AND QUALITY PERFORMANCE - AN EMPIRICAL EXAMINATION

Table B.1: Regression results for Quality score

Variable name	Description	Type	Mean	SD	Min	Max
Savings rate sav_{it}	Savings as a percentage of total benchmark expenditures computed by the CMS at the end of each performance year of ACO i in year t	Dependent	0.016	0.047	-0.288	0.311
Quality score $qual_{it}$	A composite score of all quality measures computed by the CMS at the end of each performance year of ACO i in year t	Dependent	0.944	0.066	0.174	1.000
PC by specialists $pcsp_{it}$	Ratio of primary care services provided by specialists and primary care services provided by PCPs of ACO i in year t	Independent	1.134	0.971	0.142	9.870
PC by NPs $pcnp_{it}$	Ratio of primary care services provided by NPs and primary care services provided by PCPs of ACO i in year t	Independent	0.430	0.316	0.014	3.810
Experience (years) exp_{it}	Experience of ACO i in year t	Independent	2.507	1.897	0.000	7.000
Risk model $risk_{it}$	Risk model of ACO i in year t	Independent	0.127	0.332	0.000	1.000
States $state_{it}$	Number of states ACO i operates in year t	Control	1.630	1.120	1.000	13.000
Log (Beneficiaries) $benit$	Assigned beneficiaries of ACO i in year t	Control	9.540	0.7470	5.020	12.380
PCPs pcp_{it}	Number of PCPs of ACO i in year t	Control	232.280	315.210	2.000	5697.000
Specialists sp_{it}	Number of specialists of ACO i in year t	Control	397.199	662.070	0.000	12802.000
NPs np_{it}	Number of NPs of ACO i in year t	Control	141.540	200.780	0.000	3957.000
Female gen_{it}	Number of female beneficiaries of ACO i in year t	Control	11205.390	11213.490	91.000	133423.000
outpatient out_{it}	Annualized, truncated, and weighted mean outpatient expenditure per beneficiary of ACO i in year t	Control	2383.760	756.890	772.000	7431.000
inpatient in_{it}	Annualized, truncated, and weighted mean inpatient expenditure per beneficiary of ACO i in year t	Control	3420.890	943.316	1430.000	25878.000

Table B.2: Correlation of variables

	Savings rate	Quality score	States	Experience (years)	Log (beneficiaries)	PC by SP	PC by NP	PCPs	SPs	NPs	Female	Inpatient	Outpatient
Savings rate	1.000												
Quality score	0.031	1.000											
States	-0.016	0.017	1.000										
Experience (years)	0.223	-0.230	-0.122	1.000									
Log (beneficiaries)	-0.068	0.083	0.063	0.114	1.000								
PC by SP	0.013	-0.052	0.016	0.073	-0.004	1.000							
PC by NP	-0.102	-0.001	0.027	0.001	0.030	0.651	1.000						
PCPs	-0.060	-0.039	0.121	0.108	0.658	0.087	0.081	1.000					
SPs	-0.089	-0.034	0.107	0.075	0.592	0.073	0.064	0.926	1.000				
NPs	-0.087	-0.001	0.170	0.087	0.634	0.128	0.175	0.876	0.877	1.000			
Female	-0.037	0.019	0.101	0.110	0.875	-0.014	0.002	0.792	0.670	0.749	1.000		
Inpatient	-0.158	-0.169	0.063	0.005	-0.110	0.010	-0.015	0.011	0.031	0.047	-0.080	1.000	
Outpatient	-0.260	0.046	0.051	-0.068	0.083	0.217	0.305	0.276	0.363	0.356	0.063	0.354	1.000

Table B.3: Regression results for Savings rate

Variables	Savings rate	Savings rate	Savings rate
	Model 1 (Controls only)	Model 2 (Main-effects)	Model 3 (Interactions)
States	-0.0015000 (0.0009000)	-0.0008498 (0.0009370)	-0.0008452 (0.0009370)
Log (Beneficiaries)	-0.0026000 (0.0049000)	-0.0097200 (0.0049800) **	-0.0097540 (0.0047400) **
PCPs	0.0000022 (0.0000180)	-0.0000005 (0.0000181)	0.0000005 (0.0000181)
Specialists	-0.0000040 (0.0000097)	0.00000563 (0.0000093)	0.0000025 (0.0000093)
NPs	0.0000207 (0.0000176)	-0.0000302 (0.000017)	-0.0000256 (0.0000177)
Female	0.0000003 (0.0000004)	0.0000009 (0.0000004) **	0.0000009 (0.0000004)**
Inpatient expenditures	-0.0000061 (0.00000227) ***	-0.0000037 (0.0000022)	-0.0000036 (0.0000024)
Outpatient expenditures	-0.0000007 (0.0000043)	-0.0000372 (0.0000054)***	-0.0000373 (0.0000054)***
PC by specialists		0.0046200 (0.0024000) *	0.0089900 (0.0027000) ***
PC by NPs		0.0132000 (0.0062400) **	0.0119800 (0.0063000) **
Experience (years)		0.0122000 (0.0012600) ***	0.0122000 (0.0012600) ***
Risk model		-0.0099000 (0.0050000) **	-0.0097360 (0.0049900) *
PC by specialists * Experience			-0.000392 (0.00087)
PC by NPs * Experience			0.0063840 (0.0025340) **
PC by specialists * Risk model			0.0053134 (0.0091900)
PC by NPs * Risk model			0.0536800 (0.0220400) **
Constant	0.06045130 (0.0464000)	0.1556400 (0.0459000) ***	0.15612000 (0.0452000)***
R-sq within	0.0117000	0.1097000	0.1143000
Observations	1908	1908	1908

Standard deviations in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table B.4: Regression results for Quality score

Variables	Quality score	Quality score	Quality score
	Model 4 (Controls only)	Model 5 (Main-effects)	Model 6 (Interactions)
States	-0.0008340 (0.0014000)	-0.0010000 (0.0014000)	-0.0010000 (0.0014000)
Log (Beneficiaries)	0.0221000 (0.0043000)***	0.0215000 (0.0042000)***	0.0200000 (0.0042000) ***
PCPs	-0.0000330 (0.0001530)**	-0.0000251 (0.0000152) *	-0.0000254 (0.0000153) *
Specialists	-0.0000090 (0.0000067)	-0.0000136 (0.0000068) **	-0.0000135 (0.0000068) **
NPs	0.0000432 (0.0000179)**	0.0000467 (0.0000185) **	0.0000472 (0.0000186) **
Female	-0.0000006 (0.0000004)	-0.0000006 (0.0000004) *	-0.0000006 (0.0000004) *
Inpatient expenditures	-0.0000210 (0.0000021) ***	-0.0000210 (0.0000021) ***	-0.0000221 (0.0000021) ***
Outpatient expenditures	0.0000130 (0.0000026) ***	0.0000151 (0.0000027)***	0.0000152 (0.0000027)***
PC by specialists		-0.0044000 (0.0021000) **	-0.0070000 (0.0040000) *
PC by NPs		-0.0040000 (0.0070000)	-0.0110000 (0.0110000)
Experience (years)		-0.0017000 (0.0010000) *	0.0122000 (0.0012600) ***
Risk model		0.0130000 (0.0040000) ***	-0.0097360 (0.0049900) *
PC by specialists * Experience			0.0010300 (0.0010200)
PC by NPs * Experience			0.0041715 (0.0030000)
PC by specialists * Risk model			-0.0230000 (0.0050000) ***
PC by NPs * Risk model			-0.0470000 (0.0130000) ***
Constant	0.7782000 (0.0390000) ***	0.7917000 (0.0399999) ***	0.79540000 (0.0380000)***
Log likelihood	2212.0600000	2223.2200000	2230.8700000
Observations	1586	1586	1586

Standard deviations in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table B.5: Quantile regression for Savings rate

Savings rate	5	10	25	50	75	90
States	0.0021000 (0.0015000)	0.0022000 (0.0005300)***	-0.0000631 (0.0008000)	-0.0012000 (0.0009000)	-0.0008900 (0.0007600)	-0.0023000 (0.0010000)**
Experience (years)	-0.0011000 (0.0006278)*	0.0025400 (0.0002000)***	0.0093600 (0.0003000)***	0.0045600 (0.0005000)***	0.0056000 (0.0003000)***	0.0039000 (0.0030000)***
Risk model	0.0242500 (0.0029300)***	0.0211000 (0.0086000)***	0.0090000 (0.00460000)**	0.0079000 (0.0016000)***	-0.0092000 (0.0047000)*	-0.0006000 (0.0023000)
Log (beneficiaries)	0.0312000 (0.0037000)***	0.0077000 (0.0026000)***	-0.0014000 (0.0017000)	0.0019000 (0.0018000)	-0.0224000 (0.0028900)***	-0.0370000 (0.0017000)***
PC by SP	0.0119000 (0.0016000)***	0.0120000 (0.0013000)***	0.0060000 (0.0005000)**	0.0116000 (0.0016000)***	0.0035000 (0.0006000)***	-0.0016000 (0.0028000)
PC by NPs	-0.0279000 (0.004000)***	-0.029000 (0.0019000)***	-0.0220000 (0.0009000)***	-0.0250000 (0.0030000)***	-0.0038000 (0.0077000)	0.0063000 (0.0123000)
PCPs	0.0000281 (0.0000044)***	0.0000109 (0.0000081)	0 .0000013 (0.000007)	-0.000000026 (0.0000038)	0.0000143 (0.0000073)*	0.000023 (0.000010)**
SPs	-0.0000053 (0.0000027)***	-0.0000099 (0.0000016)***	-0.0000007 (0.0000078)	-0.0000031 (0.0000020)	-0.0000024 (0.0000067)	-0.0000093 (0.00000256)*
NPs	-0.0000216 (0.0000017)**	0.0000190 (0.0000076)**	0.0000045 (0.0000110)	-0.0000059 (0.000004)	-0.0000074 (0.0000077)	0.0000080 (0.0000100)
Female	-0.0000008 (0.0000002)***	0.-0000004 (0.0000003)	0.0000007 (0.0000001)	0.00000008 (0.00000006)	0.0000059 (0.0000001)***	0.0000009 (0.0000002)***
Inpatient	-0.0000260 (0.0000018)***	-0.0000220 (0.0000019)***	-0.0000116 (0.000000428)***	-0.0000091 (0.00000098)***	-0.0000053 (0.0000016)***	-0.0000005 (0.0000001)
Outpatient	-0.0000046 (0.0000015)***	-0.0000068 (0.0000018)***	-0.0000079 (0.0000007)***	-0.0000089 (0.0000004)***	-0.0000069 (0.0000018)***	-0.000017 (0.0000016)***

Table B.6: Bootstrapped Tobit Regression - Quality score

Variables	Coef.	Std. Err.	P value	95% Confidence	Interval
States	-0.0010976	0.0019414	0.5720000	-0.0049026	0.0027075
Experience (years)	-0.0017817	0.0010986	0.1050000	-0.0039350	0.0003716
Risk model	0.0136528	0.0040532	0.0010000	0.0057086	0.0215970
Log (beneficiaries)	0.0215665	0.0101793	0.0340000	0.0016153	0.0415176
PC by SP	-0.0042304	0.0018769	0.0240000	-0.0079091	-0.0005518
PC by NP	-0.0040224	0.0086718	0.6430000	-0.0210188	0.0012974
PCP	-0.0000251	0.0000153	0.1020000	-0.0000551	0.0000049
SP	-0.0000136	0.0000064	0.0330000	-0.0000262	-0.0000010
NP	0.0000467	0.0000190	0.0140000	0.0000942	0.0000839
Female	-0.0000006	0.0000005	0.2790000	-0.0000017	0.0000004
Inpatient	-0.0000220	0.00000529	0.0000000	-0.0000324	-0.0000116
Outpatient	0.0000151	0.0000037	0.0000000	0.00000764	0.0000225
Constant	0.7917981	0.0939737	0.0000000	0.6076130	0.9759833

Table B.7: Post-hoc Analysis for FQHC, CHC, and RHC

Variables	Savings rate	Quality score
States	-0.0008591 (0.0009377)	-0.0011000 (0.0014000)
Log (Beneficiaries)	-0.0095000 (0.0047000)**	0.0215000 (0.0047000)***
PCPs	-0.0000001 (0.0000180)	-0.0000238 (0.0000152)
Specialists	0.0000031 (0.0000094)	-0.0000142 (0.0000068) **
NPs	-0.0000256 (0.0000178)	0.0000482 (0.0000187) **
Female	0.0000009 (0.0000004)	-0.0000006 (0.0000003) *
Inpatient expenditures	-0.0000036(0.0000022)	-0.0000221 (0.0000021) ***
Outpatient expenditures	-0.0000370 (0.0000026) ***	0.0000154 (0.0000028)***
PC by specialists	0.0088200 (0.0027000)***	-0.0034000 (0.0023000)
PC by NPs	0.1604000 (0.0063000)**	-0.0044000 (0.0070000)
Experience (years)	0.0120000 (0.0012000)***	-0.0016000 (0.0010000)
Risk model	-0.0100000 (0.0049000) *	0.0130000 (0.0040000) *
PC at FQHCs	0.0020300 (0.0006100)***	0.0004000 (0.0009000)
Assoc CHC	0.0025000 (0.0038000)	-0.0064000 (0.0040000)
Assoc RHC	-0.0010000 (0.0033000)	0.0017000 (0.0013700)
Constant	0.1538000 (0.0450000) ***	0.7909000 (0.0394000) ***
Observations	1908	1586

Standard deviations in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table B.8: Main Effects Models with the CDC Data

Variables	Savings rate	Quality score
States	-0.0008600 (0.0009400)	-0.0008800 (0.0013900)
Log (Beneficiaries)	-0.0095007 (0.0047000) **	0.0217700 (0.0043100) ***
PCPs	-0.0000068 (0.0000180)	-0.0000100 (0.0000100)
Specialists	0.0000064 (0.0000097)	-0.0000130 (0.000006)*
NPs	-0.0000301 (0.0000176)*	0.0000270 (0.0000100)
Female	0.0000010 (0.0000004)**	-0.0000006 (0.0000035) *
Inpatient expenditures	-0.0000023 (0.0000022)	-0.0000218 (0.0000022)***
Outpatient expenditures	-0.0000386 (0.0000055) ***	0.0000170 (0.0000027)***
PC by specialists	0.0043100 (0.0025300) *	-0.0069700 (0.0021100) **
PC by NPs	0.0140600 (0.0064600) **	0.0006100 (0.0066600)
Experience (years)	0.0114900 (0.0017100) ***	-0.0027000 (0.0009000) ***
Risk model	-0.0103000 (0.0050300)**	0.0113100 (0.0046400) **
Checkup	-0.0018000 (0.0225000)	-0.0809300 (0.018700)***
Income	-0.0004400 (0.0005400)	0.0018700 (0.0006000)***
Genhealth	-0.0228000 (0.0367000)	-0.0058500 (0.0196000)
Medcost	-0.0235000 (0.1137000)	-0.3912500 (0.0870000)***
Constant	0.2784700 (0.249100)	1.6294000 (0.2104800) ***
Model parameters	R-sq within = 0.1020	Log likelihood = 2252.0172
Observations	1908	1908

Standard deviations in parentheses
* $p < 0.1$, *** $p < 0.05$, *** $p < 0.01$

APPENDIX C

KNOW YOUR USERS BEFORE YOU SPEND: A DATA-DRIVEN OPTIMIZATION TO ENHANCE USER ENGAGEMENT USING VISUAL ANALYTICS

C.1 Empirical Analysis

In this section, we discuss our empirical analysis. We provide more details on (i) data collection, (ii) deep learning approach, (iii) econometric approach, and (iv) discussion of the findings.

C.1.1 Data Collection and Variables

Our dataset comes from the posts published by furniture firms on Facebook and Twitter accessed through the platforms' application programming interfaces (APIs). We collected a total of 2733 recent posts from Facebook and Instagram. These APIs only allow certain features of social media posts to be extracted. We select each firm's official social media accounts that periodically generate organic posts for each platform's audience. The baseline data were collected from the social media platforms, with additional information about the firms manually gleaned from their social media accounts and official websites. We built scrapers in Python that interact with the social media APIs that collect posts with images, likes, comments, shares (shares are available for Facebook only), captions, and posting dates. Comments are common and can be measured across the two platforms. The study by Aldous et al. (2019) finds that most engagements happen during an initial couple of hours from posting time, and therefore, day normalization may not be required. Our data reveal the same property as we discussed in Section 4.3.1.1. We describe the main variables used in our empirical analysis in Table C.1.

Dependent variables: The dependent variables in this chapter are likes and comments for Instagram and shares and comments for Facebook. All the variables exhibit the properties of a count variable, and therefore, we model them under Negative Binomial regression. Note that, on average, user engagement on Instagram is substantially higher than that on Facebook. We identify these variables from the data we collected from Instagram and Facebook.

Table C.1: Variables used in empirical analysis

Variable	Description	Type
likes	number of likes (negative binomial)	dependent variable for Instagram engagement
comments	number of comments (negative binomial)	dependent variable for Instagram and Facebook engagements
shares	number of shares (negative binomial)	dependent variable for Facebook engagement
simple features	number of features extracted using the SSD algorithm	independent variable
advanced features	number of extra features extracted using the advanced Faster R-CNN algorithm	independent variable
time	number of days between post date and data collection date	control variable
post interval	time (in days) since the last post	control variable
revenue	revenue of a firm	control variable
posts (Instagram only)	total number of posts by a firm on Instagram	control variable
page active (Facebook only)	length of the official account of a firm on Facebook since page opening	control variable
saturation	saturation value of an image	control variable
log(wc)	Logarithm of wordcount of a post caption	control variable

Independent variables: The key independent variables in the empirical analysis are the image features extracted from the social media posts. SSD is a simpler algorithm that only extracts prominent or focal features from images. We refer to them as simple features. On the other hand, the Faster R-CNN extracts the additional features in addition to the focal feature(s) given its algorithmic complexity and performance. These additional features are advanced features. It is noteworthy to mention here that we apply the deep learning algorithms in our empirical analysis to illustrate the use of social media analytics. Given the high accuracy and several applications of pre-trained deep learning algorithms (Cheng et al. 2017), we use the pre-trained models, both SSD and the Faster R-CNN. The use of pre-trained models is prevalent in academic research given the high level of accuracies of these algorithms in classification tasks (Yim et al. 2021). The pre-trained models that we use are trained using a popularly labeled dataset called the COCO dataset. The COCO dataset is already trained with 90 different feature classes and showed a high level of prediction accuracy across many settings.[1] We use the cut-off probability of 50% to detect features from an image.

Control variables: We employ several post-level and firm-level control variables to control for the heterogeneity among posts and firms. We run a simple Python program to extract an image's color properties, including red, green, blue (RGB) channel values and hue, saturation, and lightness (HSL) values. We only use saturation as a control variable in the econometric analysis, given the high correlation. Besides extracting image features, we also extract and analyze the proper-

[1] Readers may refer to https://paperswithcode.com/sota/object-detection-on-coco

ties of captions using the Linguistic Inquiry and Word Count or LIWC tool. LIWC tool ignores irrelevant items such as punctuation and hashtags and preserves the relevant information from the textual data. Post captions are crucial in our analysis since they offer social media users valuable information about the products. We use log-transformed word count to control the additional information effect on user engagement. We also construct a time variable that refers to the duration between post and data collection dates. This variable is used for two purposes. First, it controls the length of time a post has been on the platform. Second, it also acts as a proxy that controls for the number of followers around the post time. We control for the firm effects using firms' number of posts and revenue, consistent with the literature for controlling post volume that indicates the level of active participation (Audrezet et al. 2086).

It is important to discuss the choice of the industry in this chapter. We extract social media posts of the large furniture firms. We chose the furniture industry to capture a wide range of audiences similar to many other sectors. Additionally, furniture industries focus more on the products in their posts than human brands and logos, which is more appropriate for this chapter. More importantly, product image photography is an important display for furniture retailers as customers give importance to the professionally-shot furniture images and the beautiful room scene graphics displayed on images. Therefore, our work can be extended to several other industries.

C.1.2 Applications of Deep Learning Algorithms

We use the deep learning methods to extract features or objects from images that we use as variables in our econometric model to assess the feature-engagement relationship and develop the optimization model to maximize user engagement. For that purpose, we deploy deep learning algorithms using Tensorflow in Python (All codes are available upon request). Tensorflow is an end-to-end open-source platform developed by Google that has inbuilt functionalities for the training and testing of deep learning algorithms. Since we used pre-trained models with the labeled COCO dataset and applied it to the social media data we collected, our computational tools were sufficient to run the models. Social media analytics help firms identify the antecedents of user engagement, especially via deep learning methods, since they help identify the features without large-scale experiments and inherent biases (Ciregan et al. 2012).

Deep learning methods vary based on complexity and performance in detecting objects. Depending on the applications, several popular deep learning methods are available for use. Convolutional Neural Networks or CNNs are popular deep learning algorithms for object detection with various degrees of complexity and accuracy. CNNs are primarily built with multi-layer perceptrons, a basic neural network structure with inputs, activation functions, and outputs, with fully collected layers. In other words, each neuron within a layer is connected to all neurons in the next layer. CNNs learn to optimize the network by analyzing the hierarchical patterns in the data, in this case, an image. It first assembles and analyzes simpler patterns and then advanced patterns through the layers. The higher the layers, the more computationally complex the algorithm becomes with better accuracy but lower memory efficiency. SSD uses one such layer and quickly detects prominent objects present in an image. It is significantly faster in speed but underperforms in terms of detecting features.

Within the family of CNNs, Region-based CNN (R-CNN) methods take image input and identify the features via bounding boxes or region proposals through selective search. Once the region proposals are identified, R-CNN warps the input to a standard size for processing, passes to a CNN-based feature extractor that extracts features, and finally uses a Support Vector Machine (SVM) algorithm to classify the features into the corresponding classes. This process is slow because it requires the training of three different models: (i) the CNN to generate features, (ii) the classifier that predicts the class objects, and (iii) the regression model to reduce bounding box errors (Yadav and Binay 2017). In the family of R-CNNs, Fast R-CNN is developed to expedite the process by jointly training the CNN, classifier, and bounding box regressors and replacing the SVM classifier with a softmax for object classification. However, one bottleneck with the Fast R-CNN was that the region proposals were created using the slow selective search process. The Faster R-CNN, first proposed by Ren et al. (2015), overcomes the bottleneck by using a single CNN to carry out both region proposals and classification.

Compared to other CNN methods for image analysis, the Faster R-CNN method enables the end-to-end learning of all layers. The Faster R-CNN uses a region proposal network (RPN) instead of slow selective search methods. For our optimization model, we identify features from the social media data and analyze the appropriate number of features to be used in images. The Faster R-CNN is the appropriate method in this context to identify the additional features from an image and the focal features extracted by SSD. Compared to Faster R-CNN, SSD is a simpler model that does not require regional proposal generation or feature resampling. The difference in the performance between SSD and Faster R-CNN allows us to quantify social media analytics to gather more user information efficiently. Faster R-CNN is 5-6 times slower than SSD processing an image. Ideally, more advanced analytics employ a large amount of data, more computational tools, and better technology teams, which adds to the social media analytics costs. We use the two deep learning methods to illustrate the difference. Both algorithms are pre-trained on the COCO dataset. Faster R-CNN detects more features. We use these two algorithms on all the social media posts we collected and identify the simple and advanced features in the econometric method and optimization model.

C.1.3 Econometric Approach

This section describes our econometric approach where we establish the relationship between the features extracted using the deep learning algorithms and user engagement. The summary statistics of the variables are listed in Tables C.2 and C.3 for Instagram and Facebook, respectively.

Table C.2: Summary statistics of Instagram variables

Variable	Obs	Mean	Std. Dev.	Min	Max
likes	542	2888.7600	3243.9750	96	19815
time	542	164.3358	119.2445	1	449
saturation	542	58.1697	49.0072	0	255
log(wc)	542	3.3761	0.7042	1.0980	5.6970
simple features	542	1.5572	1.4692	0	10
advanced features	542	3.8450	2.9365	0	14
post interval	542	2.2103	1.9504	0	33
revenue	542	3.2576	1.6654	1.4500	5.3300
posts	542	2120.2920	495.0938	1227	2514

Table C.3: Summary statistics of Facebook variables

Variable	Obs	Mean	Std. Dev.	Min	Max
shares	2,191	13.6020	26.6060	0	388
comments	2,191	35.8380	63.0630	0	573
time	2,191	352.3580	246.7230	1	1057.2900
saturation	2,191	56.9330	43.3854	0	255
log(wc)	2,191	3.4888	0.6033	0	5.2200
simple features	2,191	1.7122	1.6960	0	19
advanced features	2,191	3.8621	3.0050	0	18
post interval	2,191	1.2590	1.2582	0	8.0818
revenue	2,191	3.74	2.9200	1.4500	9.1200
page active	2,191	3965.4200	289.3200	3499	4344

C.1.3.1 Model Specification.

The dependent variables represent count data and are heavily skewed. Following the literature, count models such as Poisson or Negative Binomial are appropriate in our context. All four engagement metrics that we analyze exhibit high dispersion where their variances exceed their mean values. Thus, we model the engagement variables under Negative Binomial regression as suggested by Cameron and Trivedi (2013). We present our regression equation.

Generalized Regression Equation:

$$\begin{aligned}\text{user engagement}_{lmk} &= \alpha_{lm} + \beta_{lm}\text{simple features}_{lmk} + \zeta_{lm}\text{advanced features}_{lmk} \\ &\quad + \eta_{lm}\text{advanced features}_{lmk}{}^{2} + \theta \mathrm{V}_{lmk} + \gamma \mathrm{Z} + \nu \mathrm{Y}_{l} + \epsilon_{lmk}\end{aligned}$$

As discussed earlier, the primary independent variables used in the analysis are simple $\text{features}_{\text{lmk}}$ and advanced $\text{features}_{\text{lmk}}$ for post k on platform l and engagement type m. The variable simple $\text{features}_{\text{lmk}}$ comprises of the focal or prominent features of an image, conveying some product information about the advertisement to users. On the other hand, advanced $\text{features}_{\text{lmk}}$ complement simple $\text{features}_{\text{lmk}}$ and form a narrative for users by creating a story. Users obtain their information through these features. As hypothesized, the association between advanced features and engagement exhibits non-linear trends. Thus, we construct the quadratic variable for advanced $\text{features}_{\text{lmk}}$ to test the relationship.

The coefficient of simple $\text{features}_{\text{lmk}}$ represents the impact of simple features on engagement, while the coefficient for advanced $\text{features}_{\text{lmk}}$ captures the impact of advanced features extracted by the Faster R-CNN on user engagement. The coefficient of the quadratic term for advanced $\text{features}_{\text{lmk}}$ highlights the nature of the nonlinear association between advanced features and user engagement. V_{lmk} denote the post-specific controls (time, log-transformed word count of a post's caption, saturation value of the image of a post, and post interval) while Z denote the control variables for firm effects. Variables such as revenue for a firm is a fixed variable in a single period and it can be updated while running the analysis in the future periods. Revenue refers to the market positioning of a firm; the bigger the revenue, the more power a firm has. Since it is difficult to estimate the revenue generated from each each platform, we fix the firm revenue fixed across platforms. On the other hand, the number of posts (or tweets) may vary across platforms but not across the engagements within a platform. We denote such platform specific control variables as Y_{l}.

C.1.3.2 Results.

We report the empirical results for Instagram in Table C.4 with three models for likes and comments - the control only models with simple features (1 and 4), the main effects models with advanced features (2 and 5), and the full model with the quadratic advanced features (3 and 6). The log-likelihood values substantially increase from Model 1 to Model 2 and from Model 4 to Model 5, highlighting the importance of including advanced features to the regression model. The variable advanced features is positive and statistically significant for likes (0.0307, p= 0.0240). This result suggests that one unit increase in advanced features is associated with an approximately 3% increase in the number of likes. On the other hand, the effect of advanced features on comments is negatively signed but not significant. Simple features is negatively associated with comments (-0.0540, p= 0.0270), indicating that having more simple features may not improve comments on Instagram. Next, we present the discussion on the quadratic advanced features variable.

We include the quadratic term for advanced features in Models 3 and 6 for likes and comments, respectively. Not surprisingly, these two models have the best fit with the inclusion of advanced features quadratic term. The quadratic term is negative and significant (-0.0062, p= 0.0460) for

Table C.4: Negative Binomial Regression analyses of likes and comments on Instagram data

	(1) likes	(2) likes	(3) likes	(4) comments	(5) comments	(6) comments
time	-0.0007*	-0.0008**	-0.0009**	-0.0021***	-0.0020***	-0.0020***
	(0.0190)	(0.0060)	(0.0040)	(0.0000)	(0.0000)	(0.0000)
saturation	-0.0002	-0.0001	-0.0001	0.0008	0.0008	0.0008
	(0.7990)	(0.8630)	(0.9500)	(0.3210)	(0.3250)	(0.3330)
log(wc)	0.6680***	0.6840***	0.6980***	0.7050***	0.6950***	0.6810***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
simple features	-0.0320	-0.0350	-0.0340	-0.0570*	-0.0540*	-0.0530*
	(0.1460)	(0.1120)	(0.1240)	(0.0250)	(0.0270)	(0.0270)
post interval	-0.0700*	-0.0690***	-0.0680***	-0.035200*	-0.03600**	-0.03600**
	(0.0225)	(0.0000)	(0.0000)	(0.0100)	(0.0090)	(0.0080)
revenue	0.3420***	0.3470***	0.3470***	0.4262***	0.425200***	0.42700***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
posts	0.0012***	0.0012***	0.0012***	0.0017***	0.0017***	0.0017***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
advanced features		0.03070*	0.0940**		-0.0100	-0.0530
		(0.0240)	(0.0080)		(0.4820)	(0.1880)
advanced features squared term			-0.0062*			0.0041
			(0.0460)			(0.2580)
constant	2.0700***	1.8200***	1.6400***	-3.6700***	-3.5700***	-3.4400***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
log pseudolikelihood	-4645.4731	-4641.8329	-4639.7038	-2335.1494	-2334.7808	-2334.0166
Observations	542	542	542	542	542	542

p values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C.5: Negative Binomial Regression analyses of shares and comments on Facebook data

	(1) shares	(2) shares	(3) shares	(4) comments	(5) comments	(6) comments
time	-0.0001	-0.0001	-0.0001	-0.0022**	-0.0022***	-0.0021***
	(0.9910)	(0.9910)	(0.8920)	(0.0040)	(0.0000)	(0.0000)
saturation	-0.0020*	-0.0020*	-0.0019*	-0.0018*	-0.0017*	-0.0016*
	(0.0190)	(0.0190)	(0.0190)	(0.0100)	(0.0120)	(0.0180)
log(wc)	-0.3150**	-0.3140**	-0.33220**	-0.7350***	-0.7380**	-0.7760***
	(0.0050)	(0.0053)	(0.0030)	(0.0000)	(0.0060)	(0.0000)
simple features	-0.0340	-0.0350	-0.0500	0.0100	0.0120	0.0036
	(0.4790)	(0.4390)	(0.2090)	(0.7720)	(0.6250)	(0.9050)
post interval	-0.0820*	-0.0800*	-0.0079*	-0.0820**	-0.0835**	-0.0800**
	(0.03500)	(0.0330)	(0.0310)	(0.0080)	(0.0080)	(0.0080)
revenue	0.1150***	0.1150***	0.1090***	0.3090***	0.3020***	0.2970***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
pageactive	-0.0007***	-0.0007***	-0.0007***	0.0003**	0.0003**	0.0003*
	(0.0000)	(0.0000)	(0.0000)	(0.0090)	(0.0080)	(0.0210)
advanced features		0.0020	-0.0988*		-0.0060	-0.1140***
		(0.8890)	(0.0280)		(0.6250)	(0.0000)
advanced features squared term			0.0090*			0.0100**
			(0.0200)			(0.0010)
constant	6.3100***	6.3100***	6.7400***	4.1100***	4.1100***	4.7000***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
log pseudolikelihood	-7635.9460	-7635.9159	-7626.1976	-9159.3270	-9159.0695	-9147.0946
Observations	2191	2191	2191	2191	2191	2191

p values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

likes. The linear and quadratic terms are positive and negative, respectively, suggesting a concave pattern in the relationship between advanced features and the engagement (likes). In other words, likes increase initially with advanced features and then decreases after a certain switching point. We present the discussion on switching behavior of engagement types later in Section C.1.4.

Turning our attention to the results for Facebook posts, in Columns 2 and 5 of Table C.5, we find that the linear term of advanced features has no significant impact on either shares or comments. The coefficients for simple features have no association with both shares and comments. The full models in columns 3 and 6 in Table C.5 show that the quadratic term is both positive and significant on shares (0.0090, p= 0.0200) and comments (0.0100, p= 0.0010). The number of shares and comments on a Facebook post reduce initially as the number advanced features increases and starts to increase after certain thresholds.

C.1.3.3 Discussion of Empirical Findings.

Our empirical analysis reveals that Instagram users prefer less advanced or additional features as too many features hinder the clarity of an image. Facebook still enjoys a substantial amount of users from all age groups, while Instagram is most popular among users below 30. This makes Facebook reachable to a larger audience, who prefer more information through features. Having relatively older users on the platform is advantageous since these users have higher income levels and can potentially spend more money, even on additional objects. Therefore, the firm may want to signal more information to them to achieve higher engagement and future expected sales. Additionally, unlike Facebook, Instagram is concise, and users may focus exclusively on core features.

We also discuss some additional findings that are interesting in our context. The positive and significant coefficient of log word count (0.6840, p=0.0000) in Column 2 of Table C.4 suggests that Instagram users prefer more information from the textual caption than from a cluttered image. Another important finding is the negative association of post interval with both likes (-0.0690, p= 0.0000) and comments (-0.0360, p= 0.009) on Instagram. These two results suggest that there are synergistic effects between user engagement and how frequently a firm should post as infrequent postings may hurt both likes and comments. For Facebook, the impact of log word count is negative and significant for both shares (-0.3320, p=0.0030) and comments (-0.7380, p=0.0060) as reported in Columns 2 and 5 in Table C.5. Thus, on Facebook, firms should focus more on the advanced features to seek users' attention.

One plausible reason behind the insignificance of advanced features to the number of comments may relate to the smaller number of posts we extracted from Instagram compared to Facebook. Comments are a different form of engagement than likes since it requires more effort from the users. We find that simple features have a negative and significant effect on comments, suggesting that Instagram users do not prefer too many focal features on the images. Conversely, it also implies that the firm should present its information without relying on too many focal and additional features.

C.1.3.4 Robustness Checks.

We conducted two relevant robustness checks to ensure the correctness of our empirical findings. First, we perform a zero-inflated negative binomial regression (ZINB). Second, we employ quantile regression to see the effects of the coefficients at various percentiles of the data.

We chose negative binomial regression as our depedent variables include count data, which may also include zeros. There may be two possible reasons that could lead to the number of zeros: (i) a user does not engage and (ii) a user does not see the post. Thus, the number of zeros may be inflated and the standard negative binomial may not distinguish between the two. This motivates us to re-analyze our data using ZINB. ZINB utilizes a logit model using a binary method for the zero outcomes and a negative binomial as a count process to model the counts. However, ZINB does not perform better than negative binomial if the data are too overdispersed. We present the main results in Table C.6. Note that the Instagram results are exactly the same since no posts have zero likes. Facebook shares and comments, containing many posts with zero comments or shares, are also similar to our main results as reported earlier. Thus, ZINB supports the robustness of our empirical findings.

We also analyze our findings at 0.5, 0.7, and 0.9 quantiles and present the results in Tables C.7, C.8, and C.9. We chose relatively higher quantile values for the engagements since posts at the bottom have lower engagement, especially on Facebook. The squared advanced features variable is significant for both shares and comments at 0.7 and 0.9 quantiles, with values almost similar to out main findings. For Instagram, it is consistent across all quantiles since Instagram posts attract a substantially higher number of likes.

C.1.4 Discussion of Empirical Results for Optimization Framework

In this section, we analyze the empirical results for each Instagram and Facebook engagement types and present our insights for the optimization framework. We present the empirical summaries in Table C.10 and discuss their implications.

The estimation results from the Instagram analysis (Table C.10) show that the quadratic impact of the advanced features is negative and significant. The curve bends when $f_{1,1,k} = \lfloor \frac{0.0940}{2|0.0062|} \rfloor + 1$ = 8. Thus, the number of likes reduces after the number of advanced features crosses 8 in an image, on average. The relationship between user engagement and advanced features is non-monotonic as the switching point is within the range of advanced features (Table C.2). The equation for the Instagram likes can be written as following: $\hat{x}_{1,1,k} = 1.6400 - 0.0340 s_{1,1,k} + 0.0940 f_{1,1,k} - 0.0062 f_{1,1,k}^2 + \theta_1 V_{1,1,k} + \gamma_1 Z + \nu_1 \mathrm{Y}_1$. The coefficient of $f_{1,i,k}$ shows the rate of change in likes on Instagram while other variables are fixed at a constant value. We can assess the rate of change in the dependent variable $\hat{x}_{1,1,k}$ as we change $f_{1,1,k}$. However, the quadratic relationship tells us that the change is not the same at different levels and the direction of change switches after eight advanced features. We present the engagement values near the switching point by following Hayes (2017), where other variables except for simple features and advanced features are fixed at their respective means and added to α_{lm} to construct b_{lm}. We show them in Table 4.4.

- At $f_{1,1,k}$ = 7, the expected value of $\hat{x}_{1,1,k} = 7.3400 + 0.0940 * 7 - 0.0062 * 7^2 = 7.6740$.
- At $f_{1,1,k}$ = 8, the expected value of $\hat{x}_{1,1,k} = 7.3400 + 0.0940 * 8 - 0.0062 * 8^2 = 7.6960$.
- At $f_{1,1,k}$ = 9, the expected value of $\hat{x}_{1,1,k} = 7.3400 + 0.0940 * 9 - 0.0062 * 9^2 = 7.6838$.

Thus, we can observe a downward trend of likes after the number of advanced features exceeds

Table C.6: Zero-Inflated Negative Binomial Regression analyses of Instagram likes and Facebook shares and comments

	(1) likes (Instagram)	(2) shares (Facebook)	(3) comments (Facebook)
time	-0.0009**	-0.0001	-0.0021**
	(0.0040)	(0.8090)	(0.0000)
saturation	-0.0002	-0.0019**	-0.0015*
	(0.6530)	(0.0030)	(0.0110)
log(wc)	0.6708***	-0.3310***	-0.7718***
	(0.0000)	(0.0000)	(0.0000)
simple features	-0.0348**	-.0500	.0055
	(0.0213)	(0.112)	(0.698)
post interval	-0.0413**	-0.0796***	-0-.0799***
	(0.0200)	(0.0000)	(0.0000)
revenue	0.3543***	0.1088***	0.2955***
	(0.000)	(0.000)	(0.000)
posts (or page active)	0.0011***	-0.0007***	0.0003**
	(0.0000)	(0.0000)	(0.0020)
advanced features	0.0945***	-0.1011***	-0.1223***
	(0.0040)	(0.0000)	(0.0000)
advanced features squared term	-0.0062*	0.0098***	0.0105***
	(0.03500)	(0.0000)	(0.0000)
constant	1.6409***	6.7428***	4.6861***
	(0.0000)	(0.0000)	(0.0000)
Inflate advanced features	-0.1666	-1.5057	-1.3716
	(1.0000)	(0.5320)	(0.2350)
log pseudolikelihood	-4639.7040	-7626.1090	-9144.9040
Observations	542	2191	2191

p values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C.7: Quantile Negative Binomial Regression analyses of Facebook shares

	(1)	(2)	(3)
	0.5	0.7	0.9
time	-0.0002	-0.0001	-0.0003
	(0.2950)	(0.2530)	(0.1710)
saturation	-0.0015	-0.0001	-0.0002
	(0.1100)	(0.7410)	(0.4830)
log(wc)	-0.86742***	-0.4741***	-0.3157***
	(0.0000)	(0.0000)	(0.0000)
simple features	-0.1119***	-0.1119***	-0.0627
	(0.0000)	(0.0000)	(0.6980)
post interval	-0.1082**	-0.0456	-0.0515**
	(0.0500)	(0.1880)	(0.012)
revenue	0.1720***	0.1128***	0.0901***
	(0.0000)	(0.0000)	(0.0000)
page active	-0.0007***	-0.0009***	0.0009***
	(0.0000)	(0.0000)	(0.0000)
advanced features	0.0014	-0.0420	-0.1587**
	(0.9680)	(0.1030)	(0.0010)
advanced features squared term	0.0014	0.0030**	0.0105**
	(0.6400)	(0.0000)	(0.0000)
constant	7.4697***	7.8646***	8.2514***
	(0.0000)	(0.0000)	(0.0000)
Observations	2191	2191	2191

p values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C.8: Quantile Negative Binomial Regression analyses of Facebook comments

	(1)	(2)	(3)
	0.5	0.7	0.9
time	-0.0023***	-0.0023***	-0.0024***
	(0.0000)	(0.0000)	(0.0000)
saturation	-0.0013	-0.0006	-0.0008
	(0.1750)	(0.3720)	(0.1110)
log(wc)	-1.4216***	-1.0814***	-0.7039***
	(0.0000)	(0.0000)	(0.0000)
simple features	-0.0329	-0.0064	-0.0091
	(0.1430)	(0.9040)	(0.7520)
post interval	-0.0769**	-0.1017**	-0.0546
	(0.0500)	(0.0026)	(0.3750)
revenue	0.2869***	0.2909***	0.2883***
	(0.0000)	(0.0000)	(0.0000)
page active	0.0001	0.0002	0.0002
	(0.8380)	(0.7380)	(0.1790)
advanced features	-0.0368	-0.0605	-0.1283***
	(0.4140)	(0.0080)	(0.0000)
advanced features squared term	0.0020	0.0058*	0.0094**
	(0.6440)	(0.0390)	(0.0040)
constant	7.3500***	6.0865***	5.4000***
	(0.0000)	(0.0000)	(0.0000)
Observations	2191	2191	2191

p values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C.9: Quantile Negative Binomial Regression analyses of Instagram likes

	(1)	(2)	(3)
	0.5	0.7	0.9
time	-0.0005***	-0.0003***	-0.0006***
	(0.0010)	(0.0000)	(0.0000)
saturation	-0.0012***	-0.0008**	0.0004***
	(0.0000)	(0.0070)	(0.0000)
log(wc)	0.6011***	0.8481***	0.7550***
	(0.0000)	(0.0000)	(0.0000)
simple features	-0.0256***	-0.0631***	-0.0449***
	(0.0000)	(0.0000)	(0.0000)
post interval	-0.0612***	-0.0978***	-0.0533***
	(0.0000)	(0.0000)	(0.0000)
revenue	0..4019***	0.3642***	0.2768***
	(0.0000)	(0.0000)	(0.0000)
posts	.0011***	.0013***	.0014***
	(0.8380)	(0.7380)	(0.0000)
advanced features	0.1345***	0.0967***	0.0649***
	(0.0000)	(0.0080)	(0.0000)
advanced features squared term	0.1345***	-0.0072***	-0.0649***
	(0.0000)	(0.0000)	(0.0000)
constant	1.5843***	1.1802***	1.8638***
	(0.0000)	(0.0000)	(0.0000)
Observations	542	542	542

p values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C.10: Parameter Estimation Via Empirical Analysis

Coefficients for	Instagram ($l = 1$)	Facebook ($l = 2$)	
	Likes ($m = 1$)	Shares ($m = 2$)	Comments ($m = 3$)
Constant (α_{lm})	1.6400***	6.7400***	4.7000***
Simple features (β_{lm})	-0.0340	-0.0500	0.0036
Advanced features (ζ_{lm})	0.0940**	-0.0988*	-0.1140***
Squared advanced features (η_{lm})	-0.0062*	0.0090*	0.0100**
User engagement (x_{lmk})	$x_{1,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$
p values in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$			

8. The number of likes increases initially as the number of advanced features increases and starts to reduce after $f_{1,1,k}$ = 8.

Table C.10 shows that the coefficients of quadratic term are all positive and significant for Facebook shares ($0.0090, p = 0.0200$) and comments ($0.0100, p = 0.0010$). The curves are convex. For Facebook shares and comments, the curves bend when the number of advanced features exceeds 6 and 7, respectively. As compared to likes, comments and shares involve higher level of engagement from the users, allowing them to comment or share to their networks more on the images, particularly the features. Users can express their perspectives on the features of the images. For example, the user can inquire about a couch or a desk displayed on an image. We present the equations for Facebook below after setting all variables at their mean, except for simple features and advanced features.

- Facebook shares: $\hat{x}_{2,2,k} = 2.8400 - 0.0500s_{2,2,k} - 0.0980f_{2,2,k} + 0.0090f_{2,s,k}^2$.
- Facebook comments: $\hat{x}_{2,3,k} = 2.6300 + 0.0036s_{2,3,k} - 0.1140f_{2,3,k} + 0.0100f_{2,3,k}^2$.

We tabulate the engagement values for each corresponding values for advanced features for both platforms and three engagement types in the next section. Our empirical findings reveal the nonlinear association between user engagement and advanced features. However, the tabulation helps us estimate the engagement levels at several points and linearize the association between user engagement and advanced features.

C.2 Optimization Models and Tables

In this section, we present the proofs of our Lemmas and Theorem and the supporting tables. We present the results of our computational results in tabular formats and describe the relevant extensions of our primary models. In the following three tables (Tables C.11, C.12, and C.12), we also average the simple features and include that in b.

Table C.11: g_j Values, $f_k^* = 8$, $b = 7.2900$, $\zeta = 0.0940$, $\eta = -0.0062$ (Instagram likes)

j	$g_j = b + j\zeta + j^2\eta$
0	7.2900
1	7.3778
2	7.4456
3	7.5162
4	7.5650
5	7.6050
6	7.6308
7	7.6442
8	7.6452
9	7.6360
10	7.6338

Table C.12: g_j Values, $b = 2.8400$, $\zeta = -0.0980$, $\eta = 0.0090$ (Facebook shares)

j	$g_j = b + j\zeta + j^2\eta$
0	2.8400
1	2.7519
2	2.6800
3	2.6270
4	2.5920
5	2.5750
6	2.5760
7	2.5950
8	2.6320
9	2.6870
10	2.7600
11	2.8510
12	2.9600
13	3.0870
14	3.2320
15	3.3950
16	3.5760
17	3.7750
18	3.9920

C.2.1 Proofs of Lemmas and Theorems

Proof of Lemma 3: Note that $x_k = b + \beta s_k + \zeta f_k + \eta f_k^2$. By taking derivative of x_k with respect to f_k and set equal to zero, we have $\frac{d(x_k)}{d(f_k)} = \zeta - 2|\eta| f_k = 0$. As f_k is integer and the second

Table C.13: g_j Values, $b = 2.6300$, $\zeta = -0.1140$, $\eta = 0.0100$ (Facebook comments)

j	$g_j = b + j\zeta + j^2\eta$
0	2.6300
1	2.5260
2	2.4420
3	2.3780
4	2.3340
5	2.3100
6	2.3060
7	2.3230
8	2.3580
9	2.4140
10	2.4900
11	2.5860
12	2.7020
13	2.8380
14	2.9900
15	3.1700
16	3.3660
17	3.5820
18	3.8180

derivative is negative, the result follows. $\square$

According to Lemma 3, the maximum user engagement x_k^* occurs at either $f_k^* = \lfloor \frac{\zeta_{lm}}{2|\eta_{lm}|} \rfloor = 7$ or $f_k^* = \lfloor \frac{\zeta_{lm}}{2|\eta_{lm}|} \rfloor + 1 = 8$ whichever provides maximum x_k^*. Here $f_k^* = 8$.

Note that U is the upper bound on f_k quantifying the effort required to extract advanced features. The following Lemma provide an upper bound for U.

Proof of Lemma 4: The result follows from Lemma 3 as x_k is concave function in f_k. $\square$

Proof of Lemma 5: The budget Constraint (4.1) in in Problem SMM_s can be expressed as follows:

$aq_0 + af_0 + eU + C_f \leq B$. This implies that the maximum, $f_0 = \lfloor \frac{(B - C_f - eU - aq_0)}{a} \rfloor$. $\square$

Proof of Lemma 6: Note that $x_k = b + \beta s_k + \zeta f_k + \eta f_k^2$. By taking derivative of x_k with respect to f_k and set equal to zero, we have $\frac{d(x_k)}{d(f_k)} = -|\zeta| + 2\eta f_k = 0$. As f_k is integer and the second derivative is positive, the result follows. $\square$

In case of Facebook (Shares), we use the following equation:

$x_k = b + \beta s_k + \zeta f_k + \eta f_k^2$, where b and η are positive and β and ζ are negative. Using the values from Table 4.4, we can rewrite the equation as:

$x_k = 2.8400 - 0.0500 s_k - 0.0988 f_k + 0.0090 f_k^2$.

By definition of the problem $f_k \leq u_2 = 18$.

According to Lemma 6 the minimum user engagement x_k^* occurs at period k (for Facebook (Shares)), when either $f_k^* = \lfloor \frac{|\zeta_{lm}|}{2\eta_{lm}} \rfloor = 5$ or $f_k^* = \lfloor \frac{|\zeta_{lm}|}{2\eta_{lm}} \rfloor + 1 = 6$ whichever provides minimum x_k^*. Here $x_k^* = 5$.

Proof of Lemma 7: Note that $x_k = b + \beta s_k + \zeta f_k + \eta f_k^2$. By taking derivative of x_k with respect to f_k and set equal to zero, we have $\frac{d(x_k)}{d(f_k)} = -|\zeta| + 2\eta f_k = 0$. As f_k is integer and the second derivative is positive, the result follows. $\square$

In case of Facebook (Comments), we use the following equation:

$x_k = b + \beta s_k + \zeta f_k + \eta f_k^2$, where b, η and β are positive, and ζ is negative. Using the values from Table 4.4, we can rewrite the equation as:

$x_k = 2.6300 + 0.0036 s_k - 0.1140 f_k + 0.0100 f_k^2$.

By definition of the problem $f_k \leq u_2 = 18$.

According to Lemma 7 the minimum user engagement x_k^* occurs at period k (for Facebook (Shares)), when either $f_k^* = \lfloor \frac{|\zeta_{lm}|}{2\eta_{lm}} \rfloor = 5$ or $f_k^* = \lfloor \frac{|\zeta_{lm}|}{2\eta_{lm}} \rfloor + 1 = 6$ whichever provides minimum x_k^*.

Proof of Theorem 3: Since x_k is integer, Constraints (4.9) and (4.10) linearize the nonliner Constraint (4.2) in Problem SMM_s using Boolean variables $y_{k,j}$. Since we linearize f_k as $f_k = \sum_{j=0}^{f_k^*} j y_{k,j}$, Constraints (4.4) is rewritten as Constraints (4.12) in $LSMM_s(Case1)$. Similar change is made in Constraint (4.1) that becomes Constraint (4.8) to place $\sum_{j=0}^{f_k^*} j y_{k,j}$ in place of f_k. We use our regression results to compute x_k at different values of f_k. Constraints (4.11) and (4.14) remain the same in both problems. Constraint (4.13) to $LSMM_s(Case1)$ is added since optimal solution x_k^* to $LSMM_s(Case1)$ cannot be more that f_k^*. Suppose $x_k^* = f_k^* + z$ is optimal solution in $LSMM_s(Case1)$ for some k, where z is a positive integer. We set $x_k^* = f_k^*$ in $LSMM_s(Case1)$. This change does not affect any constraint in $LSMM_s(Case1)$. But it increases the objective function value of $LSMM_s(Case1)$ since the optimal value is f_k^* for the concave function $x_k = b + \beta s_k + \zeta f_k + \eta f_k^2$ in Problem SMM_s for a fixed value of s_k (Lemma 3). This contradicts with the fact that $x_k^* = f_k^* + z$ optimal. Thus, the optimal solution x_k^* to $LSMM_s(Case1)$ cannot be more that f_k^*. Similarly, this statement is also valid for Problem SMM_s.

It has to be noted that each feasible integer solution to Problem $LSMM_s$ is also feasible to Problem $SMM_s(Case1)$ with the same objective function value Π_1. Since the optimal solution x_k^* to both problem cannot be more that f_k^*, the optimal solution to Problem $LSMM_s$ is also optimal to Problem $SMM_s(Case1)$. Thus, Problem SMM_s is equivalent to solving the linear version of Problem $LSMM_s(Case1)$. $\square$

Problem $LSMM_s(Cases2-3)$**:**

$$\textbf{Max } \Pi_s \quad = \quad E_k = \sum_{k=1}^{K} w_k x_k$$

Subject to:

$$a \textstyle\sum_{k=1}^{K} s_k + a \sum_{k=1}^{K} \sum_{j=0}^{u_2} j y_{k,j} + eU + C_f \leq B \tag{C.1}$$

$$x_k = \beta s_k + \textstyle\sum_{j=0}^{u_2} g_j y_{k,j}, \;\; \forall k \tag{C.2}$$

$$\textstyle\sum_{j=0}^{u_2} y_{k,j} = 1, \;\; \forall k \tag{C.3}$$

$$s_k \leq u_1, \quad \forall k \tag{C.4}$$

$$\textstyle\sum_{j=0}^{u_2} j y_{k,j} \leq U, \quad \forall k \tag{C.5}$$

$$U \leq u_2, \quad \forall k \tag{C.6}$$

$$s_k \geq q_k, \quad \forall k \tag{C.7}$$

$$x_k, s_k, U: \;\; \text{Integer variable } (\geq 0), \;\; \forall k \tag{C.8}$$

$$y_{k,j} \in \{0, 1\} \;\; \forall k; \;\; \forall j \tag{C.9}$$

The above formulation is similar to $LSMM_s(Case1)$. However, β, g_j, u_1, and u_2 are different for Facebook and Instagram. β and g_j are different for Facebook shares and comments, which we distinguish using m in $LSMM_f$.

The following formulation represents the combined Facebook shares and comments problem of $LSMM_s(Cases2-3)$, where $m = 2$ refers to shares and $m = 3$ denotes comments. This results in a new parameter, μ_m, where we assign weights for each engagement type. We use 0.7

Table C.14: Computational results for Instagram likes (regression parameters from Table 4.4 multiplied by 5000)

Unit effort cost e	20			30			40			50			60			70			80			90		
Content	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k
1	3	8	37740	3	8	37740	3	8	37740	3	7	37690	3	6	37640	3	6	37640	3	7	37690	3	6	37640
2	4	8	37570	4	8	37570	4	8	37570	4	8	37570	4	7	37520	4	6	37470	4	7	37520	4	7	37520
3	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	7	37860	2	7	37860
4	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	7	37860	2	7	37860	2	7	37860
5	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	7	37860	2	7	37860
6	1	8	38080	1	8	38080	1	8	38080	1	8	38080	1	8	38080	1	8	38080	1	7	38030	1	7	38030
7	2	8	37910	2	8	37910	2	7	37860	2	6	37810	2	6	37810	2	6	37810	2	7	37860	2	6	37810
8	2	8	37910	2	8	37910	2	7	37860	2	6	37810	2	6	37810	2	6	37810	2	7	37860	2	6	37810
9	4	8	37570	4	8	37570	4	8	37570	4	8	37570	4	6	37470	4	6	37470	4	7	37520	4	6	37470
10	6	8	37230	6	8	37230	6	8	37230	6	8	37230	6	8	37230	6	8	37230	6	7	37180	6	7	37180
11	1	8	38080	1	8	38080	1	8	38080	1	8	38080	1	8	38080	1	6	37980	1	7	38030	1	7	38030
12	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	7	37860	2	7	37860
13	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	8	37910	2	7	37860	2	7	37860
14	2	8	37910	2	8	37910	2	7	37860	2	6	37810	2	6	37810	2	6	37810	2	6	37810	2	6	37810
Leftover budget	100			20			0			0			0			0			0			10		
Engagement (weighted)	695946			695946			695796			695596			695366			695096			694976			694766		
U	8			8			8			8			8			8			7			7		

Table C.15: Computational results for Facebook shares (regression parameters from Table 4.4 multiplied by 5000)

Unit effort cost e	20			30			40			50			60			70			80			90		
Content	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k
1	3	0	13450	3	0	13450	3	0	13450	3	0	13450	3	0	13450	3	0	13450	3	0	13450	3	0	13450
2	4	0	13200	4	0	13200	4	0	13200	4	0	13200	4	0	13200	4	0	13200	4	0	13200	4	0	13200
3	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450
4	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	0	13700	2	0	13700	2	0	13700
5	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450
6	1	18	19700	1	18	19700	1	18	19700	1	18	19700	1	18	19700	1	0	13950	1	0	13950	1	12	14550
7	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700
8	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700
9	4	0	13200	4	0	13200	4	0	13200	4	0	13200	4	0	13200	4	0	13200	4	0	13200	4	0	13200
10	6	18	18450	6	18	18450	6	18	18450	6	18	18450	6	18	18450	6	18	18450	6	18	18450	6	18	18450
11	1	18	19700	1	12	14550	1	18	19700	1	12	14550	1	0	13950	1	18	19700	1	0	13950	1	0	13950
12	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450	2	18	19450
13	2	18	19450	2	18	19450	2	0	13700	2	0	13700	2	0	13700	2	12	14300	2	18	19450	2	0	13700
14	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700	2	0	13700
Leftover budget	60			0			60			0			60			0			20			0		
Engagement (weighted)	318730			311520			310680			303470			302630			295420			294580			287370		
U	18			18			18			18			18			18			18			18		

Table C.16: Computational results for Facebook comments (regression parameters from Table 4.4 multiplied by 5000)

Unit effort cost e	20			30			40			50			60			70			80			90		
Content	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k	s_k	f_k	x_k
1	3	0	13204	3	0	13204	3	0	13204	3	0	13204	3	0	13204	3	0	13204	3	0	13204	3	0	13204
2	4	0	13222	4	0	13222	4	0	13222	4	0	13222	4	0	13222	4	0	13222	4	0	13222	4	0	13222
3	5	18	19086	2	18	19086	5	18	19140	2	18	19086	5	18	19140	2	18	19086	5	18	19140	2	18	19086
4	2	18	19086	2	12	13536	2	0	13186	2	18	19086	2	0	13186	2	18	19086	2	0	13186	2	0	13186
5	2	18	19086	2	18	19086	2	18	19086	2	18	19086	2	18	19086	2	18	19086	2	18	19086	2	18	19086
6	1	18	19068	1	18	19068	1	18	19068	1	18	19068	1	18	19068	1	0	13168	1	0	13168	1	12	13518
7	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186
8	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186
9	4	0	13222	4	0	13222	4	0	13222	4	0	13222	4	0	13222	4	0	13222	4	0	13222	4	0	13222
10	6	18	19158	6	18	19158	6	18	19158	6	18	19158	6	18	19158	6	18	19158	6	18	19158	6	18	19158
11	1	18	19068	1	18	19068	1	18	19068	1	12	13518	1	0	13168	1	0	13168	1	18	19068	1	0	13168
12	2	18	19086	2	18	19086	2	18	19086	2	18	19086	2	18	19086	2	18	19086	2	18	19086	2	18	19086
13	2	18	19086	2	18	19086	2	18	19086	2	0	13186	2	18	19086	2	12	13536	2	0	13186	2	0	13186
14	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186	2	0	13186
Leftover budget	0			0			0			0			0			0			0			0		
Engagement (weighted)	313678			305822			305418			297562			297158			289302			288898			281042		
U	18			18			18			18			18			18			18			18		

Table C.17: Computational results for Facebook shares and comments (regression parameters from Table 4.4 multiplied by 5000)

Unit effort cost e	20				30				40				50			
Content	s_k	f_k	x_{1k}	x_{2k}	s_k	f_k	x_{1k}	x_{2k}	s_k	f_k	x_{1k}	x_{2k}	s_k	f_k	x_{1k}	x_{2k}
1	3	18	19200	19104	3	18	19200	19104	3	18	19200	19104	3	0	13450	13204
2	4	18	18950	19122	4	18	18950	19122	4	18	18950	19122	4	18	18950	19122
3	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
4	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
5	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
6	1	18	19700	19068	1	18	19700	19068	1	18	19700	19068	1	18	19700	19068
7	2	13	14900	14186	2	0	13700	13186	2	0	13700	13186	2	18	19450	19086
8	2	18	19450	19086	2	18	19450	19086	2	13	14900	14186	2	0	13700	13186
9	4	18	18950	19122	4	18	18950	19122	4	18	18950	19122	4	18	18950	19122
10	6	18	18450	19158	6	18	18450	19158	6	18	18450	19158	6	18	18450	19158
11	1	18	19700	19068	1	18	19700	19068	1	18	19700	19068	1	18	19700	19068
12	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
13	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
14	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
Leftover budget	0				80				0				80			
Engagement (weighted)	349621				348481				343826				342686			
U	18				18				18				18			

Unit effort cost e	60				70				80				90			
Content	s_k	f_k	x_{1k}	x_{2k}	s_k	f_k	x_{1k}	x_{2k}	s_k	f_k	x_{1k}	x_{2k}	s_k	f_k	x_{1k}	x_{2k}
1	3	0	13450	13204	3	0	13450	13204	3	0	13450	13204	3	0	13450	13204
2	4	18	18950	19122	4	18	18950	19122	4	18	18950	19122	4	18	18950	19122
3	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
4	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
5	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
6	1	18	19700	19068	1	18	19700	19068	1	18	19700	19068	1	18	19700	19068
7	2	13	14900	14186	2	0	13700	13186	2	0	13700	13186	2	0	13700	13186
8	2	0	13700	13186	2	18	19450	19086	2	0	13700	13186	2	0	13700	13186
9	4	18	18950	19122	4	18	18950	19122	4	18	18950	19122	4	18	18950	19122
10	6	18	18450	19158	6	18	18450	19158	6	18	18450	19158	6	18	18450	19158
11	1	18	19700	19068	1	18	19700	19068	1	18	19700	19068	1	18	19700	19068
12	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
13	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086	2	18	19450	19086
14	2	18	19450	19086	2	0	13700	13186	2	13	14900	14186	2	0	13700	13186
Leftover budget	0				80				0				80			
Engagement (weighted)	338031				336891				332236				331096			
U	18				18				18				18			

and 0.3 for shares and comments, respectively. β and g_{mj} differ for each engagement type.

Problem $LSMM_f$**:**

$$\textbf{Max}\ \Pi_f \quad = \quad E_{mk} = \sum_{m=2}^{3}\sum_{k=1}^{K}\mu_m w_k x_{mk}$$

Subject to:

$$a\sum_{k=1}^{K} s_k + a\sum_{k=1}^{K}\sum_{j=0}^{u_2} j y_{k,j} + eU + C_f \le B \tag{C.10}$$

$$x_{mk} = \beta_m s_k + \sum_{j=0}^{u_2} g_{mj} y_{k,j}, \ \ \forall k;\ \ \forall m \tag{C.11}$$

$$\sum_{j=0}^{u_2} y_{k,j} = 1, \ \ \forall k \tag{C.12}$$

$$s_k \le u_1, \quad \forall k \tag{C.13}$$

$$\sum_{j=0}^{u_2} j y_{k,j} \le U, \quad \forall k \tag{C.14}$$

$$U \le u_2, \quad \forall k \tag{C.15}$$

$$s_k \ge q_k, \quad \forall k \tag{C.16}$$

$$x_{mk}, s_k, U: \ \ \text{Integer variable}\ \ (\ge 0), \ \ \forall k;\ \ \forall m \tag{C.17}$$

$$y_{k,j} \in \{0,1\}\ \ \forall k;\ \ \forall j \tag{C.18}$$

Problem $LSMM_G$**:**

$$\textbf{Max } \Pi_g \quad = \quad E_{lmk} = \sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{k=1}^{K} z_{lm} w_k x_{lmk} \tag{C.19}$$

Subject to:

$$a\sum_{k=1}^{K} s_{lk} + a\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{j=0}^{J^l} j y_{l,k,j} + \sum_{l=1}^{L} eU_l + LC_f \leq B \tag{C.20}$$

$$x_{lmk} = \beta_{lm} s_{lk} + \sum_{l=1}^{L}\sum_{j=0}^{J^l} g_{lmj} y_{l,k,j}, \ \ \forall k; \ \ \forall m, \ \ \forall l \tag{C.21}$$

$$\sum_{j=0}^{J^l} y_{l,k,j} = 1, \ \ \forall k, \ \ \forall l \tag{C.22}$$

$$s_{lk} \leq u_l, \quad \forall k, \quad \forall l \tag{C.23}$$

$$\sum_{j=0}^{J^l} j y_{l,k,j} \leq U_l, \quad \forall; \quad \forall l \tag{C.24}$$

$$U_l \leq J^l, \quad \forall l \tag{C.25}$$

$$s_{lk} \geq q_{lk}, \quad \forall k, \quad \forall l \tag{C.26}$$

$$x_{lmk}, s_{lk}, U_l : \ \ \text{Integer variable} \ \ (\geq 0), \ \ \forall k; \ \ \forall l; \ \ \forall m \tag{C.27}$$

$$y_{l,k,j} \in \{0,1\} \ \ \forall k; \ \ \forall j; \ \ \forall l \tag{C.28}$$

The generalized formulation $LSMM_G$ is a two platform extension of $LSMM_s(Cases2-3)$, where each engagement type is denoted by both l and m for Instagram ($l = 1$) and Facebook ($l = 2$). The linearized version introduces the change in the formulation by removing the non-linear Constraint (4.18) and replacing them by Constraint sets (C.21), (C.22), and (C.24).

Table C.18: Computational results for Instagram likes, Facebook shares and comments using the linearized problem (regression parameters from Table 4.4 multiplied by 5000))

Unit effort cost	20							30						
	$s_{1,k}$	$s_{2,k}$	$f_{1,k}$	$f_{2,k}$	$x_{1,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$	$s_{1,k}$	$s_{2,k}$	$f_{1,k}$	$f_{2,k}$	$x_{1,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$
1	3	3	8	18	37740	19200	19104	3	3	8	18	37740	19200	19104
2	4	4	8	18	37570	18950	19122	4	4	8	18	37570	18950	19122
3	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
4	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
5	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
6	1	1	8	18	38080	19700	19068	1	1	8	18	38080	19700	19068
7	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
8	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
9	4	4	8	18	37570	18950	19122	4	4	8	18	37570	18950	19122
10	6	6	8	18	37230	18450	19158	6	6	8	18	37230	18450	19158
11	1	1	8	18	38080	19700	19068	1	1	8	18	38080	19700	19068
12	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
13	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
14	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
Leftover budget	2980							480						
Engagements (Total, Instagram, Facebook)	506130, 309309, 196820							506130, 309309, 196820						
U1, U2,	8,18							8,18						
Unit effort cost	40							50						
	$s_{1,k}$	$s_{2,k}$	$f_{1,k}$	$f_{2,k}$	$x_{1,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$	$s_{1,k}$	$s_{2,k}$	$f_{1,k}$	$f_{2,k}$	$x_{1,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$
1	3	3	8	18	37740	19200	19104	3	3	8	18	37740	19200	19104
2	4	4	8	18	37570	18950	19122	4	4	8	18	37570	18950	19122
3	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
4	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
5	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
6	1	1	8	18	38080	19700	19068	1	1	8	18	38080	19700	19068
7	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
8	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
9	4	4	8	18	37570	18950	19122	4	4	8	18	37570	18950	19122
10	6	6	8	18	37230	18450	19158	6	6	8	18	37230	18450	19158
11	1	1	8	18	38080	19700	19068	1	1	8	18	38080	19700	19068
12	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
13	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
14	2	2	8	18	37910	19450	19086	2	2	8	18	37910	19450	19086
Leftover budget	40							20						
Engagements (Total, Instagram, Facebook)	506130, 309309, 196820							506130, 309309, 196820						
U1, U2,	8,18							8,18						
Unit effort cost	60							70						
	$s_{1,k}$	$s_{2,k}$	$f_{1,k}$	$f_{2,k}$	$x_{1,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$	$s_{1,k}$	$s_{2,k}$	$f_{1,k}$	$f_{2,k}$	$x_{1,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$
1	3	3	6	18	37640	19200	19104	3	3	6	18	37640	19200	19104
2	4	4	6	18	37470	18950	19122	4	4	6	18	37470	18950	19122
3	2	2	8	18	37910	19450	19086	2	2	6	18	37810	19450	19086
4	2	2	8	18	37910	19450	19086	2	2	7	18	37860	19450	19086
5	2	2	8	18	37910	19450	19086	2	2	6	18	37810	19450	19086
6	1	1	8	18	38080	19700	19068	1	1	7	18	38030	19700	19068
7	2	2	6	18	37810	19450	19086	2	2	6	18	37810	19450	19086
8	2	2	6	18	37810	19450	19086	2	2	6	18	37810	19450	19086
9	4	4	6	18	37470	18950	19122	4	4	7	18	37520	18950	19122
10	6	6	8	18	37230	18450	19158	6	6	6	18	37130	18450	19158
11	1	1	8	18	38080	19700	19068	1	1	7	18	38030	19700	19068
12	2	2	8	18	37910	19450	19086	2	2	7	18	37860	19450	19086
13	2	2	8	18	37910	19450	19086	2	2	7	18	37860	19450	19086
14	2	2	6	18	37810	19450	19086	2	2	6	18	37810	19450	19086
Leftover budget	0							0						
Engagements (Total, Instagram, Facebook)	505845, 309025, 196820							505498, 308678, 196820						
U1, U2,	5,18							7,18						
Unit effort cost	80							90						
	$s_{1,k}$	$s_{2,k}$	$f_{1,k}$	$f_{2,k}$	$x_{1,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$	$s_{1,k}$	$s_{2,k}$	$f_{1,k}$	$f_{2,k}$	$x_{1,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$
1	3	3	5	18	37490	19200	19104	3	3	4	18	35940	13450	19104
2	4	4	6	18	37470	18950	19122	4	4	5	18	36870	12100	19122
3	2	2	6	18	37810	19450	19086	2	2	6	18	37210	12600	19086
4	2	2	6	18	37810	19450	19086	2	2	5	18	37210	12600	19086
5	2	2	6	18	37810	19450	19086	2	2	6	18	37210	12600	19086
6	1	1	6	18	37980	19700	19068	1	1	5	18	37380	12850	19068
7	2	2	6	18	37810	19450	19086	2	2	4	18	37210	12600	19086
8	2	2	6	18	37810	19450	19086	2	2	4	18	36110	13700	19086
9	4	4	6	18	37470	18950	19122	4	4	5	18	36870	12100	19122
10	6	6	6	18	37130	18450	19158	6	6	6	18	36530	11600	19158
11	1	1	6	18	37980	19700	19068	1	1	5	18	37380	12850	19068
12	2	2	6	18	37810	19450	19086	2	2	6	18	37210	12600	19086
13	2	2	6	18	37810	19450	19086	2	2	5	18	37210	12600	19086
14	2	2	5	18	37660	19450	19086	2	2	4	18	36110	13700	19086
Leftover budget	0							10						
Engagements (Total, Instagram, Facebook)	505178,308358,196820							504156, 307336, 196820						
U1, U2,	6,18							6,18						

Table C.19: Difference of engagement between generalized linear problem and budget allocated linearized problem

	Generalized model			Budget allocated			Difference
e	Total	U1	U2		U1	U2	
20	566852	8	18	503543	8	18	11.1685
30	565742	8	18	502910	8	18	11.1061
40	564561	8	18	500324	8	18	11.3782
50	563356	5	18	499690	8	18	11.3012
60	562394	5	18	497104	8	18	11.6093
70	561431	4	18	496471	8	18	11.5704
80	560440	4	18	493885	8	18	11.8755
90	559381	3	18	493251	8	18	11.8220