A THEORY OF SECOND-ORDER WIRELESS NETWORK OPTIMIZATION

A Dissertation

by

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ABSTRACT

We introduce a new theoretical framework for optimizing second-order behaviors of wireless networks as a core result of the dissertation. Unlike existing techniques for network utility maximization, which only considers first-order statistics, this framework models every random process by its mean and temporal variance. The inclusion of temporal variance makes this framework well-suited for modeling various wireless channels. Using this framework, we sharply characterize the second-order capacity region of wireless access networks. We also propose a simple scheduling policy and prove that it can achieve every interior point in the second-order capacity region.

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1. INTRODUCTION

Many emerging applications in the information-update system, such as industrial connected and autonomous vehicles (CAVs), and Internet-of-Things (IoT), require the real-time delivery of information. In those applications, the remote sensing problem has recently attracted significant research interests due to its critical role. In remote sensing, there are multiple sensors, i.e. Lidars in CAVs or temperature sensors of a machine industrial IoT, generating information updates about their respective surveillance fields and sending these information updates to a control center. The control center then uses its received information for real-time estimation of the current system state, so as to determine the appropriate control actions. The control center needs to be able to make accurate estimates of the system states at all times to ensure the safety and the efficiency of the system. In another words, the performance of such applications is determined by their ability to accurately estimate the real-time status of their respective information sources, such as the route condition in CAVs, or temperature of a machine in industrial IoT.

In our work, we study the problem of scheduling the transmissions of information updates when the sensors and the control center communicate over a shared wireless band. We notice two important features of remote sensing: First, because the surveillance fields evolve with time, recent information updates are much more useful than stale ones. Second, the control center may need multiple recent information updates to make an accurate estimate, which is because the sensors and the surveillance fields are subject to noises, and most estimation algorithms, even simple ones like linear extrapolation, require multiple data points in the recent past. However, most existing network performance metrics, ranging from traditional Quality-of-Service (QoS) metrics such as throughput, delay, and jitter, to emerging ones like Timely-Throughput and Age-of-Information (AoI), fail to directly capture those features of the users' estimation. Therefore, network algorithms aiming at optimizing these network performance metrics may result in poor performance for these emerging applications.

To address the need for such applications, we borrow the idea of a "confidence interval" from statistics, and introduce a new metric called Confidence-in-Estimation (CiE). In statistics, a small confidence interval means that the ground truth falls in a small range around the estimate with a pre-specified probability, and therefore implies that the estimate is accurate. We then aim to define CiE to reflect whether the network performance of wireless information-update systems leads to small confidence intervals for the end-user.

In our first work, we consider that each information source, generates information updates periodically. The CiE of an information flow only depends on the number of packets that are delivered on time in a window of the recent past. If the number of timely deliveries in this window of the recent past is below a user-specified requirement, then the resulting estimate will have a wide confidence interval, and we therefore say that this flow suffers from a Loss-of-Confidence (LoC). Our goal is to minimize the system-wide LoC in a wireless network with multiple flows, each with different user requirement and channel reliability.

Using Brownian approximation and martingale theory, we show that the problem of minimizing the system-wide LoC is equivalent to an optimization problem that involves two sets of constraints: One set of constraints are related to the average of timely deliveries of each flow, and another set of constraints are related to the temporal variance of timely deliveries. The existence of constraints about the variance of timely deliveries makes this problem significantly different from other Network-Utility-Maximization (NUM) problems that only involve constraints about the average of variables, and hence cannot be solved by most existing techniques for NUM problems.

We propose a simple online scheduling algorithm for this problem. We analytically prove that the timely deliveries under our scheduling algorithm satisfy both the constraints on the average and those on the variance in the optimization problem. We also analytically prove that our algorithm is near-optimal for the optimization problem in the sense that its performance can be made arbitrarily close to a theoretical bound.

In the second work, we extend our model to even consider the *freshness* of an information update in CiE. We propose a model where each sensor sets a threshold for the *freshness* of its information updates. At any given point of time, the instantaneous estimation accuracy for the sensor's surveillance field depends on the current *quantity* of *fresh* information updates at the control center. Compared to Age-of-Information (AoI), which is a popular metric that measures the freshness of the most recent information update, our model can provide a richer characterization by considering both the quantity and the freshness of data. We further address the challenge that sensors are located at different locations and are monitoring different fields by explicitly considering that different sensors can have different thresholds for freshness, different mappings between the quantify of fresh data and the estimation accuracy, and different channel conditions.

In order to analyze this model, we first demonstrate that the quantify of fresh information updates that the control center has at a given time can be expresses as a closed-form function involving the processes of update generations and update deliveries. We then show that, similar to first work, by applying a Brownian approximation to the update delivery process, the quantity of fresh information updates can be characterized as a random variable whose distribution only depends on the mean and temporal variance of the delivery process. Again, the dependency on temporal variances makes it infeasible to apply traditional network optimization techniques that only consider the means of delivery processes. To take temporal variances into account, we analytically establish the fundamental constraints on the means and variances of the delivery processes for all sensors, given the limitations of the wireless bandwith and channel conditions. Thus, the problem of the optimal wireless scheduling can be transformed into a constrained optimization problem of finding the optimal means and variances, subject to the constraints imposed by the wireless channels.

After finding the optimal means and variances of the delivery processes, it remains to develop a scheduling policy that actually achieves them. To this end, we also propose a simple scheduling policy and theoretically prove that its resulting means and variances are indeed the optimal ones. Thus, this scheduling policy is the one that enables the control center to have the most accurate real-time estimation. An important and surprising feature of our proposed scheduling policy is that it does not require any knowledge about the freshness of each individual information update, despite the fact that the accuracy of real-time estimation depends on data freshness.

For both works, we conduct comprehensive simulations to evaluate the performance of our proposed scheduling policies. We compare our policies against two other state-of-the-art policies, one of them is provably optimal in terms of timely-throughput, and the other achieves an approximation bound in terms of Age-of-Information. Simulation results show that our policies achieves much smaller LoC than these two policies.

Additionally, to verify the practicality of our assumptions about the confidence of information-update flows, we conduct a case study in the first work on the real-time estimation problem of linear Gaussian processes. We consider the scenario where there are multiple sensors generating noisy measurements of their monitored processes, and an estimator makes real-time estimation of all processes based on its received information. We run simulations for this estimator and evaluate its resulting mean square error. The result shows that our policy receives the best performance when compared to the other two policies, in terms of the average mean square estimate error and the 95-percentile of mean square estimate error. This result also demonstrates that the concept of CiE does capture the performance of general information-update remote-estimation problems, and provides more insights than AoI-based models in the reliability of estimate in more general cases.

In the third work, we further generalize our theory of the second-order framework. This framework consists of the second-order models, that is, the means and the temporal variances, of all random processes, including the channel qualities and packet deliveries of wireless clients. The incorporation of temporal variances enables this framework to better characterize stateful fading wireless channels, such as Gilber-Elliot channels, and emerging performance metrics.

Using this framework, we sharply characterize the second-order capacity region of wireless networks, which entails the set of means and temporal variances of packet deliveries that are feasible under the constraints of the second-order models of channel qualities. As a result, the problem of optimizing emerging performance metrics is reduced to one that finds the optimal means and temporal variances of packet deliveries within the second-order capacity region. We also propose a simple scheduling policy and show that it can achieve every interior point of the second-order capacity region.

To demonstrate the utility our framework, we apply it for an important open problem: Finding the optimal scheduling policy to minimize system-wide AoI over Gilbert-Elliot channels. We theoretically derive the closed-form expressions of the second-order models for Gilbert-Elliot channels. We also show that the AoI of each wireless client can be well-approximated by the mean and the temporal variance of its packet delivery process. We compare the system-wide AoI of our scheduling policy against other policies from recent studies on AoI minimization. Simulation results show that our policy achieves a smaller system-wide AoI. These results are especially significant when one considers that our policy is a generic second-order optimization policy, while the other policies are tailor-made to minimize the system-wide AoI.

The rest of this proposal is as following order: The details of the first work and the second work in Chapter 2 and Chapter 3 respectively. Chapter 4 is the third work. Chapter 5 reviews some related works. Finally, Chapter 6 is the conclusion and future work.

2. SCHEDULING REAL-TIME INFORMATION-UPDATE FLOWS FOR THE OPTIMAL CONFIDENCE IN ESTIMATION *

2.1 System Model

We extend the model in [3], which focuses on the short-term performance for wireless networks with homogeneous links, to address the confidence in information flows estimation in real-time wireless network where different wireless links can have different channel qualities.

We consider a real-time wireless network that serves $\mathcal{N}$ clients. Time is slotted, and the duration of one time slot is the amount of time needed by a whole bluetransmission, including all overheads such as the transmission of poll packet or ACK. Hence, the AP can transmit to at most one client at each time slot, and it has the instantaneous feedback information on whether the transmission is successful. We consider that wireless transmissions are subject to effects of shadowing, multi-path, fading, interference, etc., and different clients experience different channel qualities as they are located at different positions. Hence, we assume that each transmission for client i is successful with probability p_i.

We consider that each client is associated with a real-time information-update flow, and use flow i to indicate the flow associated with client i. Specifically, we assume that each real-time flow generates one packet periodically every τ slots, that is at time slots 1, $\tau+1$, $2\tau+1$, ... Each packet has a stringent delay bound of τ slots, and is removed from the system if it cannot be delivered before its delay bound. In other words, each packet in a real-time flow is only valid for transmission until the next packet arrives. We thereby

*Reprinted with permission from [2] D. Guo and I. Hou, "Scheduling Real-Time Information-Update Flows for the Optimal Confidence in Estimation," in IEEE Journal on Selected Areas in Communications, vol. 39, no. 5, pp. 1339-1351, May 2021, doi: 10.1109/JSAC.2021.3065093. Also, Reprinted with permission from [1] D. Guo and I. Hou, "On the Credibility of Information Flows in Real-time Wireless Networks," 2019 International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOPT), 2019, pp. 1-8, doi: 10.23919/WiOPT47501.2019.9144125.

say that τ time slots form an *interval*. Packets arrive at the system at the beginning of each interval, and have deadlines at the end of the interval.

We note that this model for real-time flows applies to many emerging wireless applications. For example, consider multi-user virtual reality (VR) or augmented reality (AR), where an AP streams VR/AR contents to multiple VR/AR headsets. All headsets play VR/AR contents at the same frame rates, and therefore they generate traffic at the same frequency. Further, as the AP should always transmit the newest VR/AR content to a headset, packets that fail to be delivered on time should be removed and replaced by newer packets. Likewise, one can also consider industrial Internet of Things (IoT), where an AP polls measurements from multiple sensors monitoring different locations. Sensors have the same sampling frequency and therefore generate traffic at the same frequency. Also, stale measurements should be dropped when a new measurement is generated.

An important feature of real-time application such as VR/AR and industrial IoT is that each flow can typically tolerate a small amount of sporadic packet losses, but is very sensitive to a burst of packet losses. For example, in industrial IoT, a controller can use various estimation techniques to estimate the value of a lost sensor reading. However, the accuracy of the estimate significantly degrades if there is a burst a packet losses. Further, it is obvious that the accuracy of the estimate only depends on the deliveries of recent sensor readings, and readings in the distant past have negligible effect on the estimation accuracy. We thereby say that the *Confidence-in-Estimation (CiE)* of one information flow relies on if its delivered packets enable the controller to make an accurate estimation.

The goal of this paper is to define and optimize the *CiE* for each information flow. To capture the aforementioned feature of real-time applications, we assume that the confidence of a real-time flow estimated status at a given point of time only depends on the packet deliveries in the *window* of past T intervals. Specifically, let $X_i(t)$ be the total number of timely-deliveries for flow i in the first t intervals. We then have $X_i(t)-X_i(t-1)=1$ if a packet is delivered to client i in interval t, and $X_i(t)-X_i(t-1)=$

0 if not. The number of timely-deliveries in the window of the last T intervals can then be represented as $X_i(t) - X_i(t-T)$, and we assume that the CiE of flow i at the end of interval t only depends on the value of $X_i(t) - X_i(t-T)$.

We assume that, to make an accurate estimate, each client i requires that there are at least $q_i T$ packets being delivered in the past T intervals, i.e., $X_i(t) - X_i(t-T) \geq q_i T$. The value of q_i depends on the context of the information flow. For example, a sensor monitoring a high-frequency signal requires a larger q_i than one that is monitoring a low-frequency signal.

Due to the unreliable nature of wireless transmissions, it is obvious that it is not possible to satisfy the requirements of all clients at all time. When the AP fails to deliver $q_i T$ packets for a client i, then the estimation of current state of client i becomes less accurate, and therefore we say that flow i *loses confidence*.

We now formally define the measure of *Loss-of-Confidence* (LoC). Suppose $X_i(t) - X_i(t-T) < q_i T$ for some i and t. Recall that every transmission for client i is successful with probability p_i. Hence, the AP would have needed to, on average, schedule $\frac{q_i T - (X_i(t) - X_i(t-T))}{p_i}$ more transmissions for client i to make $X_i(t) - X_i(t-T) = q_i T$. We therefore define the *unbiased shortage* of client i at the end of interval t as $\theta_i(t) := \max\{\frac{q_i T - (X_i(t) - X_i(t-T))}{p_i}, 0\}$. At the end of each interval t, each client i suffers from a LoC of $C(\theta_i(t))$ based on its unbiased shortage, where $C(\cdot)$ is a strictly increasing, strictly convex, and differentiable function with $C(0) = 0$ and $C'(0) = 0$.

This paper aims to evaluate the long-time average total LoC of all clients in the system, which is written as $\lim_{\mathbb{T}\to\infty} \frac{\sum_{t=T+1}^{\mathbb{T}+T} \sum_{i=1}^{N} C(\theta_i(t))}{\mathbb{T}} = \lim_{\mathbb{T}\to\infty} \frac{\sum_{t=T+1}^{\mathbb{T}+T} \sum_{i=1}^{N} C(\frac{q_i T}{p_i} - \frac{X_i(t) - X_i(t-T)}{p_i})}{\mathbb{T}}$. It also aims to propose an online scheduling policy* that minimizes the total LoC.

*An online scheduling policy is a policy that determines which packet to transmit in each slot based on all system parameters and the entire history.

2.2 The Formulation of the Optimization Problem

In this section, we derive some fundamental properties about the minimization of total LoC. We then formulate an optimization problem.

Recall that $X_i(t)$ is the total number of timely-deliveries for client i in the first t intervals. Obviously, $\{X_i(1), X_i(2), \dots\}$ is a sequence of random variables whose distribution is determined by the employed packet scheduling policy. For simplicity, we only focus on ergodic scheduling policies in this paper. Thus, the random variable $\{X_i(t) - X_i(t-T)\}$ can be modeled by a positive recurrent Markov chain. By the law of large numbers, we can define $\bar{X}_i := \lim_{t\to\infty} \frac{X_i(t)}{t}$. Further, following the central limit theorem of Markov chains [4], $\hat{X}_i := \lim_{\mathbb{T}\to\infty} \frac{X_i(\mathbb{T}) - \mathbb{T}\bar{X}_i}{\sqrt{\mathbb{T}}}$ is a Gaussian random variable with mean 0 and some finite variance, which we denote by σ_i^2, with $\sigma_i \geq 0$. Hence, we can approximate $X_i(t) - X_i(t-T)$ as a Gaussian random variable with mean $T\bar{X}_i$ and variance $T\sigma_i^2$ when T is reasonably large. Let $\Phi(x)$ represents the cumulative distribution function of a random variable under standard normal distribution, then, under this approximation, we have that the CDF of $(X_i(t) - X_i(t-T)) - T\bar{X}_i)$ is $\Phi(\frac{x}{\sqrt{\sigma_i^2 T}})$.

The long-term average total LoC can now be re-written as below:

$$\begin{aligned}
&\lim_{\mathbb{T}\to\infty} \frac{\sum_{t=T+1}^{\mathbb{T}+T} \sum_{i=1}^{N} C(\frac{q_i T}{p_i} - \frac{X_i(t) - X_i(t-T)}{p_i})}{\mathbb{T}} \\
&= \lim_{\mathbb{T}\to\infty} \sum_{i=1}^{N} E[C(\frac{q_i T}{p_i} - \frac{X_i(\mathbb{T}) - X_i(\mathbb{T} - T)}{p_i})] \\
&\approx \lim_{\mathbb{T}\to\infty} \sum_{i=1}^{N} E[C(\frac{q_i T}{p_i} - \frac{\sqrt{T}\hat{X}_i + T\bar{X}_i}{p_i})] \\
&= \sum_{i=1}^{N} \int_z C(\sqrt{\frac{\sigma_i^2 T}{p_i^2}} z - \frac{(\bar{X}_i - q_i)T}{p_i}) d\Phi(z).
\end{aligned} \tag{2.1}$$

The approximation step is from the above definitions of $\bar{X}_i$ and $\hat{X}_i$. The last step is the expectation formula under the law of the unconscious statistician.

Let $[\bar{X}_i]$ and $[\sigma_i]$ be the vectors consisting of $\bar{X}_i$ and σ_i for all $1 \le i \le N$ respectively. Then, Eq. (2.1) can be viewed as has two sets of control variables: $[\bar{X}_i]$ and $[\sigma_i]$, since their values are determined by the employed policy. Below, we derive the corresponding constraints of these two sets of variables.

We first derive the constraints on $[\bar{X}_i]$. Previous work [5] has shown that, under any work-conserving policy†, we have, for all t,

$$E[\sum_{i=1}^{N} \frac{X_i(t) - X_i(t-1)}{p_i}] = \tau - I_{\{1,2,\dots,N\}}, \tag{2.2}$$

and

$$E[\sum_{i \in S} \frac{X_i(t) - X_i(t-1)}{p_i}] \le \tau - I_S, \tag{2.3}$$

for any subset $S \subseteq \{1, 2, \dots N\}$, where I_S is called the *idle time* and has been shown to be a well-defined constant under all work-conversing policies. Therefore, we have

$$\sum_{i=1}^{N} \frac{\bar{X}_i}{p_i} = \tau - I_{\{1,2,\dots,N\}}, \tag{2.4}$$

and

$$\sum_{i \in S} \frac{\bar{X}_i}{p_i} \le \tau - I_S, \forall S \subseteq \{1, 2, \dots N\}. \tag{2.5}$$

We further assume that, similar to the total resource pooling condition, the constraint $\sum_{i \in S} \frac{\bar{X}_i}{p_i} \le \tau - I_S$ is not tight and can be ignored when S is not $\{1, 2, \dots N\}$.

Now, we derive the constraint of $[\sigma_i]$. By (2.2), the sequence of random variables $\{\sum_{i=1}^{N} \frac{X_i(t)}{p_i} - t(\tau - I_{\{1,2,\dots,N\}}) | t = 1, 2, \dots\}$ is a martingale. By the martingale central limit theorem [6], $\hat{X}_{TOT} := \lim_{\mathbb{T}\to\infty} \frac{\sum_{i=1}^{N} \frac{X_i(\mathbb{T})}{p_i} - \mathbb{T}(\tau - I_{\{1,2,\dots N\}})}{\sqrt{\mathbb{T}}} = \lim_{\mathbb{T}\to\infty} \frac{\sum_{i=1}^{N} \frac{X_i(\mathbb{T})}{p_i} - \mathbb{T}(\sum_{i=1}^{N} \frac{\bar{X}_i}{p_i})}{\sqrt{\mathbb{T}}}$

†A scheduling policy is called work-conserving if it always schedules a transmission when there is at least one packet available for transmission.

is a Gaussian random variable with mean 0, and its variance is

$$\sigma_{TOT}^2 := \lim_{\mathbb{T}\to\infty} \frac{1}{\mathbb{T}}[\sum_{t=1}^{\mathbb{T}}(\sum_{i=1}^{N} \frac{X_i(t) - X_i(t-1)}{p_i})^2] \\ - (\tau - I_{\{1,2,\dots N\}})^2, \tag{2.6}$$

whose value depends on the employed scheduling policy.

Recall that $\hat{X}_i := \lim_{\mathbb{T}\to\infty} \frac{X_i(\mathbb{T})-\mathbb{T}\bar{X}_i}{\sqrt{\mathbb{T}}}$ is a Gaussian random variable with variance σ_i^2. Hence, we have $\hat{X}_{TOT} = \sum_{i=1}^{N} \frac{\hat{X}_i}{p_i}$, and the variance of $\frac{\hat{X}_i}{p_i}$ is $(\frac{\sigma_i}{p_i})^2$. By Cauchy-Schwarz Inequality, we have:

$$\begin{aligned}
&(\sum_{i=1}^{N} \frac{\sigma_i}{p_i})^2 = (\sum_{i=1}^{N} \sqrt{Var(\frac{\hat{X}_i}{p_i})})^2 \\
=&\sum_{i=1}^{N} Var(\frac{\hat{X}_i}{p_i}) + 2\sum_{l=1}^{N}\sum_{m=l+1}^{N} \sqrt{Var(\frac{\hat{X}_l}{p_l})Var(\frac{\hat{X}_m}{p_m})} \\
\geq&\sum_{i=1}^{N} Var(\frac{\hat{X}_i}{p_i}) + 2\sum_{l=1}^{N}\sum_{m=l+1}^{N} Cov(\frac{\hat{X}_l}{p_l}, \frac{\hat{X}_m}{p_m}) \\
=&Var(\sum_{i=1}^{N} \frac{\hat{X}_i}{p_i}) = \sigma_{TOT}^2,
\end{aligned} \tag{2.7}$$

where $Var(X)$ denotes the variance of X and $Cov(X,Y)$ denotes the covariance.

Although the value of σ_{TOT} may be different for different scheduling policies, we first consider the special case of minimizing the total LoC when σ_{TOT} is given and fixed. By

(2.1), (2.4), and (2.7), the optimization problem can be written as:

$$Min \quad L = \sum_{i=1}^{N} \int_z C(\sqrt{\frac{\sigma_i^2 T}{p_i^2}} z - \frac{(\bar{X}_i - q_i)T}{p_i}) d\Phi(z) \tag{2.8}$$

$$s.t. \sum_{i=1}^{N} \frac{\bar{X}_i}{p_i} = \tau - I_{\{1,2,\ldots N\}} \tag{2.9}$$

$$\sum_{i=1}^{N} \frac{\sigma_i}{p_i} \geq \sigma_{TOT}. \tag{2.10}$$

Theorem 1. *Let* $[\bar{X}_i{}^*]$ *and* $[\sigma_i^*]$ *be the optimal solution to (2.8) – (2.10). Then* $\bar{X}_i{}^* = (\frac{\tau - I_{\{1,2,\ldots N\}}}{N} - \sum_{j=1}^{N} \frac{q_j}{Np_j} + \frac{q_i}{p_i})p_i$, *and* $\sigma_i^* = \frac{\sigma_{TOT}}{N} p_i$, *for all* $1 \leq i \leq N$.

Proof. Since $C(\cdot)$ is a convex function, we have:

$$\begin{aligned} L &= \sum_{i=1}^{N} \int_z C(\sqrt{\frac{\sigma_i^2 T}{p_i^2}} z - \frac{(\bar{X}_i - q_i)T}{p_i}) d\Phi(z) \\ &\geq N \int_z C(\frac{1}{N} \sum_{i=1}^{N} (\sqrt{\frac{\sigma_i^2 T}{p_i^2}} z - \frac{(\bar{X}_i - q_i)T}{p_i})) d\Phi(z) \\ &\geq N \int_z C(\frac{1}{N} (\sigma_{TOT} \sqrt{T} z - \sum_{i=1}^{N} \frac{(\bar{X}_i - q_i)T}{p_i})) d\Phi(z), \end{aligned}$$

with equality occurs when $\frac{\bar{X}_i^*}{p_i} - \frac{q_i}{p_i} = \frac{\bar{X}_j^*}{p_j} - \frac{q_j}{p_j}$ and $\frac{\sigma_i^*}{p_i} = \frac{\sigma_j^*}{p_j}$ for any $i, j \in \{1, 2, \ldots N\}$. By (2.9) and (2.10), we have $\bar{X}_i{}^* = (\frac{\tau - I_{\{1,2,\ldots N\}}}{N} - \sum_{i=1}^{N} \frac{q_i}{Np_i} + \frac{q_i}{p_i})p_i$ and $\sigma_i^* = \frac{\sigma_{TOT}}{N} p_i$. □

Theorem 1 establishes the optimal $\{\bar{X}_i\}$ and $\{\sigma_i\}$ that minimizes the total LoC when σ_{TOT} is given and fixed. Obviously, smaller σ_{TOT} leads to smaller total LoC. Therefore, we seek to solve the optimization problem below, which aims to minimizing σ_{TOT} while satisfying the results of Theorem 1:

$$Min \quad \sigma^2_{TOT} := \lim_{\mathbb{T}\to\infty} \frac{1}{\mathbb{T}}[\sum_{t=1}^{\mathbb{T}}(\sum_{i=1}^{N} \frac{X_i(t) - X_i(t-1)}{p_i})^2] - (\tau - I_{\{1,2,...N\}})^2 \tag{2.11}$$

$$s.t. \bar{X}_i = \bar{X}_i^*, \forall 1 \le i \le N \tag{2.12}$$

$$\sigma_i = \frac{\sigma_{TOT}}{N} p_i, \quad \forall 1 \le i \le N, \tag{2.13}$$

where $\bar{X}_i^* := (\frac{\tau - I_{\{1,2,...N\}}}{N} - \sum_{j=1}^{N} \frac{q_j}{p_j N} + \frac{q_i}{p_i})p_i$.

We note that the problem (2.11) – (2.13) involves both a constraint on the average of $X_i(t)$ (2.12) and a constraint on the variance of $X_i(t)$ (2.13) for each i. Most existing studies on network utility maximization (NUM) problem only addresses constraints on the average of decision variables, and therefore cannot be applied to solve (2.11) – (2.13). In fact, no stationary randomized policies can optimally solve (2.11) – (2.13). In the following sections, we will establish the surprising result that there exists a simple online scheduling policy that is near-optimal for the problem (2.11) – (2.13).

2.3 An online scheduling policy

In this section, we propose a simple online scheduling policy for the problem (2.11) – (2.13). We first provide a brief outline of the construction of our algorithm. First, we remove the constraint on variance (2.13) and focus on the following optimization problem:

$$Min \quad \lim_{\mathbb{T}\to\infty} \frac{1}{\mathbb{T}}[\sum_{t=1}^{\mathbb{T}}(\sum_{i=1}^{N} \frac{X_i(t) - X_i(t-1)}{p_i})^2] - (\tau - I_{\{1,2,...N\}})^2 \tag{2.14}$$

$$s.t. \bar{X}_i = \bar{X}_i^*, \forall 1 \le i \le N. \tag{2.15}$$

Obviously, this optimization problem is a lower bound to the original problem (2.11) – (2.13). It is also a standard NUM problem that only involves a constraint on the average of $X_i(t)$ for each i. We can therefore derive a near-optimal online scheduling algorithm using the Drift-Plus-Penalty approach [7]. We further demonstrate the surprising result that, due to the specific choice of our Lyapunov function, our algorithm also satisfies the constraint on variance (2.13). Therefore, our algorithm is near-optimal to the original problem (2.11) – (2.13).

We now introduce some notations that are necessary for the design and analysis of our algorithm. Let $d_i(t) := \frac{\bar{X}_i^* t}{p_i} - \frac{X_i(t)}{p_i}$ be the *deficit* of client i in interval t. Obviously, we have $\bar{X}_i := \lim_{t\to\infty} \frac{X_i(t)}{t} = \bar{X}_i^*$ if and only if $\lim_{t\to\infty} \frac{d_i(t)}{t} = 0$. We also define $\Delta d_i(t) := d_i(t+1) - d_i(t) = \frac{\bar{X}_i^*}{p_i} - \frac{X_i(t+1)-X_i(t)}{p_i}$ and $D(t) := \frac{\sum_{i=1}^N d_i(t)}{N}$.

We consider the Lyapunov function $L(t) = \frac{1}{2}\sum_{i=1}^N [d_i(t) - D(t)]^2$. The drift of the Lyapunov function is $\Delta L(t) := E[L(t+1) - L(t)|[d_i(t)]]$.

Given $[d_i(t)]$, we have, under any scheduling policy,

$$\begin{aligned}
&\Delta L(t) = E[L(t+1) - L(t)] \\
=&E\big[\frac{1}{2}\sum_{i=1}^N \big(d_i(t+1) - D(t+1)\big)^2 - \frac{1}{2}\sum_{i=1}^N \big(d_i(t) - D(t)\big)^2\big] \\
=&E\big[\frac{1}{2}\sum_{i=1}^N \big(d_i(t) - D(t) + \Delta d_i(t) - \frac{\sum_{i=1}^N \Delta d_i(t)}{N}\big)^2\big] \\
&- E\big[\frac{1}{2}\sum_{i=1}^N \big(d_i(t) - D(t)\big)^2\big] \\
=&E\big[\frac{1}{2}\sum_{i=1}^N \big(\Delta d_i(t) - \frac{\sum_{i=1}^N \Delta d_i(t)}{N}\big)^2\big] \\
&+ \sum_{i=1}^N E\big[\Delta d_i(t)\big]\big(d_i(t) - D(t)\big) \\
&- E\big[\frac{\sum_{i=1}^N \Delta d_i(t)}{N}\big]\sum_{i=1}^N \big(d_i(t) - D(t)\big) \\
\le&\beta + \sum_{i=1}^N E\big[\Delta d_i(t)\big]\big(d_i(t) - D(t)\big),
\end{aligned} \tag{2.16}$$

where β is a bounded positive number. The last inequality holds since $\Delta d_i(t)$ is bounded by $\frac{\bar{X}_i^*-1}{p_i} \leq \Delta d_i(t) \leq \frac{\bar{X}_i^*}{p_i}$ and $\sum_{i=1}^{N} d_i(t) = ND(t)$.

Our scheduling algorithm is based on the Drift-Plus-Penalty approach [7]. Let

$$\begin{aligned} B(t) := &\sum_{i=1}^{N} E\big[\Delta d_i(t)\big]\big(d_i(t) - D(t)\big) \\ &+ \epsilon E[(\sum_{i=1}^{N} \frac{X_i(t+1) - X_i(t)}{p_i})^2], \end{aligned} \tag{2.17}$$

where ϵ is a positive number whose value can be arbitrary determined by the system designer. We then have

$$\Delta L(t) + \epsilon E[(\sum_{i=1}^{N} \frac{X_i(t+1) - X_i(t)}{p_i})^2] \leq \beta + B(t). \tag{2.18}$$

We aim to design an online scheduling algorithm that minimizes $B(t)$. Note that the value of $B(t)$ depends on the scheduling decisions on all time slots within the interval t, which consists of τ time slots. Minimizing an objective function over a finite horizon of τ time slots typically requires the usage of dynamic programming. However, we will show that there exists a simple online scheduling algorithm that minimizes $B(t)$.

Our algorithm is called the *Minimum-Drift-and-Variance-First* (MDVF) policy. Under the MDVF policy, the AP calculates the value of $e_i(t) := \epsilon \frac{1}{p_i} - d_i(t)$ at the beginning of each interval t. In each time slot within the interval, the AP finds the undelivered packet with the smallest $e_i(t)$ and transmits that packet, as long as there is at least one packet to be transmitted. Alg. 1 provides a detailed description of the algorithm, where we streamline some of the steps to simplify the implementation.

We now show that the MDVF policy indeed minimizes $B(t)$.

Lemma 1. *The MDVF policy minimizes $B(t)$.*

Proof. We prove this lemma by induction. First, we consider the optimal scheduling

Algorithm 1: The MDVF Policy

```
Initialization: t = 0, d_i = 0, ∀i;
while each new interval do
    for each i do
        e_i = ϵ 1/p_i − d_i;
        d_i = d_i + X̄*_i/p_i;
    end
    Sort all flows such that e_1(t) ≤ e_2(t) ≤ ...;
    i = 1; for each time slot do
        Transmit packet i;
        if transmission is successful then
            d_i = d_i − 1/p_i;
            i = i + 1;
        end
    end
    t = t + 1;
end
```

decision in the last time slot of the interval. At this time, some packets have already been delivered in the previous $\tau - 1$ slots, and we use V to denote the set of clients whose packets have already been delivered. As this is the last time slot of the interval, the scheduling decision of the AP only consists of choosing one client $u \notin V$ and transmitting its packet. Given V and u, we will calculate the value of $\sum_{i=1}^{N} E\left[\Delta d_i(t)\right]\left(d_i(t) - D(t)\right) + \epsilon E[(\sum_{i=1}^{N} \frac{X_i(t+1) - X_i(t)}{p_i})^2]$.

For this chosen client u, its packet will be delivered, that is, $X_u(t+1) - X_u(t) = 1$, with probability p_u, and $X_u(t+1) - X_u(t) = 0$, with probability $1 - p_u$. Hence, we have $E[\Delta d_u(t)] = \frac{\bar{X}_u - p_u}{p_u}$.

On the other hand, for each client $i \in V$, its packet has already been delivered. We have $X_v(t) - X_v(t-1) = 1$ and $E[\Delta d_i(t)] = \frac{\bar{X}_i - 1}{p_i}$.

Finally, for each client $i \notin V \cup \{u\}$, its packet will not be delivered, and we have $X_i(t) - X_i(t-1) = 0$ and $E[\Delta d_i(t)] = \frac{\bar{X}_i}{p_i}$.

We now have, given V and u,

$$\begin{aligned}
&\sum_{i=1}^{N} E\left[\Delta d_i(t)\right][d_i(t)-D(t)] + \epsilon E[(\sum_{i=1}^{N}\frac{X_i(t+1)-X_i(t)}{p_i})^2] \\
&= \frac{\bar{X}_u - p_u}{p_u}[d_u(t)-D(t)] + \sum_{i\in V}\frac{\bar{X}_i - 1}{p_i}[d_i(t)-D(t)] \\
&\quad + \sum_{i\notin V\cup\{u\}}\frac{\bar{X}_i}{p_i}[d_i(t)-D(t)] + \epsilon[p_u\big(\sum_{i\in V}\frac{1}{p_i}+\frac{1}{p_u}\big)^2 + (1-p_u)\big(\sum_{i\in V}\frac{1}{p_i}\big)^2] \\
&= \epsilon\frac{1}{p_u} - d_u(t) + \lambda(V), \qquad (2.19)
\end{aligned}$$

where $\lambda(V) := D(t) + \sum_{i=1}^{N}\frac{\bar{X}_i}{p_i}[d_i(t)-D(t)] - \sum_{i\in V}\frac{1}{p_i}[d_i(t)-D(t)] + \epsilon[\big(\sum_{i\in V}\frac{1}{p_i}\big)^2 + 2\big(\sum_{i\in V}\frac{1}{p_i}\big)]$ is the same regardless of the choice of u. Therefore, it is clear that an optimal scheduling algorithm that minimizes $B(t)$ will schedule the undelivered packet u with the smallest $\epsilon\frac{1}{p_u} - d_u(t)$ in the last time slot.

Now, assume that, starting from the $(s+1)$-th time slot in an interval, scheduling the undelivered packet with the smallest $\epsilon\frac{1}{p_u} - d_u(t)$ in each of the remaining time slot is optimal. We will show that, even in the s-th time slot, scheduling the undelivered packet with the smallest $\epsilon\frac{1}{p_u} - d_u(t)$ is optimal.

We prove this claim by contradiction. Let u^* be the undelivered packet with the smallest $\epsilon\frac{1}{p_u} - d_u(t)$ in time slot s. If the claim is false, then the optimal scheduling algorithm, which we denote by $\mathbb{A}$, would schedule another undelivered packet $u' \neq u^*$ in time slot s, and the value of $B(t)$ under $\mathbb{A}$ is strictly smaller than any policy that schedules u^* in the s-th time slot. By the induction hypothesis, $\mathbb{A}$ begins to schedule the undelivered packet with the smallest $\epsilon\frac{1}{p_u} - d_u(t)$ starting from the $(s+1)$-th time slot. As u^* is not scheduled by $\mathbb{A}$ is the s-th time slot, $\mathbb{A}$ needs to schedule u^* in the $(s+1)$-th time slot. In summary, $\mathbb{A}$ schedules u' in the s-th time slot, and u^* in the $(s+1)$-th time slot.

Now, we can construct another algorithm $\mathbb{B}$ by simply swapping the transmissions in the s-th time slot and the $(s+1)$-th time slot. In other words, $\mathbb{B}$ schedules u^* in the s-th time slot, u' in the $(s+1)$-th time slot, and then follows $\mathbb{A}$ starting from the $(s+2)$-th time slot. Obviously, the value of $B(t)$ under $\mathbb{A}$ and $\mathbb{B}$ is the same, which results in a contradiction.

We have established that, even in the s-th time slot, scheduling the undelivered packet with the smallest $\epsilon\frac{1}{p_u} - d_u(t)$ is optimal. By induction, scheduling the undelivered packet with the smallest $\epsilon\frac{1}{p_u} - d_u(t)$ in each time slot is optimal, and MDVF minimizes $B(t)$. $\square$

2.4 Performance Analysis of the MDVF policy

We now study the performance of the MDVF policy. We will demonstrate the surprising result that the MDVF policy satisfies both constraints on mean (2.12) and variance (2.13), and the value of σ^2_{TOT} under the MDVF policy can be made arbitrary close to a lower bound. Throughout this section, we use $\cdot|\eta$ to denote the value of $\cdot$ under a scheduling policy η. For example, $\Delta L(t)|$MDVF denotes the value of $\Delta L(t)$ under the MDVF policy.

We first establish the following property.

Theorem 2. *Under the MDVF policy, the Markov process with state vector $\{d_i(t)-D(t)\}$ is positive recurrent.*

Proof. We prove this theorem by establishing an upper bound of $\Delta L(t)|$MDVF. To simplify notations, we let Ω be the policy that schedules the undelivered packet with the maximum value of $d_i(t)$. We also sort all clients such that $d_1(t) \geq d_2(t) \geq \cdots \geq d_N(t)$. Then Ω will only transmit a packet for client i if, for each $j < i$, the packet for flow j has already been delivered. This is equivalent to the largest-debt-first policy in [5], and we have, for all $1 \le j \le N$:

$$\sum_{i=1}^{j} E[\Delta d_i(t)]|\Omega = \sum_{i=1}^{j} \frac{\bar{X}_i^*}{p_i} - E[\sum_{i=1}^{j} \frac{X_i(t+1) - X_i(t)}{p_i}]|\Omega$$
$$= \sum_{i=1}^{j} \frac{\bar{X}_i^*}{p_i} - (\tau - I_{\{1,2,\dots,j\}}). \tag{2.20}$$

By (2.4), we have $\sum_{i=1}^{N} E[\Delta d_i(t)]|\Omega = 0$. Further, as we assume that (2.5) is not tight when $S \neq \{1, 2, \dots, N\}$, there exists a positive number $\delta > 0$ such that $\sum_{i=1}^{j} E[\Delta d_i(t)]|\Omega \le -\delta$ for all $1 \le j \le N-1$. We now have

$$\sum_{i=1}^{N} E[\Delta d_i(t)]\big(d_i(t) - D(t)\big)|\Omega$$
$$= \sum_{i=1}^{N} E[\Delta d_i(t)]\big(d_i(t) - d_{i+1}(t) + d_{i+1}(t)$$
$$- d_{i+2}(t) + \dots - d_N(t) + d_N(t) - D(t)\big)|\Omega$$
$$= \sum_{i=1}^{N} E[\Delta d_i(t)]\big(d_N(t) - D(t)\big)|\Omega$$
$$+ \sum_{i=1}^{j} \sum_{j=1}^{N-1} E[\Delta d_i(t)]\big(d_j(t) - d_{j+1}(t)\big)|\Omega$$
$$\le -\delta \sum_{j=1}^{N-1} \big(d_j(t) - d_{j+1}(t)\big) = -\delta\big(d_1(t) - d_N(t)\big). \tag{2.21}$$

Next, we study $\Delta L(t)|\text{MDVF}$. By Lemma 1, the MDVF policy minimizes $B(t)$. Hence, we have

$$\begin{aligned}
&\Delta L(t)|\text{MDVF} + \epsilon E[(\sum_{i=1}^{N} \frac{X_i(t+1) - X_i(t)}{p_i})^2]|\text{MDVF} \\
\leq& \beta + B(t)|\text{MDVF} \qquad \big(\text{By (2.18) }\big) \\
\leq& \beta + B(t)|\Omega \\
\leq& \beta + \epsilon E[(\sum_{i=1}^{N} \frac{X_i(t+1) - X_i(t)}{p_i})^2]|\Omega \\
&- \delta\big(d_1(t) - d_N(t)\big) \qquad \big(\text{By (2.17) and (2.21)}\big)
\end{aligned} \tag{2.22}$$

Since $0 \leq X_i(t+1) - X_i(t) \leq 1$, there exists some constant M such that

$$\Delta L(t)|\text{MDVF} \leq -\delta\big(d_1(t) - d_N(t)\big) + M. \tag{2.23}$$

Recall that we have sorted all clients such that $d_1(t) \geq d_2(t) \geq \dots$. Hence, $\big(d_1(t) - d_N(t)\big) \geq 0$ and $\big(d_1(t) - d_N(t)\big) \geq |d_i(t) - D(t)|$, for all i. We have

$$\Delta L(t)|\text{MDVF} < -\delta, \text{if } |d_i(t) - D(t)| > \tfrac{M}{\delta} + 1, \text{ for some } i,$$

and

$$\Delta L(t)|\text{MDVF} \leq M, \text{ otherwise.} \tag{2.24}$$

By the Foster-Lyapunov Theorem, the Markov process with state vector $\{d_i(t) - D(t)\}$ is positive recurrent. □

Now we are able to show that the MDVF policy satisfies both constraints (2.12) and (2.13).

Corollary 1. *$\bar{X}_i|MDVF = \bar{X}_i^*$ and $\sigma_i|MDVF = \frac{\sigma_{TOT}|MDVF}{N} p_i$, $\forall i$.*

Proof. Recall that $d_i(t) := \frac{\bar{X}_i^* t}{p_i} - \frac{X_i(t)}{p_i}$ and $D(t) := \frac{\sum_{i=1}^N d_i(t)}{N}$. By (2.4), we have:

$$\lim_{\mathbb{T}\to\infty} \frac{D(\mathbb{T})|\text{MDVF}}{\mathbb{T}} = \lim_{\mathbb{T}\to\infty} \frac{\sum_{i=1}^N d_i(\mathbb{T})|\text{MDVF}}{N\mathbb{T}}$$
$$=\frac{1}{N}\sum_{i=1}^N \lim_{\mathbb{T}\to\infty} \frac{\mathbb{T}\bar{X}_i^* - X_i(\mathbb{T})|\text{MDVF}}{p_i\mathbb{T}}$$
$$=\frac{1}{N}\sum_{i=1}^N \frac{\bar{X}_i^*}{p_i} - \frac{1}{N}\sum_{i=1}^N \frac{\bar{X}_i(\mathbb{T})|\text{MDVF}}{p_i}$$
$$=\frac{\tau - I_{\{1,2,\dots N\}}}{N} - \frac{\tau - I_{\{1,2,\dots N\}}}{N} = 0. \tag{2.25}$$

By Theorem 2, the vector $\{d_i(t) - D(t)\}$|MDVF converges to a steady state distribution as $t \to \infty$. Hence, both $\lim_{\mathbb{T}\to\infty} \frac{d_i(\mathbb{T})-D(\mathbb{T})}{\mathbb{T}}$|MDVF and $\lim_{\mathbb{T}\to\infty} \frac{d_i(\mathbb{T})-D(\mathbb{T})}{\sqrt{\mathbb{T}}}$|MDVF converge to 0 in probability. We then have

$$\lim_{\mathbb{T}\to\infty} \frac{d_i(\mathbb{T})|\text{MDVF}}{\mathbb{T}} = \frac{\bar{X}_i^*}{p_i} - \frac{\bar{X}_i|\text{MDVF}}{p_i}$$
$$= \lim_{\mathbb{T}\to\infty} \frac{D(\mathbb{T})|\text{MDVF}}{\mathbb{T}} = 0, \tag{2.26}$$

and hence $\bar{X}_i$|MDVF$= \bar{X}_i^*$.

Next, we study σ_i|MDVF. Recall that σ_i^2 is the variance of $\hat{X}_i := \lim_{\mathbb{T}\to\infty} \frac{X_i(\mathbb{T})-\mathbb{T}\bar{X}_i}{\sqrt{\mathbb{T}}}$. We then have:

$$\lim_{\mathbb{T}\to\infty} \frac{d_i(\mathbb{T})|\text{MDVF}}{\sqrt{\mathbb{T}}} = \lim_{\mathbb{T}\to\infty} \frac{\mathbb{T}\bar{X}_i^* - X_i(\mathbb{T})|\text{MDVF}}{p_i\sqrt{\mathbb{T}}}$$
$$= -\frac{\hat{X}_i|\text{MDVF}}{p_i},$$

since $\bar{X}_i$|MDVF$= \bar{X}_i^*$. This shows that the variance of $\lim_{\mathbb{T}\to\infty} \frac{d_i(\mathbb{T})|\text{MDVF}}{\sqrt{\mathbb{T}}}$ is $\frac{\sigma_i^2|\text{MDVF}}{p_i^2}$.

Also, recall that σ^2_{TOT} is the variance of $\hat{X}_{TOT} = \sum_{i=1}^{N} \frac{\hat{X}_i}{p_i}$. We have

$$\lim_{\mathbb{T}\to\infty} \frac{D(\mathbb{T})|\text{MDVF}}{\sqrt{\mathbb{T}}} = \lim_{\mathbb{T}\to\infty} \frac{\sum_{i=1}^{N} d_i(\mathbb{T})|\text{MDVF}}{N\sqrt{\mathbb{T}}}$$
$$= \lim_{\mathbb{T}\to\infty} \sum_{i=1}^{N} \frac{\mathbb{T}\bar{X}_i^* - X_i(\mathbb{T})|\text{MDVF}}{Np_i\sqrt{\mathbb{T}}} = -\sum_{i=1}^{N} \frac{\hat{X}_i|\text{MDVF}}{Np_i},$$

and the variance of $\lim_{\mathbb{T}\to\infty} \frac{D(\mathbb{T})|\text{MDVF}}{\sqrt{\mathbb{T}}}$ is $\frac{\sigma^2_{TOT}|\text{MDVF}}{N^2}$. As $\lim_{\mathbb{T}\to\infty} \frac{d_i(\mathbb{T})-D(\mathbb{T})}{\sqrt{\mathbb{T}}}|\text{MDVF}$ converges to 0 in probability, we have $\sigma_i|\text{MDVF} = \frac{\sigma_{TOT}|\text{MDVF}}{N} p_i$. □

We have shown that the MDVF policy satisfies both constraints (2.12) and (2.13). We now show that the value of $\sigma^2_{TOT}|\text{MDVF}$ can be made arbitrarily close to a theoretical lower bound.

Consider the problem (2.14) – (2.15), which ignores the constraint on variance (2.13). Since this problem only involves a constraint on mean, there exists a stationary randomized policy that is optimal, which we denote by ω. Obviously, $\sigma^2_{TOT}|\omega$ is a lower bound of the problem (2.11) – (2.13). We have the following theorem.

Theorem 3. $\sigma^2_{TOT}|MDVF \le \sigma^2_{TOT}|\omega + \frac{\beta}{\epsilon}$.

Proof. Since ω is a stationary randomized policy that satisfies (2.15), we have $E[\Delta d_i(t)]|\omega = 0$, for all i and t. By (2.17), we have

$$B(t)|\omega = \epsilon E[(\sum_{i=1}^{N} \frac{X_i(t+1) - X_i(t)}{p_i})^2]|\omega = \epsilon\sigma^2_{TOT}|\omega.$$

Now, recall that the MDVF policy minimizes $B(t)$. Hence, for every t, we have

$$\Delta L(t)|\text{MDVF} + \epsilon E[(\sum_{i=1}^{N} \frac{X_i(t) - X_i(t-1)}{p_i})^2]|\text{MDVF}$$
$$\le B(t)|\text{MDVF} + \beta$$
$$\le B(t)|\omega + \beta = \epsilon\sigma^2_{TOT}|\omega + \beta.$$

Summing the above inequality over $t = 1$ to $t = \mathbb{T}$, and then divide both sides by $\mathbb{T}$ yields

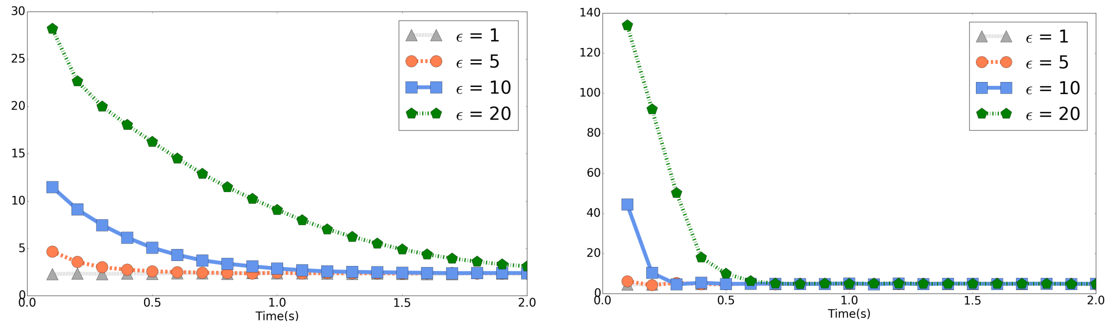
$$\begin{aligned}&\frac{E[L(\mathbb{T}+1)] - E[L(0)]}{\mathbb{T}}|\text{MDVF} + \epsilon\sigma^2_{TOT}|\text{MDVF}\\ \leq&\epsilon\sigma^2_{TOT}|\omega + \beta.\end{aligned} \tag{2.27}$$

By Theorem 2, we have $\lim_{\mathbb{T}\to\infty} \frac{E[L(\mathbb{T}+1)]-E[L(0)]}{\mathbb{T}}|\text{MDVF}= 0$, and hence $\sigma^2_{TOT}|\text{MDVF} \leq \sigma^2_{TOT}|\omega + \frac{\beta}{\epsilon}$. □

We note that Theorem 3 holds for all ϵ, which is a constant that can be arbitrarily chosen by the system designer. By choosing a large ϵ, one can make $\sigma^2_{TOT}|\text{MDVF}$ arbitrarily close to the lower bound $\sigma^2_{TOT}|\omega$. Combining Theorem 1 that gives the form of optimal solutions and Corollary 1 that shows the MDVF policy satisfies both constraints (2.12) and (2.13), the MDVF policy solves the optimization problem (2.11), (2.12) and (2.13).

2.5 Simulation Results

We present our simulation results in this section. We have implemented and tested our policy and two other state-of-the-art policies in ns-2. All simulations are conducted using the 802.11 MAC protocol with 54Mbps data rate. Simulations show that the time needed to transmit a packet and to receive an ACK is about $0.5ms$. The duration of an interval is chosen to be $10ms$, or, equivalently, 20 time slots. We evaluate the simulation for LoC in two convex functions: one is chosen to be $C(\theta_i(t)) := (\frac{\theta_i(t)}{T})^2$, which we call the *quadratic LoC function*, and the other is $C(\theta_i(t)) := e^{(\frac{\theta_i(t)}{T})} - 1$, which we call the *exponential LoC function*. We note that both functions normalize $\theta_i(t)$ by T. Recall that $\theta_i(t)$ is the unbiased shortage accumulated in last T intervals. Hence, $\frac{\theta_i(t)}{T}$ can be thought of as the *average* unbiased shortage occurred in the last T intervals. By normalization, we are able to compare the LoC across different T. All results presented in this section are the average of 1000 runs.



(a) The high-timely-throughput system

(b) The low-timely-throughput system

Figure 2.1: The convergence of $e_{max}(t) - e_{min}(t)$ in (a) The high-timely-throughput system. (b) The low-timely-throughput system.

We compare our MDVF policy against two other policies. The first policy is the largest debt first (LDF) policy in [5, 8]. In each interval t, the LDF policy sorts all clients in descending order of $q_i t - X_i(t)$, and transmits packets according to this ordering. It has been shown that LDF guarantees to deliver a long-term average timely-throughput of q_i to each client i, as long as it is feasible to do so. The second policy is a Max-Weight type of policy that aims to reduce the total age-of-information (AoI) in the network while guaranteeing some average timely-throughput policy [9]. We call this policy MW-AoI. Although the problem of minimizing AoI remains an open problem, it has been shown that the MW-AoI policy is 4-optimal in terms of AoI.

As for the network topology, we consider two different settings. In the first setting, there are 12 wireless clients. The channel reliability of client i is set to be $p_i = 0.9 - 0.05i$. We set $q_i = 0.85$ for the first 6 clients and $q_i = 0.75$ for the last 6 clients. We call this setting the *high-timely-throughput system*. In the second setting, there are 18 clients with $p_i = 1 - 0.05i$. We set $q_i = 0.35$ for all 18 clients. We call this setting the *low-timely-throughput system.*

2.5.1 The Impact of ϵ

Our MDVF policy makes scheduling decisions based on the value of $e_i(t) := \epsilon \frac{1}{p_i} - d_i(t)$ for each flow i, where ϵ is a parameter determined by the system. Theorem. 3 has shown that $\sigma^2_{TOT}|\text{MDVF} \leq \sigma^2_{TOT}|\omega + \frac{\beta}{\epsilon}$. Therefore, larger ϵ leads to better steady-state performance. On the other hand, [10], [11] and [12] have shown that larger ϵ may lead to longer convergence time. In this section, we investigate the convergence speed of the MDVF policy under different values of ϵ.

Recall that our MDVF policy sorts all flows by their $e_i(t)$ and schedules packets according to the ordering in each interval t. Hence, when the system reaches steady-state, all flows should have roughly the same $e_i(t)$. Based on this observation, we evaluate the convergence speed of the MDVF policy as follows: In each simulation run and at each interval t, we find the flow with the largest $e_i(t) =: e_{max}(t)$ and the flow with the smallest $e_i(t) =: e_{min}(t)$. We then use $e_{max}(t) - e_{min}(t)$ as the indicator of convergence. Obviously, a small value of $e_{max}(t) - e_{min}(t)$ implies that the values of $e_i(t)$ are roughly the same for all flows. We then calculate the average of $e_{max}(t) - e_{min}(t)$ over 1000 simulation runs for all t.

Simulation results for different values of ϵ and for both the low-timely-throughput system and the high-timely-throughput system are shown in Fig. 2.1. Not surprisingly, it can be easily observed that, while $e_{max}(t) - e_{min}(t)$ converges to a small value for all settings, larger ϵ leads to longer convergence time. It can also be observed that the convergence speed of the setting with $\epsilon = 5$ is reasonably fast. By setting $\epsilon = 5$, both the high-timly-throughput system and the low-timely-throughput system converge in less than 0.5 second. Hence, in the sequel, we choose $\epsilon = 5$ for our MDVF policy.

2.5.2 The Approximation Accuracy in T

Throughout the paper, we assume the CLT approximation of Markov chain that $X_i(t) - X_i(t-T)$ can be approximated as a Gaussian random variable with mean $T\bar{X}_i$

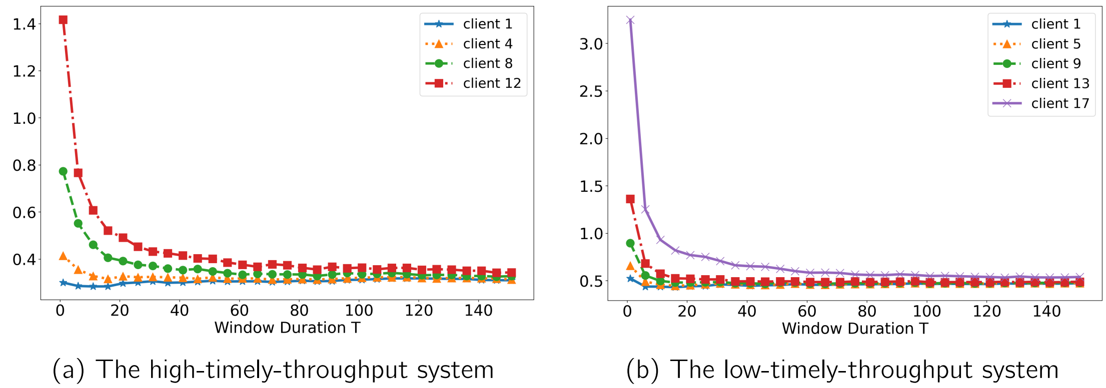
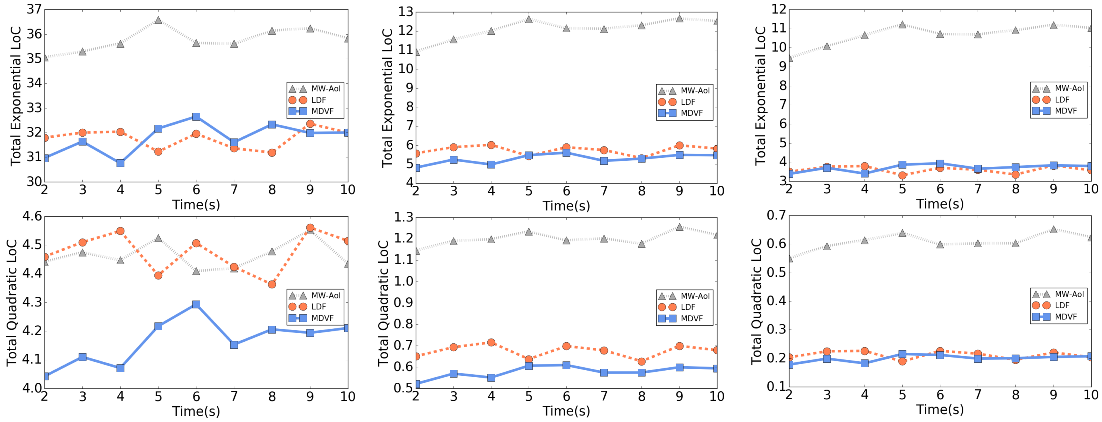


Figure 2.2: The convergence of $\frac{\sigma_i}{p_i}$ in (a) The high-timely-throughput system. (b) The low-timely-throughput system.

and variance $T\sigma_i^2$ when T reasonably large as the duration of the window. Hence, in this section, we evaluate how large T needs to be for this approximation under our MDVF policy.

Recall the Corollary. 1, $\frac{\sigma_i|MDVF}{p_i}$ should have roughly the same value as $\frac{\sigma_{TOT}|MDVF}{N}$. Although $\sigma_{TOT}|MDVF$ is a near-optimal variable determined by the value of ϵ, the values of $\frac{\sigma_i}{p_i}$ should converge across all clients to a same value when ϵ is fixed. Therefore, we design simulations in both the high-timely-throughput system and the low-timely-throughput system as follows: In each simulation run, t is set to be 500, and we obtain $X_i(500+T) - X_i(500)$ for each client i, where T is set to be increasing in increments of 5, i.e.$1, 6, 11, 16, \ldots, 151$. Consequently, we collect 1000 samples over all simulation runs, then calculate the value of $\frac{\sigma_i}{p_i}$ for all T.

Results are shown in Fig. 2.2. We plot the curve of client $1, 4, 8$ and 12 for the high-timely-throughput system in Fig. 2.2a, and the curve of client $1, 5, 9, 13$ and 17 for the low-timely-throughput system in Fig. 2.2b. The results show that, when T becomes larger, the values of $\frac{\sigma_i}{p_i}$ converge to the same value across clients. It can also be observed, the convergence speed is reasonably fast that, values converge from dramatic gaps into reasonably small gaps among clients after $T = 20$, and they keep converging slowly and



(a) $T = 10$ (b) $T = 20$ (c) $T = 30$

Figure 2.3: Two LoC Functions in the past second of high-timely-throughput system: (a) $T = 10$. (b) $T = 20$. (c) $T = 30$.

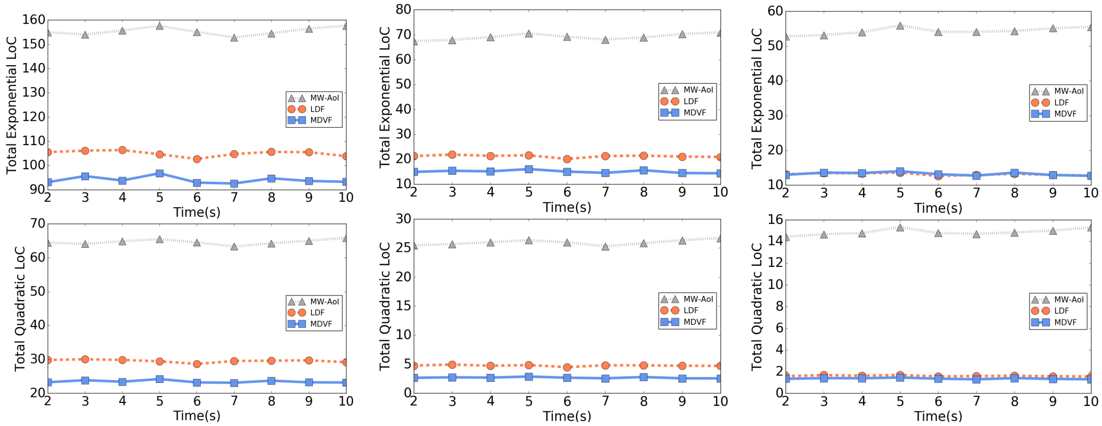
stably along with T.

2.5.3 Performance Comparison

We now present our simulation results that evaluate the LoC performance of the three policies, namely, our MDVF policy, the LDF policy, and the MW-AoI policy. We set $\epsilon = 5$ and test the three cases for $T = 10$, $T = 20$, and $T = 30$. For each simulation run, we record the total LoC incurred in the past second for up to 10 seconds.

Simulation results of the LoC in two system are shown in Fig. 2.3 and Fig. 2.4. Since LoC can only be defined after the system has run for T intervals, the first data point is at time 2 second, which is the total LoC incurred between time 1 second and 2 second. The figures clearly show that our MDVF policy achieves the smallest LoC in all settings, including both the high-timely-throughput system and the low-timely-throughput system, both the quadratic LoC function and the exponential LoC function, and the three different choices of T.

A very surprising result is that the MW-AoI policy has higher LoC than the LDF policy, even though the LDF policy only considers long-term average timely-throughput while



(a) $T = 10$ (b) $T = 20$ (c) $T = 30$

Figure 2.4: Two LoC Functions in the past second of low-timely-throughput system: (a) $T = 10$. (b) $T = 20$. (c) $T = 30$.

the MW-AoI policy considers short-term fluctuations in the form of Age-of-Information. The reason is that the MW-AoI policy focuses on optimizing AoI, which only depends on the time of the most recent packet delivery. However, many estimation techniques require more than the most recent data to make an accurate estimation. Even basic techniques like linear extrapolation needs at least two data points to make an estimate. This simulation result highlights that AoI may fail to completely capture the reliability of estimation. On the other hand, the LDF policy only aims to optimize the long-term average timely-throughputs and ignores temporal variance. This leads it to also have suboptimal total LoC.

Some interesting observations can be made by comparing the performance of the MDVF policy between different values of T. We note that the LoC of the MDVF policy decreases as T becomes larger. Under our settings, the computed value of $\bar{X}_i^*$ is always larger than q_i for each i. Therefore, we have $\frac{\theta_i(t)}{T} \to 0$ as $T \to \infty$. Our simulation results indeed demonstrate such trends.

2.6 A Case Study of Real-Time State Estimation

An important motivation of this work is emerging applications that require real-time state estimation, such as industrial IoT and VR. From an end user's perspective, the perceived performance is the user's ability to make accurate estimation. In order to demonstrate the practical value of our proposed metric, LoC, and our proposed policy, MDVF, this section studies the problem of sensing and estimating several independent linear Gaussian processes, where the performance of a flow is determined by the mean square error (MSE) of the resulting estimation.

2.6.1 Overview of the Sensing and Estimation Problem

Consider a system with one estimator and N wireless sensors. Each sensor is monitoring an independent linear Gaussian process. We number the sensors and stochastic process so that sensor i is monitoring process i. Further, we denote $z_{i,t}$ as the value of process i in interval t. The stochastic process i evolves according to the recursion:

$$z_{i,t+1} = z_{i,t} + w_{i,t}, \tag{2.28}$$

where $\{w_{i,1}, w_{i,2}, \dots\}$ is a sequence of i.i.d Gaussian random variables with mean 0 and variance W_i. We also call $w_{i,t}$ the Process Noise (PN).

In each interval t, each sensor i obtains a noisy measurement of the value of process i. The value of the measurement is denoted by $m_{i,t}$, and we assume that:

$$m_{i,t} = z_{i,t} + r_{i,t}, \tag{2.29}$$

where $\{r_{i,1}, r_{i,2} \dots\}$ is a sequence of i.i.d Gaussian random variables with mean 0 and variance R_i. We call $m_{i,t}$ the actual measurement or observation under noise, and $r_{i,t}$ the Measurement Noise (MN).

The network model is the same as that in Section 2.1. In each interval t, each sensor i generates a packet containing the value of $m_{i,t}$ and the timestamp t. The packet is discarded either when it is successfully delivered to the estimator or when the sensor generates a newer packet. The estimator, which is also the AP, schedules all transmissions. Thus, the estimator has access to the value of $m_{i,t}$ if and only if a packet is delivered for sensor i in interval t.

The goal of the estimator is to find the best estimate of the current value $z_{i,t}$ of each process i based on all the packets that it has received so far. Let $\mathcal{M}_i^t$ be the set of sensor readings, along with their timestamps that have been delivered to the estimator on or before interval t. Let $\hat{z}_{i,t}$ be the best estimate of $z_{i,t}$ and $\Sigma_{i,t}$ be the Mean Square Error (MSE) of the best estimate, with observation up to interval t. We then have:

$$\hat{z}_{i,t} = E[z_{i,t}|\mathcal{M}_i^t] \quad (2.30)$$

$$\Sigma_{i,t} = E[(z_{i,t} - \hat{z}_{i,t})^2|\mathcal{M}_i^t]. \quad (2.31)$$

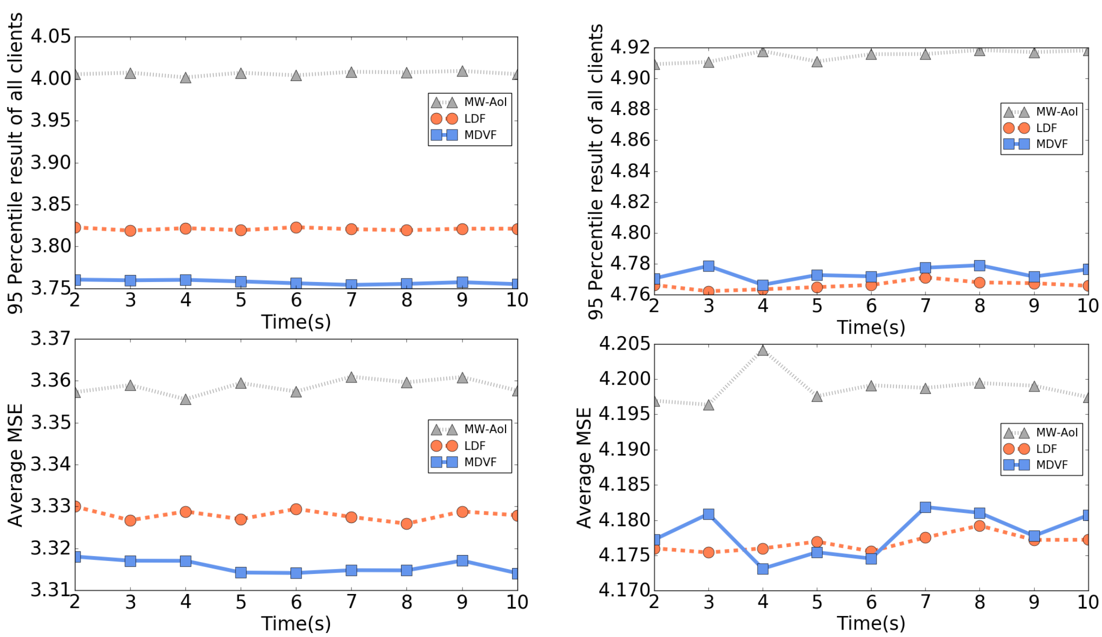
When all packets are successfully delivered on time, it is well-known that Kalman Filter [13][14][15], a recursive algorithm that calculates $\hat{z}_{i,t}$ and $\Sigma_{i,t}$ simultaneously, yields the best estimate of the underlying Gaussian linear processes. In our system, some packets may be dropped due to deadline violation, which leads to some missing samples. This scenario has been discussed in [16] and [17], where a variation of Kalman Filter has been proposed and proved to be optimal. Alg. 2 summarizes the variation of Kalman Filter.

2.6.2 Simulation of the Estimation Problem

MSE captures the error variance that occurred in the estimate process. Thus it shows the accuracy of the estimate. We design the simulation to see the performance of MSE of three policies. For all policies, we collect the average MSE and 95 percentile MSE of all clients in the past second for 10 seconds in total. The simulation result is based on the average of 1000 runs.

Algorithm 2: Kalman Filter Recursion Rule with Missing Samples

Result: The $\hat{z}_{i,t}$ and $\Sigma_{i,t}$ based on $\mathcal{M}_i^t$.
Initialization: $\hat{z}_{i,t}$, $\Sigma_{i,t}$, when $t = 0$;
while *each new interval* **do**
  for *each* i **do**
    if $m_{i,t+1}$ *is not delivered* **then**
      $\hat{z}_{i,t+1} = \hat{z}_{i,t}$;
      $\Sigma_{i,t+1} = \Sigma_{i,t} + W_i$;
    else
      $\hat{z}_{i,t+1} = \hat{z}_{i,t} + \frac{\Sigma_{i,t+1}(m_{i,t+1} - \hat{z}_{i,t})}{(\Sigma_{i,t+1} + R_i)}$;
      $\Sigma_{i,t+1} = \frac{(\Sigma_{i,t} + W_i)R_i}{(\Sigma_{i,t} + W_i + R_i)}$;
    end
  end
  $t = t + 1$;
end



(a) The 12-process system (b) The 18-process system

Figure 2.5: The average MSE and 95 percentile MSE result.

An important challenge of this simulation is to make the MSEs of different processes comparable to each other. In this simulation, we set $W_i = q_i = p_i$ for each process and $R_i = 20$ for all i. This setting is chosen based on the following two reasons. First, suppose each sensor i delivers one packet every $\frac{1}{q_i}$ intervals periodically, and therefore delivers $q_i T$ packets in every T intervals, then it can be shown that all processes have the same MSE. Second, consider the case that each sensor i delivers one packet every $\frac{1}{q_i - \delta p_i}$ intervals. In this case, the unbiased shortage of all sensors are δT. It can be shown that, under this case, all processes still have the same steady-state MSE. In summary, setting $W_i = q_i = p_i$ for each process and $R_i = 20$ for all i ensures that the MSEs between different processes are comparable. The exact calculations for the MSEs are shown in Appendix A.

We consider two different systems in our simulations. The first system has 12 processes and is called the 12-process system. The values of p_i are $\{0.5, 0.47, 0.45, 0.43, 0.4, 0.37, 0.35, 0.33, 0.3, 0.27, 0.25, 0.23\}$. The second system has 18 processes and is called the 18-process system. The values of p_i are $\{0.3, 0.29, 0.28, 0.27, 0.26, 0.25, 0.24, 0.23, 0.22, 0.21, 0.20, 0.19, 0.18, 0.17, 0.16, 0.15, 0.14, 0.13\}$.

The simulation results are shown in Fig 2.5. In the 12-process system, all three policies have roughly the same average MSE, but our MDVF policy has a smaller 95-percentile MSE. In the 18-process system, our MDVF policy and the LDF policy have almost identical performance, and both of them perform better than the MW-AoI policy, both in terms of average MSE and 95-percentile MSE. In our network model, whenever a process delivers a packet of measurement data, the AoI of that process drops to zero. However, since the measurement is noisy, the delivery of one single packet is not sufficient to make an accurate estimation. Instead, the estimator needs to have multiple recent measurements to make an accurate estimation. This is why our MDVF policy performs better than the MW-AoI policy.

This simulation result demonstrates that our MDVF policy indeed provides superior performance for real-time estimation applications. It also suggests that AoI-based solutions may not be sufficient to capture the reliability of estimation when considering many estimate techniques.

2.7 Conclusion

We have studied the problem of minimizing the total Loss-of-Confidence (LoC) in real-time wireless networks, where the LoC of each flow only depends on the timely deliveries in a window of the recent past. We have shown that, unlike most existing network utility maximization (NUM) problem, the problem of minimizing total LoC requires the precise control of the temporal variance of timely deliveries. To solve this problem, we have proposed a simple online algorithm called the MDVF policy, and have proved that the MDVF policy is near-optimal. Simulation results have demonstrated that the MDVF policy outperforms other state-of-the-art policies. Further, we have studied the application of real-time estimation of multiple independent linear Gaussian processes, where an estimator aims to make the best estimate of the current states based on all the measurements that it has received. We evaluate the performance of our policy and others by their resulting estimation error. Simulation results show that our MDVF policy achieves both the smallest average estimation error as well as the smallest 95-percentile of estimation error. This case study suggests that AoI solutions may fail to capture the performance for real-time remote estimation problems.

3. OPTIMAL WIRELESS SCHEDULING FOR REMOTE SENSING THROUGH BROWNIAN APPROXIMATION*

3.1 System Model

We consider the following network model: there is one Access Point (AP) and multiple flows, numbered as $1, 2, 3, \dots, N$, each of which is monitoring an independent and time-varying stochastic field. Time is slotted and denoted by $t = 1, 2, 3, \dots$. Each flow i generates one time-stamped information update about its monitored field every m_i slots. The AP schedules all transmissions. When the AP schedules a flow i to transmit, the AP first sends a POLL packet to flow i, and, upon receiving the POLL packet, flow i sends one of its information updates to the AP. The duration of a time slot is hence chosen to be sufficient for the transmission of one POLL packet and one information update, along with any necessary overheads. The AP then uses all the information updates that it has ever received to estimate the current status of each monitored field. We further consider the effects of shadowing, multi-paths, fading, and interference by assuming that each transmission for flow i is successful, that is, a status update is received after sending a POLL packet, with probability p_i.

Since the AP needs to make real-time estimation about each stochastic field, the performance of the network should be measured by the accuracy of the estimation. We need a model to express the accuracy of the estimation in terms of network behaviors. Our model is based on two observations of most estimation problems: First, recent information is much more useful than stale information; Second, the more recent information that the AP has, the more accurate its estimation can be. Hence, we model the accuracy of the estimation by assuming that it depends on the number of recent information updates

*Reprinted with permission from [18] D. Guo, P. -C. Hsieh and I. -H. Hou, "Optimal Wireless Scheduling for Remote Sensing through Brownian Approximation," IEEE INFOCOM 2021 - IEEE Conference on Computer Communications, 2021, pp. 1-10, doi: 10.1109/INFOCOM42981.2021.9488785.

that the AP has received. Compared to Age-of-Information (AoI), which measures the performance based on the freshness of the most recent data, our model offers a richer characterization as it considers both the freshness of data and the quantity of fresh data.

Specifically, we assume that each information update generated by flow i is only *useful* to the AP's estimation algorithm for T_i time slots. Afterwards, the information update becomes *stale* and is no longer useful. Thus, at time slot t, only information updates generated after time slot $t - T_i$ are useful. We use $U_i(t)$ to denote the number of useful packets that the AP has received from flow i at time t. For example, Fig. 3.1 illustrates the packet arrivals and deliveries histogram of a flow with $T_i = 15$. At time 40, only information updates generated after time 25 are useful, and hence we have $U_i(40) = 2$. Note that while packet 1 was delivered after time 25, it was generated before time 25 and hence is not useful at time 40.

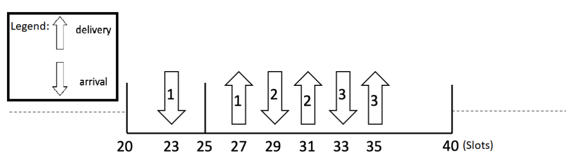


Figure 3.1: An Example for Useful Packets and Deliveries.

In order to make an accurate estimation of the stochastic field of flow i at time t, the AP needs to have a sufficient number of useful information updates. This requirement is described by a threshold q_i, and we say that the AP needs at least $q_i T_i$ useful information updates, that is, $U_i(t) \geq q_i T_i$, to make an accurate estimation. If $U_i(t) < q_i T_i$ at some time t, then the estimation is inaccurate and results in a large confidence interval. In this case, we say that the AP suffers from a *Loss-of-Confidence* (LoC) of $C_i(q_i T_i - U_i(t))$ at time t, where $C_i(\cdot)$ is a strictly increasing, convex and differentiable function over $[0, +\infty)$ with $C_i(0) = 0$ and $C_i'(0) = 0$. The goal of this paper is to minimize the long-time average LoC for the entire system of all flows, which can be written as $\lim_{k\to\infty} \frac{\sum_{i=1}^{N}\sum_{t=T_i+1}^{k+T_i} C_i(q_iT_i-U_i(t))}{k}$.

The optimization of total average LoC consists of two parts: First, the AP decides which flow to schedule in each time slot; Second, upon receiving a POLL message, the flow decides which information update to respond. For the second part, it can be shown that the Last-In-First-Out (LIFO) strategy, where the flow always responds with the newest undelivered information update, is optimal. Intuitively, the newest information update is the one that will remain useful for the longest time, and hence sending it is optimal. Recent work [19] has shown that LIFO-type strategy is optimal or near-optimal for AoI-related metrics in the queueing system under different service time. While LoC is not AoI-related, similar arguments can be used to establish the optimality of LIFO for LoC.

3.2 Fundamental Properties for LIFO Systems

To optimize the long-term average LoC problem, the first challenge is to model the behavior of the useful delivery. In this section, we derive a closed form expression for $U_i(t)$. This derivation is built on the assumption of LIFO strategy when the flow selects the information update in its buffer to transmit.

Let $a_i(t)$ be the indicator function that flow i generates a new information update at time t, and $x_i(t)$ be the indicator function that flow i successfully transmits a packet at time t. Recall that flow i generates a new information update every m_i time slots. If flow i generates the first information update at time o_i with $0 \leq o_i < m_i$, then we have $a_i(t) = 1$ if and only if $t - o_i$ is a multiple of m_i. If flow i is scheduled to transmit at time t, then we have $x_i(t) = 1$ with probability p_i, since the channel reliability of flow i is p_i. Let $A_i(t) = \sum_{\tau=0}^{t} a_i(t)$ and $X_i(t) = \sum_{\tau=0}^{t} x_i(t)$ be the accumulated number of arrivals and deliveries, respectively.

We also define $u_i(\tau, t)$ be the indicator function that flow i successfully delivers an information update at time τ that will remain useful until at least time t. For example, in Fig. 3.1, there are deliveries at times 27, 31, and 35, but the delivery at time 27 will not be useful at time 40. Thus, we have $u_i(27, 40) = 0$ and $u_i(31, 40) = u_i(35, 40) = 1$. By this definition of $u_i(\tau, t)$, we have $U_i(t) = \sum_{\tau=t-T_i+1}^{t} u_i(\tau, t)$.

Theorem 4. *For any $t > T_i$,*

$$U_i(t) = \sum_{\tau=t-T_i+1}^{t} x_i(\tau) - \sup_{t-T_i+1\le s\le t} [\sum_{\tau=t-T_i+1}^{s} x_i(\tau) - \sum_{\tau=t-T_i+1}^{s} a_i(\tau)]^+ \tag{3.1}$$

Proof. We first show that:

$$\sum_{\tau=t-T_i+1}^{d} u_i(\tau, t) = \sum_{\tau=t-T_i+1}^{d} x_i(\tau) - \sup_{t-T_i+1\le s\le d} [\sum_{\tau=t-T_i+1}^{s} x_i(\tau) - \sum_{\tau=t-T_i+1}^{s} a_i(\tau)]^+, \tag{3.2}$$

for any $t - T_i + 1 \le d \le t$ by induction.

First, consider the case $d = t - T_i + 1$. Any updates generated before time d will become stale by time t. Hence, flow i can deliver an update at time d that will remain useful at time t, and hence have $u_i(d, t) = 1$, only if both of the following conditions are satisfied: flow i generates an update at time d, that is, $a_i(d) = 1$, and flow i delivers an update at time d, that is, $x_i(d) = 1$. Hence, When $d = t - T_i + 1$, (3.2) holds.

Next, suppose (3.2) holds when $d = k$, we then consider the case when $d = k+1$. At time $k+1$, $u_i(k+1, t) = 1$ only if the following two conditions are satisfied: First, there is one successful delivery at time $k+1$, that is $x_i(k+1) = 1$; Second, there is at least one undelivered update that will be useful at time t. Since only information updates after time

$t - T_i$ will be useful at time t, the number of useful updates that flow i has generated on or before time $k+1$ is $\sum_{\tau=t-T_i+1}^{k+1} a_i(\tau)$. Before time $k+1$, flow i has delivered $\sum_{\tau=t-T_i+1}^{k} u_i(\tau, t)$ information updates. Hence, the number of undelivered updates that will be useful at time t is $\sum_{\tau=t-T_i+1}^{k+1} a_i(\tau) - \sum_{\tau=t-T_i+1}^{k} u_i(\tau, t)$. In summary, we have $u_i(k+1, t) = \min\{x_i(k), \sum_{\tau=t-T_i+1}^{k+1} a_i(\tau) - \sum_{\tau=t-T_i+1}^{k} u_i(\tau, t)\}$.

We now derive $\sum_{\tau=t-T_i+1}^{k+1} u_i(\tau, t)$ from the induction hypothesis.

$$
\begin{aligned}
&\sum_{\tau=t-T_i+1}^{k+1} u_i(\tau, t) = \sum_{\tau=t-T_i+1}^{k} u_i(\tau, t) + u_i(k+1, t) \\
&= \min\{x_i(k+1) + \sum_{\tau=t-T_i+1}^{k} u_i(\tau, t), \sum_{\tau=t-T_i+1}^{k+1} a_i(\tau)\} \\
&= \min\{\sum_{\tau=t-T_i+1}^{k+1} x_i(\tau) - \sup_{t-T_i+1 \le s \le k} [\sum_{\tau=t-T_i+1}^{s} x_i(\tau) - \sum_{\tau=t-T_i+1}^{s} a_i(\tau)]^+, \sum_{\tau=t-T_i+1}^{k+1} a_i(\tau)\} \\
&= \sum_{\tau=t-T_i+1}^{k+1} x_i(\tau) - \max\{\sup_{t-T_i+1 \le s \le k} [\sum_{\tau=t-T_i+1}^{s} x_i(\tau) \\
&\quad - \sum_{\tau=t-T_i+1}^{s} a_i(\tau)]^+, \sum_{\tau=t-T_i+1}^{k+1} x_i(\tau) - \sum_{\tau=t-T_i+1}^{k+1} a_i(\tau)\} \\
&= \sum_{\tau=t-T_i+1}^{k+1} x_i(\tau) - \sup_{t-T_i+1 \le s \le k+1} [\sum_{\tau=t-T_i+1}^{s} x_i(\tau - \sum_{\tau=t-T_i+1}^{s} a_i(\tau)]^+ \qquad (3.3)
\end{aligned}
$$

Hence, by induction, (3.2) holds for any $t - T_i + 1 \le d \le t$. Since $U_i(t) = \sum_{\tau=t-T_i+1}^{t} u_i(\tau, t)$, the theorem holds.

□

3.3 Reflected Brownian Motion Approximation

Thm. 4 has shown that $U_i(t)$ can be explicitly expressed as a function of the update arrival and delivery processes, $\{a_i(\tau)\}$ and $\{x_i(\tau)\}$. In this section, we further show that, if the employed scheduling policy is ergodic, then $U_i(t)$ can be approximated by a random variable whose distribution can be expressed in closed-form.

We first study the approximation of the accumulated number of update deliveries, $X_i(t) := \sum_{\tau=1}^{t} x_i(\tau)$. Under any ergodic scheduling policy, the delivery process $\{x_i(1), x_i(2), \dots\}$ can be modeled as a positive recurrent Markov chain with finite states. By the Law of Large Numbers, the limit $\bar{X}_i := \lim_{t\to\infty} \frac{X_i(t)}{t}$ exists. Further, by the central limit theorem of Markov chains [4], $\hat{X}_i := \lim_{t\to\infty} \frac{X_i(t)-t\bar{X}_i}{\sqrt{t}}$ is a Gaussian random variable with mean 0 and some finite variance, which we denote by σ_i^2 with $\sigma_i \geq 0$. Hence, we can approximate $X_i(t) - X_i(t - T_i) = \sum_{\tau=t-T_i+1}^{t} x_i(\tau)$ as a Gaussain random variable with mean $T_i\bar{X}_i$ and variance $T_i\sigma_i^2$ for any sufficiently large T_i. Such an approximation is called a Brownian motion process, and we denote it by $X_i(t) \approx BM(\bar{X}_i, \sigma_i^2)$.

Next, we consider the random process $Y_i(t) := A_i(t) - X_i(t)$. Recall that $A_i(t)$ is the accumulated number of update arrivals and that flow i generates one update every m_i slots. Thus, we have $\lfloor \frac{T_i}{m_i} \rfloor \leq A_i(t) - A_i(t-T_i) \leq \lceil \frac{T_i}{m_i} \rceil$, for any t and T_i. $Y_i(t) - Y_i(t-T_i) = [A_i(t) - A_i(t-T_i)] - [X_i(t) - X_i(t-T_i)]$ can then be approximated by a Gaussian random variable with mean $T_i(\frac{1}{m_i} - \bar{X}_i)$ and variance $T_i\sigma_i^2$ for any sufficiently large T_i. We express this approximation by saying $Y_i(t) \approx BM(\frac{1}{m_i} - \bar{X}_i, \sigma_i^2)$.

From Thm. 4, we have $U_i(t) = [A_i(t) - A_i(t - T_i)] - [Y_i(t) - Y_i(t - T_i)] + \sup_{t-T_i+1\leq s\leq t}[Y_i(s) - Y_i(t - T_i)]^+$. When we fix d and apply the approximation $Y_i(t) \approx BM(\frac{1}{m_i} - \bar{X}_i, \sigma_i^2)$, the random process $Y_i(t + d) - Y_i(d)$ can still be approximated by $BM(\frac{1}{m_i} - \bar{X}_i, \sigma_i^2)$, and the random process $Q_i(t) := Y_i(t + d) - Y_i(d) - \sup_{0\leq s\leq t}[Y_i(s + d) - Y_i(d)]^+$ is called a reflected Brownian process and is denoted by $RBM(\frac{1}{m_i} - \bar{X}_i, \sigma_i^2)$.

When $\bar{X}_i > \frac{1}{m_i}$, $Q_i(t)$ has a stationary distribution of an exponential variable with mean $\frac{\sigma_i^2}{2(\bar{X}_i - \frac{1}{m_i})}$ [20], and we say $Q_i(t) \sim EXP(\frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2})$. When $\bar{X}_i < \frac{1}{m_i}$, Chen and Yao [21] propose to approximate $Q_i(t)$ by a Brownian motion process $Q_i(t) \approx BM(\frac{1}{m_i} - \bar{X}_i, \sigma_i^2)$. In this case, when t is fixed, $Q_i(t)$ is approximated by a Gaussian random variable with mean $T_i(\frac{1}{m_i} - \bar{X}_i)$ and variance $T_i\sigma_i^2$, denoted by $\mathcal{N}(T_i(\frac{1}{m_i} - \bar{X}_i), T_i\sigma_i^2)$.

We note that $U_i(t) \sim [A_i(t) - A_i(t - T_i)] - Q_i(T_i)$. Thus, we can approximate $U_i(t)$

as the following:

$$U_i(t) \approx \begin{cases} \frac{T_i}{m_i} - EXP(\frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2}), & if \quad \bar{X}_i > \frac{1}{m_i} \\ \frac{T_i}{m_i} - \mathcal{N}(T_i(\frac{1}{m_i} - \bar{X}_i), T_i\sigma_i^2), & if \quad \bar{X}_i < \frac{1}{m_i} \end{cases} \tag{3.4}$$

3.4 Optimization Problem Formulation

Section 3.3 has shown that the distribution of $U_i(t)$ can be approximated by a random variable whose distribution depends on the mean and variance of the delivery process, that is, $\bar{X}_i$ and σ_i. Thus, the problem of minimizing the total long-term average LoC can be viewed as an optimization problem of choosing the optimal $[\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_N]$ and $[\sigma_1, \sigma_2, \ldots, \sigma_N]$. In this section, we first establish the fundamental constraints of $[\bar{X}_i]$ and $[\sigma_i]$. We then formulate the problem as an optimization problem and discuss finding the optimal $[\bar{X}_i]$ and $[\sigma_i]$.

3.4.1 System Constraints

We first discuss the constraints on $[\bar{X}_i]$. Hou and Kumar [5] has shown that, under any work-conserving policy that schedules a transmission in each time slot, we have for all t:

$$E[\sum_{i=1}^{N} \frac{X_i(t) - X_i(t-1)}{p_i}] = 1. \tag{3.5}$$

Thus, we have, under any work-conserving and ergodic scheduling policies,

$$\sum_{i=1}^{N} \frac{\bar{X}_i}{p_i} = 1. \tag{3.6}$$

.

Next, we derive the constraint for σ_i^2.

By (3.5), the sequence of $\{\sum_{i=1}^{N} \frac{X_i(t)}{p_i} - t | t = 1, 2, \dots\}$ is a martingale. By the martingale central limit theorem [6], $\hat{X}_{TOT} := \lim_{k\to\infty} \frac{\sum_{i=1}^{N} \frac{X_i(k)}{p_i} - k}{\sqrt{k}}$ is a Gaussian random variable with mean 0 and variance

$$\sigma^2_{[\bar{X}_i]} := \lim_{k\to\infty} \frac{1}{k} [\sum_{t=1}^{k} (\sum_{i=1}^{N} \frac{X_i(t) - X_i(t-1)}{p_i})^2] - 1. \tag{3.7}$$

Suppose the AP schedules a transmission for flow j in slot t, then $X_j(t) - X_j(t-1)$ equals 1 with probability p_j and equals 0 with probability $1-p_j$. Further, $X_i(t)-X_i(t-1) = 0$ for all other flows $i \neq j$. Hence, $\sum_{i=1}^{N} \frac{X_i(t)-X_i(t-1)}{p_i}$ equals $\frac{1}{p_j}$ with probability p_j, and equals 0 with probability $1-p_j$. Further, let $\gamma_i(t)$ be the probability that the system schedules flow i in slot t. Then we can derive that $\lim_{k\to\infty} \frac{1}{k} [\sum_{t=1}^{k} (\sum_{i=1}^{N} \frac{X_i(t)-X_i(t-1)}{p_i})^2] = \lim_{k\to\infty} \frac{1}{k} [\sum_{t=1}^{k} \sum_{i=1}^{N} \gamma_i(t) \frac{1}{p_i}]$. Note that, since every transmission for flow i is successful with probability p_i, we have $\lim_{k\to\infty} \frac{\sum_{t=1}^{k} \gamma_i(t)}{k} = \frac{\bar{X}_i}{p_i}$. Therefore, (3.7) can be written as:

$$\sigma^2_{[\bar{X}_i]} = \sum_{i=1}^{N} \frac{\bar{X}_i}{p_i^2} - 1. \tag{3.8}$$

Recall the definition of $\hat{X}_i := \lim_{t\to\infty} \frac{X_i(t) - t\bar{X}_i}{\sqrt{t}}$, we have $\hat{X}_{TOT} = \sum_{i=1}^{N} \frac{\hat{X}_i}{p_i}$, and the variance of $\frac{\hat{X}_i}{p_i}$ is $(\frac{\sigma_i}{p_i})^2$. By Cauchy-Schwarz Inequality, we have:

$$
\begin{aligned}
&(\sum_{i=1}^{N} \frac{\sigma_i}{p_i})^2 = (\sum_{i=1}^{N} \sqrt{Var(\frac{\hat{X}_i}{p_i})})^2 \\
=&\sum_{i=1}^{N} Var(\frac{\hat{X}_i}{p_i}) + 2\sum_{l=1}^{N}\sum_{m=l+1}^{N} \sqrt{Var(\frac{\hat{X}_l}{p_l})Var(\frac{\hat{X}_m}{p_m})} \\
\geq&\sum_{i=1}^{N} Var(\frac{\hat{X}_i}{p_i}) + 2\sum_{l=1}^{N}\sum_{m=l+1}^{N} Cov(\frac{\hat{X}_l}{p_l}, \frac{\hat{X}_m}{p_m}) \\
=&Var(\sum_{i=1}^{N} \frac{\hat{X}_i}{p_i}) = \sigma^2_{[\bar{X}_i]},
\end{aligned}
\tag{3.9}
$$

where $Var(X)$ denotes the variance of X and $Cov(X, Y)$ denotes the covariance. Thus, we have the constraint for σ_i as:

$$
\sum_{i}^{n} \frac{\sigma_i}{p_i} \geq \sigma_{[\bar{X}_i]} = \sqrt{\sum_{i}^{n} \frac{\bar{X}_i}{p_i^2} - 1}. \tag{3.10}
$$

3.4.2 Optimization Problem Formulation

From (3.4), the distribution $U_i(t)$ is very different in two different regimes, the regime $\bar{X}_i > \frac{1}{m_i}$ and the regime $\bar{X}_i < \frac{1}{m_i}$. When $\bar{X}_i > \frac{1}{m_i}$, then the rate of delivery is larger than the rate of update arrival. Hence, we say that the system operates in the *under-sampled regime* when $\bar{X}_i > \frac{1}{m_i}$ for all i. Conversely, we say that the system operates in the *over-sampled regime* when $\bar{X}_i < \frac{1}{m_i}$ for all i.

We first discuss the under-sampled regime. From (3.6), we know that it is possible to operate in the under-sampled regime if and only if $\sum_{i=1}^{N} \frac{1}{p_i m_i} < 1$. In this case, $U_i(t)$ is approximated by $\frac{T_i}{m_i} - EXP(\frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2})$. Let $\Theta_\lambda(z) := 1 - e^{-\lambda z}$ be the Cumulative

Distribution Function (CDF) of an exponential variable with mean $\frac{1}{\lambda}$. Then we have

$$\lim_{k\to\infty} \frac{\sum_{i=1}^{N}\sum_{t=T_i+1}^{k+T_i} C_i(q_iT_i - U_i(t))}{k} \tag{3.11}$$

$$= \lim_{k\to\infty} \sum_{i=1}^{N} E[C_i(q_iT_i - U_i(t))] \tag{3.12}$$

$$= \sum_{i=1}^{N} E[C_i(q_iT_i - \frac{T_i}{m_i} + EXP(\frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2}))] \tag{3.13}$$

$$= \sum_{i=1}^{N} \int_z C_i(z - (\frac{1}{m_i} - q_i)T_i) d\Theta_{\frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2}}(z). \tag{3.14}$$

The problem of minimizing the total LoC in the under-sampled regime is to find $[\bar{X}_i]$ and $[\sigma_i]$ that minimize (3.14), subject to (3.6) and (3.10).

Next, we discuss the over-sampled regime, which can happen when $\sum_{i=1}^{N} \frac{1}{p_i m_i} > 1$. In this case, $U_i(t)$ is approximated by $\frac{T_i}{m_i} - \mathcal{N}(T_i(\frac{1}{m_i} - \bar{X}_i), T_i\sigma_i^2)$. Let $\phi(z)$ represents the CDF of a random variable under standard Normal distribution, then the CDF of $\{\hat{U}_i(t) - T_i\bar{X}_i\}$ is $\phi(\frac{z}{\sqrt{T_i\sigma_i^2}})$. Then we have:

$$\lim_{k\to\infty} \frac{\sum_{i=1}^{N}\sum_{t=T_i+1}^{k+T_i} C_i(q_iT_i - U_i(t))}{k} \tag{3.15}$$

$$= \lim_{k\to\infty} \sum_{i=1}^{N} E[C_i(q_iT_i - U_i(t))] \tag{3.16}$$

$$= \sum_{i=1}^{N} E[C_i(q_iT_i - T_i\bar{X}_i + T_i\bar{X}_i - \mathcal{N}(T_i\bar{X}_i, T_i\sigma_i^2))] \tag{3.17}$$

$$= \sum_{i=1}^{N} \int_z C_i(\sqrt{T_i\sigma_i^2}z - (\bar{X}_i - q_i)T_i) d\phi(z). \tag{3.18}$$

The problem of minimizing the total LoC in the over-sampled regime is to find $[\bar{X}_i]$ and $[\sigma_i]$ that minimize (3.18), subject to (3.6) and (3.10).

3.4.3 Obtaining the Optimal Solution

A challenge in finding the optimal $[\bar{X}_i]$ and $[\sigma_i]$ is that the objective functions (3.14) and (3.18) both involve integrals. We propose using the Monte Carlo Method (MCM) [22] to address this challenge. For the under-sampled regime, since the rate λ_i of $EXP(\lambda_i)$ of each flow i involves both the control variables $\bar{X}_i$ and σ_i, we further convert (3.14) into the following form to obtain $EXP(1)$:

$$\begin{aligned}
&\sum_{i=1}^{N}\int_z C_i(z-(\frac{1}{m_i}-q_i)T_i)d\Theta_{\frac{2(\bar{X}_i-\frac{1}{m_i})}{\sigma_i^2}}(z) \\
&=\sum_{i=1}^{N}\int_y C_i(\frac{y\sigma_i^2}{2(\bar{X}_i-\frac{1}{m_i})}-(\frac{1}{m_i}-q_i)T_i)d\Theta_1(y), \qquad (3.19)
\end{aligned}$$

where $y=\frac{2(\bar{X}_i-\frac{1}{m_i})}{\sigma_i^2}z$. Then we can apply the Monte Carlo Method and generate K random numbers using the exponential distribution with rate 1 for each $1\le i\le N$, which are denoted by $y_{i,1}, y_{i,2}, \ldots, y_{i,K}$. The objective function (3.19) can then be approximated by $\sum_{i=1}^{N}\frac{1}{K}\sum_{k=1}^{K}C_i(\frac{y_{i,k}\sigma_i^2}{2(\bar{X}_i-\frac{1}{m_i})}-(\frac{1}{m_i}-q_i)T_i)$, and the problem of minimizing the total LoC can be written as

$$\begin{aligned}
Min \quad & \sum_{i=1}^{N}\frac{1}{K}\sum_{k=1}^{K}C_i(\frac{y_{i,k}\sigma_i^2}{2(\bar{X}_i-\frac{1}{m_i})}-(\frac{1}{m_i}-q_i)T_i) && (3.20)\\
s.t. \quad & \sum_{i=1}^{N}\frac{\bar{X}_i}{p_i}=1 && (3.21)\\
& \sum_{i=1}^{N}\frac{\sigma_i}{p_i}\ge\sigma_{[\bar{X}_i]}=\sqrt{\sum_{i=1}^{N}\frac{\bar{X}_i}{p_i^2}-1} && (3.22)\\
& \bar{X}_i\ge 0 \quad and \quad \sigma_i\ge 0, \quad \forall i. && (3.23)
\end{aligned}$$

The above problem is a well-defined optimization problem, and we can apply standard techniques to find the optimal $[\bar{X}_i]$ and $[\sigma_i]$.

Similarly, for the over-sampled regime, we generate K random numbers using the Normal distribution $\mathcal{N}(0,1)$ for each $1 \leq i \leq N$, which are denoted again by $z_{i,1}, z_{i,2}, \ldots, z_{i,K}$. Then we approximate (3.18) by $\sum_{i=1}^{N} \frac{1}{K} \sum_{k=1}^{K} C_i(\sqrt{T_i \sigma_i^2} z_{i,k} - (\bar{X}_i - q_i)T_i)$. The problem of minimizing the total LoC can be written as

$$Min \quad \sum_{i=1}^{N} \frac{1}{K} \sum_{k=1}^{K} C_i(\sqrt{T_i \sigma_i^2} z_{i,k} - (\bar{X}_i - q_i)T_i) \tag{3.24}$$

$$s.t. \quad \sum_{i=1}^{N} \frac{\bar{X}_i}{p_i} = 1 \tag{3.25}$$

$$\sum_{i=1}^{N} \frac{\sigma_i}{p_i} \geq \sigma_{[\bar{X}_i]} = \sqrt{\sum_{i=1}^{N} \frac{\bar{X}_i}{p_i^2} - 1} \tag{3.26}$$

$$\bar{X}_i \geq 0 \quad and \quad \sigma_i \geq 0, \quad \forall i. \tag{3.27}$$

3.5 Online Scheduling Policy

Section 3.4 has shown how to find the optimal $[\bar{X}_i]$ and $[\sigma_i]$ to minimize the total LoC. Let $[\bar{X}_i^*]$ and $[\sigma_i^*]$ be the optimal solution. It remains to find a scheduling policy that ensures that the mean and the variance of the update delivery process $X_i(t)$ are indeed $\bar{X}_i^*$ and σ_i^{*2}. In this section, we propose such an online scheduling policy.

We first introduce some notations before we propose and analyze our policy. Let $d_i(t) := \frac{t\bar{X}_i{}^* - X_i(t)}{p_i}$ denote the *deficit* of flow i in slot t. Consequently, we define $\Delta d_i(t) := d_i(t+1) - d_i(t) = \frac{\bar{X}_i^*}{p_i} - \frac{X_i(t+1) - X_i(t)}{p_i}$ as the change of the deficit in a slot.

We are now ready to propose our policy, which is called *Variance-Weighted-Deficit-First* (VWDF) policy. The VWDF policy assigns a weight of $v_i := \frac{p_i}{\sigma_i^*}$ to each flow i. In each time slot t, the VWDF policy schedules the client with the largest $v_i d_i(t)$ for transmission.

Let $D(t) := \frac{\sum_{i=1}^{N} d_i(t)}{\sum_{i=1}^{N} 1/v_i}$ be the weighted average of $v_i d_i(t)$. We first establish the following theorem.

Theorem 5. *Under VWDF policy, the Markov process with state vector* $\{v_i d_i(t) - D(t)\}$ *is positive recurrent.*

Proof. By the design of our VWDF policy, at the beginning of each time slot t, the flow with largest $v_i d_i(t)$ will be transmitted at this slot. We use r_t to represent the flow that has the largest $v_i d_i(t)$ at time t, hence this flow r_t has $v_{r_t} d_{r_t}(t) \geq v_i d_i(t)$ for all i. Since a transmission for flow r_t is successful with probability p_{r_t}, we have $\Delta d_{r_t}(t) = \frac{\bar{X}^*_{r_t}}{p_{r_t}} - \frac{1}{p_{r_t}}$ with probability p_{r_t} and $\Delta d_{r_t}(t) = \frac{\bar{X}^*_{r_t}}{p_{r_t}}$ with probability $1 - p_{r_t}$. For all other flows $i \neq r_t$, $\Delta d_i(t) = \frac{\bar{X}^*_i}{p_i}$. Then we have the expectation as $E[\Delta d_{r_t}(t)] = \frac{\bar{X}^*_{r_t}}{p_{r_t}} - 1$ and $E[\Delta d_i(t)] = \frac{\bar{X}^*_i}{p_i}$ for $i \neq r_t$. This also gives:

$$\sum_{i=1}^{N} E[\Delta d_i(t)] = \sum_{i=1}^{N} \frac{\bar{X}^*_i}{p_i} - 1 = 0. \tag{3.28}$$

Similarly, let $\Delta D(t) := D(t+1) - D(t)$. Following the above result, we also have:

$$E[\Delta D(t)] = E[\frac{\sum_{i=1}^{N}(d_i(t+1) - d_i(t))}{\sum_{i=1}^{N} \frac{1}{v_i}}] = 0. \tag{3.29}$$

Define the Lyapunov function $L(t) = \frac{1}{2}\sum_{i=1}^{N} \frac{1}{v_i}(v_i d_i(t) - D(t))^2$, and we have the derivation for the Lyapunov drift when given the state at t:

$$
\begin{aligned}
\Delta L(t) &= E[L(t+1) - L(t)] \\
&= E[\frac{1}{2}\sum_{i=1}^{N}\frac{1}{v_i}(v_i d_i(t+1) - D(t+1))^2 - \frac{1}{2}\sum_{i=1}^{N}\frac{1}{v_i}(v_i d_i(t) - D(t))^2] \\
&\leq \beta + E[\sum_{i=1}^{N}\frac{1}{v_i}(v_i d_i(t) - D(t)(v_i \Delta d_i(t) - \Delta D(t)))] \\
&= \beta + E[\sum_{i=1}^{N} v_i d_i(t)\Delta d_i(t) - \sum_{i=1}^{N} D(t)\Delta d_i(t)] \\
&\quad - E[\Delta D(t)\sum_{i=1}^{N} d_i(t) - \Delta D(t)\sum_{i=1}^{N} d_i(t)] \qquad (3.30)
\end{aligned}
$$

where β is a bounded positive number, and (3.30) is from the definition of $D(t)$.

Since we have $E[\Delta d_{r_t}(t)] = \frac{\bar{X}^*_{r_t}}{p_{r_t}} - 1$, $E[\Delta d_i(t)] = \frac{\bar{X}^*_i}{p_i}$ for $i \neq r_t$, $\sum_{i=1}^{N} E[\Delta d_i(t)] = 0$, and $E[\Delta d_i(t)] = 0$, we further have:

$$
\Delta L(t) \leq \beta + \sum_{i=1}^{N}\frac{\bar{X}^*_i}{p_i}\big(v_i d_i(t) - v_{r_t} d_{r_t}(t)\big). \qquad (3.31)
$$

By the design of VWDF policy, $v_{r_t} d_{r_t}(t) \geq v_i d_i(t)$, for all $i \neq r_t$. Suppose, at time t, $\max_{1\leq i\leq N} |v_i d_i(t) - D(t)| > \delta$, for some positive δ. Then, there exists a flow i'_t with $v_{i'_t} d_{i'_t}(t) - v_{r_t} d_{r_t}(t) < -\delta$, and hence $\Delta L(t) < \beta - \delta\frac{\bar{X}^*_{i'_t}}{p_{i'_t}}$. By choosing δ to be larger than $\frac{2\beta p_i}{\bar{X}^*_i}$, we have $\Delta L(t) < -\beta$ if $\max_{1\leq i\leq N} |v_i d_i(t) - D(t)| > \delta$. Therefore, by Foster-Lyapunov Theorem, we have $\{v_i d_i(t) - D(t)\}$ is positive recurrent. □

Since the Markov process $\{v_i d_i(t) - D(t)\}$ is positive recurrent, it has a stationary distribution. Hence, $\lim_{k\to\infty} \frac{v_i d_i(k) - D(k)}{k} \to 0$, and $\lim_{k\to\infty} \frac{v_i d_i(k) - D(k)}{\sqrt{k}} \to 0$, for all i.

Moreover,

$$\lim_{k\to\infty}\frac{D(k)}{k}=\lim_{k\to\infty}\frac{\sum_{i=1}^{N}\frac{k\bar{X}_i^*-X_i(k)}{p_i}}{k\sum_{i=1}^{N}\frac{1}{v_i}}$$
$$=\lim_{k\to\infty}\frac{\sum_{i=1}^{N}\frac{\bar{X}_i^*}{p_i}-\sum_{i=1}^{N}\frac{X_i(k)}{kp_i}}{\sum_{i=1}^{N}\frac{1}{v_i}}=0. \quad (3.32)$$

Then, we have the following:

$$\lim_{k\to\infty}\frac{v_i d_i(k)}{k}=\lim_{k\to\infty}v_i\frac{k\bar{X}_i^*-X_i(k)}{kp_i}$$
$$=v_i\frac{\bar{X}_i^*}{p_i}-v_i\lim_{k\to\infty}\frac{X_i(k)}{kp_i}=0, \quad (3.33)$$

and hence, $\bar{X}_i=\bar{X}_i^*$.

Next, recall the definition $\hat{X}_i:=\lim_{k\to\infty}\frac{X_i(k)-k\bar{X}_i}{\sqrt{k}}$ and $\hat{X}_{TOT}:=\lim_{k\to\infty}\frac{\sum_{i=1}^{N}\frac{X_i(k)}{p_i}-k}{\sqrt{k}}$. Hence, we have:

$$\lim_{k\to\infty}\frac{v_i d_i(k)}{\sqrt{k}}=\lim_{k\to\infty}\frac{v_i(k\bar{X}_i^*-X_i(k))}{p_i\sqrt{k}}=-\frac{v_i}{p_i}\hat{X}_i, \quad (3.34)$$

and

$$\lim_{k\to\infty}\frac{D(k)}{\sqrt{k}}=\lim_{k\to\infty}\frac{\sum_{i=1}^{N}d_i(k)}{\sqrt{k}\sum_{i=1}^{N}\frac{1}{v_i}}$$
$$=\lim_{k\to\infty}\frac{\sum_{i=1}^{N}\frac{k\bar{X}_i^*}{p_i}-\sum_{i=1}^{N}\frac{X_i(k)}{p_i}}{\sqrt{k}\sum_{i=1}^{N}\frac{1}{v_i}}$$
$$=-\lim_{k\to\infty}\frac{\sum_{i=1}^{N}\frac{X_i(k)}{p_i}-k}{\sqrt{k}\sum_{i=1}^{N}\frac{1}{v_i}}=-\frac{\hat{X}_{TOT}}{\sum_{i=1}^{N}\frac{1}{v_i}}. \quad (3.35)$$

By definition, the variance of (3.34) is $\frac{v_i^2}{p_i^2}\sigma_i^2=\frac{\sigma_i^2}{\sigma_i^{*2}}$ and the variance of (3.35) is $\frac{\sigma^2_{[\bar{X}_i]}}{(\sum_{i=1}^{N}\frac{1}{v_i})^2}=\frac{\sigma^2_{[\bar{X}_i]}}{(\sum_{i=1}^{N}\frac{\sigma_i^*}{p_i})^2}$. Since $[X_i^*]$ and $[\sigma_i^*]$ is the optimal solution to the optimization problem of minimizing either (3.14) or (3.18), subject to (3.6) and (3.10) and the objective function is increasing in $[\sigma_i^*]$, we have $\sum_{i=1}^N \frac{\sigma_i^*}{p_i} = \sigma_{[\bar{X}_i]}$, and the variance of (3.35) is 1. As (3.34) and (3.35) have the same variance, we have $\sigma_i = \sigma_i^*$, for all i.

In summary, we have:

Theorem 6. *Under the VWDF policy, $\bar{X}_i = \bar{X}_i^*$ and $\sigma_i = \sigma_i^*$, for all i.* $\square$

Since $[\bar{X}_i^*]$ and $[\sigma_i^*]$ are the optimal vectors that minimize system-wide total LoC under the Brownian approximation, Theorem 6 implies that the VWDF policy is the optimal scheduling policy. We note that the VWDF policy makes scheduling decisions only based on the deficit of each flow. In particular, the VWDF policy does not keep track of the number of undelivered useful updates that each flow has. Such a feature makes it very easy to implement the VWDF policy. It is also surprising that the VWDF policy is able to minimize the total LoC, which depends on the number of useful information updates, without any knowledge about the usefulness of individual updates.

3.6 Simulation Results

We present our simulation results in this section. We have tested our VWDF policy and compared it with with two other state-of-the-art policies in NS-2 simulation. All simulations are performed under 802.11 MAC protocol with 54Mbps data rate. Simulations show that the time needed for the AP to schedule a transmission and receive an information update is 813μs. All results presented in this paper are average in 100 runs.

We compare our VWDF policy against two other policies. The first policy is the Largest Debt First (LDF) policy from [5], [8], which schedules the flow withe largest $q_i t - X_i(t)$ in each time slot. The main difference between the LDF policy and our VWDF policy is that the LDF policy does not weigh the deficit of each flow by its variance. The second policy is a policy aiming to minimize AoI under some throughput constraints [23]. Under our model, the policy schedules the flow with the largest sum of $q_i t - X_i(t)$ and AoI in each time slot. Hence, we call this policy MW-AoI in the following simulations. In all policies, each flow sends status updates using LIFO.

In our simulations, there are eight wireless flows, which are divided into two groups. The first four flows are in the first group with $C_i(q_iT_i - U_i(t)) = (q_iT_i - U_i(t))^2$. The other four flows are in the second group with $C_i(q_iT_i - U_i(t)) = e^{q_iT_i - U_i(t)} - (q_iT_i - U_i(t)) - 1$.

We evaluate the performance under four scenarios operating in very different regimes:

1. **Over-sampled and heavily-loaded system**: This setting has $\bar{X}_i < \frac{1}{m_i}$, for all i, and $\sum_{i=1}^{N} \frac{q_i}{p_i} = 1$. Specifically, we choose $[p_i] = [0.65, 0.65, 0.7, 0.7, 0.75, 0.75, 0.8, 0.8]$, $[q_i] = [0.13, 0.065, 0.14, 0.07, 0.075, 0.075, 0.08, 0.08]$, $[m_i] = [5, 5, 5, 5, 8, 8, 8, 8]$, and $[T_i] = [200, 200, 200, 200, 500, 500, 500, 500]$.

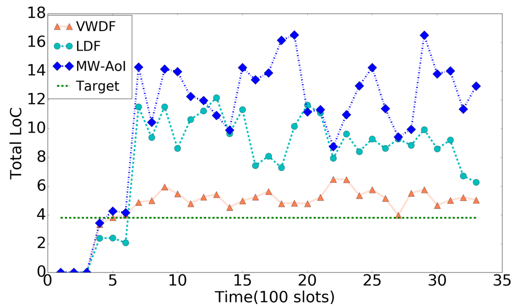
2. **Over-sampled and over-loaded system**: This setting has $\bar{X}_i < \frac{1}{m_i}$, for all i, and $\sum_{i=1}^{N} \frac{q_i}{p_i} = 1.1 > 1$. Specifically, we choose $[p_i]$ and $[m_i]$ to be the same as the first system, $[q_i] = [0.1625, 0.0975, 0.14, 0.07, 0.075, 0.075, 0.08, 0.08]$, and $[T_i] = [400, 400, 400, 400, 300, 300, 300, 300]$.

3. **Over-sampled and under-loaded system**: This setting has $\bar{X}_i < \frac{1}{m_i}$, for all i, and $\sum_{i=1}^{N} \frac{q_i}{p_i} = 0.95 < 1$. Specifically, we choose $[p_i]$ and $[m_i]$ to be the same as the first system, but different $[q_i] = [0.0975, 0.065, 0.14, 0.07, 0.075, 0.075, 0.08, 0.08]$, and $[T_i] = [400, 400, 400, 400, 300, 300, 300, 300]$.

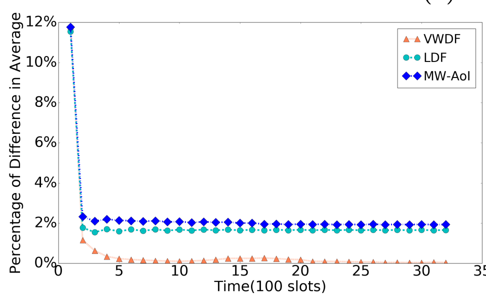
4. **Under-sampled system**: This setting has $\bar{X}_i > \frac{1}{m_i}$, for all i. Specifically, we choose $p_i = 0.52$, $q_i = 0.0625$, $m_i = 16$, and $T_i = 400$, for all i.

For all these four systems, we evaluate the average total LoC incurred in every 100 time slots. We also plot the target optimal value obtained by solving the optimization problems. Moreover, to evaluate whether our VWDF converges to the desirable $\bar{X}_i^*$ and σ_i^*, we also evaluate the total deviation from the desirable values, namely, $\frac{1}{N}\sum_{i=1}^{N} |\frac{\bar{X}_i - \bar{X}_i^*}{\bar{X}_i^*}|$ and $\frac{1}{N}\sum_{i=1}^{N} |\frac{\sigma_i - \sigma_i^*}{\sigma_i^*}|$.

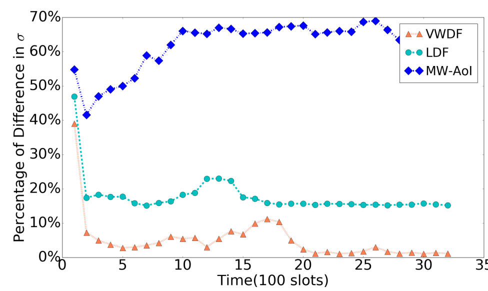
The simulation results for the four systems are shown in Fig. 3.2, 3.3, 3.4, and 3.5. It can be observed that our VWDF policy achieves the smallest total LoC among all



(a) The Total LoC



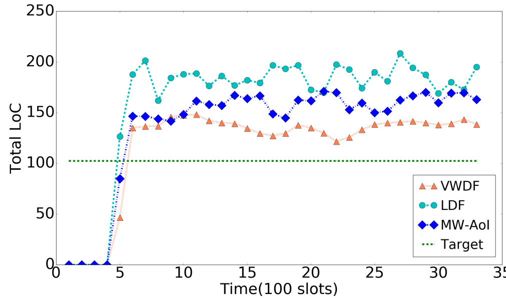
(b) The Total Error from $\bar{X}_i^*$ in Percentage



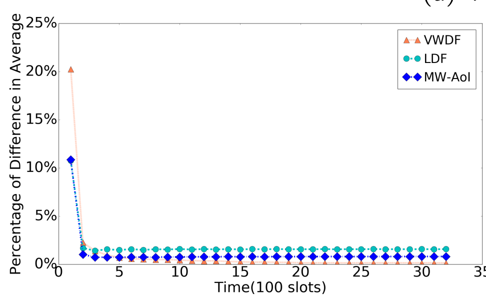
(c) The Total Error from σ_i^* in Percentage

Figure 3.2: The over-sampled and heavily-loaded system.

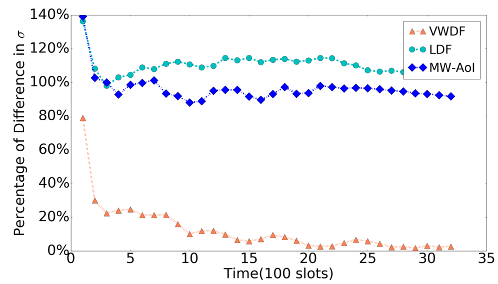
three evaluated policies in all systems. The total LoC of VWDF is also very close to the target optimal value. Moreover, it can be observed that the empirical values of $\bar{X}_i$ and σ_i converge to the target values $\bar{X}_i^*$ and σ_i^* typically within 500 time slots, which, under our network setting, is less than 0.5 second. These result suggest that VWDF not only has good performance but also fast convergence rate.



(a) The Total LoC

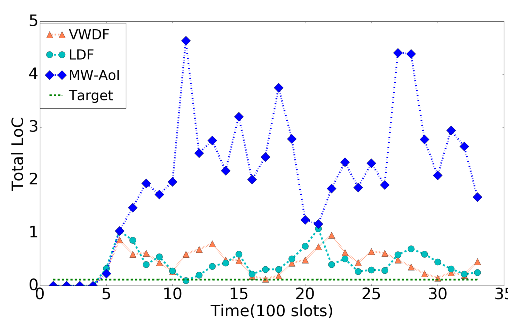


(b) The Total Error from $\bar{X}_i^*$ in Percentage

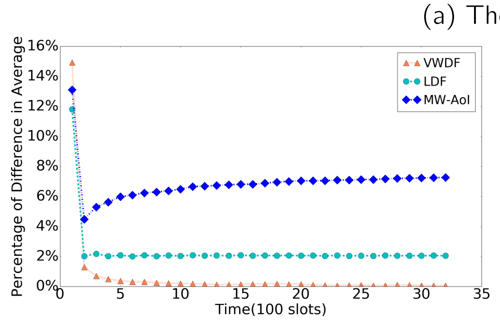


(c) The Total Error from σ_i^* in Percentage.

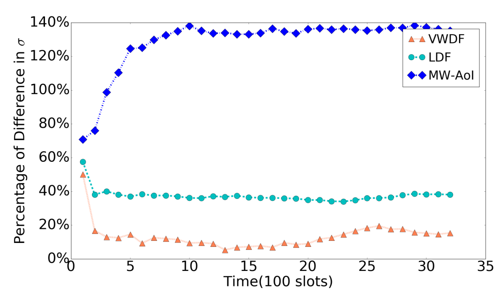
Figure 3.3: The over-sampled and over-loaded system.



(a) The Total LoC

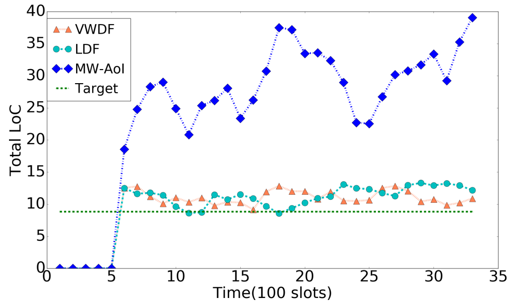


(b) The Total Error from $\bar{X}_i^*$ in Percentage

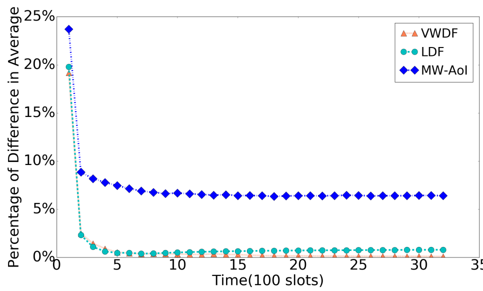


(c) The Total Error from σ_i^* in Percentage

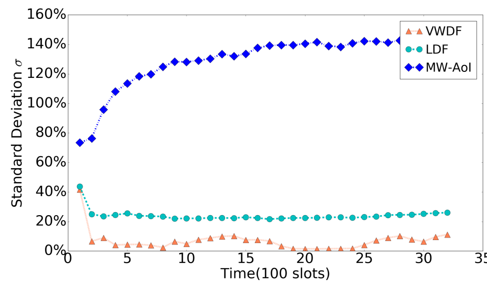
Figure 3.4: The over-sampled and under-loaded system.



(a) The Total LoC



(b) The Total Error from $\bar{X}_i^*$ in Percentage

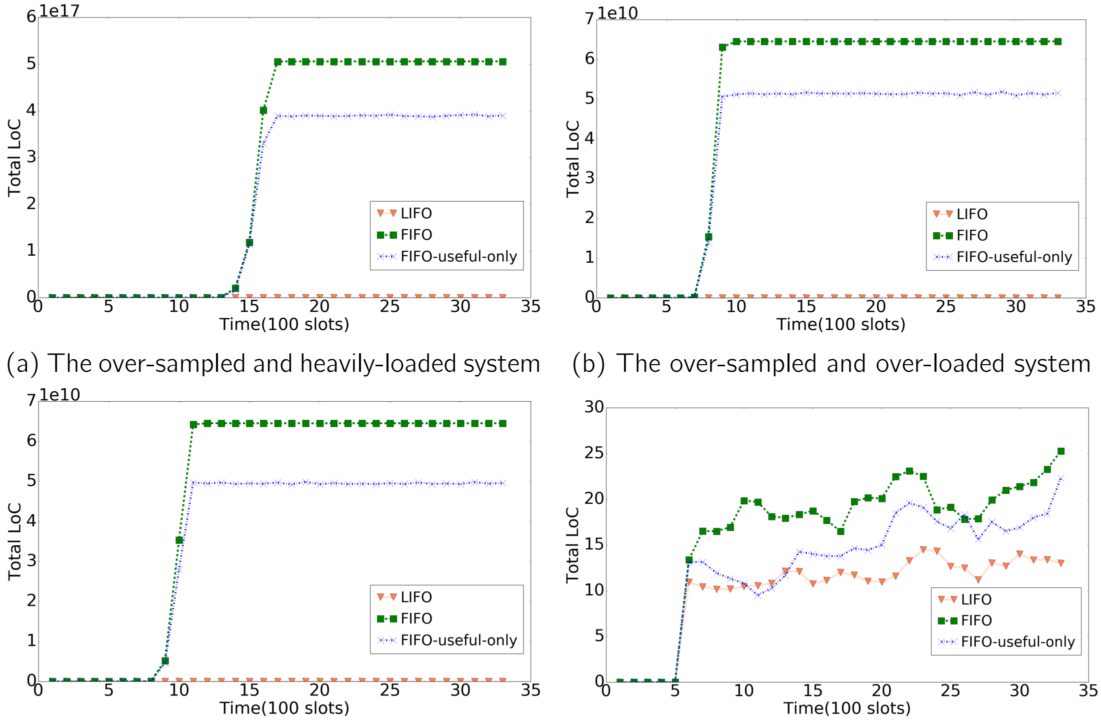


(c) The Total Error from σ_i^* in Percentage

Figure 3.5: The Under-sampled System.

Next, we evaluate the performance of VWDF under different queueing disciplines. In addition to LIFO, we also evaluate two other queueing discipline. The first one is First-In-First-Out (FIFO), where each flow sends the oldest undelivered status update every time it receives a POLL message. The second is a variation of FIFO where each flow drops status updates that have become stale, and sends the oldest useful status update every time it receives a POLL message. This discipline, which we call FIFO-useful-only, is effectively the same as the Earliest-Deadline-First (EDF) policy.

We evaluate these queueing discipline under the four systems describe above. The results are shown in Fig. 3.6. Clearly, LIFO significantly outperforms the other two queueing discipline. It is well-known that the EDF policy is optimal when the goal is to maximize the number of timely deliveries in real-time wireless networks. The result that FIFO-useful-only performs so poorly also highlight that there are fundamental differences



(a) The over-sampled and heavily-loaded system (b) The over-sampled and over-loaded system

(c) The over-sampled and under-loaded system (d) The under-sampled system

Figure 3.6: The LoC of Three Buffer Strategies.

between real-time wireless networks and information update systems.

3.7 Conclusion

We have studied a remote sensing problem and built a model to catch the estimation accuracy when the control center needs to make a estimation of current status then make a appropriate decision. This model considers both the freshness of the information update and the quantity requirements of real-time wireless flows. Through Brownian motion approximation, we approximate the process of the fresh information update as a Reflected Brownian motion. Moreover, the model of the real-time estimation accuracy is described as an optimization problem with the constrains in the averages and temporal variances of the delivery process. We then propose a simple online scheduling policy that employs the optimal averages and variances to achieve the optimal system-wide performance. We also perform comprehensive simulations to show that our scheduling policy converges fast to the optimal averages and variances and outperforms the other two state-of-the-art policies: the LDF policy and the MW-AoI policy. Moreover, our policy does not require any knowledge about the freshness of each information update, and is shown to successfully capture the estimation performance depends on data freshness. Additionally, simulations are also ran for the comparison of different buffer strategies: LIFO, FIFO and FIFO-useful-only, which proves the intuition of our work to choose LIFO.

4. OPTIMAL WIRELESS SCHEDULING FOR REMOTE SENSING THROUGH BROWNIAN APPROXIMATION

4.1 System Model for Second-Order Wireless Network Optimization

We begin by describing a generic network optimization problem. Consider a wireless system where one AP serves N clients, numbered as $\{1, 2, \dots, N\}$. Time is slotted and denoted by $t = 1, 2, 3, \dots$. We consider the ON-OFF channel model where the AP can schedule a client for transmission if and only if the channel for the client is ON. Let $X_i(t)$ be the indicator function that the channel for client i is ON at time t. We assume that the sequence $\{X_i(1), X_i(2), \dots\}$ is governed by a stochastic positive-recurrent Markov process with finite states. In each time slot, if there is at least one client having an ON channel, then the AP selects a client with an ON channel and transmits a packet to it. Let $Z_i(t)$ be the indicator function that client i receives a packet at time t. The empirical performance of client i is modeled as a function of the entire sequence $\{Z_i(1), Z_i(2), \dots\}$. We note that the performance model is very general and covers virtually all existing network performance metrics, including both traditional ones like throughput and emerging ones like AoI. The network optimization problem is to find a scheduling policy that maximizes the total performance of the network.

Solving this generic network optimization problem is difficult because it requires solving an N-dimensional Markov decision process. As a result, except for a few special cases, there remains no tractable optimal solutions for many emerging network performance metrics like AoI. To circumvent this challenge, we propose capturing each random process by its second-order model, namely, its mean and temporal variance.

We first define the second-order model for channels. With a slight abuse of notations, let $X_S(t) := \max\{X_i(t) | i \in S\}$ be the indicator function that at least one client in S has an ON channel at time t. Since all channels are governed by stochastic positive-recurrent

Markov processes, we can define the mean of X_S as

$$m_S := \lim_{T\to\infty} \frac{\sum_{t=1}^{T} X_S(t)}{T}, \tag{4.1}$$

and the temporal variance of X_S as

$$v_S^2 := E[(\lim_{T\to\infty} \frac{\sum_{t=1}^{T} X_S(t) - Tm_S}{\sqrt{T}})^2]. \tag{4.2}$$

The second-order channel model is then expressed as the collection of the means and temporal variances of all X_S, namely, $\{(m_S, v_S^2) | S \subseteq \{1, 2, \dots, N\}\}$.

The second-order model for packet deliveries is defined similarly. Assuming that the AP's scheduling policy is ergodic, we can define the mean and the temporal variance of Z_i as

$$\mu_i := \lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t)}{T}, \sigma_i^2 := E[(\lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t) - T\mu_i}{\sqrt{T}})^2]. \tag{4.3}$$

The second-order delivery model is $\{(\mu_i, \sigma_i^2) | 1 \leq i \leq N\}$. The performance a client i is modeled as a function of (μ_i, σ_i^2), which we denote by $F_i(\mu_i, \sigma_i^2)$.

Since clients want to have large means and small variances for their delivery processes, we define the second-order capacity region of a network as follows:

Definition 1 (Second-order capacity region)**.** *Given a second-order channel model* $\{(m_S, v_S^2) | S \subseteq \{1, 2, \dots, N\}\}$*, the second-order capacity region is the set of all* $\{(\mu_i, \sigma_i^2) | 1 \leq i \leq N\}$ *such that there exists a scheduling policy under which* $\lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t)}{T} = \mu_i$ *and* $E[(\lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t) - T\mu_i}{\sqrt{T}})^2] \leq \sigma_i^2, \forall i$. □

The second-order network optimization problem entails finding the scheduling policy that maximizes $\sum_{i=1}^{N} F_i(\mu_i, \sigma_i^2)$.

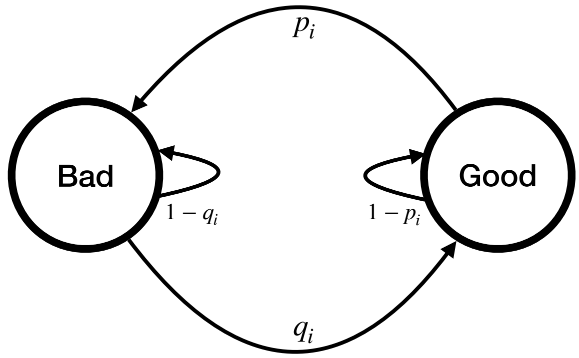


Figure 4.1: The Gilbert-Elliot Model

4.2 The Second-Order Model for AoI Optimization over Gilbert-Elliot Channels

To demonstrate the utility of our second-order models, we derive the second-order models for an important, but unsolved, problem: the optimization of AoI over Gilbert-Elliot channels.

4.2.1 The Second-Order Model of Gilbert-Elliot Channels

In Gilbert-Elliot channels [24, 25], the channel for each client i is modeled as a two-state Markov process, as shown in Fig. 4.1. The channel is ON if it is in the good (G) state, and is OFF if it is in the bad (B) state. The transition probabilities from G to B and from B to G are p_i and q_i, respectively. The channels are independent from each other.

We now show the second-order model of Gilbert-Elliot channels.

Theorem 7. *Under the Gilbert-Elliot channels, for all S,*

$$m_S = 1 - \prod_{i \in S} \frac{p_i}{p_i + q_i}, \tag{4.4}$$

$$v_S^2 = 2\sum_{k=1}^{\infty} \Big(\prod_{i \in S} G_i(k+1) - \prod_{i \in S} \frac{p_i}{p_i + q_i} \Big) \prod_{i \in S} \frac{p_i}{p_i + q_i} + \prod_{i \in S} \frac{p_i}{p_i + q_i} - (\prod_{i \in S} \frac{p_i}{p_i + q_i})^2, \tag{4.5}$$

where $G_i(k) = \frac{p_i}{p_i+q_i} + \frac{q_i}{p_i+q_i}(1 - p_i - q_i)^{k-1}$.

Proof. Let $Y_i(t) := 1 - X_i(t)$ be the indicator function that client i has an OFF channel at time t. Let $Y_S(t) := 1 - X_S(t)$ be the indicator function that all clients in the subset S have OFF channels at time t. Hence, we have $Y_S(t) = \prod_{i \in S} Y_i(t)$. Suppose the Markov process of each channel is in the steady-state at time t, then we have $Prob(Y_i(t) = 1) = \frac{p_i}{p_i+q_i}$. Hence, $E[Y_S(t)] = \prod_{i \in S} \frac{p_i}{p_i+q_i}$ and $E[X_S(t)] = 1 - E[Y_S(t)] = 1 - \prod_{i \in S} \frac{p_i}{p_i+q_i}$. This establishes (4.4).

Next, we establish (4.5). We have $(\sum_{t=1}^{T} X_S(t) - Tm_S)^2 = (\sum_{t=1}^{T} Y_S(t) - T(1 - m_S))^2$. By the Markov central limit theorem, we can calculate v_S^2 by assuming that the Markov process of each channel is in the steady-state at time 1 and using the following formula:

$$v_S^2 = Var(Y_S(1)) + 2\sum_{k=1}^{\infty} Cov(Y_S(1), Y_S(1+k)). \tag{4.6}$$

Since $Y_S(1)$ is a Bernoulli random variable with mean $\prod_{i \in S} \frac{p_i}{p_i+q_i}$, we have

$$Var(Y_S(1)) = \prod_{i \in S} \frac{p_i}{p_i + q_i} - (\prod_{i \in S} \frac{p_i}{p_i + q_i})^2. \tag{4.7}$$

Let $G_i(k) = Prob(Y_i(k) = 1|Y_i(1) = 1)$. Then,

$$\begin{aligned}
&E[Y_S(1)Y_S(1+k)] \\
=&Prob(Y_S(1+k) = 1|Y_S(1) = 1) \times Prob(Y_S(1) = 1) \\
=&Prob(Y_i(1+k) = 1, \forall i \in S|Y_i(1) = 1, \forall i \in S) \prod_{i \in S} \frac{p_i}{p_i + q_i} \\
=&\prod_{i \in S} G_i(k+1) \prod_{i \in S} \frac{p_i}{p_i + q_i},
\end{aligned} \tag{4.8}$$

and

$$\begin{aligned}
&Cov(Y_S(1), Y_S(1+k)) \\
=&E[Y_S(1)Y_S(1+k)] - E[Y_S(1)]E[Y_S(1+k)] \\
=&\Big(\prod_{i \in S} G_i(k+1) - \prod_{i \in S} \frac{p_i}{p_i + q_i}\Big) \prod_{i \in S} \frac{p_i}{p_i + q_i}
\end{aligned} \tag{4.9}$$

Combining (4.7) and (4.9) establishes (4.5).

It remains to find the closed-form expression of $G_i(k)$. We have

$$\begin{aligned}
&G_i(k) = Prob(Y_i(k) = 1|Y_i(1) = 1) \\
=&G_i(k-1)(1 - q_i) + (1 - G_i(k-1))p_i \\
=&p_i + (1 - p_i - q_i)G_i(k-1),
\end{aligned} \tag{4.10}$$

if $k > 1$, and $G_i(k) = 1$, if $k = 1$. Solving this recursive equation yields $G_i(k) = \frac{p_i}{p_i+q_i} + \frac{q_i}{p_i+q_i}(1 - p_i - q_i)^{k-1}$. This completes the proof. □

When $p_i + q_i = 1$, the Gilbert-Elliot channel reduces to the i.i.d. channel model where $X_i(t) = 1$ with probability q_i, independent from any prior events. By replacing $p_i = 1 - q_i$, we obtain the second-order model of i.i.d. channels as below:

Corollary 2. *Under the i.i.d. channels with $Prob(X_i(t) = 1) = q_i$,*

$$m_S = 1 - \prod_{i \in S}(1 - q_i), v_S^2 = \prod_{i \in S}(1 - q_i) - \prod_{i \in S}(1 - q_i)^2, \tag{4.11}$$

for all S. □

4.2.2 The Second-Order Model of AoI Optimization

Age-of-Information (AoI) has been proposed to model the performance of real-time remote sensing applications, where a controller is obtaining status updates from a number of sensors. In a nutshell, the AoI corresponding to a sensor at a given time is defined as the age of the newest information update that it has ever delivered to the controller. In terms of our network model, the AP is the controller and each client is a sensor.

Similar to the case studied in [26], we consider that each sensor i generates new updates by a Bernoulli random process. In each time slot t, sensor i generates a new update with probability λ_i, independent from any prior events. To minimize AoI, each sensor only keeps the most recent update in its memory, and it transmits the most recent update whenever it is scheduled for transmission. In other words, a sensor discards all its prior updates every time it generates a new update. The prior work [26] considers that the controller knows when each sensor generates a new update. In this paper, we further address the issue that the controller only knows λ_i but not the exact times at which sensors generate new updates. Hence, we assume that the scheduling decision is independent from update generations.

Let $A_i(n) := \min\{\tau | \sum_{t=1}^{\tau} Z_i(t) = n\}$ be the time of the n-th delivery for client i, and let $B_i(n) := A_i(n+1) - A_i(n)$ be the time between the n-th and the $(n+1)$-th deliveries. Since scheduling decisions are independent from update generations, we have the following:

Lemma 2. *If $\{B_i(0), B_i(1), \dots\}$ is independent from the update generation processes of* *sensor i, then the long-term average AoI of sensor i is*

$$\overline{AoI}_i = \frac{E[B_i^2]}{2E[B_i]} + \frac{1}{\lambda_i} - \frac{1}{2}, \tag{4.12}$$

where $E[B_i^2] := \lim_{m\to\infty} \sum_{n=1}^{m} B_i(n)^2/m$ *and* $E[B_i] := \lim_{m\to\infty} \sum_{n=1}^{m} B_i(n)/m$.

Proof. This lemma can be established by combining techniques in the proof of Proposition 2 in [26] and the fact that $B_i(n)$ is independent from update generations. The complete proof is omitted due to space limitation. □

We aim to express $\overline{AoI}_i$ as a function of the second-order delivery model of client i, (μ_i, σ_i^2). Since there can be multiple sequences of $\{Z_i(1), Z_i(2), \dots\}$ with the same (μ_i, σ_i^2), we will derive $\overline{AoI}_i$ with respect to a *second-order reference delivery process* as defined below.

Let $BM_{\mu_i,\sigma_i^2}(t)$ be a Brownian motion random process with mean μ_i and variance σ_i^2. An important property of the Brownian motion random process is that for any $t_1 < t_2$, $BM_{\mu_i,\sigma_i^2}(t_1) - BM_{\mu_i,\sigma_i^2}(t_2)$ is a Gaussian random variable with mean $(t_2 - t_1)\mu_i$ and variance $(t_2 - t_1)\sigma_i^2$. Our goal is to define a sequence $\{Z_i'(1), Z_i'(2), \dots\}$ such that $\sum_{\tau=1}^{t} Z_i'(\tau) \approx BM_{\mu_i,\sigma_i^2}(t)$.

Definition 2. *Given (μ_i, σ_i^2), the second-order reference delivery process, denoted by $\{Z_i'(1), Z_i'(2), \dots\}$ is defined to be*

$$Z_i'(t) = \begin{cases} 1 & \text{if } BM_{\mu_i,\sigma_i^2}(t) - BM_{\mu_i,\sigma_i^2}(t^-) \geq 1, \\ 0 & \text{else,} \end{cases} \tag{4.13}$$

where $t^- := \max\{\tau | \tau < t, Z_i'(\tau) = 1\}$. □

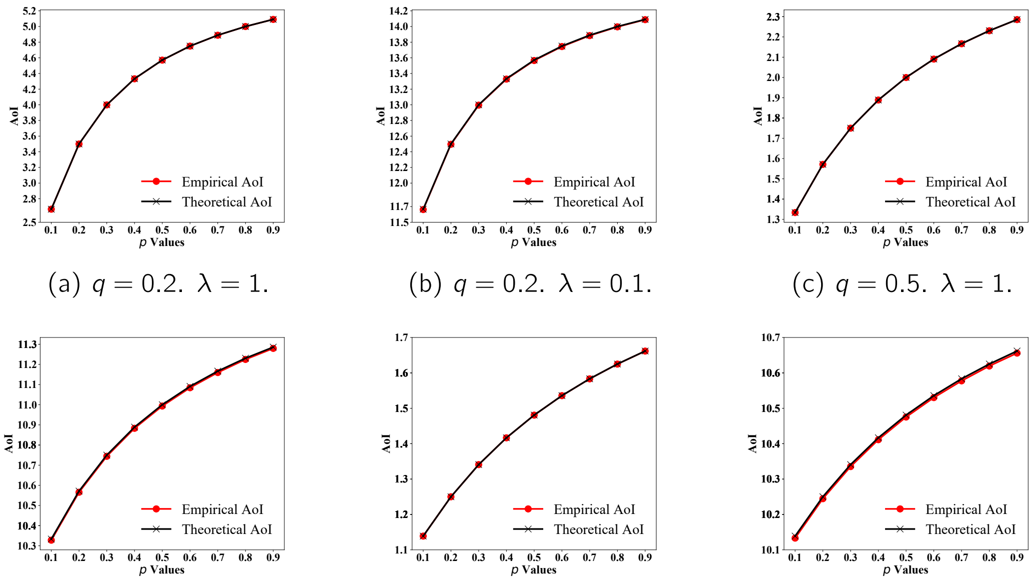
We now derive $\overline{AoI}_i$ with respect to the sequence $\{Z_i'(1), Z_i'(2), \dots\}$. Consider the time between the n-th and the $(n+1)$-th deliveries, which is denoted by $B_i(n)$, under the sequence $\{Z_i'(1), Z_i'(2), \dots\}$. From (4.13), $B_i(n)$ can be approximated by the amount of time needed for the Brownian motion random process to increase by 1, which is equivalent to the first-hitting time for a fixed level 1 and we denote it by H_i. It has been shown that the the first-hitting time for a fixed level 1 follows the inverse Gaussian distribution $IG(\frac{1}{\mu_i}, \frac{1}{\sigma_i^2})$ [27, 28]. Hence, we have $E[H_i] = 1/\mu_i$ and $E[H_i^2] = \sigma_i^2/\mu_i^3 + 1/\mu_i^2$. We now have

$$\begin{aligned}\overline{AoI}_i = &\frac{E[B_i^2]}{2E[B_i]} + \frac{1}{\lambda_i} - \frac{1}{2} \\ \approx &\frac{E[H_i^2]}{2E[H_i]} + \frac{1}{\lambda_i} - \frac{1}{2} = \frac{1}{2}(\frac{\sigma_i^2}{\mu_i^2} + \frac{1}{\mu_i}) + \frac{1}{\lambda_i} - \frac{1}{2}. \end{aligned} \quad (4.14)$$

4.2.3 Model Validation

We now verify whether the second-order model provides a good approximation of AoI over Gilbert-Elliot channels. We consider a system with only one client (sensor). The AP (controller) schedules the client for transmission whenever the client has an ON channel. Hence, we have $\mu_1 = m_{\{1\}}$ and $\sigma_1^2 = v_{\{1\}}^2$. Given, p_1, q_1, and λ_1, we can combine (4.4), (4.5), and (4.14) to obtain a theoretical approximation of the AoI. We note that (4.5) involves a summation of infinite terms $\sum_{k=1}^{\infty}(G_1(k) - \frac{p_1}{p_1+q_1})$. Since $G_1(k)$ converges to $\frac{p_1}{p_1+q_1}$ exponentially fast, we replace this term with $\sum_{k=1}^{100}(G_1(k) - \frac{p_1}{p_1+q_1})$ when calculating $v_{\{1\}}^2$.

We evaluate the accuracy of the theoretical AoI over a wide range of (p_1, q_1, λ_1). For each (p_1, q_1, λ_1), we obtain the empirical AoI by simulation the system for 1000 runs, where each run contains 50,000 time slots. The results are shown in Fig. 4.2. It can be observed that the theoretical AoI is always almost identical to the empirical AoI under all settings. The largest difference between theoretical and empirical AoI among all evaluated case is only 0.00558.



(a) $q = 0.2$. $\lambda = 1$. (b) $q = 0.2$. $\lambda = 0.1$. (c) $q = 0.5$. $\lambda = 1$.

(d) $q = 0.5$. $\lambda = 0.1$. (e) $q = 0.8$. $\lambda = 1$. (f) $q = 0.8$. $\lambda = 0.1$.

Figure 4.2: Model Validation For A Single Client.

4.3 An Outer Bound of the Second-Order Capacity Region

In this section, we derive a necessary condition for the second-order delivery model $\{(\mu_i, \sigma_i^2)|1 \le i \le N\}$ to be in the second-order capacity region.

Theorem 8. *Given a second-order channel model* $\{(m_S, v_S^2)|S \subseteq \{1, 2, \dots, N\}\}$*, a second-order delivery model* $\{(\mu_i, \sigma_i^2)|1 \le i \le N\}$ *can be in the second-order capacity region only* *if*

$$\sum_{i \in S} \mu_i \leq m_S, \forall S \subseteq \{1, 2, \dots, N\}, \tag{4.15}$$

$$\sum_{i=1}^{N} \mu_i = m_{\{1,2,\dots,N\}}, \tag{4.16}$$

$$\sum_{i=1}^{N} \sqrt{\sigma_i^2} \geq \sqrt{v_{\{1,2,\dots,N\}}^2}, \tag{4.17}$$

$$\mu_i \geq 0, \forall i. \tag{4.18}$$

Proof. We first establish (4.15). The AP can transmit a packet to a client i at time t only if the client has an ON channel, that is, $X_i(t) = 1$. Moreover, the AP can transmit to at most one client in each time slot. Hence, we have $\sum_{i \in S} Z_i(t) \leq X_S(t)$ under any scheduling policy. This gives us

$$\begin{aligned} \sum_{i \in S} \mu_i &= \lim_{T \to \infty} \frac{\sum_{i \in S} \sum_{t=1}^{T} Z_i(t)}{T} \\ &\leq \lim_{T \to \infty} \frac{\sum_{t=1}^{T} X_S(t)}{T} = m_S, \forall S \subseteq \{1, 2, \dots, N\}. \end{aligned} \tag{4.19}$$

We can similarly establish (4.16) by noting that $\sum_{i=1}^{N} Z_i(t) = X_{\{1,2,\dots,N\}}(t)$, since the AP always transmits one packet as long as at least one client has an ON channel.

Finally, we establish (4.17). Let $\hat{X}_S$ be the random variable $\lim_{T \to \infty} \frac{\sum_{t=1}^{T} X_S(t) - T m_S}{\sqrt{T}}$ and $\hat{Z}_i$ be the random variable $\lim_{T \to \infty} \frac{\sum_{t=1}^{T} Z_i(t) - T \mu_i}{\sqrt{T}}$. Since $\sum_{i=1}^{N} Z_i(t) = X_{\{1,2,\dots,N\}}(t)$

and (4.16), we have $\sum_{i=1}^{N} \hat{Z}_i = \hat{X}_{\{1,2,\dots,N\}}$. We then have

$$
\begin{aligned}
&(\sum_{i=1}^{N} \sqrt{\sigma_i^2})^2 = (\sum_{i=1}^{N} \sqrt{E[\hat{Z}_i^2]})^2 \\
=&\sum_{i=1}^{N} E[\hat{Z}_i^2] + \sum_{i \neq j} \sqrt{E[\hat{Z}_i^2] E[\hat{Z}_j^2]} \\
\geq&\sum_{i=1}^{N} E[\hat{Z}_i^2] + \sum_{i \neq j} E[\hat{Z}_i \hat{Z}_j] \quad \text{(Cauchy-Schwarz inequality)} \\
=&E[(\sum_{i=1}^{N} \hat{Z}_i)^2] = E[\hat{X}^2_{\{1,2,\dots,N\}}] = v^2_{\{1,2,\dots,N\}}.
\end{aligned}
\tag{4.20}
$$

This completes the proof. □

4.4 Scheduling Policy with Tight Inner Bound

In this section, we derive a sufficient condition for the second-order delivery model $\{(\mu_i, \sigma_i^2) | 1 \leq i \leq N\}$ to be in the second-order capacity region. We also propose a simple scheduling policy that delivers the desirable second-order delivery models as long as they satisfy the sufficient condition. We state the sufficient condition as follows:

Theorem 9. *Given a second-order channel model $\{(m_S, v_S^2) | S \subseteq \{1, 2, \dots, N\}\}$, a second-order delivery model $\{(\mu_i, \sigma_i^2) | 1 \leq i \leq N\}$ is in the second-order capacity region if*

$$\sum_{i \in S} \mu_i < m_S, \forall S \subsetneq \{1, 2, \dots, N\}, \tag{4.21}$$

$$\sum_{i=1}^{N} \mu_i = m_{\{1,2,\dots,N\}}, \tag{4.22}$$

$$\sum_{i=1}^{N} \sqrt{\sigma_i^2} \geq \sqrt{v^2_{\{1,2,\dots,N\}}}, \tag{4.23}$$

$$\mu_i \geq 0, \sigma_i^2 > 0 \forall i. \tag{4.24}$$

Before proving Theorem 9, we first discuss its implications. Comparing the conditions in Theorems 8 and 9, we note that the only difference is that the sufficient condition requires strict inequality for (4.15) for all proper subsets. Hence, the sufficient condition describes an inner bound that is almost tight except on some boundaries.

We prove Theorem 9 by proposing a scheduling that achieves every point in the inner bound. Given $\{(\mu_i, \sigma_i^2)|1 \leq i \leq N\}$, define the *deficit* of a client i at time t as $d_i(t) = t\mu_i - \sum_{\tau=1}^{t} Z_i(\tau)$. In each time slot t, the AP chooses the client with the largest $d_i(t-1)/\sqrt{\sigma_i^2}$ among those with ON channels and transmits a packet to the chosen client. We call this scheduling policy the *variance-weighted-deficit* (VWD) policy.

We now analyze the performance of the VWD policy. Let $D(t) := \sum_{i=1}^{N} d_i(t)/\sum_{i=1}^{N}\sqrt{\sigma_i^2}$. We then have

$$\Delta d_i(t) := d_i(t) - d_i(t-1) = \mu_i - Z_i(t), \tag{4.25}$$

$$\begin{aligned} &\Delta D(t) := D(t) - D(t-1) \\ =&\frac{\sum_{i=1}^{N}\mu_i - \sum_{i=1}^{N} Z_i(t)}{\sum_{i=1}^{N}\sqrt{\sigma_i^2}} = \frac{m_{\{1,2,\dots,N\}} - X_{\{1,2,\dots,N\}}(t)}{\sum_{i=1}^{N}\sqrt{\sigma_i^2}}. \end{aligned} \tag{4.26}$$

Consider the Lyapunov function $L(t) := \frac{1}{2}\sum_{i=1}^{N}\sqrt{\sigma_i^2}\Big(\frac{d_i(t)}{\sqrt{\sigma_i^2}} - D(t)\Big)^2$. Let H^t be the system history up to time t. We can derive the expected one-step Lyapunov drift as

$$
\begin{aligned}
&\Delta(L(t)) := E[L(t) - L(t-1)|H^{t-1}] \\
=&E[\frac{1}{2}\sum_{i=1}^{N}\sqrt{\sigma_i^2}\Big(\frac{d_i(t)}{\sqrt{\sigma_i^2}} - D(t))\Big)^2 \\
&-\frac{1}{2}\sum_{i=1}^{N}\sqrt{\sigma_i^2}\Big(\frac{d_i(t-1)}{\sqrt{\sigma_i^2}} - D(t-1)\Big)^2|H^{t-1}] \\
=&E[\sum_{i=1}^{N}\sqrt{\sigma_i^2}\Big(\frac{d_i(t-1)}{\sqrt{\sigma_i^2}} - D(t-1)\Big)\Big(\frac{\Delta d_i(t)}{\sqrt{\sigma_i^2}} - \Delta D(t)\Big) \\
&+\frac{1}{2}\sum_{i=1}^{N}\sqrt{\sigma_i^2}\Big(\frac{\Delta d_i(t)}{\sqrt{\sigma_i^2}} - \Delta D(t))\Big)^2|H^{t-1}] \\
\leq&B + E[\sum_{i=1}^{N}\Big(\frac{d_i(t-1)}{\sqrt{\sigma_i^2}} - D(t-1)\Big)\Delta d_i(t) \\
&-\sum_{i=1}^{N}\sqrt{\sigma_i^2}\Big(\frac{d_i(t-1)}{\sqrt{\sigma_i^2}} - D(t-1)\Big)\Delta D(t)|H^{t-1}] \\
=&B + E[\sum_{i=1}^{N}\Big(\frac{d_i(t-1)}{\sqrt{\sigma_i^2}} - D(t-1)\Big)\Delta d_i(t)|H^{t-1}], \qquad (4.27)
\end{aligned}
$$

where B is a bounded constant. The last two steps follow because $\Delta d_i(t)$ and $\Delta D(t)$ are bounded and because $\sum_{i=1}^{N} d_i(t-1) = \sum_{i=1}^{N}\sqrt{\sigma_i^2}D(t-1)$.

The VWD policy schedules the client with the largest $d_i(t-1)/\sqrt{\sigma_i^2}$, which is also the client with the largest $d_i(t-1)/\sqrt{\sigma_i^2} - D(t-1)$, among those with ON channels. Hence, under the VWD policy, the system can be modeled as a Markov process whose state consists of the channel states and $d_i(t-1)/\sqrt{\sigma_i^2} - D(t-1)$ of all clients. Further, the VWD policy is the policy that minimizes $E[\sum_{i=1}^{N}\Big(\frac{d_i(t-1)}{\sqrt{\sigma_i^2}} - D(t-1)\Big)\Delta d_i(t)|H^{t-1}]$ for all t. We first show that the Markov process is positive-recurrent.

Lemma 3. *Assume that (4.21) – (4.24) are satisfied. Then, under the VWD policy, the system-wide Markov process, whose state consists of the channel states and* $d_i(t-$

$1)/\sqrt{\sigma_i^2} - D(t-1)$ *of all clients, is positive-recurrent.*

Proof. Due to (4.21), we can define

$$\delta := \min\{m_S - \sum_{i\in S}\mu_i | S \subsetneq \{1, 2, \dots, N\}\} > 0. \tag{4.28}$$

Further, since the channel of each client follows a positive-recurrent Markov process with finite states, there exists a finite number $\mathbb{T}$ such that

$$\mathbb{T}m_S - \frac{\delta}{2} \le E[\sum_{t=\tau+1}^{\tau+\mathbb{T}} X_S(t)|H^\tau] \le \mathbb{T}m_S + \frac{\delta}{2}, \tag{4.29}$$

for any H^τ.

Let $L^V(t)$ and $\Delta d_i^V(t)$ be the values of $L(t)$ and $d_i(t)$ under the VWD policy. From (4.27), we can bound the $\mathbb{T}$-step Lyapunov drift by

$$\begin{aligned}
&E[L^V(\tau+\mathbb{T}) - L^V(\tau)|H^\tau] \\
\le& B\mathbb{T} + E[\sum_{t=\tau+1}^{\tau+\mathbb{T}}\sum_{i=1}^{N}\Big(\frac{d_i(t-1)}{\sqrt{\sigma_i^2}} - D(t-1)\Big)\Delta d_i^V(t)|H^\tau] \\
\le& B\mathbb{T} + E[\sum_{t=\tau+1}^{\tau+\mathbb{T}}\sum_{i=1}^{N}\Big(\frac{d_i(t-1)}{\sqrt{\sigma_i^2}} - D(t-1)\Big)\Delta d_i^\eta(t)|H^\tau] \\
\le& A + E[\sum_{i=1}^{N}\Big(\frac{d_i(\tau)}{\sqrt{\sigma_i^2}} - D(\tau)\Big)(\sum_{t=\tau+1}^{\tau+\mathbb{T}}\Delta d_i^\eta(t))|H^\tau],
\end{aligned} \tag{4.30}$$

for any other scheduling policy η, where $d_i^\eta(t)$ is the value of $d_i(t)$ under η and A is a bounded constant. The last inequality follows because $\mathbb{T}$, $|d_i(t) - d_i(\tau)|$, and $\Delta d_i(t)$ are all bounded for all $t \in [\tau+1, \tau+\mathbb{T}]$.

We now consider the scheduling policy η that schedules the flow with the largest $d_i(\tau)/\sqrt{\sigma_i^2}$ among those with ON channels in all time slots $t \in [\tau+1, \tau+\mathbb{T}]$.

Without loss of generality, we assume that $d_1(\tau)/\sqrt{\sigma_1^2} \ge d_2(\tau)/\sqrt{\sigma_2^2} \ge \dots$. Under

η, a client i will be scheduled in time slot t if it has an ON channel and all clients in $\{1, 2, \dots, i-1\}$ have OFF channels, that is, $X_{\{1,2,\dots i\}}(t) = 1$ and $X_{\{1,2,\dots i-1\}}(t) = 0$. We hence have $\sum_{t=\tau+1}^{\tau+\mathbb{T}} Z_i(t) = \sum_{t=\tau+1}^{\tau+\mathbb{T}} X_{\{1,2,\dots i\}}(t) - \sum_{t=\tau+1}^{\tau+\mathbb{T}} X_{\{1,2,\dots i-1\}}(t)$. Therefore,

$$
\begin{aligned}
&E[\sum_{i=1}^{N} \Big(\frac{d_i(\tau)}{\sqrt{\sigma_i^2}} - D(\tau)\Big)(\sum_{t=\tau+1}^{\tau+\mathbb{T}} \Delta d_i^{\eta}(t))|H^{\tau}] \\
=&E[\sum_{i=1}^{N-1} \Big(\frac{d_i(\tau)}{\sqrt{\sigma_i^2}} - \frac{d_{i+1}(\tau)}{\sqrt{\sigma_{i+1}^2}}\Big)(\mathbb{T}\sum_{j=1}^{i} \mu_j \\
&- \sum_{t=\tau+1}^{\tau+\mathbb{T}} X_{\{1,2,\dots,i\}}(t)) + \Big(\frac{d_N(\tau)}{\sqrt{\sigma_N^2}} - D(\tau)\Big) \\
&\times (\mathbb{T}\sum_{j=1}^{N} \mu_j - \sum_{t=\tau+1}^{\tau+\mathbb{T}} X_{\{1,2,\dots,N\}}(t))|H^{\tau}] \\
\leq&\sum_{i=1}^{N-1} \Big(\frac{d_i(\tau)}{\sqrt{\sigma_i^2}} - \frac{d_{i+1}(\tau)}{\sqrt{\sigma_{i+1}^2}}\Big)(-\delta/2) + \Big(\frac{d_N(\tau)}{\sqrt{\sigma_N^2}} - D(\tau)\Big)(-\delta/2) \\
=&\Big(\frac{d_1(\tau)}{\sqrt{\sigma_1^2}} - D(\tau)\Big)(-\delta/2),
\end{aligned}
\tag{4.31}
$$

where the inequality holds due to (4.22), (4.28), and (4.29).

Combining (4.30) and (4.31), and we have

$$
E[L^V(\tau + \mathbb{T}) - L^V(\tau)|H^{\tau}] < -\delta, \tag{4.32}
$$

if $\max_i \Big(\frac{d_i(\tau)}{\sqrt{\sigma_i^2}} - D(\tau)\Big) > 2(A/\delta + 1)$, and

$$
E[L^V(\tau + \mathbb{T}) - L^V(\tau)|H^{\tau}] \leq A, \tag{4.33}
$$

if $\max_i \Big(\frac{d_i(\tau)}{\sqrt{\sigma_i^2}} - D(\tau)\Big) \leq 2(A/\delta + 1)$. Recall that $\sum_i \Big(\frac{d_i(\tau-1)}{\sqrt{\sigma_i^2}} - D(\tau - 1)\Big) = 0$ and the channel of each client follows a Markov process with finite states. Hence, all states of the system with $\max_i \Big(\frac{d_i(\tau)}{\sqrt{\sigma_i^2}} - D(\tau)\Big) \leq 2(A/\delta + 1)$ belong to a finite set of states. By the

Foster-Lyapunov Theorem, the system-wide Markov process is positive-recurrent. □

We now show that the VWD policy delivers all desirable second-order delivery models that satisfy the sufficient conditions (4.21) – (4.24), and thereby establishing Theorem 9.

Theorem 10. *Assume that (4.21) – (4.24) are satisfied. Then, under the VWD policy,* $\lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t)}{T} = \mu_i$ *and* $E[(\lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t) - T\mu_i}{\sqrt{T}})^2] \le \sigma_i^2, \forall i.$

Proof. Since the system-wide Markov process is positive recurrent under the VWD policy, we have:

$$\lim_{T\to\infty} \frac{d_i(T)/\sqrt{\sigma_i^2} - D(T)}{T} \to 0, \forall i, \tag{4.34}$$

$$\lim_{T\to\infty} \frac{d_i(T)/\sqrt{\sigma_i^2} - D(T)}{\sqrt{T}} \to 0, \forall i. \tag{4.35}$$

First, we show that $\lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t)}{T} = \mu_i, \forall i$. Recall that $d_i(t) = t\mu_i - \sum_{\tau=1}^{t} Z_i(\tau)$ and $D(t) = \sum_{i=1}^{N} d_i(t) / \sum_{i=1}^{N} \sqrt{\sigma_i^2}$. By (4.22), we have:

$$\lim_{T\to\infty} \frac{D(T)}{T} = \lim_{T\to\infty} \frac{\sum_{i=1}^{N} T\mu_i - \sum_{t=1}^{T}\sum_{i=1}^{N} Z_i(t)}{T\sum_{i=1}^{N}\sqrt{\sigma_i^2}}$$
$$= \lim_{T\to\infty} \frac{Tm_{\{1,2,\ldots,N\}} - \sum_{t=1}^{T} X_{\{1,2,\ldots,N\}}}{T\sum_{i=1}^{N}\sqrt{\sigma_i^2}} = 0. \tag{4.36}$$

Hence, by (4.34), we have $\lim_{T\to\infty} \frac{d_i(T)}{T} = \mu_i - \lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t)}{T} = 0$, for all i.

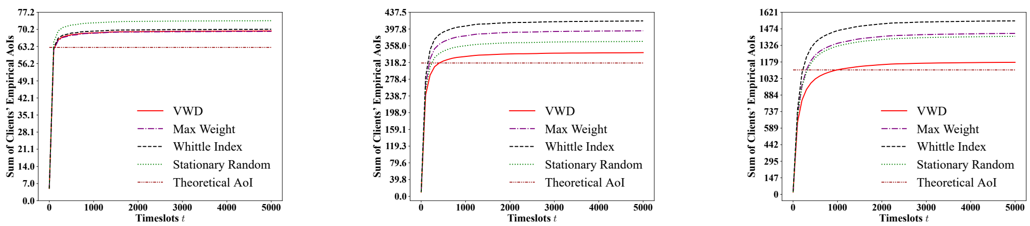
Next, we show that $E[(\lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t) - T\mu_i}{\sqrt{T}})^2] \le \sigma_i^2, \forall i$. We have, by (4.23),

$$E[(\lim_{T\to\infty} \frac{D(T)}{\sqrt{T}})^2] = \frac{v_{\{1,2,\ldots,N\}}^2}{(\sum_{i=1}^{N}\sqrt{\sigma_i^2})^2} \le 1, \tag{4.37}$$

and, hence,

$$
\begin{aligned}
&E[(\lim_{T\to\infty} \frac{\sum_{t=1}^{T} Z_i(t) - T\mu_i}{\sqrt{T}})^2] = E[(\lim_{T\to\infty} \frac{d_i(T)}{\sqrt{T}})^2] \\
=&\sigma_i^2 E[(\lim_{T\to\infty} \frac{D(T)}{\sqrt{T}})^2] \le \sigma_i^2. \qquad (4.38)
\end{aligned}
$$

□



(a) N = 5 Clients. (b) N = 10 Clients. (c) N = 20 Clients.

Figure 4.3: Total Uniformly Weighed Empirical Age of Information (AoI) Averaged Over 1000 Runs.

We conclude this section by discussing how to leverage Theorems 9 and 10 to solve the second-order network optimization problem. Recall that the performance of a client i is modeled by $F_i(\mu_i, \sigma_i^2)$. For example, when the goal is to minimize total AoI, we can define $F_i(\mu_i, \sigma_i^2) = -\frac{1}{2}(\frac{\sigma_i^2}{\mu_i^2} + \frac{1}{\mu_i}) - \frac{1}{\lambda_i} + \frac{1}{2}$. Hence, the second-order optimization problem can be written as the following:

$$
\max \sum_{i=1}^{N} F_i(\mu_i, \sigma_i^2) \qquad (4.39)
$$

$$
\text{s.t. } (4.21) - (4.24). \qquad (4.40)
$$

The condition (4.21) involves strict inequalities, which cannot be used by standard optimization solvers. We change (4.21) to $\sum_{i\in S}\mu_i \leq m_S - \delta$, where δ is a small positive number. After the change, the optimization problem can be directly solved by standard solvers to find the optimal $\{\mu_i, \sigma_i^2 | 1 \leq i \leq N\}$. After finding the optimal $\{\mu_i, \sigma_i^2 | 1 \leq i \leq N\}$, one can use the VWD policy to attain the optimal network performance.

4.5 Simulation Results

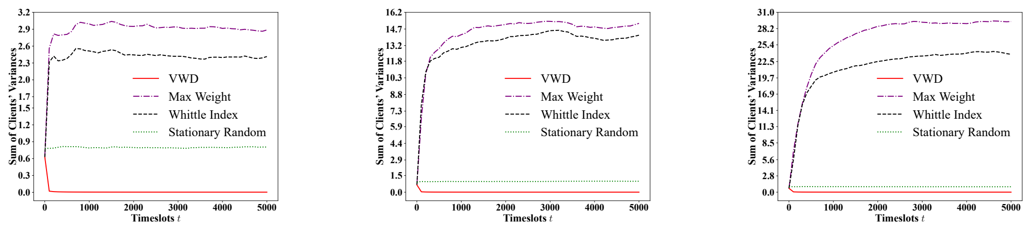
In this section, we present the simulation results for the proposed scheduler VWD. The objective is to minimize the total weighted AoI, $\sum_i \alpha_i \overline{AoI}_i$, where α_i is the weight of client i. The system model is the one discussed in Section 4.2. Each client has a Gilbert-Elliot channel with transition probabilities p_i and q_i. In each time slot, each client i generates a new packet with probability λ_i. VWD is evaluated against three recent scheduling policies on this problem. We provide a description of each policy, along with modifications needed to fit the testing setting.

- **Whittle index policy**: This policy is based on the Whittle index policy in [29]. Under our setting, the policy calculates an index for ON clients based on their AoIs as $W_i(t) = \frac{AoI_i^2(t)}{2} - \frac{AoI_i(t)}{2} + \frac{AoI_i(t)}{q_i/(p_i+q_i)}$, and then schedules the ON client with the largest index. [29] has shown that $W_i(t)$ is indeed the Whittle index of a client when the channel is i.i.d., i.e., $p_i + q_i = 1$, and $\lambda_i = 1$.

- **Stationary randomized policy**: This policy calculates a weight μ_i for each client. In each time slot, it randomly picks an ON client, with the probability of picking i being proportional to μ_i. In the setting of [26], it has been shown that, when μ_i is properly chosen, this policy achieves an approximation ratio of four in terms of total weighted AoI. In our setting, we choose μ_i to be the optimal μ_i from solving (4.39).

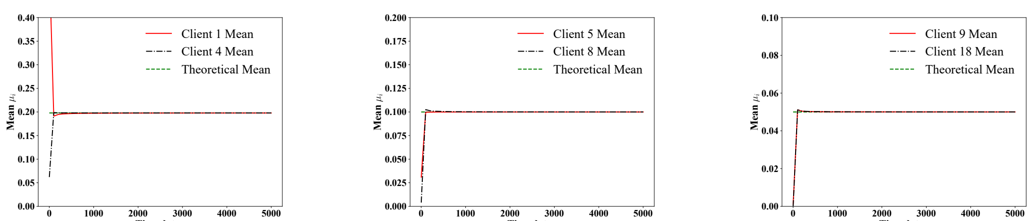
- **Max weight policy** [26]: This policy schedules the ON client with the largest

$(AoI_i(t) - z_i(t))/\mu_i$. In the setting of [26], $z_i(t)$ is the time since client i generates the latest packet. It has been shown that the total weighted AoI under this policy is no larger than that under the stationary randomized policy, and therefore this policy also achieves an approximation ratio of four. In our setting, the AP does not know when each client generates a new packet. Hence, we choose $z_i(t)$ to be $\frac{1}{\lambda_i}$, which is the expected time since client i generates the latest packet.



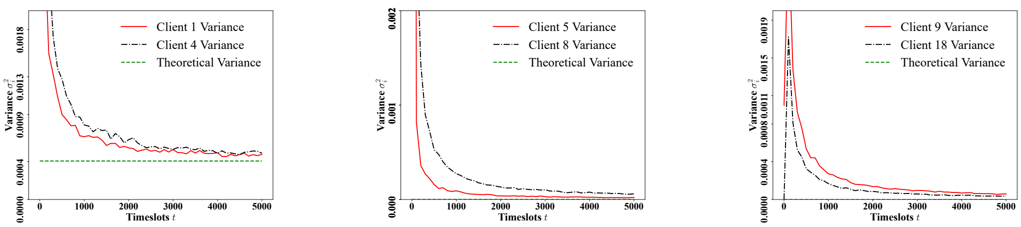
(a) N = 5 Clients. (b) N = 10 Clients. (c) N = 20 Clients.

Figure 4.4: Total Uniformly Weighed Empirical Age of Information (AoI) Averaged Over 1000 Runs.



(a) N = 5 Clients. (b) N = 10 Clients. (c) N = 20 Clients.

Figure 4.5: Mean Convergence of Two Randomly Selected Clients.



(a) N = 5 Clients.
(b) N = 10 Clients.
(c) N = 20 Clients.

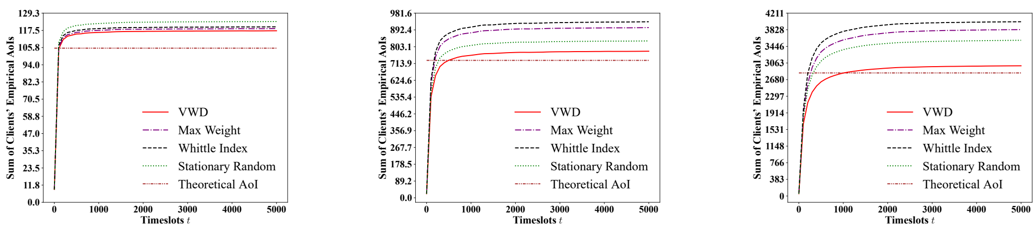
Figure 4.6: Variance Convergence of Two Randomly Selected Clients.

We consider three different systems, each with 5 clients, 10 clients, and 20 clients, respectively. For each system, p_i and q_i are randomly chosen from the range $(0.05, 0.95)$, and $\{\lambda_i\}$ is randomly chosen from $(\frac{0.1}{N}, \frac{1}{N})$. After determining the values of p_i, q_i and λ_i, we generate 1000 independent traces of channels and packet arrivals. The performance of each policy is the average over these 1000 independent traces. We consider both the unweighted case, i.e., $\alpha_i \equiv 1, \forall i$, and the weighted case. In addition to the evaluated policies, we also include the numerical solutions from solving the problem (4.39), which is referred to as the Theoretical AoI.

4.5.1 Empirical AoI Performance With Equal AoI Weights

Fig. 4.3 shows the average total AoI for different network sizes $N = \{5, 10, 20\}$ when $\alpha_i \equiv 1$. It can be observed that VWD achieves the smallest total AoI in all systems, with max weight performing virtually the same as VWD when $N = 5$. VWD's superiority becomes more significant as N increases. It can also be observed that the empirical AoI under VWD is very close to the theoretical AoI based on the solution to (4.39), and the difference decreases as N increases. The differences between the empirical AoI under VWD and the theoretical one are 10.7%, 7.8%, and 6.1% for $N = 5, 10, 20$, respectively.

To understand why VWD performs much better than the other three policies, we evaluate the total empirical variance under each policy. Specifically, let $d_i(t)$ be the total number of packet deliveries for client i from time 1 to time t. The empirical variance of a client i at time t is defined as the variance of $\frac{d_i(t)}{\sqrt{t}}$ across all 1000 independent runs. The total empirical variance is then the sum of the empirical variances of all clients. Fig. 4.4 shows that VWD has much smaller variances than the other three policies. The ability to properly control variance enables VWD to achieve small AoIs.



(a) N = 5 Clients. (b) N = 10 Clients. (c) N = 20 Clients.

Figure 4.7: Total Weighted Empirical Age of Information (AoI) Averaged Over 1000 Runs.

We also evaluate the convergence time of VWD. For each system, we randomly select two clients and plot their empirical means, i.e., the average of $\frac{d_i(t)}{t}$ across all independent runs, and empirical variances. Since the objective is to minimize the unweighted sum of AoIs, the optimal solution to (4.39) has $\mu_i = \mu_j$ and $\sigma_i^2 = \sigma_j^2$ for all $i \neq j$. We call the optimal μ_i and σ_i^2 obtained from solving (4.39) the theoretical mean and the theoretical variance, respectively. The results are shown in Figs. 4.5 and 4.6. It can be observed that both the empirical means and the empirical variances of clients indeed converge to their respective theoretical values. The empirical means converges to the theoretical ones very fast. On the other hand, it takes up to 355 slots for the empirical variances to be within 0.001 from the theoretical variances. This convergence time may be the reason why the empirical AoI is larger than the theoretical one.

4.5.2 Weighed Total AoI Evaluation

We now present the results for the weighted AoI. The weights $\alpha_1, \alpha_2, \ldots$ are randomly chosen from the range $(1, 5)$ and independently from each other. All other parameters are the same as in the unweighted case. Fig. 4.7 shows results for network sizes $N = \{5, 10, 20\}$. VWD still outperforms other policies for all tested systems. Similar to the unweighted case, it can be observed that the superiority of VWD becomes more significant, and the gap between VWD and theoretical AoI becomes smaller, with more clients in the system.

4.6 Conclusion

In this paper, we presented a theoretical second-order framework for wireless network optimization. This framework captures the behaviors of all random processes by their second-order models, namely, their means and temporal variances. We analytically established a simple expression of the second-order capacity region of wireless networks. A new scheduling policy, VWD, was proposed and proved to achieve every interior point of the second-order capacity region. The framework utility is demonstrated by applying it to the problem of AoI optimization over Gilbert-Elliot channels. We derived closed-form expressions of second-order models for both Gilbert-Elliot channels and AoIs, and formulated the problem of minimizing weighted total AoI as an optimization problem over the means and temporal variances of delivery processes. The solution of this optimization problem can then be used as parameters for VWD. Simulation results show that VWD achieves much smaller weighted total AoI than other policies.

5. RELATED WORK

Information update systems have gained a lot of research interests in the recent years, as many emerging wireless applications require real-time status updates. One state-of-the-art performance metric is Age-of-Information (AoI), which focuses on the elapsed time of last delivery. There have been a lot of studies on the optimizaiton of AoI. For the wireless information-update system, Kadota et.al have built the model of AoI optimization problem in [23] when considering unreliable channel and multiple information flows. Both Hsu et.al in [30] and Kadota et.al in [26] have recently proposed the scheduling algorithm when considering random arrivals to minimize the average age. Zheng, Zhou and Niu [31] model the estimation error along timeliness and propose the Urgency of Information (UoI) as a new performance metric that can be viewed as the non-linear AoI problems. Some work of AoI has research the remote estimation problem. Sun, Polyanskiy and Uysal-Biyikoglu [32] show that, under signal-independent sampling policies, the minimization of mean square sampling error problem can be written as the minimization problem of AoI when sampling the Wiener process. One further work is shown for the Gauss-Markov process sampling in [33]. Li, Li and Hou [34] analyze different sampling behavior and propose a guideline for AoI minimizing policy in general sampling and remote estimation problems. Yin et al. [35] introduce the concept of Effective AoI to capture the proactive information update and timely information delivery under some user‘s request pattern. Tsai and Wang [36] propose a framework for controller side AoI problem and sensor side remote estimation problem under random 2-way delay. Some extended area are also researched with AoI, such as [37] in vehicular network, [38] in multi-hops wireless network, and [39] for link-scheduling optimization in wireless system. However, all AoI-related works only focus on the information from the last delivery. On the contrary, our work considers not only the freshness of the information but also the quantity of fresh information updates.

There have been many works on scheduling in wireless networks for minimizing AoI. In [40], the Tripathi and Moharir schedule over multiple orthogonal channels and propose Max-Age Matching and Iterative Max-Age Scheduling, which they show to be asymptotically optimal. Hsu, Modiano and Duan [41] studied the problem of scheduling updates for multiple clients where the updates arrive i.i.d. Bernoulli, and formulate the Markov decision process (MDP) and prove structural results and finite-state approximations. In [29], Hsu follows up this work by showing that a Whittle index policy can achieve near optimal performance with much lower complexity. Sun et al. [42] studied scheduling for multiple flows over multiple servers, and show that maximum age first (MAF)-type policies are nearly optimal for i.i.d. servers. In [43], Talak, Karaman and Modiano study scheduling a set of links in a wireless network under general interference constraints. The optimization of AoI and timely-throughput were studied in [9, 44]. All of these works assume i.i.d channels.

Another related performance metric is timely-throughput, which is defined as the long-term average of timely deliveries. Hou, Borkar and Kumar [5] first propose a frame-based model for the real-time wireless networks and captures the delay constraints of wireless flows. Under this model, the performance of each flow is determined by its timely-throughput, which is the long-term average number of timely deliveries. Jaramillo, Srikant, and Ying [45] have studied wireless flows with heterogeneous delay and timely-throughput requirements. Kang et. al.[46] have studied the performance of timely-throughputs in ad hoc wireless networks with stochastic packet arrivals. Meko and Seid [47] have proposed a randomized scheduling algorithm for real-time flows. Zhang et. al. [48] have studied timely-throughputs in heterogeneous cellular networks with mobile nodes. Lashgari and Avestimehr [49] have looked for the additive gap of maximal timely throughput in a relaxed problem under the time-varying channel states. Tsanikidis and Ghaderi [50] recently propose a randomized policy to improve the deliver ratio in the frame-based model. Chen and Huang [51] derive a Markov Decision Process solution for optimizing the timely-throughput and quantify the improvement when applying the predictive scheduling policy. An important limitation of these studies is that they only consider the long-term average of timely deliveries and ignore short-term fluctuations. Capturing short-term fluctuations is in particular relevant to information update systems. Singh, Hou and Kumar [52] study the fluctuation of the timely-throughput. Hou [3], and Guo and Hou [1] consider systems where the instantaneous performance of a flow depends on the number of recent timely deliveries, and propose scheduling policies that aim to optimize the system. These two studies are most relevant to this work, but they rely on a frame-based structure where all flows generate packets at the same time and each flow can only store the most recent packet.

There have been a limited number of works on Markov channel and source models related to AoI. In the recent work [53], Pan et al. study scheduling a single source and choosing between a Gilbert-Elliott channel and a deterministic lower rate channel. Buyukates and Ulukus [54] study the age-optimal policy for a system where the server is a Gilbert-Elliott model and one where the sampler follows a Gilbert-Elliott model. In [55], Nguyen et al. analyze the Peak Age of Information (PAoI) of a two-state Markov channel with differing cases of channel state information (CSI) knowledge. Kam et al. [56] study the remote estimation of a Markov source, and they propose effective age metrics that capture the estimation error. Our work differs in that we focus on scheduling for multiple clients from a single AP over parallel non-i.i.d. channels.

There have been some recent efforts on studying short-term performance through Brownian motion approximation [57, 58, 59, 18], but each of them is limited to a specific channel model and a specific application.

6. CONCLUSION

In the first work, we have studied the problem of minimizing the total Loss-of-Confidence (LoC) in real-time wireless networks with heterogeneous links quality, where the LoC of each flow only depends on the timely deliveries in a window of the recent past. In the second work, we have studied a remote sensing problem and built a model that considers both the freshness of the information update and the quantity requirements of real-time wireless flows.

In the third work, we generalized and presented a theoretical second-order framework for wireless network optimization. This framework captures the behaviors of all random processes by their second-order models, namely, their means and temporal variances. A new scheduling policy, VWD, was proposed and proved to achieve every interior point of the second-order capacity region. The framework utility is demonstrated by applying it to the problem of AoI optimization over Gilbert-Elliot channels. We derived closed-form expressions of second-order models for both Gilbert-Elliot channels and AoIs, and formulated the problem of minimizing weighted total AoI as an optimization problem over the means and temporal variances of delivery processes. The solution of this optimization problem can then be used as parameters for VWD. Simulation results show that VWD achieves much smaller weighted total AoI than other policies.

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APPENDIX A

The Steady-State MSE of Real-Time Flows with Periodic Deliveries

In this section, we consider the real-time estimation problem as described in Section 2.6. We calculate the MSE of a real-time flow when the corresponding sensor i delivers one packet every $\frac{1}{q_i - \delta p_i}$ intervals periodically.

Let t_0 be the first interval in which sensor i delivers a packet. Sensor i therefore delivers one packet in each of the intervals $t_0, t_0 + \frac{1}{q_i-\delta p_i}, t_0 + 2\frac{1}{q_i-\delta p_i}, \dots$. Recall that $\Sigma_{i,t}$ is the MSE of sensor i in interval t. We then define the *steady-state MSE* of sensor i as $\Sigma_i := \lim_{k\to\infty} \Sigma_{i,t_0+k\frac{1}{q_i-\delta p_i}}$.

To calculate Σ_i, we note that, by Alg. 2, we have:

$$\begin{aligned}
&\Sigma_{i,t_0+k\frac{1}{q_i-\delta p_i}} \\
=&(\Sigma_{i,t_0+k\frac{1}{q_i-\delta p_i}-1} + W_i)R_i/(\Sigma_{i,t_0+k\frac{1}{q_i-\delta p_i}-1} + W_i + R_i),
\end{aligned}$$

since there is a packet delivery in interval $t_0 + k\frac{1}{q_i-\delta p_i}$. In addition, we also have

$$\begin{aligned}
\Sigma_{i,t_0+k\frac{1}{q_i-\delta p_i}-1} &= \Sigma_{i,t_0+k\frac{1}{q_i-\delta p_i}-2} + W_i \\
&= \Sigma_{i,t_0+k\frac{1}{q_i-\delta p_i}-3} + 2W_i \\
&\dots \\
&= \Sigma_{i,t_0+(k-1)\frac{1}{q_i-\delta p_i}} + (\frac{1}{q_i-\delta p_i} - 1)W_i.
\end{aligned}$$

Combining these two equations and setting $W_i = p_i = q_i$ and $R_i = 20$, as used in Section

2.6, yield

$$\begin{aligned}
\Sigma_i &= \lim_{k\to\infty} \Sigma_{i,t_0+k\frac{1}{q_i-\delta p_i}} \\
&= \lim_{k\to\infty} \frac{20(\Sigma_{i,t_0+(k-1)\frac{1}{q_i-\delta p_i}} + \frac{W_i}{1-\delta})}{(\Sigma_{i,t_0+(k-1)\frac{1}{q_i-\delta p_i}} + \frac{W_i}{1-\delta} + 20)} = \frac{20(\Sigma_i + \frac{W_i}{1-\delta})}{(\Sigma_i + \frac{W_i}{1-\delta} + 20)},
\end{aligned}$$

and hence

$$\Sigma_i = \frac{-\frac{1}{1-\delta} + \sqrt{\frac{1}{(1-\delta)^2} + \frac{80}{1-\delta}}}{2},$$

for all i. In particular, when $\delta = 0$, we have $\Sigma_i = 4$. This justifies the simulation settings in Section 2.6.