

ALGEBRA: A META ANALYTIC REPORT OF INSTRUCTIONAL  
INTERVENTIONS

A Dissertation

by

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## ABSTRACT

The present study is a systematic meta-analytic study of 38 primary sources investigating the effectiveness of algebra interventions for students who are struggling. We systematically searched and screened over 1,300 records for studies to include. We computed summary effect sizes for each of the 12 interventions included. We conducted 4 separate meta-regressions for potential moderators found to influence the effectiveness of intervention. Results indicate interventions to improve algebra performance on post-test measures do appear to have evidence of effectiveness. Summary effect sizes at post-intervention were medium to large for group design studies ( $k = 18$ ,  $g = 0.71$ , 95% CI, [0.50, 0.93],  $p < .0001$ ) and large to very large for single case studies ( $k = 20$ ,  $Tau-U = 0.93$ , 95% CI, [0.76, 1.11],  $p < .0001$ ). Data was considered unbalanced due to multiple interventions being used within the same study, so individual effect sizes for specific interventions were unable to be compared and accurately calculated without significant limitations. Sample sizes were considered small and therefore results should be cautiously interpreted. Moderator analyses provided minimal information due to unbalanced data, but showed minimal change in effect sizes despite legal and systematic changes through Common Core (2010). Follow-up data did not seem to moderate effect size, nor did length of intervention. Training of the interventionist suggested minor impact on effectiveness of intervention; however, small sample size and unbalanced data limited the application of this information. We discuss interpretations and implications

of these results, as well as limitations and future directions for algebra intervention research.

## DEDICATION

This work is foremost dedicated to my grandfather who inspired me to become a school psychologist, I miss you greatly and appreciate everything you taught me.

Secondly, this work is dedicated to all those who help struggling learners; you are true heroes and are underappreciated.

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The data analyzed as a part of this meta-analysis was coded by the student and 30% of the studies were also coded by a fellow graduate student. The analyses depicted in Chapter IV were guided in part by Christopher Thompson, Ph.D. of the Department of Educational Psychology. All other work conducted for this dissertation was completed by the student independently.

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# CHAPTER I

## INTRODUCTION

Expectations for students in mathematics have become increasingly more demanding in recent years due to the federal Science, Technology, Engineering, and Mathematics (STEM) education efforts and the adoption of the Common Core State Standards (CCSS). For example, CCSS implements more rigorous and greater conceptual understanding of basic math skills than are currently found in most state standards (Watt, Watkins & Abbit, 2016). Further, the STEM initiative has placed a greater emphasis on mathematics in the K-12 curriculum, challenging schools to broaden the involvement of minority populations in STEM programming (Watt et al., 2016).

The National Assessment of Educational Progress (NAEP, 2013) identified a large gap in achievement and expectation for students in math, particularly struggling learners and those with specific learning disabilities (SLD), and in relation to algebra-based concepts. Research in this area supports the notion that early expertise and mastery of math skills predicts future academic achievement more than any other skill (Duncan et al., 2007). These successes are linked to achievement after high school (Ketterlin-Geller, Chard, & Fien, 2008). Results show those who have not mastered the basic computational fluency skills by the end of elementary school are at a significantly higher risk for future difficulties in mathematics and problem solving, including algebra (Axtell, McCallum, Bell, & Poncy, 2009; Ketterlin-Geller, Chard, & Fien, 2008). With

the completion of Algebra I being mandatory in most school districts to receive a diploma, the importance to understand this math skill cannot be undervalued

### **Educational Reform**

Reform efforts at the state and national levels, including the CCSS and the National Council of Teachers of Mathematics (NCTM), call for increased rigor in curriculum and improved performance in the field of mathematics for students (National Center for Education Statistics [NCES], 2013). In looking at reform, efforts can be considered from multiple perspectives, including the state of performance in mathematics, the implementation of certain legal guidelines, state and national standards, and the effects on special education.

### **State of Performance**

Recent data from the NAEP (2015) revealed only 40% of fourth grade and 33% of eighth grade students meet proficiency standards in mathematics. Moreover, students who possess a specific learning disability (SLD) perform worse on academic measures in mathematics (NAEP, 2015). Math skill deficits are identified as the second highest factor in the identification of a student learning disability (Kavale & Reese, 1992) or among the top three (Cortellia & Horowitz, 2014). In fact, Burns, Appleton, and Steuhouwer (2005) found an average of 20% of students require general supplementary academic supports outside what the typical classroom instruction provides.

Approximately 5-8% of school-aged students evidence a skill deficit of some sort in the field of mathematics (Geary, 2004; Kosci, 1974). Success in algebra requires a solid

foundation and understanding of basic math computations in addition to a strong conceptual framework and problem-solving ability (Thornton, Langrall, & Jones, 1997).

### **Difficulties for Students with Specific Learning Disabilities (SLD).**

Under federal law, students are eligible to receive special education services if they have a disability that adversely affects academic and/or functional performance (Individuals with Disabilities Education Improvement Act [IDEIA], 2004). One of the disability categories is specific to those students who demonstrate psychological processing deficits that result in lower than expected achievement (IDEIA, 2004). Students with SLD experience challenges in solving problems, recognizing and selecting appropriate strategies, organizing information, monitoring their problem-solving method, generalizing strategies to appropriate situations, and assessing problems for accuracy (Miller & Mercer, 1997). Teachers reported they do not feel students with SLD demonstrate the adequate, nor consistent, knowledge of these facts nor do they demonstrate the ability to apply them in conceptual-based problems, like algebra (Bottge, 1999). Students who have SLD experience challenges in procedural and conceptual aspects of math, both of which are necessary for success in algebra (Watt, 2013). As problems become more challenging and require the use of multiple operations (e.g., algebra and fractions), students with SLD make more procedural errors and have difficulty detecting errors after they have been committed (Geary, 2003). Due to the nature of algebra, a subgroup of mathematics that demands multiple computations and procedures, it is understandable why algebra is among the most difficult for students

who struggle academically (Jordan, Miller, & Mercer, 1999). The prevalence of these deficits has increased with the amplified rigor of the mathematics curriculum through the implementation of the CCSS and the push for STEM education efforts (NCES, 2013; Watt et al., 2016). These efforts are largely exemplified through the CCSS.

### **Common Core and Math Instruction in Special Education.**

Based on the numerous academic, functional, and social needs of the diverse population of students with disabilities, it is vital that educational practitioners implement interventions that are both empirically validated and capable of being tailored to fit the individual's needs (Watt et al., 2016). The CCSS (2010) set a high bar for success, requiring students to develop a depth of understanding and the ability to apply mathematics to novel situations. The standards stress profound conceptual understanding to certify that students understand the vital information needed to succeed at higher levels, both in and out of school. High school standards require students to reason mathematically and apply mathematical ways of thinking to real world issues and challenges while elementary standards require students to grasp the underlying concepts and begin to think critically about how these skills can be applied both inside and outside the classroom environment (CCSS, 2010). Due to the increased demands of the more challenging curriculum, not all students meet the expectations of the CCSS; some require more intensive supports.

Mathematics instruction in special education classrooms, in most cases, however, emphasizes rote memorization of computational skills and facts, rather than encouraging the students to develop a deeper, conceptual understanding of the higher-level concepts



(Marita & Hord, 2017; Rivera, 1997). Educators explained that, in special education classrooms, they do not focus on developing logical thinking or applying the mathematical skills to real-world situations, but rather limit themselves to promoting rote memorization (Marita & Hord, 2017). This is counterintuitive to current trends in research and academics, as national standards are promoting earlier integration of problem-solving skills (Bottge, 1999; United States Department of Education, 2006; Woodward & Montague, 2000) and integrating classroom content with real-world application (Bottge, Rueda, & Skivington, 2006; Haselbring, Lott, & Zydney, 2006). In order to understand how to provide effective intervention for students with SLD, other disabilities, or those who struggle learning mathematics, it is first important to understand the means and sequences by which they learn mathematics.

### **Developmental Trajectory of Math Skill Acquisition**

The progression of math skills is variable for every student, with typical development including a series of four phases: allegorization, integration, analysis, and synthesis (Knisley, 2001). In the first phase, the concept is figuratively described in a context that is familiar to the student. Integration allows for comparison, measurement, and exploration as the student is able to differentiate the particular skill from other skills. Analysis permits the new concept and information to add itself to the existing knowledge base. Lastly, synthesis allows the concept to form its' own identity and can be used to help understand future concepts that may be in one of the prior stages. Skills in mathematics are slow to develop and inaccurate initially, yet as the student learns the skill, the responses become more accurate while task completion remains slow (Haring

& Eaton, 1978). As learning continues and mastery begins to emerge, the speed at which they are able to complete the task accurately increases and they can begin to apply their knowledge to foreign stimuli and learn to solve problems. The sequential learning process of mathematics, suggests concepts and skills build upon one another (Haring & Eaton, 1978). Basic skills with numbers are important for a variety of everyday uses, providing a foundation for learning higher-level mathematics, as well as succeeding in work life (Aunio & Rasanen, 2016; Ball et al., 2005).

Research supports the notion that early expertise and mastery of math skills predicts future academic achievement more than any other skill (Duncan et al., 2007). Additionally, these successes were linked to achievement after high school (Ketterlin-Geller, Chard, & Fien, 2008). Students who have not mastered the basic computational fluency skills by the end of elementary school are at a significantly higher risk for future difficulties in mathematics and problem solving, including algebra (Axtell, McCallum, Bell, & Poncy, 2009; Ketterlin-Geller et al., 2008).

The Matthew Effect describes the increased difference in knowledge between higher functioning and lower functioning students and how it grows with time - “the rich get richer and the poor get poorer” (Stanovich, 1986, p. 360). This is true for mathematical difficulties as well. Growth rates of students in the lower percentiles are slower than those in the higher percentiles, indicating the gap between the two groups widens as time progresses (Morgan, Farkas, & Wu, 2009). Greenstein and Strains (1977) found that mathematics abilities of students with SLD plateaued at the 4th-grade level; these students rarely achieved higher-level problem-solving skills. Warner, Alley,

Schumaker, Deshler, and Clark (1980) discovered adolescents with SLD reached a mathematics plateau after 7th grade and made on average only one year of growth in mathematics between Grades 7 and 12. These students who present early difficulties that are not addressed continue to have difficulties in the future (Calhoon, Wall, Flores, & Houchins, 2007). This has been demonstrated across a variety of mathematical concepts, from arithmetic (Steel & Funnell, 2001), to computation (Steel & Funnell, 2001), to problem solving (Axtell et al., 2009; Ketterlin-Geller et al., 2008).

### **Importance of Algebra**

An important notion of mathematics is being able to mentally represent the concepts and translate those schemas into useful information (Jitendra, DiPipi, & Perron-Jones, 2002). By doing so, students are better able to represent the problem presented and have been shown to improve academic performance (Jitendra & Xin, 1997; Xin & Jitendra, 1999). While it is important to understand the process by which students learn mathematics, the focus of this paper will center primarily on the intervention efforts and pedagogical techniques for algebra in the classroom. Algebra is a critical area of mathematical knowledge that sets the foundation for future math success both inside and outside the classroom (Fennell, 2008). The Nation's Report Card (2015) explained that two of the three assessed algebra skills in 4<sup>th</sup> graders (create a pattern of shapes given a verbal description, and solve a one variable linear equation) fell below the proficiency standards and two of the five skills assessed for 8<sup>th</sup> graders (complete a table from a description of a linear relationship, and describe the location of a line in the plane from its equation) fell below the proficiency standards.

The Third International Mathematics and Science Study (TIMSS) results positioned the scores of the United States 12th-grade students lower than 11 of 16 participating countries on the algebra subscore (U.S. Department of Education, 1998), and the most recent Programme for International Student Assessment (PISA) placed the United States in 38<sup>th</sup> place out of 71 countries (OECD, 2016). Additionally, the National Mathematics Advisory Panel (NMAP, 2008) and the RAND Mathematics Study Report (2002) called for improvement in students' learning of algebra because of this gap in achievement. With the gap in algebra achievement and expectations (NAEP, 2015), intervention efforts are vital to providing empirically supported techniques and strategies for remedying these problems.

Due to a conceptual framework that requires abstract thinking and the representation of quantities with non-numeric symbols, algebra is considered the gateway skill for higher order mathematics (Watt et al., 2016). Algebra failure is a key predictor of high school dropout (Silver, Saunders, & Zarate, 2008), has been deemed a “central concern” for mathematics by the NMAP (2008, p. xii), and is a critical concept linked to future success in higher level mathematics, entrance into college, as well as financial equity in the labor force (Fennel, 2008; Watt et al., 2016). Algebra teaches students the language of math while continuing to develop important critical thinking skills, logic, and problem-solving ability (Fennel, 2008).

Not all students graduate from high school with the aspiration to attend college or advance further in a post-secondary education. The nature of algebra spans across a variety of domains; it can be utilized calculating one's income tax, determining statistics

for athletes, or estimating a car's gas mileage. Don Davis, the executive director of the Electrical Training Institute, which runs apprenticeship programs for union electricians in Los Angeles, said, "If you want to work in the real world, if you want to wire buildings and plumb buildings, that's when it requires algebra" (Helfand, 2006, p.1).

### **Prevalence of Algebra Failure**

The ability to master algebraic concepts and skills is a critical step to success in college mathematics courses; however, many students find themselves underprepared and fail the first time they take higher level algebra courses (Balfanz, McPartland, & Shaw, 2002; Finkelstein, Fong, Tiffany-Morales, Shields, & Huang, 2012; Huang, Snipes, & Finkelstein, 2014). In a California study, 44% of the student population repeated algebra I, and 69.6% of students in special education repeated algebra I (Fong, Jaquet, & Finkelstein, 2014). 82% of students in a Montgomery, Alabama school district failed the Algebra I exam (St. George, 2014). Across the United States the prevalence of algebra failure penetrates the educational system.

### **Associated Negative Outcomes**

Passing Algebra I by the end of freshman year has been linked to a 75% improved likelihood of on-time graduation in a California study (Silver, Saunders, & Zarate, 2008). On the other hand, failing algebra can lead to considerable negative consequences, such as continued mathematics difficulties, higher rates of disengagement, higher suspension rates, and higher absenteeism (Finkelstein et al, 2012; Schiller & Muller, 2003; Schiller, Schmidt, Muller, & Houang, 2010; Spielhagen, 2006). Finkelstein and colleagues (2012) found only one in five students who take algebra in

ninth grade after initially failing it in eighth grade achieved proficiency by the end of ninth grade; illustrating that four out of five continue to fail algebra. Those students not achieving proficiency standards by the end of ninth grade were shown to have little chance of completing, much less succeeding in advanced college preparatory mathematics courses by the end of their high school education (Schiller & Muller, 2003; Schiller, Schmidt, Muller, & Houang, 2010; Spielhagen, 2006).

### **Addressing Algebra Failure**

While there are discrepant opinions between researchers about the underlying cause of low algebra achievement, there is some consensus that the problem stems from a series of origins: pedagogical and instructional techniques, curriculum design, and individual student deficits. These will be further discussed in subsequent paragraphs.

### **Pedagogical and Instructional Techniques**

The initial tier of instruction to help remedy low algebra achievement is found at the classroom instructional level. Classrooms, particularly special education classrooms, often do not focus on developing logical thinking or applying the mathematical skills to real-world situations, but rather promote rote memorization, thus limiting the possibilities to achieve greater outcomes (Marita & Hord, 2017). Research has produced many intervention techniques and modes to implement those interventions; however, much more research is needed to assess these techniques and their effectiveness for various populations (Haas, 2005; Hughes et al., 2014; Impeccoven-Lind & Foegen, 2010; Rakes, Valentine, McGatha, & Ronau, 2010; Steele & Steele, 2003; Watt et al., 2016).

These techniques include: modeling and schematic approaches, cognitive strategy instruction (CSI), explicit inquiry routine (EIR), and graphic organizers.

Modeling and schematic approaches derive from the evidence that solutions, particularly in algebra, are generated through mental representations (Jitendra, Griffin, Deatline-Buchman, & Szczesniak 2007; Jonassen, 2003). Modeling and schema-based instruction emphasize both the mathematical structure and the semantic structure of a problem and focus on understanding key words and being able to represent those key words in alternate forms (Seel, 2012). CSI is an instructional technique used to teach students cognitive and metacognitive processes that are employed by skillful students to solve problems and complete tasks (Impeccoven-Lind & Foegen, 2010). When implementing CSI, the student aims to explore his or her various strategies through flexible thinking and self-reliant learning processes as he or she attempts to solve problems. EIR is an instructional technique that implores the student to follow explicit, sequential instruction to solve a problem while aiming to develop a deeper conceptualized understanding of the content through experiential learning and a scaffolded approach (Scheuermann, Deshler, & Schumaker, 2009). EIR is a pedagogical technique that serves as an umbrella term for many common interventions, such as a) general problem-solving strategies in problem representation and problem solution; b) self-monitoring; the concrete-representational-abstract methodology; and c) the teaching of prerequisite skills (Strickland & Maccini, 2010). Lastly, graphic organizers (e.g., diagrams and charts) are empirically supported techniques for improving algebra skills

(Ives, 2007). Graphic organizers are visual representations of information that depict relationships between facts or ideas within a learning task (Hall & Strangman, 2002).

### **Curriculum Design**

Recent efforts to reform the mathematics curriculum have produced curricula that call for more intense understanding and increased rigor as well as performance in the field of mathematics (NCES, 2013). The arithmetic to algebra gap is believed to be one of the contributing causes in poor algebra performance in students, particularly in students with disabilities (Witzel, Smith, & Brownell, 2001). This gap analyzes the differences in the concreteness of arithmetic and the abstract concepts that come into play when introducing algebra. Many students, both with and without SLD, demonstrated problems when they first experienced algebraic concepts due to the abstract or symbolic nature of the field and reasoning involved (Miles & Forcht, 1995; Vogel, 2008). Often times this is the first encounter they have with abstract reasoning and problem solving. In order to remedy this situation, policy makers have attempted to implement curriculums that span algebra instruction across all grade levels, thus allowing appropriate time to adequately understand the concepts (NCTM, 2000, 2006; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010). It is important to note algebra instruction, according to the NCTM standards found in Appendix C, begins early in education. For example, students are learning and practicing algebraic skills when they use a box or a blank space to represent a value. Often times educators do not view this as algebra, but the representation of the unknown value is a crucial algebraic concept.



### **Individual Student Deficits**

In addition to the aforementioned areas of concern, specific student deficits contribute to the underlying problems associated with low algebra achievement. Research has suggested deficits stem from poor problem-solving ability (Jitendra, DiPipi, & Perron-Jones, 2002; Seel, 2012), difficulty with mental representations (Jonassen, 2003; van Garderen, 2006), poor self-monitoring skills (Day & Connor, 2017; Maccini, McNaughton, & Ruhl, 1999), memory deficits (Day & Connor, 2017; Geary, 2004; Tolar, Lederberg, & Fletcher, 2009), and reading difficulties (Adams, 2003).

With increased emphasis on the use of evidence-based practices to maximize mastery, in this case, of algebraic concepts and skills, it is important to consider the approaches and models for delivery that have been used in the existing research base. The purpose of this study is to identify effective interventions and techniques that can be used for students with special needs in algebra by conducting a meta-analysis of the existing body of research.

### **Statement of the Problem**

Research suggests students often struggle with mastery of algebra, and this is particularly true for students with special needs (Morgan, Farkas, & Wu, 2009). There is limited available research specific to addressing achievement gaps in algebra (Geary, 2003); therefore, it is important to understand the magnitude of the problem at hand.

### **The Current Study**

The purpose of this study is to investigate the overall effectiveness of algebra interventions on students' achievement in mathematics. The study will utilize meta-

analysis on a group of primary studies that individually investigated the effectiveness of algebra interventions on students' mathematics achievement and explored the extent to which this overall effectiveness of interventions was moderated by various studies' characteristics. This analysis will employ similar parameters as previous meta-analyses and aim to capture all relevant studies, analyze them and provide practical discussions about effective interventions for algebra. The gaps in achievement previously mentioned will be addressed in this meta-analysis. Bridging these gaps will aid in the answering of the following research questions:

1. What effect have algebra interventions had on students who are struggling or at-risk for algebra failure? It is hypothesized that modes of intervention that utilize technology and peer mentoring will demonstrate larger effects on algebra achievement outcomes than those not including such interventions. In addition, approaches that utilize modeling and schema-based instruction, and CSI will help remedy the difficulty understanding abstract material by demonstrating larger effects than their counterparts. Similarly, it is hypothesized that interventions targeting self-regulation, problem solving skills, and content knowledge will provide support for students requiring additional help.
2. What algebra intervention demonstrates the largest effect for students who are struggling or at-risk for algebra failure? It is hypothesized that the most effective intervention technique for students requiring special services will be interventions that implement CSI, target problem solving ability, and employ technology.

3. What factors moderate the effectiveness of algebra interventions? We hypothesized that multiple factors, including length of intervention, training of interventionists, alignment with NCTM standards, and if follow-up data was collected.
4. How have changes in federal and state standards impacted the effect of algebra interventions on student performance? It is hypothesized that changes in standards have increased the necessity of rigorous intervention and improved the effectiveness of interventions on student outcomes.

#### **Definition of Terms**

- Algebra – “any of various systems or branches of mathematics or logic concerned with the properties and relationships of abstract entities (such as complex numbers, matrices, sets, vectors, groups, rings, or fields) manipulated in symbolic form under operations often analogous to those of arithmetic”  
*(Meriam-Webster.com, definition 2)*
- Explicit instruction – an instructional technique where the students are directly presented the information and are engaged in the learning process. Explicit instruction builds on prior foundation of skills and is planned. Lecturing is an example of explicit instruction. Also known as direct instruction (Strickland & Maccini, 2010).
- Specific Learning Disability (SLD) – “a disorder in one or more of the basic psychological processes involved in understanding or in using language that is spoken or written, that may manifest itself in the imperfect ability to listen, think,

speaking, reading, writing, spelling, or doing mathematical calculations. The term includes conditions such as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia; the term does not include a learning problem that is primarily the result of visual, hearing, or motor disabilities, of an intellectual disability, or emotional disturbance, or of environmental, cultural, or economic disadvantage” (IDEA, 2004, section 602, paragraph 30)

- Explicit Inquiry Routine (EIR) – an instructional technique that encourages the student to follow explicit, sequential instruction in order to solve a problem while simultaneously trying to develop a deeper conceptualized understanding of the content. This is accomplished through experiential learning and a scaffolded approach (Scheuermann, Deshler, & Schumaker, 2009)
- Cognitive Strategy Instruction (CSI) – an instructional technique used to teach students cognitive and metacognitive processes to aid in the solving of problems and completion of tasks (Impeccoven-Lind & Foegen, 2010)
- Schema based instruction – a method of teaching problem solving that emphasizes both the semantic structure of the problem and its mathematical structure. It utilizes recognition of key words but goes further than simple recognition to stress understanding of the situation represented in the problem (Seel, 2012)

- Self-regulation – a system of conscious personal management that involves the process of guiding one's own thoughts, behaviors, and feelings to reach goals (Baumeister, Heatherton, & Tice, 1994)
- At-risk students – “students or groups of students who are considered to have a higher probability of failing academically or dropping out of school. The term may be applied to students who face circumstances that could jeopardize their ability to complete school, such as homelessness, incarceration, teenage pregnancy, serious health issues, domestic violence, transiency (as in the case of migrant-worker families), or other conditions, or it may refer to learning disabilities, low test scores, disciplinary problems, grade retentions, or other learning-related factors that could adversely affect the educational performance and attainment of some students.” (Partnership, 2013, <https://www.edglossary.org/at-risk/>).

## CHAPTER II

### LITERATURE REVIEW

The Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines (2009) specifically require each meta-analysis to have a thorough description of the previous literature and discussion about how the topic of analysis fits into the current context of the existing literature. The purpose of this chapter is to review the existing literature, identify particular gaps in the algebra intervention literature, and to present research questions to address these gaps.

#### **What is algebra?**

The definition of algebra has been fluid and dynamic throughout the years. Many researchers have attempted to pinpoint the definition of this mathematical concept and key to success, but there is difficulty settling on a consensus. Researchers agree on the importance of variables and concepts that integrate variables into the structure of the math (Briggs, Demana, & Osborne, 1986; Graham & Thomas, 2000; Kalchman & Koedinger, 2005; Kieran, 2007). These variables, however, often lead to increased difficulty for many students who misunderstand them. The links between the symbolic nature of the variable and assigning meaning to variables identified are challenges faced by students with special needs (Torigie & Gladding, 2006). Küchemann (1978) developed a model that describes the six progressive phases of variable comprehension: (a) as a single value, through trial and error evaluation; (b) as irrelevant (i.e., students

ignoring the variable in a contextual situation); (c) as an object or label; (d) as a specific unknown; (e) as a generalized number; and (f) as a functional relationship.

The NCTM has refined its definition of algebra numerous times. Originally, in 1989, their definition emphasized equations, inequalities, and matrices. In 2000, they reformed algebra by four all-encompassing concepts and skills: (a) functions, (b) algebraic symbols, (c) mathematical modeling, and (d) analyzing change. In a more recent statement by the NCTM (2008), algebra was defined as not only a way of thinking, but also as a set of concepts and skills that facilitate students to generalize, model, and analyze mathematical situations. NMAP (2008) suggested that the instruction and study of algebra be categorized into six primary topics: (a) symbols and expressions, (b) linear equations, (c) quadratic equations, (d) functions, (e) polynomials, and (f) combinatorics and finite probability. For the purposes of this analysis, the NMAP topics will be used to categorize content areas pertaining to algebra. With a greater understanding of how educational reform has impacted the structure and implementation of algebra instruction, it is also important to understand what previous research has demonstrated to be effective interventions and instructional techniques.

### **Algebra**

The inability to master algebraic concepts and skills is a critical step to success in college mathematics courses and a key predictor of high school dropout (Silver, Saunders, & Zarate, 2008); however, many students find themselves underprepared and fail the first time they take higher level algebra courses (Balfanz, McPartland, & Shaw, 2002; Finkelstein, Fong, Tiffany-Morales, Shields, & Huang, 2012; Huang, Snipes, &

Finkelstein, 2014). Failing algebra can lead to considerable negative consequences: only one in five students who take algebra in grade 9 after initially failing it in grade 8 achieve proficiency by the end of grade 9, illustrating that four out of five continue to fail algebra (Finkelstein et al., 2012). Those students not achieving proficiency standards by the end of grade 9 have little chance of completing, much less succeeding in advanced college preparatory mathematics courses by the end of their tenure in high school (Schiller & Muller, 2003; Schiller, Schmidt, Muller, & Houang, 2010; Spielhagen, 2006).

Algebra is considered the gateway skill for higher order mathematics, particularly in the areas of abstract thinking and representing quantities with non-numeric symbols, as it has been proven to be one of the critical concepts to master for future success in mathematics and outside of the classroom (Watt et al., 2016). Fennell (2008) discovered that algebra achievement was an important factor in predicting students' future achievement in math courses, entrance into college, as well as financial equity in the workforce.

The instruction of algebra has undergone a significant transformation over the last decade (Hiebert et al., 2005), with the implementation of the CCSS (2010) and failure to prepare students adequately for future mathematics. The CCSS scope and sequence, which is the prototypical mathematics instruction pathway, places algebraic concepts in the curriculum from kindergarten through the 8th grade and into high school (CCSS, 2010).



One of the primary arguments for the reformation of algebra pedagogy has been that the arithmetic-then-algebra approach (i.e., having arithmetic curriculum in elementary grades, followed by formal algebra in secondary grades) does not allow enough time for the student to fully develop his/her algebraic thinking (Kaput 2008; Moses & Cobb, 2001; Schoenfeld, 1995). The implementation of CCSS is aimed to help with this problem while increasing the rigor of the knowledge base for deeper understanding (CCSS, 2010). This led to extensive failure in these realms, as the students were not able to fully develop their skills in these areas and when later mathematics courses required in-depth analysis, the students were unable to perform (Kaput 2008; Moses & Cobb, 2001; Schoenfeld, 1995).

Reformation efforts that began in the early 2000s have yielded a new style of algebra pedagogy, where algebra instruction spans across the Kindergarten through Grade 12 curriculum and allows the students necessary time and experience to comprehend the concepts (e.g., NCTM, 2000, 2006; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010). Due to this shift, research has now provided insights into how children think algebraically and understand algebra. Through this research, we have been able to understand children's capabilities to cultivate a relational understanding of the equal sign (Carpenter, Franke, & Levi, 2003; Carpenter, Levi, Berman, & Pilgge, 2005), notice regularity and patterns among arithmetic situations through generalization of mathematical structures (Bastable & Schifter, 2008; Schifter, Monk, Russell, & Bastable, 2008), use advanced tools to explore, generalize, and symbolize functional

relationships (Cooper & Warren, 2011; Moss, Beatty, Shillolo, & Barkin, 2008); build mathematical arguments that reflect more generalized forms than the empirical, case-based reasoning often used (Carpenter et al., 2003; Schifter, 2009); and reason about abstract quantities (e.g., area, length) to symbolize algebraic relationships (Dougherty, 2003, 2008). Research suggests successful performance in algebra requires mastery of (a) basic skills and terminology, (b) problem representation, (c) problem solution, and (d) self-monitoring strategies (Hutchinson, 1987).

### **Previous Meta-Analyses, Literature Reviews and Systematic Reviews**

Several meta-analyses have been conducted and published on algebra interventions and pedagogy. Rakes, Valentine, McGatha, and Ronau (2010) piloted a systematic review and meta-analysis that investigated algebra instruction for students with studies published between the years of 1968 and 2008. Their review located 82 studies, including published and grey literature (e.g., theses and dissertations). Their findings generated five chief approaches to intervention and instructional improvement: (a) technology curricula, (b) nontechnology curricula, (c) instructional strategies, (d) manipulatives, and (e) technology tools with an emphasis on conceptual understanding.

In another meta-analysis, Haas (2005) investigated algebra teaching methods for secondary students. Haas (2005) located 35 published and grey literature sources published between the years 1980 and 2002 and discovered that direct instruction had the largest impact for students who were considered to be high-ability and low-ability, alike. Students who struggle with mathematics, but are not identified with a disability or do not receive an official diagnosis, often portray comparable difficulties with

mathematics as those identified with a disability (Haas, 2005). In the past, research has been synthesized to evaluate mathematics research for students with disabilities (Browder, Spooner, Ahlgrim-DeLzell, Harris, & Wakeman, 2008; Gersten et al., 2009). While both of these reviews included students with varying ability levels, neither one specifically included students with identified disabilities.

Maccini, McNaughton, and Ruhl (1999) conducted a review of literature that analyzed the effects of different instructional interventions on pre-algebra or algebra achievement of students who had diagnoses of specific learning disabilities (SLD). Despite their search criteria spanning from 1970 to 1996, their search produced six studies that qualified based on their inclusion criteria. These findings support explicit review, orientation to the strategy to be learned, modeling by the teacher, guided practice, feedback and reinforcement, mastery learning, opportunities for independent practice, assessment, and cumulative review and closure as methodologies and interventions that improved algebra performance and achievement.

Steele and Steele (2003) reviewed studies on teaching algebra to students with SLD. Their findings suggested that difficulties in processing, memory, and language were factors in students with SLD struggling in algebra. In their analysis, they noted there is a lack of interventions that are empirically supported for these students and for algebra. While they provided some guidance and suggestions for these approaches to intervention (e.g., teacher directed instruction, self-monitoring, stepwise approaches, mnemonics, and visual representations), there was little scientific evidence to support these claims. Impeccoven-Lind and Foegen (2010) published an article that explored two

approaches to intervention (CSI and EIR) and one intervention (class-wide peer tutoring) for students who have SLD and are struggling with algebra; however, they did not provide substantial statistical comparisons and empirically supported data to back these claims.

Another meta-analysis of algebraic interventions was conducted by Hughes, Witzel, Riccomini, Fries, and Kanyongo (2014). The Hughes et al. (2014) meta-analysis was unclear in their methodology, as they did not explicitly report how they located their articles. They mentioned they used major databases, but did not specify which ones in particular they used and only provided examples of major databases. Perhaps the ones listed in their article are the ones they used for gathering articles; however, they did not specify so it cannot be confirmed. Their initial search yielded 168 articles from peer-reviewed and non-peer reviewed sources. Their inclusion criteria were clear, as the studies needed to have (a) implemented experimental or quasi-experimental designs, reported on academic outcomes and effect size (ES) or had sufficient information to determine ES, and were published in peer-reviewed journals or as dissertations that (b) included students at-risk and those with disabilities, and (c) were published between the years of 1983 and 2013. After their inclusion criteria, their study utilized 12 articles, but due to lack of information, only eight were able to be used in the weighted analysis. The four studies that were excluded did not provide sufficient information to run the statistical analyses the researchers desired to compute. The study looked at three primary research questions: (a) would targeted instruction (e.g., explicit or direct instruction) or intervention in algebra improve math achievement for students with

disabilities, (b) what are the effects of algebra interventions when aimed at increasing achievement of students who may be at-risk or have a disability, and (c) what is the most effective algebra intervention for students who may be at-risk or have a disability? Their analysis found that students with SLD benefitted from targeted instruction.

Additionally, the interventions used in the studies were effective in improving algebra achievement (Hughes et al., 2014). In their discussion, Hughes and colleagues (2014) mentioned the intervention style with the highest effect size was cognitive/modeling-based instruction, followed by concrete-representational-abstract, followed by technology. The authors noted that technology has the largest independent effect sizes, but due to the limited amount of research, they were unable to conclude that it was the most effective intervention. Some limitations associated with this article included the small number of articles and the fact that the most recent article was published in 2012; their literature base could be updated to see if any new instruction or intervention techniques would impact the results.

The most recent meta-analysis covering algebra interventions was conducted by Watt et al. (2016). Their review produced a total of 15 studies, including five single-subject design and ten experimental studies; however, these 15 produced different studies than those that were discovered in the Hughes, et al. (2014) meta-analysis, despite their close temporal proximity and similar search criteria. Additionally, the Hughes et al. (2014) article produced some studies that were not included in the Watt et al. (2016) article. Potentially, combining the articles from these two meta-analyses, as well as additional search criteria would increase the number of studies for further study.

Watt et al. (2016) did a commendable job describing their search criteria, methodology, and inclusionary criteria for their analysis. Inclusionary criteria for this analysis required the study to include (a) students with a SLD, (b) have algebra content that fell under the specified algebra domains, (c) examine instructional interventions, (d) use an experimental, quasi-experimental, or single-subject design, and (e) been published between 1980 and 2014. Their overall findings suggested the following interventions or pedagogical techniques were found to be effective for teaching students with SLD algebra: (a) concrete-representational-abstract methodology, (b) tutoring, (c) CSI, (d) enhanced anchored instruction, and (e) graphic organizers.

**Table 1 Previous Meta-Analyses of Algebra Instruction/Intervention**

Study	Years	Search Terms Used	Sample Characteristic	Studies Included	General Findings	Missing Components
Maccini, McNaughton, & Ruhl, 1999	1970-1996	Algebra and learning disabilities (including other labels, such as learning handicapped, exceptional, and mildly handicapped)	Students with SLD	6	Explicit review, orientation to the strategy to be learned, modeling by the teacher, guided practice, feedback and reinforcement, mastery learning, opportunities for independent practice, assessment, and cumulative review and closure are effective interventions	Limited to SLD, no quality analysis
Haas, 2005	1980-2002	Not provided	Study conducted at the secondary level with algebra as the focus; experimental design with achievement as an outcome measure; teaching method had to deal with algebra	35	Direct instruction had the largest impact for high and low ability students	Weak methodology, little information on effect size, no publication bias
Rakes et al., 2010	1968-2008	Algebra, function, equation, expression, quadratic, polynomial, exponent, and rational	Had to target algebra concepts; assess for student achievement; experimental design with comparison group; comparison group had to receive “usual instruction”	82	Identified five areas for instructional improvement: (a) technology curricula, (b) nontechnology curricula, (c) instructional strategies, (d) manipulatives, and (e) technology tools with an emphasis on conceptual understanding	No quality analysis, no publication bias
Hughes et al., 2014	1983-2013	Algebra, learning disabilities, at-risk, intervention, disabilities	Students at-risk and with disabilities	8	Cognitive/modeling-based instruction yielded highest ES	Weak methodology, no publication bias
Watt et al., 2016	1980-2014	Algebra, learning disability, learning disorder, math, pre-algebra, and special education	Students with LD	15	CRA, tutoring, EAI, CSI, and graphic organizers are all highly effective techniques	Not limited to SLD, questionable search criteria, no quality analysis

*Notes.* SLD = Specific Learning Disability; LD = Learning Disability; CRA = Concrete-Representational-Abstract; EAI = Enhanced Anchored Instruction; CSI = Cognitive Strategy Instruction

## **Weaknesses of Existing Meta-analyses and Need for a Current Meta-analysis**

Prior meta-analyses have provided the field of algebra research important information regarding what works and what does not work when providing algebra interventions for students. Upon researching this area of mathematics and reviewing the existing meta-analyses, there are several areas of need that warrant this study. One of these weaknesses is prior meta-analyses restricted their inclusion criteria to 2014, so there may be additional studies that have not been included and are more recent. There has been a shift in national and state standards for what is expected to be learned in mathematics, so classroom content has changed, and it is believed newer studies will reflect those changes and students' responses. As explained, rates of student failure in algebra continue to remain high (Balfanz, McPartland, & Shaw, 2002; Finkelstein, Fong, Tiffany-Morales, Shields, & Huang, 2012; Huang, Snipes, & Finkelstein, 2014), indicating the interventions discussed in prior meta-analyses are not working to their full potential and there is room for improvement; this meta-analysis aims to identify additional strategies that may help mediate the gap. Second, cursory searches indicate several articles are missing from previous studies. There are suggestions of publication bias when looking at prior meta-analyses with similar methodology and inconsistent findings and inclusion of articles. Third, the methodology of prior studies does not consistently align with more recent meta-analytic standards. Finally, there have been minimal components of quality analysis included in the prior studies.

One weakness in particular is the lack of rigor in the methodology of the search for some of the previous meta-analyses. Ambiguous, or weak, methods may have



impacted the ability of the authors to locate and include all of the work available in the field, which has some implications for publication bias. External standards of meta-analysis require certain criteria to be included (PRISMA, 2009). Notably, Hughes et al. (2014) did not follow all the criteria, nor did Haas (2005). They did not explicitly state how they retrieved the articles used in their analysis, so it is possible additional relevant articles were not used. In the Haas (2005) meta-analysis, several important features of a good systematic review were missing (e.g., no rationale for inclusion criteria and limited information about ensuring the reliability of data extraction). Haas (2005) did not explain the methodology for calculating the ES, did not account for non-independent observations, and did not investigate publication bias. Previous meta-analyses have located articles limiting the search criteria to individuals who are currently diagnosed with SLD or some other variation of a diagnosis, thus eliminating those who do not have a diagnosis and are still at-risk for developing mathematics problems. While there is no widely accepted definition of “at-risk,” this population continues to be studied and is excluded from the previous meta-analyses. Watt et al. (2016) and Hughes et al. (2014), while written only two years apart, utilized similar search timeframes and similar inclusion criteria, but yielded different studies. This poses the threat that they may not have retrieved all the relevant studies.

While there is some consensus about intervention techniques that have proved to be effective for algebra, there are various ideologies and theories about the derivation of algebraic misunderstanding and where the underperformance originates. Several of these propositions are discussed in the following paragraphs, including: the arithmetic to

algebra gap, deficits in modeling and schemas, problem solving challenges, and trouble self-regulating.

### **Identification of the Problem**

#### *Arithmetic to Algebra Gap*

Witzel, Smith, and Brownell (2001) discussed a concept referred to as the arithmetic to algebra gap. The arithmetic to algebra gap is believed to be one of the contributing causes in poor algebra performance in students, particularly in students with disabilities. This gap analyzes the differences in the concreteness of arithmetic and the abstract concepts that come into play when introducing algebra. Miles and Forcht (1995) as well as Vogel (2008) explained many students, both with and without SLD, demonstrated problems when they first experienced algebraic concepts due to the abstract or symbolic nature of the field and reasoning involved. Often times this is the first encounter they have with abstract reasoning and problem solving. Witzel et al. (2001) provided a series of remedies teachers could utilize. They included (a) teaching through stories that connected the instruction to real life; (b) assessing for necessary prerequisite knowledge prior to introducing foreign concepts; and (c) using explicit instruction when modeling. Understanding the core components of algebra, researchers are better able to develop, and teachers or tutors are better able to implement effective interventions. Over the years, several researchers have attempted to synthesize and analyze these interventions on a large-scale basis. These studies have largely analyzed different pedagogical interventions and requisite skills needed for algebra competence.

### *Modeling and Schema*

Research supports the notion schema-based instruction can improve the problem-solving skills of students with SLD (Jitendra, Griffin, Deatline-Buchman, & Sczesniak 2007). Schema based instruction is “a method of teaching problem solving that emphasizes both the semantic structure of the problem and its mathematical structure. It utilizes recognition of key words but goes further than simple recognition to stress understanding of the situation represented in the problem” (Seel, 2012, p. 2945).

Schema-based instruction and modeling often parallel one another in a sense that schema-based instruction allows the student to create a mental representation of the problem and modeling allows the student to create both mental representations and physical products (e.g., charts, diagrams, projects; Seel, 2012). An area students with SLD struggle in is modeling. Modeling the problem situation requires mentally arranging the information presented in the word problem, or any other problem that requires this skill, and fabricating mental representations of the situation. Modeling has been shown to be an effective approach to intervention in a variety of mathematics subjects, including algebra (Blanton, et al., 2015).

Mathematical solutions are typically derived from the mental representation, so the ability to manufacture the representation of the problem affects the likelihood of accuracy for the problem (Jonassen, 2003). Students with SLD have more challenges with generating mental representations (van Garderen, 2006) and even if one is formed, research shows it is more likely to be a visual image of the problem, rather than a schematic representation that models the relationships among the problem elements (van

Garderen, 2006; van Garderen & Montague, 2003). This poses complications for the student because, in word problems, students who utilize schematic representations are more successful in arriving at the correct answer than those who do not use schematic representation (Hegarty & Kozhevnikov, 1999; Lesh & Harel, 2003). The implementation of schematic and modeling skills is often found in situations in which students' need to solve a problem (e.g.,  $2x + y = 25$ ). Deficits in these areas correlate with challenges in problem solving abilities (van Garderen, 2006). Word problem solving is not the only area where a schema-based approach is validated; it has been shown to be effective in algebra as well, specifically in quadratic equation solving (Lopez, Robles, & Martinez-Planell, 2016). In this study, improved outcomes were found in students who utilized a three-stage schema approach to understand and solve quadratic equations.

### *Problem Solving*

One particular area that recent research has addressed looks into the realm of problem-solving ability (Jitendra, DiPipi, & Perron-Jones, 2002). As mentioned before, the increased rigor of standardized assessments and desired learning outcomes, has pioneered the shift from regurgitation and rote memorization of facts to more intensive problem-solving approaches and questions. While this broadband term of problem solving relates to many mathematics fields, algebra in particular utilizes problem solving ability (Hutchinson, 1987). As students' progress through the educational system and enter into secondary education, this shift becomes even more apparent as schools attempt to generate more "real-world" applications of mathematics. With this introduction of

higher-level mathematics and challenging topics, word problems and problem-solving approaches that require higher order thinking skills become more obvious (Jitendra, DiPipi, & Perron-Jones, 2002).

Despite the increased prevalence of difficulty implementing effective problem-solving techniques, this remains an area of difficulty for an extensive number of students (Verschaffel et al., 2000), especially those who struggle with learning disabilities (Bryant, Bryant, & Hammill, 2000). Many students fail the first time they take higher level algebra courses (Balfanz, McPartland, & Shaw, 2002; Finkelstein et al., 2012; Huang, Snipes, & Finkelstein, 2014). Only one in five students who take algebra in grade 9 after initially failing it in grade 8 achieve proficiency by the end of grade 9. Those students not achieving proficiency standards by the end of grade 9 have little chance of completing, much less succeeding in advanced college preparatory mathematics courses by the end of their tenure in high school (Schiller & Muller, 2003; Spielhagen, 2006; Schiller, Schmidt, Muller, & Houang, 2010). A high prevalence of students with SLD struggle in problem-solving skills and higher-order reasoning, skills that are necessary for higher-level mathematics courses (e.g., algebra; Bryant, Bryant, & Hammill, 2000; Tolar et al., 2012; Verschaffel, Greer, & De Corte, 2000).

The NCTM (2000) endorsed the idea that fluency in mental computation of basic number skills is critical to efficient problem-solving processes. Fluency helps free up cognitive space, which leads to more efficient problem solving. These approaches can be even more challenging for individuals with disabilities, as they have a difficult time generalizing their learning to settings different from the framework in which they

originally learned those skills (Binder, 1996; DuVall, McLaughlin, & Senderstrom, 2003; Heward, 2006). Realistically, these problems with fluency are addressed through the use of technology (e.g., calculators) which falls under the category of technology that will be coded for in this analysis. Perhaps this increased struggle stems from the cognitive processing deficits that underlie the SLD and how these deficits map onto critical skills that are necessary for problem solving to occur (Fuchs et al., 2004).

Mathematics is a language that consists of special symbols and terms that are unique to its field and possess meaningful affiliations with related terms (Duru & Koklu, 2011). Mathematical reading requires both linguistic comprehension skills and knowledge of mathematics linguistics, which consists of letters and symbols (Adams, 2003). There are prominent links between reading comprehension and word problem solving ability (Boonen, van der Schoot, van Wesel, de Vries, & Jolles, 2013; Vilenius-Tuohimaa, Aunola, & Nurm, 2008) as well as algebra (Adams, 2003). In a field in which symbols are used to represent alternative meanings, often times algebraic equations are treated as a sentence (Duru & Kolku, 2011). Many students who have a learning disability also encounter challenges in reading comprehension, potentially offering another explanation for the high occurrence of students with SLD having difficulty in problem solving. When tackling a word problem, the student must first comprehend the problem, then form a mental model of the problem situation, and finally, analyze the information on the grounds of that particular model that has been generated (Hegarty & Kozhevnikov, 1999; Lesh & Harel, 2003; van Garderen, 2006).

### *Self-Regulation*

Two primary self-regulated learning (SRL) processes that facilitate academic success are: (a) monitoring and (b) self-reflection (Bol, Riggs, Hacker, Dickerson, & Nunnery, 2010). Monitoring, defined as the mental tracking of one's performance processes and outcomes, permits individuals to evaluate shifting task demands, focus awareness on their mistakes, and generate internal feedback (Zimmerman & Moylan, 2009). Self-reflection, when learners judge their performance and react to these judgments, helps individuals interpret feedback, learn from their mistakes, and make decisions that enhance subsequent learning and performance (Zimmerman, 2000). In an academic setting, when a student does not possess appropriate monitoring and reflection skills, their ability to regulate themselves and make adaptive decisions during academic pursuits is stalled (Dunlosky & Rawson, 2012; Hacker, Bol, & Keener, 2008).

Research shows students who are able to self-regulate their learning processes effectively have a better chance of succeeding academically (DiGiacomo & Chen, 2016; Dignath, Buettner, & Langfeldt, 2008). Self-regulation strategies have been shown to improve mathematics problem-solving ability in individuals who have specific learning disabilities in mathematics (Case, Harris, & Graham, 1992; Cassel & Reid, 1996). Self-regulation plays an important role in being able to translate the skills learned in the intervention phase to maintenance phase so the student is able to continue the success he or she observes during the intervention (Gersten et al., 2008). As research has shown, math skills deficits that are not remedied at early stages manifest themselves into continuing, if not more intense, challenges in the future (Axtell et al., 2009; Duncan et

al., 2007; Haring & Eaton, 1978; Ketterlin-Geller et al., 2008). Similar ideologies exist in the field of algebra, explaining that early difficulties in algebra predict future mathematics difficulties (Fennell, 2008; Watt et al., 2016).

This has been explained by a variety of researchers (Balfanz, McPartland, & Shaw, 2002; Finkelstein, Fong, Tiffany-Morales, Shields, & Huang, 2012; Huang, Snipes, & Finkelstein, 2014) one of which demonstrated 80% of the students who failed algebra I did not meet the proficiency standards by the end of high school (Finkelstein, et al., 2014). In order to mediate these negative factors associated with poor algebra performance, intervention efforts have targeted specific areas to identify and clarify the root of these difficulties.

### **Summary**

Several areas have been identified that need additional research and analysis specific to algebra interventions. Following up Hughes et al. (2014) and expanding the literature with more recent studies, addressing fidelity of the interventions, locating studies that provide information on the maintenance of the intervention, and running a funnel plot analysis to check for publication bias will add to the research base. Expanding the literature is a task that will aid in determining a more reliable conclusion to further advance the current knowledge base of algebra interventions. As addressed previously in this chapter, there is a need for continuing research in secondary mathematics classrooms, not just algebra. The Haas (2005) meta-analysis revealed little research has been conducted specifically addressing algebra instructional techniques and interventions for students with identified disabilities. This meta-analysis will look at



research studies pertaining to this group of individuals in order to present the research community with more detailed evidence about how to implement effective interventions for this population. In their analysis, Steele and Steele (2003) explained the lack of interventions that are empirically supported for these students and for algebra.

## CHAPTER III

### METHODS

The purpose of this study is to investigate the overall effectiveness of algebra interventions on students' achievement in mathematics. The study utilized meta-analysis on a group of primary studies that individually investigated the effectiveness of algebra interventions on students' mathematics achievement and explored the extent to which this overall effectiveness of interventions was moderated by various studies' characteristics. Typically, a meta-analysis includes the following: (a) decide on the area of interest to be studied and defines the association to be analyzed; (b) identification of the dependent variables and the characteristics of the study that will be analyzed; (c) operationally define all variables to be included in the study; (d) a decision about the inclusion criteria is made by the researcher; (e) studies that meet the inclusion criteria are then located by the researcher; (f) the studies are coded; (g) effect sizes are computed from the studies by converting the statistics provided in the articles; (h) the average effect size is calculated and determined to be significantly different from zero or not; (i) the independent studies are occasionally examined by the researcher to determine if the degree and direction for the effect sizes is consistent across studies; and (j) if any sources of variation arise, the researcher then attempts to explain any sources of variation or heterogeneity (Glass, 1976; Houston et al., 1983; Rosenthal & Rubin, 1986; Wolf, 1986).

The current study applied the meta-analytic procedure supported by Glass (1976), Cohen and Dacanay (1992), Kulik et al. (1980) and Azvedo and Bernard (1995). This method required the researcher locate studies, define study features, code the characteristics of the studies, calculate effect sizes, find the mean effect size, and explain sources of error variance. Previous meta-analyses included approximately 20 studies in their sample size; therefore, this study aimed to include at least 20 as well.

### **Location and Selection of Studies**

Initial studies were located through a combination of electronic and ancestral searches. An ancestral search consists of examining the reference list of cited articles in a paper for the purpose of locating additional studies to include. Online search engines (e.g., What Works Clearinghouse, [dyscalculia.org](http://dyscalculia.org)), dissertation/thesis abstracts, electronic journals and research databases will include JSTOR, EBSCO, Academic Search Ultimate, Educational Resources Information Center (ERIC), PSYC INFO, Education Full Text, ProQuest, and Google Scholar were searched using combinations of the following key words: *algebra, algebra education, intervention, math, mathematics, math disability, instruction, cognitive strategy instruction, explicit inquiry routine, schema-based instruction, modeling, technology, graphic organizer, learning disability, learning problem, special needs, disability, special education, and at-risk*. A total of 188 citations were identified in the initial search. Appendix B displays the flow in which these articles were located.

These studies were screened to meet identified inclusion criteria. In order to be included in the meta-analysis, the study must have been published between 1975 and

2018. These dates were selected because in 1975 IDEA was enacted by the United States Congress, which implemented the idea of special education and receiving special services for those struggling academically. Studies for this meta-analysis must include students enrolled in a Kindergarten through 12<sup>th</sup> grade classroom. Early meta-analyses on similar topics did not include the entirety of K-12, but more recent meta-analyses utilized the totality of the grade levels. Additionally, the study must have implemented an experimental or quasi-experimental design. Single case or single subject designs were included in this meta-analysis due to the commonality of this body of research for educational interventions. The study needed to provide sufficient information to calculate effect size or information needed was made available through contact with the corresponding author. The study had to be published in a peer-reviewed journal or as a dissertation and be available in English. Finally, the studies had to include an intervention addressing algebra skills that included academic dependent variables. For the purposes of this analysis, algebra skills will include the standards endorsed by the NCTM, which can be found at <http://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Algebra/> (See Appendix C).

**Table 2 Inclusion and Exclusion Factors**

Factor	Inclusion	Exclusion
Type of Publication	Refereed Journal Article Dissertation	Literature Review Position Paper
Study Design	Single Subject Randomized Control Group Pre-Post Design	Qualitative Only

**Table 2 continued**

Factor	Inclusion	Exclusion
Study Population	Students must be classified as at-risk or struggling with algebra	Not classified as a K-12 student No mathematical deficits
Intervention Focus	Algebra skills (NCTM)	Other math skills but not algebra
Outcome Measure	Math achievement (standardized or CBM)	Teacher report or another qualitative indicator only
Data Available	Effect size for design or sufficient data to compute in the article or from the author Written in English	Insufficient data and not able to obtain data from

### **Screening and Coding of Studies**

#### *Preliminary Screening.*

In this initial phase, each article was screened to ensure it met the inclusion criteria in Table 3. Studies where the only issue may be obtaining additional information from the authors, the study was initially retained. All retained studies will advance to the second phase and be coded for inclusion in the study. See the coding sheet in Appendix A.

As suggested by Wolf (1986), the initial studies in this meta-analysis were coded to clarify the different sources of error variance (i.e., grade level, specific type of algebra skills, type of intervention). The study utilized a variety of independent variables to account for sources of error variance including: type of publication; location of the intervention; type of intervention (modeling and schema; problem solving skills; self-regulation; technology; explicit instruction; guided practice; feedback and reinforcement; peer tutoring; cognitive strategy instruction; concrete-representational-

abstract; enhanced anchored instruction; and graphic organizers) study design, evaluation method, educational level, length of intervention, number of sessions, duration of each session, presence and type of disability, and sex. The final phase of the study involved calculation of the effect sizes.

In addition to the independent variables, the quality of studies is an important characteristic that informs the reliability of the finding. The articles selected for analysis were coded for quality according to the What Works Clearinghouse Procedures and Standards Handbook (WWCPSH) Version 3.0. Quality indicators looked at the randomization of samples, accuracy and relevance of intervention descriptions, and how well variables were measured. Consideration of quality (rigor) of the study was considered as an additional confounding variable, a possible source of error variance. Study quality considered recruitment of subjects, randomization of subjects, demographic data of subjects, who implemented the intervention, intervention characteristics, fidelity checks (i.e., was the intervention implemented as designed), and reliability of the measurement method.

Initially, the coding process was checked by comparing the doctoral student (JM) and a faculty member (CR) coding results on some studies and coding terms clarified. Inter-rater reliability of the coding was addressed by having 30% of the included studies coded independently by another graduate student, who has a graduate degree and background in Psychology. All parties who participated in coding participated in a training seminar. Training included a review of the coding sheet, and then a practice coding session that lasted 15 minutes. Each coder received three test articles and coded

the articles according to the coding sheet. These were checked by the author of this dissertation. If 80% agreement on these test articles was not achieved, additional training was provided until agreement was met and the coding sheet was clarified as needed. Only then was the coder given 30% of the final studies that were randomly selected and independently coded.

### **Planned Analyses**

There are three common goals of data analysis when conducting a meta-analysis: (a) to obtain an index that measures the overall effect size for a group of studies, (b) to determine whether the studies are homogeneous, and (c) to identify sources of heterogeneity if the studies are found not to be homogeneous (Huedo-Medina, Sánchez-Mecca, & Marin-Martinez, 2006). Additionally, meta-regression can be used to examine potential confounds. A random-effects model was utilized to allow for within-study and between-study variation.

There are three types of designs included in this study: single subject, single group pre- post-test, and group designs. Each of these used a different effect size (see Table 4). There are two primary schools of thought for combining effect sizes when dealing with meta-analyses. One method, utilized by Hedges and Stock (1983) and Slavin (1984), involves the researcher combining all the effect sizes from a single study and using one value for the analysis. The second method consists of extracting multiple effect sizes from each study (based on the number of comparisons of interest) and using each effect size in the meta-analysis. A core reason for implementing this approach stems from the work by researchers (e.g.; Ahmad and Shashaani, 1994; Glass et al.,

1981; Kulik & Kulik, 1991) who have contended that too much information is lost when effect sizes are combined using the first method described. The second method was used in the present study. With no empirically supported metric for converting and comparing these different effect sizes, this analysis did not combine all effect sizes, but rather interpreted them by the effect size used.

**Table 3 Effect Sizes by Research Design**

Study Design	Effect Size Calculated
Single Case	Tau-U
Single group pre-/post-test	Cohen's <i>d</i>
Group design	Hedges' <i>g</i>

### **Effect Size – Single Case Study**

Due to the variety of study designs included, for this study, multiple effect sizes were calculated. When analyzing single case studies, the effect sizes calculated were *Tau-U* and Baseline Corrected *Tau*. *Tau-U* is a statistical technique that combines nonoverlap between phases with trend from within the intervention phase (Parker, Vannest, Davis, & Sauber, 2011). This technique, while new, was tested over a series of 382 studies involving AB and ABA single-case designs and performed well as compared to other statistical techniques (Parker et al., 2011). Baseline Corrected *Tau* utilizes a two-step, nonparametric method to address the problems of baseline trend in AB designs. First, the monotonic baseline trend is estimated, and corrected using Kendall's *Tau* rank if needed. If the baseline trend is statistically significant, then it may be



corrected across A and B phases using the Theil-Sen estimator. Secondly, using a dummy code variable (where A phase=0 and B phase=1), an effect size is calculated as a *Tau* correlation between the dummy code and the original or corrected data (Tarlow, 2017).

### **Effect Size Two-Group Design**

For two-group designs, Hedges' *g* was calculated as the effect size rather than Cohen's *d*. For smaller sample sizes ( $n < 20$ ), Hedges' *g* offers a more reliable effect size, whereas for sample sizes greater than 20, both statistics are believed to be equally effective (Ellis, 2010). With the classroom being the anticipated location of intervention implementation for many of the studies in this meta-analysis, sample sizes are expected to be smaller than 20, which is why Hedges' *g* will be used. Calculations and data analysis for this study were conducted using R Version 4.0.2, a statistical software program developed for statistical and data analysis. The researcher has prior experience with this statistical program and there are pre-existing codes for meta-analyses to help expedite the analytical process. When calculating Hedges' *g*, the difference between the means is divided by the pooled and weighted standard deviation (Hedges' & Olkin, 1985). Since Hedges' *g* is a correction of Cohen's *d*, we provided the formulas below for calculating Cohen's *d* (Borenstein, Hedges', Higgins, & Rothstein, 2011).

$$\text{Cohen's } d = \frac{M_1 - M_2}{SD_{pooled}^*}$$

$M1 - M2$  is the differences in means and  $SD^*pooled$  is the pooled and weighted standard deviation. This statistic yields a reliable effect size for smaller, classroom-sized populations.

$$S * pooled = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

The following formula is how Hedges'  $g$  was calculated for the purposes of this paper:

$$\text{Hedges' } g = c(m)d$$

where  $c(m)$  is the biased correction of the overestimated  $d$  value in larger samples. This statistic yields an unbiased effect size for smaller, classroom-sized populations.

$$c(m) = \frac{\Gamma\left(\frac{m}{2}\right)}{\sqrt{\frac{m}{2}} \Gamma\left(\frac{m-1}{2}\right)} \approx 1 - \frac{3}{4m-1} : m = n^T + n^C - 2$$

The following correction formula was used to convert  $d$  to Hedges'  $g$  to avoid overestimation (Borenstein et al., 2011):

$$J = 1 - \frac{3}{4df - 1}$$

### **Effect Size Single Group Studies**

Single group studies that have pre- and post-analyses were not excluded; they served as their own control group. For single group designs with pre- and post-test scores, Cohen's  $d$  was used with the following formulas where  $r$  is the pre-post test correlation (Borenstein, Hedges', Higgins, & Rothstein, 2011).

$$\text{Cohen's } d = \frac{Y_{post} - Y_{pre}}{S_{within}} = \frac{\sqrt{2(1-r)} (Y_{post} - Y_{pre})}{S_{gain}}$$

$S_{within}$  is the mean difference between gain scores of the pre- and post-test. This statistic yields a reliable effect size for smaller, classroom-sized populations.

$$S_{within} = \frac{S_{gain}}{\sqrt{2(1 - r)}}$$

### **Heterogeneity in Effect Sizes**

Heterogeneity across studies was examined in several ways. First, we examined overall heterogeneity for each outcome by calculating the  $Q$  statistics,  $I^2$  statistics,  $\tau^2$ , and interpreting forest plots. We reported summary effect sizes along with 95% confidence intervals and estimates of  $\tau^2$  and  $I^2$ , which we computed using the *metafor* package in R (Möbius, 2014; R Version 4.0.2). Moderator analyses also were conducted to explore potential causes of systematic variance using the *metafor* package for R (Viechtbauer, 2010). When estimating values in R, we utilized the Restricted Maximum Likelihood (REML). We chose to use REML as the method for calculating heterogeneity estimates using meta-regressions in an attempt to avoid over-estimating the heterogeneity variance accounted for by the model.

The  $Q$  statistic is the weighted sum of square deviations from the mean effect size. It is computed by multiplying each study's squared deviation from the mean by its inverse variance weight, and then summing those values (Borenstein et al., 2011). The  $Q$  statistic can be difficult to interpret; it is a sum and is impacted by the number of studies. A statistically significant  $p$ -value for the  $Q$  statistic does provide evidence that true effects appear to vary across studies; however, a non-statistically significant  $p$ -value

does not mean heterogeneity is low; it may simply be a small number of studies are included or within-study variance is large (Borenstein et al., 2011).

It  $I^2$  statistic helps understand what proportion of the observed variance explains real differences in effect sizes (Borenstein et al., 2011).  $I^2$  has a range of 0-100%; values of approximately 25% are considered low, whereas 50% might be considered moderate and 75% may be considered high.  $I^2$  values of 100% indicate only most of the observed variance is likely to be true variance rather than spurious variance; high values do not mean heterogeneity is high nor do low values mean between study variation is low (Borenstein et al., 2011).

### **Publication Bias**

To detect any publication bias, a funnel plot analysis was used for each of the design types. A funnel plot analysis is a scatter plot of individual studies, sometimes their precision, and results. The y-axis represents the standard error and studies with higher power are placed towards the top of the funnel plot. The x-axis represents the results of the study. If the funnel plot appears asymmetrical, then the likelihood of publication bias is higher than if the funnel plot is symmetrical. The method for generating the funnel plot involves writing code in the statistical program *R*. This methodology was specifically geared for meta-analyses and has the ability to work with both group and single-case designs together through a funnel plot. A Trim-and-Fill method will be applied as well as Egger's Regression Test and the Fail-safe N methodology. Running multiple publication bias will help account for error due to small sample sizes.

## Meta-regression

In order to consider the extent to which moderator variables affect the effect size, meta-regression techniques will be employed. Meta-regression assesses the relationships between a dependent variable and one, or more, of predictor variables (Borenstien et al., 2009). There are several meta-regression techniques that could be used: simple regression, fixed-effect, and mixed-effects (Borenstein et al., 2011). For the purposes of this study, a random-effects model was utilized to allow for within-study and between-study variation. In particular, each of the 12 intervention types were analyzed, as well as several moderators including: training of the interventionist, length of the intervention, alignment to NCTM standards, and follow-up data.

We used  $Q$ -tests and goodness of fit tests to test our models. The  $Q$  statistic is the weighted sum of squares and reflects the total variability of studies. For a fixed effect model, the  $Q$ -test partitions  $Q$  into its component parts,  $Q_{\text{resid}}$  ( $Q_R$ ) and  $Q_{\text{model}}$  ( $Q_M$ ) such that  $Q_R$  and  $Q_M$  are additive; however, for a random effects model, the weights assigned for each study incorporate between study variance; thus, the variance components are not additive for the random effects model (Borenstein et al., 2009). The random effects model assumes that for any value of a moderator, there is a distribution of true slope coefficients and the true coefficient depends on the subgroup of the population; the slope coefficient ( $B$ ) found for each moderator is assumed to be the mean, not the “true” coefficient (Borenstein et al., 2009).

For each moderator examined we present the results of the  $Q$  test and the goodness of fit test, which produced a  $Q_M$  and a  $Q_R$  that indicate between-study

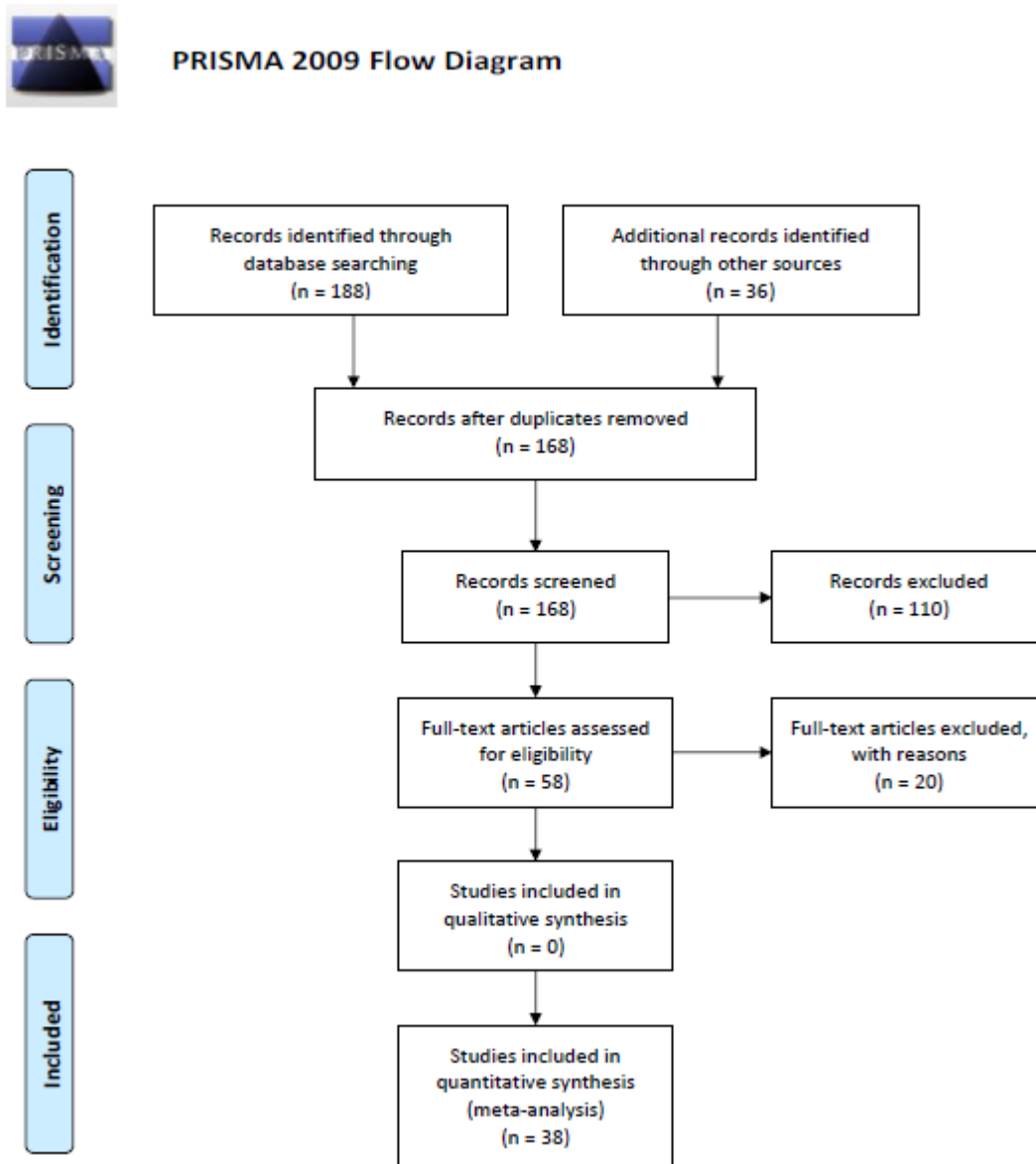
variability and within-study variability respectively. For the  $Q$  test, we examine if the p-value that corresponds to  $Q_M$  is statistically significant at  $\alpha = .05$ . For the goodness of fit test, we examine  $Q_R$ , and its corresponding p value, as well as  $\tau^2$ , to examine heterogeneity not explained by the model. In this context,  $\tau^2$  refers to the estimated population value of between-studies variance. We also examine  $I^2$ , which in the context of random effects meta-regression, refers to the proportion of the unexplained variance that is likely true variance as opposed to error. We also examine  $R^2$ , the proportion of the variance that is explained by the model.

## CHAPTER IV

### RESULTS

The planned analyses describe in Chapter III were completed. For the 30% of studies that were coded by two coders, overall interrater reliability was acceptable at 95%. All effect sizes were computed such that a positive value indicates improvement for the treatment group. Initial considerations were for heterogeneity. Heterogeneity across studies was examined by calculating the  $Q$  statistics,  $I^2$  statistics, and interpreting forest plots. Moderator analyses also were conducted to explore potential associations with any systematic variance impacting the effectiveness of algebra intervention. Study quality and increased adherence to national standards (i.e., Common Core) were examined as a continuous moderator following the testing of hypotheses. The following sections present the results. Figure 1 shows the PRISMA flow diagram of studies and how we arrived at the 38 coded studies used in this meta-analysis.

Figure 1 PRISMA Flow Diagram



### Heterogeneity Analyses

Heterogeneity across studies was examined by calculating the  $Q$  statistics,  $I^2$  statistics, the estimate of the between-studies variance component ( $\tau^2$ ) and interpreting forest plots. The  $Q$  statistics,  $I^2$  statistics,  $\tau^2$  estimates were computed using the rma



function in the metafor package for R. Moderator analyses were also conducted to explore potential causes of systematic variance using the rma function in the *metafor* package for R (Viechtbauer, 2010).

### *Heterogeneity of Intervention Results*

Results are presented first for effect sizes at post-intervention. Table 3 provides an overview of relevant statistics for examining heterogeneity at post-intervention.

**Table 4 Effect Sizes of Intervention**

Outcome variable	<i>k</i>	Hedges' <i>g</i> /Tau-U	95% CI		<i>t</i>	$\tau^2$	<i>Q</i>	<i>I</i> <sup>2</sup>
Algebra Skills Single Case	20	0.93*** (Tau-U)	0.76	1.11	0	0	0.89	0.00%
Algebra Skills Group Design	18	0.71*** (Hedges' <i>g</i> )	0.50	0.93	0.37	0.14	55.41	74.34%

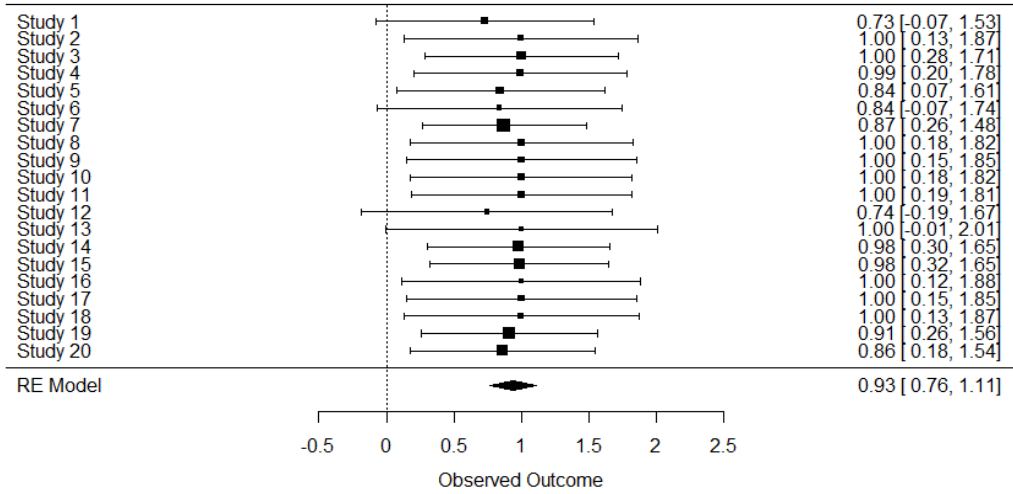
Note. \* statistically significant at  $p < .05$ ; \*\* statistically significant at  $p < .01$ ; \*\*\* statistically significant at  $p < .001$ , *k* is the number of studies included in the analysis.

### **Heterogeneity for Algebra Skills Intervention**

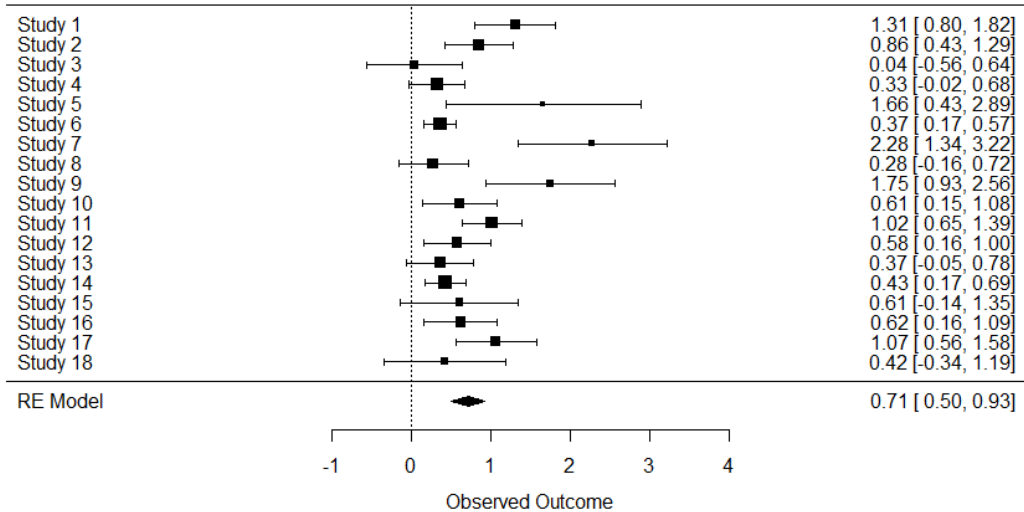
The individual study effect sizes for algebra skills at post-intervention are reported in the forest plots seen in Figure 2 and Figure 3. As seen in Table 4, the *Q* statistic for single case studies was statistically significant,  $Q(19) = 0.89, p = <.0001$ . This result, combined with examining the *I*<sup>2</sup> statistic (0%) and the variance of the effect sizes ( $\tau^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra

interventions have small levels of heterogeneity. The  $Q$  statistic for group design studies was also statistically significant,  $Q(17) = 55.41, p = <.0001$ . This result, combined with examining the  $I^2$  statistic (74.34%) and the variance of the effect sizes ( $\tau^2 = 0.14$ ) together suggest the effect sizes for algebra interventions with group designs have high levels of heterogeneity. Overall, both study designs produced statistically significant effects; however, data should be interpreted with caution due to higher levels of heterogeneity within the group design studies. Visual analysis of Figure 2 notes consistent effect sizes and confidence intervals across studies. Visual analysis of Figure 3 shows more variability across studies. One particular study to note is Study 7 which demonstrated a large effect of  $g = 2.28$ . Quality analysis will be discussed in future sections, but it is important to note this study received a quality score near the lower threshold out of the included studies. Perhaps the diminished quality of the study impacted the higher level of effect demonstrated as having weaker methodology could lead to over-estimated, or inflated, effect sizes.

**Figure 2 Algebra Skills Post-Intervention Forest Plot for Single Case Studies**



**Figure 3 Algebra Skills Post-Intervention Forest Plot for Group Design Studies**

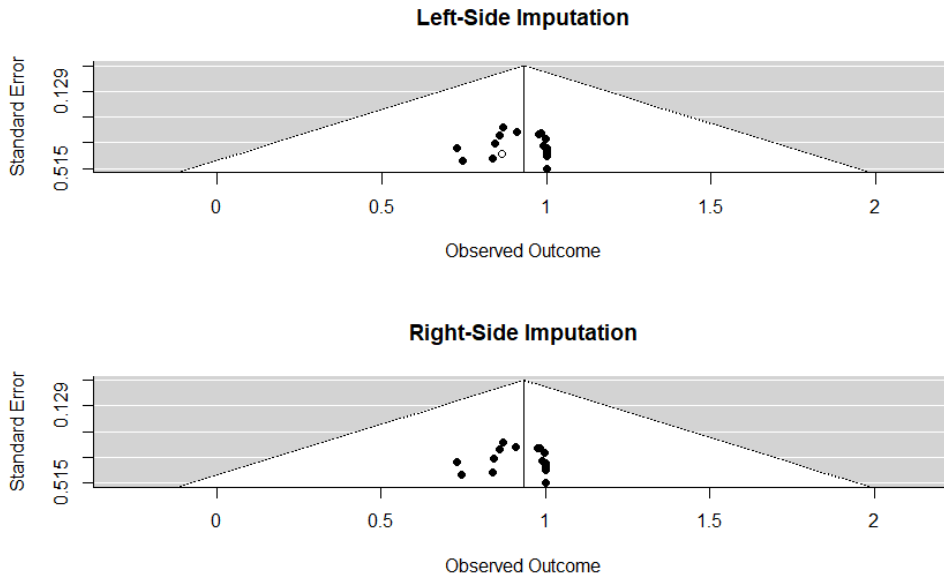


## Analyses of Publication Bias

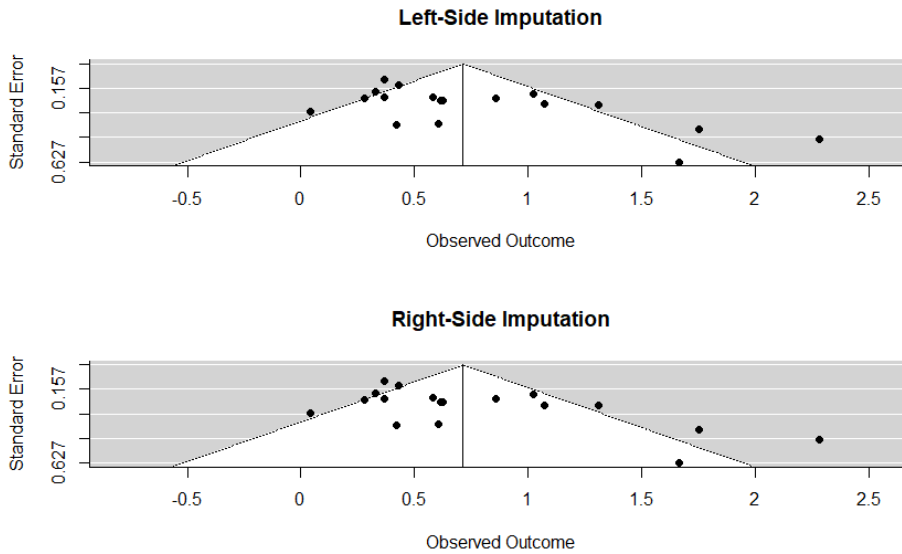
We examined publication bias by conducting several analyses including: Egger's Regression Test, a Trim-and-Fill investigation and a Fail-safe N calculation. Overall results were not indicative of publication bias; however, due to small sample size, these results should be interpreted with caution. Egger's regression test is a sensitivity analysis that essentially assesses for the slope of the regression line with a Z-score. Typically, we expect the slope of that line to be zero, and statistically significant results indicate plot asymmetry. Results yielded a Z-score of 3.09 with a  $p\text{-value} = .0020$  for group designs and a Z-score of 0.11 with a  $p\text{-value} = .91$  for single case designs, thus indicating plot symmetry for single case studies; however, there is evidence of publication bias in group designs.

A Trim-and-Fill investigation was conducted to further assess the possibility of publication bias. The Trim-and-Fill method is an iterative approach that assesses asymmetry and re-estimates the mean effect size while creating a new funnel plot. This is accomplished by removing smaller studies that may be responsible for asymmetry. Both left and right sides are imputed during the analysis. The Trim-and-Fill method for group design studies yielded a significant point estimate of 0.71. The calculation estimated the number of missing studies on the left side to be 0. For single case design studies, the Trim-and-Fill method yielded a significant point estimate of 0.93. The calculation estimated the number of missing studies on the left side to be 1. These results, shown in Figure 4 and Figure 5, are indicative of plot symmetry and minimal publication bias.

**Figure 4 Trim-and-Fill Plot for Single Case Studies**



**Figure 5 Trim-and-Fill Plot for Group Design Studies**



A final sensitivity analysis was conducted to address the likelihood of publication bias. A Fail-safe N analysis was computed to determine how many additional studies would need to make a statistically significant result become non-statistically significant. The results from the single case study investigation yielded a Fail-safe N of 787 to reach a significance p-value level of 0.05 and the group design study investigation yielded a Fail-safe N of 1000 to reach the same level of significance. This data provides evidence of minimal publication bias.

### **Research Question 1**

Research question 1 asked: what effect have algebra interventions had on students who are struggling or at-risk for algebra failure? We analyzed this by calculating the average weighted effect size for all coded studies, while separating them by study design. They were separated by study design because it is not best practice to compare effect sizes from group designs to single case designs. It was hypothesized modes of intervention that utilize technology and peer mentoring would demonstrate large gains than those not including such interventions. In addition, approaches that utilize modeling and schema-based instruction, and CSI will help remedy the difficulty understanding abstract material by demonstrating larger effects than its counterparts. Similarly, it is hypothesized that interventions which target self-regulation, problem solving skills, and content knowledge will provide support for students requiring additional help. We hypothesized that overall, interventions would produce a medium to

large effect in improving algebra performance, as defined using Cohen's conventions (1988) and Tau-U research.

Tau-U effect size scores can be interpreted as follows:  $<0.20$  = small change,  $0.20$  to  $0.60$  = moderate change,  $0.60$  to  $0.80$  = large change, and  $>0.80$  = large to very large change (Vannest & Ninci, 2015). We tested this hypothesis by calculating the summary effect size statistics for all included studies, using a random effects model, using the metafor package in R. The hypothesis for research question 1 was supported in that both group study designs and single case designs produced moderate to large effects when considering Cohen's conventions (1988) and Tau-U standards. It is important to note the data set used in this analysis were unbalanced, which made it challenging to analyze moderators. There were several coded studies that implemented multiple types of intervention, which led to some interventions being used with high frequency, and others being used at low frequency. This caused the data to be highly unbalanced. Additionally, since the coded studies had to be separated by design (i.e., single case and group design), there were small sample sizes ( $n=18$  and  $n=20$ ). It is also important to note Tau-U is a bounded variable and operates within the interval of  $-1$  to  $+1$ , thus creating more limitations on analyses.

The mean effect sizes at post-intervention for each type of study design, are presented in Table 3. The mean effect size for single case designs (Tau-U =  $0.94$ , 95% CI,  $[0.76, 1.11]$ ,  $p < .0001$ ) and the mean effect size for group designs ( $g = 0.71$ , 95% CI,  $[0.50; 0.93]$ ,  $p < .0001$ ) are both considered large or better by Cohen's conventions (1988) and Tau-U research. For the purposes of this study, our interest lies, not in the

statistical significance of the effect sizes, but rather the magnitude of the effect sizes and our interpretation of the practical significance of these effect sizes. Regardless of what alpha is set at, Hedges'  $g$  and Tau-U and the 95% confidence interval around each effect size remains the same and this information is what we weigh most heavily in our interpretation of the mean effect sizes.

As mentioned previously, effect size conventions must be used with caution (Durlak, 2009; Thompson, 2002). Durlak (2009) recommends interpreting effect sizes in the context of the larger literature as well as for the clinical meaningfulness of the effect. These standardized mean difference effect sizes and non-parametric effect sizes produced in this research are similar to previous meta-analyses.

## **Research Question 2**

Research question 2 asked: do different types of interventions have stronger effects? Our results are described in the following section. It is hypothesized the most effective intervention technique for students requiring special services will be approaches that implement CSI, interventions that target problem solving ability, and intervention modes that employ technology.

To answer this question, we employed a meta-regression of each intervention and compared the difference between the mean effect size of the coded studies not including that intervention to the studies that included the specific intervention. This provided us information regarding the impact of the intervention. It is important to note the various limitations that come with this methodology. First, the data collected in this study were



determined to be unbalanced. Due to the unbalanced nature of the data, moderator analyses were difficult to conduct. Secondly, the researcher understands a multivariate approach would be more aligned with the provided variables; however, given the imbalance of the data, there was no guarantee a multivariate analysis would have produced more accurate results. Each moderator had to be separated and ran independently because of the unbalanced data. This separation created individual null hypotheses which computed its own point estimate. Comparing these point estimates is considered a limitation because as we compared one null hypothesis significance test to another, we, in turn, generated a new null hypothesis significance test.

Second, a network meta-analysis may have fit the nature of the study more effectively; however, similar to the multivariate discussion, there was no guarantee the unbalanced data would have provided accurate and reliable information through the network meta-analysis. As a result, the researcher elected to analyze the differences between each intervention technique and the mean effect size of the other studies not including the intervention, which provides at least minimal information regarding the effectiveness of each intervention. This methodology makes it challenging to compare various intervention styles to one another, so the information in this section should be interpreted with these limitations in mind.

Table 4 presents the mixed-effects moderator results of the various intervention types and also displays the difference between the mean effect size of the coded studies and the specific intervention.

**Table 5 Intervention Moderator Results for Group Designs**

<b>Intervention</b>	<b><i>k</i></b>	<b>Hedges' <i>g</i></b>	<b>Difference</b>	<b>95% CI</b>		<b><i>t</i></b>	<b><math>\tau_x^2</math></b>	<b><i>Q</i><sub>within</sub></b>	<b><i>Q</i><sub>between</sub></b>	<b>Residual <i>I</i><sup>2</sup></b>
<b>Modeling and Schema</b>	18	0.62	-0.11	-0.08	1.31	0.40	0.16	55.37	40.24	76.93%
<b>Problem Solving</b>	18	0.70**	-0.02	0.23	1.17	0.40	0.16	55.19	39.88	76.61%
<b>Self-Regulation</b>	18	0.04	-0.70	-0.88	0.97	0.36	0.13	52.22	47.17	73.54%
<b>Technology</b>	18	0.72***	0.01	0.35	1.10	0.40	0.16	52.15	39.88	75.42%
<b>EI</b>	18	0.65**	-0.10	0.22	1.08	0.40	0.16	54.79	39.61	77.03%
<b>Guided Practice</b>	18	0.61*	-0.13	0.07	1.15	0.40	0.16	55.37	40.12	76.76%
<b>Feedback and Reinforcement</b>	18	0.46	-0.29	-0.21	1.13	0.39	0.15	54.63	42.03	75.80%
<b>Peer Tutoring</b>	18	0.35	-0.44	-0.14	0.85	0.36	0.13	53.06	47.01	73.26%
<b>CSI</b>	--	--	--	--	--	--	--	--	--	--
<b>CRA</b>	18	0.88**	0.19	0.35	1.41	0.40	0.16	55.15	40.78	75.26%
<b>EAI</b>	18	0.65*	-0.08	0.15	1.15	0.10	0.16	54.76	39.61	75.12%
<b>Graphic Organizer</b>	--	--	--	--	--	--	--	--	--	--

Note. \* statistically significant at  $p < .05$ ; \*\* statistically significant at  $p < .01$ ; \*\*\* statistically significant at  $p < .001$ ,  $k$  is the number of studies included in the analysis. EI = Explicit Inquiry; CSI = Cognitive Strategy Instruction; CRA = Concrete-Representational-Abstract; EAI = Enhanced Anchored Instruction.

### *Modeling and Schema*

As seen in Table 5, group design studies including a modeling and schema-based intervention yielded a Hedges'  $g$  of 0.62, which is found to not be statistically significant with a  $p$ -value = .0816, and a  $Q_{\text{within}}$  of 55.37 and a  $Q_{\text{between}}$  of 40.24, both of which are statistically significant with  $p$ -values <.0001. These results combined with examining the  $I^2$  statistic (76.93%) and the variance of effect sizes ( $\tau_x^2 = 0.16$ ) together suggest the effect sizes for group design studies with algebra interventions have high levels of heterogeneity and the intervention explains some, but not all, of the variance within and between groups. Thus, alluding to other potential moderators accounting for the remaining portion of the effect.

Single case studies including modeling and schema-based interventions yielded a Tau-U effect size of 0.89, which is found to be statistically significant with a  $p$ -value <.0001, and a  $Q_{\text{within}}$  of 0.68 and a  $Q_{\text{between}}$  of 110.99; which are found to be statistically significant and not statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains most of the between-studies difference.

The difference between studies coded using the modeling and schema approach and studies not using the modeling and schema approach yielded a difference of 0.62 for group designs and -0.08 for single case studies. These differences suggest for each

additional unit of intervention, the Hedges'  $g$  and Tau-U effects will decrease by 0.11 and 0.08 respectively.

### *Problem Solving*

As seen in Table 5, group design studies including a problem-solving intervention yielded a Hedges'  $g$  of 0.70, which is found to be statistically significant with a  $p$ -value = .0033, and a  $Q_{\text{within}}$  of 55.19 and a  $Q_{\text{between}}$  of 39.88, both of which are statistically significant with  $p < .0001$ . These results combined with examining the  $I^2$  statistic (76.61%) and the variance of effect sizes ( $\tau_x^2 = 0.16$ ) together suggest the effect sizes for group design studies with algebra interventions have high levels of heterogeneity and the grouping variable explains some, but not all, of the variance within and between groups.

Single case studies including problem-solving interventions yielded a Tau-U effect size of 0.89, which is found to be statistically significant with a  $p$ -value  $< .0001$ , and a  $Q_{\text{within}}$  of 0.72 and a  $Q_{\text{between}}$  of 110.94, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains little of the between-studies difference.

The difference between studies coded using the problem-solving approach and studies not using the problem-solving approach yielded a difference of -0.02 for group designs and -0.07 for single case studies. These differences suggest for each additional

unit of intervention, the Hedges'  $g$  and Tau-U effects will decrease by 0.02 and decrease by 0.07 respectively.

### *Self-Regulation*

As seen in Table 5, group design studies including a self-regulation intervention yielded a Hedges'  $g$  of 0.04, which is found to not be statistically significant with a  $p$ -value = 0.93, and a  $Q_{\text{within}}$  of 52.22 and a  $Q_{\text{between}}$  of 47.17, both of which are statistically significant with  $p < .0001$ . These results combined with examining the  $I^2$  statistic (73.54%) and the variance of effect sizes ( $\tau_x^2 = 0.13$ ) together suggest the effect sizes for group design studies with algebra interventions have high levels of heterogeneity and the grouping variable explains some, but not all, of the variance within and between groups.

Single case studies including self-regulation interventions yielded a Tau-U effect size of 0.89, which is found to be statistically significant with a  $p$ -value  $< .0001$ , and a  $Q_{\text{within}}$  of 0.84 and a  $Q_{\text{between}}$  of 110.82, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains all of the between-studies difference.

The difference between studies coded using the self-regulation approach and studies not using the self-regulation approach yielded a difference of 0.04 for group designs and -0.05 for single case studies. These differences suggest for each additional unit of intervention, the Hedges'  $g$  and Tau-U effects will increase by 0.04 and decrease by 0.05 respectively.

### *Technology*

As seen in Table 5, group design studies including a technology intervention yielded a Hedges'  $g$  of 0.72, which is found to be statistically significant with a  $p$ -value = .0001, and a  $Q_{\text{within}}$  of 52.15 and a  $Q_{\text{between}}$  of 39.88, both of which are statistically significant with  $p < .0001$ . These results combined with examining the  $I^2$  statistic (75.42%) and the variance of effect sizes ( $\tau_x^2 = 0.16$ ) together suggest the effect sizes for group design studies with algebra interventions have high levels of heterogeneity and the grouping variable explains some, but not all, of the variance within and between groups.

Single case studies including technology-based interventions yielded a Tau-U effect size of 0.96, which is found to be statistically significant with a  $p$ -value  $< .0001$ , and a  $Q_{\text{within}}$  of 0.87 and a  $Q_{\text{between}}$  of 110.79, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains all of the between-studies difference. The difference between studies coded using the technology approach and studies not using the technology approach yielded a difference of 0.01 for group designs and 0.03 for single case studies. These differences suggest for each additional unit of intervention, the Hedges'  $g$  and Tau-U effects will increase by 0.01 and increase by 0.03 respectively.

### *Explicit Instruction (EI)*

As seen in Table 5, group design studies including an EI intervention yielded a Hedges'  $g$  of 0.65, which is found to be statistically significant with a  $p$ -value = .0029,

and a  $Q_{\text{within}}$  of 54.79 and a  $Q_{\text{between}}$  of 39.61, both of which are statistically significant with  $p < .0001$ . These results combined with examining the  $I^2$  statistic (77.03%) and the variance of effect sizes ( $\tau_x^2 = 0.16$ ) together suggest the effect sizes for group design studies with algebra interventions have high levels of heterogeneity and the grouping variable explains some, but not all, of the variance within and between groups.

Single case studies including EI interventions yielded a Tau-U effect size of 0.94, which is found to be statistically significant with a  $p\text{-value} < .0001$ , and a  $Q_{\text{within}}$  of 0.84 and a  $Q_{\text{between}}$  of 110.82, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains all of the between-studies difference.

The difference between studies coded using the EI approach and studies not using the EI approach yielded a difference of -0.10 for group designs and 0.06 for single case studies. These differences suggest for each additional unit of intervention, the Hedges'  $g$  and Tau-U effects will decrease by 0.10 and decrease by 0.06 respectively.

#### *Guided Practice*

As seen in Table 5, group design studies including a guided practice intervention yielded a Hedges'  $g$  of 0.61, which is found to be statistically significant with a  $p\text{-value} = .0280$ , and a  $Q_{\text{within}}$  of 55.37 and a  $Q_{\text{between}}$  of 40.12, both of which are statistically significant with  $p < .0001$ . These results combined with examining the  $I^2$  statistic (76.76%) and the variance of effect sizes ( $\tau_x^2 = 0.16$ ) together suggest the effect sizes for

group design studies with algebra interventions have high levels of heterogeneity and the grouping variable explains some, but not all, of the variance within and between groups.

Single case studies including guided practice interventions yielded a Tau-U effect size of 0.93, which is found to be statistically significant with a *p-value* <.0001, and a  $Q_{\text{within}}$  of 0.88 and a  $Q_{\text{between}}$  of 110.78, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains all of the between-studies difference.

The difference between studies coded using the guided practice approach and studies not using the guided practice approach yielded a difference of -0.13 for group designs and -0.02 for single case studies. These differences suggest for each additional unit of intervention, the Hedges' *g* and Tau-U effects will decrease by 0.13 and 0.02 respectively.

#### *Feedback and Reinforcement*

As seen in Table 5, group design studies including a feedback and reinforcement intervention yielded a Hedges' *g* of 0.46, which is found to not be statistically significant with a *p-value* = 0.18, and a  $Q_{\text{within}}$  of 54.63 and a  $Q_{\text{between}}$  of 42.03, both of which are statistically significant with  $p < .0001$ . These results combined with examining the  $I^2$  statistic (75.80%) and the variance of effect sizes ( $\tau_x^2 = 0.15$ ) together suggest the effect sizes for group design studies with algebra interventions have high levels of



heterogeneity and the grouping variable explains some, but not all, of the variance within and between groups.

Single case studies including feedback and reinforcement interventions yielded a Tau-U effect size of 0.91, which is found to be statistically significant with a *p-value* <.0001, and a  $Q_{\text{within}}$  of 0.83 and a  $Q_{\text{between}}$  of 110.83, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains all of the between-studies difference.

The difference between studies coded using the feedback and reinforcement approach and studies not using the feedback and reinforcement approach yielded a difference of -0.29 for group designs and -0.04 for single case studies. These differences suggest for each additional unit of intervention, the Hedges' *g* and Tau-U effects will decrease by 0.29 and decrease by 0.04 respectively.

#### *Peer Tutoring*

As seen in Table 5, group design studies including a peer tutoring intervention yielded a Hedges' *g* of 0.35, which is found to not be statistically significant with a *p-value* = .1642, and a  $Q_{\text{within}}$  of 53.06 and a  $Q_{\text{between}}$  of 47.01, both of which are statistically significant with  $p < .0001$ . These results combined with examining the  $I^2$  statistic (73.26%) and the variance of effect sizes ( $\tau_x^2 = 0.13$ ) together suggest the effect sizes for group design studies with algebra interventions have high levels of

heterogeneity and the grouping variable explains some, but not all, of the variance within and between groups. There were no single case studies included in this analysis that incorporated a peer tutoring component.

The difference between studies coded using the peer tutoring approach and studies not using the peer tutoring approach yielded a difference of -0.44 for group designs. These differences suggest for each additional unit of intervention, the Hedges'  $g$  will decrease by 0.44.

#### *Cognitive Strategy Instruction (CSI)*

Single case studies including CSI interventions yielded a Tau-U effect size of 0.95, which is found to be statistically significant with a  $p$ -value  $<.0001$ , and a  $Q_{\text{within}}$  of 0.88 and a  $Q_{\text{between}}$  of 110.79, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains all of the between-studies difference. There were no group design studies that included a CSI intervention.

The difference between studies coded using the CSI approach and studies not using the CSI approach yielded a difference of 0.02 for single case studies. This difference suggests for each additional unit of intervention, the Tau-U effect will increase by 0.02.

### *Concrete-Representational-Abstract (CRA)*

As seen in Table 5, group design studies including a CRA intervention yielded a Hedges'  $g$  of 0.88, which is found to be statistically significant with a  $p$ -value = .0012, and a  $Q_{\text{within}}$  of 55.15 and a  $Q_{\text{between}}$  of 40.78, both of which are statistically significant with  $p < .0001$ . These results combined with examining the  $I^2$  statistic (75.26%) and the variance of effect sizes ( $\tau_x^2 = 0.16$ ) together suggest the effect sizes for group design studies with algebra interventions have high levels of heterogeneity and the grouping variable explains some, but not all, of the variance within and between groups.

Single case studies including CRA interventions yielded a Tau-U effect size of 0.91, which is found to be statistically significant with a  $p$ -value  $< .0001$ , and a  $Q_{\text{within}}$  of 0.85 and a  $Q_{\text{between}}$  of 110.82, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains all of the between-studies difference.

The difference between studies coded using the CRA approach and studies not using the CRA approach yielded a difference of 0.19 for group designs and -0.04 for single case studies. These differences suggest for each additional unit of intervention, the Hedges'  $g$  and Tau-U effects will increase by 0.19 and decrease by 0.04 respectively.

### *Enhanced Anchored Instruction (EAI)*

As seen in Table 5, group design studies including an EAI intervention yielded a Hedges'  $g$  of 0.65, which is found to be statistically significant with a  $p$ -value = .0103,

and a  $Q_{\text{within}}$  of 54.76 and a  $Q_{\text{between}}$  of 39.61, both of which are statistically significant with  $p < .0001$ . These results combined with examining the  $I^2$  statistic (75.12%) and the variance of effect sizes ( $\tau_x^2 = 0.16$ ) together suggest the effect sizes for group design studies with algebra interventions have high levels of heterogeneity and the grouping variable explains some, but not all, of the variance within and between groups.

Single case studies including EAI interventions yielded a Tau-U effect size of 0.95, which is found to be statistically significant with a  $p\text{-value} < .0001$ , and a  $Q_{\text{within}}$  of 0.88 and a  $Q_{\text{between}}$  of 110.78, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains all of the between-studies difference.

The difference between studies coded using the EAI approach and studies not using the EAI approach yielded a difference of -0.08 for group designs and 0.02 for single case studies. These differences suggest for each additional unit of intervention, the Hedges'  $g$  and Tau-U effects will decrease by 0.08 and increase by 0.02 respectively.

#### *Graphic Organizer*

Single case studies including graphic organizer interventions yielded a Tau-U effect size of 0.90, which is found to be statistically significant with a  $p\text{-value} < .0001$ , and a  $Q_{\text{within}}$  of 0.77 and a  $Q_{\text{between}}$  of 110.89, which are found to be not statistically significant and statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_x^2 = 0$ ) together suggest

the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains all of the between-studies difference. There were no group design studies that included a graphic organizer intervention.

The difference between studies coded using the graphic organizer approach and studies not using the graphic organizer approach yielded a difference of -0.06 for single case studies. This difference suggests for each additional unit of intervention, the Tau-U effects will decrease by 0.06.

**Table 6 Intervention Moderator Results for Single Case Designs**

<b>Intervention</b>	<b><i>k</i></b>	<b>Tau-U</b>	<b>Difference</b>	<b>95% CI</b>		<b><i>t</i></b>	<b><math>\tau_x^2</math></b>	<b><i>Q</i><sub>within</sub></b>	<b><i>Q</i><sub>between</sub></b>	<b><i>I</i><sup>2</sup></b>
<b>Modeling and Schema</b>	20	0.89***	-0.08	0.61	1.16	0	0	0.68	110.99	0.00%
<b>Problem Solving</b>	20	0.89***	-0.07	0.63	1.16	0	0	0.72	110.94	0.00%
<b>Self-Regulation</b>	20	0.89***	-0.05	0.45	1.33	0	0	0.84	110.82	0.00%
<b>Technology</b>	20	0.96***	0.03	0.58	1.34	0	0	0.87	110.79	0.00%
<b>EI</b>	20	0.94***	0.06	0.76	1.13	0	0	0.84	110.82	0.00%
<b>Guided Practice</b>	20	0.93***	-0.02	0.66	1.19	0	0	0.88	110.78	0.00%
<b>Feedback and Reinforcement</b>	20	0.91***	-0.04	0.61	1.20	0	0	0.83	110.83	0.00%
<b>Peer Tutoring</b>	--	--	--	--	--	--	--	--	--	--
<b>CSI</b>	20	0.95***	0.02	0.62	1.28	0	0	0.88	110.79	0.00%
<b>CRA</b>	20	0.91***	-0.04	0.64	1.18	0	0	0.84	110.82	0.00%
<b>EAI</b>	20	0.95***	0.02	0.57	1.33	0	0	0.88	110.78	0.00%
<b>Graphic Organizer</b>	20	0.90***	-0.06	0.64	1.17	0	0	0.77	110.89	0.00%

Note. \* statistically significant at  $p < .05$ ; \*\* statistically significant at  $p < .01$ ; \*\*\* statistically significant at  $p < .001$ ,  $k$  is the number of studies included in the analysis. EI = Explicit Inquiry; CSI = Cognitive Strategy Instruction; CRA = Concrete-Representational-Abstract; EAI = Enhanced Anchored Instruction.

### Research Question 3

Research question 3 asked: what factors influence, or moderate, the effectiveness of algebra interventions? We hypothesized multiple factors, including length of intervention, training of interventionists, alignment with NCTM standards, and if follow-up data was collected. The specific hypotheses are described below. Note the same limitations discussed in the prior analysis apply to these moderators as well.

#### Hypothesis 3a

We hypothesized that training the person providing the intervention would lead to greater effects on the intervention outcome. We tested this by running a moderator analysis with intervention training being coded as a dichotomous variable (1=yes; 0=no). As seen in Table 7, both single case and group design studies produced a large effect size when using their respective conventions.

The  $Q$  statistic for single case studies was statistically significant,  $Q(6) = 0.93, p = <.0001$ . This result, combined with examining the  $I^2$  statistic (0%) and the variance of the effect sizes ( $\tau_M^2 = 0$ ) together suggest the effect sizes for single case design studies with trained interventionists have small levels of heterogeneity. The  $Q$  statistic for group design studies was also statistically significant,  $Q(5) = 11.86, p = <.001$ . This result, combined with examining the  $I^2$  statistic (71.53%) and the variance of the effect sizes ( $\tau_M^2 = 0.20$ ) together suggest the effect sizes for algebra interventions with group designs have higher levels of heterogeneity. Overall, both study designs produced statistically significant effects; however, data should be interpreted with caution due to higher levels of heterogeneity within the group design studies.

**Table 7 Interventionist Training**

Moderator	<i>k</i>	Hedges' <i>g</i> / <i>TauU</i>	95% CI		<i>t</i>	$\tau_M^2$	<i>Q</i>	<i>I</i> <sup>2</sup>
<b>Single Case</b>	6	0.93***	0.60	1.26	0	0	0.37	0.00%
<b>Group Design</b>	5	0.73**	0.25	1.20	0.45	0.20	11.86	71.53%

Note. \* statistically significant at  $p < .05$ ; \*\* statistically significant at  $p < .01$ ; \*\*\* statistically significant at  $p < .001$ , *k* is the number of studies included in the analysis.

### Hypothesis 3b

We hypothesized that interventions lasting longer would produce a greater outcome for students. We tested this hypothesis by running a moderator analysis and analyzing the difference between coded studies that reported intervention length, which was coded as a continuous variable. For single case designs, the  $R^2$ , which explains the amount of heterogeneity that is explained by the moderator, was 0% and for group designs it was 32.1%.

As seen in Table 8, both single case and group design studies produced a medium to large effect sizes when using their respective conventions. Group design studies reported a large amount of heterogeneity as denoted by the elevated  $I^2$  statistic (63.38%). Upon analyzing the differences between studies that reported length of intervention and those who did not, the data shows minor differences in effect size as one unit of intervention length increases, the effect only improves by 0.02 for group designs and decreases by 0.01 for single case designs. Overall, while the intervention length is statistically significant for both variables, the moderator does not explain much of the variance when considering the  $R^2$ , *Q* values, and  $I^2$  statistics.



**Table 8 Intervention Length (weeks)**

Moderator	<i>k</i>	Hedges' <i>g</i> /Tau-U	Diff.	95% CI		$\tau$	$R^2$	$Q_{within}$ <i>n</i>	$Q_{bet}$ <i>ween</i>	$I^2$
Single Case	18	0.96***	-0.01	0.69	1.24	0	0%	0.74	0.07	0.00%
Group Design	17	0.50*	0.02	0.00	0.22	0.32	32.1%	38.85	5.06	63.38%

Note. \* statistically significant at  $p < .05$ ; \*\* statistically significant at  $p < .01$ ; \*\*\* statistically significant at  $p < .001$ , *k* is the number of studies included in the analysis.

### Hypothesis 3c

We hypothesized that interventions aligning to NCTM standards would produce greater effects on algebra skills because they would follow specific protocols and align with mathematical standards. For this analysis, NCTM alignment was coded as a dichotomous variable (1=aligned; 0=not aligned). All studies that were group designs were coded as aligning with NCTM standards, so a moderator analysis was not able to be run. When analyzing single case studies, the overall Tau-U was 0.94, which was statistically significant with a  $p$ -value  $< .0001$ . There was minimal heterogeneity within studies, as denoted by the  $I^2$  (0%). Similar to other analyses, the difference between point estimates of NCTM-aligned studies and non-NCTM-aligned studies was calculated and produced a value of 0.10. Overall, studies aligned with NCTM standards produced a large effect and demonstrated low levels of heterogeneity.

**Table 9 NCTM**

Moderator	<i>k</i>	Hedges' <i>g</i> /Tau-U	Diff.	95% CI		$\tau$	$R^2$	$Q_{within}$	$Q_{between}$	$I^2$
Single Case	20	0.94***	0.10	0.76	1.13	0	0%	0.78	110.88	0.00%

**Table 9 continued**

Moderator	<i>k</i>	Hedges' <i>g</i> /Tau-U	Diff.	95% CI		$\tau$	$R^2$	$Q_{within}$	$Q_{between}$	$I^2$
<b>Group</b>	--	--	--	--	--	--	--	--	--	--
<b>Design</b>										

Note. \* statistically significant at  $p < .05$ ; \*\* statistically significant at  $p < .01$ ; \*\*\* statistically significant at  $p < .001$ , *k* is the number of studies included in the analysis.

### Hypothesis 3d

We hypothesized studies that provided follow-up data would demonstrate a larger effect than studies who did not provide follow-up data. For this analysis, follow-up was coded as a dichotomous variable (1=follow-up data collected; 0=no follow-up). As seen in Table 10, the  $Q$  statistic for single case studies was statistically significant,  $Q(19) = 34.96, p = <.0141$ . This result, combined with examining the  $I^2$  statistic (46.16%) and the variance of the effect sizes ( $\tau_F^2 = 0.14$ ) together suggest the effect sizes for single case design studies with algebra interventions have high levels of heterogeneity. The  $Q$  statistic for group design studies was also statistically significant,  $Q(17) = 126.57, p = <.0001$ . This result, combined with examining the  $I^2$  statistic (92.13%) and the variance of the effect sizes ( $\tau_F^2 = 0.55$ ) together suggest the effect sizes for algebra interventions with group designs have high levels of heterogeneity. Overall, both study designs produced statistically significant effects; however, data should be interpreted with caution due to higher levels of heterogeneity within the studies. This is likely due to small sample sizes and the unbalanced data that was discussed previously.

**Table 10 Follow-up**

Moderator	<i>k</i>	Hedges' <i>g</i> / <i>TauU</i>	95% CI		<i>t</i>	$\tau_F^2$	<i>Q</i>	<i>I</i> <sup>2</sup>
<b>Single Case</b>	20	0.94***	0.66	1.22	0.37	0.14	34.96	46.16%
<b>Group Design</b>	18	0.74*	0.16	1.33	0.74	0.55	126.57	92.13%

Note. \* statistically significant at  $p < .05$ ; \*\* statistically significant at  $p < .01$ ; \*\*\* statistically significant at  $p < .001$ , *k* is the number of studies included in the analysis.

#### Research Question 4

Research question 4 asked: how have changes in federal and state standards impacted the effect of algebra interventions on student performance? It was hypothesized changes in standards have increased the necessity of rigorous intervention and improved the effectiveness of interventions on student outcomes. Common Core was implemented in 2009. As a result, we used 2009 as a cut-off year to separate pre- and post-Common Core studies. To determine the answer to this question, we ran a moderator analysis using post-2009 as the variable.

As seen in Table 11, group design studies post-2009 yielded a Hedges' *g* of 0.93, which is found to be statistically significant with a *p-value* < .0001, and a  $Q_{\text{within}}$  of 55.37 ( $p < .0001$ ) and a  $Q_{\text{between}}$  of 43.40 ( $p < .0001$ ). These results combined with examining the  $I^2$  statistic (74.39%) and the variance of effect sizes ( $\tau_c^2 = 0.15$ ) together suggest the effect sizes for group design studies post-2009 have high levels of heterogeneity and the intervention explains some, but not all, of the variance within and between groups. Thus, alluding to other potential moderators accounting for the remaining portion of the effect. Single case studies post-2009 yielded a *Tau-U* effect size of 0.95, which is found to be statistically significant with a *p-value* < .0001, and a  $Q_{\text{within}}$  of 0.83 and a  $Q_{\text{between}}$  of 110.83; which are found to be not statistically significant and

statistically significant respectively. These results combined with examining the  $I^2$  statistic (0%) and the variance of effect sizes ( $\tau_c^2 = 0$ ) together suggest the effect sizes for single case design studies with algebra interventions have small levels of heterogeneity and the grouping variable explains most of the between-studies difference.

**Table 11 Post-2009**

Moderator	<i>k</i>	Hedges' <i>g</i> /Tau-U	95% CI		<i>t</i>	$\tau_c^2$	$Q_{within}$	$Q_{between}$	$I^2$
Post 2009 – Group	18	0.93***	0.58	1.28	0.39	0.15	55.37	43.40	74.39%
Post 2009 – Single	20	0.95***	0.74	1.16	0	0	0.83	110.83	0.00%

Note. \* statistically significant at  $p < .05$ ; \*\* statistically significant at  $p < .01$ ; \*\*\* statistically significant at  $p < .001$ , *k* is the number of studies included in the analysis.

### Study Quality

Study quality was measured by the study quality indicator tool in Appendix A; this measure was developed for this study and was based on standards developed by What Works Clearinghouse Procedures and Standards Handbook (WWCPSH) Version 3.0. Interrater reliability for the study quality indicator alone was initially 88.2%. The first author determined the final codes for each item before conducting analyses.

We elected not to conduct a meta-regression on algebra interventions with study quality scores as a continuous moderator due to disagreement in the research field regarding using quality indicators as moderators (Ahn & Becker, 2011). Instead, we decided to analyze quality score on a qualitative level. Total quality scores of coded studies ranged from 4 to 18 with a total possible score of 23 for group designs and a total possible score of 17 for single case designs.

Upon visual analysis, there does not appear to be a correlation between effect size and total quality score. Single case studies appeared to have similar Tau-U values throughout the spectrum of total quality scores, and group study designs had variability among quality scores and Hedges' *g*. Quality indicators benefit the research in that higher quality scores produce more reliable research and, in the opinion of this examiner, permit "safer" conclusions. Of note, it was more challenging for single case studies to receive higher quality scores because some of the questions to assess quality focused on randomized control trials which is not permissible in single case research.

Where many studies failed to gain quality score points were in the reporting of demographic information. Studies often only reported 3-4 demographic characteristics. Another area where studies did not gain points, was in reporting the reliability and validity of the outcome measures, likely due to many of the instruments being researcher-created. Lastly, many studies only reported construct validity or criterion validity; only one study in this meta-analysis produced both.

Of the 20 single case studies in this meta-analysis, 12 (60%) generated a quality score that was at or above a 9, which is above the 50% margin of total points possible. Of the 18 group design studies in this meta-analysis, 8 (44.4%) generated a quality score that was at or above a 12, which is above the 50% margin of total points possible. Generally speaking, single case research appeared to provide higher quality research when using the quality indicators associated with this study.

## CHAPTER V

### DISCUSSION AND CONCLUSIONS

The purpose of the present study was to perform a systematic meta-analysis on interventions to improve algebra skills in students who are classified as struggling or at-risk for algebra failure. The present study has expanded search criteria compared to Watt, Watkins and Abbitt (2016), measured study quality in a different way (Appendix A), and examined additional potential moderators of intervention effectiveness. These four potential moderators included factors associated with interventionist training, length of intervention, alignment with NCTM standards, and follow-up data. We also compared the effect sizes for 12 different intervention techniques and analyzed the differences between studies who used the intervention and those who did not.

Overall, interventions to improve algebra performance in students with difficulties do appear to have evidence of efficacy, as demonstrated by the consistent medium to large effects when using the ranking scales described in previous sections. Summary effect sizes for overall algebra interventions fell on the upper cusp of the medium range for group designs and in the large range for single case designs; however, heuristics such as these ranking systems are not the best method by which to judge an effect size and it is difficult to interpret the practical significance of these effect sizes (Durlak, 2009). These conclusions are congruent with previous research (Haas, 2005; Hughes et al., 2014; Maccini, McNaughton & Ruhl, 1999; Rakes et al., 2010; and Watt, Watkins & Abbitt, 2016).

Summary effect sizes for the individual interventions themselves varied significantly. For group design studies, modeling and schema, problem-solving, technology, explicit inquiry (EI),

guided practice, concrete-representational-abstract (CRA), and enhanced anchored instruction (EAI) all produced medium effects by Cohen's (1988) conventions. Self-regulation produced a small, almost non-existent, effect, and cognitive strategy instruction (CSI) and graphic organizers were not used in group studies. For single case designs, all 12 interventions previously described produced a large to very large effect by Vannest and Ninci's (2015) standards. While we are unable to rank the various interventions because of unbalanced data, small sample sizes, and the intermingling of interventions within the same study, the difference calculated between studies using the intervention and those not using the intervention provide some insight about the effectiveness of the intervention(s). Results when analyzing the differences discussed previously, show minimal to no change in single case designs and only minor changes in group designs, with the greatest differences found for self-regulation, peer tutoring, and feedback and reinforcement. Notably, these were also the interventions that were used the least amount. As such, they have a higher level of heterogeneity and robustness and are more impacted by the small sample size and unbalanced data than other interventions that had more data points. These conclusions are consistent with previous research, particularly Maccini, McNaughton and Ruhl (1999) who discovered self-regulation, peer tutoring and feedback and reinforcement were among several highly effective interventions for this population. Overall, the results of positive change in algebra performance was consistent with existing meta-analyses (Haas, 2005; Hughes et al., 2014; Maccini, McNaughton & Ruhl, 1999; Rakes et al., 2010; and Watt, Watkins & Abbitt, 2016).

Results of the moderator analysis demonstrated studies where the interventionist was trained yielded medium to large effects and produced significant results. However, there were limited studies which reported interventionist training; therefore, the results should be interpreted

with caution due to unbalanced data and small sample size. Despite these limitations, this moderator analysis indicates training interventionists leads to improved effects; however, the significance of that effect is unknown.

Contrary to our hypothesis, we did not find length of intervention (i.e., span of time over which direct contact occurred) to moderate intervention effectiveness. While studies yielded statistically significant results, there were high levels of heterogeneity and the  $R^2$  did not explain the variance. While length of intervention appears to be an important aspect of improving algebra skills, the data collected in this study did not show it to be a significant moderator. Haas (2005) addressed length of intervention; however, it did not analyze length of intervention as a potential moderator.

Regarding alignment to NCTM standards, only single case studies were analyzed because all group designs were aligned to NCTM standards, therefore not making it a possible moderator. The single case analysis produced a statistically significant effect with little heterogeneity, implying NCTM alignment plays a role in effective algebra intervention; however, it does not act as a moderator due to an  $R^2$  of 0. It should be noted the data were unbalanced and likely skewed the results in a positive direction. Therefore, results should be interpreted cautiously.

A follow-up moderator analysis was conducted. Both group and single case designs produced a statistically significant effect that demonstrated medium to large effects. However, there was a large amount of heterogeneity in both analyses, which is a large limitation in interpreting the results. Generally speaking, follow-up data is likely to aid in documenting maintenance of the acquired skills, but does not appear to significantly help continue growth.

Lastly, data were collected to address the legal implementation of the Common Core (2010) standards. It was hypothesized after 2009, when Common Core was implemented,



intervention effects would show greater gain than prior to 2009 due to increased rigor and adherence to a curriculum. Contrary to our hypothesis, there was no significant difference in post-intervention performance. Perhaps this could be attributed to improved general education instruction because of Common Core.

It is important to note that while our aims were similar to those of Watt, Watkins and Abbitt (2016), Hughes et al. (2014), Rakes et al. (2010), Haas (2005) and Maccini, McNaughton and Ruhl (1999) our methods were not identical. While each used a meta-analytic approach, we included quality analysis and addressed grey literature. Grey literature was considered dissertations and theses that were not published in peer-reviewed journals but were reviewed by a university-level committee. Interventions were categorized differently in each study and we expanded our search criteria to gather more studies than previous meta-analyses.

### **Implications**

Much of the experimental and quasi-experimental research on algebra interventions have evidence of producing statistically significant effect sizes which produce medium to large effects by Cohen's (1988) and Vannest and Ninci's (2015) conventions. The practical significance of this study lies within the school system. Understanding what interventions work and which ones produce greater outcomes or differences is critical in educational reform and pedagogical techniques. As mentioned previously, algebra is a critical skill for post-secondary success; therefore, having effective interventions for students who struggle with it has strong societal implications. While some interventions produced smaller effects, or had less difference between studies who used it and those who did not, the practical nature of growth cannot be undervalued. For a student who is struggling with a particular concept, even small growth in that area can lead to large gains down the road, especially in a field such as mathematics where foundations build

upon one another. While the results may not be statistically significant, they are educationally, or instructionally, significant. Inclusively, it is our belief that “minimal” effect sizes or growth should not dissuade school systems from incorporating potentially efficacious interventions to support students’ academic needs.

While examining study quality, we found single case research appeared to be slightly stronger than group design. In an academic field where students’ futures are at stake, it is critical to employ empirically sound and reliable research. Study quality may be an important factor for improving algebra intervention and should be continuously addressed in future research.

### **Limitations and Future Directions**

There were several major limitations that have been identified throughout this study, which provide opportunity for improvement and future research. The most obvious and important pertains to the statistical analyses used in this research. First, the current study employed a univariate modeling approach, when a multivariate model was more appropriate provided the overlap of interventions used within studies and shared variance among different variables (Becker, 2000; Jackson et al., 2010). The present analysis did not implement correlations between related variables to create a multivariate model; therefore, the overlap between variables was not accounted for in the meta-regression. Research in the future should attempt to satisfy this qualm and address algebra intervention effectiveness from a multivariate mindset.

Secondly, a random-effects model was implemented to address the moderating factors between interventions when a network meta-analysis would likely have fit the model more effectively. A network meta-analysis is a quantitative synthesis of multiple outcomes from studies that span multiple treatments. Of note, due to the unbalanced data, it was unknown if

utilizing this methodology would have mediated the impact of skewed data; however, future research should implement this method to address these questions.

As previously mentioned, the data set for this study was largely unbalanced. This was attributed to multiple studies implementing multiple interventions within a treatment package for a study. This made it difficult to run moderator analyses and tease out the true effect of each intervention. Future research needs to address these problems and the occurrence of multiple interventions being utilized simultaneously. One possible way to alleviate this issue would be to collapse the intervention terms into smaller categories, as many of them were similar in nature and could be combined under a larger, more encompassing term. Additionally, studies could be grouped by number of interventions utilized. For example, studies using only one of the coded interventions would be in a group and studies using two or more coded interventions would be in another group. These could be compared in an effort to address the importance of integrating a variety of pedagogical techniques to enhance all styles of learners.

Lastly, this research was relatively small in terms of sample size, as it only included 38 studies. These groups became smaller ( $n=18$ ;  $n=20$ ) as they were separated by design due to different effect sizes. Due to small sample size, effects were interpreted with caution and more sensitive to change and error. Future research should aim at incorporating more studies both at the single subject level and group subject level.

### **Conclusion**

In conclusion, the present study suggests interventions to improve algebra skills have medium to large effects across a variety of studies. It is difficult to interpret the practical significance of these effects because of the unbalanced data and intermingling effects of interventions within the same study. Heterogeneity for all facets of intervention fluctuated at

post-intervention. Single case design studies appeared to have higher quality of research and more consistent effects across various interventions, as demonstrated by the lower levels of heterogeneity, while group design studies yielded more variable effect sizes across interventions and had overall less high-quality studies. Common Core (2010) implementation did not appear to have significant impacts on the effectiveness of algebra interventions as hypothesized.

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*\*indicates a study used in the meta-analysis*

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School characteristics: 1 – Urban 2 – Suburban 3 – Rural 5 – Not reported

School Characteristics (SES) 1- high poverty/low SES 2- low/middle 3-middle 4- middle/upper 5- upper 6-not reported

Participant information:

Reported Ethnicity: 1 – American Indian or Alaska Native 2 – Asian 3 – Black or African American 4 – Hispanic or Latino 5 – Native Hawaiian or Other Pacific Islander 6 – White Non-Hispanic 7 - Other 8 – Not reported

Primary Gender: 1 – Male 2 – Female 3 – Both 4 – Other: \_\_\_\_\_ 5 – Not reported

Inclusion of students with disability in sample? 1 – Yes 2 – No

Who identified/diagnosed disability? 1 – Clinician 2 – Not provided 3 – IEP Team 4 – Other: \_\_\_\_\_

What type of disability: 1 – Specific Learning Disability 2 – ADHD 3 – Autism 4 – OHI 5 – Emotional Disturbance 6 – Visual Impairment 7 – Deafness 8 – Hearing Impairment 9 – Orthopedic Impairment 10 – Intellectual Disability 11 – TBI 12 – Multiple Disabilities 13 – Deaf-Blindness 14 – Not indicated 15 – Other: \_\_\_\_\_

**Intervention Characteristics**

Type of Intervention in this article(check all that apply)

1 – Modeling and Schema	Yes	No
2 – Problem Solving Skills	Yes	No
3 – Self-regulation	Yes	No
4 – Technology	Yes	No
5 – Explicit Instruction	Yes	No
6 – Guided Practice	Yes	No
7 – Feedback and Reinforcement	Yes	No
8 – Peer Tutoring	Yes	No
9 – Cognitive Strategy Instruction	Yes	No
10 – Concrete-Representational-Abstract	Yes	No



11 – Enhanced Anchored Instruction      Yes      No  
12 – Graphic Organizers      Yes      No

Content Area      1 – symbols and      2 – linear equations      3 – quadratic  
Intervention      expressions      equations  
Addresses:  
4 – functions      5 – polynomials      6 – combinatorics  
and finite probability

Total Length of Intervention (# of weeks): \_\_\_\_\_

Frequency of Intervention (# of sessions per week): \_\_\_\_\_

Number of Treatment Sessions total: \_\_\_\_\_

Length of Intervention Session in minutes: \_\_\_\_\_

Who delivered the      1 – Gen      2 –      3 – Researcher      4 – Clinician      5 Parent:  
intervention:      ed.Teacher      Special Ed  
Teacher

6- Other

Were they trained?      1 – Yes      2 – No

Training time (# of hours): \_\_\_\_\_

Type of Training      1 – Lecture      2 – Modeling      3- Role play  
(check all that apply):      4- Other\_\_\_\_\_

Who conducted the      1 – Researcher      2 – Graduate student      3 – Other: \_\_\_\_\_  
training:

Structure of the      1 – Individual      2 – Group      3 – Classroom  
session:

Intervention implemented as      1 – yes      2 – no  
described:

Fidelity checks on the      1 –yes      2 – no  
intervention?:

If yes, who conducted      1 – Researcher      2 – Graduate student      3 – Other: \_\_\_\_\_  
fidelity checks?:

If yes, how often did they check? (# of weeks): \_\_\_\_\_

Inter-rater reliability: \_\_\_\_\_

## Outcome Measure

Dependent Variable:	1 – Standardized (norm-referenced) Test	2 – State Test
3 – End of Unit Test	4 – CBM	5 – experimenter designed
Who administered the measure:	1 – parent	2 – teacher
3 – researcher	4 – clinician	5 – Other: _____
Instrumentation reported?	1 – Yes	2 – No
Reliability of test reported?	1 – Yes	2 – No
Alignment with NCTM reported?	1 – Yes	2 – No

**Any Follow-Up  
Yes/No**

If yes, how many days after intervention?  
 With same measure(s)?  
 With different measures ? (If so, specify)

**Effect Size Information:**

Baseline for each the dependent variables in the study:

Group Single or 2 group:	Mean (SD)	_____	Single Case baseline	_____
Group Single or 2 group:	Mean (SD)	_____	Single Case baseline	_____
Group Single or 2 group:	Mean (SD)	_____	Single Case baseline	_____
Group Single or 2 group:	Mean (SD)	_____	Single Case baseline	_____
Group Single or 2 group:	Mean (SD)	_____	Single Case baseline	_____

**Results by dependent variable**

Group Single or 2 group:	Mean (SD)	_____	Single Case outcome	_____
Group Single or 2 group:	Mean (SD)	_____	Single Case outcome	_____
Group Single or 2 group:	Mean (SD)	_____	Single Case outcome	_____
Group Single or 2 group:	Mean (SD)	_____	Single Case outcome	_____
Group Single or 2 group:	Mean (SD)	_____	Single Case outcome	_____

Effect size (if reported):

Type _____	ES _____	Dependent Variable _____
Type _____	ES _____	Dependent Variable _____
Type _____	ES _____	Dependent Variable _____
Type _____	ES _____	Dependent Variable _____
Type _____	ES _____	Dependent Variable _____

**Quality of Study**

Is means of recruitment to participate in the study clearly explained?  
 0=No, does not specify how the participants were recruited  
 1= Yes, clearly describes the recruitment process

Were appropriate procedures used to **increase the likelihood** that relevant characteristics of participants in the sample were comparable across conditions?

0= No (does not specify assignment to groups or is based on convenience, randomly assigned by district or by school)

1 = Yes; random assignment or matching design

Was sufficient information given characterizing the interventionists provided?

0= No; interventionists unknown, or may have systematically varied across conditions

1= Yes; interventionists clearly specified (level of training, profession, etc.)

2= Interventionists were the teachers

Was sufficient demographic information provided (age, gender, race/ethnicity, disability status, SES, grade, ELL status) for the participants?

0= No (demographic data limited to 1-2 of these factors)

2 = Demographic data limited to 3-4 of these factors

3= Demographic data included all 7 factors

Did participants in the control and intervention group differ on any of the demographic factors?

2= No between group differences were present at baseline

1= Minimal between group differences were present or any differences were accounted for

0= Insufficient information to determine if between group differences existed at baseline

Was the intervention clearly described and specified?

0 = No; intervention not clearly described

1= Yes; intervention clearly described; you could implement it from the description

Is there a record of how many minutes/hours of intervention each participant received?

0= No; no record of minutes/hours of intervention received by each participant

1= Yes; there is a clear record of how many minutes/hours of intervention each participant received

Was the nature of services provided in comparison conditions described?

0= No; comparison condition was not described or was described as “business as usual”

1= Professional development of the control group was described (quarterly in services, etc.)

Was data available on attrition rates among intervention samples?

0= No; attrition not documented in report

1= Yes; attrition documented in report, but not comparable across groups

2= Yes; attrition documented in report, and comparable across groups

Were data collectors and/or scorers blind to study conditions and equally (un)familiar to examinees across study conditions?

0= No; data collectors and/or scorers were not blind to conditions, were not equally familiar to examinees across conditions, or no information was provided on this

1= Yes & No; data collectors and scorers were blind to condition or equally familiar/unfamiliar to examinees across conditions

2= Yes; data collectors and scorers were blind to condition AND equally familiar/unfamiliar to examinees across conditions

Were outcomes for capturing the intervention’s effect measured beyond an immediate posttest? (follow-up data collected?)

0= No; no follow up data

1= Yes; follow up data were collected for the Treatment groups, but not control

2= Yes; follow up data were collected for BOTH Treatment AND Control groups

Did the study provide not only internal consistency reliability but **also test–retest reliability and interrater reliability** (when appropriate) for outcome measures?

0= No reliability statistics provided

1= only internal consistency provided

2= Internal consistency assessed as well as test-retest and/or interrater reliability

Was evidence of the criterion-related validity **and** construct validity of the measures provided?

0= no clearly presented evidence for criterion related validity **OR** construct validity for outcome measures

1= evidence of **ONLY** criterion-related **OR** construct validity (for all measures)

2= clear evidence of criterion related validity **AND** construct validity for outcome measures

Was more than one norm-

1 – Yes

2 – No

referenced or CBM used to

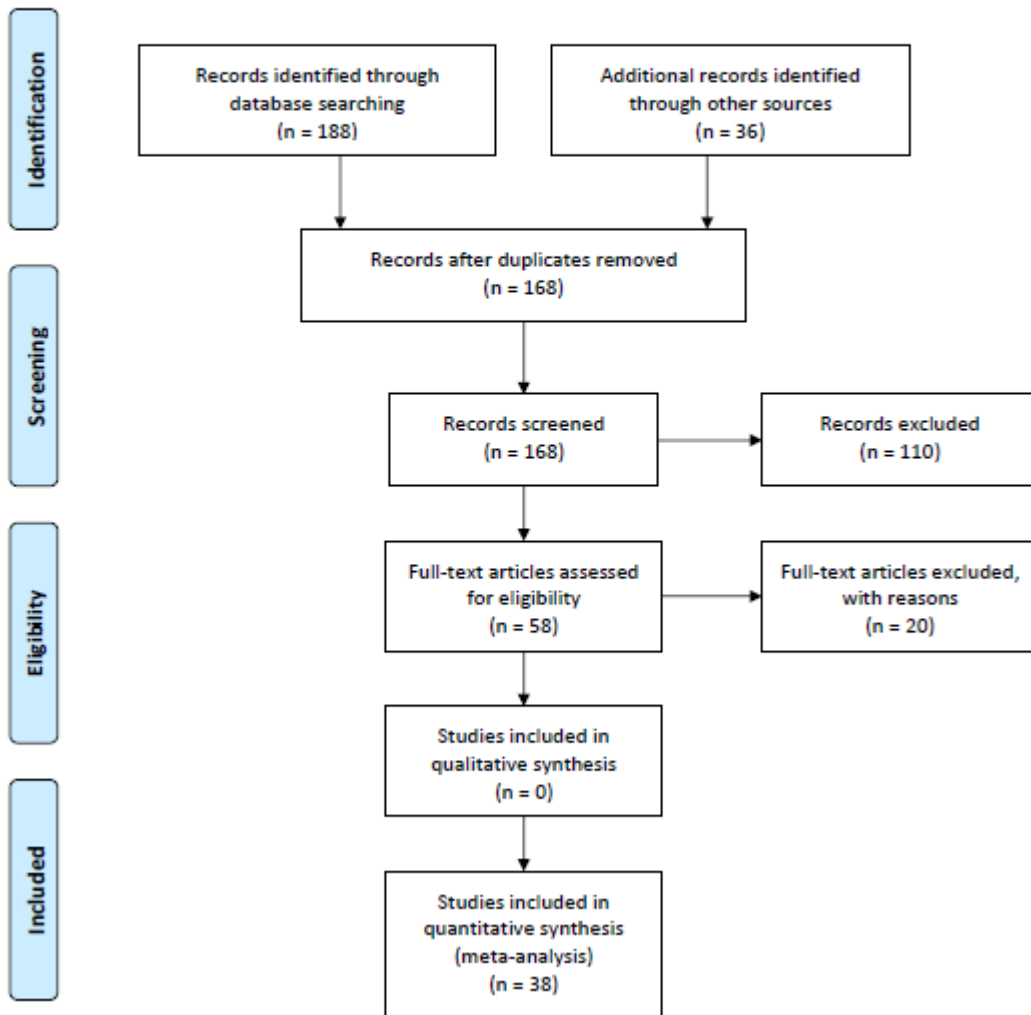
measure Algebra skills?

Total Quality Score (sum of all these items)

APPENDIX B  
PRISMA DIAGRAM



PRISMA 2009 Flow Diagram



## APPENDIX C

### NCTM STANDARDS FOR ALGEBRA

Adapted from National Council of Teachers of Mathematics Standards(2000)

Learning Objectives	Understand patterns, relations, and functions	Represent and analyze mathematical situations and structures using algebraic symbols	Use mathematical models to represent and understand quantitative relationships	Analyze change in various contexts
Pre-K – 2	<ul style="list-style-type: none"> <li>• sort, classify, and order objects by size, number, and other properties;</li> <li>• recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another;</li> <li>• analyze how both repeating and growing patterns are generated.</li> </ul>	<ul style="list-style-type: none"> <li>• illustrate general principles and properties of operations, such as commutativity, using specific numbers;</li> <li>• use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.</li> </ul>	<ul style="list-style-type: none"> <li>• model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols</li> </ul>	<ul style="list-style-type: none"> <li>• describe qualitative change, such as a student's growing taller;</li> <li>• describe quantitative change, such as a student's growing two inches in one year.</li> </ul>
Grades 3 – 5	<ul style="list-style-type: none"> <li>• describe, extend, and make generalizations about geometric and numeric patterns;</li> <li>• represent and analyze patterns and functions, using words, tables, and graphs.</li> </ul>	<ul style="list-style-type: none"> <li>• identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers;</li> <li>• represent the idea of a variable as an unknown quantity using a letter or a symbol;</li> <li>• express mathematical relationships using equations.</li> </ul>	<ul style="list-style-type: none"> <li>• model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.</li> </ul>	<ul style="list-style-type: none"> <li>• investigate how a change in one variable relates to a change in a second variable;</li> <li>• identify and describe situations with constant or varying rates of change and compare them.</li> </ul>
Grades 6 - 8	<ul style="list-style-type: none"> <li>• represent, analyze, and generalize a variety of patterns with tables, graphs, words, and,</li> </ul>	<ul style="list-style-type: none"> <li>• develop an initial conceptual understanding of different uses of variables;</li> </ul>	<ul style="list-style-type: none"> <li>• model and solve contextualized problems using various representations,</li> </ul>	<ul style="list-style-type: none"> <li>• use graphs to analyze the nature of changes in quantities in</li> </ul>

	<p>when possible, symbolic rules;</p> <ul style="list-style-type: none"> <li>• relate and compare different forms of representation for a relationship;</li> <li>• identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations</li> </ul>	<ul style="list-style-type: none"> <li>• explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;</li> <li>• use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;</li> <li>• recognize and generate equivalent forms for simple algebraic expressions and solve linear equations</li> </ul>	<p>such as graphs, tables, and equations</p>	<p>linear relationships.</p>
<p>Grades 9 - 12</p>	<ul style="list-style-type: none"> <li>• generalize patterns using explicitly defined and recursively defined functions;</li> <li>• understand relations and functions and select, convert flexibly among, and use various representations for them;</li> <li>• analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;</li> <li>• understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on</li> </ul>	<ul style="list-style-type: none"> <li>• understand the meaning of equivalent forms of expressions, equations, inequalities, and relations;</li> <li>• write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases;</li> <li>• use symbolic algebra to represent and explain mathematical relationships;</li> <li>• use a variety of symbolic representations, including recursive and parametric equations, for functions and relations;</li> </ul>	<ul style="list-style-type: none"> <li>• identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;</li> <li>• use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;</li> <li>• draw reasonable conclusions about a situation being modeled.</li> </ul>	<ul style="list-style-type: none"> <li>• approximate and interpret rates of change from graphical and numerical data.</li> </ul>

- more-complicated symbolic expressions;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
  - interpret representations of functions of two variables
- judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.