HYBRID HADRONIZATION WITH COLOR FLOW

A Thesis

by

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MASTER OF SCIENCE

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ABSTRACT

Hybrid Hadronization is an approach that combines two well-established models, string fragmentation and recombination, to describe the hadronization of QCD jets. It is available as a code written and maintained by Professor Fries' group at Texas A&M university and it is a part of the JETSCAPE framework [1]. My project improves the physics of the current Hybrid Hadronization code to be closer to realistic Quantum Chromo Dynamics (QCD) by working on two related issues.

The first one is about utilizing known information about the color states of quark-antiquark pairs or quark triplets in determining the probability of recombination. Previously, the probability for quark-antiquark pairs to form color singlet state was set to $\frac{1}{9}$ as default, and that for three quarks to form color singlet was set to $\frac{1}{27}$. These are the values appropriate if color quantum numbers are completely randomized. The improved code now modifies these values based on the color states of partons chosen.

The second issue concerns the string repair process. Based on the result of the recombination step, the code organizes remnant strings and temporary junctions by tracing color flow, which is given by the color tags that were generated by jet shower modules, and utilizes these information for repairing strings after recombination. Subsequently it processes temporary junctions to be compatible with PYTHIA by cutting problematic junctions, checking conservation laws, and assigning the history tags to define final partons in junctions.

My improved Hybrid Hadronization code can, for the first time, run with MATTER and LBT parton shower input and add thermal partons in a way that utilizes both the space-time information and the color information provided by these codes correctly. We have studied the systematic behavior of jet hadronization for different jet energies and medium sizes. We find typical recombination signatures, like baryon / meson ratios and flow effect as expected with medium size. This opens the door for further explorations of hadronization and jet physics using JETSCAPE and the Hybrid Hadronization code.

ACKNOWLEDGMENTS

It was my great honor to work in Professor Fries' group. I really want to express my gratitude for Professor Fries' considerate attention to me and his guidance. Even though I have ceased studying for five years while serving as a military officer, he believed in me to do well and permitted me to join the group, and I have done my best to fulfill for his belief.

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Contributors

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The computational background to conduct the research was provided by JETSCAPE framework. The previous Hybrid Hadronization code was treated by Michael Kordell in Professor Fries' group.

All the other work conducted for the thesis was completed student independently.

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1. INTRODUCTION AND PREVIOUS WORK

1.1 QCD Lagrangian for color flow

The QCD Lagrangian is essential for discussing QCD color flow and string structure. Formulating the QCD Lagrangian starts from the Lagrangian density of free quarks without interaction with the gauge boson, which is the gluon that mediates strong interaction. The free Lagrangian is [2]

$$\mathcal{L}_0 = \bar{\psi}_q^i (i\gamma^\mu \partial_\mu - m_q)\delta^{ij}\psi_q^j$$

In the definition above, ψ_q^i is a quark field with color (fundamental) index i, starting from 1 to 3, which means three color charges (red, blue, green). γ^μ signifies the Dirac matrix, where μ is the four-vector index. m_q is the quark mass. To consider the interaction with a gauge boson using this basic Lagrangian, a partial derivative should be substituted by the covariant form as [3]

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - ig_s t^a_{ij} A^a_\mu \ ,$$

where g_s is the strong coupling ($g_s^2 = 4\pi\alpha_s$), and A^a_μ is the gluon field with the color (adjoint) index. t^a_{ij} are half Gell-Mann matrices ($t^a_{ij} = \frac{1}{2}\lambda^a_{ij}$, where λ^a_{ij} is a Gell-Mann matrix), which are the generators of the SU(3) group. Along with considering the interaction with a gauge boson, considering the local gauge invariance for the Lagrangian is needed. Since the phase factor to apply the gauge transformation includes a generator of the SU(3) group, the fact that Gell-Mann matrices do not commute with each other (i.e. $[t_i, t_j] = if_{ijk}t_k$, where f_{ijk} are the structure constants of the group) requires an additional term for the Lagrangian, and the term to be added is regarding the kinetic energy of the gluon fields ($-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$). In summary, the complete form of the QCD Lagrangian is [3]

$$\mathcal{L}_{QCD} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_q^j - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \ .$$

There is an important property of the gluon fields that follows from this Lagrangian, which gives us a more intuitive picture of allowed interactions in QCD [2]. If we write the gluon field strength tensor in terms of the fields A^a_μ , i.e. ,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f_{abc} A^b_\mu A^c_\nu$$

we can rewrite the QCD Lagrangian explictly in terms of the gluon fields. By abbreviating quark fields as ψ^i and gluon field as A, we can find the following terms with at least one power of the coupling constant g_s: $g_s \bar{\psi} A \psi, g_s AAA$, and $g_s^2 AAAA$. They correspond to the coupling of quarks to gluons and of three and four gluons with each other. Then, the symbolic form of the Lagrangian is [2]

$$\mathcal{L} = \text{"}\bar{q}q\text{"} + \text{"}(Gluon^2)\text{"} + g_s \text{"}\bar{q}q(Gluon)\text{"} + g_s \text{"}(Gluon^3)\text{"} + g_s^2 \text{"}(Gluon^4)\text{"}.$$

The last two terms show clear differences from Quantum Electro Dynamics (QED), where photons only couple with leptons or antileptons and do not couple with each other. We can see that gluons can couple with each other in the form of three or four coupled gluons structures, and this means gluons carry color charges. While carrying the color charge, gluons have octet color states from the combination of color charges and anticolor charges, and, when carrying color charges, color flow should be conserved [4]. For example, if a gluon in the color state of anti-blue and red color charge decays into quark and antiquark, the color charge of the antiquark should be anti-blue and that of the quark should be red as shown in Fig.1.1. Not only does the decay of gluons obey this, but the other interactions in QCD should as well. This give us a concept for color flow in the QCD string structure.

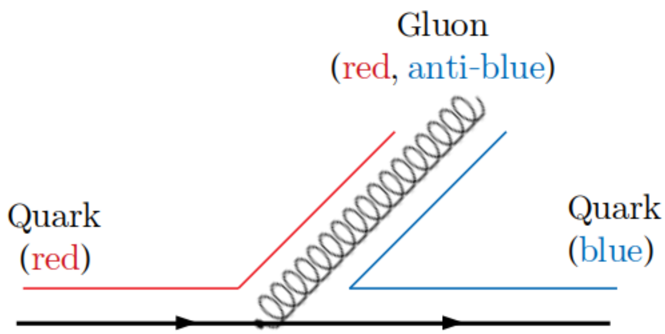


Figure 1.1: Example interaction and color flow where a quark with red color charge interacts with a gluon and is changed into a blue charged quark

1.2 Hadronization and QCD strings

We can calculate the probability for partons (quarks and gluons) to interact and radiate by applying perturbative QCD (pQCD) for sufficiently large momentum transfers. However, hadronization, the act of forming bound states from quarks and gluons, proceeds with low momentum transfer and is non-perturbative. One way to deal with this is by using measured fragmentation functions. Fragmentation functions $D_i^h(z_i, Q_{hard}^2)$ are defined phenomenologically by fitting experiment data, and they represent the differential probability for a parton i, which could be quark or gluon, to fragment into a hadron h that carries a fraction z of its momentum. With the factorization theorem, which enables the separation of cross section of hard processes [5], and the universality of fragmentation functions [6], we can express the differential cross section in the hadronization of hadron h that carries energy E_h in the $a + b \to h + X$ process as [7]

$$\frac{d\sigma}{dE_h}(ab \to hX) = \sum_i \int \frac{d\sigma}{dE_i}(ab \to iX) D_i^h(z_i, Q_{hard}^2) \frac{dE_i}{E_i}$$

where $z_i = E_h/E_i$ $(0 \leq z_i \leq 1)$, and $\frac{d\sigma}{dE_h}$ represents the differential cross section to produce hadron h with energy E_i in the collision of particles a and b. Q_{hard}^2 is the virtuality scale for the hard process. This formalism provides the basic background for treating the hadronization from strings.

Next, we discuss the origin of QCD strings to fully explain the background for this project. In considering the string structure, the project follows the Lund model, which is widely used and forms the basis of the PYTHIA code [3]. We can start from a quark-antiquark pair in a color singlet state (e.g. red + anti-red). From the QCD Lagrangian, using lattice QCD and other methods, we find the potential between them, which is also called the Cornell potential [8],

$$V(r) = -\frac{\chi}{r} + \sigma r$$

where χ is the strength of Coulomb-like interaction, and σ is the string tension. As we can see above, even at a large distance, the potential between the particles still grows. This long range interaction is reminiscent of strings. The gluon field lines between the quark-antiquark pair are shown in Fig.1.2 for small and large distances.

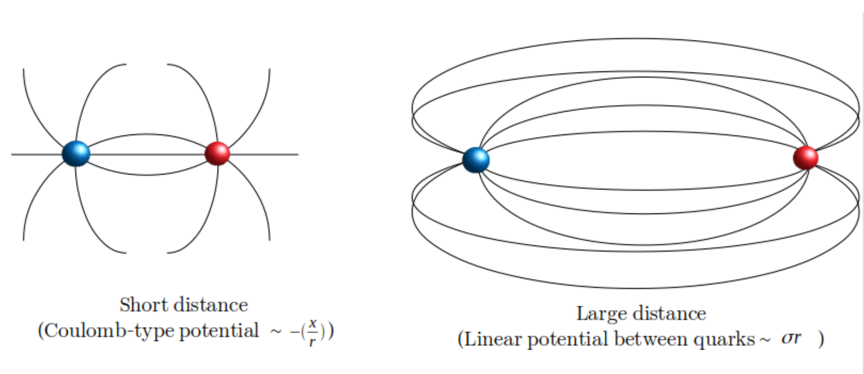


Figure 1.2: Description of the fields between quark and antiquark at small and large distances.

In the Lund model, perturbative gluons are considered as kinks in strings between quarks and antiquarks, which carry momentum and energy of the gluons [9]. According to the Lund model, hadrons can be produced from fluctuations in the string field. See Fig. 1.3. This process is called string fragmentation, and this is what is implemented as the default hadronization in the codes like PYTHIA.

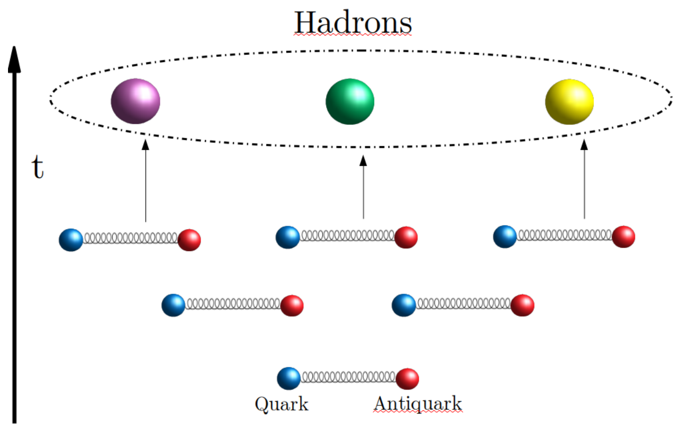


Figure 1.3: Pictorial description of string fragmentation.

A different hadronization channel is available when quarks are close enough in phase space. In that case they can form bound states by recombining into hadrons. This process is called quark recombination, and it has been succesfully applied to heavy ion collision. [7, 10, 11]. There are typical signatures in recombination that distinguish it from string fragmentation. The most important ones are charged hadron chemistry, in particular a larger baryon / meson ratio, and collective effects, like collective flow of particles. Fig. 1.4 shows different recombination channels for a string embedded in a hot quark-gluon plasma background medium which serves as a resorvoir of thermal partons.

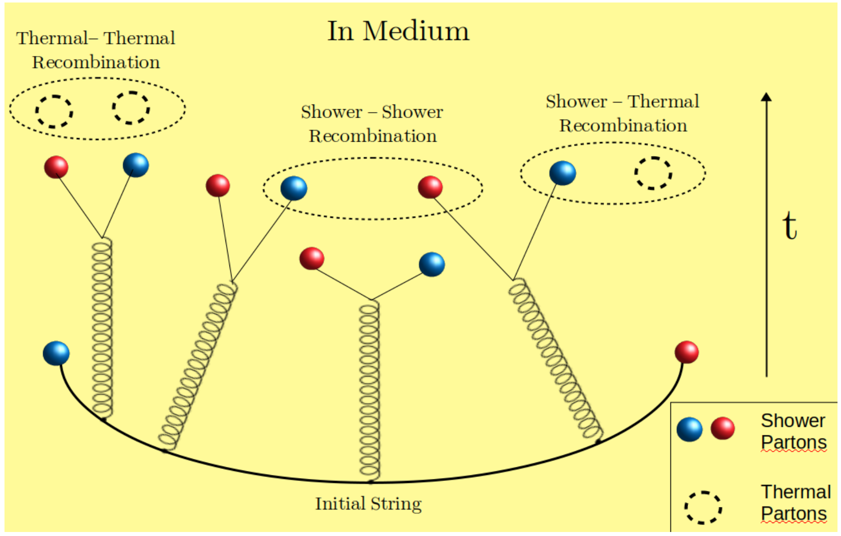


Figure 1.4: Three types of recombination in close distance in phase space.

1.3 QGP medium and evolution of jets

We are interested in hadronization and string structure of jets in the Quark-Gluon Plasma (QGP). Quarks and gluons cannot exist on their own, but only exist in the form of color singlet hadrons [2, 4]. This is the confinement property of QCD. However, with enough collision energy, hadrons or nuclei can be broken up into quarks and gluons. They can exist on their own without forming color-singlet hadrons as long as the temperature is large enough, roughly above 160 MeV. As a result, quarks and gluons can exist deconfined, and we call this fluid-like phase of quarks and gluons the Quark-Gluon Plasma [12]. In hard scatterings in collisions of hadrons or nuclei, highly collimated bunches of particles called jets can be produced. Since jets provide large enough virtuality scales to apply perturbative QCD, we can calculate or compute the evolution of jets in QGP [13, 14, 15].

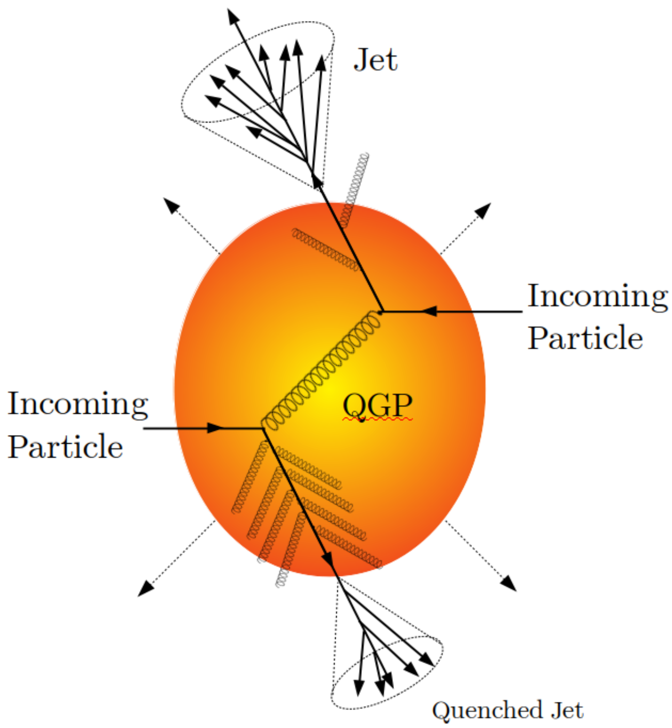


Figure 1.5: Formation of QGP and Jet interacting with medium.

A jet starts out as a quark or gluon created in a hard process which possesses both large energy and large virtuality. Virtuality is defined as $Q^2 = E^2 - p^2 - m^2$ and characterizes intermediate quantum states. High virtuality particles will radiate to transition into two particles with smaller virtuality. This process is repeated until all particles have sufficiently small virtuality, building a "shower" of particles. The DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) equation determines the probability for a parton a to split into two partons b and c. It is given by [13, 14, 16]

$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{dQ^2}{Q^2}P_{a\to bc}(z)dz \quad . \tag{1.1}$$

where z is the momentum fraction of parton b with respect to parton a. For $P_{a\to bc}(z)$ in Eq.(1.1) three channels, $q \to qg$, $g \to q\bar{q}$. and $g \to gg$, are possible, and the splitting functions of these channels are

$$P_{q\to qg}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$
$$P_{g\to gg}(z) = 3\frac{(1-z(1-z))^2}{z(1-z)}$$
$$P_{g\to q\bar{q}}(z) = \frac{3}{2}(z^2 + (1-z)^2)$$

In a shower Monte Carlo (MC) code like PYTHIA, the jet initiating parton is given a maximum possible virtuality Q_{max}. Therefore, for the splitting of the parton to happen with a virtuality Q_1, we need to ensure that another splitting does not happen during this evolution between virtualities Q_{max} and Q_1. To fulfill this requirement, the Sudakov form factor, which provides the probability for partons not splitting in certain vertuality range, has to be applied by defining the variable t [17].

$$t = \ln(Q^2/\Lambda^2), dt = d\ln(Q^2) = dQ^2/Q^2$$

The Sudakov form factor is defined as

$$\mathcal{P}_{NoSplitting}(t_{max}, t_1) = \text{exp}(-\int_{t_{max}}^{t_1} dt \frac{d\mathcal{P}_{a\to bc}(t)}{dt})$$

The actual virtuality in the first splitting is then determined in shower MC codes by using random number generator. MATTER (the Modular All Twist Transverse-scattering Elastic drag and Radiation) is the shower Monte Carlo module we utilize most here [18]. It determines the virtuality evolution of jets with this method. When jets evolve in a medium, the splitting process is modified but the basic picture of virtuality ordered splitting remains the same. In vacuum, particles go directly into hadronization after MATTER shower, but in a medium, jets can undergo another evolution process if partons are still inside the QGP and have low enough virtuality to apply further shower MC. For this we use the shower MC, Linear Boltzmann Transport (LBT). LBT focuses on the evolution of the phase distribution of partons when the jets evolves further by solving the Boltzmann equation [18]. The important difference from the vacuum shower is that partons in medium keep evolving until their ambient temperature becomes lower than the critical temperature of phase transition from QGP into the other phases (T_c). After cooling down under T_c, jets start to hadronize. To simulate this, we add sampled thermal partons on the $T = T_c$ hypersuface around jets for hadronization.

1.4 Color states

Based on SU(3) algebra we can choose the color basis states of red ($|R\rangle$), blue ($|B\rangle$), and green ($|G\rangle$) color charges. First, we can calculate the color states of two quarks. By applying irreducible representations of SU(3) group [19], we can express the combination of two color charges as a sextet and anti-triplet color state [3].

$$3 \otimes 3 = 6 \oplus \bar{3}$$

$$6 = \begin{pmatrix} |RR\rangle \\ |GG\rangle \\ |BB\rangle \\ \frac{1}{\sqrt{2}}(|RG\rangle + |GR\rangle) \\ \frac{1}{\sqrt{2}}(|GB\rangle + |BG\rangle) \\ \frac{1}{\sqrt{2}}(|BR\rangle + |RB\rangle) \end{pmatrix} \qquad \bar{3} = \begin{pmatrix} \frac{1}{\sqrt{2}}(|RG\rangle - |GR\rangle) \\ \frac{1}{\sqrt{2}}(|GB\rangle - |BG\rangle) \\ \frac{1}{\sqrt{2}}(|BR\rangle - |RB\rangle) \end{pmatrix} \tag{1.2}$$

Since we are interested in hadronization, we shall also deduce the color states of mesons which are combinations of a quark and an antiquark. Thus, we consider color charge and anticolor charge. In this case, $q\bar{q}$ color states can be separated into octet and singlet states. Color octet states correspond to the color states of gluons which are not color neutral, so gluons carry color charges. The color singlet state corresponds to the color neutral mesons. Because of color confinement which forbids quarks to exist by themselves, all color charges must be arranged into color neutral bound states upon hadronization. The $q\bar{q}$ color states above are [4]

$$3 \otimes \bar{3} = 8 \oplus 1$$

$$8 = \begin{pmatrix} \frac{1}{\sqrt{2}}(|R\bar{B}\rangle + |B\bar{R}\rangle) \\ -\frac{i}{\sqrt{2}}(|R\bar{B}\rangle - |B\bar{R}\rangle) \\ \frac{1}{\sqrt{2}}(|R\bar{R}\rangle - |B\bar{B}\rangle) \\ \frac{1}{\sqrt{2}}(|R\bar{G}\rangle + |G\bar{R}\rangle) \\ -\frac{i}{\sqrt{2}}(|R\bar{G}\rangle - |G\bar{R}\rangle) \\ \frac{1}{\sqrt{2}}(|B\bar{G}\rangle + |G\bar{B}\rangle) \\ -\frac{i}{\sqrt{2}}(|B\bar{G}\rangle - |G\bar{B}\rangle) \\ \frac{1}{\sqrt{6}}(|R\bar{R}\rangle + |B\bar{B}\rangle - 2|G\bar{G}\rangle) \end{pmatrix} \qquad 1 = \left(\frac{1}{\sqrt{3}}(|R\bar{R}\rangle + |G\bar{G}\rangle + |B\bar{B}\rangle) \right) \tag{1.3}$$

For baryons which are bound states of three quarks, we need to use the result in Eq (1.2). After adding a third color state, we can separate color states of three quarks into decuplet, octet and singlet states:

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

Based on the number of possible color states above, in a system of randomized color charges the probability for a color-anticolor (quark-antiquark) pair to form a color singlet meson is $\frac{1}{9}$, and the probability for three color charges (quarks) to form a baryon is $\frac{1}{27}$. These are the default values set in the previous version of Hybrid Hadronization. However, the jet evolution discussed above leads to systems which are not color randomized. We will discuss these effects in the next two chapters.

1.5 Previous work

The Jet Energy-loss Tomography with a Statistically and Computationally Advanced Program Envelope (JETSCAPE) [1, 20] framework is a computational framework that pursues building a complete Monte-Carlo event generator for heavy-ion collisions. The processes done in this framework can be sorted by the categories of function of each module. The Initial State module, called TRENTO works to set the initial conditions such as spatial distribution, energy, and entropy distribution of the soft sector of the collision (p+p, Pb+Pb, Au+Au). Second, the Hydro module determines the evolution of hydrodynamic variables such as energy momentum tensor, temperature, and flow-velocity. They form the background for jet evolution in the medium. Third, the PartonGun module determines the distribution of hard partons based on the parton distribution functions and cross sections of each hard collision [1]. After these processes are completed by the modules above, the essential conditions for the evolution of jets are set. Each jet is then evolved by the shower and energy loss modules (MATTER, LBT, MARTINI). Among the jet-energy loss modules, the Modular All Twist Transverse-scattering Elastic drag and Radiation takes the important role of the evolution of high-energy, high-virtuality partons. Specifically in MATTER, the dominant mechanism that simulates the jet evolution is the virtuality-ordered splitting of jet partons [18]. Parton showers in MATTER are modified in QGP medium. MATTER serves as a unique tool because we can test the jet in vacuum and a medium. MATTER applies integer color tags to follow the flow of color charges in jet evolution following the schematic picture in of Fig 1.6.

Professor Fries' group at Texas A&M University has worked on the issue of recombination during hadronization in the past. The philosophy of Hybrid Hadronization is that quarks close to each other in phase space recombine into hadrons, while quarks far from each other form strings and are fragmented to hadrons [7, 10, 11]. Based on this approach, the Hybrid Hadronization code has been developed. The workflow of the current version can be separated into three parts: recombination, string repair, and string fragmentation. After reading in the original parton information from parton shower modules, gluons in the parton shower are decayed into quark and antiquark pairs. During the subsequent recombination step, two or three partons are randomly chosen to be evaluated for their recombination probability. The code computes the probability from their distance in phase space, their spin, and color state [10]. The algorithms for evaluating the probability in phase space are already well established using Wigner Functions. [7, 11]. To evaluate the color factor, however, the code currently applies a purely statistical approach where the probability is set as $(\frac{1}{3})^N$, where N is the number of quarks in the bound state. In other words, color tags are currently not utilized for color factors although in principle available. In the string repair process, remnant partons are rearranged based on the distance in phase space and the color tags are assigned based on distance so again in this step existing color flow information is not utilized. At last, all partons, strings and recombined hadrons are transferred into PYTHIA. Recombined hadrons are not affected by this module, but strings are fragmented into hadrons. We can see the entire description of the procedures in the code in Fig. 1.6.

It is possible to improve the process of determining color factors to evaluate the probability of recombining hadrons and the process of repairing the strings when we consider the color tags of the participating partons. Those color tags have been properly calculated by previous modules, like MATTER. We propose to consider the color flow read from the initial string configuration to improve the Hybrid Hadronization code.

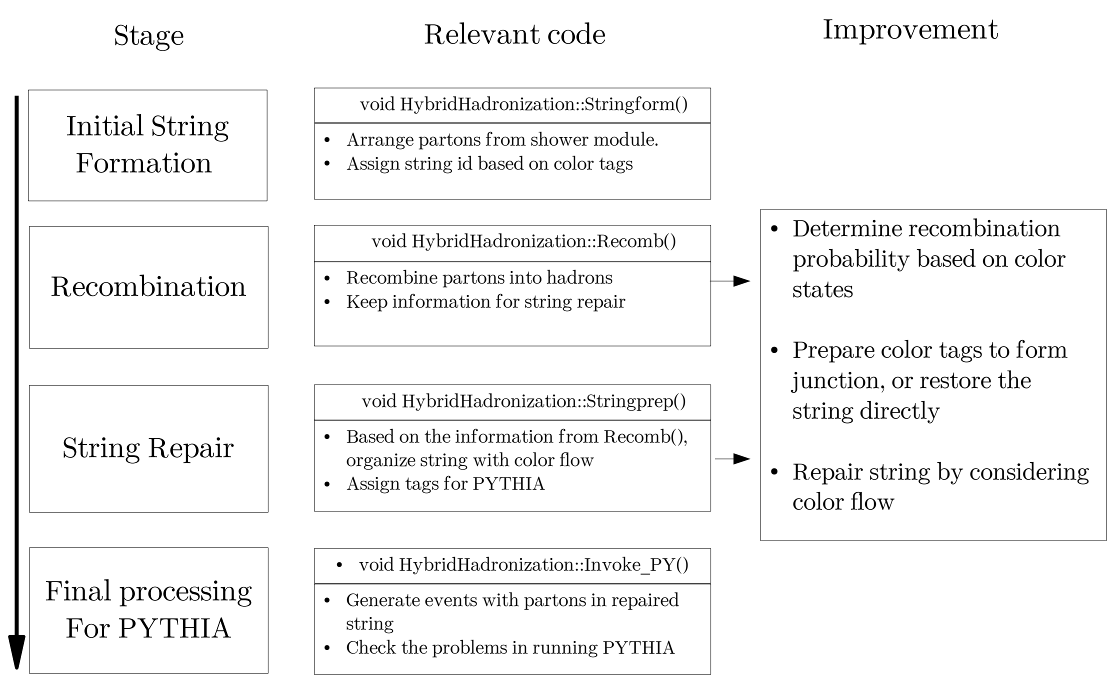


Figure 1.6: Workflow of the Hybrid Hadronization code and possible improvements.

2. RECOMBINATION IN IMPROVED HH CODE

2.1 Workflow

The improvement in recombination starts by sorting color tags read into Hybrid Hadronization. Parton configurations read into Hybrid Hadronization typically contain one or several strings or string fragments. In this procedure, repeated color tags and zero elements are excluded from sorting. A color tag of zero indicates a thermal parton or a shower parton that has interacted with the medium. Its color state is randomized. Sorted color tags are accumulated into a vector which forms the rows and columns of a correlation matrix which is used to store the probability to form color singlets. The algorithm to randomly choose two or three particles to evaluate recombination is unchanged from the existing code. Determining the probability of recombination in phase space and spin by considering Wigner functions [10, 11] also follows the previous code version. After sorting color tags and creating the initial color correlation matrices for mesons and baryons, the color factor contribution to the recombination is determined and can be read off the correlation matrix which is updated after each successful recombination. Finally, preparation for the string repair follows by updating color tags after the meson formation, and by keeping track of the color tag information to form junctions for conservation of baryon number. Junctions are Y-shaped string configurations, i.e. they have 3 strings as legs, and they carry baryon number +1 or -1. See Fig. 2.1. that is the description of these processes.

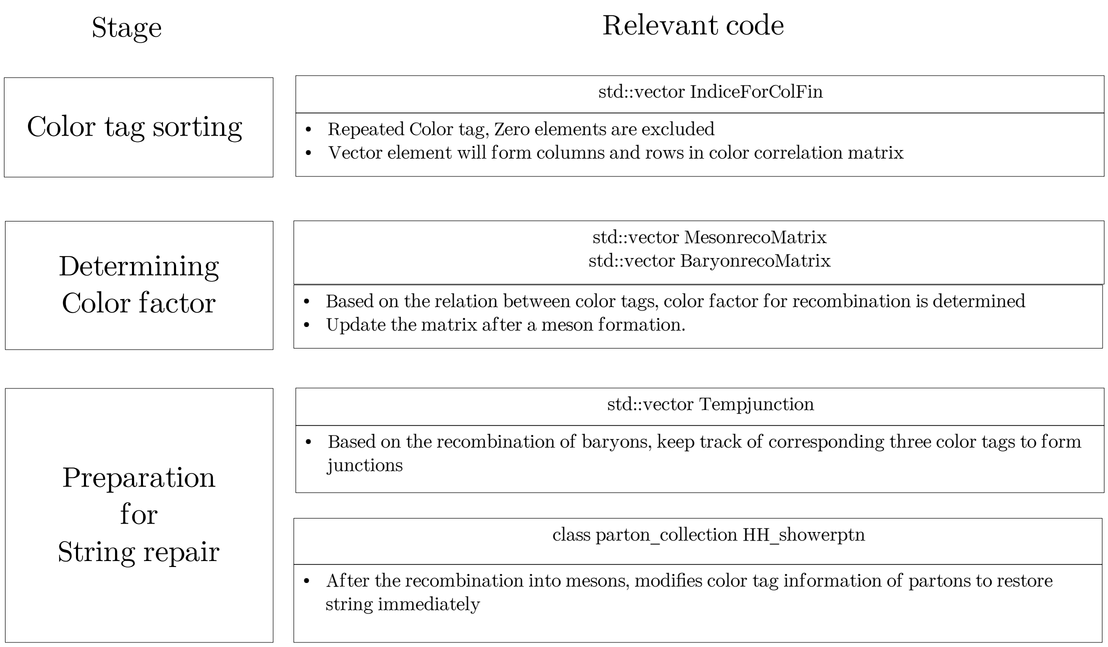


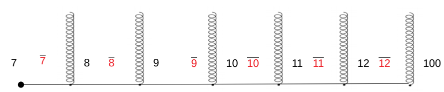
Figure 2.1: Workflow of the updated recombination step and relevant code in HH.

2.2 Color factor determination

For meson formation, the code maintains the meson color correlation matrix. An initial color correlation matrix is considered from the initial parton system based on adjacency of color tags in a string. The elements of the matrix are updated when two color tags are chosen to form a color singlet. In other words, the code updates elements when mesons are recombined from chosen partons. What is stored in the color correlation matrix is the probability P_C for a quark-antiquark pair to form a color singlet. The total recombination probability for the $q\bar{q}$ is $P = P_C \times P_S \times P_W$ where P_S and P_W are the probabilities from spin and distance in phase space which are already provided by the previous version of the code. The scheme to compute the initial P_C from the given correlations is simple and can be further improved in the future. For the same color tags, the probability to form a color singlet is 1. When two color tags are in a known color octet state, the probability is zero. For all other possible combinations of color states of quarks and antiquarks, the probability is $\frac{1}{9}$.

When $q\bar{q}$ pair is chosen for recombination, P_C for the color tags is updated to 1. Based on these color tags, the code finds corresponding elements in the color correlation matrix by matching the color tags with columns and rows in the matrix. Then the code changes the matrix element to 1. Example string and color correlation matrix are given in Fig. 2.2.

Example String :



Color Reconnection Matrix :

tags	$\bar{7}$	$\bar{8}$	$\bar{9}$	$\overline{10}$	$\overline{11}$	$\overline{12}$	$\overline{100}$
7	1	0	1/9	1/9	1/9	1/9	1/9
8	0	1	0	1/9	1/9	1/9	1/9
9	1/9	0	1	0	1/9	1/9	1/9
10	1/9	1/9	0	1	0	1/9	1/9
11	1/9	1/9	1/9	0	1	0	1/9
12	1/9	1/9	1/9	1/9	0	1	0
100	1/9	1/9	1/9	1/9	1/9	0	1

Color tag 8 and $\overline{11}$ are considered to be independent on each other. Therefore, the factor is the statistical value 1/9

Color tag 9 and $\overline{10}$ forms a gluon, and are in a color octet states are allowed. Thus, the factor is zero. The factor for $\overline{10}$ and 9 is the same

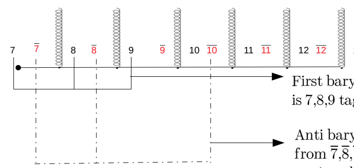
Color tag 12 and $\overline{12}$ always form color singlet. Thus, the factor is 1

Figure 2.2: Example of the initial color correlation matrix for mesons.

For baryon formation, the color correlation matrix element contains two pieces of information. The first one is the color factor to form a baryon, another one is the color tag which is used to form a baryon. MATTER and LBT do not provide junctions (baryons) in their output. Therefore initially the probability to form a color singlet from three quark or antiquarks are set to the statistical value of $\frac{1}{27}$. However, correlation from the meson correlation matrix can be important for baryons. Let us assume that three quarks have formed a baryon and three antiquarks are considered for an antibaryon, and two color tags are in common between the baryon and the anti-baryon candidate.

We need to check the relation between the color tags which are not same. In this case, we go back to the meson color correlation matrix. If the non-interacting color tags are in a color octet state, the probability to form the baryon is zero. Possible string configuration and treatment for this are given in Fig. 2.3.

Example String and Recombination :



First baryon is formed from 7,8,9. That is 7,8,9 tags form color singlet

Anti baryon is about to being made from $\overline{7}$,$\overline{8}$,$\overline{10}$, but since color tags 7, 8, 9 are in color singlet state, we need to consider the relation between 9 and 10.

Consideration in Matrix :

tags	$\overline{7}$	$\overline{8}$	$\overline{9}$	$\overline{10}$	$\overline{11}$	$\overline{12}$	$\overline{100}$
7	1	0	1/9	1/9	1/9	1/9	1/9
8	0	1	0	1/9	1/9	1/9	1/9
9	1/9	0	1	0	1/9	1/9	1/9
10	1/9	1/9	0	1	0	1/9	1/9
11	1/9	1/9	1/9	0	1	0	1/9
12	1/9	1/9	1/9	1/9	0	1	0
100	1/9	1/9	1/9	1/9	1/9	0	1

Factor zero in meson recombination matrix means they are in color octet states. Thus the color factor to be recombined into baryon is zero.

e.g. if tag 7 = red, 8 = green, 9 = blue, color tag 10 should be anti blue to form anti baryon of anti red($\overline{7}$), anti green($\overline{8}$), anti blue(which corresponds to $\overline{9}$), 10 and 9 cannot be same because they are in gluon, which allows only color octet states.

Figure 2.3: Interaction between meson formation and baryon formation.

2.3 Preliminary string repair

To repair strings after recombination, the code updates color tags and keeps track of the color tags needed to form the junction structure. After each meson recombination, the color correlation matrix is updated, and since the two color tags involved have been determined to be in a color singlet, they are set to be equal. Since PYTHIA forms strings based on color tags, we end up with valid string structures linked by color flow. We can see how this works more clearly in the Fig 2.4 below.

Baryon recombination is followed up by storing the color tags which form junctions. This happens in a vector which has 4 components; first component is a flag indicating whether a junction or antijunction should be formed to conserve baryon number, and the other elements are the corresponding three color tags. This information is used for evaluating the color factor to form baryons by providing the information about color tags determined to be in singlet state. This information also provides starting point for the next string repair process.

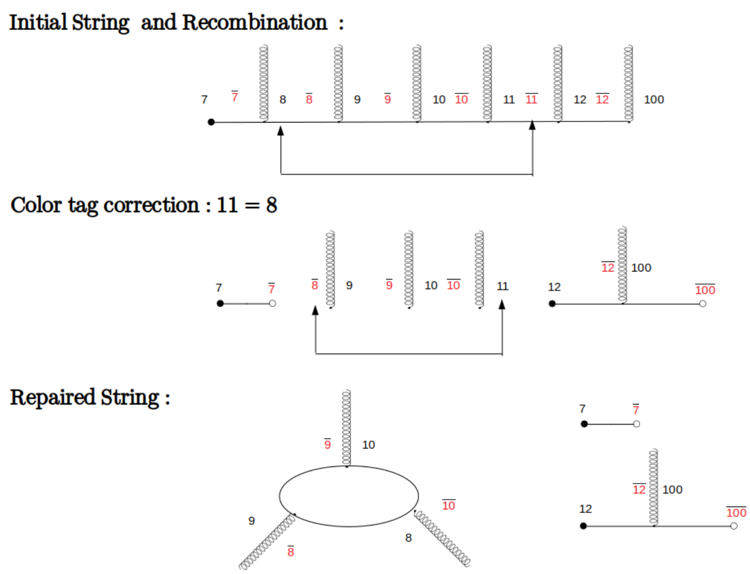


Figure 2.4: Immediate string repair after meson formation by correcting relevant color tags.

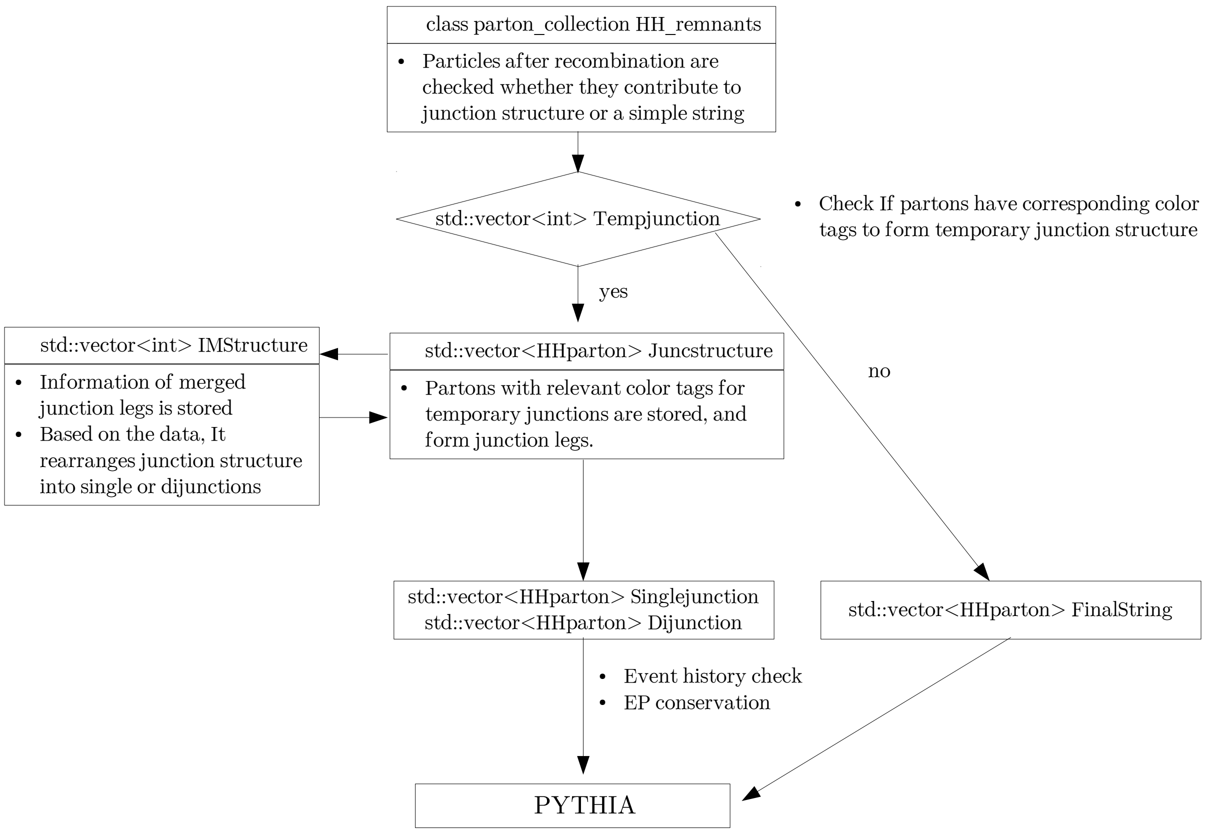


Figure 3.1: Flow of partons data and relevant code in the string repair process.

3.1 Workflow

As we can see in Fig. 3.1, Forming temporary junction is the first step for string repair. The purpose of the first step is to check whether partons are relevant to junction structure. Partons which are not related to junctions are not considered in this step. The code assembles partons into temporary junctions if one of their color tags appears in the list of junction color tags. Temporary junctions might have partons in common. A vector keeps track of these overlapping partons which indicates that there are multiple junctions in one string. Unfortunately, PYTHIA can not handle complicated multi-junction systems beyond simple dijunctions. Therefore, some junction legs have to be chosen to be cut and junctions which have more than two shared legs are forced to recombine into baryons. To finish up, the code adds baryons to the history. They are not present as actual particles and only act as mothers for junction quarks. They have to be used for letting PYTHIA know about junctions. Along with this step, the code assigns mother and daughter tags to exactly point out the relation of partons with those mother baryons as required by PYTHIA. Then the code takes final step to transfer the partons in these structures into PYTHIA, for string fragmentation.

3.2 Temporary junction formation

Before forming temporary junctions the function which executes string repair receives lists of remnant partons from the function which has executed recombination. Along with this list, the vectors including color tag information for junctions are also transferred into string repair function. The vectors of color tags which form junctions contain four elements. The first element is the junction / antijunction flag (± 1), and the other elements are three color tags which form the junction.

Going through this vector, the code finds partons which have corresponding color tags. First, the code finds the particle forming the junction itself. Possibly some junction color tags might not be assigned to partons anymore because they have been used to form mesons or baryons. Only when the junction has three adjacent partons, the code tries to trace the color flow to them by checking color tags of remnant partons and attaching relevant partons onto the junctions. This process is repeated until there are no partons with relevant color tags to form junction legs. This color flow tracing procedure does not consider whether partons are used for the formation of the other junctions to prevent skipping partons for junction formation. For each temporary junction, three vectors of partons are assigned and each vector corresponds to one junction leg.

We can see the example for temporary junctions and vectors of overlapping partons in the event printout in Fig. 3.2. There are three temporary junctions. For convenience, we call the first junction leg in the first temporary junction leg 1-1. When we check the color tags of partons in junction legs, we can find that leg 1-1 and leg 3-3 are overlapping. We can also find leg 2-3 and leg 3-2 are overlapping. Informations about these overlapping legs are stored to deal with the overlapping partons among temporary junctions. In the last three lines in Fig 3.2, we can check this information. The elements in these lines indicate the location of information in certain vectors, so according to C++ convention the numbering starts from zero. See the first line of these vectors. For the 4 elements in first parenthesis with elements 0, -1, 0, 0, the first one indicates the temporary junction number, the second one means junction/antijunction flag, which can be ± 1, the third one means temporary junction number, the last one is the flag for either dijunction or single junction, where 0 means single junction and 1 means dijunction structure. Hence, the first parenthesis means that the 0th (first) temporary junction is not connected to another junction. When we see the second parenthesis in the first line, which is 0, 0, 2, 2, this parenthesis means that leg 1-1 and leg 3-3 are overlapping. similarly, the other lines store the information about the additional overlapping legs. With the information about the junction structure and the participating partons understood we can prepare the system for string fragmentation.

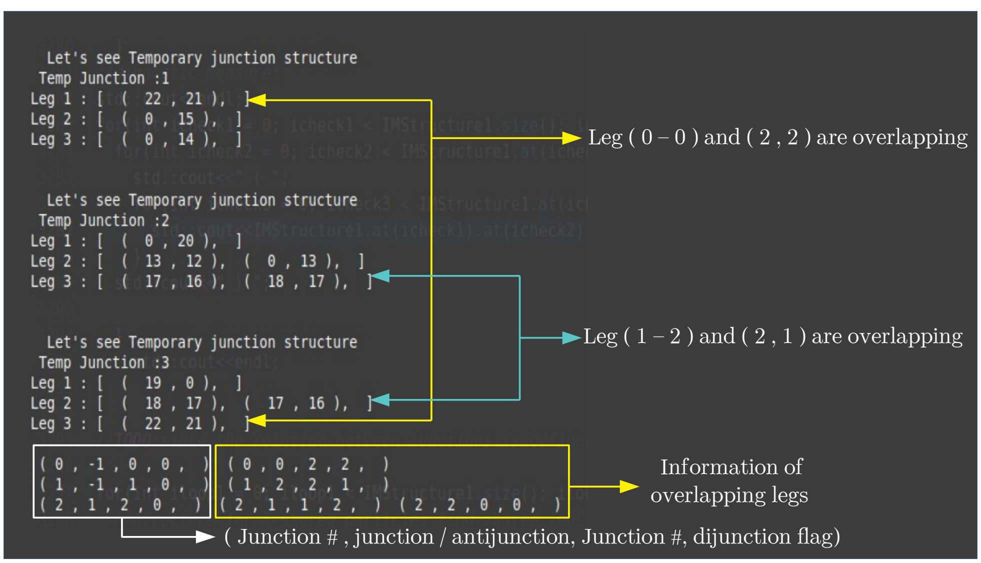


Figure 3.2: Example for temporary junctions and their information in the updated Hybrid Hadronization code. Particles are described by their color and anti color tags, the lsat three line indicates informations about overlapping legs in temporary junctions

3.3 Preparation for PYTHIA

Prior to discussing string repair based on our temporary junctions, we want to point out some important aspects of sending parton and string data to PYTHIA. Generally, PYTHIA can detect string structure by tracing color flow. For example, If we assume one particle with 101 color tag, 102 anticolor tag, and another one with 102 color tag, 103 anticolor tag, PYTHIA will interpret the gluons as part of a string linked by color tag 102.

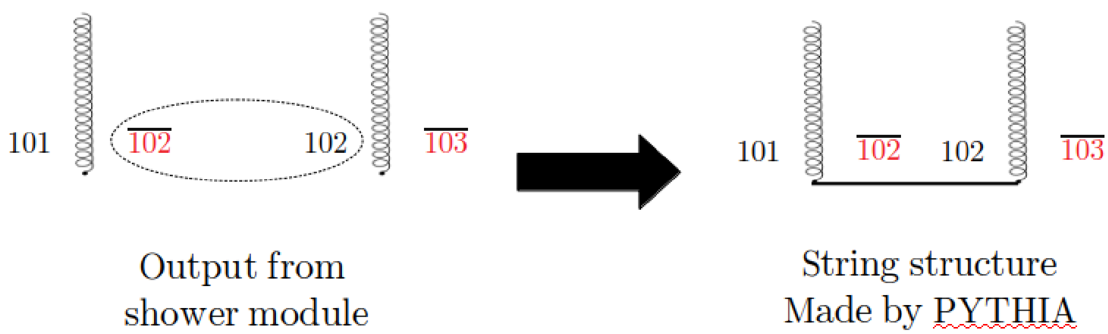


Figure 3.3: Procedure of PYTHIA to check color flow and form string.

In this way PYTHIA can recover string structure from the color tag information. However, for junctions, where three colors are relevant, PYTHIA needs more information to detect the junction structure. The additional requirements are mother and daughter tags for the particles. These are integer tags and used by PYTHIA to record the history of particles evolving in PYTHIA. Mother tags refers to a previously existing particle in the event record which created the particle with these tags. In reverse, daughter tags of the particles indicate subsequent particles in the event related to it. The default value for those tags is zero, because in the event record in PYTHIA, zero refers to the whole event [21]. The mother/daughter tagging system can be used as a general referencing system even if the mother particle was never actually in the event. For every event in PYTHIA, the virtual particle to represent the whole event is always declared first to control the particles with default mother and daughter tag. In our case we create fake baryons in the event history to act as mothers for the junctions. "Fake" here means that those baryons are not part of the final particle output. In some case additional fake mother partons need to be added as well. There are more sophisticated classification about mother and daughter tag usage in the official website for PYTHIA by Lund University (http://home.thep.lu.se/). For convenience, we discuss here a simple example to understand how these tags work in the code.

-------- PYTHIA Event Listing (complete event) --

no	id	name	status	mothers		daughters		colours		p_x	p_y	p_z	e	m
0	90	(system)	-11	0	0	0	0	0	0	877.224	-0.000	0.000	1003.284	486.885
1	2114	(Delta0)	-11	0	0	3	6	0	0	11.920	-0.548	-6.672	15.330	6.935
2	-1114	(Deltabar+)	-11	0	0	7	10	0	0	8.662	3.180	-1.663	10.684	5.122
3	1	(d)	-23	1	1	29	29	18	0	4.961	0.705	0.075	5.023	0.340
4	21	(g)	-23	1	1	30	30	30	29	2.366	-1.615	-1.858	3.447	0.477
5	2	(u)	-23	1	1	31	31	29	0	2.919	-0.195	-2.364	3.776	0.336
6	1	(d)	-21	1	1	11	11	17	0	1.674	0.557	-2.525	3.100	0.340
7	-1	(dbar)	-21	2	2	11	11	0	16	1.674	0.557	-2.525	3.100	0.340
8	-1	(dbar)	-23	2	2	32	32	0	22	1.122	0.273	0.068	1.205	0.340
9	21	(g)	-23	2	2	33	33	24	23	5.866	2.350	0.794	6.397	0.598
10	-1	(dbar)	-23	2	2	34	34	0	24	0.000	0.000	0.000	0.340	0.340
11	21	(g)	-23	6	7	35	35	17	16	1.674	0.557	-2.525	3.085	0.154

Diagram of Mother, daughter tags

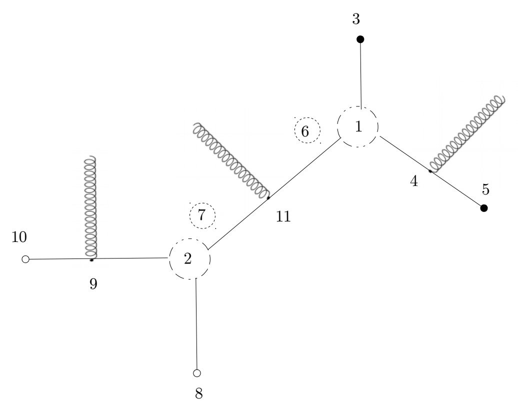


Diagram of color tags

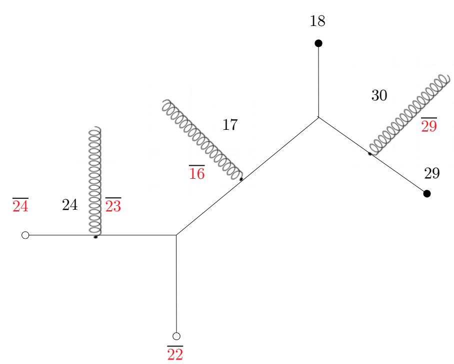


Figure 3.4: Example of a PYTHIA event log (top panel) and diagrams for event record (lower left) and color tags (lower right). numbers in first column labelled "no" are used for mother daughter tags. E.g. Particle in the 3rd line, starting from 1, is the fake mother junction baryon, and It has daughter tags from 3 to 6. This means that the particles in the lines starting from 3, 4, 5, 6 are daughter particles of it.

Fig. 3.4 shows an event with 3 gluons (particle numbers 4, 9, 11), 2 quarks (3, 5), and 2 antiquarks (8, 10) which form a junction-antijunction system. The particles with dotted circles are fake particles. Their status flags are set not to be engaged with the physical process. So they are not treated as physical particles. Two fake baryons (1, 2) are shown in place of the junctions. They are the mother junctions. Partons in adjacent legs refer to their event number as the mother tag. In addition, two fake quarks (6, 7) have been placed on the shared leg between junctions. They mediate the two mother junctions with the gluons between them. Without these mediating partons, PYTHIA cannot detect the structure and breaks internally.

In Fig 3.4, the numbers in the first column represent the particle number that are referred to by mother and daughter tags. For example, the baryon with number 1 has daughter tag from 3 to 6 associated with it. Similarly, the anti-baryon with number 2 has particles with event record 7 to 10 as daughter particles to form the antijunction. Meanwhile, partons with numbers 6 and 7 have a common daughter particle, given number 11.

Because of the internal consistency checks module in PYTHIA, the conservation laws of energy and momentum should be fulfilled. These laws should be respected by fake partons. Basic principle for the conservation is that the energy and momentum of mother particles should be the the same as sum of energy and momentum of all daughter particles. Fake baryon junction mothers must have the same momentum and energy as the sum of those quantities of all subordinate particles. Fake partons in legs connecting junctions are treated differently. Their energy and momentum are half of the sum of those values from their daughter particles because they need to share those values.

The internal consistency check in PYTHIA also checks color neutrality, before fragmenting strings. This is fulfilled by all color tags being either part of a junction or paired with their anticolor tags. Hence the code checks whether all color tags in the event are paired. MATTER showers usally are not singlets since they represent a single jet initiated by a single colored parton. Worse, adding thermal partons or partons from LBT guarantees that the total system is a already not a color singlet. In the medium, thermal partons and shower partons from LBT can be added for color neutrality. They are chosen based on the distance in phase space. Proper color tags are assigned to them if they are used for color neutrality. In the vacuum case, fake partons with low energy and momentum or along the beam direction are added to the event.

In conclusion, the code establishes the string structure by adding particles to the history for junctions. It enforces color neutrality by adding partons if needed.

3.4 Further string processing

The previous step has given us string structures which are phrased precisely in the way PYTHIA can understand. They are perfectly acceptable color singlets. However, there is one more complication. The physics of string systems with several junctions is difficult to handle for PYTHIA. In reality, strings more complicated than a dijunction with one shared leg is likely too challenging for current PYTHIA versions. Let us discuss examples.

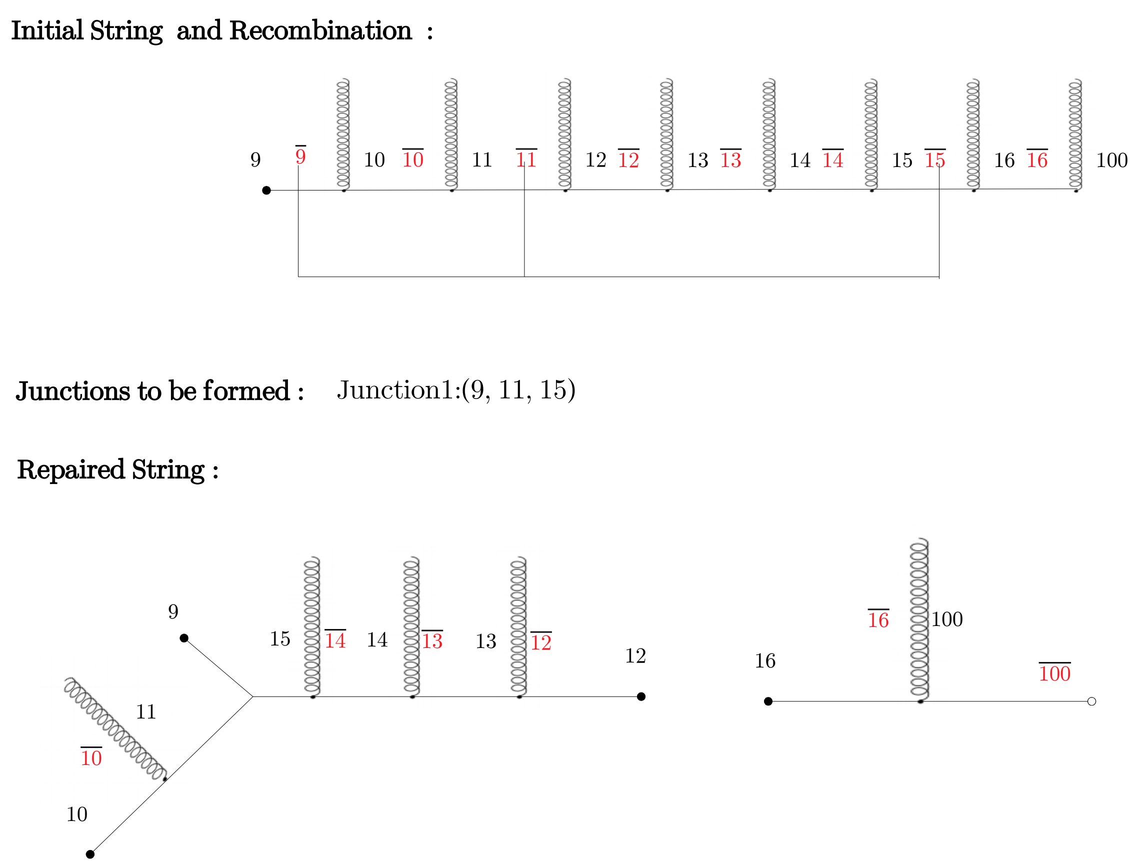


Figure 3.5: Example 1: string repair into single junction structure.

We can start from the structure of single junction in a single jet, which is the simplest example of string repair after baryon recombination. In the example in Fig 3.5, an antibaryon recombines using anticolor tags $\bar{9}$, $\bar{11}$, and $\bar{15}$. It requires a junction of color tags (9, 11, 15) for baryon number conservation. In this case, since there are no overlapping junction legs, the code simply assigns a baryon mother. See the event record in Fig 3.6. This single junction string can be easily processed by PYTHIA. No additional modifications are needed.

-------- PYTHIA Event Listing (complete event) --

no	id	name	status	mothers		daughters		colours		p_x	p_y	p_z	e	m
0	90	(system)	-11	0	0	0	0	0	0	628.865	-8.371	27.803	745.306	398.958
1	2214	(Delta+)	-11	0	0	2	4	0	0	100.552	2.280	4.731	152.873	115.030
2	2	(u)	-23	1	1	13	13	10	0	0.914	0.469	1.110	1.549	0.336
3	1	(d)	-23	1	1	14	14	13	0	-24.701	-1.237	10.248	26.773	0.340
4	2	(u)	-23	1	1	15	15	15	0	124.338	3.048	-6.626	124.553	0.336
5	2	(u)	-23	0	0	16	16	11	0	0.599	0.536	1.240	1.515	0.336
6	-2	(ubar)	-23	0	0	17	17	0	11	0.407	0.788	0.883	1.296	0.336
7	2	(u)	-23	0	0	18	18	16	0	3.127	-0.332	-0.790	3.260	0.336
8	21	(g)	-23	0	0	19	19	100	16	1.515	1.313	-1.233	2.355	0.085
9	-1	(dbar)	-23	0	0	20	20	0	100	0.000	0.000	0.000	0.340	0.340

Figure 3.6: Example 1 : Event log for a single junction event. We can distinguish particles with numbers in the first column. See the third line of the log. Particle 1 is a mother junction and it has daughter tags from 2 to 4. Therefore, we can confirm the particle 2,3, and 4 are linked to the particle 1 and they form single junction. For the particles with mother tag 0, they are directly integrated into the simple quark-antiquark strings.

Our second example leads to a dijunction structure for which additional modification in temporary junctions is required for temporary junctions. Fig 3.7 shows a long string with 2 baryons and 2 anti-baryons recombining. There are two temporary junctions as described in Section 3.2, with one overlapping leg. The other two junction / antijunction candidates have created several smaller strings. See center panel of Fig 3.7. Two temporary junctions are combined into a dijunction structure as shown in the bottom panel in Fig 3.7 including partons color tags. The event log including fake particles is shown in Fig 3.8.

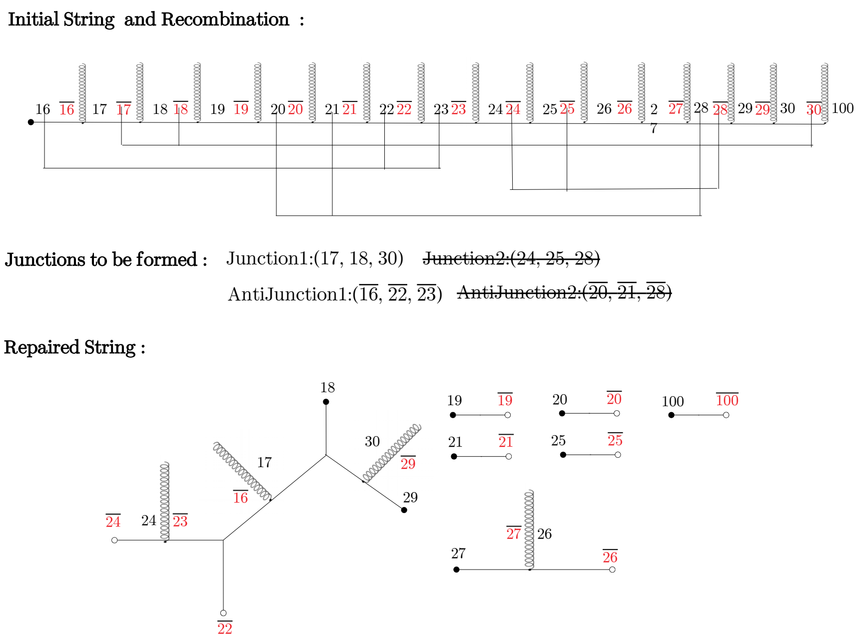
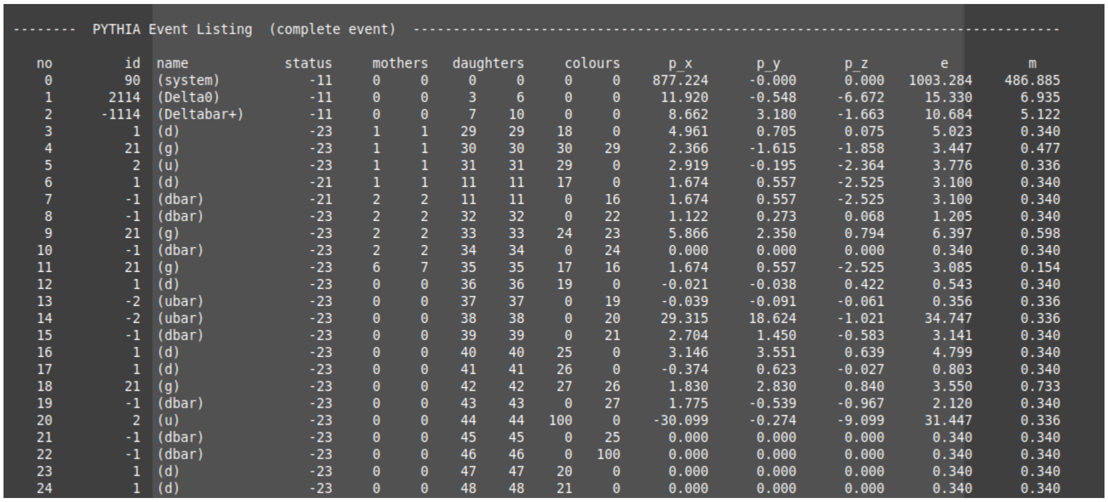


Figure 3.7: Example 2 : String repair into dijunction structure. Two temporary junctions(striped in the junction list) vanish since the partons with corresponding color tag are used for other recombination



-------- PYTHIA Event Listing (complete event) --------

no	id	name	status	mothers		daughters		colours		p_x	p_y	p_z	e	m
0	90	(system)	-11	0	0	0	0	0	0	877.224	-0.000	0.000	1003.284	486.885
1	2114	(Delta0)	-11	0	0	3	6	0	0	11.920	-0.548	-6.672	15.330	6.935
2	-1114	(Deltabar+)	-11	0	0	7	10	0	0	8.662	3.180	-1.663	10.684	5.122
3	1	(d)	-23	1	1	29	29	18	0	4.961	0.705	0.075	5.023	0.340
4	21	(g)	-23	1	1	30	30	30	29	2.366	-1.615	-1.858	3.447	0.477
5	2	(u)	-23	1	1	31	31	29	0	2.919	-0.195	-2.364	3.776	0.336
6	1	(d)	-21	1	1	11	11	17	0	1.674	0.557	-2.525	3.100	0.340
7	-1	(dbar)	-21	2	2	11	11	0	16	1.674	0.557	-2.525	3.100	0.340
8	-1	(dbar)	-23	2	2	32	32	0	22	1.122	0.273	0.068	1.205	0.340
9	21	(g)	-23	2	2	33	33	24	23	5.866	2.350	0.794	6.397	0.598
10	-1	(dbar)	-23	2	2	34	34	0	24	0.000	0.000	0.000	0.340	0.340
11	21	(g)	-23	6	7	35	35	17	16	1.674	0.557	-2.525	3.085	0.154
12	1	(d)	-23	0	0	36	36	19	0	-0.021	-0.038	0.422	0.543	0.340
13	-2	(ubar)	-23	0	0	37	37	0	19	-0.039	-0.091	-0.061	0.356	0.336
14	-2	(ubar)	-23	0	0	38	38	0	20	29.315	18.624	-1.021	34.747	0.336
15	-1	(dbar)	-23	0	0	39	39	0	21	2.704	1.450	-0.583	3.141	0.340
16	1	(d)	-23	0	0	40	40	25	0	3.146	3.551	0.639	4.799	0.340
17	1	(d)	-23	0	0	41	41	26	0	-0.374	0.623	-0.027	0.803	0.340
18	21	(g)	-23	0	0	42	42	27	26	1.830	2.830	0.840	3.550	0.733
19	-1	(dbar)	-23	0	0	43	43	0	27	1.775	-0.539	-0.967	2.120	0.340
20	2	(u)	-23	0	0	44	44	100	0	-30.099	-0.274	-9.099	31.447	0.336
21	-1	(dbar)	-23	0	0	45	45	0	25	0.000	0.000	0.000	0.340	0.340
22	-1	(dbar)	-23	0	0	46	46	0	100	0.000	0.000	0.000	0.340	0.340
23	1	(d)	-23	0	0	47	47	20	0	0.000	0.000	0.000	0.340	0.340
24	1	(d)	-23	0	0	48	48	21	0	0.000	0.000	0.000	0.340	0.340

Figure 3.8: Example 2 : PYTHIA event log of dijunction structure.

When running Hybrid Hadronization with physically reasonable parameters, it is relatively rare to create trijunction and higher systems, or junctions with multiple legs overlapping. However, for stability of the code when running large event numbers, one needs a strategy to deal with those pathological cases. Since PYTHIA cannot stably handle general N-junction structures, we need to modify them. When junction structures are hadronized, each leg is fragmented, similar to the string fragmentation of a simple quark-antiquark string, and finally the proper junction remains and hadronizes into a baryon. In the Hybrid Hadronization code, we detect junctions with two or three overlapping legs, then force recombination of these junctions into baryons. Applied repeatedly if neceessary, only single junction or dijunction systems remain. The following examples (Fig 3.9 + 3.10) depict such complicated systems and also show the way how the code treats these system to make them fit for PYTHIA.

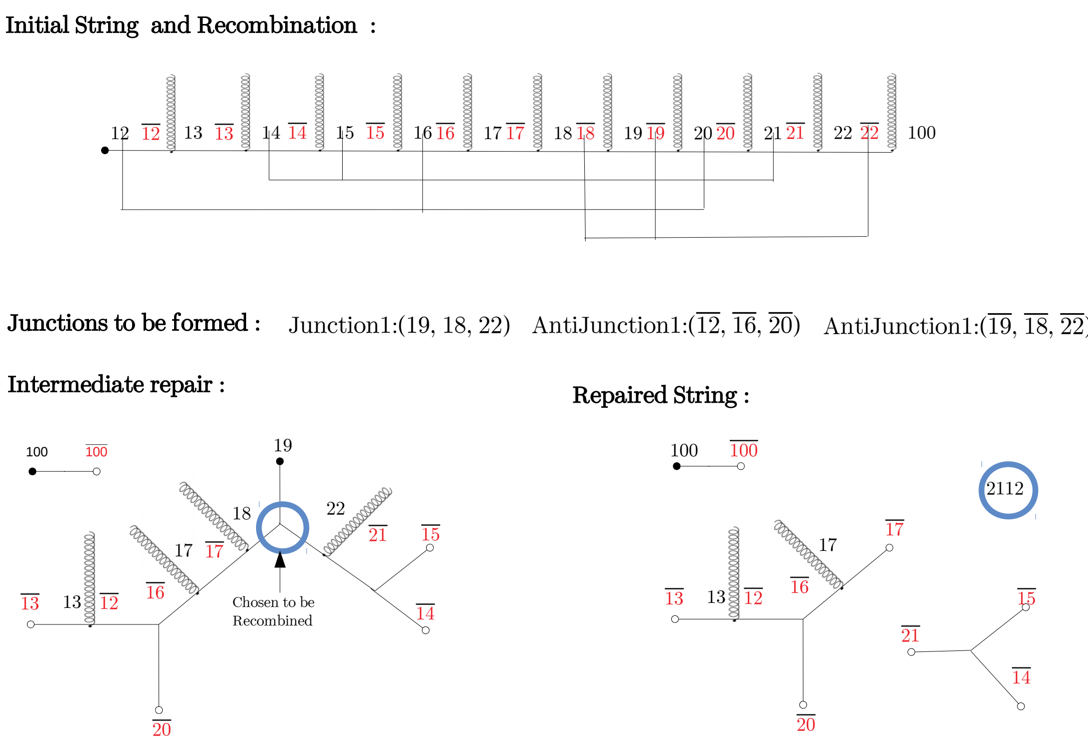


Figure 3.9: Example 3 : String repair of trijunction, modified into dijunction and a baryon.

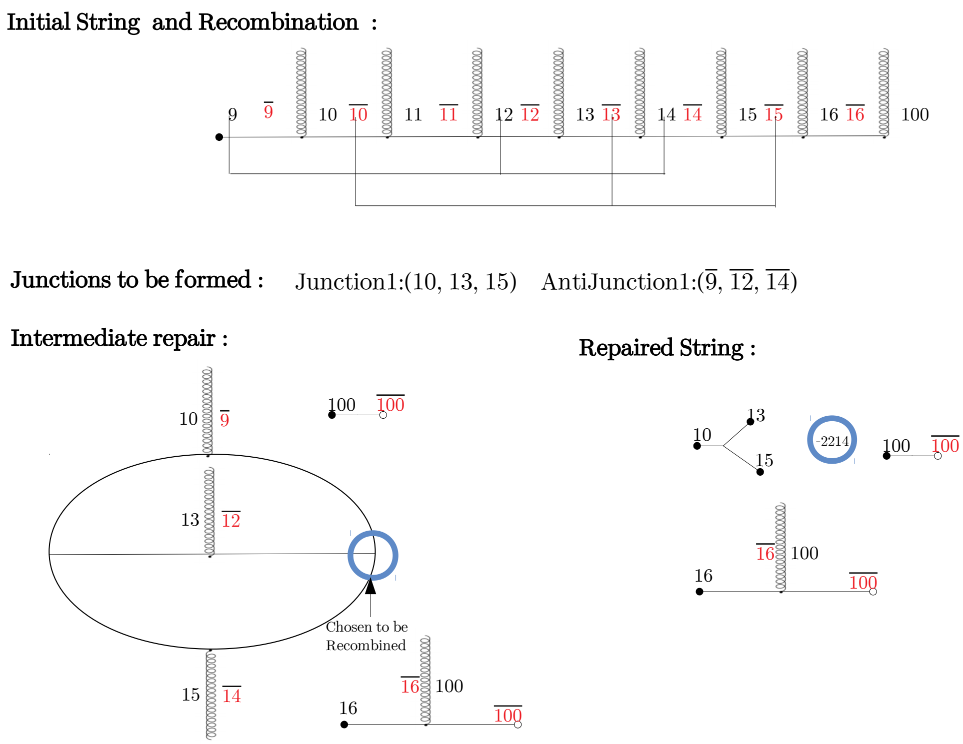


Figure 3.10: Example 4 : Junction-antijunction system with 3 overlapping legs, modified into an junction and a anti-baryon.

Through the improvements in the string repair process above, the Hybrid Hadronization code becomes able to deal with junction systems by utilizing parton color flow data. At the same time, the code allows PYTHIA to run without errors by checking energy-momentum conservation, color neutrality, and by the pre-processing of multi-junction systems.

4.1 Comparison with the data without recombination

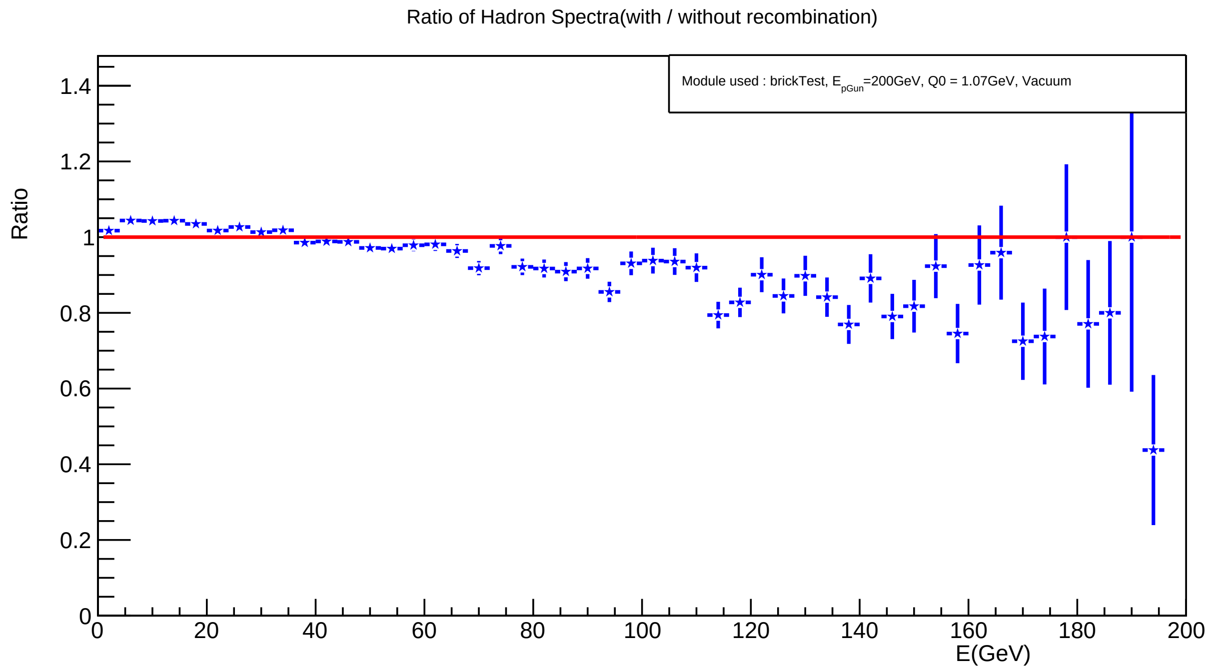


Figure 4.1: Ratio of fragmentation functions D of 200 GeV partons in Hybrid Hadronization with and without recombination.

As a first test we evaluate the effect of recombination by comparing Hybrid Hadronization to the same code with the recombination probability forcibly set to zero. We compare the hadron spectrum of a single jet in the vacuum. The number of events is 100,000 for both runs. The lower virtuality cut off for MATTER is set as $Q_0 = 1.07$ GeV. This means that partons split until their virtuality is smaller than Q_0. As we can see form the Fig. 4.1, the spectrum of hadrons in the vacuum is slightly modified from pure string fragmentation. This is desirable because string fragmentation is well-tuned to experimental data in the vacuum case and recombination is not supposed to disrupt this situation. Recombination is most active in the low energy region, especially under 5 GeV. This is because the value of the Wigner function which evaluates recombination probability is based on the distance in phase space, and partons in low energy tend to be close to each other [10].

4.2 Comparison with the results of previous HH code

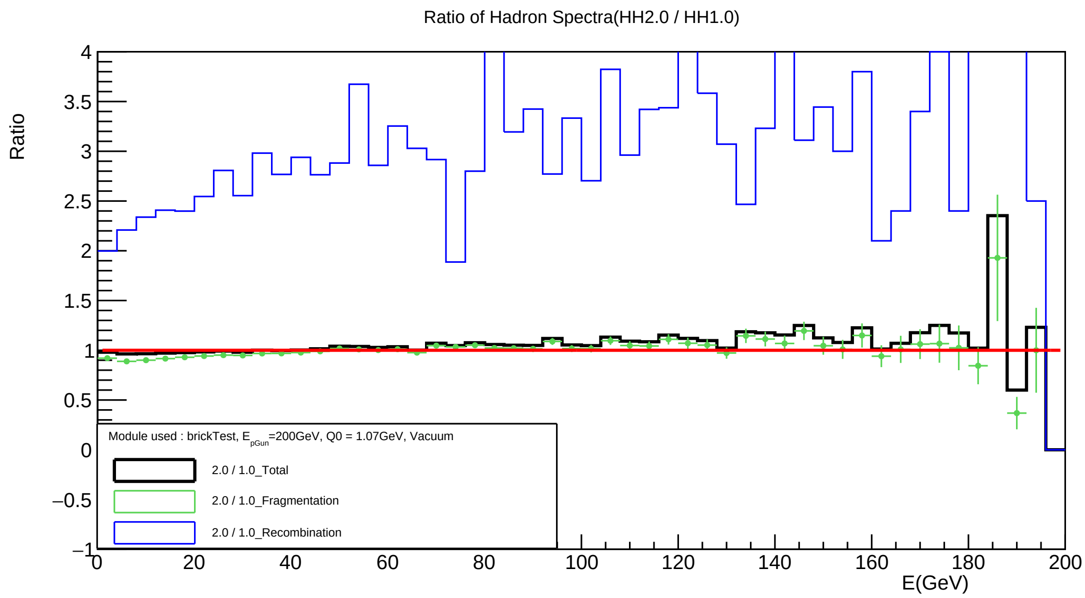
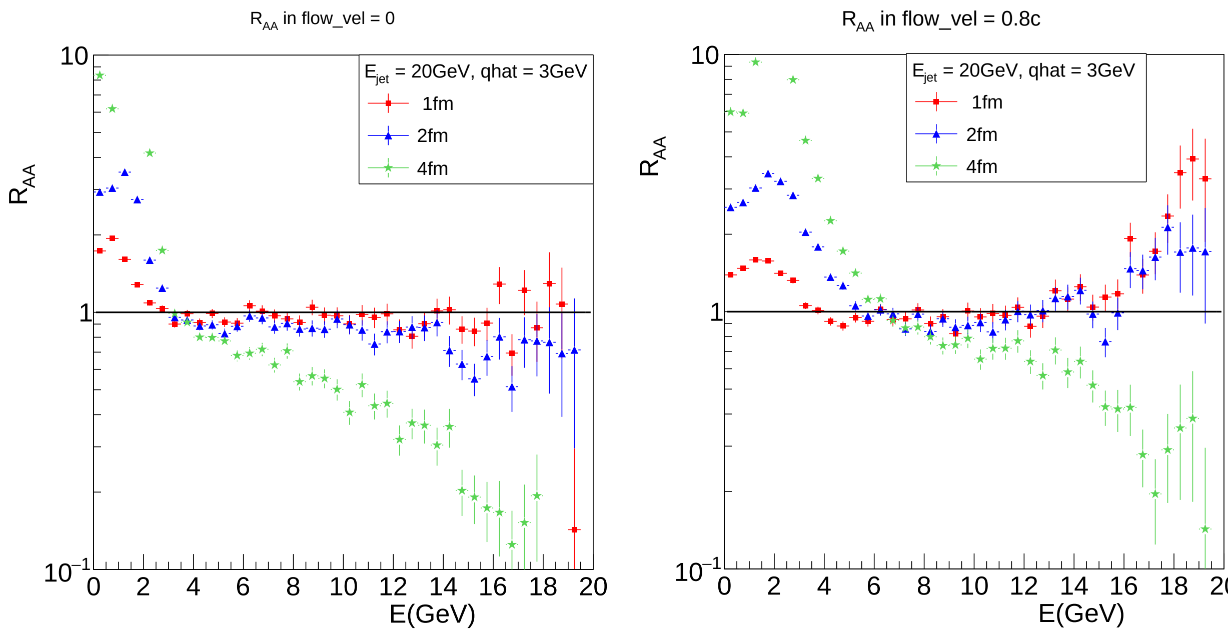


Figure 4.2: Comparison with extant version of Hybrid Hadronization code.

Next, we compare the hadron spectra of the improved Hybrid Hadronization code, which is labelled as "2.0" here with that of the previous version that is labelled as "1.0" in Fig. 4.2. This comparison is done using the same setup as that of Fig. 4.1. We can see that recombination is enhanced in improved Hybrid Hadronization code. That is, including realistic color flow in determining the recombination probability affects total recombination. However, when we check the ratio of total hadron spectra, the effect is very small as vacuum jets are still dominated by fragmentation. Overall, the previous two results show that there are modest changes to hadron spectra in the vacuum. Note that this result is not unexpected. It is beyond the scope of this thesis to explore more vacuum observables.

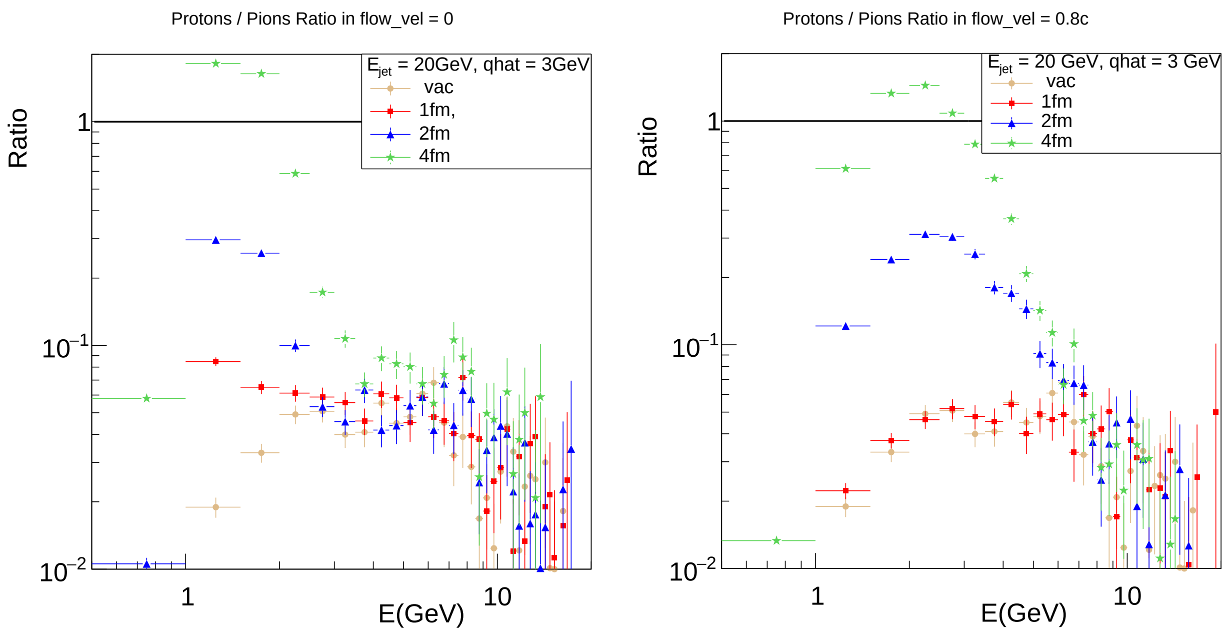
4.3 Events in medium



(a) flow velocity = 0 (b) flow velocity = 0.8c

Figure 4.3: R_{AA} in different medium sizes and flow velocity.

Finally we present a systematic study of medium effects. In this case, we use 20 GeV jets either in vacuum or in a QGP brick of lengths 1, 2 and 4 fm/c. The thermal partons sampled from the brick can be given a flow velocity longitudinal to the jet to mimic radial flow in heavy ion collisions. Fig 4.3 shows the ratio of fragmentation functions in the brick and in the vacuum. One can see the expected jet quenching at energies larger than 6 GeV, depending on medium size. At low energy an enhancement is seen that scales with medium size and comes mostly from shower-thermal recombination. This is confirmed by the existance of a flow effect. As we can see from Fig 4.3, for all medium sizes the peaks of the ratios are shifted from 1 GeV to 2 GeV by increasing thermal parton flow velocity from zero to 0.8c. This indicates that the application of flow velocity is implemented well and the effect is consistent with expectations.



(a) flow velocity = 0 (b) flow velocity = 0.8c

Figure 4.4: Protons / Pions ratio in different medium sizes and flow velocity

We can also confirm the similar effects by looking at protons / pions ratios in the same study in Fig 4.4. The p/π ratio is an indicator for recombination with thermal partons. We observe a large enhancement at small energies, which scales with medium size. Again a strong flow effect is observed. These observations are qualitatively in agreement with what can be seen in data from heavy-ion collisions. In the near future JETSCAPE calculations using Hybrid Hadronization will be compared directly to data.

5. SUMMARY AND OUTLOOK

This project is conducted for improving the existing Hybrid Hadronization code to be realistic in the determination of recombination probabilities and in repairing strings by considering color flow. Hybrid Hadronization combines two kinds of hadronization models, string fragmentation and recombination. The probability for recombination is determined by the color factor to form color singlet, the spins and the Wigner function which evaluates the recombination probability based on the distance in phase space [10, 11]. The goal is to have a hadronization code which can hadronize jet showers in vacuum and also in a QGP medium.

My goal was to improve the Hybrid Hadronization code. This was done by using color factors reflecting true color states of quark-antiquark pair, which are singlet and octet, and possible 27 color states of three quarks or anti quarks. Previously, color factors were simply set to $\frac{1}{9}$ for meson formation, and $\frac{1}{27}$ for baryon formation. In the improved code, these factors are determined by initial color flow, and by updates as recombination proceeds. After each recombination, the code directly repairs strings by correcting the color tags after meson formation, and by preparing lists of color tags that should be used for forming junction structures.

In the string repair stage, temporary junctions are made based on the information after recombination process. Some of them could have legs shared with other junctions. Since the maximum capacity for PYTHIA to stably handle junction structure is the dijunction structure, we cut the strings if there are junctions with multiple shared legs by recombining problematic junctions. We restore color singlets if needed, and add fake mother particles for junctions along with checking energy-momentum conservation before handing the system to PYTHIA

Comparison of hadron spectra with previous Hybrid Hadronization shows that the improvements enhances recombination. When we run jets with medium, we can confirm typical recombination signatures. The peak values of R_{AA} and protons / pions ratio are enhanced by medium size. Flow of thermal partons shifts these peaks. This result indicates that we will most likely be able to describe these effects in real experimental data.

This thesis has made the Hybrid Hadronization code more physical and had improved its comtatibility with PYTHIA. Vacuum results have not changed dramatically and results in medium look very promising to compare to experimental data.

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