DESIGN OF ULTRA-WIDEBAND (2-18 GHz) BUTLER MATRIX

A Thesis

by

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ABSTRACT

This thesis presents the design of an ultra-wide band hybrid coupler, crossover and phase shifters for the design of four-input, four-output (4x4) stripline Butler matrix in order to feed an antenna array in 2-18 GHz frequency range. The goal of this thesis is to develop an antenna-array feeding passive microwave network based on Butler matrix with a ultra-wide bandwidth which works as ground work for realization of eight input, eight output Butler matrix. Further the 4x4 and 8x8 Butler matrix can be used in a row-card configuration to realize 16x16 and 64x64 Butler matrix. In order to meet the ultra-wide band requirements, wide band passive microwave components such as hybrid coupler, crossovers and phase-shifters are designed, which operate from 2 to 18 GHz. The Butler matrix can be used as a beam-forming network produces orthogonal beams which can be steered in different directions.
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1. INTRODUCTION

Beam-forming or spatial filtering techniques have an extensive range of applications including but not limited to defense [1, 2], automotive communications [3] and cellular communications [4]. The upcoming 5G technology for cellular communication has strong dependence on beam-forming to form a direct link between the receiver and sender to counter the attenuation of millimeter waves [5]. A beam-forming network generates the required amplitude and phase excitations, to steer the radiated beam to a specific spatial direction without changing the physical locations of the antenna elements. Examples include, the digital beam-forming networks [6], the circuit based beam-forming networks such as Butler Matrix [7] and Blass Matrix [8] and the Microwave lens beam-forming networks such as Rotman lens [9] and Luneberg lens [10].

While digital beam-forming networks provide better performance with very low phase errors and more flexible amplitude tapering, they are not suitable for high frequencies and requires large hardware and enormous power as the aperture size or the number of antenna elements increase. Butler matrix network is easy to construct and implement on printed circuit boards. By varying the input current amplitude and input phase to the Butler matrix the beam can be scanned to the required direction. Sheleg discusses the design and implementation of Butler matrix based scanning using circular array [11]. Tayeb et al. described a 4x4 Butler matrix design for linear microstrip antenna array operating at 1.9 GHz [12]. Mourad et al. have designed 4x4 Butler matrix
using coplanar waveguides at 5.8 GHz [13]. In this work ultra-wideband (2-18 GHz) stripline Butler matrix design is proposed. Individual building blocks, hybrid coupler, crossover, phase shifter are first designed for 2-18 GHz. Further they are put together to form 4x4 Butler matrix.

1.1. Butler Matrix

A butler matrix is a passive microwave beamforming network used to feed a phased antenna array. It was first proposed by Butler and Lowe in 1961 [14]. To steer the beam using a phased antenna array, current phase needs to be varied. Each antenna element in the phased array can be fed with progressive phase shifts to steer the beam in a specific direction. Butler matrix facilitates this operation by feeding current to the antenna elements in a progressive phase shift manner.

A Butler matrix is an N-input N-output passive microwave network, where N is generally some power of 2. Power is applied at the N-input ports and the progressive phase shifts are obtained at the N-output ports to which N antenna elements are connected. The N-input ports are also called the beam ports and the N-output ports are called the antenna ports. The beam direction can be controlled by which input or beam port is excited. The input at any one beam port results in current of equal amplitude and linearly varying phase at the antenna ports. This kind of operation is generally called switched beam, where the input is switched to one of the beam ports. Multiple beam ports can also be simultaneously fed. For example, if two ports are fed simultaneously, the antenna array radiates dual beams simultaneously, superimposed on one another.
The phase difference between the antenna ports varies depending on which beam port is excited. In general for an N-input port Butler matrix if $K^{th}$ ($K = \pm 1, \pm 2, \pm 3.. \pm N/2$) port is excited the phase difference between output ports is $\pm (2K-1)/N$. For the simple case of 2x2 Butler matrix the phase difference between output ports is $\pm 90^\circ$. Therefore a 2x2 Butler matrix is simply a 90° hybrid coupler. **Figure 1-1** shows a block diagram of such a Butler matrix. In general N-port Butler matrix is a passive network formed by a combination of 90° hybrid couplers, crossovers and phase shifters.

**Figure 1-1 : 2x2 Butler Matrix Schematic**

**Figure 1-2** shows the block diagram of a 4x4 Butler matrix. It requires a crossover, four hybrid couplers and two 45° phase shifters. Extending the idea to 8x8 Butler, it requires 12 hybrid couplers, ten crossovers, two 67.5° phase shifters, two 22.5° phase shifters and four 45° phase shifters. **Figure 1-3** shows the block diagram for the same.
Figure 1-2: 4x4 Butler matrix schematic

Figure 1-3: 8x8 Butler matrix schematic
1.2. 90° Hybrid Coupler

A hybrid coupler is a four port passive microwave device, which splits the input signal into two equi-power signals at outputs and the fourth port is isolated. It is a special case of directional coupler such that the coupling is 3 dB or half power split at the outputs. For a 90° hybrid coupler the outputs differ in phase by 90°. Figure 1-4 shows the block diagram of a hybrid coupler. Port 1 is the “input” port; coupled power goes to port 3 or the “coupling” port. Rest of the power goes directly to port 2 or the “through” port while port 4 is the “isolation” port. Ideally no power shows up at isolation port. So, for a 90° hybrid coupler power at port 1 splits into equally to port 2 and 3, with a phase difference of 90° between port 2 and 3.

\[
\begin{bmatrix}
0 & -j \cos \alpha & \sin \alpha & 0 \\
-j \cos \alpha & 0 & 0 & \sin \alpha \\
\sin \alpha & 0 & 0 & -j \cos \alpha \\
0 & \sin \alpha & -j \cos \alpha & 0 
\end{bmatrix} \tag{1-1}
\]

Figure 1-4 : Hybrid Coupler

S-matrix of a general coupler is written as:
where \( \sin \alpha = k \) is the voltage coupling coefficient. For the hybrid coupler, since coupled power is \( \frac{1}{2} \) therefore the voltage coupling coefficient, \( k \) becomes \( \sqrt{\frac{1}{2}} \) or \( \alpha = \pi/4 \).

Based on the application, the frequency of operation, 90° hybrid coupler can be realized as coupled line coupler, branchline coupler, Lange coupler or interdigitated coupler etc. in transmission line structures such as microstrip, stripline, waveguide etc. Section 2 covers the design choices available for this particular application where 3 dB coupling is required over 2-18 Ghz and the finalized stripline design with its simulated results in HFSS.

1.3. Crossover

Crossover can be realized by connecting two 90° hybrid couplers in tandem, such as shown in Figure 1-5 where port 2 of first coupler is connected to port 4’ of second coupler and port 3 of first coupler to port 1’ of second coupler.
When input port of the first coupler is excited by a unity wave voltage, the voltage waves amplitude and phase at the other ports can be obtained using the S-matrix of the 90° hybrid. Using voltage coupling coefficient of \( k = \sqrt{\frac{1}{2}} \),

\[
\begin{bmatrix}
V^-_1 \\
V^-_2 \\
V^-_3 \\
V^-_4
\end{bmatrix} = \begin{bmatrix}
0 & -j\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\
-j\sqrt{\frac{1}{2}} & 0 & 0 & \sqrt{\frac{1}{2}} \\
\sqrt{\frac{1}{2}} & 0 & 0 & -j\sqrt{\frac{1}{2}} \\
0 & \sqrt{\frac{1}{2}} & -j\sqrt{\frac{1}{2}} & 0
\end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\] (1-2)

which gives,

\( V^-_1 = 0 \) (1-3)
\( V^-_2 = -j\sqrt{\frac{1}{2}} \) (1-4)
\( V^-_3 = \sqrt{\frac{1}{2}} \) (1-5)
\( V^-_4 = 0 \) (1-6)

Now, \( V^-_2 \) and \( V^-_3 \) are the incident voltage waves at port 1 and port 4 of the second coupler, therefore the reflected voltages of coupler 2 can be obtained as:

\[
\begin{bmatrix}
V'^-_1 \\
V'^-_2 \\
V'^-_3 \\
V'^-_4
\end{bmatrix} = \begin{bmatrix}
0 & -j\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\
-j\sqrt{\frac{1}{2}} & 0 & 0 & \sqrt{\frac{1}{2}} \\
\sqrt{\frac{1}{2}} & 0 & 0 & -j\sqrt{\frac{1}{2}} \\
0 & \sqrt{\frac{1}{2}} & -j\sqrt{\frac{1}{2}} & 0
\end{bmatrix} \begin{bmatrix} 0 \\ -j\sqrt{\frac{1}{2}} \end{bmatrix}
\] (1-7)

which leads to,

\( V'^-_1 = 0 \) (1-8)
\( V'^-_2 = -j \) (1-9)
\( V'^-_3 = 0 \) (1-10)
\( V'^-_4 = 0 \) (1-11)
According to the analysis above, when two hybrids are connected in tandem all of the input power comes out through port 2 of the second coupler.

Hence design of crossover for the realization of Butler matrix is straightforward once design topology of 90° hybrid coupler is fixed.

1.4. Phase Shifters

Phase shifters are critical components in numerous RF systems including linear power amplifiers (Doherty amplifier) and phased array systems as in this application. For Butler Matrix the phase shift required as shown in Figure 1-2 and Figure 1-3 are with respect to the crossovers and any extra line lengths that can be attributed to the layout of Butler Matrix. For broadband designs, challenge lies in designing phase shifters that can provide a flat phase shift throughout the bandwidth of interest, ie. 2-18 GHz.

Ideally, the power at the output of phase shifter should be equal to the input power, which implies an insertion loss of 0 dB. But due to the finite non-ideal transmission line length, there are dielectric and conductor losses which degrade the insertion loss. These losses are more significant at higher frequencies.
2. HYBRID COUPLER

As described on last section, hybrid coupler is one of the key building blocks for the design of Butler matrix. For this design, the frequency of operation is 2-18 Ghz, hence the an ultra-wideband 90° hybrid coupler is required with 3dB coupling and 90° phase difference between the through and coupled ports over the entire frequency range. The challenge of the design inherently lies in the ultra-wideband operation. Such wide bandwidth of operation is practically not possible with a single section coupler.

2.1. Coupling Structures

When two or more transmission lines are present in close proximity to each other, due to the interaction of electric and magnetic fields, power from one line is coupled to another. Specifically, in a two transmission line system, if the primary transmission line is excited, power couples to the secondary transmission line. The coupled power is a function of the dimensions of the transmission lines, dielectric media, mode of propagation and frequency of operation. Such coupling structures could either be edge coupled, broadside coupled or offset coupled as shown in Figure 2-1. Since microstrip lines lie in the same plane they can only be design as edge coupled.
Figure 2-1 : Coupled transmission lines  (a) edge coupled microstrip,  (b) edge coupled striplines,  (c) broadside coupled striplines,  (d) offset coupled striplines

Closer the two lines, stronger is the interaction between the electromagnetic fields of the two lines, hence higher coupling can be achieved. Similarly, large interfacing area of the lines results in higher coupling. Which results in broadside coupled lines being coupled tighter as compared to edge coupled lines. For edge-coupled transmission lines, practical spacing limitations between the two lines limit the maximum coupling achievable using a single quarter-wave ($\lambda/4$) section to around 8dB. Whereas, a broadside coupled single section of $\lambda/4$ length can result in 3dB or even tighter coupling.

The separation and the dimensions of the two transmission lines can be also be variable throughout the length of the coupled section. If the dimensions of the lines are equal, and they have constant separation between them, such lines are called symmetric and uniformly coupled lines. A structure with varying separation is called non-uniformly
coupled lines and structure with varying widths of lines is called asymmetric coupled lines.

The coupling between the two coupled transmission lines is described in terms of even and odd modes of excitation. In even-mode excitation the two transmission lines are at equal potentials and in odd-mode the lines are at equal but opposite polarity potential. The coupling is defined in terms of characteristic impedances of these two modes. When both lines are excited in phase with equal amplitude (even-mode), the impedance from one line to the ground is the even-mode characteristic impedance \( Z_{0e} \). Similarly, when the lines are excited in opposite phase but equal amplitude (odd-mode), the impedance from one line to ground is the odd-mode characteristic impedance \( Z_{0o} \).

Voltage coupling coefficients of the coupled line structures are expressed in terms of these even and odd-mode characteristic impedances, length of the coupled structures and the effective dielectric constant. For homogenous transmission lines of \( \lambda/4 \) length, the voltage coupling coefficient, \( k \) is

\[
k = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}
\] (2-1)

2.2. Even and Odd Mode Impedances

Let’s consider two coupled transmission lines with common ground, three types of capacitances are associated with such a system as shown in Figure 2-2. \( V_1, Q_1 \) and \( V_2, Q_2 \) are the voltages and charges on the two coupled lines respectively. The relationship between the capacitances, voltages and charges can be written as,

\[
Q_1 = C_{11}V_1 + C_{12}(V_1 - V_2)
\] (2-2)
\[ Q_2 = C_{22}V_2 + C_{12}(V_2 - V_1) \]  \hspace{1cm} (2-3)

\[ V_1, Q_1 \quad \begin{array}{c} \text{C}_{12} \end{array} \quad V_2, Q_2 \]
\[ \text{C}_{11} \quad \begin{array}{c} \text{C}_{12} \end{array} \quad \text{C}_{22} \]

**Figure 2-2 : Representation of capacitances on coupled lines**

For even mode excitation equal amplitude and in phase voltages \((V_1 = V_2 = V_e)\) are applied to the two lines. Due to the symmetrical structure, if equal voltages are applied the charges would also be equal, \(Q_1 = Q_2 = Q_e\). This will result in the two capacitances \(C_{11}\) and \(C_{22}\) to be equal. \(C_{12}\) can be divided into two series capacitances \(2C_{12}\) and a virtual open or magnetic ground is created at the plane of symmetry as shown in **Figure 2-3**. Thus the two series capacitance can be “disconnected”. For even mode, the capacitance of one line in even mode, \(C_e\) can be written as,

\[ C_e = C_{11} = C_{22} = \frac{Q_e}{V_e} \]  \hspace{1cm} (2-4)

\[ V_e, Q_e \quad \begin{array}{c} \text{C}_{12} \end{array} \quad C_{12} \quad \begin{array}{c} \text{C}_{12} \end{array} \quad V_e, Q_e \]
\[ \text{C}_{11} \quad \begin{array}{c} \text{C}_{12} \end{array} \quad \begin{array}{c} \text{C}_{22} \end{array} \quad \text{C}_{22} \]

**Figure 2-3 : Even mode excitation of coupled lines**

For odd mode excitation equal amplitude and opposite phase voltages \((V_1 = -V_2 = V_o)\) are applied to the two lines. Again due to the symmetry, the charges would also be equal and opposite in polarities, \(Q_1 = -Q_2 = Q_o\). \(C_{12}\) can be divided into two series
capacitances $2C_{12}$ and a virtual ground or electric ground is created the plane of symmetry as shown in Figure 2-4. Thus,

$$Q_o = (C_{11} + 2C_{12})V_o$$

(2-5)

or

$$C_o = (C_{11} + 2C_{12}) = \frac{Q_o}{V_o}$$

(2-6)

where $C_o$ is the capacitance of one line for odd mode excitation.

![Figure 2-4: Odd mode excitation in coupled lined](image)

The magnetic wall and electric wall plane of symmetry for even and odd modes respectively make the analysis of calculating capacitances $C_e$ and $C_o$ easier by just considering half the structure.

Characteristic impedance of transmission line is related to capacitance per unit length,

$$Z_0 = \frac{1}{v_p C}$$

(2-7)

Where $v_p$ is the phase velocity,

$$v_p = \frac{c}{\sqrt{\varepsilon_r}}$$

(2-8)

Therefore even and odd mode characteristic impedances are,

$$Z_{0e} = \frac{1}{v_{pe} C_e}$$

(2-9)

and
\[ Z_{0o} = \frac{1}{v_{po} \epsilon_o} \]  

(2-10)

For homogeneous transmission lines such as striplines, the phase velocities of even and odd modes are equal,

\[ v_{pe} = v_{po} = \frac{c}{\sqrt{\epsilon_r}} \]  

(2-11)

For inhomogeneous transmission lines, for instance microstrip transmission lines, the even and odd mode phase velocities are generally unequal,

\[ v_{pe} = \frac{c}{\sqrt{\epsilon_{re}}} \]  

(2-12)

and

\[ v_{po} = \frac{c}{\sqrt{\epsilon_{ro}}} \]  

(2-13)

Where \( \epsilon_{re} \) and \( \epsilon_{ro} \) are effective even and odd mode dielectric constants.

### 2.3. Braodband Couplers

Multiple sections of quarter wave coupled lines can be cascaded, in order to achieve near constant coupling over a wide bandwidth. It involves appropriate selection of even and odd-mode impedances of each section. The relation between even- and odd-mode impedance of each section is related by,

\[ Z_{0}^2 = Z_{0e} Z_{0o} \]  

(2-14)

where \( Z_0 \) is the impedance of terminating ports of the coupler.

A symmetrical broadband coupler has odd number of sections. In such a coupler the \( ith \) section is identical to the \( N+1-ith \) section and thus is symmetric around the middle section. An asymmetric coupler lacks such symmetry and can employ even or odd number of sections. In the case of symmetrical couplers the through and coupled ports are 90° apart in phase, a property required for the design of Butler matrix.
In [15] Cristal and Young tabulated even and odd mode impedances, of each section of multi-section coupler for number of sections, coupling coefficients and fractional bandwidth for symmetrical TEM mode coupled lines. One way to realize the required 90° hybrid coupler could be in microstrip multi-section edge coupled lines. But the amount of coupling required for the middle sections is too high to be physically realizable. For a practical realization of edge coupled microstrip lines, it is very hard to achieve less than 8 dB of coupling since it requires too small spacing between the lines. Moreover, transmission mode in microstrip lines is not purely TEM due to inhomogeneous dielectric (air and the dielectric medium). The phase velocities of odd and even mode are not equal, which leads to phase errors between the through and coupled ports. [16] Employed wiggly lines in order to get equal phase velocities for even- and odd- mode. Due to restriction in achievable coupling requirement of the tightly coupled middle section, edge coupled micro-strip lines are not suitable for this work.

For tighter coupling [17] proposed Lange or interdigitated couplers by increasing the mutual capacitance between the lines. Interdigitated couplers are sensitive to small gaps between the conductors. Also, it requires bond wires which can cause manufacturing issues.

More recently [18] used slot coupled microstrip lines to achieve tighter coupling over 3.1 to 10.6 GHz. It was followed by [19] where three sections were used to achieve even higher bandwidth from 2.3 to 12.3 GHz. Using this design paradigm, a broadband coupler was designed for our application for 2-18 GHz operation. Figure 2-5
shows the single section design and **Figure 2-6** shows its magnitude response.

![Figure 2-5: Slot Coupled Microstrip Coupler](image)

**Figure 2-5**: Slot Coupled Microstrip Coupler

![Figure 2-6: Magnitude response of single section slot coupled microstrip coupler](image)

**Figure 2-6**: Magnitude response of single section slot coupled microstrip coupler
Figure 2-7 shows the designed 3-section coupler followed by its magnitude and phase response in Figure 2-8. Although the phase response of these couplers are decent with error within 5 degrees, but the coupling achieved is not sufficient. It ranges from 1.8 to 5.8 dB, which ideally should have been 3 dB.

Figure 2-7 : 3-Section slot coupled microstrip coupler
Figure 2-8: Magnitude and phase response of 3-section slot coupled microstrip coupler (a) Magnitude Response (b) Phase Response
This design was further extended to 5 sections as shown in Figure 2-9 in order to achieve a tighter coupling, but the desired coupling could not be achieved at higher frequencies as shown in Figure 2-10.

Figure 2-9 : 5-Section slot coupled microstrip coupler

Figure 2-10 : Magnitude Response of 5-section slot coupled microstrip coupler
Another way to design a tight coupler is to connect two loose couplers in tandem. By connecting two 8.34 dB couplers in tandem a tight coupling of 3 dB is achieved. This can be shown using a similar analysis as done in Section 1.3. The S-matrix of first hybrid coupler is,

\[
\begin{bmatrix}
0 & -j \cos \alpha_1 & \sin \alpha_1 & 0 \\
-j \cos \alpha_1 & 0 & 0 & \sin \alpha_1 \\
\sin \alpha_1 & 0 & 0 & -j \cos \alpha_1 \\
0 & \sin \alpha_1 & -j \cos \alpha_1 & 0
\end{bmatrix}(2-15)
\]

And similarly S-matrix of second hybrid coupler is given by

\[
\begin{bmatrix}
0 & -j \cos \alpha_2 & \sin \alpha_2 & 0 \\
-j \cos \alpha_2 & 0 & 0 & \sin \alpha_2 \\
\sin \alpha_2 & 0 & 0 & -j \cos \alpha_2 \\
0 & \sin \alpha_2 & -j \cos \alpha_2 & 0
\end{bmatrix}(2-16)
\]

Where \(\sin \alpha_1\) and \(\sin \alpha_2\) are the voltage coupling coefficients of the two hybrid couplers. When input port of the first coupler is excited by a wave voltage of amplitude one, the reflected voltages obtained using the S-parameter are,

\[
\begin{bmatrix}
V_1^- \\
V_2^- \\
V_3^- \\
V_4^-
\end{bmatrix} = \begin{bmatrix}
0 & -j \cos \alpha_1 & \sin \alpha_1 & 0 \\
-j \cos \alpha_1 & 0 & 0 & \sin \alpha_1 \\
\sin \alpha_1 & 0 & 0 & -j \cos \alpha_1 \\
0 & \sin \alpha_1 & -j \cos \alpha_1 & 0
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}(2-17)
\]

\(V_1^- = 0\)  \hspace{1cm} (2-18)

\(V_2^- = -j \cos \alpha_1\)  \hspace{1cm} (2-19)

\(V_3^- = \sin \alpha_1\)  \hspace{1cm} (2-20)

\(V_4^- = 0\)  \hspace{1cm} (2-21)

For the second hybrid \(V_2^-\) and \(V_3^-\) are the incident voltage waves at port 1 and port 4, and using S-parameters of the second hybrid coupler,
\[
\begin{bmatrix}
V'_{1'}
V'_{2'}
V'_{3'}
V'_{4'}
\end{bmatrix} =
\begin{bmatrix}
0 & -j \cos \alpha_2 & \sin \alpha_2 & 0 \\
-j \cos \alpha_2 & 0 & 0 & \sin \alpha_2 \\
\sin \alpha_2 & 0 & 0 & -j \cos \alpha_2 \\
0 & \sin \alpha_2 & -j \cos \alpha_2 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\sin \alpha_1 \\
0 \\
0 \\
-j \cos \alpha_1 \\
\end{bmatrix}
\]

(2-22)

\[V'_{1'} = 0 \] (2-23)

\[V'_{2'} = -j \cos \alpha_2 \sin \alpha_1 - j \sin \alpha_2 \cos \alpha_1 = -j \sin(\alpha_1 + \alpha_2) \] (2-24)

\[V'_{3'} = \sin \alpha_2 \sin \alpha_1 - \cos \alpha_2 \cos \alpha_1 = -\cos(\alpha_1 + \alpha_2) \] (2-25)

\[V'_{4'} = 0 \] (2-26)

If \( \alpha_1 = \alpha_2 = \frac{\pi}{8} \), \( \sin \left(\frac{\pi}{8}\right) = 0.3827 \) is the voltage coupling coefficient, which equals

\[-20 \log(0.2827) = 8.34 \, dB.\]

Using these values of voltage coupling coefficients,

\[V'_{2'} = -j \sin \left(\frac{\pi}{8} + \frac{\pi}{8}\right) = -j \sin \left(\frac{\pi}{4}\right) = -j\sqrt{\frac{1}{2}} \] (2-27)

\[V'_{3'} = -j \cos \left(\frac{\pi}{8} + \frac{\pi}{8}\right) = -j \cos \left(\frac{\pi}{4}\right) = -\sqrt{\frac{1}{2}} \] (2-28)

which is equivalent to 3 dB coupling in terms of power.

**Figure 2-11** shows a block diagram of such a coupler.

**Figure 2-11 : 3 dB coupler as a tandem connection of two 8.34 dB couplers**
2.4. Coupler Design

For this work, two multi-section 8.34 dB couplers are designed and then connected in tandem as discussed in the last section to achieve 3 dB or the hybrid coupler.

The theoretical design equations and tables given in [15] were used as reference for designing the 8.34 dB coupler. The work in [15] is based on finding an insertion loss function which is unity plus square of an odd polynomial function which follows from the previous work in [20]. Thus the theoretical design of a multi-section coupler reduces to finding the optimum polynomial function followed by extracting the even and odd-mode transmission-line impedances from the polynomial function. Following this analysis, theoretical values of odd and even mode impedances have been tabulated in [15] according to the required coupling coefficient, bandwidth and number of sections.

To minimize the area of design, least number of sections should be used. At the same time ultra-wideband operation must be achieved. The required bandwidth of 2-18 GHz was achieved for 8.34 dB coupler using a symmetric 7-section design having normalized even mode impedances 1.045, 1.122, 1.3014 and 2 Ohms. For a 50 Ohm design, these are multiplied by 50 to get the even mode impedance to be realized. Odd mode impedance can be calculated using [21],

$$Z_0^2 = Z_{0e} Z_{0o}$$  \hspace{1cm} \text{(2-29)}

Where $Z_0 = 50$ Ohms, $Z_{0e}$ and $Z_{0o}$ are even and odd mode impedances respectively.

The voltage coupling coefficient is given by, $\frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$, which is equivalent to
$$20 \log \left( \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \right)$$ in dB. Using these equations even mode impedances for the symmetric 7-section design are found to be 52.27, 56.10, 65.07 and 107.82 Ohms and the odd mode impedances are 47.83, 44.56, 38.42 and 23.19 Ohms. The coupling factors are 27.06 dB, 18.81 dB, 11.78 dB and 3.80 dB.

The 3.80 dB tight coupling is still hard to realize as edge coupled microstrip lines as discussed in previous sections. Hence homogenous stripline structure has been designed which also helps in maintaining equal odd and even mode phase velocities since pure TEM mode propagates. The center section which requires tight coupling of 3.80 dB is designed as broadside coupled since it provides large area for coupling and other six sections are designed as offset coupled. The geometries of broadside and offset coupled striplines were shown in Figure 2-1.

The geometry of each section of stripline is decided according to the coupling required. For broadside coupled striplines the even and odd mode characteristic impedances are given by [22] (assuming negligible strip thickness),

$$Z_{0e} = \frac{60 \pi}{\sqrt{\varepsilon_r}} \frac{K'(k)}{K(k)}$$ (2-30)

$$Z_{0o} = \frac{296.1}{\sqrt{\varepsilon_r} \frac{k}{\tanh^{-1}(k)}}$$ (2-31)

where $K$ is complete elliptic function of the first kind, and $K'$ is the complementary function given by,

$$K'(k) = K(k') = K \left( \sqrt{1 - k^2} \right)$$ (2-32)

where k is related to the dimensions of the structure as follows:

$$\frac{W}{b} = \frac{1}{\pi} \left[ \ln \left( \frac{1+R}{1-R} \right) - \frac{S}{b} \ln \left( \frac{1+R/k}{1-R/k} \right) \right]$$ (2-33)
\[
R = \left[ \left( \frac{k b}{S} - 1 \right) / \frac{a b}{k S} - 1 \right]^{1/2}
\]

(2-34)

where, \( b, W, S \) are shown in Figure 2-1 (c).

For voltage coupling coefficient, \( C \) and terminal characteristic impedance \( Z_0 \),

the odd even mode characteristic impedance is be given by \([19]\),

\[
Z_{0e} = Z_0 \left( \frac{1+C}{1-C} \right)^{1/2}
\]

(2-35)

\[
Z_{0o} = Z_0 \left( \frac{1-C}{1+C} \right)^{1/2}
\]

(2-36)

and,

\[
\frac{K}{K'} = \frac{60\pi}{Z_0\sqrt{e_r}} \left( \frac{1-C}{1+C} \right)^{1/2}
\]

(2-37)

[23] presented formulas for \( K/K' \) can be used to calculate \( k \) for given coupling and

substrate.

\[
\frac{K}{K'} = \frac{1}{\pi} \ln \left( 2 \frac{1+\sqrt{K}}{1-\sqrt{K}} \right) \quad \text{for } 0.707 < k < 1
\]

(2-38)

\[
\frac{K}{K'} = \frac{\pi}{\ln(2 \frac{1+\sqrt{K}}{1-\sqrt{K}})} \quad \text{for } 0.0 < k < 0.707
\]

(2-39)

For \( K/K' > 1 \) or \( C < (1.0 - P e_r)/(1.0 + P e_r) \), \( k \) is expressed as,

\[
k = \left( \frac{0.5 e^{(K'/K)} - 1}{0.5 e^{(K'/K)} - 1} \right)^2
\]

(2-40)

And for the case of \( K/K' < 1 \) or \( C > (1.0 - P e_r)/(1.0 + P e_r) \), \( k \) is expressed as,

\[
k = \left[ 1 - \left( \frac{0.5 e^{(K'/K)} - 1}{0.5 e^{(K'/K)} - 1} \right)^4 \right]^{1/2}
\]

(2-41)

where \( P = (Z_0/60\pi)^2 \) and in the geometry the ratio,

\[
\frac{S}{b} = 0.0017 Z_0\sqrt{e_r} \left( \frac{1-C}{1+C} \right)^{1/2} \ln \frac{1+k}{1-k}
\]

(2-42)
These elliptical integrals can also be evaluated in Python. For given $C$, $\varepsilon_r$ and $Z_0$ the geometry ratios $W/b$ and $S/b$ can be calculated using these equations. Stripline calculators can also be used to instead of solving these equations.

Analysis for offset coupled lines has been given in [24], which is skipped because modern stripline calculators can be used to find the geometries of the coupled lines according to the coupling requirement or the even and odd mode impedances.

Although the tables and equations can theoretically design any coupler but for a physically realizable structure the spacing ‘$s$’ in Figure 2-1 should be uniform throughout for each section and at the same time the widths ‘$w$’ and spacing ‘$w_c$’ should not be too large or too small. Such constraints were satisfied by diligently selecting substrate type and designing the stripline structure. Duroid 4003 ($\varepsilon_r = 3.55$) is used as substrate in this work which satisfied the above constraints. Using equations mentioned in this section, even odd mode impedances are calculated for each section and using the equations or calculators the geometry of each section was calculated for Duroid 4003.

Figure 2-12 shows the designed 7-section symmetric 8.34 dB coupler, with center section as broadside coupled and the 6 sections as offset coupled striplines. Magnitude and phase response is shown in Figure 2-14. Figure 2-13 highlights the generic stripline structure that will be followed for all the further designs discussed in this thesis.
Figure 2-12 : 7-section 8.34 dB stripline coupler (a) Top view (b) Trimetric view

Figure 2-13 : Geometry of stripline structures
Figure 2-14: Magnitude and phase response of 7-section 8.34 dB coupler (a) Magnitude response (b) Phase response
As discussed in the last subsection, two 8.34 dB couplers connected in tandem result in a 3 dB coupler. **Figure 2-15** is the designed 3 dB coupler by connecting the above 8.34 dB couplers.

**Figure 2-15**: 3 dB coupler as a tandem connection of two 8.34 dB couplers (a) Top view (b) Trimetric view
Figure 2-16: Magnitude and phase response of 3 dB coupler
(a) Magnitude response (b) Phase response
**Figure 2-16 (a)** shows it achieves tight coupling varying from 2.5 dB to 3.6 dB across 2-18 GHz range. **Figure 2-16 (b)** shows the phase difference between the through and coupled ports which is 90° with an error of ±3°.

Similar coupler was designed in [25], with 41 sections (97 mm x 46 mm) for 0.5-18 Ghz. The simulated results in [25] show coupling lowers as the frequency increases, with 5 dB coupling at 18 GHz. In this work tight coupling (2.6 dB to 3.7 dB) is achieved for 2-18 GHz with much smaller size (31.78 mm x 9.78 mm).
3. CROSSOVER DESIGN

This brief section covers the design of crossover required for Butler Matrix. As discussed in Section 1.3 crossover is designed by connecting two 3 dB couplers in tandem. Figure 3-1 shows such a crossover design in HFSS. Extra line lengths are added to assist in layout of Butler Matrix later.

Although the physical size of the crossover is small, but the electrical size is too large considering the highest frequency of operation is 18 GHz. Moreover the simulations need to run from 2 to 18 GHz to analyze the design over the full bandwidth. For running finite element method simulations in HFSS, such a design requires large computational resources. Therefore these simulations were run on supercomputing cluster, Texas A&M High Performance Research Computing (HPRC).

Figure 3-2 shows its magnitude response. Ideally all the input power should be present at output but due to long length of transmission lines we see maximum insertion loss of -2.6 dB.
Figure 3-1: Crossover design as a tandem connection of two 3 dB couplers (a) Top View (b) Trimetric View
Figure 3-2: Crossover magnitude response
4. PHASE SHIFTER DESIGN

As discussed in Section 1.1 a 4x4 Butler Matrix requires 45 degree phase shifters. For a narrow band design such a phase shift was easily obtained by designing a transmission line which is electrically 45 degrees longer than a reference line. Although it is a quick way to achieve phase shift, it cannot provide a flat phase shift over a very broad range of frequencies.

To achieve a flat phase shift over a wide bandwidth, a class of differential phase shifter called Schiffman phase shifter [26] was designed. These are four port circuits and provide a constant differential phase shift across the two output ports over a frequency range. Schiffman proposed a design of phase shifter shown in Figure 4-1, which consists of two TEM transmission lines. One of the lines is the reference line and the other is a pair of parallel coupled transmission lines connected at one end. Length of this connection is kept as small as possible. This pair of parallel coupled lines is also called a single C-section. The parallel coupled lines are each a quarter-wavelength long at the center/design frequency. The reference line is just a straight TEM line, which in the case of Butler Matrix would be the crossover discussed in last section.

![Figure 4-1: Schiffman phase shifter](image)

Figure 4-1: Schiffman phase shifter
Following expressions given by [21] determine the characteristic impedance and phase response of the Schiffman coupled section, in terms of even and odd mode impedances and the electrical line length,

\[ Z_0^2 = Z_{0e} Z_{0o} \]  

\[ \cos \phi = \frac{\rho - \tan^2 \theta}{\rho + \tan^2 \theta} \]  

where,

\[ \rho = \frac{z_{0e}}{z_{0o}} \]

and \( \theta \) is the electrical length of the each of the coupled line.

The parameter \( \rho \) is also related to coupling C, in dB,

\[ C = -20 \log \left( \frac{\rho - 1}{\rho + 1} \right) \]

The total phase shift is equal to the phase difference (\( \Delta \phi \)) between the reference line and the single C-section coupled lines. Thus,

\[ \Delta \phi = K\theta - \cos^{-1} \left( \frac{\rho - \tan^2 \theta}{\rho + \tan^2 \theta} \right) \]

\( K\theta \) is the transmission phase of the reference line.

There are a number of design parameters such as \( K, \theta, Z_{0e}, Z_{0o} \). Further, the product of even and odd mode impedances and their ratio \( \rho \) can be fixed independently.

Thus the characteristic impedance \( Z_0 \) of the coupled line network can be specified independent of phase response \( \phi \). Once the desired phase response \( \phi \), or the total phase shift \( \Delta \phi \), is specified the Schiffman phase shifter can be designed with different values of \( Z_{0e}, Z_{0o} \) and \( \theta \). It has been shown in [27] that if at the center frequency, length of each of
the coupled line equals quarter wavelength or $\theta = \pi/2$, then the phase shift $\Delta \phi$ is anti-symmetric around the center frequency, which leads to the broadest bandwidth.

**Figure 4-2** shows the phase shift for 90 degree Schiffman phase shifters designed using different values odd, even mode impedances and coupling values while keeping the product, $Z_0^2 = Z_{0e} Z_{0o}$ constant and equal to 50 Ohms. Different values of the ratio $\rho$ yields varying bandwidths. For the designs with higher bandwidth the deviation in phase response or the error is also higher. A bandwidth of 1.95:1 was reported in [26] with a phase shift of $90 \pm 2.5$ degrees for $\rho = 2.7$. Bandwidth of 2.34:1 was achieved for phase shift of $90 \pm 4.8$ degrees.

- Figure 4-2 : Phase response of single section 90° Schiffman phase shifters

For ultra-wideband design of the Butler matrix, bandwidth requirement is 9:1 (2-18GHz). The single C-section Schiffman phase shifter can be extended to multiple
sections of coupled lines to increase bandwidth over which the phase shift is flat.

Analysis for this was done in [28]. The equations extend from the phase response of the single section. For ‘n’ sections of coupled line in a Schiffman phase shifter the phase response is given by,

\[ \cos \phi_n = \frac{\rho_1 - \tan^2 \theta_1}{\rho_1 + \tan^2 \theta_1} \]  \hspace{1cm} (4-6)

where,

\[ \theta'_1 = \theta_1 + \tan^{-1}(\rho_{12} \tan \theta'_2) \]  \hspace{1cm} (4-7)

\[ \theta'_2 = \theta_2 + \tan^{-1}(\rho_{23} \tan \theta'_3) \]  \hspace{1cm} (4-8)

\[ \theta'_{n-1} = \theta_{n-1} + \tan^{-1}(\rho_{n-1,n} \tan \theta'_{n}) \]  \hspace{1cm} (4-9)

\[ \theta'_{n} = \theta_{n} \] \hspace{1cm} (4-10)

and

\[ \rho_{i,i+1} = \frac{Z_{0e \ i}}{Z_{0e \ (i+1)}} = \frac{Z_{0o \ (i+1)}}{Z_{0o \ i}} \] \hspace{1cm} (4-11)

While \( Z_0^2 = Z_{0e \ i} Z_{0o \ i} \) is true for each section.

As in the case of multi-section hybrid coupler, the equations result in a theoretical design which may or may not be physically realizable. Using the same substrate as before Duroid 4003 (\( \varepsilon_r = 3.55 \)), a 5-section 45 degree phase shifter was designed diligently to provide a constant phase shift over 2 to 18 GHz. Figure 4-3 shows the initial design of 5-section Schiffman phase shifter. A straight transmission line is used as reference. To shorten the simulation time, the straight line was designed to be short but extra length added to it in terms of de-embed as shown in Figure 4-44. Figure 4-5 shows the constant 45 degree phase shift with an error of \( \pm 5^\circ \).
Figure 4-3 : 5-section Schiffman phase shifter with a straight reference line

Figure 4-4 : Schiffman phase shifter with reference line showing the de-embed
In the Butler matrix design the reference line is not a straight line, rather it’s the crossover and any extra line lengths that are added due to the layout. Therefore the phase shifter design was modified accordingly and added extra lengths were added to match the slope of phase response of the crossover. Figure 4-66 shows the design of such a phase shifter with crossover. The phase shift is 45 degrees with maximum error of ±7° as shown in Figure 4-77.

A 6-section phase shifter was also designed to flatten the phase shift even more and hence reduce the error. The phase response of such a phase shifter is shown in Figure 4-88 it didn’t significantly reduce the phase error, but there was an improvement of 1°.
Figure 4-6: Phase shifter with extra lengths to compensate for crossover phase response
Figure 4-7: Phase response of 5-section 45 degrees Schiffman phase shifter with respect to crossover.

Max. phase error is ±7 degrees

Figure 4-8: Phase response of 6-section 45 degrees Schiffman phase shifter with respect to crossover.

Max. phase error is ±6 degrees
Using the same design procedure two more phase-shifters were designed. These are 22.5 degrees and 67.5 degrees phase shifters for the design of 8-input, 8-output (8x8) Butler matrix. A schematic of 8x8 Butler matrix was shown in Figure 1-3. Phase response of these phase shifters are shown in Figure 4-9 and Figure 4-10. As with the case of 45 degree phase shifter, there are some errors amounting upto ±8°.

**Figure 4-9**: Phase response of 6-section 67.5 degrees Schiffman phase shifter with respect to crossover
Figure 4-10: Phase response of 6-section degrees Schiffman phase shifter with respect to crossover
5. 4X4 BUTLER MATRIX DESIGN AND RESULTS

The previous three sections described the design and results of the three building blocks of the ultra-wideband butler matrix, hybrid coupler, crossover and a phase shifter. Comparing the block diagram of Figure 1-2 and the design in Figure 4-6 it’s only a matter of adding the four hybrid couplers to get to the design of Butler matrix. Figure 5-1 and Figure 5-2 show the complete layout/design of ultra-wideband 4-input, 4-output Butler Matrix. It uses the 3-dB coupler designed as a tandem connection of two 7-section 8.34 dB hybrid couplers, the crossover and 6-section 45 degrees phase shifters. It follows the same general structure as shown in Figure 2-13.
Figure 5-1: Layout of 4x4 ultra-wideband Butler matrix (Top View)
Figure 5-2: Layout of ultra-wideband Butler matrix (Trimetric view)

Figure 5-3 and Figure 5-4 show the magnitude response with input at Port 1 and Port 2 respectively. Power is equally distributed among all the output ports. The insertion loss is more at higher frequencies. Due to long lengths of transmission lines from each input to the output port we see a high insertion loss. Phase response is shown in Figure 5-5 and Figure 5-6. With respect to Port 1 we see a progressive phase shift of 45 degrees and 135 degrees with respect to Port 2. The phase errors have accumulated in the complete design and as a result, at a few frequency points the errors reach ±15°. But the phase shifts are centered at 45 degrees and 135 degrees for the entire frequency range from 2-18 GHz. Since the Butler matrix is symmetrical and the layout is also symmetrical the magnitude and phase response for Port 3 and Port 4 are also similar, with progressive phase shifts of -135 degrees and -45 degrees respectively.
Figure 5-3: Magnitude Response of Butler matrix with respect to Port 1

Figure 5-4: Magnitude response of Butler matrix with respect to Port 2
Figure 5-5: Butler Matrix progressive phase shifts of 45° with respect to Port 1
Figure 5-6: Butler Matrix progressive phase shifts of 135° with respect to Port 2
6. FUTURE WORK

6.1. TRL Calibration Kit

Once fabricated, the above designed circuits would require de-embedding techniques for measurement with VNA. This subsection covers the design of TRL (Through – Reflect - Line) calibration kit for measurement across 2-18 GHz. Only two-port kits have been designed. Through is designed as just back to back connection of the ports shown in Figure 6-1. Reflect standard can be designed as open or short. In this case it has been designed as short, by connecting the stripline with the ground planes at top and bottom as shown in Figure 6-2.

Since the circuits in this work are ultra-wideband (2-18 GHz), having a bandwidth ratio of 9:1, two Line standards have been designed. One has length equal to quarter wavelength at 6 GHz and the other at 14 GHz as shown in Figure 6-3.

![Figure 6-1: Through standard](image)
Figure 6-2: Reflect standard

Figure 6-3: Line standards
(a) Quarter-wavelength long at 6 GHz (b) Quarter-wavelength long at 14 GHz
For verification, that the designed TRL calibration kit works, two-port magnitude data of the designed 3 dB coupler is compared with and without the TRL kit. This can be done quickly in Python using scikit-rf library. As can be seen from Figure 6-44 and Figure 6-55, the application of calibration kit does not affect the two-port response of the coupler appreciably. Hence the designed calibration kit can be used for measurement.

![Two-port response of hybrid coupler without calibration kit](image)

**Figure 6-4 : Two-port response of hybrid coupler without calibration kit**
6.2. 8X8 Butler Matrix

The block diagram of 8-input, 8-output (8x8) Butler matrix was shown in Figure 1-3. Each of the building blocks of the 8x8 Butler matrix, i.e., hybrid coupler, crossover, 22.5 degrees and 67.5 degrees phase shifter has been designed in this work. Unfortunately the full layout of 8x8 Butler matrix is electrically too large, which make its analysis a long process. For instance one instance of FEM simulation for the 4x4 Butler matrix takes around a week to run. An 8x8 Butler matrix being electrically large would take even more time for a single simulation.

Another task before moving onto the design of 8x8 Butler matrix is to reduce the phase errors. As seen in the case of 4x4 Butler matrix the phase errors accumulate resulting in too large phase errors across the required bandwidth. Since the smallest
progressive phase shift for an 8x8 Butler matrix is 22.5 degrees, phase errors need to be reduced in the full design.

6.3. 16X16 Butler Matrix

The layout of 16-input, 16-output Butler matrix can get too tedious if same approach is followed as for 4x4 and 8x8 Butler matrices. Moreover the board size would be too large. Another way to design 16x16 Butler matrix is to use a row card configuration as shown schematically in Figure 6-6. Each board is a 4x4 Butler matrix. Figure 6-7 shows how this configuration would look like with the actual design.

Inputs are on the side where each 4x4 board is oriented horizontally and outputs are on the vertical side. The first output port of each of the 4x4 horizontal board is connected to the 4 inputs of the first 4x4 vertical board. Similarly the second output port of each of the horizontal board is connected to the inputs of the second vertical board and so on.
Figure 6-6: Schematic view of row card configuration of 16x16 Butler matrix from 4x4 Butler matrix

Figure 6-7: Designed 4x4 Butler matrix stacked in row-card configuration to form 16x16 Butler matrix
An ideal circuit simulation was run for such a design of 16x16 Butler Matrix as shown in Figure 6-8. Magnitude response with respect to Port 1 in Figure 6-9 shows equal distribution of power and the phase response with respect to Port 1 in Figure 6-10 shows constant progressive phase shift of 11.25 degrees.

**Figure 6-8**: Circuit schematic of 16x16 Butler matrix from 4x4 Butler matrices
Figure 6-9: Ideal magnitude response of 16x16 Butler matrix designed as row card configuration of 4x4 Butler matrix

Figure 6-10: Ideal phase response of 16x16 Butler matrix designed as row card configuration of 4x4 Butler matrix
6.4. 64X64 Butler Matrix

A 64-input, 64-output Butler matrix can be designed following similar design paradigm as in last sub-section. Building blocks for this Butler matrix would be the 8x8 Butler matrix. A schematic construction similar to 16x16 Butler matrix is shown in Figure 6-11.

Figure 6-11 : Schematic view of row card configuration of 64x64 Butler matrix from 8x8 Butler matrix
Design of ultra-wideband hybrid coupler, crossover and phase shifters has been presented in this thesis, as a requirement for the design of ultra-wideband Butler matrix. Hybrid coupler achieves tight coupling of 2.6 dB to 3.7 dB across the required bandwidth of 2-18 GHz with a phase error of ±3 degrees between the coupled and through port. The hybrid coupler is extremely small in terms of physical size (3.1mm x 9.8 mm) as compared to previously designed coupler for such an ultra-wideband range. The phase shifter has been designed as a multi-section Schiffman phase shifter which was shown to have a constant phase shift of 45 degrees across 2-18 GHz with phase error of ±5 degrees.

Using the designed hybrid coupler, crossover and phase shifter an ultra-wideband 4-input, 4-output (4x4) Butler matrix has been designed and phase response shows progressive phase shifts.

This work also lays down the foundation for the designs of ultra-wideband 8x8, 16x16 and 64x64 Butler matrices.
REFERENCES


