

REDUCING THE PEAK-TO-AVERAGE POWER RATIO IN OFDM BY USING NEURAL  
NETWORK

A Thesis

by

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## ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is an approach for realizing high speed transmission of data. The OFDM modulation scheme has the advantage of providing high spectral efficiency in multicarrier transmission of high data rate. One disadvantage of OFDM is that it has a high Peak-to-Average Power Ratio (PAPR) which leads to stricter linearity requirements for power amplifiers. This necessary increase in the linear range of the power amplifier results in low power efficiency. Many PAPR reduction techniques use optimization algorithms to iteratively find a locally optimal point. In this project, we focus on the task of training neural networks which can produce signals with close performance to PAPR reduction methods using convex optimization. Also, we use unsupervised learning to train neural networks to find its own algorithm to achieve PAPR reduction.

## CONTRIBUTORS AND FUNDING SOURCES

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## 1. INTRODUCTION

With the development of modern communication techniques, Orthogonal Frequency Division Multiplexing (OFDM) becomes an attractive multi-carrier modulation technology that satisfies the requirement of high quality wireless communication. By dividing the channel into several orthogonal sub-channels, it reduces inter-symbol interference (ISI), provides immunity to frequency selective fading and improves Quality of Service. So far, OFDM has been widely implemented in 4G (fourth generation) mobile Communication, Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), wireless networks and World wide Interoperability for Microwave Access (WiMAX) [1, 2].

However, an OFDM system produces a high Peak-to-Average Power Ratio (PAPR) of the transmitted signal. High PAPR values require a large linear range for power amplifiers. There are many techniques for PAPR reduction in OFDM systems. Clipping and filtering[3, 4], peak insertion (PI)[5], selective mapping (SLM)[6, 7], partial transmit sequence (PTS)[8, 9], linear block coding (LBC)[10] are some of the many algorithms used for PAPR reduction at transmitter. However, those techniques still suffer from problems. In particular, it is expensive computationally for them to perform an exhaustively search to find the optimal solution, especially when the number of sub-carriers increases. The search complexity increases rapidly with increased number of sub-carriers [11].

To overcome those disadvantages, in this project, we focus on the task of training a neural network that requires less computational complexity to perform peak suppression. First, we train the neural network with a data set of PAPR suppressed signal generated by convex optimization technique. Then, we use unsupervised learning and train the neural network with randomly generated input signal and a self-defined cost function.

In Section 2, we provide detailed introduction about OFDM systems, and give an example with a specific number of input data. In Section 3, we compare our method with other neural network based PAPR reduction methods. The proposed research is shown in Section 4, we introduce the

training data used for neural network training and we describe how the network model is applied in PAPR reduction including network structure and training procedure. Experiments and simulation results are shown in Section 5.



## 2. OFDM SYSTEM AND PAPR PROBLEM

In this chapter we briefly introduce an OFDM modulation system and its relationship with fast Fourier transform(FFT) and inverse fast Fourier transform(IFFT). We also give the mathematical formula for PAPR.

### 2.1 OFDM System

In an OFDM system, as shown below, a number of  $n$  input data are first modulated. For example, we use Quadrature Phase Shift Keying (QPSK) modulated signal. We use  $n$  orthogonal sub-carriers as each input data is modulated by a sub-carrier. All those sub-carriers are mutually independent. Then all modulated signals are combined together for transmission. Because the sub-carriers are orthogonal, the transmitted data can be separated and demodulated at the receiver without distortion. The procedure including modulation by sub-carriers and combining can be replaced with an IFFT, as well as the procedure of separating and restoring can be replaced by an FFT, which greatly simplifies the system.

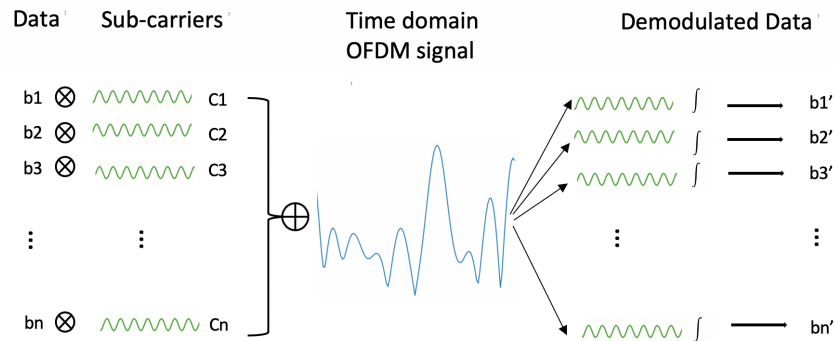


Figure 2.1: OFDM System.

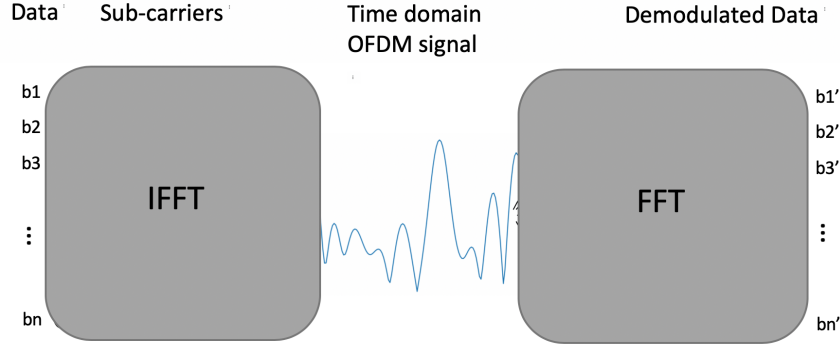


Figure 2.2: IFFT and FFT in OFDM System.

To clearly understand the transformation from the original input data to the combined OFDM signal, suppose we have a OFDM signal consisting of  $N$  sub-carriers with a frequency interval of  $\Delta f$ . The bandwidth  $B$  of the entire system is divided into  $N$  equally spaced sub-channels. Within one symbol duration length  $\frac{1}{\Delta f}$ , all sub-carriers are orthogonal to each other. The original input data are first modulated (e.g., QPSK), and the combination is achieved by IFFT.

Suppose we have a set of data

$$X = [X_0, X_1, \dots, X_{N-1}]^T.$$

The duration for each symbol from each sub-carrier is  $T$ . Then the OFDM block for the set of input is represented by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp j2\pi n \Delta f t, \text{ for } 0 \leq t \leq NT.$$

## 2.2 PAPR

Peak-to-average power ratio (PAPR) is a time domain parameter, which is defined as the maximum power of an transmitted OFDM signal divided by its average power. In an OFDM system, the transmitted signal is the sum of signals each modulated with a sub-carrier. In some cases, the peak

power of the transmitted signal will be much higher than its average power. This requires the high power amplifier (HPA) at the transmitter have a large linear range, resulting in a low efficiency of the HPA. As a power ratio, PAPR is normally expressed in decibels (dB).

$$PAPR = \frac{\max |x(n)|^2}{E[|x(n)|^2]}.$$

### 3. NEURAL NETWORK BASED PAPR REDUCTION APPROACHES

PAPR reduction has been a main challenge for OFDM systems in the field of communications for a long time. There are many techniques proposed to solve the problem, most of them are divided into three groups. (1) Clipping and filtering.[3, 4] (2) Coding, like linear block coding (LBC), Golay Sequences and Reed-Muller Codes.[10, 12] (3) probabilistic techniques, like Selected Mapping (SLM)[6, 7], Partial Transmit Sequence (PTS)[8, 9], Tone Reservation (TR)[13, 14], Tone Injection (TI) [13]and Active Constellation Expansion (ACE)[15].

However, there is no technique that is fully effective in reducing PAPR, they all have different advantage and disadvantages. In the industry, the clipping and filtering technique is often used as one of the simplest. Although this method is simple, it increases the bit error rate of the system and introduces out-of-band noise. Coding techniques obtain the best PAPR reduction, but their decoding complexity are usually high. They are only applicable to the case where the number of sub-carriers is small. Probabilistic techniques do not focus on reducing the maximum value of the signal amplitude, but rather reduce the probability of peak occurrence. They also require high computational complexity and memory, as it's hard to find the optimal combination by exhaustive searching. SLM techniques require multiple IFFT operations. PTS techniques suffer an exponentially increase of complexity with the number of OFDM signal blocks in order to obtain an optimal phase rotation factor. And both methods need to send side information. For TR technique, the selection of the optimal cutting tone is difficult in order to obtain the optimal peak reduction. And it is also hard for TI technology to select the optimal signal constellation representation.

In recent years, few attempts related to neural networks have been made as machine learning technology has benefited human life and realized great application in many areas. Neural network is a useful way to decrease computation complexity and achieve good result when combining with some traditional PAPR reduction techniques. So far the idea has been used in many works relating to TR approach[16], clipping and filtering technique[17], SLM technique[18, 1], TI scheme[19] and ACE[20].

In[16], a network structure is proposed based on PRC scheme, a kind of TR technique. The TR technique [60] is a PAPR reduction method without distortion. This method uses a part of the sub-carriers called Peak Reduction Tones (PRT) to generate a peak-reduced signal, and this part of the sub-carriers does not carry any data information. These Peak Reduction Tones are added to the original OFDM signal to produce a new signal which has a lower PAPR value than the original OFDM signal. The PAPR reduction performance of the TR method is mainly determined by the selection of the PRT set. PRC scheme reserves a number of tones and reduces PAPR by optimizing phases of pilot tones which are not used for carrying data. The network in [16] consists of several sub-nets, each with the same number of pilot tones. Every sub-net shares the same structure that contains one input layer, one hidden layer and a single output node. The neural network is pretrained by large enough training data to establish the relation between OFDM symbol and suitable pilot phases. Produced pilot phases are combined with data tones. After doing zero padding and IFFT, we can get the final transmitted data with a lower PAPR. It is shown that the network performs a satisfactory result that reduces the complementary cumulative density function (CCDF) of PAPR for  $2.4\text{dB}$  at a probability of  $10^{-4}$  with one hidden layer consisting of 60 neurons and 10000 training data.

In[17], a simplified clipping and filtering (SCF) technique and artificial neural network based method is proposed. SCF is a novel iterative clipping and filtering (ICF) algorithm[21]. By adding a frequency domain clipping operation, SCF obtains the same PAPR with less iteration time. The proposed network establishes the map between the input as the time domain OFDM signal and the output which is the time domain SCF signal. There are two modules that can generate real part and imaginary part of the SCF signal separately. Each network model is based on a multiple-layer perceptron (MLP) neural network and has two hidden layers with two neurons in the first layer and one in the second. For the proposed scheme, it shows a close performance of PAPR reduction with  $0.1\text{dB}$  loss and a  $1.5\text{dB}$  performance loss on bit-to-error ratio (BER) compared to SCF technique.

The authors in [18, 1] proposed SLM based PAPR reduction algorithms using neural network.

The basic idea of the SLM method [6, 7] is to multiply the original statistical signal by several statistically independent vectors to obtain several transformed signals having the same information as the original one. Then performing IFFT for each multiplied signal to obtain several corresponding time domain signals. The one with the smallest PAPR is selected for transmission. The probability of a PAPR exceeding a given threshold is greatly reduced by using the SLM method. The SLM scheme is simple but the computational complexity increases when the number of vectors increases. And the receiver needs side information to correctly decode. In [18], the scheme is proposed as two neural network (NN) units employed at both the transmitter and the receiver side. The transmitter NN is pretrained with the input of the time domain OFDM signals and with the output of the time domain selected SLM signals. The receiver NN is pretrained with the input of the time domain transmitted signals and with the output of the time domain OFDM signals. Each unit is based on a MLP neural network. The authors in [1] propose a novel framework based on NN and SLM with a kernel called modified Novel Kernel Based – Radial Basis Function (MNKB-RBF). Radial Basis Function (RBF) is an activation function for NN and is often used for solving optimization problems. Conventional RBF is represented as the Euclidean distance between the input and the centers of every neuron [1]. To better separate features, the proposed MNKB-RBF kernel combines the Euclidean RBF and cosines RBF linearly with adaptive weights. The framework based on MNKB-RBF and SLM block is proved to obtain a higher probability of selecting carriers with low PAPR than NKB-RBF based technique in most cases.

In [20], the NN based model is proposed based on ACE. In ACE technique, some external signal constellation points in the data block expand substantially to the outside of the original constellation to reduce the PAPR of the data block. For QPSK modulation, there are four possible constellation points in each sub-carrier, which are located in each quadrant of the complex plane and are equidistant from the real axis and the imaginary axis. To maintain the BER performance, constellation points are allowed a little adjustment. ACE technique can reduce PAPR by doing time domain clipping and frequency domain constellation point extension iteratively. The proposed model replaces conventional ACE algorithm by NN structure to overcome large computation

complexity of discrete Fourier transform (DFT). The neural network (NN) units are employed at both the transmitter and the receiver side. The transmitter NN is pretrained with the input of the time domain OFDM signals and with the output of the time domain selected ACE signals. The receiver NN is pretrained with the input of the time domain transmitted signals and with the output of the time domain OFDM signals. With a structure of two hidden layers with each layer of two neurons, the proposed approach achieves low computation complexity and 3dB PAPR reduction, which is only 0.75dB higher than ACE scheme. Also, the BER performance is proved to be better than ACE scheme.

All of those NN based PAPR reducing models learn the relationship between original time domain OFDM signal and optimized time domain signal and improve the conventional technique with low computation complexity. Trained with a collection of data set generated by the conventional approaches, those NN can achieve a satisfactory PAPR and BER performance close to the conventional one. However, the performance mostly depends on training data set, as those are used to renew the parameters of the NN.

In this research, it is proposed to train the neural network that learn the relationship between original frequency domain OFDM signal and the PAPR suppressed frequency domain signal with a given input-output data set. Furthermore, with a self-defined renewing strategy, the neural network is supposed to learn the relation between the OFDM signal and its PAPR independently.

## 4. PROPOSED RESEARCH

In this chapter, we will present specific discussions on how we are going to train the neural network to reduce PAPR for supervised learning and unsupervised learning, including the network structure and the training procedure.

### 4.1 Dataset

We select Quadrature Phase Shift Keying(QPSK) modulated data for training. QPSK is a form of quadratic modulation that every two bits (00, 01, 10, 11) are translated into one complex number.

Input signal:	Modulated signal:	Signal phase
00	$0.7071 + 0.7071j$	$\pi/4$
01	$-0.7071 + 0.7071j$	$3\pi/4$
10	$0.7071 - 0.7071j$	$7\pi/4$
11	$-0.7071 - 0.7071j$	$5\pi/4$

Figure 4.1: Transforming Format of QPSK Modulation.

As shown in Figure 4.2, for each symbol in each sub-carrier there are four possible constellation points. Each point in the complex plane equals to  $x \in \pm 0.7071 \pm 0.7071j$ . We have thousands of examples that suppressed PAPR to a lower value by using convex optimization technique. We are going to train the neural network that learn the relationship between the original frequency domain OFDM signal and the PAPR reduced frequency domain optimized signal. Finally, we will obtain a neural network(NN) that find the algorithm between PAPR reduced signal and the original OFDM signal without a given training data set.

Figure 4.2 shows the constellation plot for the original frequency domain OFDM symbol.



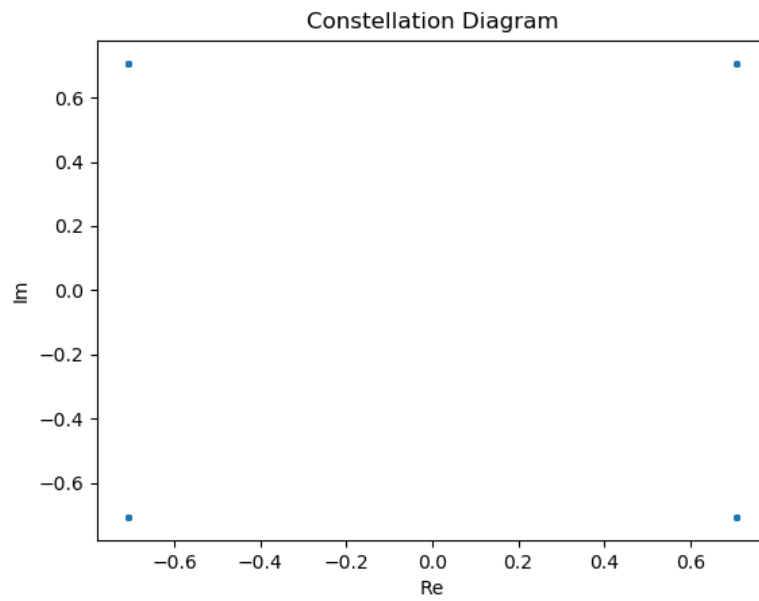


Figure 4.2: Constellation Plot of Original QPSK Modulated Data.

Figure 4.3 shows the constellation plot for the PAPR reduced frequency domain optimized OFDM signal.

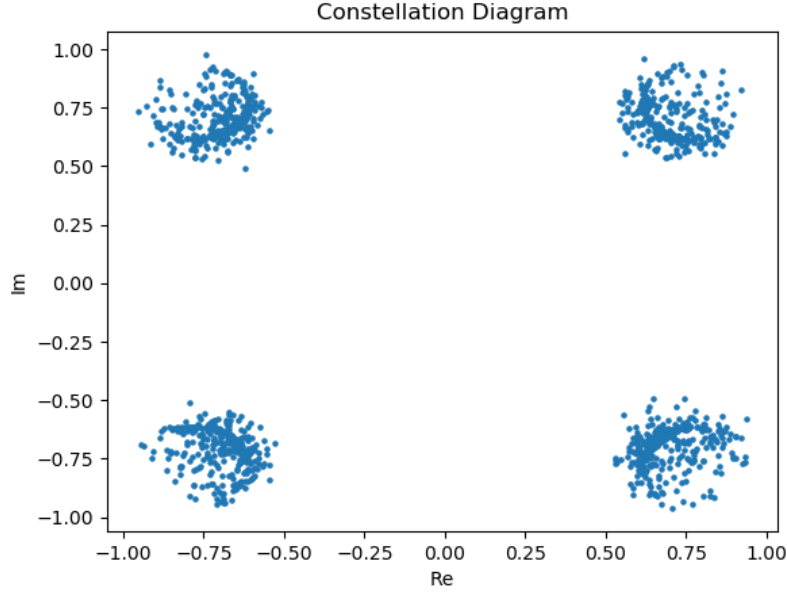


Figure 4.3: Constellation Plot of PAPR Suppressed QPSK Modulated Data.

## 4.2 Network Structure

The OFDM symbols are QPSK modulated with different number of sub-carriers. For different number of sub-carriers, we need to train different networks to fit the number of input and output neurons. For example, an OFDM signal with 12 sub-carriers contains 12 complex numbers, we need 24 neurons for the input layer to express real and imaginary part separately. The same as the output layer. Hidden layers can have many different sets of combinations and different structures.

PAPR is a time domain feature for OFDM signal. In order to make it easier for the neural network to understand features of signal in time domain, we use hidden layers to mimic fast Fourier transform algorithm. The IFFT expression below shows an example for OFDM signal with  $N$  sub-carriers.

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_N \exp j2\pi n \Delta f t, \text{ for } 0 \leq t \leq NT$$

Both  $X_N$  and  $x(t)$  are complex numbers, we need to consider both real and imaginary parts.

We can expand the equation as following,

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \{(Re[X_N]Re[\exp j2\pi n\Delta ft] - Im[X_N]Im[\exp j2\pi n\Delta ft]) + j(Re[X_N]Im[\exp j2\pi n\Delta ft] + Im[X_N]Re[\exp j2\pi n\Delta ft])\}, \text{ for } 0 \leq t \leq NT$$

Instead of separating real part and imaginary part and operate two NN units, we double the number of neurons to express one complex number with two neurons in order to simplify NN module. In this case, we use  $2N$  as the number of input neurons. With a oversampling factor  $T = 4$ , the original signal is extened by  $N(T - 1)$  zeros. The matrix expression of IFFT is shown as

$$\begin{pmatrix} x_0 \\ \vdots \\ x_{NT} \end{pmatrix} = \begin{pmatrix} W_N^0 & \cdots & W_N^0 \\ \vdots & \ddots & \vdots \\ W_N^0 & \cdots & W_N^{-(NT-1)(N-1)} \end{pmatrix} \begin{pmatrix} X_1 \\ \vdots \\ X_N \end{pmatrix} \quad (4.1)$$

where  $W_N = \exp j2\pi n\Delta ft$ . The expression can be simplified as  $\vec{x} = W_N \vec{X}$ .

We use a fully connected neural network to mimic the map of IFFT by setting the first hidden layer with  $2NT$  neurons. The proposed neural network structure is shown as below.

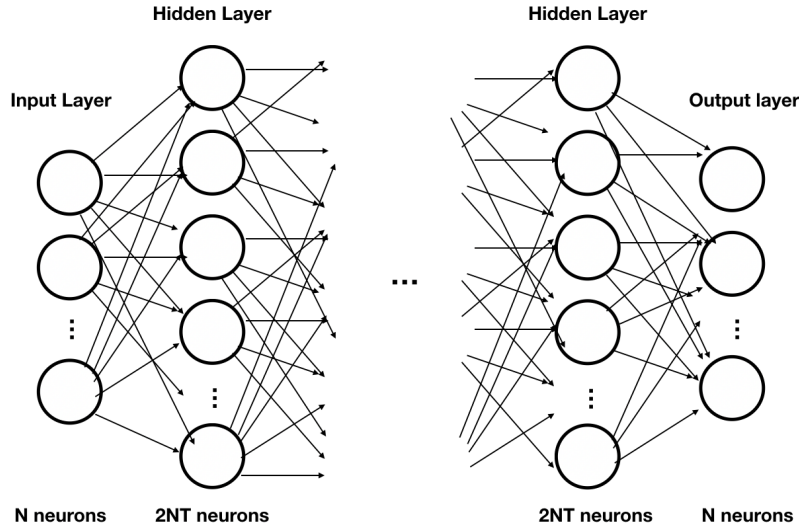


Figure 4.4: Network Structure.

### 4.3 Training Procedure

The main idea of renewing neural network parameters is the Back-propagation (BP) algorithm. BP algorithm reflects how parameters like weights and biases effect the loss function. With the help of the chain rule, we can obtain the partial derivative of the loss function with respect to any weight or bias by deriving the partial derivative between loss function with respect to the network output.

Suppose the data  $D = \{(X_i, Y_i)\}_{i=1}^N$  are given for training and  $\hat{D} = \{(X_i, \hat{Y}_i)\}_{i=1}^N$  is the NN input-output data set. First, we train the NN to mimic the performance of the give data set. Here the loss function is:

$$L = \text{Mean Square Error}(MSE) = \frac{1}{2} \sum_i (Y_i - \hat{Y}_i)^2$$

The partial derivative between loss function with respect to the network output is:

$$\frac{\partial L}{\partial \hat{Y}_i} = \frac{\partial \frac{1}{2} \sum_i (Y_i - \hat{Y}_i)^2}{\partial \hat{Y}_i} = \hat{Y}_i - Y_i$$

After doing the supervised learning. We are going to train a neural network without the help of other PAPR reduction techniques. Here we propose a self-defined loss function to renew weights and biases in the network. The loss function contains two parts, MSE and PAPR with mixing parameters  $\alpha, \beta$ . The MSE between NN output layer and input layer constrains signal deviation to prevent BER distortion. The PAPR part is the main composition for reducing PAPR. For each sample, the output vector is  $\vec{Y} = (Y_1, Y_2, \dots, Y_N)$ . Suppose the IFFT for the output vector is expressed as  $\vec{y} = IFFT(\vec{Y}) = W_N \vec{Y}$ . So the loss function is:

$$Loss = \alpha MSE + \beta PAPR = \alpha \frac{1}{2} \sum_i (Y_i - \hat{Y}_i)^2 + \beta \frac{\max |\vec{y}|^2}{E[|\vec{y}|^2]}.$$

To infer the partial derivative of the loss function with respect to every output variables, we analyze the formula for PAPR as the MSE part is similar to above. Suppose the partial derivative of PAPR with respect to output layer is

$$\begin{aligned} \frac{\partial PAPR}{\partial \hat{Y}_i} &= \frac{\partial \frac{\max |\vec{y}|^2}{E[|\vec{y}|^2]}}{\partial \hat{Y}_i} \\ &= \frac{\partial \frac{\max |IFFT(\vec{Y})|^2}{E[|IFFT(\vec{Y})|^2]}}{\partial \hat{Y}_i} \\ &= \frac{\partial \frac{\max |W_N \vec{Y}|^2}{E[|W_N \vec{Y}|^2]}}{\partial \hat{Y}_i} \end{aligned}$$

Max function is not differentiable, so we use soft-max function instead that is differentiable. Also, we ignore the average power part because the average power of OFDM signal remains same for different data samples. The approximate formula is proposed as

$$\frac{\partial PAPR}{\partial \hat{Y}_i} \approx \frac{\partial softmax |W_N \vec{Y}|^2}{\partial \hat{Y}_i}$$

$$\begin{aligned}
&= \frac{\partial \ln \sum_j \exp |W_{Nj}^{\vec{}} \hat{Y}^{\vec{}}|^2}{\partial \hat{Y}_i} \\
&= \frac{\partial \ln \sum_j \exp \{(Re[W_{Nj}^{\vec{}}]Re[\hat{Y}^{\vec{}}] - Im[W_{Nj}^{\vec{}}]Im[\hat{Y}^{\vec{}}]) + j(Re[W_{Nj}^{\vec{}}]Im[\hat{Y}^{\vec{}}] + Im[W_{Nj}^{\vec{}}]Re[\hat{Y}^{\vec{}}])\}^2}{\partial \hat{Y}_i} \\
&= \frac{\partial \ln \sum_j \exp \{(Re[W_{Nj}^{\vec{}}]Re[\hat{Y}^{\vec{}}] - Im[W_{Nj}^{\vec{}}]Im[\hat{Y}^{\vec{}}])^2 + (Re[W_{Nj}^{\vec{}}]Im[\hat{Y}^{\vec{}}] + Im[W_{Nj}^{\vec{}}]Re[\hat{Y}^{\vec{}}])^2\}}{\partial \hat{Y}_i}
\end{aligned}$$

We have doubled the number of neurons of the output layer to express one complex number with two real neurons. So the formula above need to be separated into two cases. Here is the example for the real part. For simplicity, assume  $Z_j = (Re[W_{Nj}^{\vec{}}]Re[\hat{Y}^{\vec{}}] - Im[W_{Nj}^{\vec{}}]Im[\hat{Y}^{\vec{}}])^2 + (Re[W_{Nj}^{\vec{}}]Im[\hat{Y}^{\vec{}}] + Im[W_{Nj}^{\vec{}}]Re[\hat{Y}^{\vec{}}])^2$ , so

$$\begin{aligned}
\frac{\partial PAPR}{\partial Re[\hat{Y}_i]} &\approx \frac{\partial \ln \sum_j \exp Z_j}{\partial Re[\hat{Y}_i]} \\
&= \frac{1}{\sum_j \exp Z_j} \sum_j \exp Z_j \frac{\partial Z_j}{\partial Re[\hat{Y}_i]}
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial Z_j}{\partial Re[\hat{Y}_i]} &= 2Re[W_{Nj,i}^{\vec{}}](Re[W_{Nj}^{\vec{}}]Re[\hat{Y}^{\vec{}}] - Im[W_{Nj}^{\vec{}}]Im[\hat{Y}^{\vec{}}]) \\
&\quad + 2Im[W_{Nj,i}^{\vec{}}](Re[W_{Nj}^{\vec{}}]Im[\hat{Y}^{\vec{}}] + Im[W_{Nj}^{\vec{}}]Re[\hat{Y}^{\vec{}}])
\end{aligned}$$

Furthermore, we combine supervised learning and self-defined loss to see if it is possible to surpass the existing performance of suppressed data used for training. We test for different mixing parameters to find the optimal combination. For different data sizes the optimal parameter com-

Operations	Convex OPT(N=12)	Convex OPT(N=36)	Proposed NN(N=12)	Proposed NN(N=36)
(I)FFT	48	144	-	-
Multiplications	437,400	3,696,840	13,920	124,704
Additions	249,120	1,991,520	14,016	124,992
Exponential	7,920	23,760	96	288

Table 4.1: Computational Complexity Comparison of the Convex Optimization and the Proposed Neural Network with N Sub-carriers

binations are different. In future research, we will propose an adaptive optimization algorithm for those mixing parameters.

#### 4.4 Complexity Analysis

The computational complexity of the convex optimization method and proposed NN is compared in Table 4.1. The table shows the number of (I)FFT, multiplication, addition and exponential operations due to convex optimization algorithm and NN models. For each N point (I)FFT, the number of multiplications and additions are  $\frac{N}{2} \log_2(N)$  and  $N \log_2(N)$ . The NN models based on a fully connected structure with the number of neurons in each layer of  $\{N, h1, h2, N\}$  requires  $(N \times h1 + h1 \times h2 + h2 \times N)$  times multiplications and additions if not considering activation functions.

## 5. EXPERIMENTS AND RESULTS

### 5.1 Determine Mixing Parameters of Loss Function

In our loss function

$$Loss = \alpha MSE + \beta PAPR,$$

$\alpha$  and  $\beta$  are mixing parameters that constrain signal deviation and signal PAPR respectively. It's a trade-off problem. In this section, we compare the results between different parameter combinations. The goal is to find a set of mixing parameters that can reduce PAPR under an acceptable derivation.

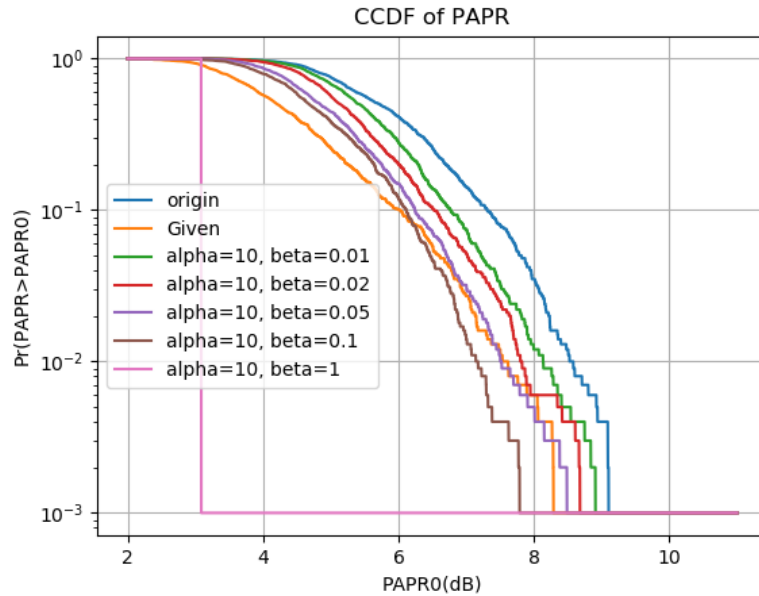


Figure 5.1: PAPR CCDF Comparison of Different Parameter Combinations for 12 Sub-carriers.



	Mean Deviation	Max Deviation	$10^{-3}$ CCDF PAPR(dB)
Original	0	0	9.37
Convex OPT	0.007	0.09	8.64
$\alpha = 10, \beta = 0.01$	0.002	0.08	9.01
$\alpha = 10, \beta = 0.02$	0.006	0.18	8.73
$\alpha = 10, \beta = 0.05$	0.016	0.25	8.37
$\alpha = 10, \beta = 0.1$	0.027	0.54	7.98
$\alpha = 10, \beta = 1$	0.059	0.82	4.0

Table 5.1: Deviation and PAPR Performance of Different Parameter Combinations with 12 Sub-carriers

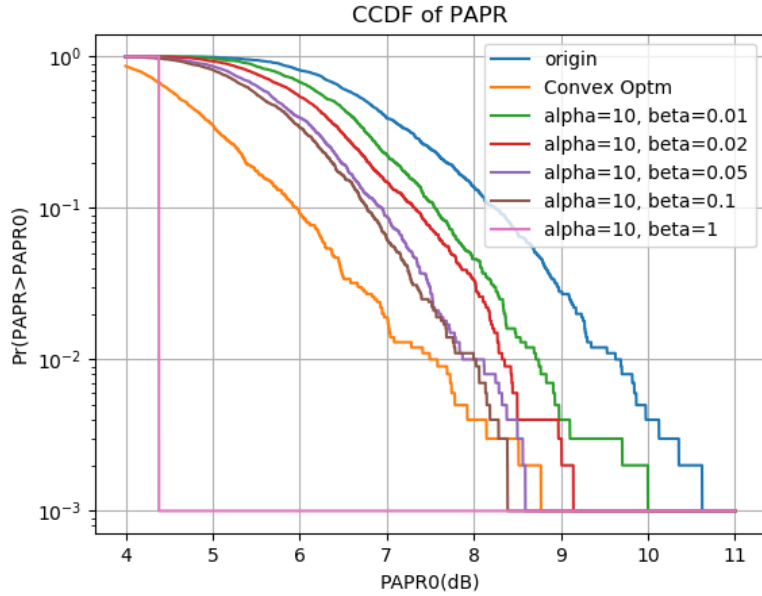


Figure 5.2: PAPR CCDF Comparison of Different Parameter Combinations for 36 Sub-carriers.

Figure 5.1 and 5.2 compare the complementary cumulative distribution function (CCDF) of PAPR for different parameter combinations. By fixing  $\alpha = 10$ , PAPR changes as expected when the value of  $\beta$  changes.

As shown in Table 5.1 and 5.2, we use the results of the convex optimization method for comparison. In order to have a fair comparison, we want to keep the same degree of average deviation with the convex optimization method. So we choose the parameter set with  $\alpha = 10$  and

	Mean Deviation	Max Deviation	$10^{-3}$ CCDF PAPR(dB)
Original	0	0	10.48
Convex OPT	0.007	0.17	8.60
$\alpha = 10, \beta = 0.01$	0.005	0.12	9.52
$\alpha = 10, \beta = 0.02$	0.016	0.31	9.20
$\alpha = 10, \beta = 0.05$	0.031	0.64	8.59
$\alpha = 10, \beta = 0.1$	0.053	1.26	8.55
$\alpha = 10, \beta = 1$	0.51	1.27	4.38

Table 5.2: Deviation and PAPR Performance of Different Parameter Combinations with 36 Sub-carriers

$\beta = 0.02$  for 12 sub-carriers and the set with  $\alpha = 10$  and  $\beta = 0.01$  for 36 sub-carriers.

## 5.2 Results

The simulations are performed for 12 and 36 sub-carriers. They are all QPSK modulated. In this section, our experiments illustrate the performance of signal PAPR and deviation for proposed three NN based methods. In method 1, we train our neural network based on a given data set generated with convex optimization method. In method 2, we use unsupervised learning and train with our loss function  $Loss = \alpha MSE + \beta PAPR$ . Mixing parameters are determined in the previous section. In method 3, we combine method 1 and 2 to improve the performance. We replace the original signal in the MSE part of the loss function by OFDM reduced signal and use the same mixing parameter with method 2. All those three methods share the same network structure.

	Mean Deviation	Max Deviation	$10^{-3}$ CCDF PAPR(dB)
Original	0	0	9.37
Convex Optimization	0.007	0.09	8.64
Method 1	0.005	0.09	8.10
Method 2	0.006	0.11	8.73
Method 3	0.009	0.17	7.95

Table 5.3: Deviation and PAPR Performance of Different Methods with 12 Sub-carriers

### 5.2.1 12 Sub-carriers

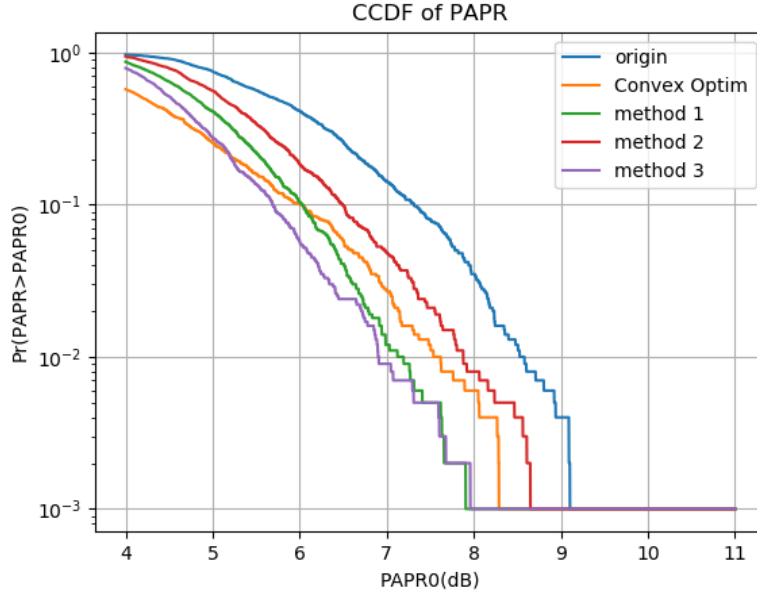


Figure 5.3: PAPR CCDF Comparison of Different methods for 12 Sub-carriers.

Figure 5.3 illustrates the CCDF of PAPR for original OFDM, convex optimization and proposed schemes with 12 sub-carriers and QPSK modulated OFDM signals. It is shown that the PAPR performance of proposed methods are close to the convex optimization method. They all achieve a significant PAPR reduction compared to the original signal.

Table 5.4 gives an explicit comparison of the PAPR and deviation performance of different

methods. With a close average deviation, proposed methods show a good approximation to the given PAPR suppressed signal. For a  $10^{-3}$  CCDF, method 1 achieves  $1.27dB$  PAPR reduction, method 2 achieves  $0.64dB$  PAPR reduction and method 3 achieves  $1.42dB$  PAPR reduction. Method 1 and method 3 outperform the convex optimization method. Method 3 shows the best performance as taking advantages of both method 1 and method 2.

### 5.2.2 36 Sub-carriers

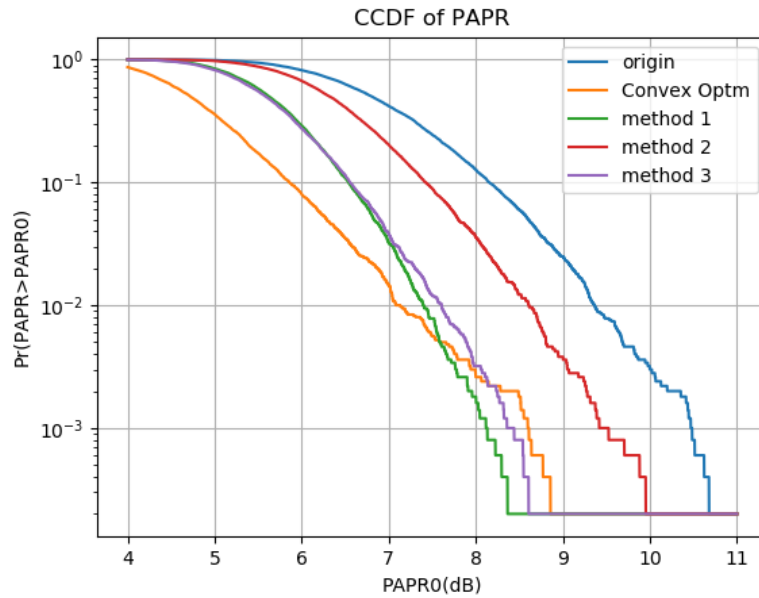


Figure 5.4: PAPR CCDF Comparison of Different methods for 36 Sub-carriers.

Figure 5.4 illustrates the CCDF of PAPR for original OFDM, convex optimization and proposed schemes for 36 sub-carriers. Same as the results for 12 sub-carriers, the PAPR performance of proposed methods are close to the convex optimization method. Both method 1 and method 2 achieve a significant PAPR reduction compared to the original signal.

Table 5.4 gives an explicit comparison of the PAPR and deviation performance of different methods. With a close average deviation, we can make a fair comparison of the PAPR performance.

	Mean Deviation	Max Deviation	$10^{-3}$ CCDF PAPR(dB)
Original	0	0	10.48
Convex Optimization	0.007	0.17	8.60
Method 1	0.004	0.08	8.13
Method 2	0.005	0.12	9.52
Method 3	0.009	0.19	8.43

Table 5.4: Deviation and PAPR Performance of Different Methods with 36 Sub-carriers

For a  $10^{-3}$  CCDF, method 1 achieves  $2.35dB$  PAPR reduction, method 2 achieves  $0.96dB$  PAPR reduction and method 3 achieves  $2.05dB$  PAPR reduction. Method 1 and method 3 outperform the convex optimization method.

## 6. CONCLUSION AND FUTURE WORK

In this research, we have proposed neural network based PAPR reduction methods to solve the important issue for the attractive multi-carrier transmission technique called OFDM. We propose both supervised and unsupervised schemes and try to figure out its ability in learning the relationship between original frequency domain OFDM signal and PAPR suppressed frequency domain optimized OFDM signal. Our methods are simple for implementation and can work for cases lacking training data set.

From the simulation results, it is shown that we achieve a significant PAPR reduction for both 12 and 36 sub-carriers cases with a much lower computational complexity than convex optimization method. For a  $10^{-3}$  CCDF, method 1 and method 3 can outperform the convex optimization method. The reduction technique can be further enhanced by using different variables and constraints on loss function that can optimize signal features benefit communication quality.

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