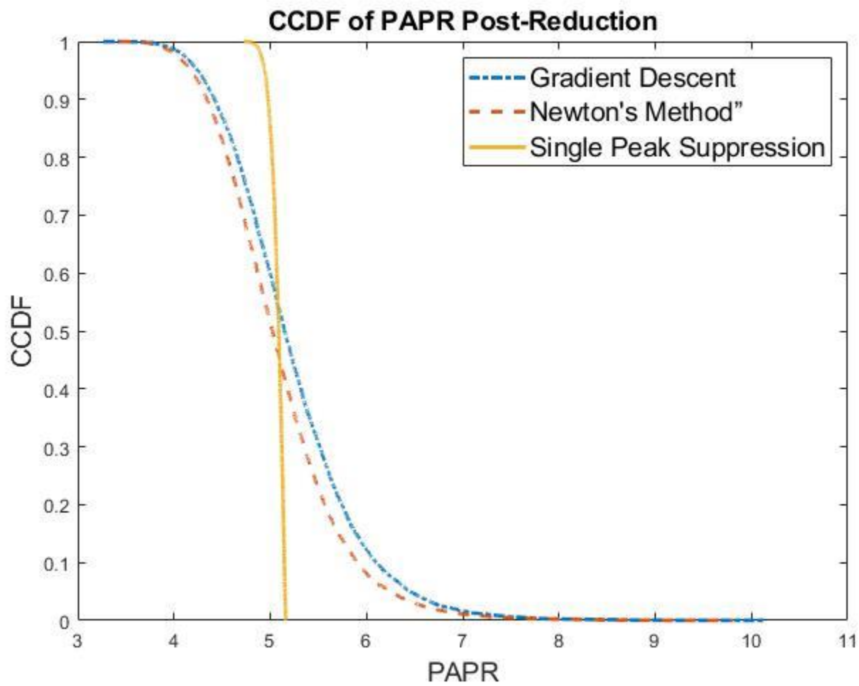

Figure 3 CCDF of the PAPR of the 10,000 signals shown in Table 4.

Reduction Efficiency

Gradient Descent and Newton's Method follow the same principle. However, their difference in the search directions makes Newton's Method more efficient at finding the optimal solution with each iteration. This can be shown in Figure 4, where the PAPR after each iteration for the two methods is recorded. Newton's Method descent to a low PAPR in fewer iterations, whereas Gradient Descent's reduction is smoother and steadier after the initial iteration.

SPS cannot be compared properly with Gradient Descent and Newton's Method because it does not produce a desirable PAPR until all peaks are suppressed, therefore it is not included in this figure.

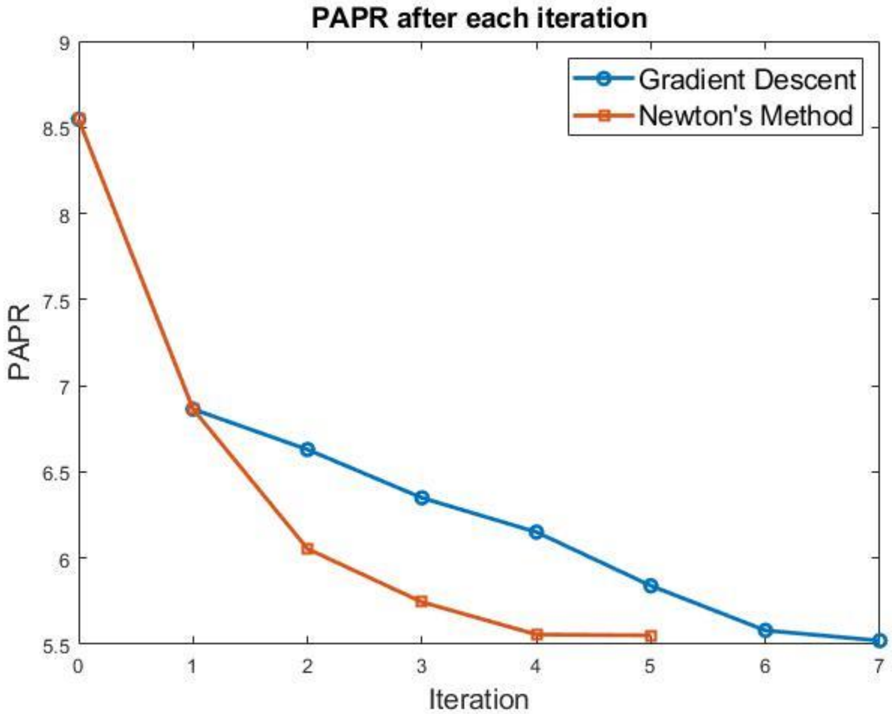

Figure 4 PAPR after each iteration of Gradient Descent and Newton's Method.

Method Complexity

To fully understand the efficiency of each method, their complexities need to be evaluated. The complexity is measured in the number of operations per iteration. 3 operation types are tallied up and compared in Table 5. Complexity for typical clipping and filtering (assuming 12 iterations) [22] is included in the table to cast a comparison of the complexity between these proposed methods and an existing method.

	Add./Sub.	Mul./Div.	Exp./Log./Other
Gradient Descent	$1440N^2+3480N$	$2760N^2+3330N$	660N Exp.
Newton's Method	$380N^3+580N^2$	$460N^3+1110N^2$	220N Exp.
Single Peak Suppression	$72N^2+108N$	$144N^2+216N$	27 square roots
Clipping and Filtering	192NF+96N	192NF+96N	N.A.

Table 5 Approximated number of operations for each category (real addition/subtraction; real multiplication/division; exponential/logarithmic and square roots) for all methods (including clipping and filtering). N = number of subcarriers, F = Finite Impulse Response (FIR) Filter Length

Assuming IFFT length is 4 times the number of subcarriers. The complexity for some calculation parts is summarized below

1) To calculate the gradient of (24), $24N^2 +30N$ multiplications/divisions and $16N^2+20N$ additions/subtractions are needed.
2) The Hessian matrix and its inversion, assuming gaussian elimination method is used to invert the matrix, need about $92N^3+38N^2$ multiplication/divisions and $76N^3+20N^2$ additions/subtractions.
3) To find the best step size results requires $160N^2+192N$ multiplications/divisions and $80N^2+212N$ additions/subtractions.
4) Calculate the signal requires $16N^2+4N$ multiplications/divisions and $8N^2$ addition/subtractions.

Gradient Descent requires the calculation of 1) and 3), and the total complexity assumes 15 iterations. Newton's Method requires the calculation of 1), 2), and 3), and the total complexity assumes 5 iterations. The complexity for SPS is dominated by the renormalization of the signal as in 4), 20N multiplications/divisions and 12N additions/subtractions are needed to calculate its gradient and finding the new constellation points. The total complexity for SPS assumes 9 iterations.

The relative speed between the proposed three methods' iterations can be roughly proven by the average for total method execution time recorded in MATLAB shown in Table 6. However, this is not a true indicator as it depends on program and machine efficiency, but the MATLAB functions were written to be as efficient as possible so this is a relatively good representation of the three methods' speed.

	Average Total Time(s)
Gradient Descent	0.0256
Newton's Method	0.7821
Single Peak Suppression	0.0013

Table 6 Average time for each method's total time recorded in MATLAB.

Error Rate

The symbol error rate is computed by summing the likelihood of each symbol decoded as a wrong symbol when white additive white Gaussian noise (AWGN) is present. In addition, Error Vector Magnitude (EVM) are calculated with each error rate graph. All three method's error rate graphs are show in the Figure 5.

The same data sequence that produced the result in Figure 2 was used. The error rate for all three methods are very similar. As noise level decreases, their difference becomes more noticeable, with Newton's Method being slightly better than Gradient Descent which is slightly better than SPS. The SNR increase at 1% error rate is about 0.5dB. This is consistent with the parameters used across all three methods.

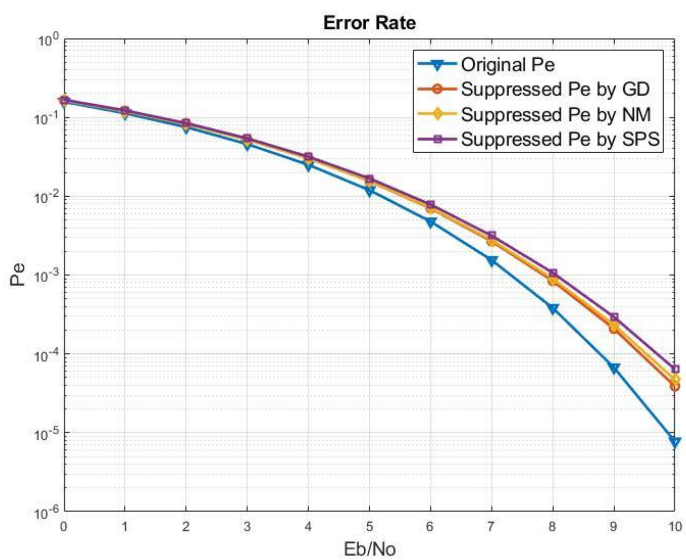

Figure 5 Error rate of the constellation points before and after reduction (72 subcarriers with QPSK). EVMs: -19.44dB (GD), -18.68dB (NM), -18.12dB (SPS).

Additional Results

All three methods are not specified to any constellation size. To illustrate that, 16QAM will be used instead of QPSK. Without changing any other parameters, the final PAPRs for all three methods are shown in Figure 6. And their respective error rates are shown in Figure 7.

To keep the error rates at 1%, signal power needs to be increased by 3dB, which makes the PAPR reduction (roughly 3dB) ineffective overall. This huge increase in error rate is due to the tight spacing between the constellation points, there are less room for the modified points to be affected by noise before they are decoded as wrong data symbols.

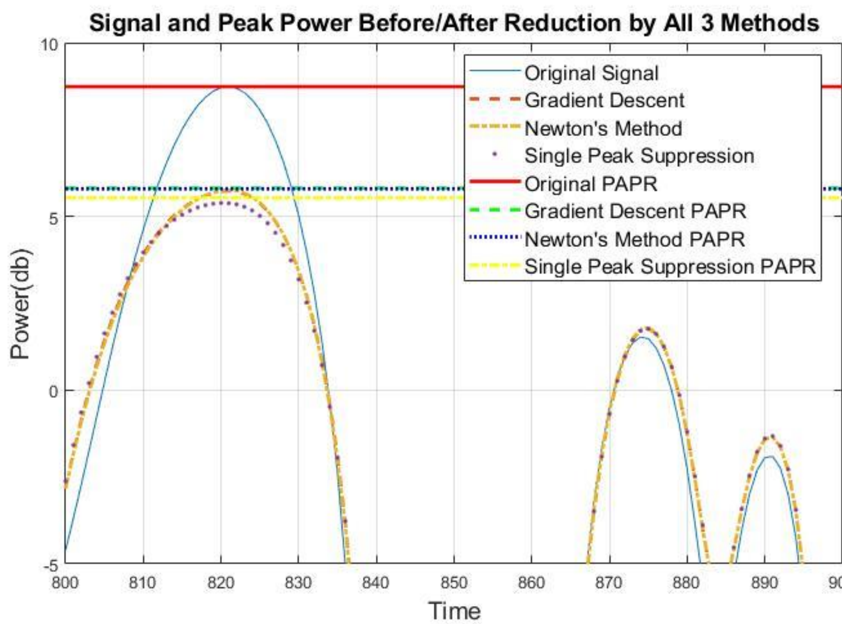

Figure 6 Part of a randomly generated signal before and after reduction (72 subcarriers with 16QAM). The respective PAPR is also shown.

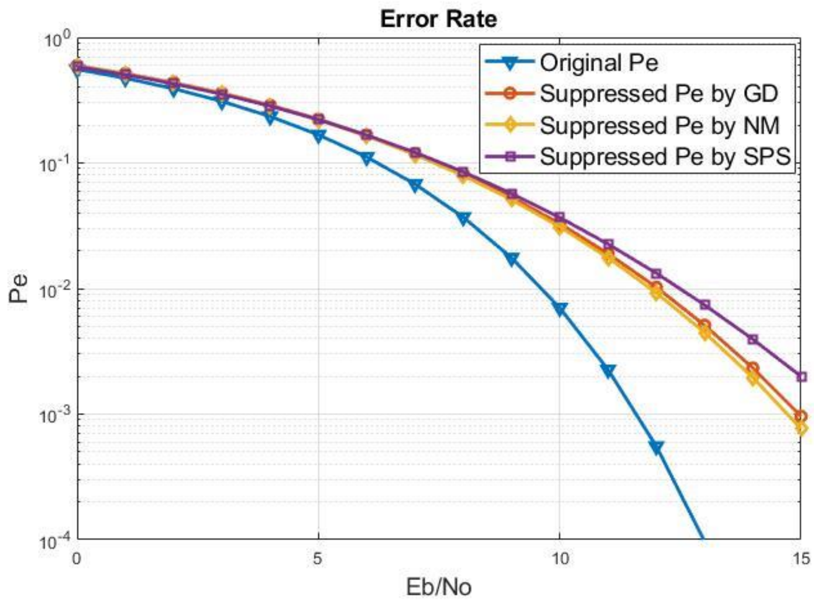

Figure 7 Error rate of the constellation points before and after reduction (72 subcarriers with 16QAM). EVMs: -18.57dB (GD), -18.69dB (NM), -17.50dB (SPS)

By increasing the number of subcarriers, each constellation point is allowed to move less which could potentially decrease the error rates. First, 180 subcarriers are considered. The reduced PAPRs and signals with 180 subcarriers are shown in Figure 8. The error rate shows improvement in Figure 9. But it can still be improved further (1dB signal increase at 1% error rate).

Next, 300 and 1200 subcarriers are used, and their respective error rates are shown in Figure 10. It can be assumed that all three methods would produce similar error rates as the number of subcarriers increase. Thus, only error rates from Gradient Descent is shown. Both curves exhibit acceptable SNR at 1% error rate.

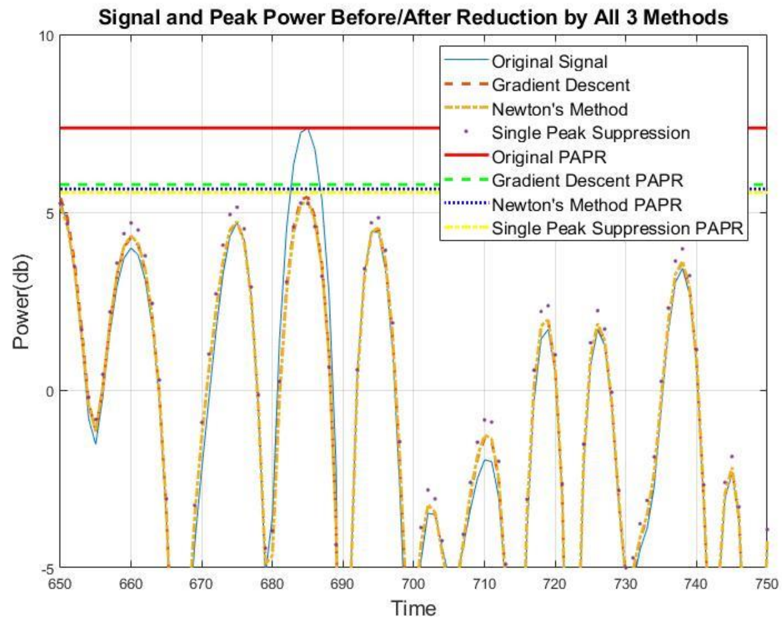

Figure 8 Part of a randomly generated signal before and after reduction (180 subcarriers with 16QAM). The respective PAPR is also shown.

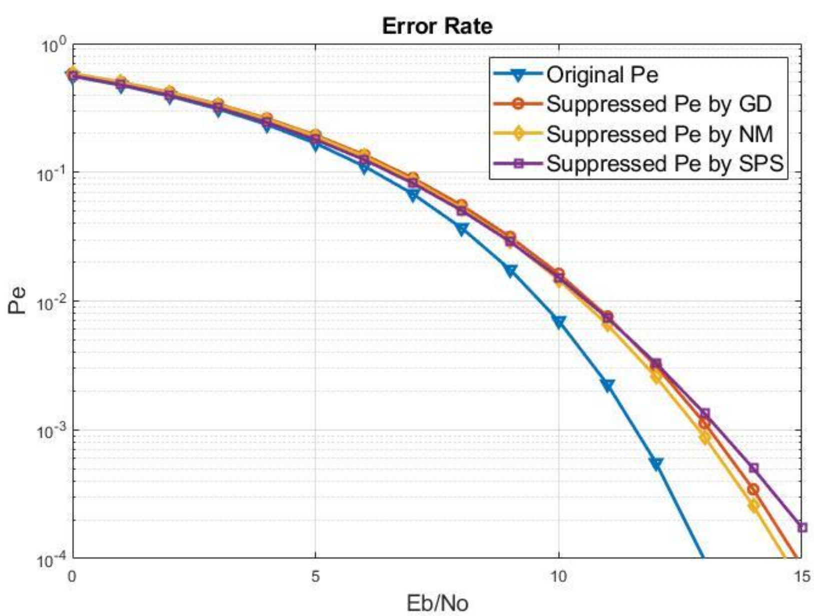

Figure 9 Error rate of the constellation points before and after reduction (180 subcarriers with 16QAM). EVMs: -22.56dB (GD), -23.20dB (NM), -21.11dB (SPS).

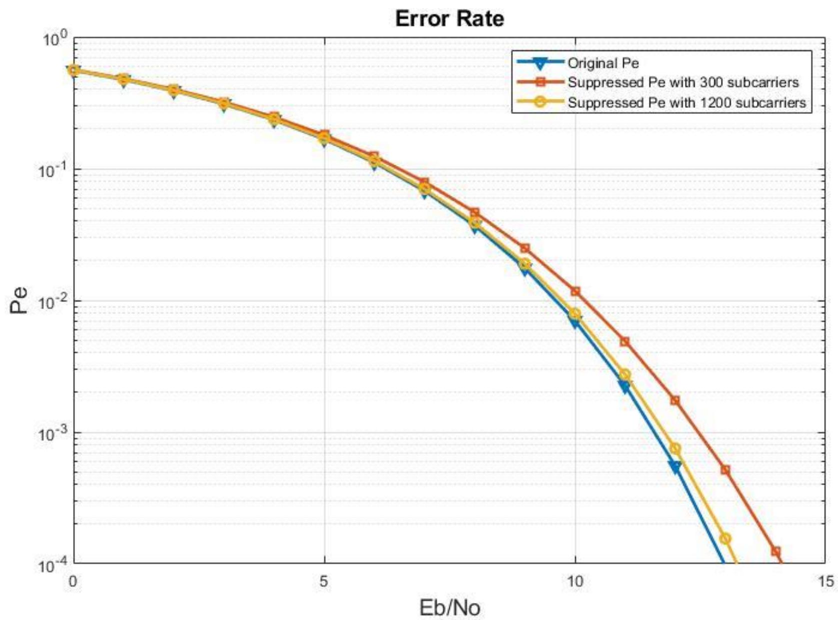

Figure 10 Error rate of the constellation points before and after Gradient Descent (300 and 1200 subcarriers with 16QAM). EVMs: -24.78dB (300), -30.80dB (1200).

Instead of changing the number of subcarriers, the total deviation ϵ can be lowered to achieve the same effect. The result with $\epsilon = 0.2$ is shown in Figure 11 and Figure 12.

The PAPR reduction is expectedly smaller. But the error rates' improvement can be easily seen. Following the same reasoning, higher order constellation maps (64QAM and 256QAM) can be used and reduced by these methods with the appropriate number of subcarriers and deviation limit.

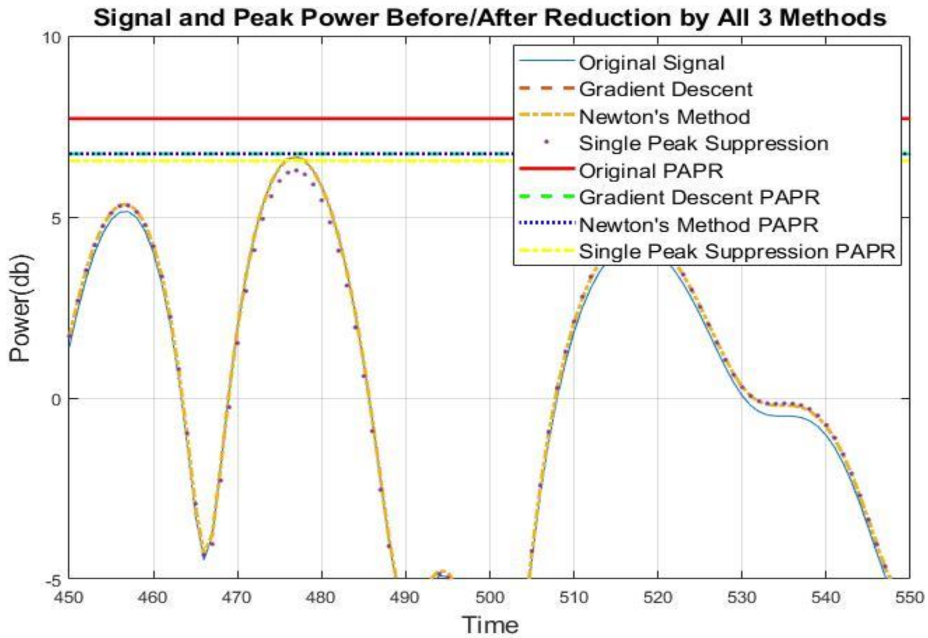

Figure 11 Part of a randomly generated signal before and after reduction (72 subcarriers with 16QAM, $\epsilon = 0.2$). The respective PAPR is also shown.

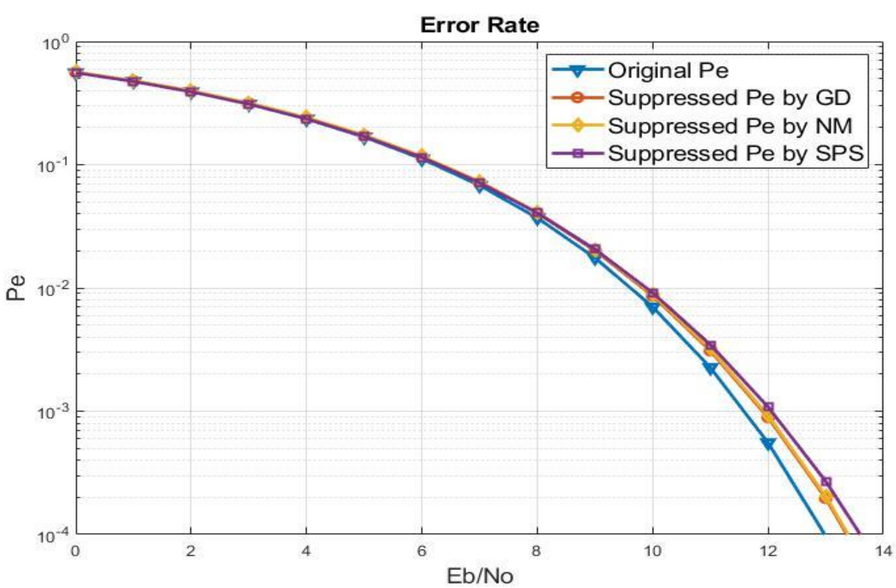

Figure 12 Error rate of the constellation points before and after reduction (72 subcarriers with 16QAM, and $\epsilon = 0.2$). EVMs: -28.58dB (GD), -28.67dB (NM), -26.13dB (SPS).

CHAPTER V

CONCLUSION

In this thesis, 3 PAPR reduction methods for the OFDM systems are provided. The most traditional methods cannot be realistically implemented in existing standard as most require extra side information. These methods focus on changing the mapped constellation points in order to manipulate the peak power. The 3 methods' algorithms are described in detail, including the modification to the problem in order to apply Gradient Descent and Newton's Method. Their simulation is done in MATLAB, and their reduction results are discussed.

To measure the method's effectiveness, a large number of samples are generated and their post reduction PAPR are compared. In addition, each method's complexity is summarized and roughly examined via the timing function in MATLAB. Each method's effect on error rate is also compared.

In conclusion, all three methods can be used to suppress any OFDM system. However, if the system has a large number of subcarriers, Newton's Method is not recommended due to its intense computational complexity. Gradient Descent has much less complexity and it can reach similar results as Newton's Method. But Single Peak Suppression can both suppress the signal most efficiently as it takes the least amount of operations in most scenarios.

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