MULTIPLE HYPOTHESIS TRACKING FILTER WITH LAMBERT POST FILTERING FOR MULTIPLE SPACE OBJECT TRACKING

A Thesis

by

UTKARSH RANJAN MISHRA

Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Chair of Committee,	Manoranjan Majji
Committee Members,	John L. Junkins
	Iman Borazjani
Head of Department,	Rodney Bowersox

May 2019

Major Subject: Aerospace Engineering

ABSTRACT

The data association problem occurs when there is uncertainty and ambiguity in assessing the origin of measurements especially when the measurements are being made from multiple locations at multiple times. For example, in the Space Situational Awareness (SSA) problem, of tracking catalogued Resident Space Objects, it is important to discern the objects being tracked from other debris, non-catalogued objects, maneuvering satellites and even random or spurious measurements. In situations with multiple targets in the field of view, sensor generates multiple measurements where the origin of each measurement is not readily evident. The target to track association problem thus forms an integral preprocessing step for catalog maintenance and uncertainty quantification of the space objects of interest. Most filter implementations assume that the measurements to be of the same physical target being tracked. This scheme can lead to wrong estimates and lost tracks due to incorrect target-measurement assignments. A Multi Hypothesis Tracking (MHT) filter, which combinatorically performs feasible target-measurement assignments is implemented to solve the data association problem. The MHT algorithm, has been augmented with Lambert problem based Initial Orbit Determination (IOD) technique. IOD has been used for target initializations and also as a post filtering sanity check, on batches of measurements which have been assigned to a target, in order to arrest the combinatorial growth of the possible assignment hypothesis.

DEDICATION

To my former teachers, Mr. Manish Kumar Singhal and Dr. Manoranjan Sinha, for their kindness and support in my academic journey.

ACKNOWLEDGMENTS

I would like to thank my advisor Dr. Manoranjan Majji. In addition to his guidance in research I have looked up to him and learned perseverance, patience, clarity, academic integrity and commitment to his student's growth as a researcher. His insights which are borne from a willingness to work from the very fundamentals have been the source of innovation throughout my research. I have benefited from learning his brand of 'learning by doing' in classes as well as research-work. I would also like to thank him for financially supporting me through graduate school. I am deeply indebted to him for all the effort he has been putting into making a good researcher out of me, and sometimes even hand-holding me in overcoming flaws in my methods.

I would also like to thank Dr. John Junkins for being on my committee and his valuable insights. I have benefited greatly from his comments and ideas on my research. With his depth of knowledge and willingness to support a young researcher like myself, he has been a constant source of deep physical intuition about approaching a problem and formulating the right questions. I look forward to even more frequent exchanges and meetings with him in future. His happiness at seeing his students succeed is very apparent and brings about a sense of positivity and motivation to produce good research.

I would like to thank Dr. Nagavenkata Adurthi for helping me grow professionally. I would also like to acknowledge his kindness in letting me use a large library of his codes on filtering and IOD. I would also like to thank Dr. Iman Borazjani for being on my committee and Dr. Vadali, who had brought me to TAMU, for financially supporting me and helping me adjust to the pace of life in Graduate school. I would like to express gratitude to my colleague Dr. Dipanjan Saha. His wisdom, advice, genuine company, and patience have greatly helped me. I would also like to thank my colleague Mr. Raman Goel for his company and friendship. Finally, a thank you to my dear friends Subham and Sankalp, my parents, my brother and my partner for always being there for me.

CONTRIBUTORS AND FUNDING SOURCES

Contributors

This work was supported by a thesis committee consisting of Dr. Manoranjan Majji and Dr. John Junkins of the Department of Aerospace Engineering and Professor Iman Borazjani of the Department of Mechanical Engineering.

The code for Conjugate Unscented Transforms Filter and Lambert solver were provided by Dr. Nagavenkat Adurthi.

All other work conducted for the thesis was completed by the student independently.

Funding Sources

There are no outside funding contributions to acknowledge related to the research and compilation of this document.

TABLE OF CONTENTS

LIST OF FIGURES

FIGURE Page

LIST OF TABLES

1. INTRODUCTION AND LITERATURE REVIEW*

In Space Situational Awareness (SSA) applications, the target to track association problems form an integral pre-processing step for catalog maintenance and uncertainty quantification of the space objects of interest. In addition to this another variation of this problem namely track to track association also plays an important role. The central theme for this thesis is to exploit the special dynamics of the two body problem and leverage its well developed initialization schemes such as Lambert problem solver to reduce the computational effort involved in the association architecture.

This thesis is laid out as follows: We briefly look at the basics of Orbital debris tracking problem and then introduce multi-target tracking algorithms and their role in generating target-track and track-track associations. We then move on to a brief overview of state estimation of nonlinear dynamical systems using quadrature and cubature methods. In addition, the recently developed Conjugate Unscented Transformation[3] (CUT) is briefly discussed along with various elements of the filter implementation. Next we give a brief overview of special least squares methods for orbital debris tracking collectively called Initial Orbit Determination (IOD) algorithms. We introduce the Lambert problem which will be the IOD algorithm that we will be using in this work. This is followed by a detailed discussion of the mathematical formulation of the Multi Hypothesis Tracking (MHT) algorithm, that is agnostic to the state estimation approach.We show that multiple hypotheses allowing all possible data associations in time are considered by the Multi Hypotheses Tracking algorithm as a part of a Bayesian network. This allows for the measurements arriving at various instances of time are utilized to resolve ambiguities in the current frame. That is to say that the association decisions are made based upon the residual error of the observed data in conjunction with the candidate.

These elements are then combined to outline the process for implementing a quadrature based MHT filter. Numerical examples are used to evaluate the utility of the quadrature based MHT in tracking multiple targets simultaneously and even initializing new tracks which is not possible

* The material in this chapter previously appeared in [2]

with Joint Probabilistic Data Association filter covered in previous work [4]. We will close with a discussion on issues encountered during practical implementation and their solutions.

1.1 Space Situational Awareness (SSA)

In simple terms SSA refers to the ability to track the status, location, movement, and characteristics of a space based system. A worldwide network of 20 American ground based optical sensor systems and radars(mechanical and phased array) along with space based assets together constitute the Space Surveillance Network (SSN). The SSN is tasked with maintaining a catalog of object around the earth. The Catalog uses a truncated Brouwer theory with drag-added/lunar effect added-SDP4 for maintaining 2-line elements sets.

1.1.1 Orbital Debris Tracking

Orbital debris in earth orbit includes the debris from the mass of defunct, artificially created objects in space like old satellites, spent rocket stages, fragments from their disintegration, erosion and collision. Currently SSN tracks approximately 18,000 artificial objects in orbit above the Earth. But with new radar based sensing sensors like the New Space Fence and optical systems like the DARPA SST telescope this number is expected to grow to 150,000. Determining orbits from uncorrelated tracks (UCTs) particularly those with angles only observations is a major challenge. In the past correlating UCTs has been a manual process that needs to be completely automated in order to successfully maintain a large number of tracked objects in the catalog.

1.2 Data Association

The problem of Data Association (DA) occurs when there is uncertainty or ambiguity in assessing the origin of the measurement. For example, in a typical Space Situational Awareness (SSA) problem of tracking cataloged Resident Space Objects (RSOs) with ground based sensors, such as a radar or camera, it is important to discern the objects being tracked from other debris, non cataloged object and even random or spurious measurement. In situations where multiple targets are in the field of view, the sensor generates multiple measurements where the origin of each measurement is not readily evident. The target to track association problems form an integral pre-processing step for catalog maintenance and uncertainty quantification of the space objects of interest[5]. Many filter implementations, such as the Kalman Filter[6] and Particle Filter[7], often assume the measurements to have been originated from the same physical target being tracked. As a result, any incorrect associations can lead to wrong estimates and eventually lost tracks.

1.2.1 Multi Hypothesis Tracking Filter and Joint Probabilistic Data Association Filter

Assigning multiple measurements to multiple targets is essentially a combinatorial problem, the complexity of which grows exponentially with the number of measurements and targets. This computational complexity grows even faster when the data association has to be done collectively over several time steps. The Multiple Hypothesis Tracker (MHT)[8, 9, 10] filter is an ideal data association approach as it keeps track of every possible assignment over multiple time steps. As the name suggests, the MHT filter generates plausible hypotheses as part of a Bayesian network as shown in Figure 1.1. Each hypothesis contains feasible assignments of measurements to targets or equivalently targets to measurements. Subsequently, each hypothesis is weighted by the probability or likelihood of describing the sequence of observed measurements. Heuristic approaches are often utilized to prune and contain the potential hypotheses from all the tracks, as the growth is exponential in nature. MHT retains and propagates multiple hypotheses, after processing each set of concurrent measurements, in the hope of resolving the ambiguity with subsequent measurements[11]. MHT can also incorporate higher-order information such as long-term motion and appearance models as the entire track hypothesis can be considered when computing the likelihood[12]. In implementing Reids algorithm[13] for MHT, measurement gates or validation region are used to eliminate impractical hypothesis and thus reducing computational complexity. Further, a pruning/deletion operation is used to remove less likely hypotheses and finally the measurement update is performed. In addition to this clustering and track merging can also be used to make the implementation of MHT computationally tractable.

Several approaches with varying degree of heuristic approximations have been developed to address the inherent computational complexity in data association problems. Among these, the Joint Probabilistic Data Association Filter[14] is a widely used data association filter with several

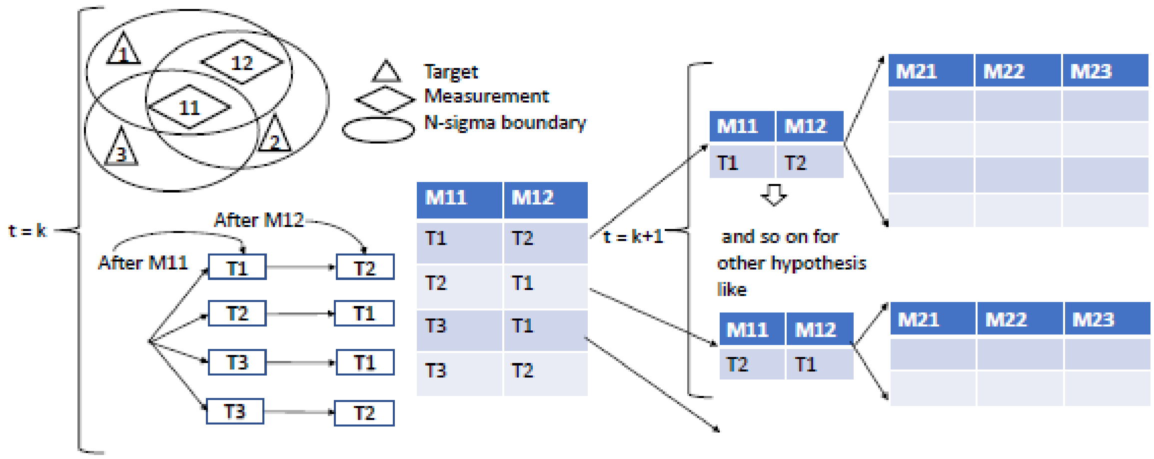

Figure 1.1: Hypotheses tree for MHT

applications in real time computer vision tracking [15, 16, 17]. Joint Probability Data Association (JPDA) is an elegant method of associating the detected measurements in each time frame with existing targets using a joint probabilistic score. Figure 1.2, shows the processing steps of a conventional filter where measurements are directly used to update the state of the target, regardless of where the measurement originated from. Figure 1.3 depicts a data association sub block that assesses the best assignment of measurements to the targets before the conventional measurement update is performed. However, a straightforward implementation of the JPDA algorithms suffer from combinatorial complexity. This is owing to the fact that at each measurement time, one has to consider all possible assignments to all the targets in the catalog to calculate the score associated with the joint probability density [18, 19, 20]. Appropriate approximations that are either greedy or local assignment approaches are typically used to associate the measurements to targets. Recent approaches use Markov Chain Monte Carlo (MCMCDA) data association technique [21]. JPDA also suffers from the problems of coalescing[22], whereby the tracks of closely spaced targets tend to come together, as well as increase in number of false observation in target gate[23] due to a larger Kalman filter covariance matrix.

The key distinction between the JPDA and the MHT approach is that while JPDA maintains a weighted average of contributions from all potential hypotheses, the MHT framework prunes out

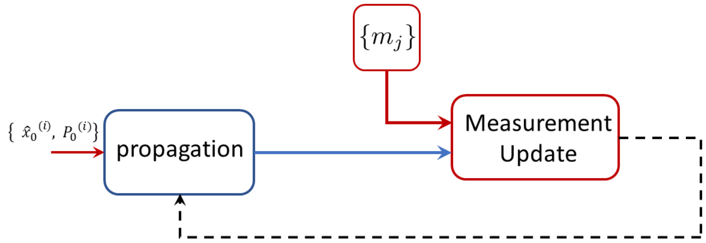

Figure 1.2: Conventional filter

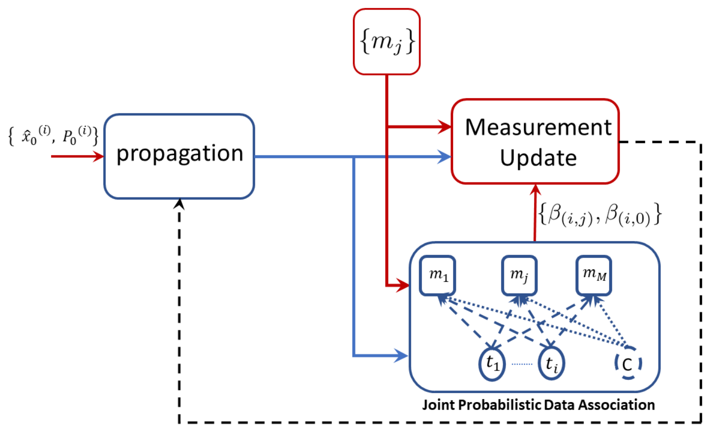

Figure 1.3: Data Association filter

invalid hypotheses for each track independently and deleted terms are disregarded entirely from subsequent association possibilities. MHT has been popular in the radar target tracking community[6]. However, in visual tracking problems, it is generally considered to be slow and memory intensive, requiring many pruning tricks to be practical. MHT is in essence a breadth-first search algorithm, hence its performance strongly depends on the ability to prune branches in the search tree quickly and reliably, in order to keep the number of track hypotheses manageable. In the early work on MHT for visual tracking [12], target detectors were unreliable and motion models had limited utility, leading to high combinatoric growth of the search space and the need for efficient pruning methods.

2. ESTIMATION AND FILTERING

2.1 Basics of Estimation

In order to determine any system's performance and ultimately to control the system, we need to know the states of the system. For an electrical system the states may be the voltages and phase angles at nodes, while in a attitude control problem the states may be the Euler angles and angular velocities. The physical systems are subject to random disturbances and as a result the states may be random.

To find (estimate) the states of a system we rely on measurements or observations, which are themselves generally corrupted by noise due to electronic and mechanical components of measuring devices. The problem of determining the states of a system from noisy measurements is called estimation or filtering. There are two ways of estimating the states, one is batch estimation which uses a bunch of measurements, received at different time steps, the other is sequential estimation which processes measurements one at a time as and when they arrive. Suppose the nonlinear-dynamics of the system is given by the following

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) + \mathbf{\Gamma}(\omega) \tag{2.1}$$

where x is the state vector, $\mathbf{\Gamma}$ is a white noise sequence of normal random variables with zero mean and covariance Q. Measurement model is given by:

$$\mathbf{z} = \mathbf{H}(\mathbf{x}) + \nu \tag{2.2}$$

where H is the measurement model and ν is a white noise sequence of normal random variables with zero mean and covariance R.

We set out with the following goal in estimation problem. Given, the dynamics of the system and a measure of how well we have modeled the dynamics, a guess on the initial states vector, at t = 0 and a measure of how good is our initial guess compared to actual initial conditions, a set of successive measurements and a measure of how good our measurements are we want to give the best estimate of states at any time after t = 0. This is accomplished in a two step process, first is propagation(time-update) and the second is (measurement)update step.

2.1.1 Conjugate Unscented Kalman Filter

In this work the Conjugate Unscented Kalman Filter is being used to do the time and measurement updates. The equations for the same are listed below. The time update equations to get the prior mean $\bar{\mathbf{x}}(k+1)$ and covariance $\bar{\mathbf{P}}(k+1)$ from the posteriori mean $\hat{\mathbf{x}}(k)$ and covariance $\hat{\mathbf{P}}(k)$ at time k are given by :

$$\bar{\mathbf{x}}(k+1) = \sum_{i=1}^{q} w^{(q)}\mathbf{F}(\mathbf{x}(k)^{(q)}) \tag{2.3}$$

$$\bar{\mathbf{P}}(k+1) = \sum_{i=1}^{q} w^{(q)} \left(\mathbf{F}(\mathbf{x}(k)^{(q)}) - \bar{\mathbf{x}}(k+1)\right) \left(\mathbf{F}(\mathbf{x}(k)^{(q)}) - \bar{\mathbf{x}}(k+1)\right)^T \tag{2.4}$$

where the UT sigma points set is taken as $\{\mathbf{x}(k)^{(q)}, w^{(q)}\} \sim \mathcal{N}\left(\hat{\mathbf{x}}(k), \hat{\mathbf{P}}(k)\right)$. After receiving the measurement the conditional mean $\hat{\mathbf{x}}(k+1)$ and covariance $\hat{\mathbf{P}}(k+1)$ are given by :

$$\hat{\mathbf{x}}(k+1) = \bar{\mathbf{x}}(k+1) + K_{k+1}(\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1}) \tag{2.5}$$

$$\hat{\mathbf{P}}(k+1) = \bar{\mathbf{P}}(k+1) - K_{k+1}B_{k+1}K_{k+1}^T \tag{2.6}$$

$$K_{k+1} = P_{xz}B_{k+1}^{-1} \tag{2.7}$$

where

$$\hat{\mathbf{z}}_{k+1} = E[\mathbf{H}(\mathbf{x}_{k+1})] = \sum_{q=1}^{n} w_{k+1}^{(q)}\mathbf{H}(x_{k+1}^{(q)}) \tag{2.8}$$

$$B_{k+1} = E[(\mathbf{H}(\mathbf{x}_{k+1}) - \hat{\mathbf{z}}_{k+1})\,(\mathbf{H}(\mathbf{x}_{k+1}) - \hat{\mathbf{z}}_{k+1})^T] \tag{2.9}$$

$$= \sum_{q=1}^{n} w_{k+1}^{(q)} \left(\mathbf{H}(x_{k+1}^{(q)}) - \hat{\mathbf{z}}_{k+1}\right)\left(\mathbf{H}(x_{k+1}^{(q)}) - \hat{\mathbf{z}}_{k+1}\right)^T \tag{2.10}$$

$$P_{xz} = E[(\mathbf{x}_{k+1} - \bar{\mathbf{x}}(k+1))\left(\mathbf{H}(x_{k+1}^{(q)}) - \hat{\mathbf{z}}_{k+1}\right)^T] \tag{2.11}$$

$$= \sum_{q=1}^{n} w_{k+1}^{(q)} \left(x_{k+1}^{(q)} - \bar{\mathbf{x}}(k+1)\right)\left(\mathbf{h}(x_{k+1}^{(q)}) - \hat{\mathbf{z}}_{k+1}\right)^T \tag{2.12}$$

where the UT sigma points set is now taken from the apriori pdf at time $k+1$ as $\{\mathbf{x}(k+1)^{(q)}, w^{(q)}\}$ $\sim \mathcal{N}\left(\bar{\mathbf{x}}(k+1), \bar{\mathbf{P}}(k+1)\right)$

3. INITIAL ORBIT DETERMINATION TECHNIQUES

Initial orbit determination or IOD refers to the whole family of techniques used to find out the accurate orbit elements (or initial state vector). This is a well investigated problem in astrodynamics, whose first solutions came at the time of Kepler's attempt at predicting the Sun-Mercury conjunction in 1631.

3.1 Sensors and Measurements

The most common sensors used in a Initial Orbit Determination Problem are the telescope(which measure azimuth and elevation) and the radar(which measures the range, azimuth and elevation). The telescopes record data by pointing at the target and recording the azimuth and elevation from the mount. Radars also get a target's azimuth and elevation similarly but in addition to this they can also 'shine' electromagnetic radiation on the target and receive 'reflected' electromagnetic radiation. This additional capability allows radars to calulate the range to the target using Eq. 3.1

$$\rho = \frac{c \times (t_{received} - t_{transmitted})}{2} \tag{3.1}$$

Different techniques exist for finding out the initial orbits depending on the type of data available. For example, if only angular measurements i.e. elevation and azimuth measurements are available we can use GIBBS/Herrick-GIBBS method using a set of three measurements. For brevity we shall only focus on one initial orbit determination technique which can determine the initial state vector(initial orbit) given the range, azimuth and elevation data at two time steps and the time elapsed between them. This will be of particular use in the case of measurements from radars because we can convert the range, azimuth, and elevation data into Cartesian position vectors. A pair of these position vectors and the time elapsed between recording them can be used to for solving the IOD technique known as the Lambert problem.

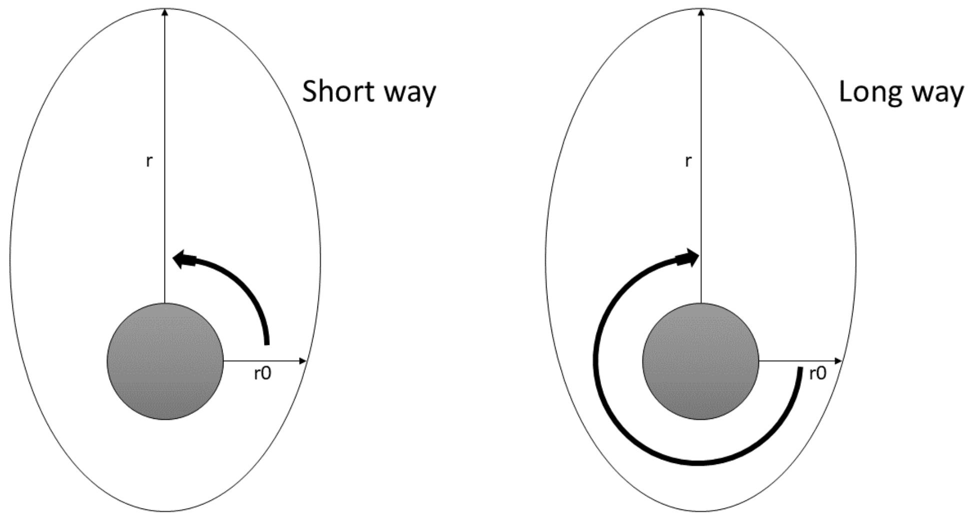

Figure 3.1: Transfer methods for lambert problem

3.2 The Lambert Problem

Lambert problem is orbital boundary value problem, many algorithms to solve the Lambert problem exist. In the simulations presented later, Gooding's algorithm[**?**] has been used. Transfers between two points can take two paths, the long way and the short way as shown in Figure 3.1. Lambert original solution interpreted the problem geometrically to provide the minimum energy transfer between two endpoints. Lambert's theorem ,which forms the basic premise of his solution, says the following:

The orbital transfer time depends only upon the semi-major axis, sum of the distances of initial and final points of the arc from the center of force, and the length of the chord joining these points. Chord length c is given by

$$c = \sqrt{r_0^2 + r^2 - 2r_0rcos(\Delta v)} \tag{3.2}$$

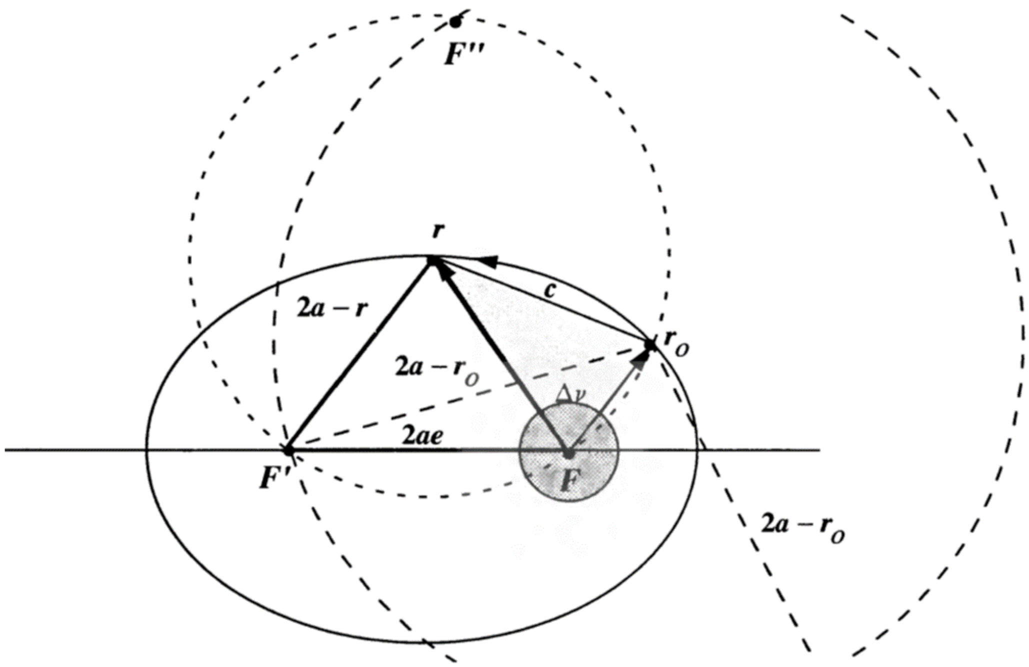

Figure 3.2: Geometry of the Lambert problem - 1 (Reprinted with permission from [1] Fundamentals of Astrodynamics and Applications 1^{st} ed. by David A. Vallado, 1997)

where

$$cos(\Delta v) = \frac{\mathbf{r_0}.\mathbf{r}}{r_0 r} \tag{3.3}$$

Semiperimeter, s, is defined

$$s = \frac{r_0 + r + c}{2} \tag{3.4}$$

A result from conic geometry, for an ellipse which is also of use here is the following

$$2a = r + (2a - r) \tag{3.5}$$

This result is used in Figure 3.2 to locate the other focus and determine the orbit's semi-major

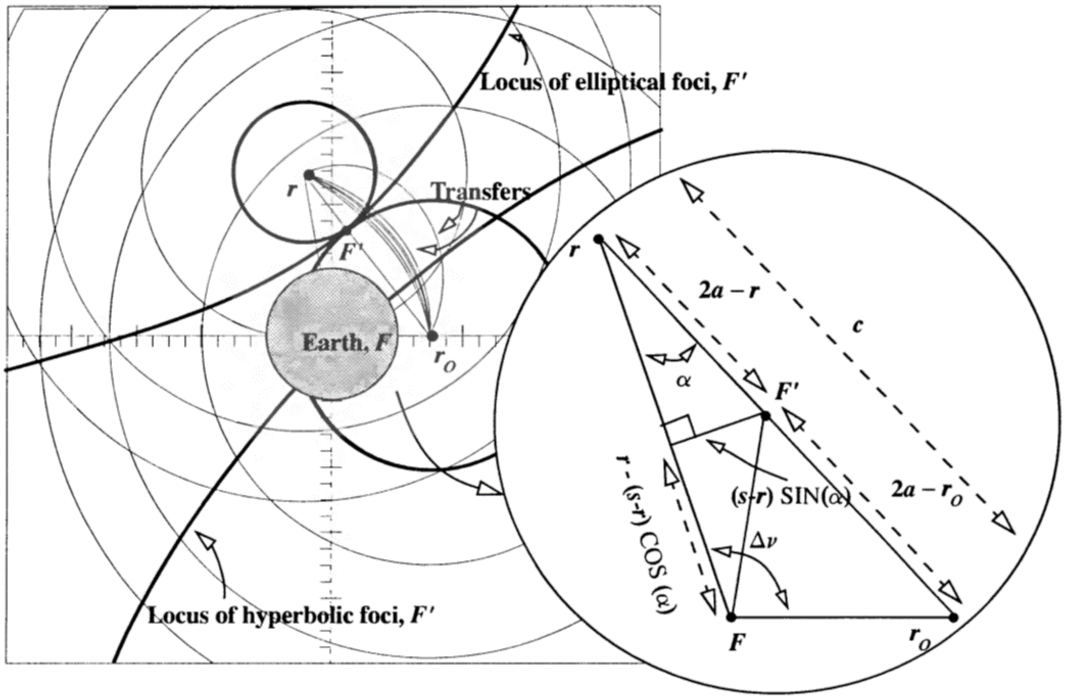

Figure 3.3: Geometry of the Lambert problem - 2 (Reprinted with permission from [1] Fundamentals of Astrodynamics and Applications 1^{st} ed. by David Vallado, 1997)

axis. In Figure 3.2 circles about each position with radius $2a_i - r$ and $2a_i - r_0$, the intersection of these circles mark the location of the other focus. In Figure 3.3 this same result is used to find out many circles representing different values of a_i. The reader is encouraged to go through Reference [] to get a detailed derivation of many results like minimum delta-v transfers.

4. MULTIPLE HYPOTHESIS TRACKING

In order to explain Multi Hypothesis Tracking, let us look at the following dummy example. Suppose you are being shot at by three cannons simultaneously as shown in Figure 4.1 you don't know anything about their positions and launch velocities. Also you are equipped with a radar that can find the bearing angle(elevation) and range of incoming projectiles once every 5 seconds. The first measurement is made at t = 5 and successively the measurements are made at t = 10 and t = 15 seconds. The measurements (range-elevation pairs) are presented in Figures 4.2 and Figure 4.3. Assuming a uniform gravity model and no drag, the dynamics of the cannon balls can be written as following:

$$\frac{d\mathbf{X}}{dt} = \mathbf{AX} + \mathbf{Bu} \tag{4.1}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.2}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{4.3}$$

$$\mathbf{X} = \begin{bmatrix} x \\ y \\ \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} \tag{4.4}$$

$$\mathbf{u} = -g = -9.81 m/s^2 \tag{4.5}$$

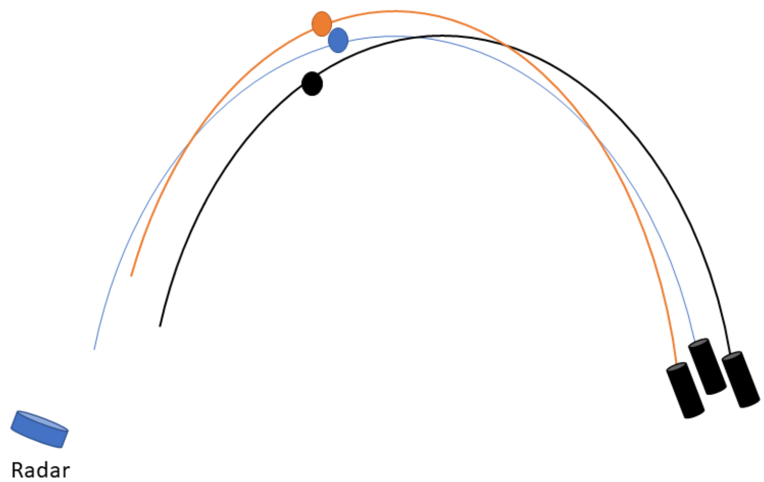

Figure 4.1: Cannonball trajectory initialization scenario

The measurement model is given by,

$$\mathbf{Y} = \mathbf{H}(\mathbf{X}) + \nu \tag{4.6}$$

where ν is a white noise sequence of normal random variables with zero mean and covariance $\mathbf{R}$. $\mathbf{H}(\mathbf{X})$ is the measurement model where,

$$\mathbf{H}(\mathbf{X}) = \begin{bmatrix} elevation \\ range \end{bmatrix} = \begin{bmatrix} tan^{-}(\frac{y}{x}) \\ \sqrt{x^2 + y^2} \end{bmatrix} \tag{4.7}$$

The question being posed here is given sets of three measurement (set of [elevation; range]) at three time steps. Find out the location of the cannonballs (or equivalently where the projectiles will land). Let us look at what we would have done if we knew uniquely knew about successive measurements from one cannonball. We could have started out with some guess and done recursive nonlinear least squares/filtering or we could have done a batch estimate for finding out the coefficients which would have been the initial conditions. But let us pose the problem differently,

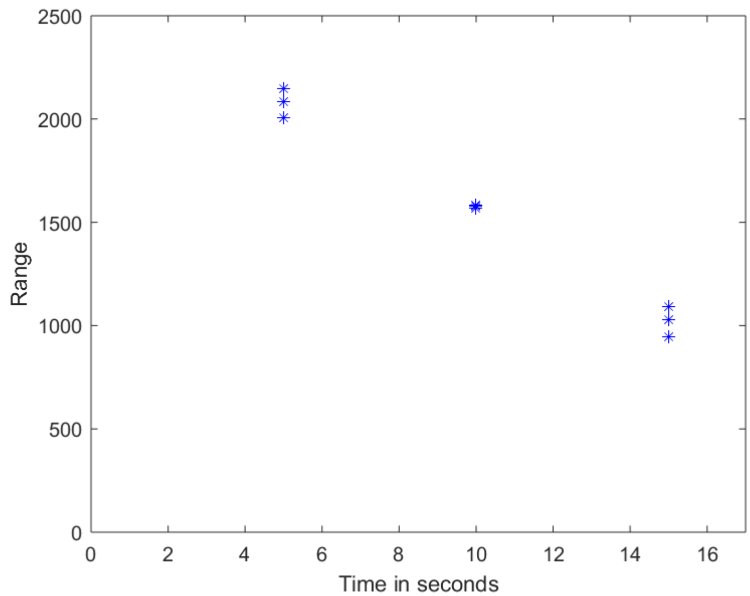

Figure 4.2: Range Measurements

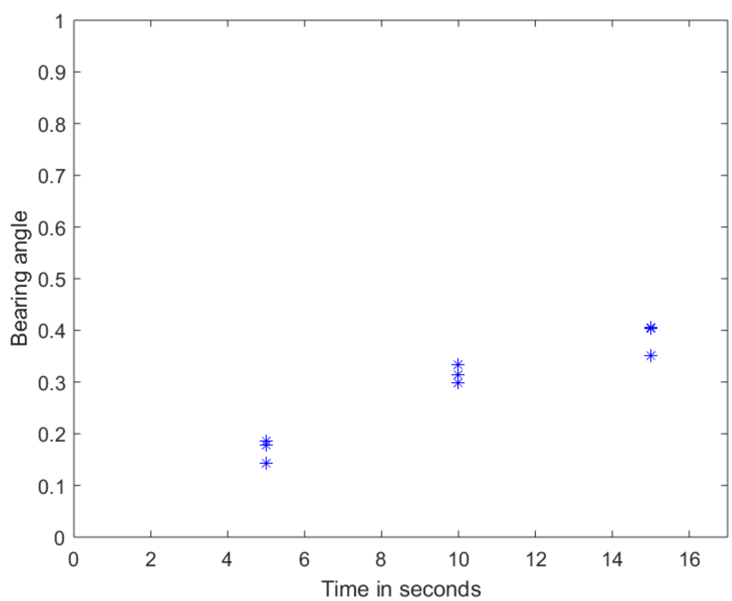

Figure 4.3: Elevation Measurements

suppose we wanted to generate even the initial conditions from the measurements. Stated differently, suppose we did not have any guess on what the initial conditions were, all we have are the measurements. This would have put us in a situation similar to the initial orbit determination problem. So let us try to find a trajectory(ballistic in this case) that can satisfy the measurements. Each measurement set provides two pieces of information, elevation and range, which can be converted to get x and y coordinates.

$$\frac{dx}{dt} = u_0 \tag{4.8}$$

$$\frac{dy}{dt} = v_0 \tag{4.9}$$

$$x = x_0 + u_0 \times \Delta t \tag{4.10}$$

$$y = y_0 + v_0 \times \Delta t - g \times \frac{(\Delta t)^2}{2} \tag{4.11}$$

if $\Delta t = t_2 - t_1$ bwtween two time steps t_1 and t_2, and the corresponding Cartesian coordinates of position is given we can write

$$x_2 = x_1 + u_1 \times \Delta t \tag{4.12}$$

$$y_2 = y_1 + v_1 \times \Delta t - g \times \frac{(\Delta t)^2}{2} \tag{4.13}$$

u_1 and v_1 can be found from Equation 4.12 and Equation 4.13, and this completes our initial 'trajectory' determination. With x_1, y_1, u_1, v_1 at hand we know everything required to initialize a recursive least squares scheme from $t = t_3$ onwards. Now, we address the issue that we don't know x_2, y_2, x_1, y_1, all we know are the mean and covariance for $[elevation_1; range_1]$, $[elevation_2; range_2]$. We do this by generating sigma points and corresponding weights for the joint pdf of the vector $[elevation_1, range_1, elevation_2, range_2]$ whose covariance is given by diagonally concatenating two measurement error covariance matrices $diag[R, R]$. Now we solve for $u1, v1$ for each sigma point and finally do a weighted average to get mean of the final solution, we can also do a weighted average of the individual covariance matrices to get the weighted covariances. This

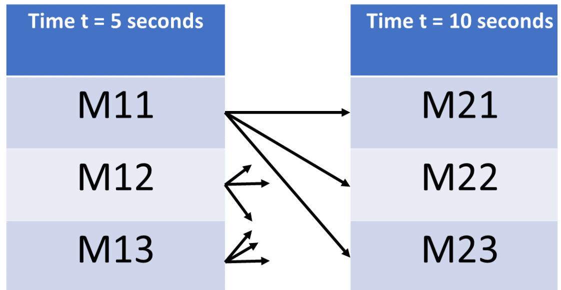

Figure 4.4: Initial 'trajectory' determination

is exactly how we can proceed in solving the 'uncertain' Lambert problem where all we know are measurements $[azimuth; elevation; range]$ that themselves are random vectors. We generate a bunch of sigma points of the joint distribution of starting and final measurements. Convert the $[azimuth; elevation; range]$ for each sigma point into the $[x1, y1, x2, y2]$, we can use this to solve a Lambert problem and get the orbital elements for each sigma point. These bunch of sigma points can be used to find out the mean and covariance of the orbital elements.

4.1 Addressing the association complication

Everything works well for the case in which we had successive measurements from one target. But our original problem demands deeper investigation. Now we have 3 measurements at each time step and we don't know which of these three measurements belong to which target. This problem is at the heart of Data association. One logical way to start is to look at all the possible combinations of measurements at $t_1 = 5$ seconds and $t_2 = 10$ seconds to initialize states of the targets,as shown in Figure 4.4 and use subsequent measurements to clarify which set of compatible combinations makes the most sense. Let us assume that two targets cannot be at the same place at the same time, i.e. one measurement cannot be attributed to two targets. The opposite of this is trivially true that one target cannot generate two separate measurements at the same time.

Let us name the three measurements made at t = 5 seconds as M11, M12 and M13. The three measurements we make at t = 10 seconds as M21, M22, M23 and so on (M31, M32 ...) for t=15

	M21	M22	M23
M11	u1= -95.1 m/s v1= 2.1 m/s	u1= -111.1 m/s v1= 7.1 m/s	u1= 123.2 m/s v1= 1.5 m/s
M12	u1= -95.5 m/s v1= -3.6 m/s	u1=-111.5 m/s v1= 1.3 m/s	u1= -123.6 m/s v1= -4.3 m/s
M13	u1= -94 m/s v1= 7.6 m/s	u1= 110 m/s v1= 12.6 m/s	u1= -122.2 m/s v1= 7m/s

Figure 4.5: Measurements combinations

	Target 1	Target 2	Target 3
1	M11-M21	M12-M22	M13-M23
2	M11-M22	M12-M22	M13-M23
3	M11-M32	M12-M22	M13-M23
4	M11-M21	M12-M23	M13-M22
5	M11-M22	M12-M23	M13-M22
6	M11-M32	M12-M23	M13-M22

Figure 4.6: Valid sets of target initializations

seconds. Using Equation 4.12 and Equation 4.13 we can populate the mean values of u_1 and v_1 in Figure 4.5. Note, that here we are discounting the possibility that any target generates measurement at only one time steps. Initializing the targets by this association matrix implicitly assumes that each target has been observed twice. If we make an additional assumption that each there are three targets in total we can get 6 possible sets of initializations of the three targets such that no two target in the same set are initialized using the same measurement(s). The valid sets of 3 targets are given in Figure 4.6. Note that each measurement is used only once in each row of Figure 4.6, this means that each no two targets in a valid target set are initialized using a common measurement.

M31	M32	M33
T1	T2	T3
T1	T3	T2
T2	T1	T3
T2	T3	T1
T3	T1	T2
T3	T2	T1

Figure 4.7: Measurement oriented hypothesis matrix AT t = 15 seconds

4.1.1 Moving to time step three, new association problem

Now we have 6 mutually exclusive valid 3 target sets as shown in Figure 4.6. How do we find out which amongst them is the correct initialization set to find out the place where shells will land? We do that by using the set of measurements that are made at step t=15 seconds. First we pick one row at a time in Figure 4.6 to initialize the three targets at t = 5 seconds. Now use Equation 4.1 to propagate each target to t = 15 seconds. Let us call the new target states 'prior states'. At t = 15 seconds, we have 3 measurements and 3 targets. This leads to 6 possible target-measurement assignments which are shown in Figure 4.7. Corresponding to each row in Figure 4.6 there will be a measurement-target assignment matrix like Figure 4.7. In total we will have 36 possible 'hypothesis' i.e. valid target-measurement assignments after processing data through time step t = 3 seconds. If we solve the uncertain trajectory initialization problem with two measurements whose mean and covariance are given we can find out the mean and covariance of the initial conditions of the three targets. Then we could have propagated the states and covariance using Equations 2.3 and Equation 2.4. Each row of Figure 4.7 can be used to do measurement update in measurement using Equation 2.5 and Equation 2.6. But all we know so far is that there are 36 possible combinations where cannon balls can land/originated. What we need is a scoring scheme to find quantify likelihood of each of these 36 combinations. A likely scoring scheme can be based on using measurement likelihoods. We can write the covariance of $\nu = \mathbf{Z_m} - \mathbf{H}(\bar{\mathbf{x}})$ as Equation 4.14, while the probability normalized over all 36 possible associations is given by Equation 4.15

$$\mathbf{B} = \mathbf{H}\mathbf{P}\mathbf{H}^T + \mathbf{R} \tag{4.14}$$

$$P = \frac{\left[\prod_{m=1}^{3} N(\mathbf{Z}_m - \mathbf{H}(\bar{\mathbf{x}}), \mathbf{B})\right]}{c} \tag{4.15}$$

This likelihood scoring scheme uses a physically intuitive measure, using measurement likelihoods, to quantify how likely is each of the 36 assignments. But it suffers from too many limiting assumptions. In a more general example there could have been more(or less) than 3 targets that along with clutter(false measurements) could have generated the measurements at t = 5, 10, and 15 seconds. It is not necessary that each target could have been initialized successfully in such a scheme, ie some association hypotheses might have had targets which received only one measurement so far, implying that they cannot be initialized yet. The problem only grows in computational complexity when we introduce two body dynamics in three dimensions. One problem that comes immediately to mind is the fact that given boundary values at two measurements and time of transit between them, the Lambert problem solver can generate multiple solutions. In the next section we shall be introducing a more robust and general algorithm to deal with the Data-association, Mutli-target tracking problem.

4.2 The Multi Hypothesis Tracking Filter

The MHT filter was developed by Reid[8] in 1979. It accounts for false targets (clutter), new (introduced) targets and keeps multiple assignment hypotheses. The reader is referred to Reference [8] for a detailed overview how logic behind the algorithm. Here we shall be presenting only details of the algorithm which are relevant for and skip the derivations. The MHT algorithm generates a set of data association hypothesis, at each time step, to account for all the possible origins of every measurement. The MHT implementation follows the flow diagram of the tracking algorithm as shown in Figure 4.8 and uses a measurement-oriented hypothesis generation in contrast to a target-oriented approach. The measurement-oriented approach lists every possible target for each

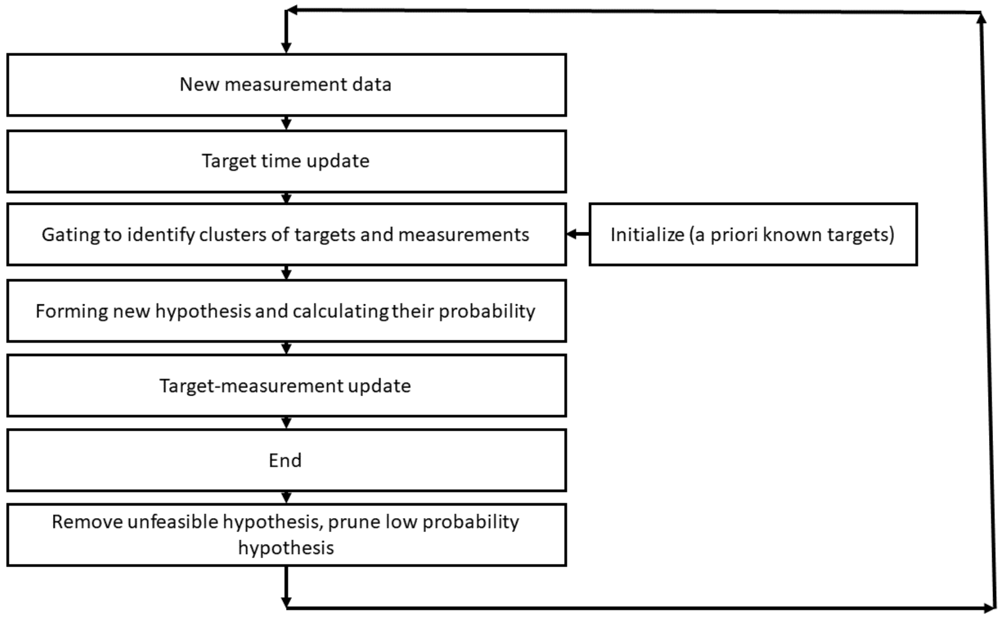

Figure 4.8: Flow algorithm for MHT

listed measurement and vice versa for a target oriented approach. The key advantage of using a measurement-oriented approach is in the ease of initiating new targets at some time-step while processing measurements over multiple time steps.

4.3 Gating and Hypothesis Generation for MHT

Let $\mathbf{Z}(k) \triangleq \{\mathbf{Z_m}(k), m = 1, 2, 3.....M_k\}$ be the set of measurements in data set at time step k; $\mathbf{Z^k} \triangleq \{\mathbf{Z}(1), \mathbf{Z}(2), ..., \mathbf{Z}(k)\}$ denote the set of measurements through time step k; $\mathbf{\Omega}^k \triangleq \{\mathbf{\Omega}_i^k, i = 1, 2, ...I_k\}$denote the set of hypotheses after processing measurements at time step k.

For the simple case where no new target initiations are considered, two conditions have to be met for hypothesis generation. First, for a hypothesis to be valid, a target cannot be associated with more than one measurement.The second condition is that a target is associated with a measurement only if the measurement lies within the validation region or gate of the prior states of the target. If $\mathbf{x}$ and $\mathbf{P}$ are the mean and covariance of the target estimate for the prior hypothesis, then the covariance of $\nu = \mathbf{Z_m} - \mathbf{H}(\bar{\mathbf{x}})$ is given by

$$\mathbf{B} = \mathbf{H}\mathbf{P}\mathbf{H}^T + \mathbf{R} \tag{4.16}$$

and the measurement $\mathbf{Z}_m$ lies within an "η-sigma" validation region if

$$(\mathbf{Z}_m - \mathbf{H}(\bar{\mathbf{x}}))^T \mathbf{B}^{-1} (\mathbf{Z}_m - \mathbf{H}(\bar{\mathbf{x}})) < \eta^2 \tag{4.17}$$

The gating is used to form clusters of targets that can be associated with a measurement as shown in Figure 1.1. There are 3 known targets and 2 measurements, measurement M11 can be associated with target 1, 2 and 3 while measurement M12 can be associated with target 1 and 2. In absence of including new targets or considering false alarms, this leads to the 4 possible association hypothesis as shown. Note that this is a static time picture and all the measurements are being received and processed simultaneously. This gives rise to 4 new hypothesis stemming from a previously known hypothesis. The fact that each hypothesis has only 2 measurement means that only two of the targets will get a measurement update for states and covariance. At next time step the process repeats with each of the 4 hypothesis at the current time step giving birth to multiple new hypotheses(as per the gating and possible associations), leading to a tree data structure with hypothesis as the nodes.

4.4 Probability calculation for hypothesis

This paper uses the hypothesis probability formula developed in Reference [8] to calculate the likelihood of hypotheses.

$$P_i{}^k = \frac{1}{c} P_D{}^{N_{DT}} (1 - P_D)^{(N_{TGT} - N_{DT})} \beta_{FT}^{N_{FT}} \beta_{NT}^{N_{NT}} \left[\prod_{m=1}^{N_{DT}} N(\mathbf{Z}_m - \mathbf{H}(\bar{\mathbf{x}}), \mathbf{B}) \right] P_g^{k-1} \tag{4.18}$$

where P_i^k is the probability of the i^{th} hypothesis at the k^{th} time step Ω_i^k, c is the probability normalization term, P_D is the probability of detection, N_{TGT} is the number of targets known before the measurement was processed, N_{DT} is the number of known targets that were detected according to the current hypothesis, N_{FT} is the number of measurements that are deemed false alarms according to the hypothesis, and N_{NT} is the number of new targets whose existence is implied by the current hypothesis. The probability of individual measurements given the association hypothesis with a known target is calculated by the normally distributed pdf $N(\mathbf{Z}_m - \mathbf{H}(\bar{\mathbf{x}}), \mathbf{B})$ and the P_g^{k-1} term is the probability of the parent hypothesis of the current hypothesis at time step k-1 . In order to derive this equation it has been assumed that the number of previously known targets that are detected is given by a binomial distribution, the number of false targets follows a Poisson distribution and the number of new targets also follows Poisson distribution.

4.5 Numerical Example

In the example problem that has been presented 12 sets of 5-7 measurements,each comprised of range, elevation and azimuth information, made over a 3.5 hour period, are processed using MHT filters, initialized with 10 targets.

The measurements are made from a radar and include the Range, Elevation and Azimuth information. The filters are initialized by the initial states i.e. Cartesian positions, velocities, the state error covariance matrices of the targets and measurement error covariance matrix. Dynamics is given by :

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) + \mathbf{\Gamma}(\omega) \tag{4.19}$$

where x is the state vector consisting the three Cartesian position elements and corresponding velocity vector elements.$\mathbf{\Gamma}$ is a white noise sequence of normal random variables with zero mean and covariance Q. Measurement model is given by:

$$\mathbf{z} = \mathbf{H}(\mathbf{x}) + \nu \tag{4.20}$$

$$\mathbf{H}(\mathbf{x}) = \begin{bmatrix} r & el & az \end{bmatrix}' \tag{4.21}$$

$$r = \sqrt{\mathbf{x}(1)^2 + \mathbf{x}(2)^2 + \mathbf{x}(3)^2} \tag{4.22}$$

$$el = tan^{-1}(\frac{\mathbf{x}(3)}{\sqrt{\mathbf{x}(1)^2 + \mathbf{x}(2)^2}}) \tag{4.23}$$

$$az = tan^{-1}(\frac{\mathbf{x}(2)}{\mathbf{x}(1)}) \tag{4.24}$$

where H is the measurement model and ν is a white noise sequence of normal random variables with zero mean and covariance R. The measurement model returns $\begin{bmatrix} r & el & az \end{bmatrix}'$, where r denotes the range, el denotes the elevation and az denotes the azimuth.

The MHT filter is initialized with the states $\mathbf{X}$ of the ten targets in low earth orbit and their state error covariance matrices $\mathbf{P}$ along with the measurement error covariance matrix $\mathbf{R}$.

$$P = diag(\begin{bmatrix} 1^2 & 1^2 & 1^2 & .1^2 & .1^2 & .1^2 \end{bmatrix}) \tag{4.25}$$

$$R = diag(\begin{bmatrix} .1^2 & .02^2 & .02^2 \end{bmatrix}) \tag{4.26}$$

Equation 2.3 is used to update the states and Equation 2.4 is used to update the state error covariance matrices for all the 10 targets till time step 1. At time t = 1, the first batch of measurements is received. As per the flow algorithm shown in Figure 4.8 clustering of measurements and targets is performed. The clusters of targets associated with a target are then arranged into a hypothesis matrix. It is ensured that in any hypothesis Ω_i^k, where $k = time = 1$, no two measurements are associated with the same target. After the all the possible hypotheses $\Omega_i^k, k = time = 1, i = 1, 2...n$ at time step 1 have been formed, Equation 2.5 and Equation 2.6 are used to update the states $\mathbf{x}$, and state error covariances $\mathbf{P}$, of the targets. If a target has not been associated with any measurement the measurement update is skipped. At this point the probabilities of each of the possible hypotheses are computed using Equation 4.18. This is followed by a pruning step,in this paper for MHT, at the end of processing all the measurements at each time step, only 10 of the hypothesis with the highest probability,of the thousands of hypothesis (most of which have very low probability score) are allowed to survive to the next time step and their probabilities are re-normalized. This pruning strategy will be augmented in future work with a Lambert problem based post-filtering to restrict the exponential growth in the number of hypothesis. In the suggested pruning scheme Uncertain Lambert problem's solutions will be used in order to get bounds on orbital elements from just the measurement data. These Orbital elements will be compared to the orbital elements that of the targets that have received same measurements for state update. Difference in these two sets of orbital elements will be used to remove unfeasible hypothesis. The suggested method will use the physics of the two body problem to prune the hypothesis tree.

The performance of the MHT filter is presented in Figure 4.9, Figure 4.10, Figure 4.11 and Figure 4.12. Figure 4.9 and 4.10 show the estimation errors in the Cartesian Position(x,y,z) and Velocity(u,v,w) of the 10 targets being tracked versus time. The 3 sigma bounds used here is the 3 times the average standard deviation of the states. It can be seen that the errors remain bounded. The occasional growth of error outside the average 3-sigma bound can be explained by realizing that the bounds are averaged bounds and if errors of an individual target that is getting out of average bounds are plotted along with its standard deviation its errors still remain bounded. This can be seen in Figure 4.13 and Figure 4.14 which shows the estimation error in states of the ninth target, which is the target that was exceeding the 3-sigma bounds in the Figure4.9 and 4.10. Also note that the Figure 4.9 does not show a Monti Carlo over initial conditions of one targets, it is showing the estimation errors in the states of 10 targets being tracked simultaneously by MHT. Figure 4.11 and Figure 4.12 show the Root mean square error of the 10 targets and reaffirms that MHT is tracking all the targets successfully. For a more detailed treatment refer to [2].

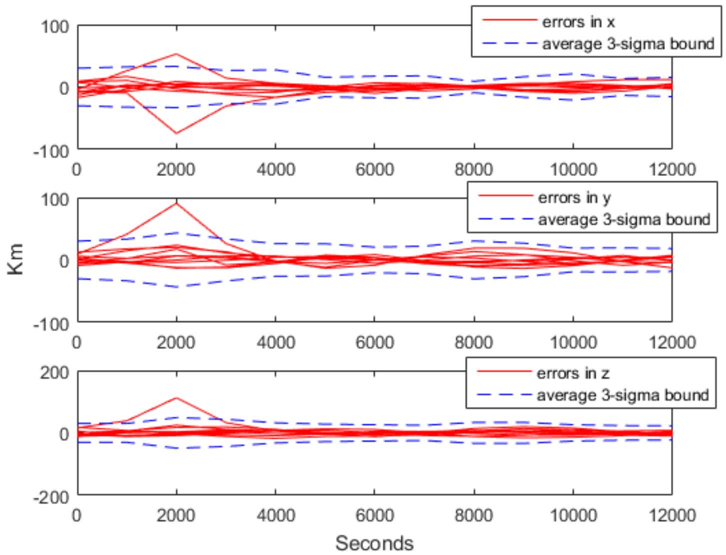

Figure 4.9: Estimation error in position in 10 targets being tracked by MHT Filter vs Time (Reprinted with permission from [2])

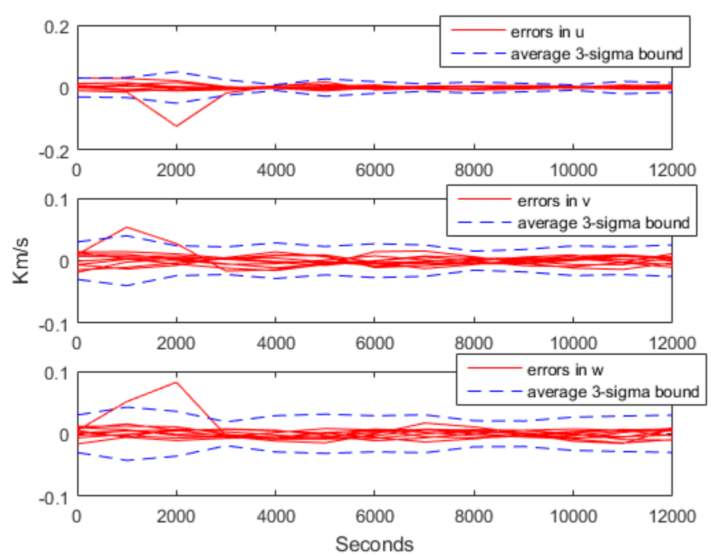

Figure 4.10: Estimation error in velocity in 10 targets being tracked by MHT Filter vs Time Reprinted with permission from [2])

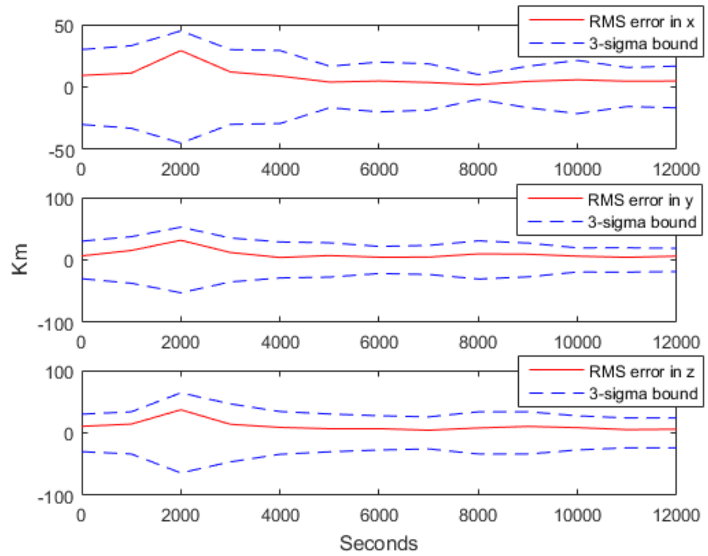

Figure 4.11: Root Mean Square of the estimation error in position for 10 targets being tracked by MHT Filter Reprinted with permission from [2])

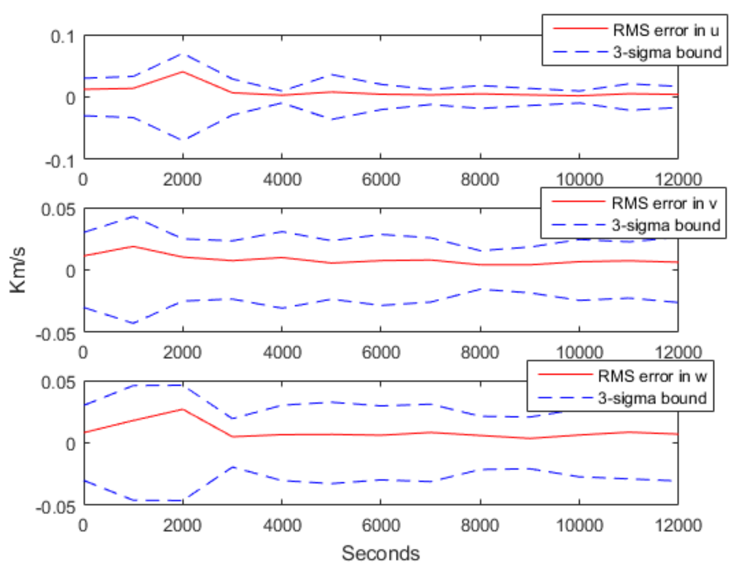

Figure 4.12: Root Mean Square of the estimation error in velocity for 10 targets being tracked by MHT Filter Reprinted with permission from [2])

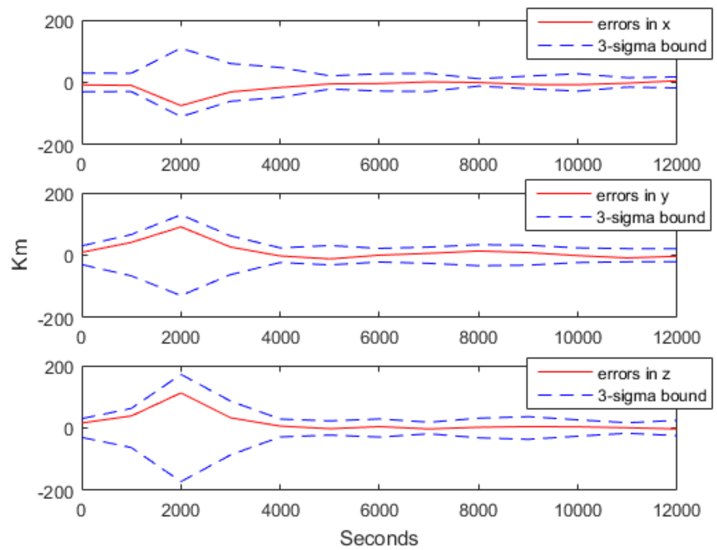

Figure 4.13: Estimation error in position for the ninth target being tracked by MHT Filter vs Time Reprinted with permission from [2])

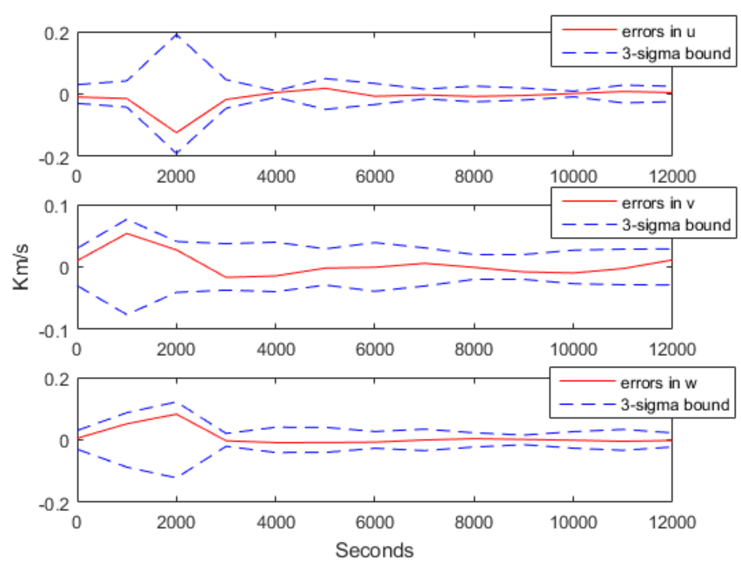

Figure 4.14: Estimation error in velocity for the ninth target being tracked by MHT Filter vs Time Reprinted with permission from [2])

5. MHT WITH TIME DELAYED INITIAL ORBIT DETERMINATION AND TRACK MERGING

The previous chapters introduced Initial Orbit Determination and the standard mathematical formulation for a Multi-Hypothesis Tracking filter. Broadly speaking MHT helps us keep track of multiple consistent target-measurements assignments by treating them as nodes on a data tree. The motivation for maintaining multiple 'hypotheses' is delaying decision making about correct association with the expectation that future information will help resolve ambiguity. We also mentioned the normally used likelihood function that gave us the likelihood for each node.

The numerical results for multi-target tracking problem shown in previous chapter demonstrated MHT's ability to track multiple previously known targets simultaneously. But in practice MHT is most useful while initializing new, previously unknown targets from measurements. This is usually accomplished by simply initializing a 'new target' in a measurement's cluster or in simpler terms making up a new target and claiming that the measurement came from this new target. But there is a obvious problem with this approach. For the individual targets inside a hypothesis MHT is just a wrapper that tells which measurement goes (or doesn't goes) to a target. For a target (it's mean states and state error covariance) it just boils down to time update followed by possible measurement update and so on, just like in the case of any normal filter. The problem arises due to the fact that this 'new target' cannot usually be initialized. In practice a partially guessed state vector with a very large state error covariance in used. The large covariance takes care of the error in the guessed mean state vector. This is because the state space is usually larger than measurement space and it is not possible to initialize a larger initial state vector from just one measurement vector. The way we go around this in the case of measurements coming from a single target is to do some sort of least squares approximation for measurements over multiple time steps and then feeding that initial guess into the filter.

The Initial Orbit Determination techniques we discussed earlier are essentially variations of the least squares. In this chapter we will show a consistent method of leveraging IOD techniques to initialize targets and make necessary changes to MHT Algorithm to accomodate this multi-timestep initialization. We will focus our attention on Lambert's problem and use it to initialize new targets that have been assigned two measurements. Later on we will also demonstrate the efficacy of suggested MHT scheme in initializing new targets successfully with a numerical problem.

5.1 Probability calculation for hypothesis

Earlier we had followed the MHT likelihood function given in Equation 4.18 originally from Reference [8]. We make changes to this formulation and introduce two terms N_{DT1} and N_{DT2} to replace N_{DT}. N_{DT1} denotes the number of previously known targets that had already been initialized. N_{DT2} denotes all the previously known targets that were fully initialized (second measurement assigned and an elliptic orbit possible between the two measurements) at the current time step in the given hypothesis. N_{TGT} like earlier remains as the total number of previously known targets that should have been in the field of view, all other definitions remain the same. This changes the likelihood function to the following:

$$\begin{aligned} P_i^k = &\frac{1}{c} P_D^{(N_{DT1}+N_{DT2})}(1-P_D)^{(N_{TGT}-(N_{DT1}+N_{DT2}))} \beta_{FT}^{N_{FT}} \beta_{NT}^{N_{NT}} \beta_{DT2}^{N_{DT2}} \\ &\times \left[\prod_{m=1}^{N_{DT1}} N(\mathbf{Z}_m - \mathbf{H}(\bar{\mathbf{x}}), \mathbf{B}) \right] P_g^{k-1} \end{aligned} \tag{5.1}$$

5.1.1 Derivation

The derivation follows the one in Reference [8] whose end result was given in Equation 4.18. Let P_i^k denote the probability of hypothesis Ω_i^k, given measurements up to time k,

$$P_i^k = P(\Omega_i^k | Z^k) \tag{5.2}$$

Where Ω_i^k is the joint hypothesis formed by Ω_g^{k-1} which is the prior hypothesis and ψ_h which is the association hypothesis for just the current data set.Using Bayes rule the following recursive relationship can be obtained.

$$P(\Omega_g^{k-1}, \psi_h | Z(k)) = \frac{1}{c} P(Z(k) | \Omega_g^{k-1}, \psi_h) \times P(\psi_h | \Omega_g^{k-1})(\Omega_g^{k-1}) \tag{5.3}$$

The first term is given by

$$P(Z(k) | \Omega_g^{k-1}, \psi_h) = \prod_{m=1}^{M_k} f(m) \tag{5.4}$$

where

$$f(m) = \frac{1}{V} \tag{5.5}$$

if m^{th} measurement is from clutter or the very first or second sighting of a new target. Or,

$$f(m) = N(\mathbf{Z}_m - \mathbf{H}(\bar{\mathbf{x}}), \mathbf{B}) \tag{5.6}$$

if measurement is assigned to come from a confirmed target from prior hypothesis Ω_g^{k-1}. The second term takes care of the probability of current hypothesis given the prior hypothesis. For more details please refer to [8]. This term can be broken into three different terms that take care of three levels of complexity in assignments namely Numbers, Configuration and specific Assignment.

Numbers refers to a set of 4 numbers. N_{DT1} which refers to the number of measurements that have been assigned to already known (initialized) targets being observed for the very first time. N_{DT2} which refers to the number of measurements that have been assigned to uninitialized targets being observed for exactly the second time and are also getting successfully initialized as a result of having received two measurements. N_{NT} refers to the number of uninitialized targets being observed for the very first time.N_{FT} refers to the number of false targets

Configuration takes care of which measurements are from previously known targets, which are from first time observed(uninitialized) targets, which are from second time observed(uninitialized) targets and which are from false targets.

Assignment refers to the specific target that was associated with each measurement. Prior hypothesis already has information about total number of known targets $N_{TGT}(g)$ within coverage of sensor. $N_{TGT}(g)$ includes all the targets that had not yet been initialized and had received only one measurement assignment. Of these only $N_{DT1}+N_{DT2}$ are detected by the sensor. It is assumed that the number of previously known targets that are detected is given by binomial distribution,number of false targets is given by a Poisson distribution, the number of new targets follows a Poisson distribution and the number of uninitialized targets that are being observed for the second time is also given by a Poisson distribution. With these assumptions we may write:

$$P(N_{DT1}, N_{DT2}, N_{FT}, N_{NT}|\Omega_g^{k-1}) = \binom{N_{TGT}}{(N_{DT1}+N_{DT2})} P_D^{N_{DT}} (1-P_D)^{(N_{TGT}-(N_{DT1}+N_{DT2}))} \times F_{N_{FT}}(\beta_{FT}V) F_{N_{NT}}(\beta_{NT}V) F_{N_{DT2}}(\beta_{DT2}V) \tag{5.7}$$

where $P_D =$ probability of detection, β_{FT} is the density of new targets, β_{DT2} is the density of a new target being detected for the second time , β_{NT} is the density of a new previously unknown target being detected for the first time and $F_n(\lambda)$ denotes the Poisson PDF for n events when average rate of arrival is λ. The rest of the derivation trivially mirrors the one given in [8] to arrive at equation 5.1.

In order to derive Equation 5.1 a new assumption was made in addition to the ones mentioned alongside Equation 4.18. We assumed that the arrival of a new target for a second time is Poisson distributed with a rate of arrival β_{DT2}. This implicitly means that we are assuming that un-initialized targets (ie targets that have so far only been assigned only one new measurement) can show up anywhere in the field of view with this given rate of arrival. The tuning of these β's in real world cases would be informed by many different factors. But one simple way of looking at them is that they are tied to how many new targets or false targets the sensor expect to record. It can be intuitively reasoned that it is good to keep the following $\beta_{FT} < \beta_{NT}$ relationship because it will force victory of a hypothesis with a new target being generated only after successive measurements have supported a new target initiation. Very similarly it can also be argued that keeping $\beta_{NT} << \beta_{DT2}$ will arrest the tendency to initiate new targets and basically force the algorithm to wait for successive time steps. In addition to this it is also advisable that all the β's should be of low value. Even if we miss multiple new targets as some time steps and end up assigning those measurements to false alarms they can still be recovered later on when they come back in the field of view. This slowly picking up new targets approach is much more practical than initializing a bunch of new targets that weren't real targets in the first place and then trying to remove them later when they receive no measurement updates later on.

5.2 Numerical Example

This time we simulate the added complication of persistently supplying measurements that do not come from any of the known targets. The modified MHT filter is shown to successfully track multiple targets simultaneously as well as initialize new targets from the measurements that to the filter appear to be coming from none of the known targets. The two body dynamics and a radar-like measurement model as given in 4.19 through 4.24 is again used. The MHT filter is initialized with the states $\mathbf{X}$ of the ten targets in low earth orbit and their state error covariance matrices $\mathbf{P}$ along with the measurement error covariance matrix $\mathbf{R}$.

$$P = diag(\begin{bmatrix} 1^2 & 1^2 & 1^2 & .01^2 & .01^2 & .01^2 \end{bmatrix}) \tag{5.8}$$

$$R = diag(\begin{bmatrix} .1^2 & .02^2 & .02^2 \end{bmatrix}) \tag{5.9}$$

One set of radar measurements, having 4 to 6 measurements, arrives once every 1000 seconds, 20 such sets of measurements were processed during the simulation. First, we present the number of targets in the most likely hypothesis vs time. Because we kept $\beta_{FT} < \beta_{NT} << \beta_{DT2}$ the new targets are getting initialized one at a time. The other thing of importance here is that it takes 6 time steps for the first new target to be initialized. The reason for this is two fold, firstly, it takes time for the hypothesis that initializes a new target to grow its probability with successive correct assignments of measurements. Secondly, a correct new target can be initialized (and eventually

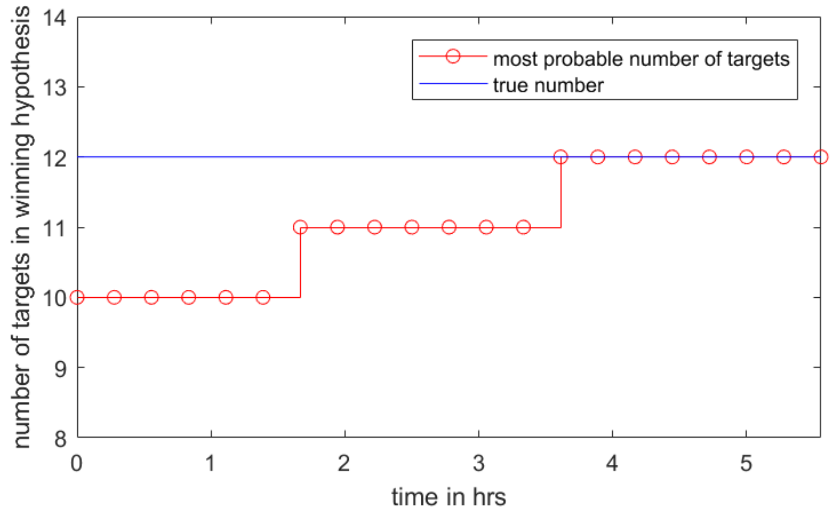

Figure 5.1: Estimated number of objects within the most probable hypothesis through time

survive) only when it is actually a real target not just a fictitious target suggested by combinatorially solving least squares. And because it is not guaranteed that this 'real' but as yet un-initialized target will be seen often, we cannot put a number on when this new target will show up in the winning hypothesis. Also of note is the fact that when it does show up it does so after having been sighted multiple times and thus increasing the likelihood of the corresponding hypothesis enough to make it the winning hypothesis.

Next we look at the bounded-ness of estimated states errors. For this purpose we plot the errors in estimated states vs time and the 3 sigma bounds for each of the 12 targets. Obviously the plots for new targets are shown only after they get initialized in the winning hypothesis. The state estimation errors remain bounded as shown in figure 5.2 through figure 5.25. One concern that can be raised is the growth of the standard deviation for position for some of the targets like target number 6 grows to a large value, Similar growth in standard deviations for the elements of error in velocity vector of some of the targets like target 9 can be raised. But this growth has got largely to do with the combination of lack of new measurement updates and bad initial condition guess, and would have happened even if we had a perfect target measurement association. This will be elaborated in greater detail in following sections.

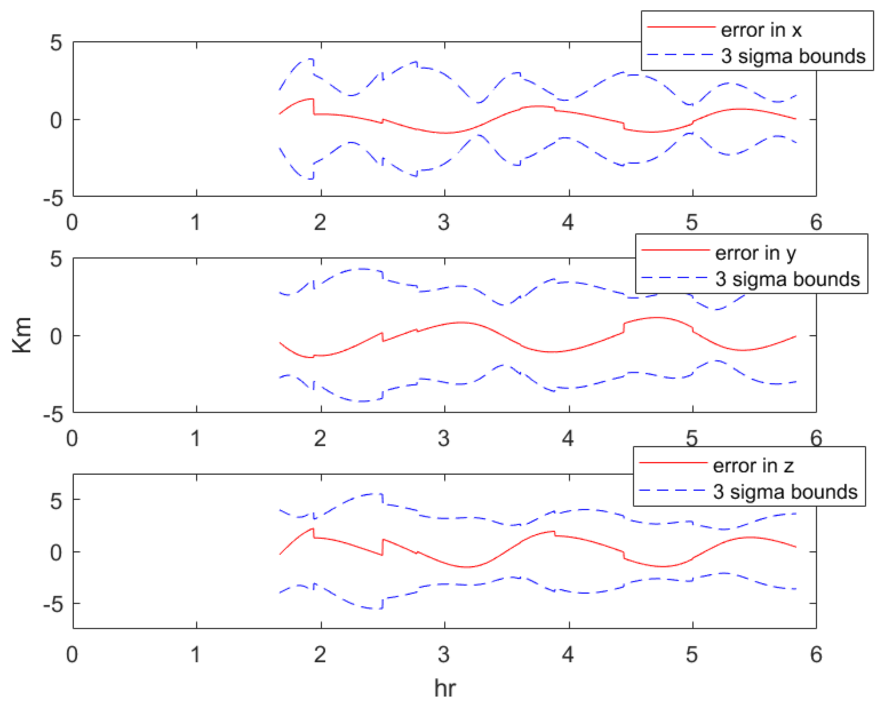

Figure 5.2: Estimation error in position for new target number 1

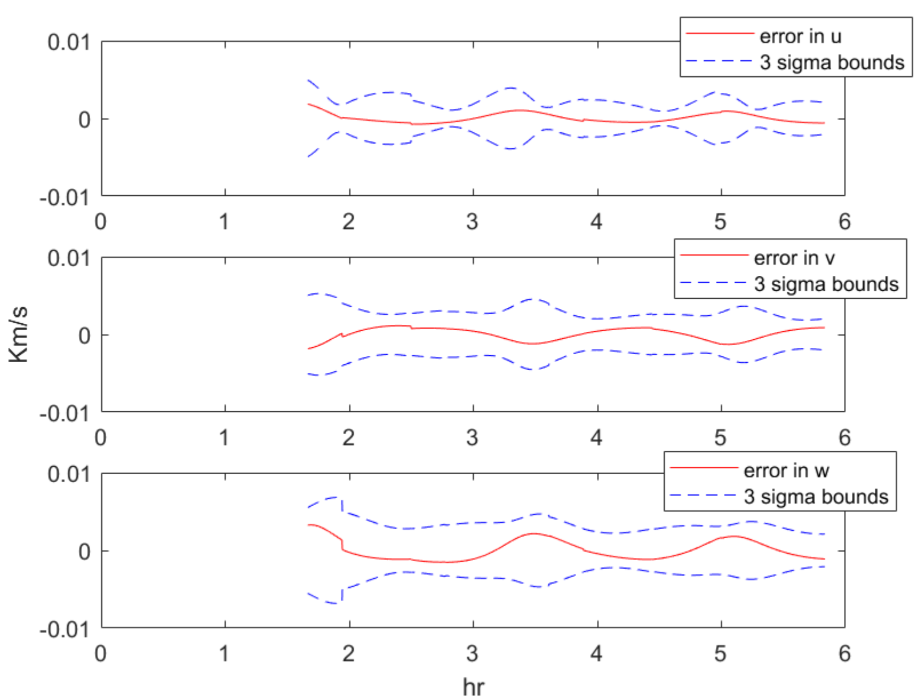

Figure 5.3: Estimation error in velocity for new target number 1

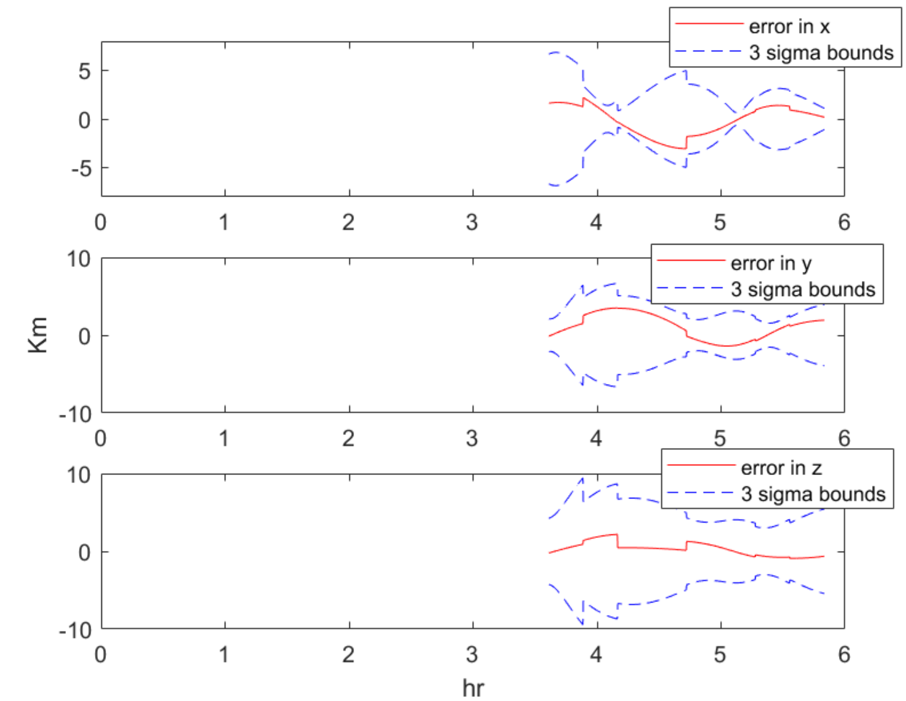

Figure 5.4: Estimation error in position for new target number 2

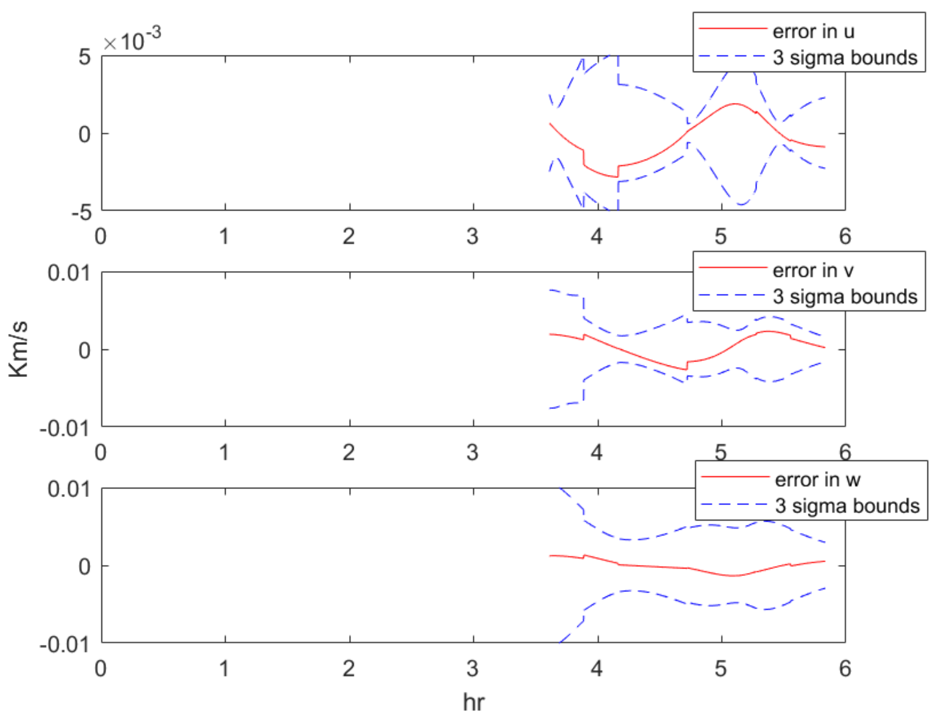

Figure 5.5: Estimation error in velocity for new target number 2

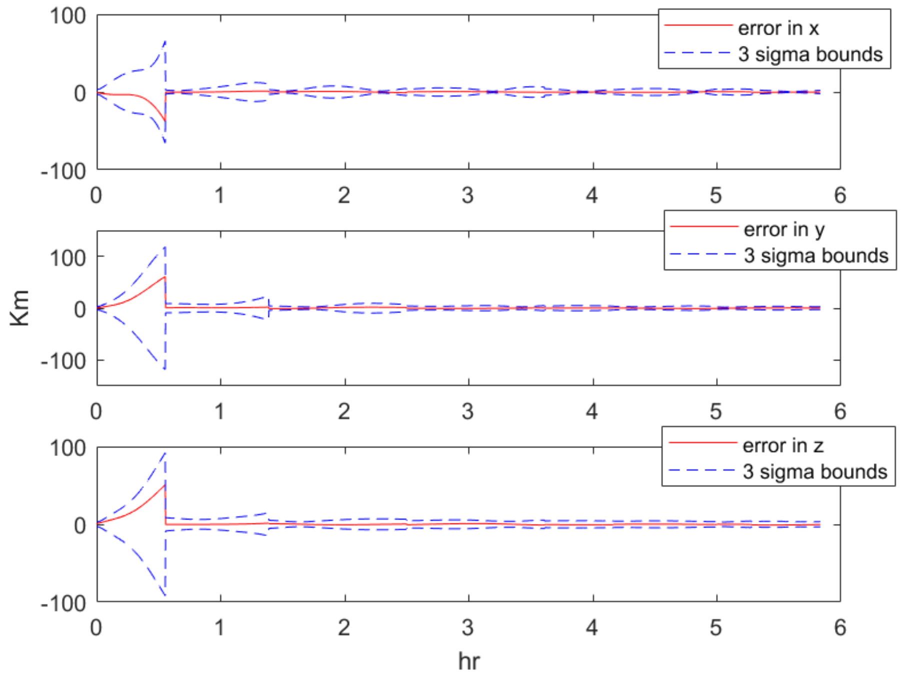

Figure 5.6: Estimation error in position for target number 1

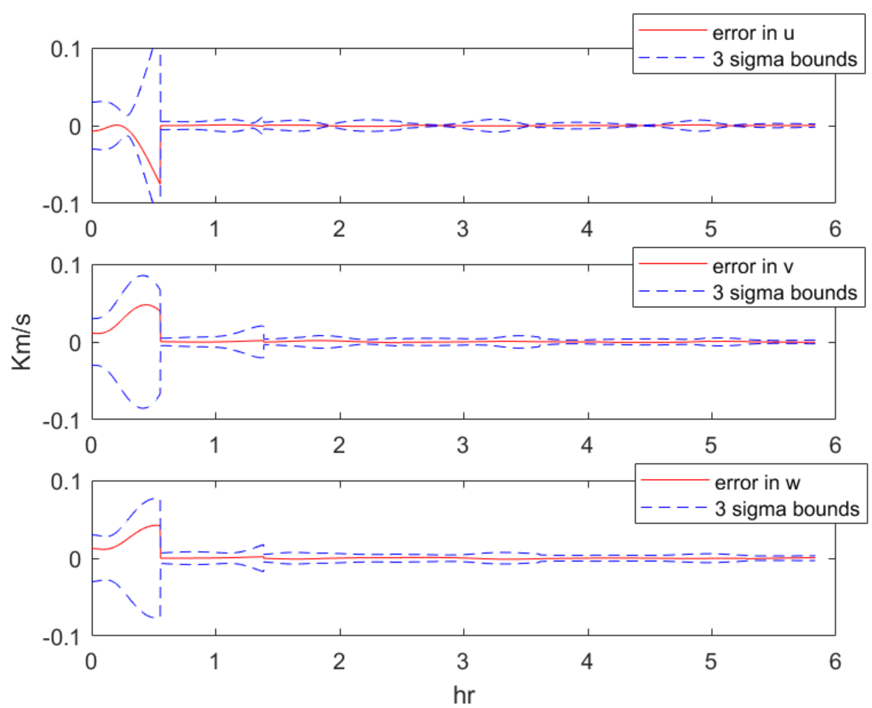

Figure 5.7: Estimation error in velocity for target number 1

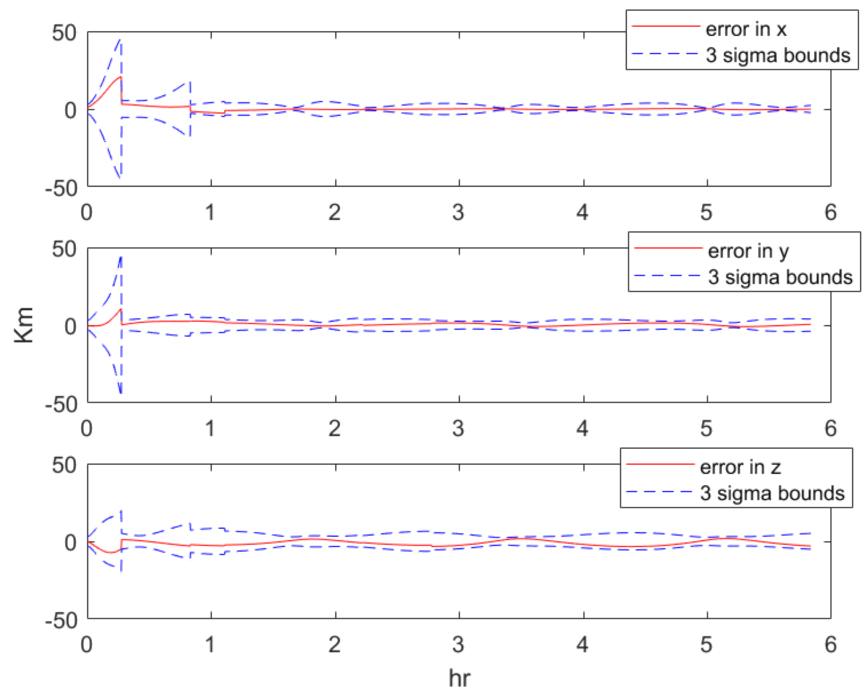

Figure 5.8: Estimation error in position for target number 2

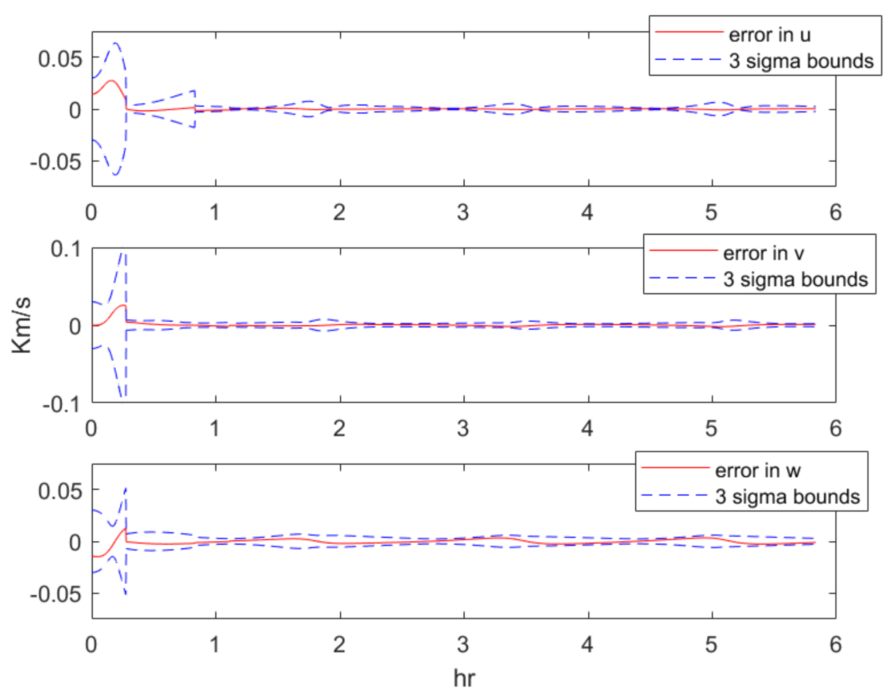

Figure 5.9: Estimation error in velocity for target number 2

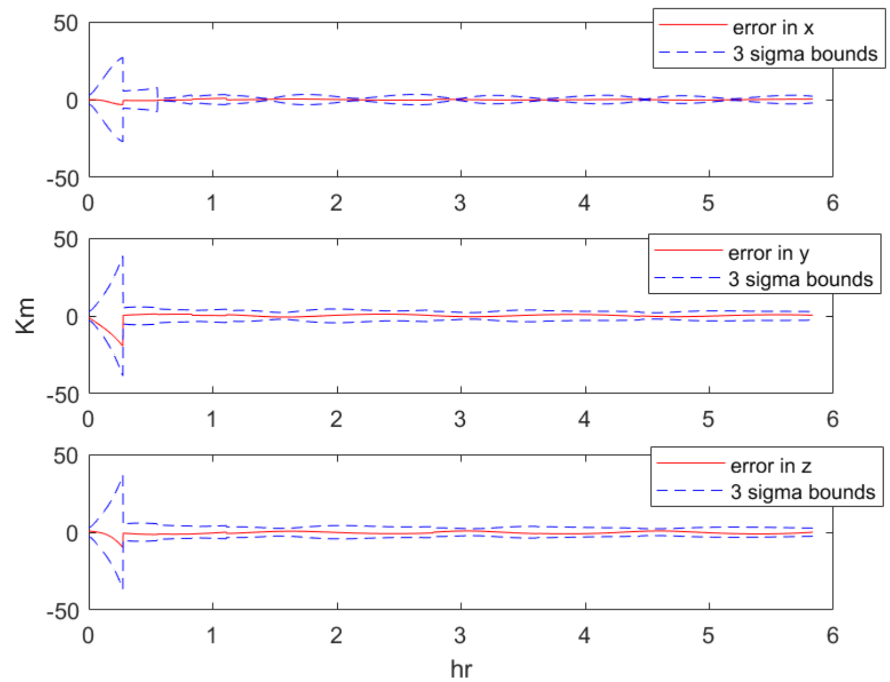

Figure 5.10: Estimation error in position for target number 3

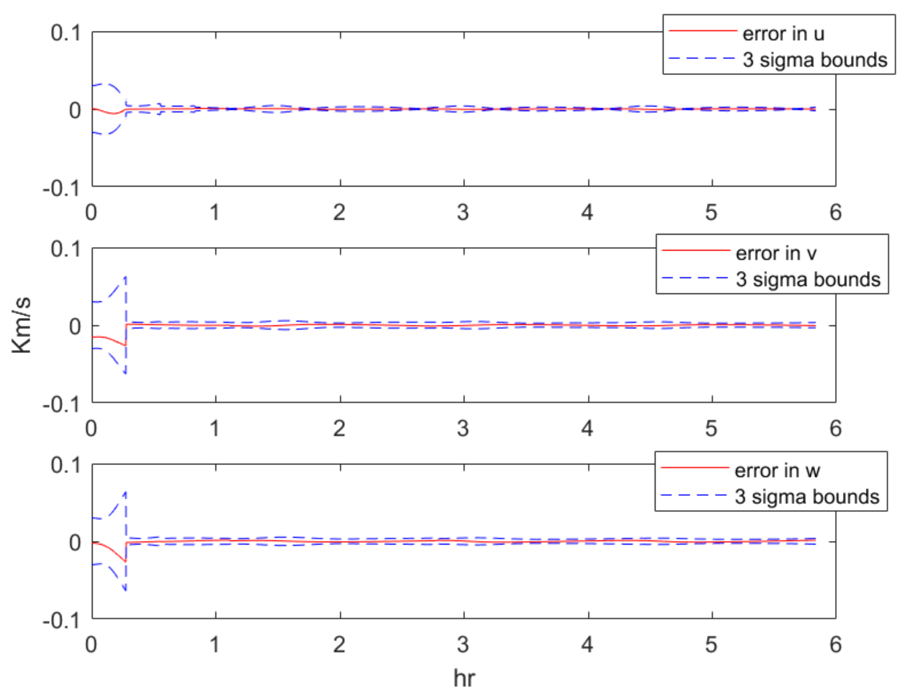

Figure 5.11: Estimation error in velocity for target number 3

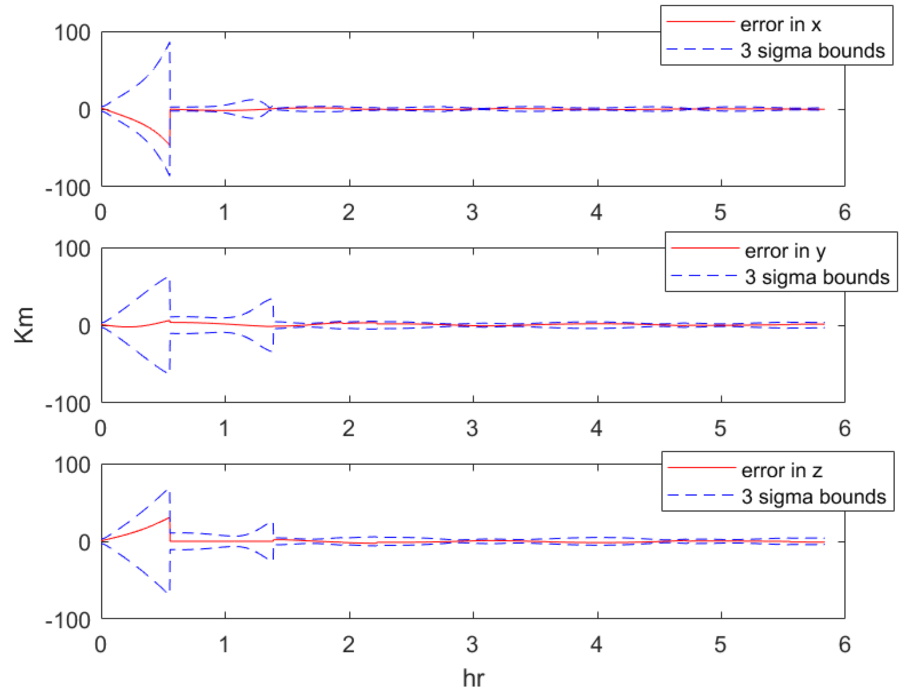

Figure 5.12: Estimation error in position for target number 4

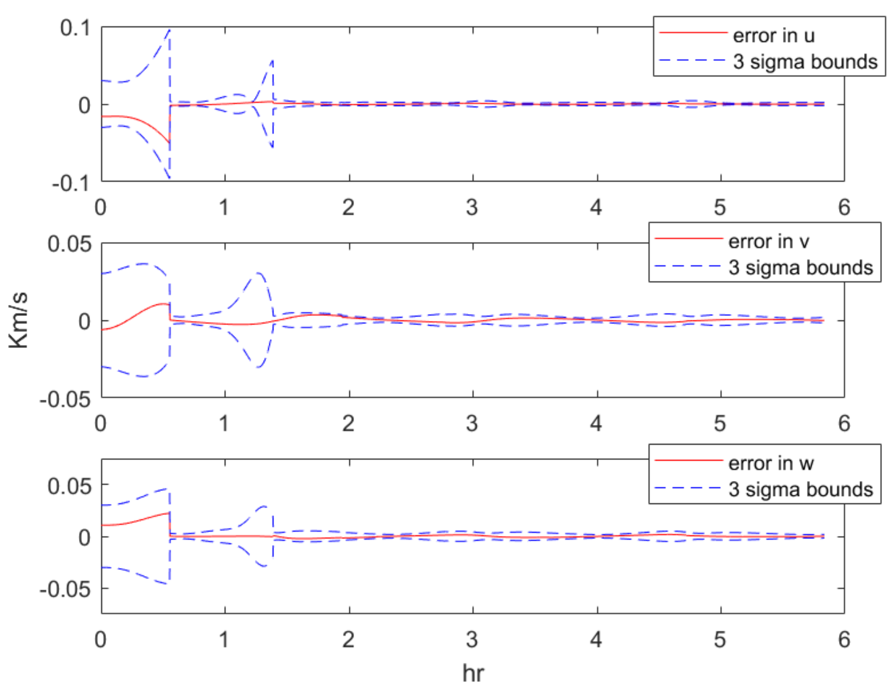

Figure 5.13: Estimation error in velocity for target number 4

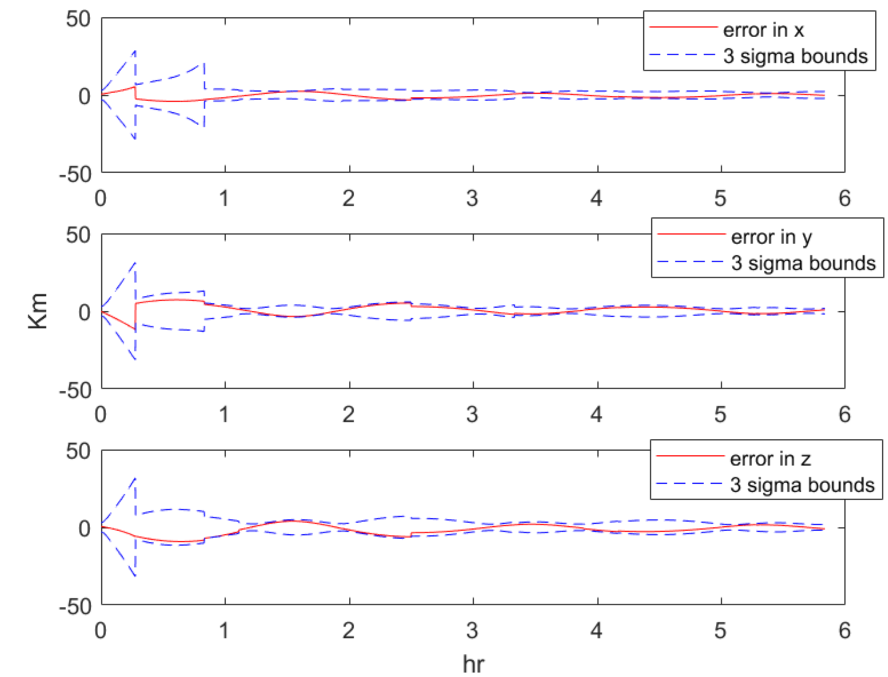

Figure 5.14: Estimation error in position for target number 5

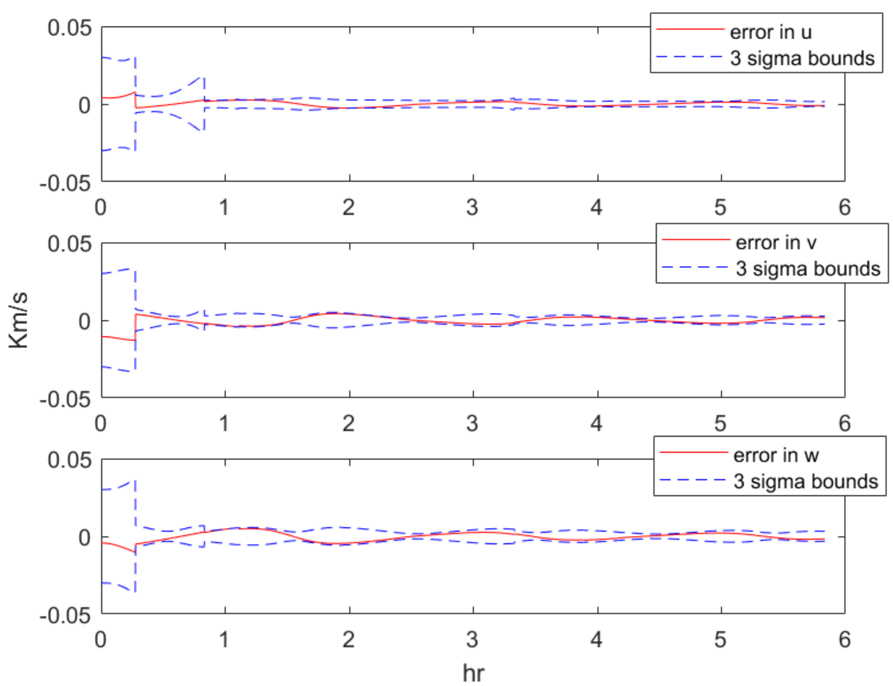

Figure 5.15: Estimation error in velocity for target number 5

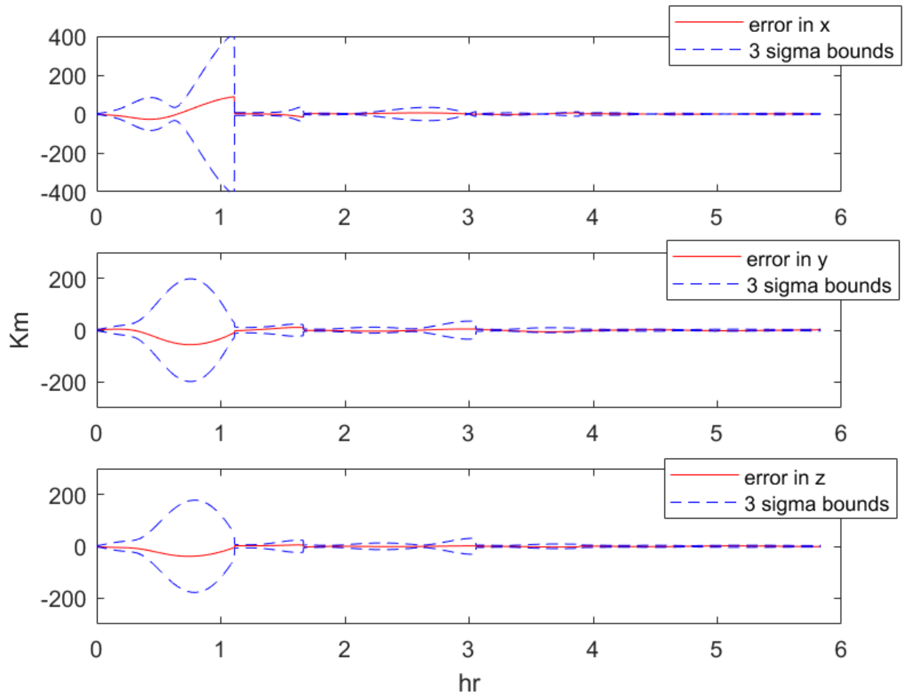

Figure 5.16: Estimation error in position for target number 6

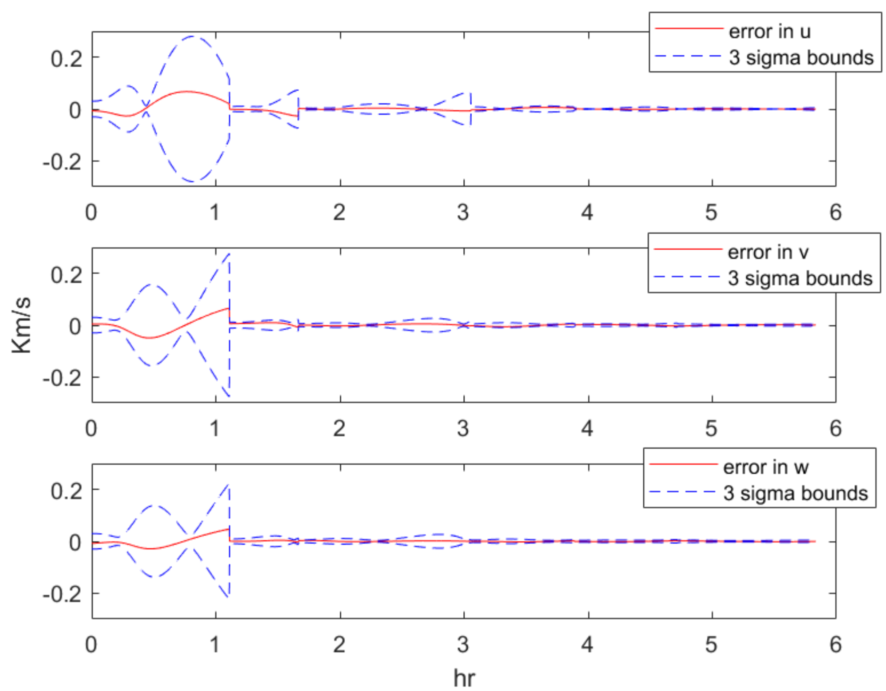

Figure 5.17: Estimation error in velocity for target number 6

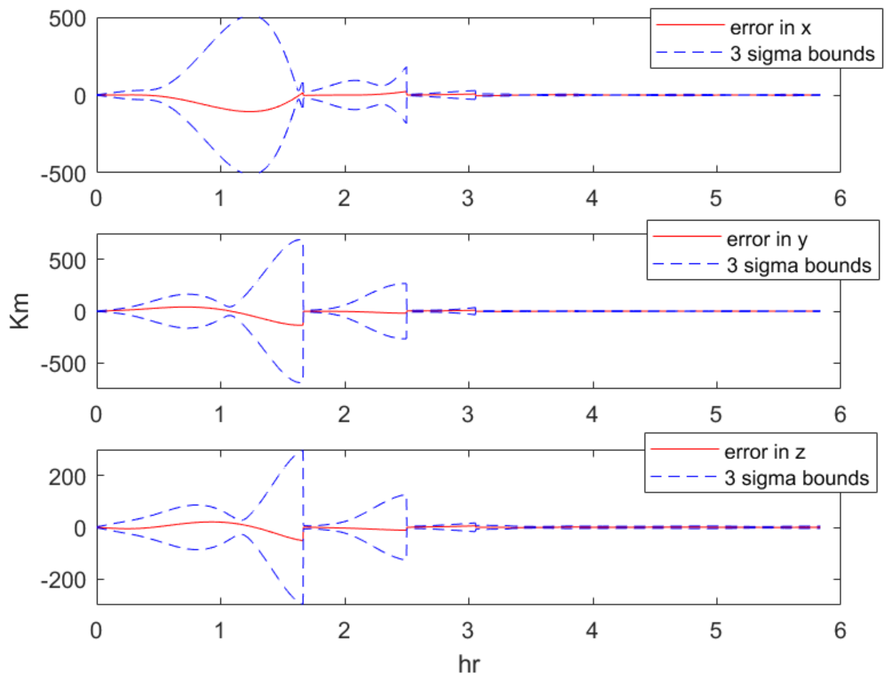

Figure 5.18: Estimation error in position for target number 7

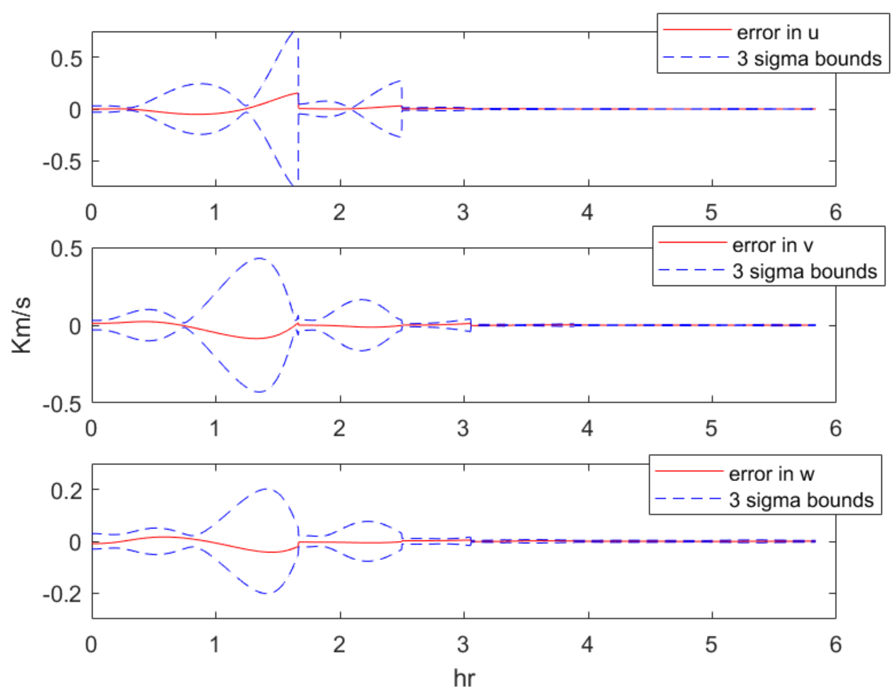

Figure 5.19: Estimation error in velocity for target number 7

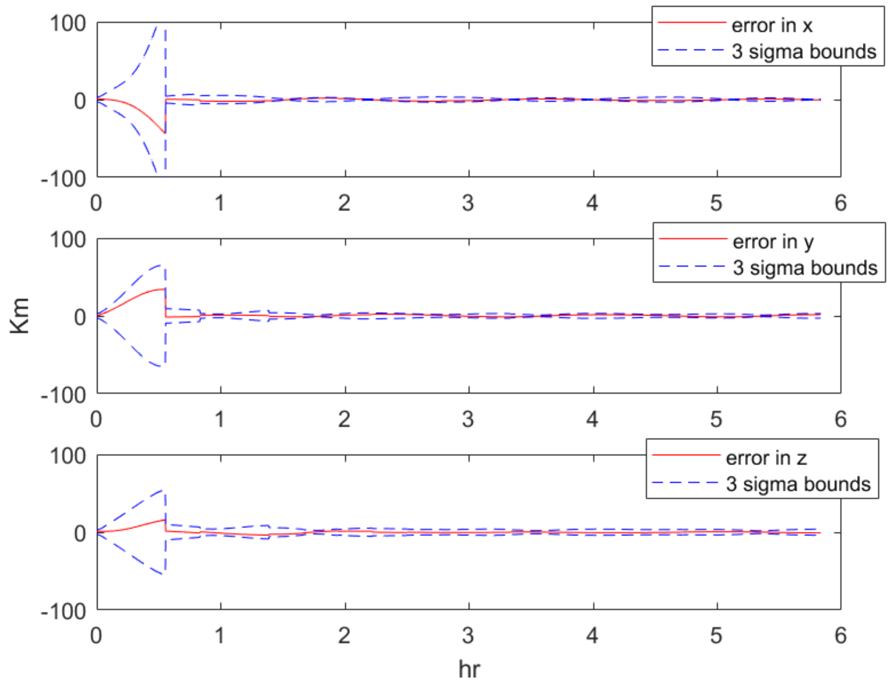

Figure 5.20: Estimation error in position for target number 8

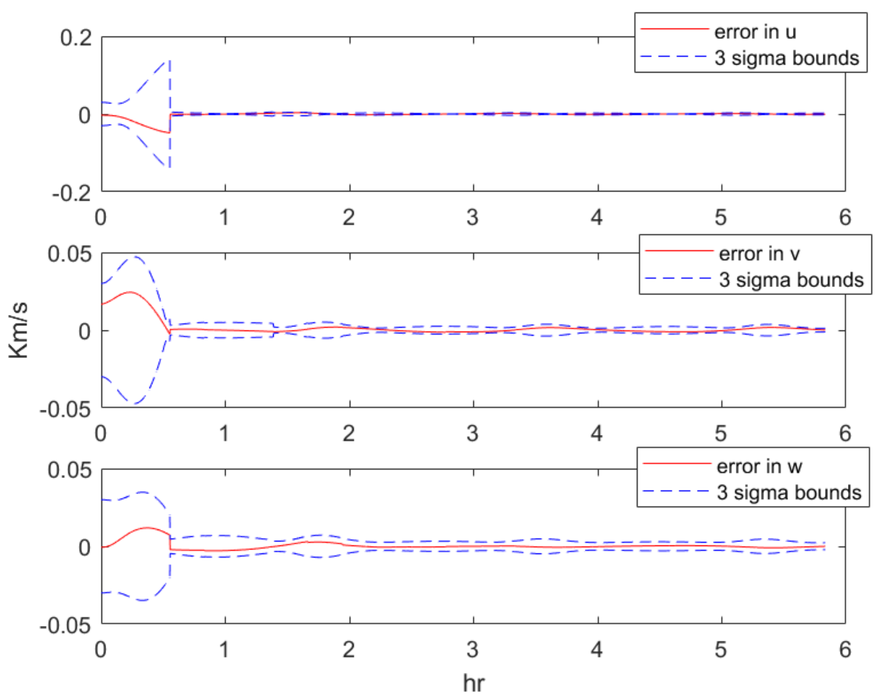

Figure 5.21: Estimation error in velocity for target number 8

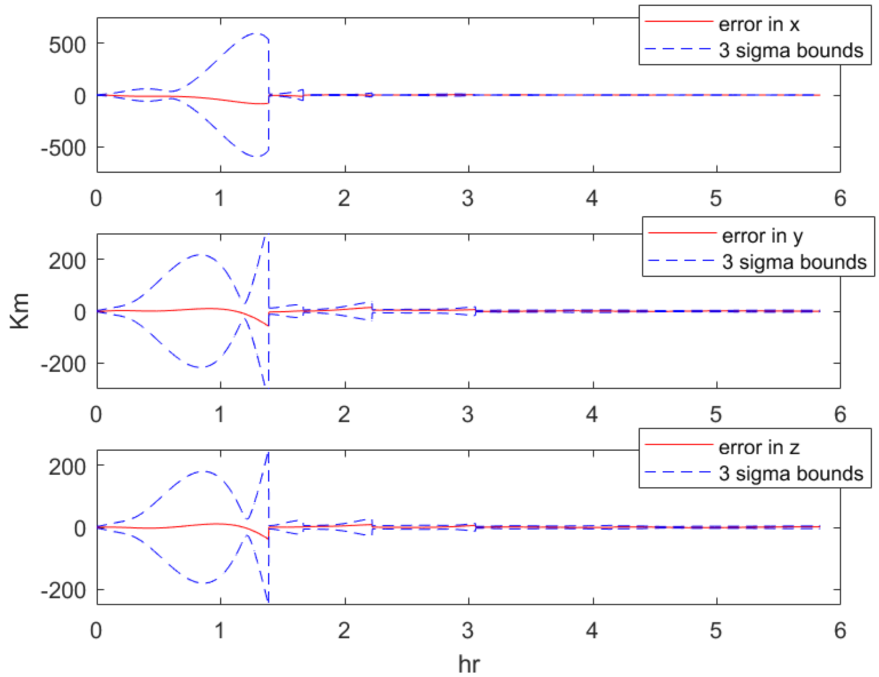

Figure 5.22: Estimation error in position for target number 9

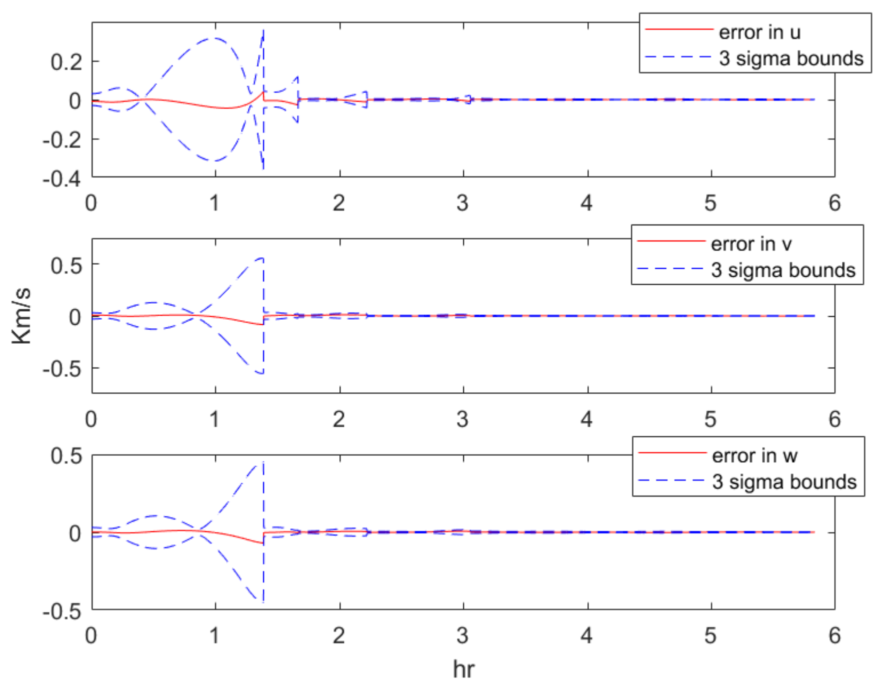

Figure 5.23: Estimation error in velocity for target number 9

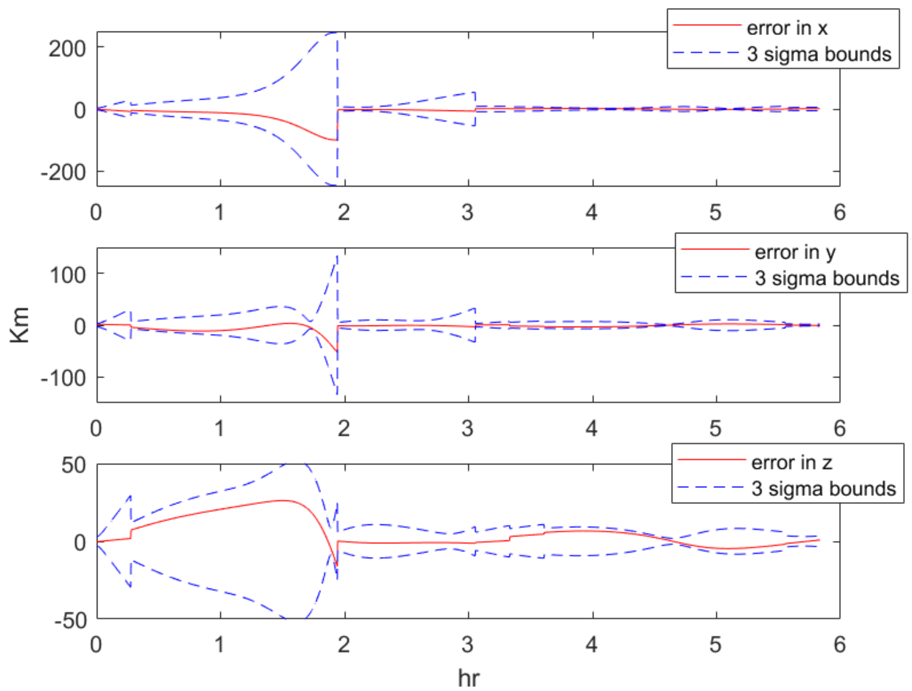

Figure 5.24: Estimation error in position for target number 10

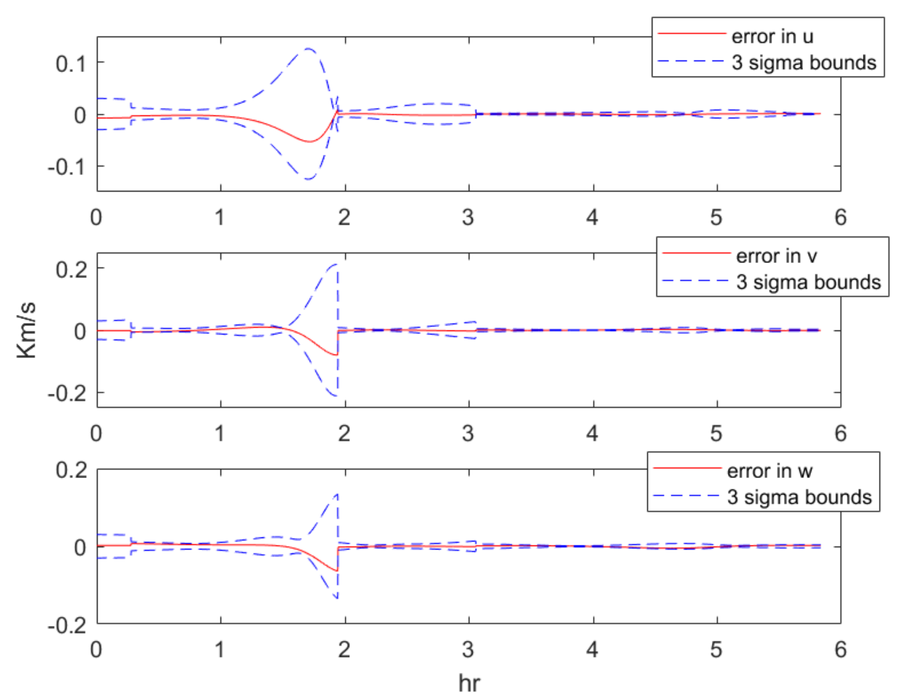

Figure 5.25: Estimation error in velocity for target number 10

Measurement set arriving at first timestep					
Measurement number	1	2	3	4	5
Assigned target	Target 5	Target 2	Target 10	Target 3	False alarm
True origin	Target 5	Target 2	Target 10	Target 3	New Target 1
True-False	T	T	T	T	F

Table 5.1: Assignments for winning hypothesis at time step 1

Measurement set arriving at first time step					
Measurement number	1	2	3	4	5
Assigned target	Target 5	Target 2	Target 10	Target 3	False alarm
True origin	Target 5	Target 2	Target 10	Target 3	New Target 1
True-False	T	T	T	T	F
Measurement set arriving at second time step					
Measurement number	1	2	3	4	5
Assigned target	Target 3	Target 1	Target 4	Target 8	False alarm
True origin	Target 3	Target 1	Target 4	Target 8	New Target 1
True-False	T	T	T	T	F
Measurement set arriving at third time step					
Measurement number	1	2	3	4	5
Assigned target	Target 3	Target 8	Target 5	Target 2	False alarm
True origin	Target 3	Target 8	Target 5	Target 2	New Target 2
True-False	T	T	T	T	F

Table 5.2: Assignments for winning hypothesis at time step 3

5.3 Differences in assignments over time

Now let us look at how well each winning hypothesis at any given time step was able to associate all the preceding measurements up-to its current time-step. It is important to list out all the prior measurement-target assignments because the current winning hypothesis does not always

Measurement set arriving at first time step					
Measurement number	1	2	3	4	5
Assigned target	Target 5	Target 2	Target 10	Target 3	False alarm
True origin	Target 5	Target 2	Target 10	Target 3	New Target 1
True-False	T	T	T	T	F
Measurement set arriving at second time step					
Measurement number	1	2	3	4	5
Assigned target	Target 3	Target 1	Target 4	Target 8	New Target 1
True origin	Target 3	Target 1	Target 4	Target 8	New Target 1
True-False	T	T	T	T	T
Measurement set arriving at third time step					
Measurement number	1	2	3	4	5
Assigned target	Target 3	Target 8	Target 5	Target 2	New Target 2
True origin	Target 3	Target 8	Target 5	Target 2	New Target 2
True-False	T	T	T	T	T

Table 5.3: Assignments up to time-step 3 for winning hypothesis at time step 7

have the prior time step winning hypothesis as its parent node. This usually implies that the the best target-measurement assignments at previous time-steps keeps on changing as more information arrives. For the sake of brevity let us do this exercise over a few time steps. In the following tables we show the assignments for 3 winning hypothesis at different time steps. In table 5.1 we have shown the assignments for the only available set of measurements. In table 5.2 we have shown the assignments for the three available sets of measurements while for winning hypothesis at time step 7 in table 5.3 we have shown assignments only up to time step 3. Remember, for the winning hypothesis at time step 3 there are actually 7 sets of measurements but we are showing assignments only for first 3 of them brevity.

A cursory look at Tables 5.1,5.2, and 5.3 show us that for the winning hypothesis almost always the associations are correct. This is largely because of the penalty imposed on the likelihood function which punishes associations with measurements that are not close by and also due to gating as we discussed in the previous chapter. Of course if two targets come really close then it would be difficult to differentiate them so easily and in such case a mis-assignment may be possible but we discussed such scenarios in previous chapter. Here, let us focus on the measurement number 5 from the first set of measurements. In all the three tables it is wrongly associated with false alarm even though it comes from a new as yet unknown target. But see what happens to measurement number 5 in measurement set 2 and 3. For table 5.2 they are assigned to false alarm while for table 5.3 they assigned to two separate new targets. This demonstrates how with further information MHT may recover from incorrect assignments.

5.4 Hypothesis merging/grafting

Figure 5.26 shows how a MHT filter grows with multiple new hypotheses being born from each prior hypothesis, with each having a unique association history. Even for trivial problems number of hypothesis quickly grows to unmanageable numbers. The motivation to develop hypothesis likelihood equations like Equation 4.18 and 5.1 was to prune this growth by removing hypotheses with low likelihood as well as pick the most likely hypothesis at each time step. But this approach runs into a roadblock when a bunch of hypothesis end up getting equal likelihoods. In other words the likelihood becomes uniformly distributed over a bunch of hypotheses. Careful observation reveals this happens because the hypotheses with equal likelihoods are very similar and differ only in few trivial mis-associations from each other. This essentially means that these similar hypothesis have almost the same state information about targets. We had discussed earlier that for a hypothesis introduction of new targets pays off and ensures its survival when the new target initiation proves to be valid with future measurements. But with a bunch of uniformly likely hypothesis their prior probabilities are already low and hypothesis new target will almost never be picked up unless a very large number of hypothesis are propagated and processed every time step. And the unnecessary competition between very similar hypotheses carries a computational burden. One easy way around this is to merge similar hypotheses and assign the new hypotheses the cumulative probability of all the merging hypotheses. The suddenly increased weight of this hypothesis helps ensure that its children hypotheses that have a new target assignment survive for

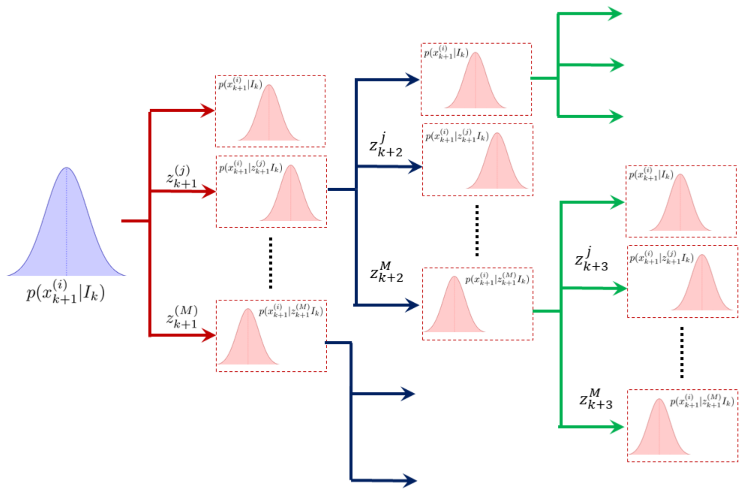

Figure 5.26: Multi-Hypothesis 'tree'

a few time steps instead of getting washed off within one time step. This is exactly what was done in this simulation where at time step 10 a bunch of hypothesis, which had exactly 11 targets and each of those targets had similar states, were combined to form a single hypotheses. This yielded quick results with the initialization of New target 2 within the next few time steps as can be seen in Figure 5.1.

6. CONCLUSION

A modified Multi-Hypothesis Tracking algorithm that accommodates a two time-step target initialization using a Lambert problem solver was presented in this thesis. The approach uses an extension of Reid's algorithm for MHT. The efficacy of the algorithm in tracking multiple targets simultaneously and identifying, initializing and tracking as yet unknown targets was demonstrated via a numerical example.

Important issues in implementing algorithm namely growth in number of hypotheses and uniform probability distribution over many hypotheses were discussed and their solutions were suggested, implemented and shown to work satisfactorily.

The approach presented in this thesis can easily be extended to other multi-target tracking problems whose measurement models and dynamics allow for special initialization techniques. An obvious extension of this work would be Resident Space Object tracking using telescope measurements which requires three successive measurements to initialize a target. In a broader sense the basic MHT architecture can also be used as a wrapper around any multi-target tracking problem.

It is felt that this body of work would benefit from applying better sampling algorithms to sample the hypotheses. Any improvement in sampling more likely hypotheses would go a long way in reducing the computational complexity. On the theoretical side, MHT suffers from inability to allow for targets being lost/dying off. Improvements are possible by moving on to R-FISST which is a more general and mathematically more rigorous framework than MHT.

REFERENCES

[1] D. A. Vallado and W. D. McClain, *Fundamentals of Astrodynamics and Applications*. Space Technology Library, Microcosm Press, 2001.

[2] U. Mishra, N. Adurthi, M. Majji, and P. Singla, "Multiple hypothesis tracking and joint probabilistic data association filters for multiple space object tracking," in *2018 AIAA/AAS Astrodynamics Specialist Conference, Snowbird, UT*, 2018.

[3] N. Adurthi, P. Singla, and T. Singh, "The conjugate unscented transform - an approach to evaluate multi-dimensional expectation integrals," in *Proceedings of the IEEE American Control Conference*, (Montreal, Canada), 2012.

[4] N. Adurthi, M. Majji, U. Mishra, and P. Singla, "Conjugate unscented transform based joint probability data association," in *2017 AIAA/AAS Astrodynamics Specialist Conference, Stevenson, WA*, 2017.

[5] N. Adurthi and P. Singla, "Conjugate unscented transformation based approach for accurate conjunction analysis," *Journal of Guidance Control and Dynamics*, vol. 38, no. Battin Special Issue, pp. 1642–1658, 2015.

[6] R. Kalman, "A new approach to linear filtering and prediction problems," *ASME Journal of Basic Engineering*, vol. 82, pp. 35–45, 1960.

[7] M. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking," *Signal Processing, IEEE Transactions on*, vol. 50, pp. 174 –188, feb 2002.

[8] D. B. Reid, "An algorithm for tracking multiple targets," *IEEE Transactions on Automatic Control*, vol. 24, no. 6, pp. 843–854, 1979.

[9] N. Chenouard, I. Bloch, and J.-C. Olivio-Marin, "Multiple hypothesis tracking for cluttered biological image sequences," *IEEE Transactions of Pattern Analysis and Machine Intelli-*

gence, vol. 35, no. 11, pp. 2736–2750, 2013.

[10] I. J. Cox and S. L. Hingorani, "An efficient implementation of reid's multiple hypothesis tracking algorithm and ites evaluation for the purpose of visual tracking," *IEEE Transactions of Pattern Analysis and Machine Intelligence*, vol. 18, no. 2, 1996.

[11] S. S. Blackman, "Multiple hypothesis tracking for multiple target tracking," *IEEE Aerospace and Electronic Systems Magazine*, vol. 19, pp. 5–18, Jan 2004.

[12] C. Kim, F. Li, A. Ciptadi, and J. M. Rehg, "Multiple hypothesis tracking revisited," in *Proceedings of the IEEE International Conference on Computer Vision*, pp. 4696–4704, 2015.

[13] D. Reid, "An algorithm for tracking multiple targets," *IEEE transactions on Automatic Control*, vol. 24, no. 6, pp. 843–854, 1979.

[14] Y. Bar-Shalom, F. Daum, and J. Huang, "The probabilistic data association filter," *IEEE Control Systems*, vol. 29, pp. 82–100, Dec 2009.

[15] P. Li and Y. Wang, "Object tracking based on particle filter with data association," in *Advances in Image and Graphics Technologies* (T. Tan, Q. Ruan, S. Wang, H. Ma, and K. Di, eds.), (Berlin, Heidelberg), pp. 104–112, Springer Berlin Heidelberg, 2015.

[16] A. Milan, K. Schindler, and S. Roth, "Detection and trajectory-level exclusion in multiple object tracking," in *Proceedings of the 2013 IEEE Conference on Computer Vision and Pattern Recognition*, 2013.

[17] A. Milan, S. Roth, and K. Schindler, "Continuous energy minimization in multitarget tracking," *IEEE Transactions of Pattern Analysis and Machine Intelligence*, vol. 36, no. 1, pp. 58–72, 2014.

[18] J. Roecker, "Multiple scan joint probabilistic data association," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 31, no. 3, pp. 1204–1210, 1995.

[19] J. Roecker and G. Phillis, "Suboptimal joint probabilistic data association," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 29, no. 2, pp. 510–518, 1993.

[20] T. E. Fortmann, Y. Bar-Shalom, and M. Scheffe, “Sonar tracking for multiple targets using joint probabilistic data association,” *IEEE Journal of Oceanic Research*, vol. OE-8, pp. 173–184, 1983.

[21] S. Oh, S. Russell, and S. Sastry, “Markov chain monte carlo data association for multi-target tracking,” *IEEE Transactions on Automatic Control*, vol. 54, no. 3, pp. 481–497, 2009.

[22] R. J. Fitzgerald, “Track biases and coalescence with probabilistic data association,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-21, pp. 822–825, Nov 1985.

[23] W. Fleskes and G. van Keuk, “On single target tracking in dense clutter environment - Quantitative results,” in *Radar - 87; Proceedings of the International Conference*, pp. 130–134, 1987.