ESSAYS IN FINANCIAL CRISIS AND THE RESOLUTIONS

A Dissertation

by

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ABSTRACT

This dissertation includes three sections, all study the issues related to the financial crisis.

The Section Two challenges the conventional wisdom that government spending is effective and labor tax cuts are not effective, when interest rate is at lower bound (LB). I use a dynamic stochastic general equilibrium (DSGE) New Keynesian model with labor search and matching frictions to evaluate the effectiveness of fiscal policy. Labor search and matching frictions generate a steep aggregate supply curve, which changes the comparative curvature to the aggregate demand curve at the lower bound. Labor tax cuts are expansionary rather than contractionary, while government spending is not effective. The policy implication is that firms' supply side factors should be considered when conducting fiscal policy.

After the Financial Crisis in 2008, the Gross Domestic Product (GDP) of the United States recovered only slowly to its pre-crisis level. There are two notable phenomena after the Great Recession: the fall in the labor force participation rate and the decline of the growth rate of Total Factor Productivity (TFP). In Section Three, I build a Dynamic Stochastic General Equilibrium (DSGE) model that includes endogenous growth and dynamic labor market components (including labor force participation), the first of its kind in the literature. A Bayesian estimation is applied to this model. I find four shocks (a monetary policy shock, a government spending shock, a financial related shock, and a labor productivity shock) can explain most of the variation in GDP that occurred after the financial crisis.

Section Four evaluates and backtest commonly used Value-at-Risk(VaR) methods in practice, for three portfolios of assets: equities, bonds and currencies. Specifically, I compute one-day-ahead forecasts for the time period from 2001 to 2018, and compare them to the realized daily profits and losses of these portfolios. Portfolio-based methods like Variance-Covariance produce very conservative estimates in general. The performances of VaR methods are related to the VaR confidence interval, underlying portfolio and the sample periods. The underlying properties of the distribution of a portfolio should be taken into consideration when choosing the VaR method.

DEDICATION

To my mother, who supported me for the pursuit of knowledge.

To my family, who raised me up.

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1. INTRODUCTION

This dissertation dicusses issues related Financial Crisis, these issues do not arise so significant in previous recessions and have new features that need empirical studies.

In the first paper, I discuss the effectiveness of different fiscal policies when the nominal interest rate has a lower bound. Particular attention is paid to study the government spending multiplier, that is how much output increases when one dollar is spent on government spending. Some literature finds that government spending can be excessively large when there is a lower bound on interest rate. I use a New Keynesian DSGE model with labor search and matching frictions to show the effects of labor market frictions on fiscal policy effectiveness. The additional employment and production stickiness changes firm's responsiveness to an increase in aggregate demand, as represented by government spending. The firm competes intensely on prices and reluctant to adjust prices back to pre-recession level, as a result government spending does not stimulate the economy out of recession. I find labor tax cuts to be the most effective fiscal policy at the lower bound. It directly gives incentive for firms to increase production and increase prices, and the economy is boosted.

In the second paper, I look at the scenario during the Financial Crisis in 2008, the Gross Domestic Product (GDP) of the United States recovered only slowly to its pre-crisis level. There are two notable phenomena after the Great Recession: the fall in the labor force participation rate and the decline of the growth rate of Total Factor Productivity (TFP). I build a Dynamic Stochastic General Equilibrium (DSGE) model that includes endogenous growth and dynamic labor market components (including labor force participation), the first of its kind in the literature. A Bayesian estimation is applied to this model. I find four shocks (a monetary policy shock, a government spending shock, a financial related shock, and a labor productivity shock) can explain most of the variation in GDP that occurred after the financial crisis.

In the third paper, I evaluate and backtest commonly used Value-at-Risk(VaR) methods in practice, for three portfolios of assets: equities, bonds and currencies. Specifically, I compute one-day-ahead forecasts for the time period from 2001 to 2018, and compare them to the realized daily profits and losses of these portfolios. Portfolio-based methods like Variance-Covariance produce very conservative estimates in general. The performances of VaR methods are related to the VaR confidence interval, underlying portfolio and the sample periods. The underlying properties of the distribution of a portfolio should be taken into consideration when choosing the VaR method.

2. FISCAL POLICY AT THE LOWER BOUND WITH LABOR SEARCH AND MATCHING FRICTIONS

2.1 Introduction

The history of a zero lower bound (ZLB)[1] on the interest rate can be dated back to the Great Depression, when the U.S. economy went through a catastrophic setback. The exact ZLB period lasted longer than the recession, from 1931 to 1945[2]. For a recent example, from December 2008 to December 2015, the Federal Reserve kept the Federal Funds Rate around zero in response to the 2007 Financial Crisis. The LB for interest rate is also a common modern phenomenon internationally. With the bursting of Japan's bubble in equities and real estate in the early 90s, the Bank of Japan has kept its interest rate around zero from 1998. In Europe, the European Central Bank has implemented zero interest rate for a variety of E.U. countries from September 2014. Reoccurrences of a binding on the interest rate are thought to be possible when future recessions hit the economy.

While the nominal interest rate is at the LB, there are several of questions concerning economists and policy makers during economic recessions. What kind of monetary and fiscal policy should be implemented? Should we stimulate aggregate demand or stimulate aggregate supply, or even restrict aggregate supply? When the U.S. and Europe faced a liquidity trap as the interest rate fell to the lower bound (LB)[3], they adopted unconventional monetary policy like quantitative easing to provide extra liquidity. At the same time, governments implemented expansionary fiscal policy like massive government spending [4]. What is the effect of government spending on the economy, especially at the lower bound? What are the potential effects of other fiscal policy? This paper

[1] I consider a general lower bound rather than a specific zero lower bound in this article. For various reasons, monetary authorities often commit to a lower bound which can be higher or lower than zero. For example Bank of Japan, European Central Bank in Sweden, Switzerland and Denmark have adopted negative interest rate in 2016.

[2] See Ramey and Zubairy (2017) for the identified ZLB period using historical data.

[3] Another example is for countries in a currency union or states within a country. They can lose the independence of monetary policy for the obvious reason. (see Farhi and Werning 2016 for a discussion)

[4] In 2009 the U.S. Congress passed The American Recovery and Reinvestment Act measured at $787 billion. On average European countries saw a rise in government spending in year 2009 and 2010, then a decline afterwards.

attempts to answer these questions.

In this paper I analyze the efficiency of fiscal policy in the framework of a New Keynesian model [5]. For a relevant example, Eggertsson (2011) finds that government spending generates an output multiplier more than three times that of labor tax cuts in normal times; while at the lower bound, government spending is expansionary and effective while labor tax cuts are contractionary. In this paper, I adopt the same New Keynesian framework as Eggertsson, but extend it with labor search and matching frictions. I find labor tax cuts have similar effects as government spending in normal time; but at the lower bound, labor tax cuts are expansionary and inflationary, and much more effective than government spending.

Labor search and matching frictions are shown in the literature to have important applications in business cycle models. Christiano, Eichenbaum and Trabandt (2016) develop a general equilibrium labor search and matching model without wage inertia, and show it outperforms the standard New Keynesian Calvo sticky wage model in accounting for key business cycle properties of macroeconomic aggregates. In this paper I show that including labor search and matching generates insights about the effectiveness of fiscal policy at the lower bound. Labor search and matching frictions were initially used by Diamond (1982), Mortensen (1982) and Pissarides (1985) to account for labor flows and unemployment dynamics. They are also used in Shimer (2005) and Hall (2005) to show that combining them with wage rigidity can help explain the business cycle properties of unemployment. In this paper, by extending Eggertsson (2011) with a Diamond-Mortensen-Pissarides (DMP) labor search and matching process, I show that labor market frictions play a key role in analyzing fiscal policy effectiveness in the New Keynesian model. This labor market friction generates a strong channel of employment stickiness, as firms adjust labor less than price when facing an exogenous shock. This results in a steeper aggregate supply (AS) curve, so that the elasticity of price is much bigger than the elasticity of labor. At the lower bound, when the aggregate demand (AD) curve becomes upward sloping, a steeper AS curve gives results starkly in contrast to Eggertsson's (2011) case.

[5] See Christiano, Eichenbaum and Rebelo (2011), Eggertsson (2011), Woodford (2011) and Eggertsson and Krugman (2012) for illustrations of effective government spending at the lower bound

In addition to finding a novel performance of government spending and labor tax cuts, my model also suggests a new direction of fiscal policy at the lower bound. An increase in government spending stimulates AD greatly, but its effect on output is largely mitigated by the steep slope of the AS curve. Instead, I find in this paper that most of any increase in output comes from stimulating aggregate supply. This can happen due to a cut in the labor tax rate that directly reduces the firms' marginal costs, or due to the crowding out of private consumption by government spending which decreases the marginal rate of substitution between consumption and labor, and increases labor supply.

This is not the only paper to find that labor tax cuts are more effective than government spending at the lower bound. Mertens and Ravn (2015) assume a simple New Keynesian model but with an exogenous confidence shock to the households. They show that this exogenous confidence shock can push the aggregate demand curve into a much flatter slope. As a result of this, government spending has deflationary effects that reduce the output, while a cut in marginal labor tax rate is expansionary at the case of lower bound[6]. The net result is similar in the present paper, but is driven by the structure of the economy, rather than a specific choice. The driving force here is labor search and matching frictions, and its effect rests on aggregate supply instead of aggregate demand.

There is a mixture of results for the effectiveness of fiscal policy from the empirical literature. Barro (1981) argues that the output multiplier of government spending is around 0.8, while Ramey (2008) estimates the multiplier to be close to 1.2. There are few empirical studies on government spending multiplier for the zero lower bound period. For the U.S., Ramey and Zubairy (2017) find the multiplier to be below unity irrespective of the amount of slackness of the economy. For the zero lower bound, the results are more mixed with a few specifications implying multipliers as high as 1.5. For the zero lower bound period in Japan, Miyamoto et. al (2016) find an on-impact output multiplier to be 1.5 and 0.6 out of the recession state. Mertens and Ravn (2013) use a

[6]Some paper find inefficient government spending but do not focus on labor tax cuts. See Baxter and King (1993) for a Real Business Cycle model with a distortionary government tax on households. Monacelli, Perotti and Trigari (2010) use a similar labor search and matching setting, (without a lower bound constraint), and find that it is hard to reconcile a government spending multiplier above unity even with additional components

narrative approach with structural vector autoregressions, and find that a 1 percentage point cut in the average personal income tax rate raises real GDP per capita by 1.4 percent on impact and by up to 1.8 percent after three quarters.

The contribution of this paper is to study all four components together: i) the New Keynesian Model; ii) labor search and matching frictions; iii) a lower bound on interest rate; and iv) fiscal policy. To my knowledge, this is the first time in the literature that a study considers all those components together. I follow Eggertsson (2011) closely, reducing the model to a two-dimensional diagram with output and inflation. The dynamics of an exogenous fiscal policy shock can be seen from the shift of the AS and AD curves. One advantage of this approach as comparing to the normal DSGE model with data application is that it gives additional channel of AS/AD to analyze the economy. Taking the model to the data can be challenging because quantitative easing was often implemented with fiscal policy when the interest rate fell to zero after the financial crisis. By extending labor search and matching frictions, I can accompany the employment stickiness in this model and show its effect on fiscal policy. This paper also adds another story to the conventional New Keynesian model, where researchers often find stimulating aggregate demand from government spending can be very effective.

In addition to the striking difference in the results of labor tax cuts and government spending at the LB, as compared to Eggertsson's (2011) work, when the AS curve is steeper than the AD curve, the rightward shift of the AD curve leads to an contractionary output change. In this framework, stimulating aggregate supply is much more important than stimulating aggregate demand. This is in contrast to the widely-held belief that aggregate demand is vital in recessions.

To isolate the effects of labor search and matching frictions, I set the labor search cost parameter to zero and observed how the results change in the model. This reverts the model to that of Eggertsson's (2011). Without labor search and matching frictions, the government spending multiplier is larger than 1, effective, and expansionary. This also leads to the labor tax cut becoming contractionary and generating a negative multiplier at the lower bound.

To test the sensitivity of the results to different parameters, I follow Eggertsson and Singh

(2018) to estimate the parameters using a Bayesian maximum-likelihood algorithm. Then I compute the fiscal policy multipliers using each set of maximized parameters, and evaluate the robustness of the results from the computed fiscal policy multipliers distributions. I find small and negative government spending multipliers, and positive labor tax cuts multipliers in the distribution. I conclude that with the same goal of approximating the Great Depression scenario, when output falls by 30 percent and there occurs an annual deflation of 10 percent, the fiscal policy multipliers have the same qualitative performances as suggested in this paper.

The remainder of this paper is outlined as follows: Section 2 gives out the model of this paper, section 3 defines the equilibrium and solution of the model, section 4 shows how the model is calibrated, section 5 discusses the main results of the paper, section 6 presents the sensitivity test and section 7 explains the determinacy of the model, section 8 outlines the Bayesian estimation performed in this paper, section 9 gives the results followed from previous section and section 10 concludes.

2.2 Model

This paper extends the backbone New Keynesian model with price stickiness to include the labor search and match frictions, which establish distinction from Eggertsson (2011). I consider varieties of fiscal policies like distortionary taxes on labor, consumption, capital and profits, as well as government spending. The discussion is composed of two cases, when the nominal interest rate can adjust freely or is fixed at the lower bound.

2.2.1 Consumer's Problem

I assume this economy is populated by a large number of identical households. Each household is made up of a continuum of members, each specialized in a different labor service indexed by $j \in [0, 1]$. Income is pooled within each household, which acts as a risk sharing mechanism. A typical household seeks to maximize

$$E_0 \sum_{t=1}^{\infty} \beta^t \xi_t [u(C_t) + g(G_t) - \int_0^1 v(N_t(j))dj] \tag{2.1}$$

Where C_t is the consumption choice of the household, G_t is the amount of government spending exogenously determined by the government and N_t is the labor supplied by the household. Specifically I assume a constant relative risk aversion (CRRA)[7] utility function for u and g: $u(C_t) = \frac{(C_t)^{1-\sigma}-1}{1-\sigma}$, $g(G_t) = \frac{(G_t)^{1-\sigma}-1}{1-\sigma}$. $v(N_t(j)) = \frac{N_t(j)^{1+\phi}}{1+\phi}$ is the disutility function for labor. Accordingly σ is the CRRA parameter, which measures the degree of risk aversion and ϕ is the Frisch elasticity of labor supply, which measures the elasticity of labor supply to the wage rate.

I assume that in each period the household is subject to a periodic preference shock ξ_t, which can be interpreted as a shock to the discount factor β. Apart from consumption, the government spending G_t affects consumer's utilities directly, but in a seperable form. I disavow the substitutable government spending as in Eggertsson (2011) because it would be irrelevant for the household's problem eventually in this setting.[8]

The period budget constraint of the household is as follows, it is subject to consumption tax τ_t^s, capital tax τ_t^A, profit tax τ_p and payroll tax τ_t^w. I assume the government uses a lump-sum tax T_t to clear the deficits and surplus in government budget in every period.[9] $Z_t(i)$ is the profit earned by firm i.

$$\begin{aligned} (1+\tau_t^s)P_t C_t + B_t = (1-\tau_{t-1}^A)(1+i_{t-1})B_{t-1} + (1-\tau_t^p)\int_0^1 Z_t(i)di \\ +(1-\tau_t^w)P_t \int_0^1 W_t(j)N_t(j)dj - T_t \end{aligned} \tag{2.2}$$

Since consumers are choosing consumption level and labor decision in every period, I have the

[7] I assume this utility function as it is simple to solve, and also to be comparable to Eggertsson(2011)

[8] The author is fully aware of the importance of this kind of government spending. As shown in Oh and Reis (2012), during the Obama administration, a large amount of government spending is transferable and targeted.

[9] Assume a lump-sum tax can simplify the computation of this model. As shown in Baxter and King (1997) and known in the literature, a distortionary tax would lead to smaller output multiplier for government spending.

following F.O.C.s with respect to consumption, savings and labor:

$$C_t^{-\sigma} = (1+i_t)(1-\tau_t^A)\beta E_t C_{t+1}^{-\sigma} \frac{\xi_{t+1}}{\xi_t} \frac{P_t}{P_{t+1}} \frac{1+\tau_t^s}{1+\tau_{t+1}^s} \tag{2.3}$$

$$\frac{1-\tau_t^w}{1+\tau_t^s} W_t(j) = N_t(j)^\phi C_t^\sigma \tag{2.4}$$

I can combine the resource constraint $Y_t = C_t + G_t$ with the Dynamic Euler equation (3) to solve for the consumer's side "IS" curve or the aggregate demand(AD) equation. The AD equation from Eggertsson (2011), is in the following log-linearized version:

$$\begin{aligned} -\sigma \frac{Y}{Y-G}(\hat{Y}_t - \hat{G}_t) = i_t - E_t\pi_{t+1} - r_t^e + \sigma \frac{Y}{Y-G} E_t(\hat{Y_{t+1}} - \hat{G_{t+1}}) \\ -\chi^A \hat{\tau_t^A} + \chi^s E_t(\hat{\tau_t^s} - \hat{\tau_{t+1}^s}) \end{aligned} \tag{2.5}$$

Where the letter without subscript denotes the steady state value of a variable. $\hat{Y}_t \equiv (Y_t - Y)/(Y)$, $\hat{G}_t \equiv (G_t - G)/(Y)$, $\hat{\tau_t^s} \equiv \tau_t^s - \tau^s$, $\hat{\tau_t^A} \equiv (\tau_t^A - \tau^A)/(1-\beta)$, $\chi^A \equiv (1-\beta)/(1-\tau^A)$ and $\chi^s \equiv 1/(1+\tau^s)$. Following the same definitions for parameters and log-linearized variables, I arrive at the same "AD" equation as Eggertsson (2011)[10]:

$$\hat{Y}_t = E_t\hat{Y_{t+1}} - \hat{\sigma}(i_t - E_t\pi_{t+1} - r_t^e) + (\hat{G}_t - E_t\hat{G_{t+1}}) + \hat{\sigma}\chi^s E_t(\hat{\tau_{t+1}^s} - \hat{\tau_t^s}) + \hat{\sigma}\hat{\chi^A}\hat{\tau_t^A} \tag{2.6}$$

2.2.2 Firm's Problem

As for a conventional New Keynesian model, I assume a continuum of firms indexed by $i \in [0,1]$. Each monopolistically competitive firm produces a differentiated good with a technology represented by the following production function:

$$Y_t(i) = N_t(i) \tag{2.7}$$

The production function is assumed to be constant returns to scale, and the only input is labor.

[10] $\hat{\sigma}$ is defined as $\frac{Y-G}{\sigma Y}$.

I assume labor is unique in a continuum between 0 and 1. Each household is endowed with one unit of labor and can supply in any labor market. Thus the firm i has an index of labor input $N_t(i)$ defined as follows:

$$N_t(i) = \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\phi}} dj \right)^{\frac{\phi}{\phi-1}} \tag{2.8}$$

Where $N_t(i,j)$ denotes the quantity of type-j labor employed by firm i in period t. The parameter ϕ represents the elasticity of substitution among labor varieties. The above equation just states that firm i's labor demand is a summation of labor over the space of labor type.

I can define the aggregate real wage as an indexation of sectoral real wage as follows:

$$W_t \equiv \left(\int_0^1 W_t(j)^{1-\phi} dj \right)^{\frac{1}{1-\phi}} \tag{2.9}$$

Further, I can define the wage bill of a firm as the product of the wage index times that firm's employment index:

$$\int_0^1 W_t(j) N_t(i,j) dj = W_t N_t(i) \tag{2.10}$$

Because of the production function given in equation 6, the labor demand schedule for firm i is determined by the firm's price for its own product relative to the average price level. This demand schedule is the same as the consumption goods demand schedule:

$$N_t(i) = (\frac{P_t(i)}{P_t})^{-\epsilon_p} N_t \tag{2.11}$$

Similarly, I can define the aggregate price index as a summation of different varieties of prices for goods for different firms:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_p} di \right)^{\frac{1}{1-\epsilon_p}} \tag{2.12}$$

The consumption indexation problem can be written as:

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{2.13}$$

Where ϵ_p governs the elasticity of substitution between different varieties of consumption goods.

So the consumption maximization problem gives consumption demand schedules:

$$C_t(i) = (\frac{P_t(i)}{P_t})^{-\epsilon_p} C_t \tag{2.14}$$

2.2.2.1 Firms and the labor market: Nash Bargaining

In this section I describe the firm's interactions with the labor market. The assumptions mainly follow from Blanchard and Gali (2006) and Abbritti, Boitani and Damiani (2007).

In these studies the employment dynamics are determined by a job separation rate δ, where $\delta \in (0, 1)$ defines the proportion of employers from the previous period who get separated from their jobs. Also, a pool of workers h_t^i is restored to a representative firm $i's$ employment pool . Thus:

$$N_t^i = (1 - \delta) N_{t-1}^i + h_t^i \tag{2.15}$$

As an aggregation, the total employment $N_t \equiv \sum_0^1 N_t^i di$ evolves according to the following:

$$N_t = (1 - \delta) N_{t-1} + H_t \tag{2.16}$$

Where $H_t \equiv \sum_0^1 h_t^i di$ denotes the aggregate hiring level.

labor is assumed to be involuntarily supplied from the household. Those who are currently not occupied with a job form the pool of jobless. As labor force is normalized to 1, the jobless pool U_t is defined as follows:

$$U_t = 1 - (1 - \delta) N_{t-1} \tag{2.17}$$

This is also referred to as the beginning-of period unemployment according to Blanchard and Gali (2006). An alternative measure of unemployment is given by:

$$u_t = 1 - N_t \tag{2.18}$$

The hiring cost of firm i is determined by the following:

$$F_t^i = \frac{\gamma H_t}{U_t} h_t^i \tag{2.19}$$

Where γ is a positive scaling constant, and the ratio $x_t = \frac{H_t}{U_t}$ governs the tightness of the labor market. The searching cost function is directly proportional to the number of workers recruited H_t from the jobless pool and is inversely proportional to the overall level of the jobless pool U_t. The former makes the firm search with higher intensity, and the latter smooths the matching process between firms and potential workers.

The searching cost is incurred to firms as a prerequisite for production. It can be thought of as rent charged from an outside matching institution that has superior information than firms in the labor market.

In addition, I assume all the searching profits are spent by this institution in the goods market. In sum, the output is consumed by either the public sector, the government, or two private sectors, the household and the labor matching institution. In other words:

$$Y_t = C_t + G_t + \frac{\gamma H_t}{U_t} H_t \tag{2.20}$$

Note that in the above equation, the hiring cost is assumed to be small[11] compared to other components in the output. Thus I can ignore it, which is also assumed in Blanchard and Gali (2006).

Next I discuss the real wage determination under Nash Bargaining, which follows the standard

[11] In the calibration, the hiring cost is calibrated to be two percent of total output, which is small enough to be ignored.

literature and my base papers.

I assume that the worker and the intermediate firm bargain in each period to maximize the weighted product of both parties' surpluses:

$$\max \omega_W = (V_t^E - V_t^U)^s S_t^{1-s} \tag{2.21}$$

Where V_t^E is the surplus of workers associated with being employed and V_t^U is the surplus of workers associated with being unemployed. Both are expressed in terms of consumption units. s and $1-s$ are below 1 unit and represent workers' and the firm's bargaining powers, respectively. The intuition is that the associated surplus ω_W is split between the bargaining parties. The surplus S_t is the firm's surplus from the matching, which represents the opportunity cost of searching again in the labor market. The amount equals our unit hiring cost:

$$S_t = \frac{\gamma H_t}{U_t} \tag{2.22}$$

I can use recursive formulation to solve for V_t^E and V_t^U. The workers' marginal value from employment is the current after-tax real wage minus the disutility of labor in terms of marginal consumption (marginal rate of substitution), plus discounted next period marginal employment and unemployment values. The probability of being employed next period is the sum of the probability of remaining employed and getting into the jobless pool (but finding a job immediately): $[(1-\delta)+(\delta\frac{H_{t+1}}{U_{t+1}})]$. The expression for employment value is given below:

$$\begin{aligned} V_t^E = (1-\tau_t^w)W_t^{Nash} - C_t^\sigma N_t(i)^\phi \\ +\beta E_t\left\{(\frac{C_{t+1}}{C_t})^{-\sigma}[(1-\delta(1-\frac{H_{t+1}}{U_{t+1}}))V_{t+1}^E + \delta(1-\frac{H_{t+1}}{U_{t+1}})V_{t+1}^U]\right\} \end{aligned} \tag{2.23}$$

The value of being unemployed is similarly written. The unemployed worker receives the discounted benefits of next period employment and unemployment values, where $\frac{H_{t+1}}{U_{t+1}}$ is the probability of finding a job next period in the jobless pool:

$$V_t^U = \beta E_t\Big\{(\frac{C_{t+1}}{C_t})^{-\sigma}[\frac{H_{t+1}}{U_{t+1}}V_{t+1}^E + (1-\frac{H_{t+1}}{U_{t+1}})V_{t+1}^U]\Big\} \tag{2.24}$$

Combining both conditions, we get the net value of being employed:

$$\begin{aligned} V_t^E - V_t^U = (1-\tau_t^w)W_t^{Nash} - C_t^\sigma N_t(i)^\phi + \\ \beta(1-\delta)E_t\Big\{(\frac{C_{t+1}}{C_t})^{-\sigma}[(1-\frac{H_{t+1}}{U_{t+1}})(V_{t+1}^E - V_{t+1}^U)]\Big\} \end{aligned} \tag{2.25}$$

The Nash solution would give the following condition, where $\eta \equiv (s/(1-s))$ denotes the workers' relative bargaining power:

$$V_t^E - V_t^U = \eta S_t \tag{2.26}$$

Thus, I can solve for the Nash wage as follows using the above two equations:

$$(1-\tau_t^w)W_t^{Nash} = C_t^\sigma N_t(i)^\phi + \eta S_t - \beta(1-\delta)E_t\Big\{(\frac{C_{t+1}}{C_t})^{-\sigma}[(1-\frac{H_{t+1}}{U_{t+1}})\eta S_{t+1}]\Big\} \tag{2.27}$$

Note that in this paper I log-linearize the marginal rate of substitution(MRS) terms in the above equation directly. Blanchard and Gali (2010) deviate from this approach by assuming a form of real wage rigidity right away. In their paper the MRS term is a combination of a constant term and the exponent form of technology. In this paper I prefer not to consider technology, as I focus on reduced form solution.[12]

2.2.2.2 Firm's optimizing problem under price stickiness

In this paper I consider price rigidity of setting prices by intermediate firms. Following the Calvo sticky price assumption, each intermediate firm has a random probability of $1-\theta$ to adjust

[12]Another way to model wage rigidity is to assume a linear relationship between MRS and the current level of real wage, but the parameter of wage rigidity plays a key role in determining the size of real wage. It seems not to be a perfect way of modeling real wage rigidity either and is subject to debate.

its price in every period.

The firm's price decision in every period is to maximize the expected discounted profits. A representative firm maximizes the following profits function:

$$\Sigma_t = \max_{P_t^*} E_t \sum_{k=0} \theta_p^k \Lambda_{t,t+k}(1-\tau_{t+k}^p)[P_t^* Y_{t+k|t} - P_{t+k}(W_{t+k}^{Nash} N_{t+k|t} + \gamma x_{t+1} H_{t+k|t})] \tag{2.28}$$

Where τ_{t+k}^p is a tax on profits, P_t^* is the reset price set by the firm in time t, $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$ is the stochastic discount factor and $Y_{t+k|t}$ is the time t+k demand schedule for an intermediate firm setting the time t optimal price P^*:

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon_p} (C_{t+k} + G_{t+k} + \gamma \frac{H_{t+k}}{U_{t+k}} H_{t+k}) \tag{2.29}$$

Similarly, in my model the expression of $N_{t+k|t}$ is the same as $Y_{t+k|t}$. Accordingly, the expression of $H_{t+k|t}$ will follow similarly as in equation 16. In addition, I can express the price equation as:

$$P_t = [(1-\theta_p)(p_t^*)^{1-\epsilon_p} + \theta_p P_{t-1}^{1-\epsilon_p}]^{1/(1-\epsilon_p)} \tag{2.30}$$

I solve firm's price setting problem by taking first order condition to the reset price in the profit function. The labor market variables can be related to the labor market tightness variable x_t. The following equation describes its derivation in relation to output.

From equations (15) and (16), I can describe x_t as a relationship between the current employment N_t and the previous employment rate N_{t-1}. Also I use the strict relationship between employment and output in my model such that $Y_t = N_t$. Then I arrive at the following log-linearized relationship between labor market tightness and the output gap:

$$\delta \hat{x}_t = \hat{Y}_t - (1-\delta)(1-x)\widehat{Y_{t-1}} \tag{2.31}$$

From the above equation, the tightness of the labor market can be interpreted as how much labor is demanded in the market. The previous employment level measures the current stock of labor by firms, which enters negatively in the above equation, which means firms need to hire less workers in this period, thus a looser labor market. A higher employment level in the current period would make firms compete harder for new workers, thus resulting in a tighter labor market.

Finally, I can express the New Keynesian Phillips Curve(NKPC) as follows. This is also the final AS equation I have[13]:

$$\pi_t = a_\pi E_t \pi_{t+1} + a_w \hat{\tau_t^w} + a_{2y} \hat{Y_{t-1}} + a_{1y} \hat{Y}_t + a_{3y} E_t \hat{Y_{t+1}} + a_{2g} E_t \hat{G_{t+1}} + a_{1g} \hat{G}_t \tag{2.32}$$

2.2.3 Monetary Policy

A Taylor type monetary policy by the Central bank is assumed. The nominal interest rate would react to the output gap and inflation in addition to the real interest rate of the economy. An approximation of the interest rate would be given in the following equation:

$$i_t = max(LB, r_t^e + \phi_\pi \pi_t + \phi_y \hat{Y}_t) \tag{2.33}$$

Where ϕ_π is the coefficient on inflation and is larger than 1, and ϕ_y is the corresponding weight on output gap and is larger than 0. Note that the nominal interest rate cannot take a value less than the lower bound given great deflation or negative output gap. When the interest rate takes the LB, it is one scenario I study intensively later. Overall, the monetary policy has implications on the aggregate demand (AD) of the economy.

2.3 Approximated Equilibrium

2.3.1 Short-Run and Long-Run Equilibrium Definition

For a given policy rule for taxes and government spending, equations 2.6, 2.32, and 2.33 close the model. An approximate equilibrium can now be defined as a collection of stochastic processes for $\{\hat{Y}_t, \pi_t\}$ that satisfy equations 2.6, 2.32 and 2.33 given an exogenous path for $\{r_t^e\}$, a monetary policy specifying the process $\{i_t\}$ that satisfies equation 2.33, and fiscal rules that determine the

[13]The exact expressions of the coefficients are complicated functions of the deep parameters, see the appendix for the detail.

path for $\{\hat{\tau_t^w}.\hat{\tau_t^s}, \hat{\tau_t^A}, \hat{G}_t\}$, which would be more specific in upcoming policy experiment.

In the long-run, it is assumed that r_t^e has gone to the steady state, and other exogenous variables and endogenous variables are at their steady states as well. The situation can be regarded as the economy being at the expansionary phase of the business cycles, i.e. the normal time.

The short-run is the period in which the economy is subject to temporary disturbance. More precisely, in the short run the real interest rate shock has a specific equilibrium value such that $r_t^e = r_S^e$. The size and sign of this real interest rate shock is calibrated in the context of this model, such that it induces the economy into a recession situation close to the Great Depression. Under the short-run equilibrium, I classify the recession states to be the Non-Binding situation when interest rate is above the LB, and the Binding situation when interest rate is fixed at the LB.

In the short-run, the shock to the real interest rate reverts back to long run steady state, $\bar{r}$, with probability $1-\mu$ in each period of the short-run. So let T^e be the period in which the shock is back to the steady state. Then $t < T^e$ is the short run and $t > T^e$ is the long run. I assume the monetary policy follows 2.33, and the fiscal policy is perfectly correlated with the shock. So that $(\hat{\tau_t^w}, \hat{\tau_t^s}, \hat{\tau_t^A}, \hat{G_S}) = (\hat{\tau_S^w}, \hat{\tau_S^s}, \hat{\tau_S^A}, \hat{G_S})$ in the short-run and $(\hat{\tau_t^w}, \hat{\tau_t^s}, \hat{\tau_t^A}, \hat{G_t}) = (0,0,0,0)$ in the long-run.

2.3.2 Short-Run and long-Run Equilibrium Analysis

This section gives a brief Short-Run and Long-Run equilibrium analysis, which follows closely from the previous section. These two sections give the basis for the rest of the paper, which discusses the allocations of the equilibrium under the exogenous Fiscal and Monetary shocks.

The above definitions of long-run and short-run equilibrium follow Eggertsson (2011). This type of analysis is very convenient to compute and solve analytically, and it also has straightforward interpretations of the effects of fiscal policy changes in different monetary policy circumstances.

I impose a short run equilibrium when the shock to the real natural interest rate is big enough, it pushes the natural interest rate under some negative threshold. The nominal interest rate is at the lower bound $i_t = i_s^z = 0$, the output deviates from steady state at some fixed level $\hat{Y_t} = \hat{Y_s^z} = -0.30 \quad \forall t < T^e$, and price inflation is at some short run level $\pi_t = \pi_s^z = -0.025$.

Following Eggertsson (2011), I want to focus on the short-run equilibrium circumstance when output collapse is associated with interest rate binding at the LB, because I can derive some fiscal policy effects from this case. I consider the model when there is no fiscal intervention initially; that is, each of the fiscal variables is at the long run steady state. Then the shock to the real natural interest rate r_t^e generates a recession that pushes the nominal interest rate to the LB. As Eggertsson (2011) explains, the source of this shock can be interpreted as a preference shock or a banking crisis. [14] I also assume this shock follows a Markov chain process with probability μ to stay the same and probability $1-\mu$ to revert back to long-run equilibrium. Once the economy reverts back to the long run equilibrium, it would stay there forever.

In the short run, $t < T^e$, I consider two cases:

1. The nominal interest rate is above the LB in the short run. I assume the locally bounded equilibrium is such that

$\pi_t = \pi_s^p \quad \forall t < T_e, \hat{Y}_t = \hat{Y_s^p} \quad t < T^e, i_t = i_s^p = r_s^e + \phi_\pi \pi_s + \phi_y \hat{Y}_s > LB.$

2. The interest rate is at the LB in the short run. I assume the locally bounded equilibrium is such that

$\pi_t = \pi_s^z \quad \forall t < T^e, \hat{Y}_t = \hat{Y_s^z} \quad t < T^e, i_t = i_s^z = LB.$

The main idea behind the allocations of these equilibrium is that the implementation of fiscal policy at the short-run can still affect their allocations, but in a one time manner. The equilibrium is achieved in the short-run when the shock hits the economy, this is due to the assumption of perfect correlations of Fiscal policy and the real interest rate shock[15].

2.4 Calibration

Table 2.1 shows the calibration of this paper[16], in which I choose the value of the deep parameters to be in line with previous literature and empirical findings. The preference parameters

[14]seen as an increase in the probability of default by borrowers, specifically the shock is defined as $r_t^e \equiv log\beta^{-1} + E_t(\hat{\xi}_t - \hat{\xi_{t+1}})$, where $\hat{\xi}_t \equiv log\xi_t/\xi$.

[15]I am aware of the ideas of continuous or cumulative Fiscal policy multipliers, but they are not related in this model or circumstance.

[16] Eggertsson (2011) estimates some of the parameters following a Bayesian exercise for the Great Depression data points, I replicate the same exercise in later section.

describe the household's and firm's problem. Labor market parameters describe the labor market problem specifically. Monetary and Fiscal policy parameters describe Monetary authority and government targets. AS related parameters are computed as functions of the above mentioned parameters' values, these functions are model implied from the log-linearization process. Finally the equilibrium parameters govern the short run equilibrium of the model.

σ is the constant relative risk aversion parameter, which by definition is equal to the Arrow-Pratt measure of relative risk aversion. I let $\sigma = 1$, the case of log utility function. Accordingly I define $\hat{\sigma}^{-1} \equiv -(\bar{u_{cc}}\bar{Y}/\bar{u_c}) = \sigma\frac{Y_{ss}}{Y_{ss}-G_{ss}}$, this definition helps $\hat{G}_t$ to be interpreted as percentage change of government spending relative to the steady state output level (instead of steady state government spending). This makes the government spending multiplier have dollar to dollar interpretation. G_{ss} is computed by solving for its steady state value in this model, which equals to 15.99 percent of the output level. This results in the defined $\hat{\sigma} = 0.9744$.

The price stickiness parameter θ_p is set to be equal to 0.75. The interpretation is on average three quarters of the firms do not reset their price in three months, and it takes 1 year for a firm to change price on average. The elasticity of substitution between consumption goods parameter ϵ_p is set to be 7, this automatically implies a mark up value M of 1.1667, which means that firms charge a price mark up over their marginal cost of 16.67 percent. I define $\phi \equiv \bar{v_{nn}}\bar{N}/\bar{v_n}$, which is the Frisch elasticity of labor supply; the value is set to be 1. The definitions and values for profit and sales tax related parameters are $\chi^A \equiv (1-\beta)/(1-\bar{\tau^A}) = 0.003$ and $\chi^s \equiv 1/(1+\bar{\tau^s}) = 0.9524$. I choose these definitions to have the tax multipliers to be interpreted as a percentage point cut in tax rate to have certain percentage change on output, which is the conventional interpretation for tax multiplier in the literature. τ^w, τ^A and τ^s are the steady state labor, capital and sales tax rate implied by the Fiscal Authority, they are 20 percent, 0 percent and 5 percent, accordingly. They do respect the state of U.S. economy in practice.

I follow Blanchard and Gali (2010), and calibrate the labor market as the case of the U.S. The steady state unemployment rate u is 5 percent, the job market tightness parameter x is 0.7, the model implied job separation rate δ is computed from these two values in the steady state as 0.1228.

The steady state labor supply amount N is 0.95 out of 1, and the labor search cost parameter γ is equal to 0.22 to have an equilibrium search cost approximately equal to 2 percent of GDP, which is small enough to be ignored. I assume a Nash Bargaining worker's relative bargaining power η equal to 0.5, which is standard in the labor search and matching literature[17]. W is the model implied steady state real wage rate.

The Central bank responds to price inflation and the output gap in a Taylor type of rule. It is assumed that the central bank responds to inflation by 1.5 (ϕ_π) which is a common parameter for the determinace of monetary policy. The response to output gap is 0.125.

The AS related parameters as explained before are functions of previous parameters; I do not choose any values in particular. The interpretation should be that these parameters carry both the firm's price setting and the labor search and matching frictions as implied in this model.

Finally, the two equilibrium parameters are the exogenous shock r_s^e and the probability of getting out of recession in the short run every quarter μ. The exogenous real interest rate shock is chosen from the estimated real interest rate distribution, i.e. two standard deviations from the mean. The value is chosen in a way to have the short run recession to be close to the Great Depression state: 30 percent drop in output and 10 percent annual deflation. μ has the interpretation that every quarter the economy has a fixed probability of 0.85 staying in the recession next period. As implied by the rational expectation, the household will have the same expectation as the state of the economy. On average the economy needs 6.67 quarters to get out of the recession.

2.5 Fiscal Policy analysis in the recession

2.5.1 Output collapse in the absence of Fiscal Intervention

In periods $t \geq T^e$, the solution is $\pi_t = \hat{Y}_t = 0$. In periods $t < T^e$, the assumption about the shock implies that inflation next period is either zero (with probability $1-\mu$) or the same as at time t, that is, $\pi_t = \pi_s$ (with probability μ). Hence the solution of a lower bound on the interest rate in

[17]The Hosios condition is satisfied. And the interpretation of this parameter is that the worker has half of the bargaining power of the firm in determining wage.

Table 2.1: Calibration for Parameters

Preference Parameters	$\hat{\sigma}$	β	ϕ	θ_p	ϵ_p
	0.9744	.9905	1	.75	7
	M	λ			
	1.1667	0.0677			
Labor Market Parameters	W	u	γ	N	H
	1.0189	0.05	0.22	0.95	0.1148
	x	δ	η		
	0.7	0.1228	0.5		
Monetary/Fiscal Policy Parameters	ϕ_π	ϕ_y	$\bar{\tau^s}$	$\bar{\tau^w}$	$\bar{\tau^A}$
	1.5	0.125	0.05	0.2	0.003
AS Related Parameters	a_π	a_w	a_{1y}	a_{2y}	a_{3y}
	1.0034	0.0134	0.0321	-0.0047	0.0142
	a_{1g}	a_{2g}			
	0.0131	-0.000032			
Equilibrium Parameters	r_s^e	μ			
	0.106	.85			

$t < T^e$ satisfies the following AD and AS equations:

$$AD: \qquad \hat{Y}_s = \mu \hat{Y}_s + \hat{\sigma}\mu\pi_s + \hat{\sigma} r_s^e \tag{2.34}$$

$$AS: \qquad \pi_s = a_\pi \mu \pi_s + (a_{1y} + \mu a_{3y})\hat{Y}_s \tag{2.35}$$

The AD equation just states that the current output depends on expected future output and inflation. The New Keynesian Phillips (AS) curve also has inflation increasing in the output gap, given that the parameters $(a_{1y} + \mu a_{3y})$ takes a positive value.

From the above equations both curves are positively sloped, the aggregate demand becomes upward sloping because when there is persistent deflationary pressure. When the nominal interest rate is set at the lower bound, the Central bank is unable to decrease the interest rate further to react to deflation or a negative output gap, and the real interest rate falls further which worsens the output collapse. This phenomenon is commonly referred to in the literature as the 'Liquidity Trap', in the sense that monetary policy cannot provide additional liquidity to the economy to push it out of a recession.

It is helpful to plot the AD and the AS curves in the output gap and inflation diagram. Supposing the probability of getting out of recession equals to 1 ($\mu = 0$), the AD curve would be a vertical line and the AS curve would be positively sloped. Then the output level would solely be determined by the AD curve. For μ to take a positive value, which means that there is some probability that the recession lasts for more than one quarter, then both the AS and the AD curve have positive slope. More precisely the AD curve has a slope equal to $\frac{1-\mu}{\hat{\sigma}\mu}$, and the AS curve has the slope equal to $\frac{(a_{1y}+\mu a_{3y})}{1-a_{\pi}\mu}$.

Even though the relative steepness of AS and AD curves depend on the calibration of the parameters, but the labor market frictions will make AS curve steeper, means that firms adjust their prices relatively more than their employment or production. The intuition is that firms need to pay search cost to hire new workers, so that this friction makes employment more sticky from the firm's perspective. This situation is shown in Figure 1, in the short run, AS curve is steeper than AD when AD becomes upward sloping due to the LB. This differs fundamentally from Eggertsson's (2011) scenario. And in latter analysis, the implications for the effectiveness of fiscal policy can be illustrated.

2.5.2 The case of labor tax cut

I want to ask the same question as in Eggertsson (2011): Can government implement fiscal policy that help resolve the issue of output collapse in a crisis? In particular, can a labor tax cut generate positive and larger than 1 output multiplier?

First I want to analyze the labor tax cut under normal circumstances when the interest rate is above the lower bound. I assume a temporary labor tax cut $\hat{\tau_t^w} = \hat{\tau_s^w} < 0$, which is reversed with probability $1 - \mu$ in each period to the steady state $\tilde{\tau_s^w} = 0$. And I let all other fiscal variables be silent such that $\hat{G_t^N} = \hat{\tau_t^s} = \hat{\tau_t^A} = 0$. The model has forward looking parts in both the AD and the AS equations, and I can divide the expectations into two states: the long run equilibrium state such

Figure 2.1: An illustrating short run equilibrium for the lower bound

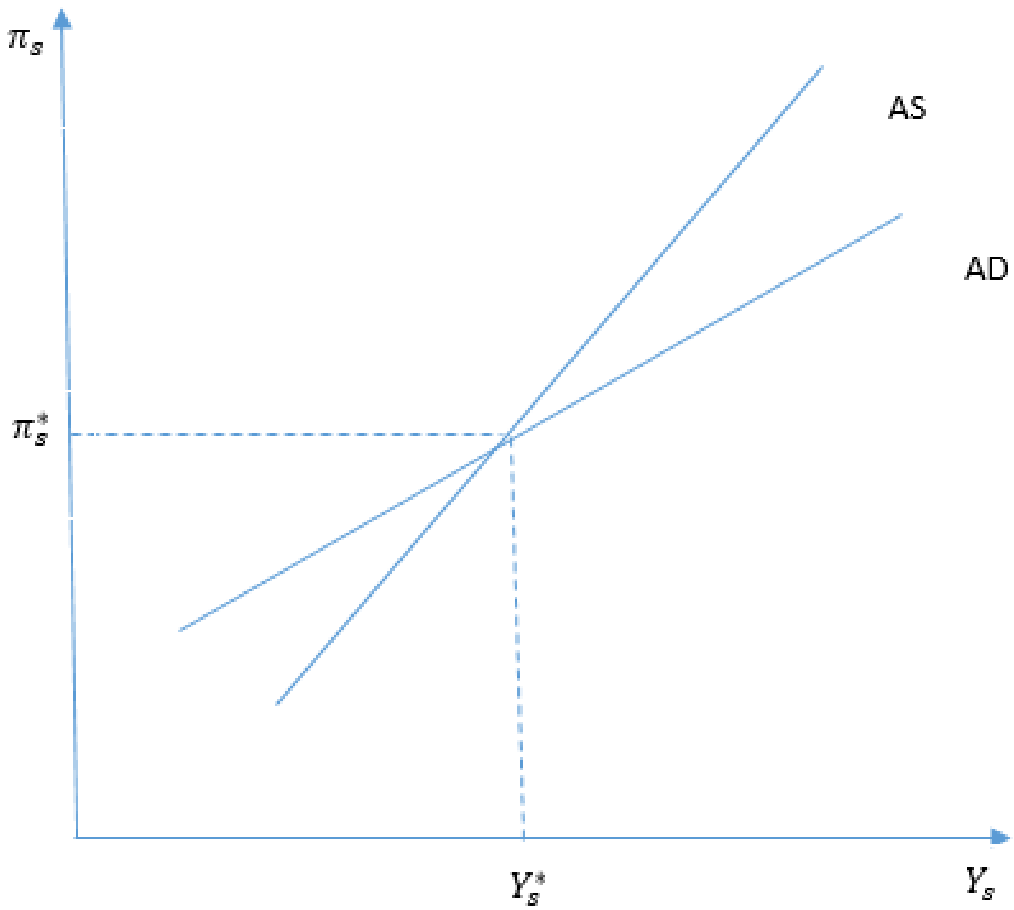
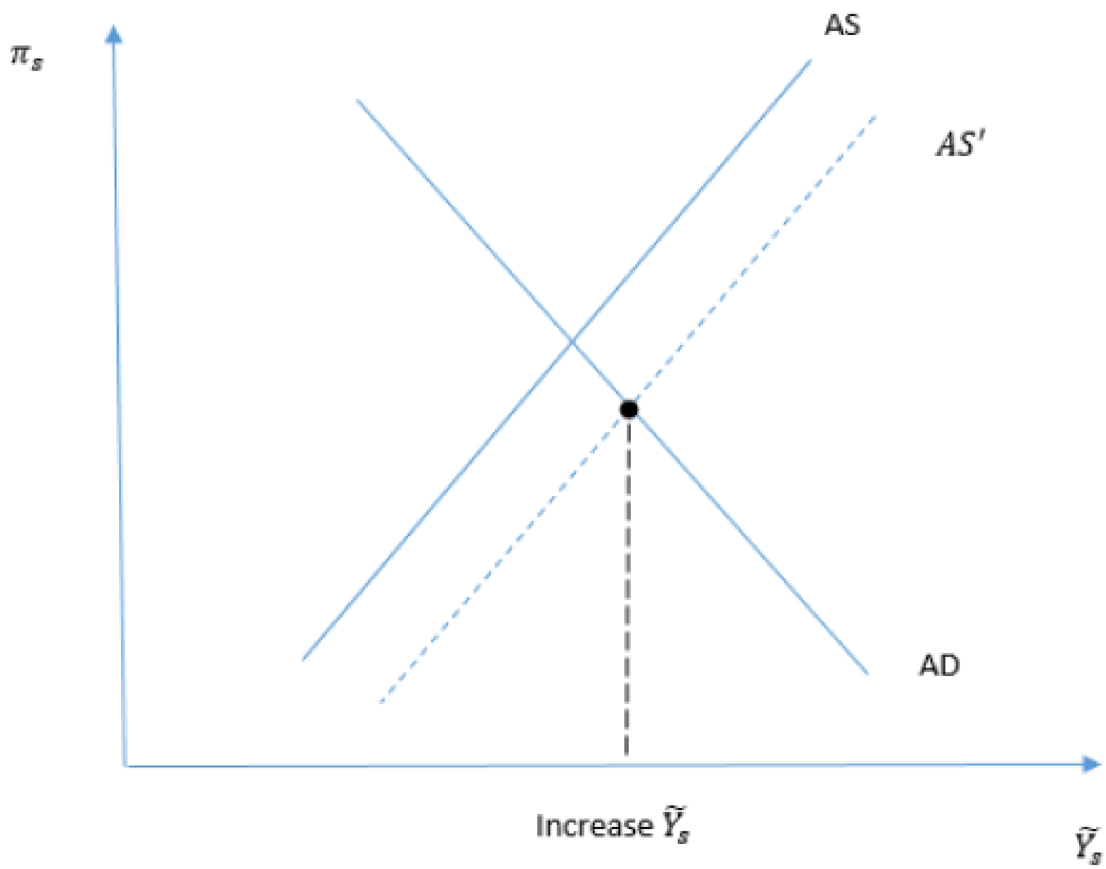


Figure 2.2: Labor tax cuts: the non-binding case



that the output gap, inflation and labor tax rate would take long run equilibrium values of 0; and the other state when all variables take corresponding short run equilibrium values. For the backward looking part in the AD curve, I assume that the previous period output gap is just 0. For monetary policy, I assume the central bank can respond to output gap and inflation by setting the nominal interest rate as a standard Taylor rule. So I have the following new AD and AS equations relating output gap, inflation and labor tax cut:

$$AD: \qquad \hat{Y}_s = -\hat{\sigma}\frac{\phi_\pi - \mu}{1 - \mu + \hat{\sigma}\phi_y}\pi_s \tag{2.36}$$

$$AS: \qquad (1 - a_\pi \mu)\pi_s = (a_{1y} + \mu a_{3y})\hat{Y}_s + a_w \hat{\tau_s^w} \tag{2.37}$$

By substitution of the two equations I get the following expression:

$$\hat{Y}_s = \frac{a_w \hat{\sigma}(\mu - \phi_\pi)}{(\phi_\pi - \mu)(a_{1y} + \mu a_{3y})\hat{\sigma} + (1 - a_\pi \mu)(1 - \mu + \hat{\sigma}\phi_y)} \hat{\tau_s^w} \tag{2.38}$$

As shown in Figure 2.2, aggregate supply curve shifts rightward when the firms experience a labor tax cut. This is represented as a positive exogenous shock to the aggregate supply. When the nominal interest rate is not binding, i.e. the aggregate demand curve is downward sloping, this leads to a decrease in the price level and an increase in output. The slicing of real wage payments from a labor tax cut transmits to lower marginal cost for the firms, so that they are willing to charge lower prices for their products and produce more. The Central bank cuts the nominal interest rate as suggested by the Taylor Rule when deflationary pressure is generated, and as a result the real interest rate falls. Households react by decreasing saving and increasing consumption, this is seen as the rightward movement along the aggregate demand curve.

The multiplier is computed as 0.116. If the government cuts the labor tax by 1 percentage point in a given period, the output increases by 0.116 percent. The interpretation is if labor tax rate decreases by one percentage point, output would increase by about .116 percent. Compared to the multiplier of 0.0816 in Eggertsson (2011), the calculated labor tax cut multiplier above is bigger. Given a less than unit labor tax cut multiplier, government should not necessarily implement labor tax cut in normal circumstances.[18]

The only difference between Eggertsson (2011) and this paper is the labor search and match frictions. The AD curve is exactly the same in both papers, however in this paper the AS curve takes a much steeper slope. Thus, for a rightward shift in the AS curve, the variation in output is bigger than Eggertsson's (2011) case, which results in a larger output multiplier for labor tax cuts. The intuition is that due to labor market frictions, firms tend to vary prices more often than labor decisions. In this case, a direct stimulus in labor market incentive as represented as a labor tax cut pushes firms to hire more workers than the friction-less Eggertsson's (2011) case.

[18]For a dollar to dollar multiplier, meaning for one dollar decrease in tax revenue, how many dollars does output increase, the results are not changed dramatically but slightly raised.

Then I show the effects of labor tax cuts when the lower bound is binding for the nominal interest rate. The goal of implementing this fiscal policy is to push the economy out of a recession when further change to monetary policy is impossible. When the Central bank cannot respond to deflationary pressure caused by a decrease in prices, the real interest rate drops further, which generates deflationary pressure to the economy. In this case, both the AD curve and the AS curve become upward sloping. I have the AD and AS equations as follows:

$$AD: \qquad (1-\mu)\hat{Y}_s = \hat{\sigma}\mu\pi_s + \hat{\sigma}r_s^e \tag{2.39}$$

$$AS: \qquad (1-a_\pi\mu)\pi_s = (a_{1y}+\mu a_{3y})\hat{Y}_s + a_w\hat{\tau_s^w} \tag{2.40}$$

Given I set $\mu = 0.85$, the AS curve would have a steeper slope than the AD curve, as shown in the figure 2.3.

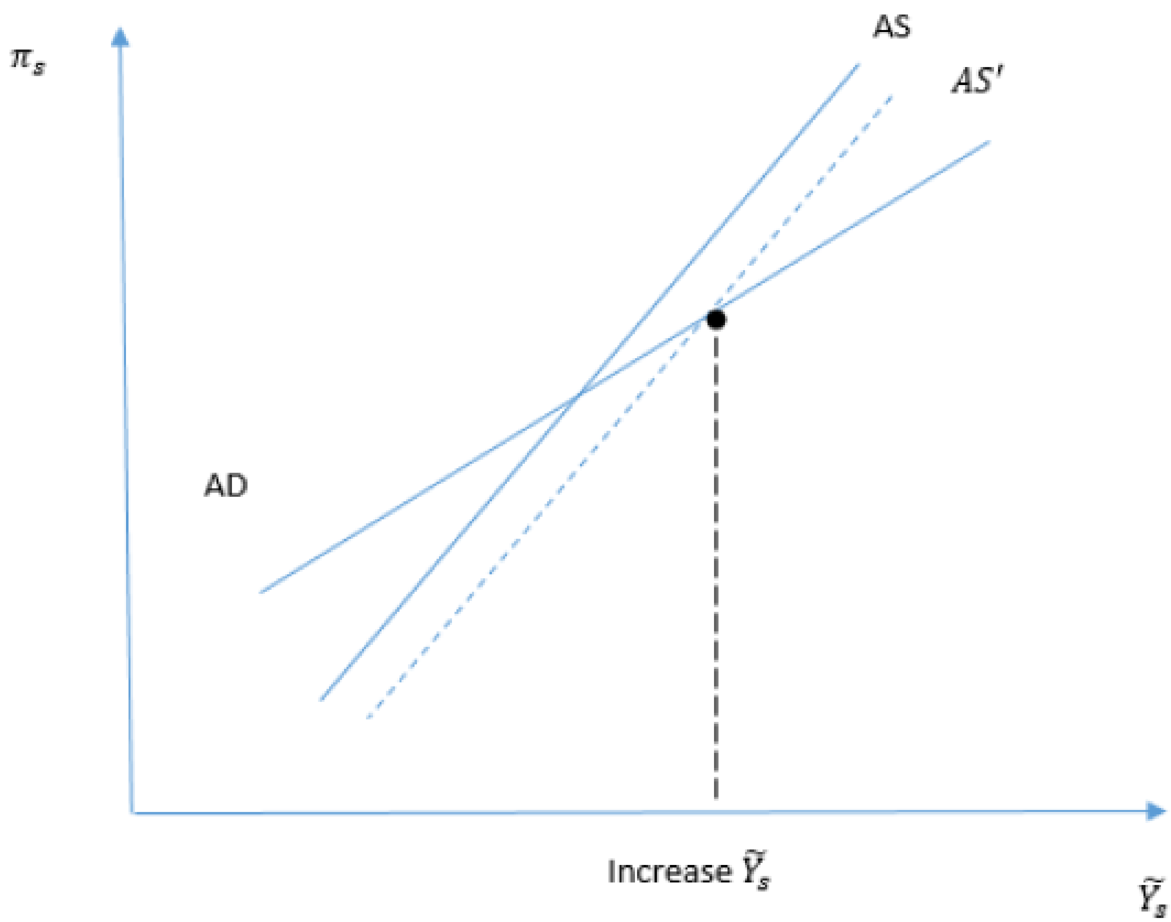
As labor tax cuts shift out the AS curve, for a steeper slope of the AS curve, this results in an increase in output combined with a higher inflation rate. This result is contrary to what Eggertsson (2011) finds in the case of labor tax cut at the LB; he has an ineffective and negative multiplier (-1.0301). As shown below, in my case the multiplier is positive:

$$\frac{\triangle\hat{Y}_s}{-\triangle\hat{\tau_s^w}} = \frac{a_w\hat{\sigma}\mu}{(a_{1y}+\mu a_{3y})\hat{\sigma}\mu - (1-a_\pi\mu)(1-\mu)} = 1.0866 \tag{2.41}$$

Compared to the non-LB interest rate case the multiplier is bigger and above 1. The interpretation is for a 1 percentage point cut in labor tax rate, output increases by 1.0866 percent.

The positive labor tax cut multiplier at the LB comes from a steeper AS curve, so that a rightward shift of the AS curve leads to an increase in output and inflation. The economic intuition behind this is that due to the labor market frictions, the price elasticity of supply is bigger than the labor elasticity of supply. After an increase in aggregate supply caused by labor tax cuts, firms naturally react to a positive supply shock by increasing production; in this situation price increases more than output. The upward sloping aggregate demand curve suggests real interest rate falls

Figure 2.3: Labor tax cuts: the lower bound case



when price level goes up, households increase private consumption and decrease saving, this is seen as a rightward movement along the aggregate demand curve. The key is an inflationary equilibrium is expected, and the economy is stimulated. The labor tax cuts multiplier at the LB is much bigger than the non-LB case, and larger than 1. An effective labor tax cut is found in this case.

2.5.3 The case of government spending

Similar to the case before, I first consider an increase in G_t when the nominal interest rate is not binding. For a probability of μ, government spending stays at the short-run equilibrium level; otherwise, it reverts back to the long-run equilibrium level. The AD and AS equations are as follows:

$$AD: \qquad \hat{Y}_s = -\hat{\sigma}\frac{\phi_\pi - \mu}{1 - \mu + \hat{\sigma}\phi_y}\pi_s + \frac{1-\mu}{1-\mu+\hat{\sigma}\phi_y}\hat{G}_s, \tag{2.42}$$

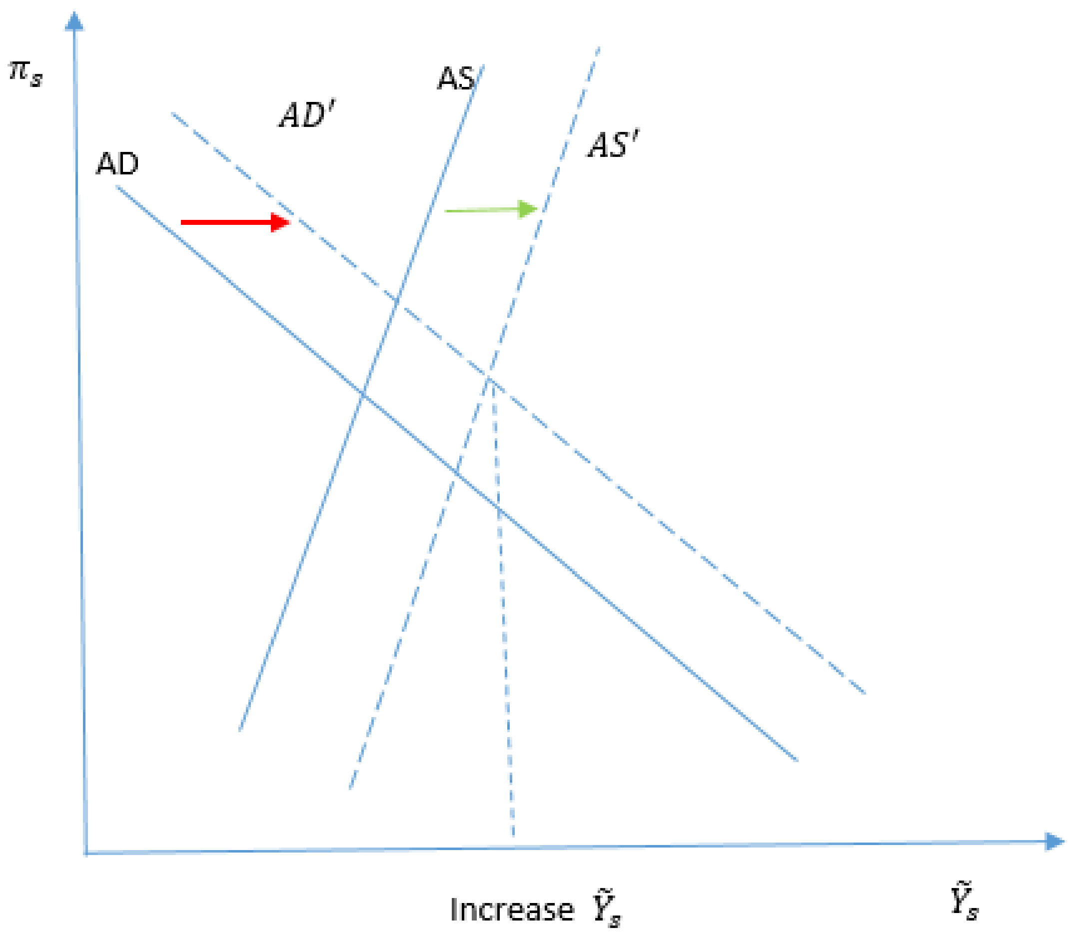
$$AS: \qquad (1 - a_\pi\mu)\pi_s = (a_{1y} + \mu a_{3y})\hat{Y}_s + (a_{1g} + \mu a_{2g})\hat{G}_s \tag{2.43}$$

From these two equations, an increase in government spending would shift both the AS and AD curves to the right. This would generate a higher level of output. The dynamics are shown in Figure 2.4.

$$\hat{Y}_s = \frac{(1-\mu)(1-a_\pi\mu) + \hat{\sigma}(\mu - \phi_\pi)(a_{1g} + a_{2g}\mu)}{(a_{1y} + \mu a_{3y})\hat{\sigma}(\phi_\pi - \mu) + (1 - a_\pi\mu)(1 - \mu + \hat{\sigma}\phi_y)}\hat{G}_s \tag{2.44}$$

The multiplier is computed to be 0.2509. Compared to Eggertsson's (2011) case of 0.46, this multiplier is slightly smaller. The interpretation is that for one dollar of government spending, output increases by 25 cents. When at the non-LB interest rate, an increase of government spending would stimulate aggregate demand because of the effect of crowding out private consumption. A lower marginal utility of consumption would lead to a higher demand from consumers. For a higher level of aggregate demand, firms charge higher prices and increase their production, this is seen as the rightward movement along the aggregate supply curve. This crowding out also induces workers to supply their labor at lower wages and firms take advantage of this lower marginal cost by slicing price when they can. This is seen as an exogenous positive shock to the aggregate supply in Figure

Figure 2.4: Government spending: the non-binding case



2.4. When nominal interest rate is not binding, central bank reacts to deflationary expectation by decreasing nominal rate further, real interest rate falls and private consumption is stimulated. This is seen as the rightward movement along the aggregate demand curve. Both positive shocks to AD and AS increase output.

Consider now the effect of increasing government spending at the lower bound. Under this specification, I have the following AD and AS equations:

$$AD: \qquad (1-\mu)\hat{Y}_s = \hat{\sigma}\mu\pi_s + \hat{\sigma}r_s^e + (1-\mu)\hat{G}_s \tag{2.45}$$

$$AS: \qquad (1-a_\pi\mu)\pi_s = (a_{1y}+\mu a_{3y})\hat{Y}_s + (a_{1g}+\mu a_{2g})\hat{G}_s \tag{2.46}$$

Again, the aggregate demand curve has an upward slope for the same reason analyzed before at the lower bound of interest rate. From equation 2.48, government spending is expansionary and moves the AD curve rightward. The AS curve is also pushed rightward, which can be seen from equation 2.49. The graph is shown below:

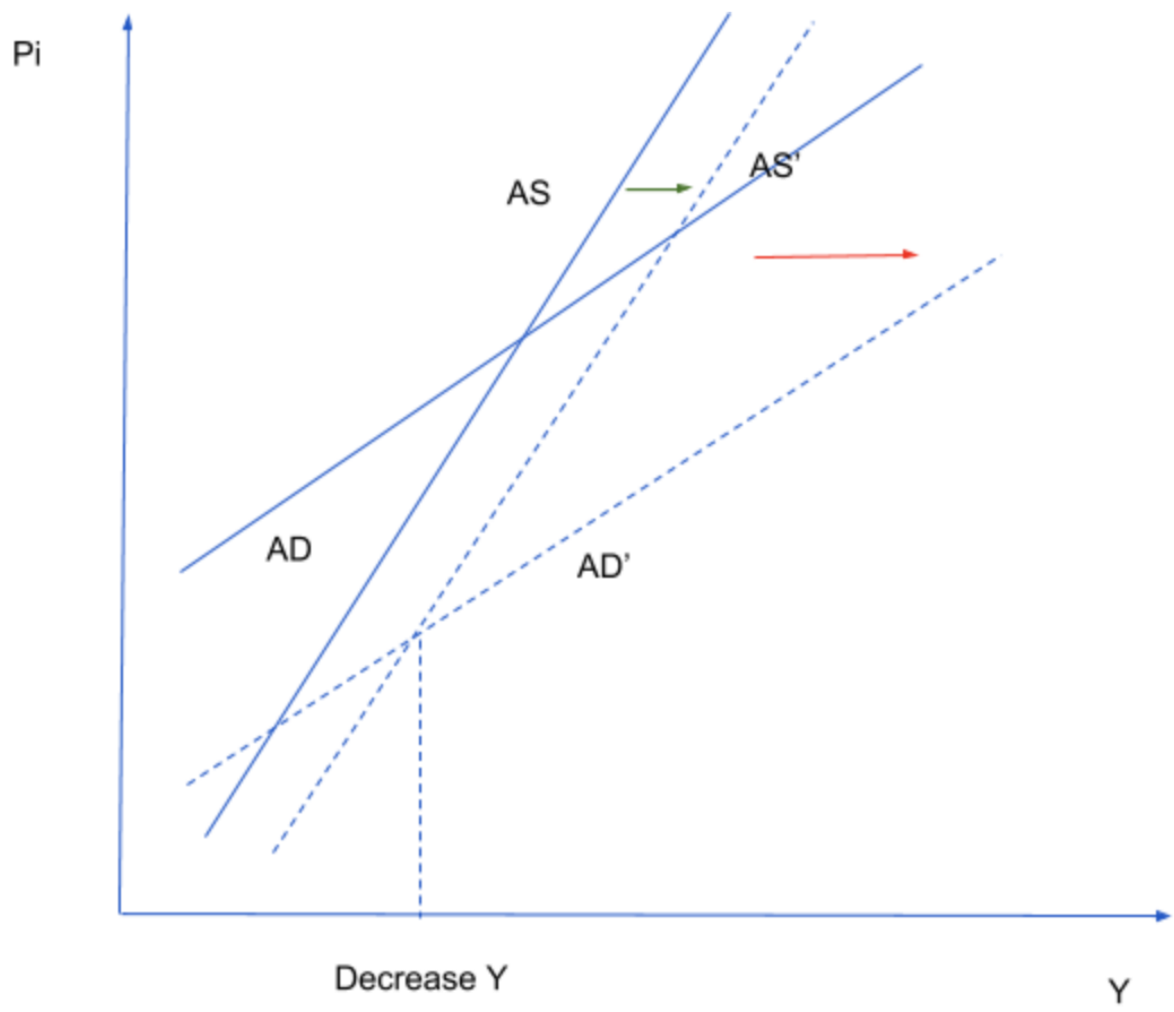
$$\frac{\triangle\hat{Y}_s}{\triangle\hat{G}_s} = \frac{(1-\mu)(1-a_\pi\mu)+\hat{\sigma}\mu(a_{1g}+\mu a_{2g})}{(1-a_\pi\mu)(1-\mu)-(a_{1y}+\mu a_{3y})\hat{\sigma}\mu} = -3.7223 \tag{2.47}$$

The multiplier is computed to be -3.7223, which exemplifies that each dollar of government spending decreases output by 3.72 dollars. Eggertsson (2011) finds a very big multiplier of 2.2931, while my case here is negative. The main reason is that the steeper slope of the AS curve largely mitigates the effect of stimulating aggregate demand. For a steeper AS curve, a rightward shift of the AD curve leads to a decrease in output.

In my analysis, the rightward shift of the AD curve leads to a decrease in output by 0.0616 percent. The rightward shift of the AS curve contributes to an increase of output by 0.3806 percent. A combination of the two effects gives a final positive output multiplier, although the AS curve shift plays a bigger role, which means that stimulating aggregate supply is much more important in this case. Government spending cannot stimulate the economy through changing aggregate demand.

Overall, I cannot find multipliers larger than 1 in both cases for government spending. It is

Figure 2.5: Government spending: the lower bound case



much less efficient at the LB case. From the decomposition analysis, most of the stimulation in output comes from stimulating aggregate supply, by giving direct incentives for firms.

2.5.4 The case of consumption tax cut

The effect of consumption tax cuts only affects the aggregate demand of the consumers, thus only the AD curve would shift to the right. Given a steeper AS curve, its effect is mitigated. This reinforces previous intuition that stimulating AD alone would be very inefficient.

The AD and AS equations in this case are as follows:

$$AD: \qquad \hat{Y}_s = -\hat{\sigma}\frac{\phi_\pi - \mu}{1-\mu+\hat{\sigma}\phi_y}\pi_s + \frac{\hat{\sigma}\chi^s(\mu-1)}{1-\mu+\hat{\sigma}\phi_y}\hat{\tau_s^s}, \tag{2.48}$$

$$AS: \qquad (1-a_\pi\mu)\pi_s = (a_{1y}+\mu a_{3y})\hat{Y}_s \tag{2.49}$$

For the interest rate above LB, the multiplier is computed as follows:

$$\frac{\triangle\hat{Y}_s}{-\triangle\hat{\tau_s^s}} = \frac{\hat{\sigma}\chi^s(1-\mu)(1-a_\pi\mu)}{(a_{1y}+\mu a_{3y})\hat{\sigma}(\phi_\pi-\mu)+(1-a_\pi\mu)(1-\mu+\hat{\sigma}\phi_y)}\hat{\tau_s^s} = 0.2865 \tag{2.50}$$

This generates a positive multiplier but still less than one. The interpretation is that for a one percentage point cut in consumer tax rate, output would increase by 0.2865 percent.

For the case with the LB, the AS curve is steeper than the AD curve, thus shifting the AD curve to the right would lead to a negative multiplier. The AD and AS equations are:

$$AD: \qquad (1-\mu)\hat{Y}_s = \hat{\sigma}\mu\pi_s + \hat{\sigma}r_s^e + (\mu-1)\hat{\sigma}\chi^s\hat{\tau_s^s} \tag{2.51}$$

$$AS: \qquad (1-a_\pi\mu)\pi_s = (a_{1y}+\mu a_{3y})\hat{Y}_s \tag{2.52}$$

The negative multiplier generated for the consumption sales tax has the same intuition as the government spending multiplier at the LB. Stimulating AD only generates deflationary expectation for the agents, thus consumption drops further.

$$\frac{\triangle \hat{Y}_s}{-\triangle \hat{\tau}_s^s} = \frac{\hat{\sigma}\chi^s(1-\mu)(1-a_\pi\mu)}{(1-a_\pi\mu)(1-\mu)-(a_{1y}+\mu a_{3y})\hat{\sigma}\mu} = -1.8669 \tag{2.53}$$

To sum up, consumption tax cuts have a small multiplier above LB, and a negative and inefficient multiplier at the LB.

2.5.5 The case of dividend tax and capital tax cuts

The dividend tax τ_t^p would drop out during the calculation of the equilibrium. But capital tax τ_t^A still remains. The capital tax cuts have very similar effects on output and inflation as the case of sales tax cuts; the multiplier is negligible due to a steeper AS curve.

The following tables summarize all fiscal policy multipliers found in this paper, with comparison to Eggertsson's (2011) results.

See the footnote for explanations.[19]

Table 2.2: Summary Table for multipliers, compared to Eggertsson (2011)

Type of Fiscal policy	This paper	Eggertsson's(2011) result
Labor (non-binding)	0.116	0.16
Labor (LB)	1.0866	-1.02
Government Spending (non-binding)	0.2509	0.46
Government Spending (LB)	-3.7223	2.2931
Sales Tax cuts (non-binding)	0.2446	larger than 0.46
Sales Tax cuts (LB)	-1.8669	larger than 2.2931
Capital Tax cuts (non-binding)	negligible	-0.0013
Capital Tax cuts (LB)	negligible	-0.1

[19] Labor(non-binding) refers to labor tax cuts multiplier with non-binding interest rate. Labor(LB) refers to the case of labor tax cuts multiplier with LB at the interest rate. Government Spending(non-binding) refers to government spending multiplier at the non-binding interest rate. Government Spending(LB) refers to government spending multiplier at the LB case. Sales Tax cuts (non-binding) refers to sales tax cuts multiplier at the non-binding case. Sales Tax cuts (LB) refers to sales tax cuts multiplier at the LB case. Capital Tax cuts (non-binding) refers to capital tax multiplier at the non-binding case. Capital Tax cuts (LB) refers to capital tax cuts multiplier at the LB case.

2.6 Sensitivity Analysis

2.6.1 Discussion of deep parameters

It is curious to know the changes in the computed multipliers when changing some parameters of this model. Since the labor tax cuts and government spending are the most obvious and important ones, this section will only focus on these two. As shown in previous parts, the effectiveness of fiscal policy at the lower bound depends largely on the relative steepness of the two curves. The discussion as follows focuses on how the parameters' values affect the slopes of the two curves.

The expectation parameter μ determines how long the recession will last, a smaller value means on average the economy gets out of the recession in a shorter period of time. Under the rational expectation assumption, the household and firms form the same expectation of the state of the economy. If the household expects the recession to last shorter (smaller μ) period of time, they expect price level to revert back to steady state (zero inflation instead of deflation) more likely in the future. The real interest rate is smaller given the lower bound, and consumption and output falls less; this is represented as a steeper AD curve. On the firm's side, because of price stickiness, for those firms who can adjust their prices, they will look at the expected price level in the future. A higher expected price level in the future means less aggressive price changes, or a wait and see strategy for the firms; this is represented as a flatter AS curve.

The price stickiness parameter θ_p gives on average the proportions of firms who can adjust prices in a period. For a bigger price stickiness parameter value, more firms cannot adjust prices in a given period; this is represented as a flatter AS curve.

The elasticity of substitution parameter ϵ_p determines directly the markup of a firm charged over its marginal cost; the bigger its value, the less the markup. The size of the markup decides how much the change in marginal cost goes in price/inflation. So a bigger ϵ_p gives a flatter AS curve.

The Frisch elasticity of labor supply captures the elasticity of wage rate on labor supply, thus a larger elasticity means the firms can attract labor by varying wage rate more easily. So a larger

ϕ gives a flatter AS curve since the firms can set a lower wage, i.e. a lower marginal cost, and employ more workers to produce goods and services.

For the labor market tightness parameters, a less tight labor market index x, more unemployed population u, and a smaller labor search cost parameter γ will give less labor market frictions. For the above intuition, the firm gets to adjust its labor more easily; this is represented as a flatter AS curve. I show the results of the multipliers for the model without labor market frictions in the next section.

Overall, I discuss how the slopes of the AS and/or the AD curve change, with respect to some deep parameters of the model. First, I do not compute in each case the exact fiscal policy multipliers, because the results are very sensitive to the setting of parameters, especially when the two curves are nearly parallel to each other. Second, I do not focus on any specific values of the parameters, but rather to give intuition behind the direction of change. The purpose here is to give intuitions for how these parameters affect the decision making of households or firms, and thus the AD or AS curve.

2.6.2 Results without labor market frictions

What do labor search and matching frictions do in this paper explicitly? To answer this question, I set the labor search and matching frictions to zero, which reverts the model back to Eggertsson's (2011) model. This is done by setting the labor search cost parameter γ to zero, so that the implied parameters would change accordingly. Then I compute explicitly the fiscal policy multipliers and compare to the base paper's results. Table 2.3 gives the results.

Table 2.3: Summary Table for multipliers, without labor search and matching frictions

Type of Fiscal policy	Without labor frictions	Eggertsson's result
Labor (non-binding)	0.1377	0.16
Labor (LB)	-1.377	-1.02
Government Spending (non-binding)	0.3152	0.46
Government Spending (LB)	4.7190	2.2931

The results are very similar to Eggertsson's case; the minor difference is due to the differences in the values of parameters. Especially at the lower bound, the labor tax cut is contractionary, and the government spending multiplier is larger than one. The pattern of the results can be seen from Figure 2.6. Without labor search and matching frictions, firms can adjust labor more freely than prices, which is seen as a flatter AS curve. In this case, a positive AS shock generates deflationary expectation for the households. As the real interest rate drops further, and the consumption and production decrease, the leftward movement along the AD curve leads to a negative labor tax cuts multiplier as shown in Table 2.3.

In general, when I silent the labor search and matching frictions, I go back to Eggertsson's (2011) model and have the same qualitative implications for the fiscal policy multipliers. This shows the importance of labor search and matching frictions, which generate different behavior pattern for the firms. This change of expectation of next period inflation from the households is key to tell the dramatic difference in fiscal policy multipliers.

2.7 Determinacy

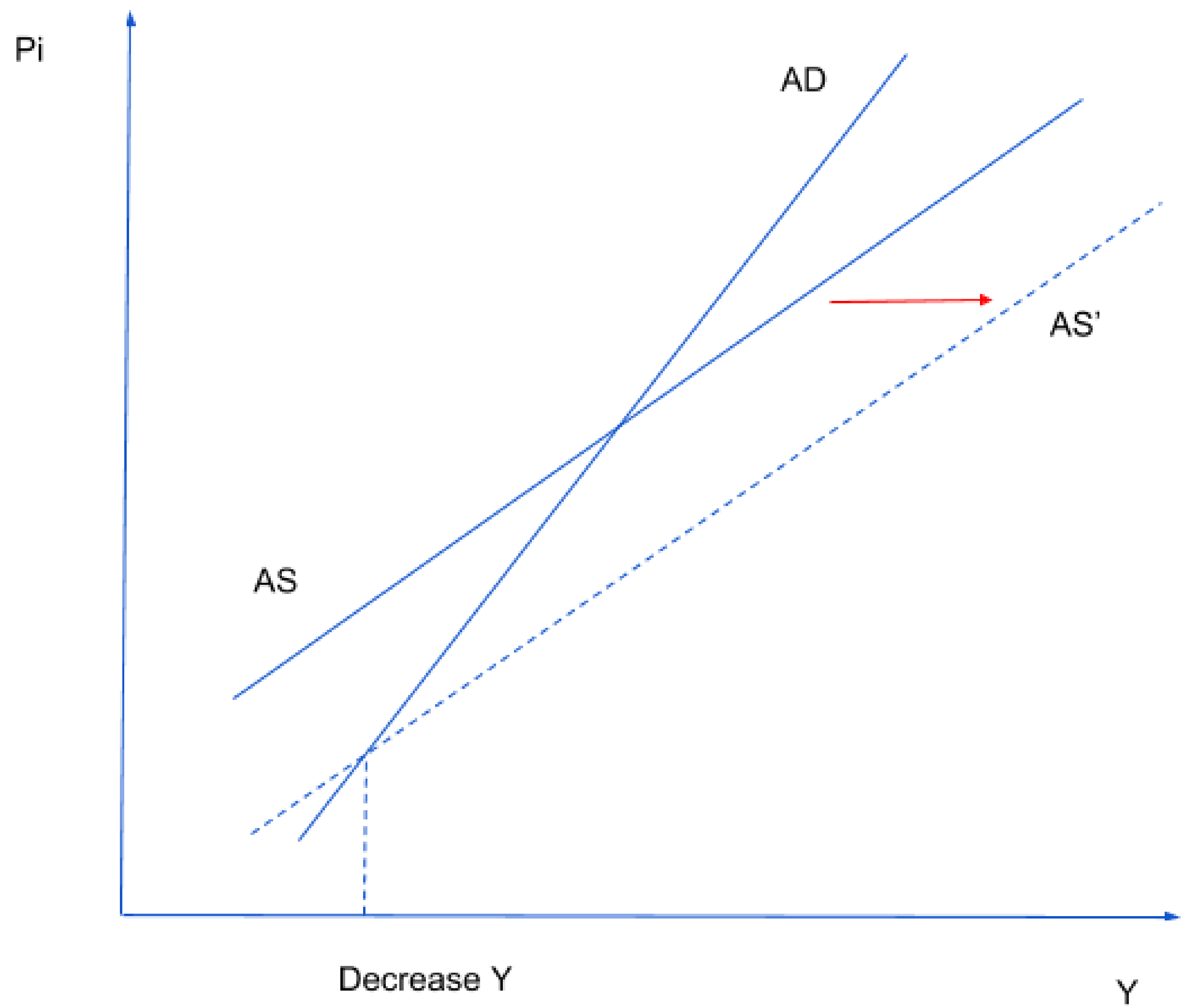
Starting from the AS and AD equations, under the assumption of two equilibrium, the model represents a regular rational expectation system. It can be solved via standard methods, e.g. using Blanchard and Kahn (1980).

As shown in the appendix, the conditions needed for unique equilibrium are spelled out. I discussed the two cases when nominal interest rate is binding or not bidning and their associated conditions for unique equilibrium. But the algebraic conditions are found to be hard to satisfy in this model, which means the short-run equilibrium found in the model can still be locally maximum but not unique.

2.8 The Bayesian estimation

In this section, I replicate Eggertsson and Singh (2018)'s goal of estimating the parameters according to the Great Depression scenario, when output falls by 30 percent and an annual deflation of 10 percent. I focus on the linear model, and use the same Bayesian maximum likelihood

Figure 2.6: Labor tax cuts, without labor market frictions



algorithm. The estimation philosophy and results are shown below.

The three equations below can describe the simple system with output and inflation as the two endogenous variables, and fiscal policy and natural rate of interest shock as the exogenous variables.

$$\hat{Y_t} = E_t\hat{Y_{t+1}} - \hat{\sigma}(i_t - E_t\pi_{t+1} - r_t^e) + (\hat{G_t^N} - E_t\hat{G_{t+1}^N}) + \hat{\sigma}\chi^s E_t(\hat{\tau_{t+1}^s} - \hat{\tau_t^s}) + \hat{\sigma}\chi^A\tau_t^A \tag{2.54}$$

$$\pi_t = a_\pi E_t\pi_{t+1} + a_w\hat{\tau_t^w} + a_{2y}\hat{Y_{t-1}} + a_{1y}\hat{Y_t} + a_{3y}E_t\hat{Y_{t+1}} + a_{2g}E_t\hat{G_{t+1}} + a_{1g}\hat{G_t} \tag{2.55}$$

$$i_t = max(LB, r_t^e + \phi_\pi\pi_t + \phi_y\hat{Y_t}) \tag{2.56}$$

To further match to the Depression scenario, I assume no exogenous fiscal interventions and the nominal interest rate is binding. The only shock driving the economy to recession is the exogenous real interest rate shock. With the previous three equations, the change in inflation and output can be described in the following expressions of the parameters:

$$\pi_s = \frac{(a_{1y} + \mu a_{3y})\hat{\sigma}}{(1-\mu)(1-a_w\mu) - (a_{1y} + \mu a_{3y})\hat{\sigma}\mu} r_L^e \tag{2.57}$$

$$\hat{Y_s} = \frac{(1-\mu a_\pi)\hat{\sigma}}{(1-\mu)(1-a_w\mu) - (a_{1y} + \mu a_{3y})\hat{\sigma}\mu}\hat{\sigma} r_L^e \tag{2.58}$$

The theoretical derivations of the Bayesian estimation method in Denes and Eggertsson (2009) is as follows. The true observation in the 1933 Great Depression is assumed to be observed with errors, so that $\pi_{1933}^{data} = \pi_L + \epsilon^\pi, \hat{Y_{1933}^{data}} = \hat{Y_L} + \epsilon^Y$, where ϵs are normally distributed measurement errors. The simple Bayes rule applies, the posterior $p(\Pi/X) = p(X/\Pi)p(\Pi)$. By taking logs of both sides and ignoring the constant terms, the estimated posterior likelihood function is as follows:

$$logp(\Omega/X) = -\frac{(\pi_L(\Omega) - (-\frac{0.1}{4}))^2}{2\sigma_\pi^2} - \frac{(Y_L(\Omega) - (-0.3))^2}{2\sigma_Y^2} + \sum_{\Phi_s \in \Omega} f(\Phi_s) \tag{2.59}$$

The previous two parts are derived above as expressions of the priors. The third part is a sum of the probability density functions given by the associated prior distributions. Denes and Eggertsson (2009) uses gamma and beta distributions for the parameters, depending on whether the upper bound of the parameter is below 1 or not. I follow the same rule and apply it to the labor market parameters as well.

Table 2.4 describes the prior distribution. It gives out the type of distribution, the mean and the standard deviation for the priors.

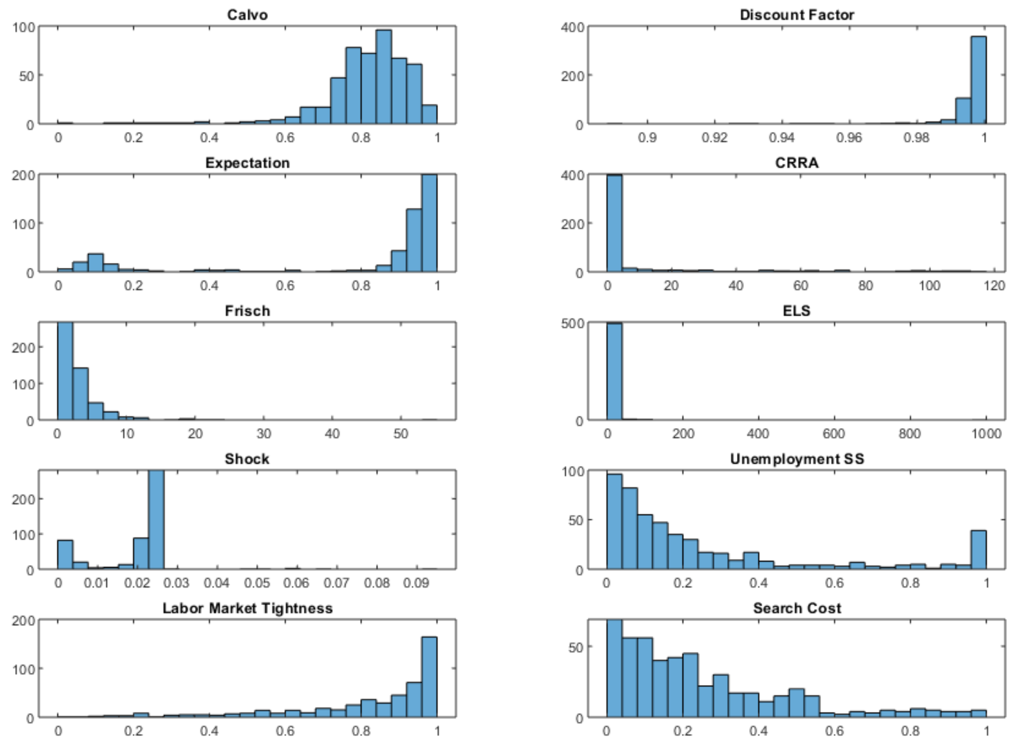
Table 2.4: Prior table for the estimated parameters

Parameter	Distribution Type	Mean	S.D.
α	Beta	0.66	0.05
β	Beta	0.99669	0.001
$1-\mu$	Beta	1/12	0.05
σ^{-1}	Gamma	2	0.5
ω	Gamma	1	0.75
θ	Gamma	8	3
r^e	Gamma	-0.010247	0.005
u	Beta	0.1	0.05
x	Beta	0.7	0.1
γ	Gamma	0.11	0.1

As mentioned in Eggertsson and Singh (2016), two options are given in the Matlab optimization routine. One is a built in function of Matlab, the fmincon function, which is essentially the minimum of the constrained nonlinear multivariate function. The second option is Curdia and Sim's optimization routine, which uses the csminwel matlab function repeatedly. I use the second option for estimation in this paper.

I perform 500 initial random draws of the parameters from the prior distribution, and a maximization routine is used to maximize the likelihood of each set of the parameters until convergence. I think two objects are achieved in this process: to maximize the data points of inflation and output, and to maximize the likelihood function. The algorithm takes two weeks to finish 500 sets of

Figure 2.7: Posterior distributions for the estimated parameters



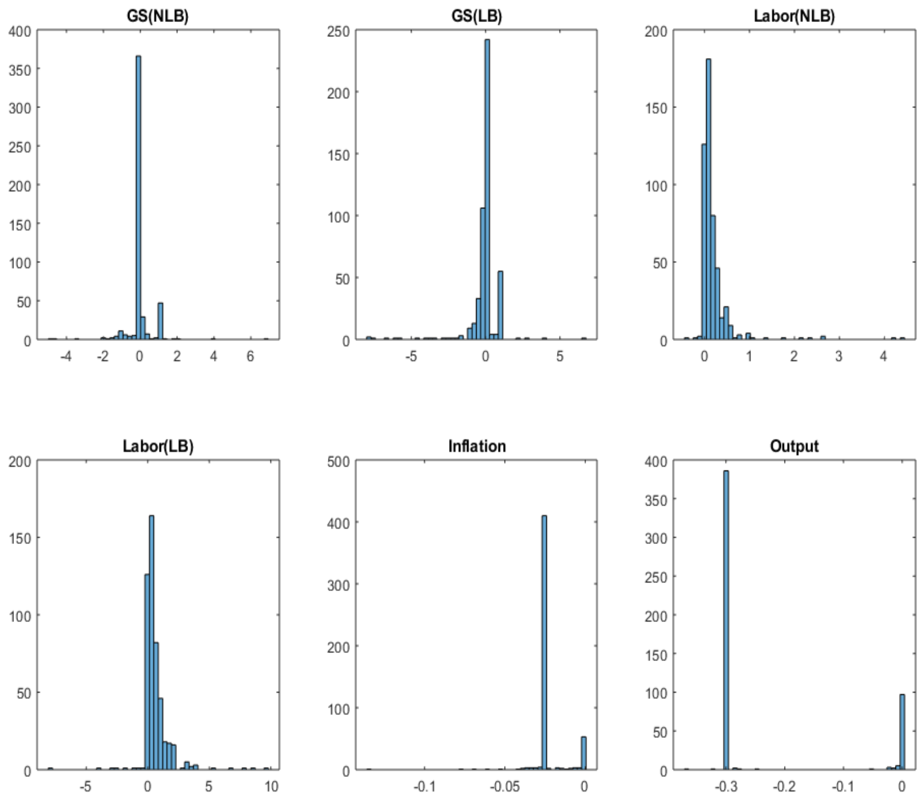
parameters. The results are presented in next section.

2.9 Result from Bayesian Estimation

Figure 2.7 gives the posterior distribution of the estimated parameters. In general, the posterior distributions are different from the prior distribution. Some parameters have a wide range of values, for example, the Frisch labor elasticity, labor search cost parameter and the expectation parameter, etc. This suggests the data gives enough information in the estimation process.

Figure 2.8 gives the estimated posterior distribution for the estimated labor tax cut and government spending multipliers using the estimated set of parameters, and the model implied inflation and output. From the last two distributions, it can be seen that the goal to match the data is quite achieved, as the inflation and output values are close to my original target. The first four distributions seem to be centered around some specific bins, especially for government spending

Figure 2.8: Posterior distributions for the estimated labor tax and government spending multipliers



multipliers. For labor tax cuts, it is more obvious that the two cases tend to be distributed toward the positive values range.

I compute the mean and median for the multipliers excluding the extreme values. They are shown in Table 2.5. I find small and positive multipliers for labor tax cuts and government spending at the non-LB case. At the lower bound, government spending has small negative multipliers while labor tax cut has positive multipliers. This result is consistent with the previous finding in the calibration exercise that labor tax cut is more effective than government spending at the lower bound.

Table 2.5: Mean and Median for the posterior multipliers distribution (excluding extreme values)

Type of Fiscal policy	Mean	Median
Labor (non-B)	0.1851	0.0965
Labor (LB)	0.5957	0.3507
GS (non-B)	0.0368	0.0018
GS (LB)	-0.0981	-0.0207

2.10 Conclusion

This paper has extended Eggertsson (2011) to consider the effectiveness of fiscal policies with labor search and matching. To do that, I add a Nash Bargaining process every period between the consumers and the firms. This changes the Aggregate Supply of the economy, which is the root of the difference in the results compared to Eggertsson (2011).

I find small multipliers for government spending and labor tax cuts. The results are surprisingly different at the lower bound. For government spending, the multiplier is negative which suggests inefficient government spending. This contrasts previous theoretical findings. I find labor tax cuts to be expansionary and effective, which is due to the labor market friction that generates inflationary expectation when facing aggregate supply stimulation. The effects of labor search and matching frictions in this paper can be seen from the sensitivity analysis. Without labor search and matching frictions, the model generates fiscal policy multipliers similar to Eggertsson (2011). I also conduct a Bayesian maximum likelihood estimation exercise for the parameters, and the same qualitative results for fiscal policy are found.

Overall, this paper contributes to combining the study of fiscal and monetary policies with the DMP model of labor search and matching. For future research, I think it is necessary to put a longer duration of data into a similar DSGE framework to estimate the parameters. Then analyzing impulse response functions of different variables to an exogenous shock would be interesting.

3. ENDOGENOUS GROWTH, LABOR FORCE PARTICIPATION AND FISCAL POLICY IN A DSGE MODEL

3.1 Introduction

There are two general macroeconomic trends after the Financial Crisis; both are unexpected, and they come from the supply side. First, the labor force participation rate dropped sharply and unexpectedly after the financial crisis, from 66 percent to 63 percent. Mishel, Bivens, Gould, and Shierholz (2012) find that 2/3 of this drop can be explained by cyclical factors due to the financial crisis, while the rest results from demographic trends. Second, Fernald (2014) computes TFP and utilization-adjusted TFP time series using simple growth accounting, and finds signs of decline before the financial crisis but a large drop after the financial crisis. These two phenomena coincide with the general fall in U.S. GDP after the financial crisis and they can potentially contribute to the slow recovery of the economy as a weak sign of the supply side.

The previous literature has investigated both endogenous economic growth and labor force participation, but separately. Anzoategui, Comin, Gertler and Martinez (2017) attempt to model R&D activity as a knowledge creation and adoption process. By using historical R&D data in the model, they find the return from investing in the technology sector exhibites an endogenous decline long before the 2010s. In their model, this endogenous TFP decline during the Great Recession can be tracked back to the 90s. They show that endogenous movement in TFP can explain the majority of the volatility of macro time series. Guerron-Quintana and Jinnai (2018) propose a model featuring endogenous productivity a la Romer and a financial friction a la Kiyotaki-Moore. They find that the financial disturbances in the Great Recession result in a contraction of output since they slow down the firm creation process. Garga and Singh (2016) use a real business cycle model and investigate the optimal monetary policy.

There are a few papers that explain the underlying slow recovery of GDP from the perspective of the labor force participation rate. Christiano, Eichenbaum, and Trabandt (2015) factor the labor force participation problem into the DSGE model. They argue that the vast bulk of movements in aggregate real economic activity during the Great Recession were due to financial frictions. According to their model, the observed fall in TFP and the rise in the cost of working capital played critical roles in accounting for the small drop in inflation that occurred during the Great Recession. Erceg and Levin (2014) examine the behavior of the labor force participation rate (LFPR) prior to the Great Recession and then focus more specifically on analyzing the sources of its post-2007 decline based on disaggregate demographic patterns and evidence from state-level panel data. They also assess the extent to which the fall in the LFPR may reflect accelerated enrollment in the Social Security Disability Insurance (SSDI) program.

In this paper, I ask the following question: what roles do these two empirical supply side factors play in explaining the performance of general macroeconomic variables like GDP, the labor force participation rate, and the unemployment rate? To answer this question, I build a New Keynesian-style DSGE model with an endogenous growth component and a dynamic labor market component, extended with the labor force participation problem. These two components are combined into the DSGE framework to shed some light on the importance of these supply side factors in macro modelling.

I also consider the effects of monetary policy during the Great Recession. When the interest rate was at the zero lower bound (ZLB), the Federal Reserve conducted quantitative easing (QE) to decrease the long-term interest rate, hoping to affect the short-term interest rate. In this paper I use Wu and Xia's (2015) shadow Federal Funds rate, which comes from a term-structure model, to evaluate the unconventional monetary policy during the 2009 and 2015 ZLB period. In a historical variance decomposition analysis I find that unconventional monetary policy played a big role in reviving GDP.

In addition to the importance of monetary policy, I find that a labor market shock is central to explain the fall in GDP. The financial friction type of shock, such as to the marginal efficiency of investment (MEI), explains part of the GDP variance, as well. The Endogenous growth shock does not contribute significantly to the variation of GDP, but I argue this is because the R&D process has only short-lived effects on the endogenous variables in my model. This pattern can be seen from the impulse response functions of R&D shock.

To identify the fiscal policy shocks, I follow Zubairy (2014). The identification assumption is also spelled out in Leeper et al. (2010a, 2010b). In Zubairy (2014), government spending is found to have a big impact on the economy with on impact multiplier of 1.07, while labor and capital tax have impact multipliers of 0.13 and 0.34. In this paper, the government spending shock plays a big role in explaining the volatility of observed variables.

Zhang (2017) uses a similar fiscal policy identification but models the unemployment benefits of American Recovery and Reinvestment Act. Her focus is to explain the high and persistent unemployment rate in the United States during and after the Great Recession. She finds that unemployment benefits play an important role in the cyclical movement of unemployment through their effects on labor demand. But she does not model the labor force explicitly, nor consider endogenous growth.

3.2 The model

3.2.1 Households and Labor Force Dynamics

Following Christiano, Eichenbaum, and Trabandt (2015), the household has a unit measure of family members to allocate between the labor market and home production sectors. Let L_t represents the fraction allocated in the labor market, with $1 - L_t$ denoting the fraction out of the labor force and working in the home production. L_t is therefore the labor force participation rate. In the labor market, a fraction l_t is currently employed, so $u_t \equiv L_t - l_t$ are unemployed. u_t is also the unemployment rate.

There are three labor statuses for the family members: unemployed, employed and out of the labor force. There are dynamic flows in and out of each of these three. For employed workers, each period there is a fixed job separation probability $1 - \rho$. The separated workers have a fixed probability s of staying in the labor force, thus going to the unemployed pool; otherwise, they exit the labor force. For the unemployed, the same probability of staying in the labor force s applies. If they choose to stay in the labor force, there is an endogenous job finding probability f_t, otherwise they stay unemployed. For non-participating workers, there is an endogenous probability e_t that they join the labor force as unemployed facing the same job finding probability f_t in every period.

Let r_t be the amount of family members the household chooses in period t to transfer from the nonparticipation pool to the labor force. The labor force law of motion is described by the equation below:

$$L_t = s(L_{t-1} - \rho l_{t-1}) + \rho l_{t-1} + r_t \tag{3.1}$$

In equation 1, the first term on the right hand side is the unemployed population who choose to stay in the labor force, the second term is the employed unseperated workers, and the third term is the population newly joining the labor force from the non-participation pool. The sum of the three population makes up the total current period labor force.

The endogenously determined probability e_t for a nonparticipation worker to enter the labor force can be given as follows:

$$e_t = \frac{r_t}{1 - L_{t-1}} = \frac{L_t - s(L_{t-1} - \rho l_{t-1}) - \rho l_{t-1}}{1 - L_{t-1}} \tag{3.2}$$

Denote x_t as the proportion of new hires compared to the previous employment pool. The law of motion for employment can be simply stated as follows:

$$l_t = \rho l_{t-1} + x_t l_{t-1} \tag{3.3}$$

It is natural to define the job finding probability as follows:

$$f_t = \frac{x_t l_{t-1}}{L_t - \rho l_{t-1}} \tag{3.4}$$

The sum of separated and unemployed workers is given by:

$$(1-\rho)l_{t-1} + u_{t-1}L_{t-1} = (1-\rho)l_{t-1} + \frac{L_{t-1} - l_{t-1}}{L_{t-1}} L_{t-1} = L_{t-1} - \rho l_{t-1} \tag{3.5}$$

3.2.2 Household Maximization

The household follows a simple log utility function of the composite consumption good:

$$\mu = ln\tilde{C}_t \tag{3.6}$$

The composite consumption good is a weighted average of market consumption goods C_t, which are purchased in the goods market, and the home consumption goods $C_{H,t}$ which are produced by family members out of the labor force. The mathematical expression is as follows:

$$\tilde{C}_t = ((1-\omega)(C_t - bC_{t-1})^{\chi} + \omega(C_{H,t} - bC_{H,t-1})^{\chi})^{1/\chi} \tag{3.7}$$

where ω is the weight on market consumption goods and χ is the constant elasticity of substitution parameter that is between 0 and 1. Note that consumption follows habit formation; the parameter b denotes the degree of habit formation from previous period consumption.

The home good, C_t^H, is produced using the labor of the $1 - L_t$ individuals who are not in the labor force:

$$C_t^H = \eta_t^H(1 - L_t). \tag{3.8}$$

η_t^H is the productivity of home production. I assume η_t^H to have a steady state value close to the general productivity growth rate of GDP, which is similar to the steady state value in Christiano et al. (2016). Additionally I model an AR(1) process for home productivity as follows:

$$\eta_t^H = \rho_{nH}\eta_{t-1}^H + (1-\rho_{nH})\eta_{SS}^H + \epsilon_h \tag{3.9}$$

The household budget constraint is as follows:

$$P_tC_t + P_tI_t + \frac{B_{t+1}}{R_t} = R_{K,t}P_tK_t + (L_t - l_t)P_t\eta_t^D D + l_tW_t(1-\tau_{w,t}) - T + B_t + \Sigma_t \tag{3.10}$$

where $\frac{B_{t+1}}{R_t}$ is the interest bearing bond, T is the lump-sum taxes net of transfers, Σ_t is the firm's aggregate profits and $\tau_{w,t}$ is the labor tax rate.

3.2.3 The Monopolistic Retailers

I assume there is one retailer in each sector producing with the following representative production function:

$$Y_t = \eta_{tech,t} K_t^{\alpha} l_t^{1-\alpha} K_{G,t}^{\alpha G} \tag{3.11}$$

where $\eta_{tech,t}$ is the neutral technological shock to the production function which follows an AR(1) process:

$$\eta_{tech,t} = \rho_{tech}\eta_{tech,t-1} + (1-\rho_{tech})\eta_{tech,SS} + \epsilon_{tech,t} \tag{3.12}$$

K_t is the physical capital stock, l_t is the labor input, K_t^G is the government spending capital stock. α is the Cobb-Douglas production function weight on capital. And following Leeper et al. (2010a), the government capital is production augmenting, thus α_G is a small positive number. The following two law of motions are assumed for the two capital accumulation process:

$$K_t = (1-\delta_K)K_{t-1} + I_t[1 - S(I_t/I_{t-1})] \tag{3.13}$$

$$K_t^G = (1-\delta_G)K_{t-1}^G + G_t \tag{3.14}$$

where I_t is the investment in physical capital and G_t is the amount of government spending spent on government capital. An investment adjustment function S(.) is assumed to be increasing and convex to capture increasing costs of adding capital.

Given this production function, the return of physical capital $R_{K,t}$ is equal to the marginal product of physical capital.

The stochastic discount factor is given by:

$$Q_{t,t+s} = \beta \frac{\lambda_{t+s}}{\lambda_t} \frac{1}{g_{A,t+1}} \tag{3.15}$$

where λ_t is the marginal utility of consumption and $1 + g_{A,t+1}$ is the endogenous growth rate between time t and t+1.

There is only one leader in each sector who can produce at a lower marginal cost such that they can charge a mark-up over the competitive marginal cost. As a result, the profit function is just the mark-up times the overall production level:

$$\Sigma_t = (\xi_t - 1)Y_t \tag{3.16}$$

where Σ_t is the profit function and ξ_t is an exogenous mark up variable that follows an AR(1) process:

$$\xi_t = \rho_{xi}\xi_{t-1} + (1 - \rho_{xi})\xi_{SS} + \epsilon_{xi} \tag{3.17}$$

3.2.4 Perfectly competitive composite Good Production and the labor market

Each of the intermediate good composites are produced by a perfectly competitive firm using only labor. Consequently, all intermediate good firms are identical. As discussed in section 3.2.1, every perfectly competitive intermediate goods firm has to meet the vacancies of separated workers every period by posting vacancies and matching with new employees. To hire $x_t l_{t-1}$ workers, $x_t l_{t-1}/\tilde{Q}_t$ vacancies are posted, where $\tilde{Q}_t$ denotes the aggregate vacancy filling rate, which the representative firm takes as given. A standard matching function is proposed as follows:

$$x_t l_{t-1} = \eta_{Me,t}\sigma_m(L_t - \rho l_{t-1})^\sigma \tilde{Q}_t^{\,1-\sigma} \tag{3.18}$$

where $\eta_{ME,t}$ is the matching efficiency shock and follows an AR(1) process, and σ_m and σ are related matching parameters.

The pool of workers l_t engages in a Nash Bargaining process with a representative of the firm. To be more specific, I assume that the worker and the intermediate firm bargain in each period to maximize the weighted product of both parties' surpluses:

$$\max \omega_W = (V_t - U_t)^\eta J_t^{1-\eta} \tag{3.19}$$

where V_t is the surplus of workers associated with being employed and U_t is the surplus of workers associated with being unemployed. Both are expressed in terms of consumption units. Both η and $1-\eta$ are below 1 and represent the workers' and the firm's bargaining powers, respectively. The intuition is that the associated surplus ω_W is split between the bargaining parties. The surplus J_t is the firm's surplus from the match, which is the difference between the present values of marginal products and wage payments:

$$J_t = vp_t - wp_t \tag{3.20}$$

where vp_t is the present value of hiring the worker. Since the production function of the firm produces one unit of product using one unit of labor, the current value of hiring is just the price of intermediate goods pm_t. Also wp_t is the present value of current and future after-tax wage payments to the worker.

The value functions of family members in the three different labor market situation all follow their own dynamic processes. Thus there are three value functions: employment, unemployment and non-participation. The general form of these value functions are the current benefits of this status, plus expected future discounted sums of the value functions. The second part would take into account the dynamic flows in and out of each status as described in section 3.2.1.

Let V_t denote the value to a worker of being employed in period t. The expression for this value function is:

$$V_t = wp_t + E_t m_{t+1}[\rho V_{t+1} + (1-\rho)s(f_{t+1}V_{t+1} + (1-f_{t+1})U_{t+1}) + (1-\rho)(1-s)(N_{t+1} + \Upsilon_{t+1})]. \tag{3.21}$$

In the next period, with probability ρ, the worker would remain employed and have next period value of employment V_{t+1}. With probability $(1-\rho)s$, the worker would be separated from work but remain in the labor force; then f_{t+1}, the endogenous job finding probability, would determine whether the worker remains unemployed or gets matched with another firm. With probability $(1-\rho)(1-s)$ the worker exits the labor force. The value function of unemployment and non-participation are U_t and N_t, respectively. The labor adjustment cost, Υ_t, is contributed by people in the labor force to people out of the labor force.

The next expression gives the value of unemployment. The only benefit of staying unemployed is the unemployment benefits η_t^D paid out from the government:

$$U_t = \eta_t^D D + E_t m_{t+1}[s f_{t+1} V_{t+1} + s(1 - f_{t+1}) U_{t+1} + (1 - s)(N_{t+1} + \Upsilon_{t+1})]. \tag{3.22}$$

The above expression reflects the assumption that an unemployed worker finds a job in the next period with probability $s f_{t+1}$, remains unemployed with probability $s(1 - f_{t+1})$ and exits the labor force with probability $1 - s$. The parameter D is set such that the total unemployment benefits is equal to 31.51 percent of the wage compensation to the worker. The unemployment benefits shock has a unique process explained in section 3.2.6 blow that describes government functions.

The value of non-participation is:

$$N_t = \frac{\lambda_{H,t}}{\lambda_t} \eta_t^H + E_t m_{t+1}[e(f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} - \Upsilon_{t+1}) + (1 - e) N_{t+1}]. \tag{3.23}$$

For non-participating family members, the benefit is the utility gain from home produced goods. The first term is the marginal utility of home production in terms of market consumption goods multiplies by the home productivity shock. The expression for the marginal contribution of home produced goods is as follows:

$$\lambda_{H,t} = \frac{\omega}{1 - \omega} \Big(\frac{C_t - b\bar{C}_{t-1}}{C_t^H - b\bar{C}_{t-1}^H} \Big) \tag{3.24}$$

C_t^H is the home goods consumption, and C_t is the market consumption goods.

With probability e, the non-participating family members would join the labor force and earn the employment and unemployment value, respectively. Note that the labor adjustment cost is incurred when joining the labor force. Otherwise, this family member would continue to earn the non-participation value.

3.2.5 Innovation and research arbitrage

The endogenous growth component is very similar to Garga and Singh (2017). Their model follows Aghion, Akcigit and Howitt (2014)'s classic Schumpeterian idea of innovating to replace a leader in one industry.

It is assumed that only the monopolistic retailer as described in section 3.2.3 is producing the final goods in that sector of the industry, and this monopolist can be the sole producer because it has the most advanced technology. At the same time, there is one single entrepreneur in each sector that engages in research and development using the final good. With probability z_{it}, the entrepreneur is successful with the innovation and advances current technology by $A_{it+1} = \gamma A_{it}$. The research intensity z_{it} is chosen by the entrepreneur in order to maximize the discounted next period profit function:

$$max_{z_t \in [0,1]} \{E_t z_{it} Q_{t,t+1} V_{t+1}(\gamma A_{it}) - P_t R_{it}\} \tag{3.25}$$

The first part is the expected discounted revenue value if the innovation is successful; the second part is just the current price multiplied by the research good spent by the entrepreneur.

The revenue value function has the following form:

$$V_t(A_{it}) = \Sigma_t(A_{it}) + (1 - z_{it})(1 + g_{A,t}) E_t m_{t+1} V_{t+1}(A_{it}) \tag{3.26}$$

This expression states the recursive form for the value function: the current revenue Σ_t of producing, plus the expected discounted value if not replaced during the next period. The current revenue function is assumed as in equation 3.15.

The research good is assumed to have the following function:

$$R_{it} = \delta z_t^{\rho_{RD}} A_{it} \tag{3.27}$$

Where $\rho_{RD} > 1$ is the inverse of the R&D elasticity.

The interior solution of the maximization problem above is solved by taking the first order condition. It produces the following condition:

$$\rho_{RD} z_t^{\rho_{RD}-1} \delta = E_t m_{t+1}(1 + g_{A,t}) V_t \tag{3.28}$$

The total amount of the final good used in research and innovation:

$$RD_t = \int_0^1 RD_{it} di = c(z_t) A_t = \delta z_t^{\rho_{RD}} A_t \tag{3.29}$$

The Growth rate of endogenous TFP has the following process:

$$g_{A,t+1} = z_t(\eta_{RD,t} - 1) \tag{3.30}$$

Where γ_t is the innovation shock which would decide the jump in TFP. This innovation shock also has the following AR(1) process:

$$\eta_{RD,t} = \rho_{RD} \eta_{RD,t-1} + (1 - \rho_{RD})_{RD,SS} + \epsilon_{RD} \tag{3.31}$$

3.2.6 Market Clearing, Monetary and Fiscal Authority

The market clearing condition for this economy is $C_t + I_t + RD_t + G_t = Y_t$, where G_t denotes government expenditures, and Y_t is measured GDP.

The Monetary Authority follows a simple Taylor rule to set its monetary policy as follows:

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(R_{SS} + r_y ln(Y_t - Y_{t-1}) + r_\pi \pi_t) + \epsilon_r \tag{3.32}$$

Where r_y, r_π are the Taylor rule coefficients on output deviation and inflation, respectively. ϵ_r is the monetary policy innovation.

The fiscal authority has the following budget constraint that government issuing debt as follows:

$$B_tR_t = B_{t-1} + G_t + T - \tau_t^w w_t l_t - \eta_{D,t} D(L_t - l_t) \tag{3.33}$$

where $\epsilon_{g,t}$ is the innovation to government spending. B_t denotes the government bond issued in a period.

I follow the literature and propose the following ways to identify fiscal policy shocks. Government expenditure follows an exogenous law of motion:

$$ln(g_t) = \rho_g ln(g_{t-1}) + (1 - \rho_g) ln(g_{SS}) - \rho_{gb} ln(B_t - B_{t-1}) - \rho_{gy} ln(Y_t - Y_{t-1}) + \epsilon_t^{g,t} \tag{3.34}$$

The unemployment benefits follow the following exogenous process:

$$ln(nD_t) = \rho_{nD} ln(nD_{t-1}) + (1 - \rho_{nD}) ln(nD_{SS}) - \rho_{nDb} ln(B_t - B_{t-1}) - \rho_{nDy} ln(Y_t - Y_{t-1}) + \epsilon_t^{nD,t} \tag{3.35}$$

The above two processes show that government spending and unemployment benefits are increasing in recession (counter-cyclical), and they are decreasing in the amount of government debt such that deficits make financing these two types of spending harder. The coefficients before the endogenous variables are estimated using data.

The labor tax rate has the following process:

$$ln(\tau_{w,t}) = \rho_w ln(\tau_{w,t-1}) + (1 - \rho_w) ln(\tau_{w,SS}) + \rho_{wb} ln(B_t - B_{t-1}) + \rho_{wy} ln(Y_t - Y_{t-1}) + \epsilon_t^{w,t} \tag{3.36}$$

The above process show that the tax rate is increasing in the debt level. This helps to maintain a balanced budget, and increases in output growth level. It is reasonable to assume tax cuts in recession instead in economic boom. The specific coefficients before the endogenous variables are also estimated using data.

3.3 The estimation methods

I state all equations in non-log-linear form. To solve the model, all real variables (e.g. GDP, investment etc.) are detrended by the technology growth rate, resulting in real variables having constant steady state values. The idea behind this is that the R&D sector is the sole determinant of the endogenous growth rate of the economy. I extract the endogenous growth rate of the economy given observed variables.

Labor market variables (e.g. the labor force and unemployment rate) are assumed to have steady state values. The underlying assumptions on these parameters can be found in Table B.1. The basic idea is to match to the labor market condition of the U.S. economy in normal time.

Given the assumptions on the steady state values of the variables and the values of parameters, the initial steady state values are computed for all other parameters and variables to reach an equilibrium.

A Bayesian Markov Chain Monte Carlo estimation method is applied on some parameters, with a combination of 12 observation data explained in the next section. I use Dynare for the estimation, shock decompostion and impulse response functions computation. 2 chains of 25000 replications are drawn, and half of the initial parameters are dropped as burns before using posterior estimation. The acceptance rate for each chain is between 20 and 25 percent.[1]

3.4 Data used in estimation

The data used here is quarterly frequency starting from 1955 Q1 and ending in 2017 Q4. Most of the time series are constructed as real per-capita values. Nominal data is divided by GDP deflator (given by Table 1.1.9. Implicit Price Deflators for Gross Domestic Product, line 1), and per capita values are divided using the civilian non-institutional population over the age of 16 (given by LNU00000000Q, at Bureau of Labor Statistics). The table and line numbers otherwise specified refer to the NIPA tables on the Bureau of Economic Analysis website. The data for GDP,

[1]Estimating even a medium scale model using Dynare is challenging. There are a number of options in Mode compute algorithms to estimate the model. I tried to use mode compute=6, which has a more restricted computing requirement. I tried to change the initial conditions of the prior parameters but still keep running into the bounds. In the end, I used mode compute=9 to get the estimation results.

consumption, investment, government spending and debt are linearly detrended by the average GDP growth rate. The labor tax rate is detrended by its long-term average level, respectively. The Federal Funds rate and Shadow rate are detrended by the average level of FFR. The purpose of detrendning is to get the stationary series to reflect the percentage deviations to their steady states.

To summarize, I use data on labor force participation rate, unemployment rate, inflation rate as computed by GDP deflator, GDP, consumption, investment, effective Federal Funds rate with shadow rate, unemployment benefits, labor tax rate, and debt as observables.

I state the exact source of the data in the appendix.

3.5 Results

The effects of different shocks on the volatility of observed variables can be evaluated from the historical variance decomposition graphs. I include the whole sample period from 1955 to 2017 in the estimation, and the expectation is that the initial sample periods (before the 80s, for instance) can give information in the Bayesian estimation. I do focus on the sample period after the Great Recession since this is the target period of this paper. Robustness checks can be done to subset the sample period, for example, exclude before 80s when the monetary policy is determined in a different fashion. Since the initial values of the variables are not stabilized, the initial periods of variance decomposition should not receive too much attention. But my analysis is based on the sample period after 2007.

From Figure B.1, the historical variance decomposition is performed on the level output, i.e. the observed output including the model generated endogenous growth rate. As explained before, I compare the level output to detrended output, the latter refers to the output level excluding the model generated endogenous growth rate. Since the endogenous growth rate is model generated, I expect the variance decomposition to be different among the level variable and the detrended variable. From this exercise, I can tell the effects of research and innovation on the output, consumption, investment, etc. at a more clear angle. In Figure B.1, it can be seen that during the first 100 periods, the initial values part of variance decomposition plays a sizable role, which is due to the fact that the steady states are not stabilized. After 2007, as the output starts to decline, initially the investment specific shock drives it down, then the labor productivity shock dominates the downward trend. At the same time, when the recession hits the economy, the monetary policy shock and government spending shock pull output upward, which captures the active policy orientation of the monetary authorities and fiscal policy. The technology related shocks do not play a big role in the model, I argue that this is due to the assumptions on the R&D sector, where the innovation intensity has immediate short lived effects on the endogenous growth rate but not a long term effect. As a result, the overall effects of technology on the real variables are very small.

Very similar results can be found for the detrended output variance decomposition from Figure B.2. For the labor force participation rate variance decomposition, in Figure B.3, after the Financial Crisis, negative home productivity shock and negative investment shock explain large portions of the total downward trend, while monetary policy shock and government spending shock explain most of the upward variation. In Figure B.4, for the unemployment rate variance decomposition, the story is a bit different. After 2007, initially the investment related shock drives up the unemployment rate, which comes from the Financial Crisis itself. Then the matching efficiency shock is the main upward force of the high unemployment rate. At the same time, government spending shock seems to be the main force driving down the unemployment rate, mark up shock and monetary policy shock also play some roles. The home productivity shock plays an important role for the upward force at the later period of time. This decomposition suggests that government spending shock helps the unemployment rate falls.

The effects of different shocks on some macroeconomic variables can be evaluated from the impulse response functions, which are computed using the estimated posterior parameters. For example, in Figure B.5, this is the case of a positive monetary policy shock. Federal Funds Rate reacts by increasing to a higher nominal level. This generates contractionary effects on all real variables and increase the price level. Investment increases for first several periods but falls eventually, the output falls and wages drop, consumption also drops. This is consistent with the contractionary monetary policy orientation. In Figure B.6, the impulse response functions for labor market are shown. Employment falls and labor force participation drops, these can be seen as the reaction to the real wage rate, since the real wage does not recover in first 20 periods, the effects on the labor market variables are quite persistent.

Figure B.7 and B.8 show the impulse response functions for an expansionary government spending shock, initially the increase in government spending crowds out the private sector's investment level, but since it expands the economy output and consumption level, later the investment also goes up. Inflation drops initially and triggers the cut of Federal Funds rate. For the labor market, the labor force participation rate and employment both increase due to the expansionary government spending shock, unemployment increases as a result but this can be related to the rise of wage rate.

In figure B.9, a positive neutral technology shock leads to more investment, inflation increases and nominal interest rate increases. Consumption and output also increase. In figure B.10, labor force participation rate drops and employment level drops, this is because of a decrease in wage paid to the worker. This kind of technology shock only expands the aggregate production level and even gives firms a higher opportunity to charge higher prices. It contracts the labor market.

The effects of R&D related innovation shock can be seen in Figure B.11 and B.12, for a positive shock it crowds out private investment, but expands production. This generates a temporary higher inflation but as Federal Funds rate rises, the inflation rate quickly drops. Consumption has a hump shape that drops initially then increases. For labor market, this type of technology shock increases the wage paid to the worker, it increases employment and the labor force participation rate. The effects of R&D related shock as compared to the neutral technology shock generates totally different implications to the labor market.

3.6 Conclusion

In this paper, I consider a DSGE model extended with an explicit modelling of the labor market and labor force participation within it, endogenous growth and fiscal policy. To evaluate the driving force for the Great Recession and its recovery, I find the investment related shock played a key role in the output slowdown. After the recession, monetary policy and fiscal policy helped the U.S. economy recover while the labor force participation rate related shock is a major reason for slowing down economic recovery.

The endogenous growth shock does not show a big impact from historical variance decomposition, but I argue that this is due to the current structure of the model. From the impulse response functions, a positive R&D related shock expands the labor market and a positive neutral technology shock contracts the labor market.

I do find that monetary and fiscal policies after the Great Recession played a big role in the economic recovery.

4. EVALUATING AND BACKTESTING PORTFOLIO VALUE-AT-RISK

4.1 Introduction

Risk management becomes an essential role in modern finance. Financial institutions need to closely monitor daily transaction risks due to internal and outside regulating authorities. In practice value-at-risk is the common methods financial institutions use to monitor risk. It gives a simple answer to the question: given a portfolio, how much money is at risk to lose for a certain probability. Even though this seems straightforward, there are a few empirical methods to reach the answer.

After the Financial Crisis, risk management becomes a more common practice among financial institutions. Institutions are required to perform backtesting for their positions for financial assets, central banks also require financial institutions to pass stress testing according to some hypothetical economic crisis scenario. Understanding the accuracy of different value-at-risk methods is key for researchers in the industry, research institutions and regulation institutions. In this paper, I intend to test the accuracy of commonly used value-at-risk methods using real financial asset data. I am also curious to see the performance of forecasting of different methods when the underlying portfolio is different. The assumption of data generating process for portfolio of equity, fixed income securities and currencies differ in nature, thus would affect the forecasting performance of different value-at-risk methods.

I find the performance of these VaR methods are strongly related to the underlying portfolio, the VaR confidence interval that the forecasters is focusing on and the time periods. It is quite often to find the Variance-Covariance method and a Monte-Carlo simulation based on multivariate normal distributions of the portfolio to overestimate VaR. Different methods seem to apply to different portfolio, RiskMetrics seem to apply to equity portfolio and Currencies portfolio, while simple variance and historical simulation are more applicable to bonds and currencies portfolios. For sub-samples, the performance of VaR methods depends on whether the economy is in a recession or not. For instance during the Financial Crisis, the underlying assets went through extreme negative returns, so that the VaR methods underestimate the loss of the portfolio. However, in this exercises I don't find one VaR method perform reasonably well in all circumstances, the back-testing should be conducted more seriously and the nature of statistical distributions of return should be considered more.

4.2 literature review

Fallon (1996) reviews the four standard methods used in VaR: Delta-Normal or Variance-Covariance method in Garbade (1986), Delta-Weighted Normal, a.k.a. "RiskMetrics" in J.P. Morgan (1994), Delta-GARCH in Hsieh (1993), and Gamma-Normal in Wilson (1994). A competing Gamma-Normal model similar to Wilson (1994) is proposed in Fallon (1996). Three evaluating methods are used: the mean percentage error (MPE), defined as the average percentage difference between the true VAR and an approximate VAR; mean absolute percentage error (MAPE), defined as the average difference between the absolute values of the percentage difference between the true and approximate; and the square root of the percentage difference between true and approximate. He finds Gamma method always outperforms the delta method, usually by wide margins.

Manganelli and Engle (2001) survey and evaluate the performance of the most popular univariate VaR methodologies, paying particular attention to their underlying assumptions and to their logical flaws. The performance of these VaR methods is evaluated using a Monte Carlo simulation. The data is generated using GARCH processes with different distributions and compare the estimated quantiles to the true ones. The results show that CAViaR models are the best performers with heavy-tailed DGP. But they do not use empirical observations of Financial assets, and do not consider a portfolio. Also the evaluations of the performance is based on bias and MSE.

Harmantzis, Miao and Chien (2006) aim to test empirically the performance of different models in measuring VaR and ES in the presence of heavy tails in returns using historical data. Daily returns of popular indices (S&P500, DAX, CAC, Nikkei, TSE, and FTSE) and currencies (US dollar vs Euro, Yen, Pound, and Canadian dollar) for over ten years are modeled with empirical (or historical), Gaussian, Generalized Pareto (peak over threshold) technique of extreme value theory

(EVT) and Stable Paretian distribution (both symmetric and non-symmetric). They experiment different modelling factors, e.g. rolling window size and confidence level. In estimating VaR, the models that capture rare events can predict risk more accurately than non-fat-tailed models. For ES estimation, the historical model (as expected) and POT method are proved to give more accurate estimation. The number of violations is modeled as a Bernoulli process, then accordingly tested, which is the evaluation method is this paper.

Gencay and Selcuk (2002) study the VaR estimation with extreme value theory (EVT) which is a parametric approach. They compare six different models for estimating one period ahead return predictions in both tails of the return distribution at different tail quantiles. These models are the variance-covariance approach with normal distribution, variance-covariance approach with Student's t distribution, historical simulation, adaptive generalized Pareto distribution (GPD) and nonadaptive GPD. The performances of these models to estimate one period ahead estimated returns are compared. GARCH estimation is excluded because the estimated returns are extremely volatile. The estimation methodology is to adopt a sliding rolling window with different sizes: 500, 1000 and 1500 days. The performance is compared by a "violation ratio". A violation occurs if the realized return is greater than the estimated one in a given day. The violation ratio is defined as the total number of violations, divided by the total number of one step ahead forecasts. The portfolios considered are the daily stock market returns for Argentina, Brazil, Hong Kong, Indonesia, Korea, Mexico, Singapore, Taiwan and Turkey.

Manganelli and Engle (2001) describes a hybrid approach proposed by Boudoukh, Richardson and Whitelaw (1998). This method combines RiskMetrics and historical simulation methodologies. The weights are assigned to selected historical returns of the portfolio, in an exponentially declining order to past returns. Then the quantile is found by taking sum of the weights to the desired significance level. The basic idea is that a low return in the far past has much less to say about future risk. For example, 2008 falls of stock price would generate a bigger VaR for 2009 or 2010, but not for the year of 2018. However, this might underestimate the business cycle effects, when a new cycle is coming forward.

They implement Monte Carlo Simulation, generate 1000 samples of 2000 observations each, for different processes. The first 5 are GARCH process with parameters [2.5, 0.04, 0.92]. The error terms are generated using random number generators with the following distributions: 1) standard normal, 2) Student-t with 3 degrees of freedom, 3) Student-t with 4 degrees of freedom, 4) Gamma with parameters (2,2), 5) Gamma with parameters (4,2). Two more process is a GARCH variance but with non-i.i.d. error terms that are alternatively drawn from a Student-t and a Gamma (2,2). The second one is a CAViaR pricess. The performance of the models is evaluated using the equivalents of the bias and MSE for vector estimates.

Chan and Gray (2006) find extreme value theory based model performs well in forecasting out-of-sample VaR, compared to a number of other parametric models and simple historical simulation based approach.

Diamandis, Drakos, Kouretas and Zarangas (2011) evaluate the performance of RiskMetrics, normal APARCH, student APARCH and skewed Student APARCH models that belong to the class of ARCH models. The focus is on long and short positions for three groups of stock market indices, Developed, Southeast Asia and Latin America. The evaluation is based on Giot and Laurent (2003), which takes advantage of the fat left and right tail using APARCH model. Kupiec's (1995) Likelihood Ratio test is also exploited. They find the skewed Student APARCH improves considerably the forecasts of one-day-ahead VaR for long and short trading positions.

4.3 Basel Framework

According to Basel (2004), there are three pillars to bank's risk management. First the minimum capital requirement, second supervisory review, and third market discipline. In Basel (2004), item 178, "The VaR models approach is available to banks that have received supervisory recognition for an internal market risk model under the Market Risk Amendment...Internal models will only be accepted when a bank can prove the quality of its model to the supervisor through the back-testing of its output using one year of historical data". In Item 180, "A bank using a VaR model will be required to backtest its output using a sample of 20 counterparties, identified on an annual basis. These counterparties should include the 10 largest as determined by the bank according to its own exposure measurement approach and 10 others selected at random. For each day and for the sample of 20 counterparties, the bank must compare the previous day's VaR estimate for the counterparty portfolio to the change in the exposure of the previous day's portfolio. This change is the difference between the net value of the previous day's portfolio using today's market prices and the net value of the that portfolio using previous day's market prices. Where this difference exceeds the previous day's VaR estimate, an exception had occurred. Depending on the number of exceptions in the observations for the 20 counterparties over the most recent 250 days (encompassing 5000 observations), the output of the VaR model will be scaled up using a multiplier as provided in the table below."

4.4 Methodologies

In this paper, I only consider profits and loss for a portfolio. Let the vector y denote the return of the set of assets used to form a portfolio. To allow for aggregation, the vector y is already in the log return form. Then y_j denotes the log return for the asset type j, and the return vector is $y = (y_1, .., y_n)$. In this paper I only consider $n = 10$, this in each portfolio there are 10 assets in total. I use $w = (w_1, ..., w_n)$ as the position of each instrument in this portfolio. For simplicity, the position of each instrument is 10 percent. This means that regardless of the initial price of the asset, the total position of each instrument is just 10 percent of the portfolio. Then the return of the portfolio is just the weighted average of each instrument return:

$$f(w, y) = \sum_{j=1}^{n} w_j y_j \tag{4.1}$$

Let the α denote the quantile of value at risk for the portfolio. That is the lowest α percentile or loss the portfolio would have. In this paper I consider the 1 percent quantile and 5 percent quantile, i.e. $\alpha = 0.01$ or $\alpha = 0.05$. The trading day I care about is just the next day.

4.4.1 Parametric Method–Multivariate Normal Distribution

If the distribution of the underlying assets is following a Normal distribution, then I only need to know the mean and standard deviation to compute its Value-at-Risk. For a portfolio, if each assets is assumed to have a univariate normal distribution, then the portfolio would follow a multivariate normal distribution. The sum of normal random variables is itself normally distributed. The portfolio mean and variance can be computed as a weighted average of all individual mean and variance.

The most straightforward way of computing value at risk is to extract some parameters from past data and make some assumption on the portfolio return distribution. In this case I assume the portfolio return has a multivariate normal distribution. That is assume, $y \sim N(\mu, \sigma)$ and depending on the sample, the next day portfolio return has the distribution $\tilde{y} \sim N(\tilde{\mu}, \tilde{\sigma})$. Where $\tilde{\mu} = \sum_{j=1}^{j=10} \frac{\mu_j}{10}$ and $\tilde{\sigma} = \sqrt{w'\tilde{Cov}w}$, $\tilde{Cov}$ is the covariance matrix of this portfolio.
The associated next trading day value at risk for this portfolio is $VaR_t(\alpha = 0.01, N = 1) = \tilde{\mu}_t - N^{-1}(\alpha)\tilde{\sigma}_t = \tilde{\mu}_t - 2.33\tilde{\sigma}_t$.

4.4.2 Parametric Method-Simple Normal distribution

The simple normal distribution method just makes simple assumption about the return of the portfolio, it is generated by a normal distribution with some mean and standard deviation. Then by choosing a sample period, I can get the sample mean $\tilde{\mu}_t$ and sample standard deviation $\tilde{\sigma}_t$ from these observations. Then the one day ahead value at risk is computed as $VaR_t(\alpha = 0.01, N = 1) = \tilde{\mu}_t - N^{-1}(\alpha)\tilde{\sigma}_t = \tilde{\mu}_t - 2.33\tilde{\sigma}_t$.

4.4.3 The Monte Carlo Approach-Multivariate Normal Distributions

The Monte Carlo Approach for Value at Risk is by extrapolating and creating future data. For instance, the future prices can be generated with a normal distribution with mean of 0, and standard deviation as the historical volatility of past prices.

For a portfolio VaR, I consider a Monte-Carlo simulation method with a multivariate normal distributions assumption on the underlying assets. That is I assume the assets have a DGP same as the multivariate normal distribution which has the sample means of the assets and sample variance-covariance matrix. Then a large sample of simulations is generated say, 5,000 simulations. Then the VaR is just the associated quantile from these 5,000 pseudosequence of next day forecasts.

4.4.4 RiskMetrics TM

J.P. Morgan introduced RiskMetricsTM, a VaR computation methods, in the 90s. This method is used to approximate GARCH volatility, that is the daily volatility involves a weighted average of past squared returns, with recent squared returns weighted more heavily. The weights are fixed for different types of assets and portfolios, and this simplifies the computation compared to a standard GARCH estimation.

The weight on the GARCH process is exponentially decaying over time, say $\lambda_i = 0.96$, then the weighting vector is $\lambda = (\frac{\lambda_i^{499}}{\sum_{j=0}^{499}\lambda_i^j}, \frac{\lambda_i^{498}}{\sum_{j=0}^{499}\lambda_i^j}, ..., \frac{1}{\sum_{j=0}^{499}\lambda_i^j})$. The matrix M of historical returns applied by the weight for the 10 assets is

$$\begin{bmatrix} y_{1,t-499}\frac{\lambda_i^{499}}{\sum_{j=0}^{499}\lambda_i^j} & y_{2,t-499}\frac{\lambda_i^{499}}{\sum_{j=0}^{499}\lambda_i^j} & \cdots & y_{10,t-499}\frac{\lambda_i^{499}}{\sum_{j=0}^{499}\lambda_i^j} \\ y_{1,t-498}\frac{\lambda_i^{498}}{\sum_{j=0}^{499}\lambda_i^j} & y_{2,t-498}\frac{\lambda_i^{498}}{\sum_{j=0}^{499}\lambda_i^j} & \cdots & y_{10,t-498}\frac{\lambda_i^{498}}{\sum_{j=0}^{499}\lambda_i^j} \\ \hdotsfor{4} \\ y_{1,t}\frac{1}{\sum_{j=0}^{499}\lambda_i^j} & y_{2,t}\frac{1}{\sum_{j=0}^{499}\lambda_i^j} & \cdots & y_{10,t}\frac{1}{\sum_{j=0}^{499}\lambda_i^j} \end{bmatrix}$$

The value-at-risk for one percent confidence interval computed as $-2.326 * \sqrt{w'M'Mw}$.

4.4.5 NonParametric- Historical VaR

Let F_n denote the empirical process of the observed profits and losses $X_1, ..., X_n$, that is:

$$F_n(t) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \leq t), \tag{4.2}$$

where $I()$ is the indicator function, and X_i is i.i.d. with (unknown) distribution F.

By standard statistical results on empirical processes (Van Der Vaart, 1988), the α quantile $F^{-1}(\alpha)$ can be estimated by:

$$F_n^{-1}(\alpha) = X_{n(i)}, \alpha \in (\frac{i-1}{n}, \frac{i}{n}), \tag{4.3}$$

where $X_{n(1)} \leq X_{n(2)} \leq ... \leq X_{n(n)}$ are the order statistics.

4.4.6 Extreme Value Theory

The mathematical foundation of EVT is the class of extreme value limit laws, first derived heuristically by Fisher and Tippett (1928) and later from a rigorous standpoint by Gnedenko (1943). Suppose $X_1, X_2, ...,$ are independent random variables with common distribution function $F(x) = Pr_X$ and let $M_n = \max X_1, ..., X_n$. For suitable normalizing constants $a_n > 0$ and b_n, we seek a limit law G satisfying

$$Pr\{\frac{M_n - b_n}{a_n} \leq x\} = F^n(a_n x + b_n) \rightarrow G(x) \tag{4.4}$$

for every x.

It can be combined into a single generalized extreme value distribution, first proposed by von Mises (1936), of form

$$G(x) = exp\bigg\{ - \bigg(1 + \xi \frac{x - \mu}{\psi}\bigg)_+ \bigg\}, \tag{4.5}$$

Thus I use Python EVT function to estimate the parameters of the generalized extreme value distribution from the sample data. Then the according VaR estimates are generated from the fitted distributions.

4.5 Back testing Methods

Conducting Value-at-Risk forecasts is not enough, the forecasts need to be evaluated with appropriate back-testing methods. Let $VaR_t(p)$ be the Value-at-Risk forecast with intended coverage rate p, such that ideally the probability of the actual realization exceeding the forecast equal to the coverage rate: $Pr_{t-1}(R_t < -VaR_t(p)) = p$. With the assumption the Value-at-Risk value is positive by convention.

4.5.1 Basel Traffic Light Approach

Banks are required by the Basel Committee (1996) to report back-testing results for daily forecasts compared to realized profit-and-loss of given asset. This so called Basel Traffic Light Approach focuses on 99% Value-at-Risk Confidence level and 250 trading days in one year. For 250 trading observations, the expected number of cases that exceed the VaR forecasts is 2.5 on average. The Basel Committee denote three zones based on the number of violations. 0 to 4 violations fall into Green zone, 5 to 9 violations fall into Yellow Zone, and 10 or more exceptions fall into Red zone. Then the scaling factor is computed as follows:

$$S_t = \begin{cases} 3.0 & \text{if } N <= 4 \text{ green} \\ 3.0 + 0.2(N-4) & \text{if } 5 <= N <= 9 \text{ yellow} \\ 4.0 & \text{if } 10 < N \text{ red} \end{cases} \tag{4.6}$$

Then accordingly the required market risk capital is defined as follows:

$$MRC_t = max\Big(VaR_t(0.01), S_t\frac{1}{60}\sum_{i=0}^{59} VaR_{t-i}(0.01)\Big) + c \tag{4.7}$$

The capital requirement is the maximum of estimated value at risk or a scaled volume. Where S_t is the multiplication factor that is applied to the 60 days average of previous and current estimated value at risk. c is just the cash requirement.

Basel Committee (1996) provides Table 1 that gives the cumulative probability for certain number of violations to happen. This table gives the trade off of making Type I and Type II errors. For 0 number of exceptions the cumulative probability is only 8.11% but it is in the Green zone, accepting the null hypothesis is very likely to make type I error. The same intuition for making type I error can be applied to small number of exceptions as well. While some low yellow zone numbers get penalties but the type I error is small. Overall the Traffic Light Approach cannot tell the accuracy of the model since it strongly favors model which generates bigger loss forecasts. At the same time, other properties like the independence of the forecasts are not evaluated (Hass, 2001).

In this paper, since I have much longer trading data and I compare to out-of-sample much longer than one year, so I don't follow this traffic light approach.

Zone	Number of Exceptions	Cumulative Probability
Green Zone	0	8.11%
	1	28.58%
	2	54.32%
	3	75.81%
	4	89.22%
Yellow Zone	5	95.88%
	6	98.63%
	7	99.60%
	8	99.89%
	9	99.97%
Red Zone	10 or more	99.99%

Table 4.1: Traffic light approach (Basel Committee, 1996): Cumulative probability is the probability of obtaining a given number or fewer exceptions when the model is correct(i.e. the true coverage is 99%) The boundaries are based on a sample of 250 observations. For other sample sizes, the yellow zone begins at the point where cumulative probability exceeds 95%, and the red zone begins at cumulative probability of 99.99%.

4.5.2 Hit Sequence

The hit sequence of VaR_t violations can be defined as follows, with the violation situation indicated as one:

$$I_t = \begin{cases} 1 & \text{if } R_t < VaR_t(p) \\ 0 & \text{otherwise} \end{cases} \tag{4.8}$$

Note that the hit sequence appears to discard a large amount of information regarding the size of violations etc. Recall, however, that the VaR forecast does not promise violations of a certain magnitude, but rather only their conditional frequency, i.e. p.

In addition, the number of violations and no violations are classified based on previous day's violation performance. Table 4.2 gives the criteria:

The computed conditional probabilities accordingly are:

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}}, \pi_1 = \frac{n_{11}}{n_{10} + n_{11}}, \pi = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10} + n_{11}} \tag{4.9}$$

π_0 is the probability of violations given that the previous day is a non-violation day. π_1 is the

Table 4.2: Conditional number of violation table

	$I_{t-1}=0$	$I_{t-1}=1$	
$I_t=0$	n_{00}	n_{10}	$n_{00}+n_{10}$
$I_t=1$	n_{01}	n_{11}	$n_{01}+n_{11}$
	$n_{00}+n_{01}$	$n_{10}+n_{11}$	N

probability of violation given that the previous day is a violation day. π is the total fraction of violation.

The straightforward accuracy of the VaR methods can be told by looking at the Hit Rate π. If the testing sample size is large enough, then the computed Hit Rate should be close to the VaR confidence Interval. For example, for a 1 percent value-at-risk forecast and a sample size of 1000 days, the expected number of violations should be close to 10. The Hit Rate is the first pass of the accuracy of Value-at-Risk method.

4.5.3 Unconditional Likelihood Ratio Test

Kupiec (1995) suggests the POF-test (proportion of failures), which measures whether the number of exceptions is consistent with the VaR confidence level. Under null hypothesis the number of exceptions follows the binomial distribution:

$$I_t \sim i.i.d.Bernoulli(p) \tag{4.10}$$

against the alternative that

$$I_t \sim i.i.d.Bernoulli(\pi) \tag{4.11}$$

and refers to the test of correct unconditional coverage (uc)

$$H_{0,uc} : \pi = p \tag{4.12}$$

This test is an unconditional test because the occurrence of violation is not based on any previous information. The basic idea of the test is to see whether the observed number of violation π is significantly different from the failure rate suggested by the VaR confidence level. Kupiec (1995) suggests the following likelihood-ratio (LR) test, the test statistic is as follows:

$$LR_{POF} = -2ln(\frac{(1-p)^{N-\pi}p^{\pi}}{[1-(\frac{\pi}{N})]^{N-\pi}(\frac{\pi}{N})^{\pi}}) \tag{4.13}$$

Under the null hypothesis the test statistic LR_{POF} is asymptotically χ^2 (chi-squared) distributed with one degree of freedom. If the test statistic is larger than the critical value at one degree of freedom, the null hypothesis would be rejected, this also indicates a large p-value. I choose 95% percentile of the chi squared distribution as the critical value. And in the tables of results, I also report the p-values of these tests.

4.5.4 Christoffersen's Independence Test

Christoffersen's independence test just takes into consideration of the timing of violation, whether the occurrence of violation is independent of whether the previous day has incurred a violation or not. Ideally the violation should be independent of previous event. The test statistic takes the following form:

$$LR_{ind} = -2ln(\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}}) \tag{4.14}$$

A Markov Chain process can be used to describe how the violation tend to occur based on previous day's event:

$$\Pi = \begin{bmatrix} 1-\pi_0 & \pi_0 \\ 1-\pi_1 & \pi_1 \end{bmatrix} \tag{4.15}$$

The null hypothesis is just that the probability of violation given previous day is non-violation equals to the probability of violation given previous day is violation:

$$H_{0,ind} : \pi_{01} = \pi_{11} \tag{4.16}$$

Under the null hypothesis, the test statistic will follow a Chi-squared distribution with one degree of freedom.

4.5.5 Christoffersen's Interval Forecast Test

This conditional coverage test combines previous two tests, the idea is to test whether violation based probabilities are also equal to the VaR confidence interval. The test statistic is:

$$LR_{cc} = LR_{POF} + LR_{ind} \tag{4.17}$$

The null hypothesis is as follows:

$$H_{0,cc} : \pi_{01} = \pi_{11} = p \tag{4.18}$$

Under the null hypothesis, the test statistic will follow a Chi-squared distribution with two degrees of freedom.

4.5.6 Dynamic Conditional Quantile Test

Engle & Manganelli (2004) proposed the dynamic conditional quantile (DQ) test, which involves the following statistic:

$$DQ = (Hit_t' X_t [X_t' X_t']^{-1} X_t' Hit_t)/(Np(1-p)) \tag{4.19}$$

Where $Hit_t = I_t - p$ and the vector of instrument might include the lags of Hit_t. Engle & Managanelli (2004) tests the null hypothesis that Hit_t and X_t are orthogonal. Under the null, the test statistic follows a χ_q^2 distribution, in which $q = rank(X_t)$. Even though this test is designed for quantile regression but any type of VaR methodologies can also be tested.

4.6 Portfolios

In this paper I consider different kinds of portfolios and their value at risk, all portfolios are equally weighted in their assets. The equity portfolio is consisted of 10 leading companies in different sectors: Apple Inc. (AAPL), Amazon.com Inc. (AMZN), Boeing Co (BA), Duke Energy Corp (DUK), FedEx Corporation (FDX), Johnson Johnson (JNJ), JPMorgan Chase Co. (JPM), Mcdonald's Corp (MCD), Pico Holdings Inc (Pico) and Exxon Mobil Corporation (XOM). The equity portfolio has duration from 2000-01-01 to 2018-12-31. The currencies portfolio is consisted of Australian Dollar to U.S. Dollar, Canadian Dollar to U.S. Dollar, Chinese Yuan to U.S. Dollar, European Dollar to U.S. Dollar, Great British Pound to U.S. Dollar, Hong Kong Dollar to U.S. Dollar, Japanese Yen to U.S. Dollar, South Korean Wan to U.S. Dollar, Mexican Peso to U.S. Dollar, and Brazilian Real to U.S. Dollar. The currency portfolio is downloaded from Wharton and has duration from 2001-01-03 to 2018-11-02. For the Bond portfolio I obtain data from FRED, which contain 1 year, 2 year, 3 year, 4 year, 5 year, 6 year, 7 year, 8 year Treasury bonds and moody's AAA, BAA corporate bond yields. The corporate bond is assumed to mature in 10 years. The duration is from 2000-01-03 to 2018-12-31. All above portfolios are considered to be long.

4.7 Results

The tables in the Appendix show the testing results for different portfolios for different testing periods of time. I have three underlying portfolios: equities, bonds and currencies. Since the sample for the VaR method is 500 trading days (2 years), so the starting time is the beginning of 2002, and the ending testing period is late 2018. I also consider three sub testing sample periods, the time before the Financial Crisis, ending before 2007; the Financial crisis period, between 2007 and 2009; the aftermath of Financial Crisis, 2010 to 2018. I consider two VaR confidence interval: the 1 percent confidence interval and the 5 percent confidence interval. I report the Hit Rate, i.e. the violation probability, the likelihood ratio tests-unconditional, independent, and conditional coverage, and the dynamic quantile test. For the latter four tests, only p-values are reported, for instance, p-values less than 0.05 means the null hypothesis would be rejected at 95 percent confidence interval. 6 VaR methods are considered, the Variance-Covariance method, the simple portfolio variance method, Historical Simulation method, Monte-Carlo Simulation based on multivariate normal distribution of the portfolio, the Extreme Value method, and RiskMetrics.

4.7.1 Equities Portfolio

For this portfolio of equities, the Variance-Covariance method and Monte-Carlo Simulation based on the portfolio multivariate normal distribution generate similar testing results, and their forecasts are much more conservative than other methods. A potential reason is that the stocks I choose have strong positive correlations with each other, these positive covariance increase the portfolio weighted variance when taking weighted average. While the simple variance method which do not take into account the covariance will have much smaller standard deviation.

For the full sample, at 1 percent confidence interval, only historical simulation can pass the unconditional likelihood test. At 5 percent VaR confidence level, only simple variance and historical simulation can pass the conditional test but fail all others. For the pre-Financial crisis sample, RiskMetrics provide very accurate forecasts and pass all tests at 1 percent VaR confidence interval. At 5 percent confidence interval, the overall hit rate is too low for all methods. For the Financial Crisis sample period, the 1 percent confidence interval forecasts either have too large or too small hit rates, this means that VaR methods do not react to bad events quick enough but still generate opportunistic estimates. At the 5 percent confidence interval, only RiskMetrics generate good forecasts and pass all tests. For the after crisis period, at 1 percent confidence interval, Risk Metrics generate forecasts that nearlly pass all tests, historical simulation is also a good candidate. At 5 percent confidence interval, only simple portfolio variance method can pass the conditional test.

Overall, RiskMetrics method perform reasonable especially in sub-samples, simple variance method and historical simulation can also be considered in some circumstances.

4.7.2 Bonds Portfolio

The Assets' returns in the Bonds Portfolio is much less volatile than the other two assets portfolio. I have 8 Treasury bonds with different maturities and two high rating corporate bonds. Due to the low volatility, Variance-Covariance method, Monte-Carlo simulation and RiskMetrics all generate forecasts that are too conservative. For the full sample, Historical Simulation perform the best among all methods. Historical Simulation and portfolio variance method perform reasonably well for pre and post recession sample periods. EVT also perform well especially in the pre recession sample period. But for the recession sample period, they all generate quite conservative forecasts due to the extreme negative returns.

4.7.3 Currencies Portfolio

For the whole sample, the performance of different VaR methods have quite different competitive advantages to the specific percentile that I target to forecast. It is well known the return of financial assets have fat tail, so that the lower percentile is harder to forecast. For EVT, RiskMetrics, Portfolio variance methods they do not perform well for one percentile interval, historical simulation is only weakly significant, while Variance-Covariance and Monte Carlo using its covariance matrix can pass the unconditional test. For five percentile interval, it reverts the above performance as shown in table 11. This is due to the target changes to a more central quantile instead of the tail.

For pre-recession sample, RiskMetrics perform the best for both quantiles, significant for all four tests. Historical simulation performs reasonably well for both quantiles and passes majority of the tests.

For the recession period, all methods generate underestimated forecasts that have too many violations at one percentile interval. For the five percent interval, the Variance-Covariance and Monte Carlo can pass unconditional test and weakly pass conditional test. RiskMetrics and EVT can pass the some tests. Overall, in the very volatile sample period, no methods really perform so good as before.

For the post-recession period, Historical simulation and portfolio variance can pass three tests at both quantile. EVT passes four tests at one percentile interval and RiskMetrics passes four tests at five percentile interval.

In general, RiskMetrics is the best method for pre and post recession periods, Historical Simulation also performs reasonably well for these two sample periods. But for the recession sample period, surprisingly no methods perform well enough due to the extreme volatility of the underlying portfolio.

4.8 Conclusion

In this paper I evaluate and backtest commonly used value at risk methods for three portfolios, at different confidence intervals and across different periods of time. I find the performance of these value at risk methods vary across specifications and no single method really dominate the others. One caveat is the state of the economy matters for VaR computations, that I find in Financial Recession period, many standard VaR methods typically underestimate negative returns the portfolio can get. I think one limitation of this paper is not an exhaustive list of value at risk methods are considered, for example filtered historical simulation, monte-carlo simulation with different distribution assumption, t-distribution assumption. Also advanced VaR method like quantile regression should be compared and backtested, even the estimation might take longer time.

5. SUMMARY AND CONCLUSIONS

In this dissertation, I studies the resolutions when an economy is in a crisis. The condition is set to be related to the recent Financial Crisis, and these resolutions can also shed light to past and future recessions.

In the first paper, I study the effectiveness of a variety of fiscal policy when nominal interest rate is binding at the lower bound. I show when there is labor search and matching frictions, fiscal instrument like increase in government spending is not effective, because of inertia of supply side to increase production. Labor tax cuts is more effective since it directly stimulates the aggregate demand side, firms respond by increasing prices and production. This further decreases saving and increases consumption, so that the economy gets out of recession faster than usual. These economic conclusions are reinforced when taken to the Great Depression scenario. The major economic suggestion from this exercise is that the firm's supply side factors should also be considered when conducting economic policies in the crisis.

In the second paper, I study the reasons why GDP falls and recovers during the Financial crisis in the United States. I consider the household's side labor force participation decisions, and model the firm's research and development behavior. I find at the beginning of the recession, the investment related shock explains why GDP falls. During the recession, as the labor force participation rate drops, the shock related to that explains why GDP is found hard to recover. I also evaluate the roles of monetary policy like quantitative easing and fiscal policy like American Recovery and Reinvestment Act. I find fiscal policy shock stimulates the economy mostly at the beginning of the recession; monetary shock has more persistent effects over the horizon.

In the third paper, I evaluate and backtest the commonly used Value-at-Risk Methods in financial institutions all around the world. Risk Management has taken a central role after the financial crisis to prevent extreme events like bank failures. I consider three financial assets portfolios: stocks, bonds and currencies. The testing time period is from around 2001 to 2018, and further divided into pre-recession, recession and post-recession sub-sample periods. I find for different portfolios, at different percent confidence intervals and at different sample periods, VaR methods have different testing performances. This result suggests the importance of underlying econometric properties of a portfolio. One caveat is that most VaR methods underestimate risks in the recession period when the assets generate extreme negative returns.

In general, this dissertation gives some explanations and advice of an economic recession to governments, firms and individuals. To gain a better understanding of the economy, we can recover from and prevent a crisis.

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APPENDIX A

APPENDIX OF SECTION TWO

A.1 Derivation of the AD equation

This section explains the main derivation and log-linearization of this article.

The AD equation in this paper is exactly the same as Eggertsson's (2011) case by solving the consumer's problem. First I maximize the consumer's utility equation (1) by the budget constraint equation (2), to get the first order condition of consumption equation (3). Then I use the resource constraint among consumption, government spending and aggregate output $C_t = Y_t - G_t$; substitute it into equation (3) and log-linearize it to get the AD equation (5). Overall there is no difference from what Eggertsson (2011) derives.

A.2 Derivation of the AS equation

The AS equation is derived by solving the firm's profit maximization problem. Start with the following profit function:

$$\Sigma_t = \max_{P_t^*} E_t \sum_{k=0} \theta_p^k \Lambda_{t,t+k}(1-\tau_{t+k}^p)[P_t^* Y_{t+k|t} - P_{t+k}(W_{t+k}^{Nash} N_{t+k|t} + \gamma x_{t+1} H_{t+k|t})] \qquad \text{(A.1)}$$

Substitute in the expressions for $Y_{t+k|t}$, $N_{t+k|t}$ and $H_{t+k|t}$, I get the following:

$$\Sigma_t = \max_{P_t^*} E_t \sum_{k=0} \theta_p^k \Lambda_{t,t+k}(1-\tau_{t+k}^p)[\frac{P_{t+k}^{*(1-\epsilon_p)}}{P_{t+k}^{-\epsilon_p}} Y_{t+k} - P_{t+k}\frac{1}{1-\tau_{t+k}^w} \Big(C_{t+k}^{\sigma}\frac{P_{t+k}^{*-\epsilon_p(\phi+1)} Y_{t+k}^{\phi+1}}{P_{t+k}^{-\epsilon_p(\phi+1)}} + \eta\delta x_{t+k}\frac{P_{t+k}^{*(-\epsilon_p)}}{P_{t+k}^{-\epsilon_p}} Y_{t+k}\Big) - \gamma x_{t+k}(\frac{P_{t+k}^{*(-\epsilon_p)}}{P_{t+k}^{-\epsilon_p}} Y_{t+k} - (1-\delta\frac{P_{t+k-1}^{*(-\epsilon_p)}}{P_{t+k-1}^{-\epsilon_p}} Y_{t+k-1}))]$$

Then the profit maximization problem is solved by taking first order condition of above with respective to the reset price P_{t+k}^*:

$$E_t \sum_{k=0} \theta_p^k \Lambda_{t,t+k}(1-\tau_{t+k}^p)[(1-\epsilon_p)\frac{P_{t+k}^*}{P_{t+k}}^{-\epsilon_p} Y_{t+k} - P_{t+k}\frac{1}{1-\tau_{t+k}^w}\Big(C_{t+k}^{\sigma}(-\epsilon_p(\phi+1))\frac{P_{t+k}^{*(-\epsilon_p\phi-\epsilon_p-1)}}{P_{t+k}^{-\epsilon_p\phi-\epsilon_p}} Y_{t+k}^{\phi+1} - \epsilon_p\eta\delta\frac{P_{t+k}^{-\epsilon_p-1}}{P_{t+k}^{-\epsilon_p}} x_{t+k} Y_{t+k} - \gamma x_{t+k}(-\epsilon_p)\frac{P_{t+k}^{-\epsilon_p-1}}{P_{t+k}^{-\epsilon_p}} Y_{t+k}]$$

Then log-linearize the above expression, combined with the price expression equation 30 and the labor market tightness expression equation 31. I can solve for the AS equation 32. The expressions below give out the exact forms of the coefficients of the AS equations.

$$D_1 = \epsilon_p \frac{\phi+1}{1-\epsilon_p}\frac{1+\bar{\tau^s}}{1-\bar{\tau^w}} \tag{A.2}$$

$$D_2 = (\frac{Y_{ss}-G_{ss}}{Y_{ss}})^{\sigma} \tag{A.3}$$

$$D_3 = \frac{\epsilon_p}{1-\epsilon_p}\gamma x\delta \tag{A.4}$$

$$D_4 = \frac{\epsilon_p}{1-\epsilon_p}(\eta\gamma x - \beta(1-\delta)(1-x)\eta\gamma x) \tag{A.5}$$

$$D_5 = 1 + D_3 + D_4 \tag{A.6}$$

$$T_1 = \frac{\epsilon_p}{1-\epsilon_p}\gamma x(2-\delta) \tag{A.7}$$

$$T_2 = \frac{\epsilon_p}{1-\epsilon_p}\gamma x[(1-\delta)(1-x)+1-\delta] \tag{A.8}$$

$$T_3 = \frac{\epsilon_p}{1-\epsilon_p}\gamma x(1-\delta)\epsilon_p \tag{A.9}$$

$$T_4 = \frac{\epsilon_p}{1-\epsilon_p}\eta\gamma\frac{x}{\delta} \tag{A.10}$$

$$T_5 = \frac{\epsilon_p}{1-\epsilon_p}\eta\gamma\frac{x}{\delta}(1-\delta)(1-x) \tag{A.11}$$

$$T_6 = \beta(1-\delta)\frac{\epsilon_p}{1-\epsilon_p}\eta\gamma\frac{x-2}{\delta} \tag{A.12}$$

$$T_7 = \beta(1-\delta)\frac{\epsilon_p}{1-\epsilon_p}\eta\gamma\frac{x-2}{\delta}(1-\delta)(1-x) \tag{A.13}$$

$$T_8 = \beta(1-\delta)\sigma\frac{\epsilon_p}{1-\epsilon_p}(1-x)\eta\gamma x\frac{Y_{ss}}{Y_{ss}-G_{ss}} \tag{A.14}$$

$$T_\pi = \frac{\theta_p}{1-\theta_p} + T_3\frac{1-\theta_p\beta}{1+D_5\phi\epsilon_p} \tag{A.15}$$

$$a_{1y} = \frac{(\phi+\sigma\frac{Y_{ss}}{Y_{ss}-G_{ss}})D_5 - T_1 - T_4 - T_7 + T_8}{1+D_5\phi\epsilon_p}\frac{1-\theta_p\beta}{T_\pi} \tag{A.16}$$

$$a_{2y} = \frac{T_2+T_5}{1+D_5\phi\epsilon_p}\frac{1-\theta_p\beta}{T_\pi} \tag{A.17}$$

$$a_{3y} = \frac{T_6 - T_8}{1 + D_5\phi\epsilon_p}\frac{1-\theta_p\beta}{T_\pi} \tag{A.18}$$

$$a_{1g} = \frac{\sigma\frac{Y_{ss}}{Y_{ss}-G_{ss}}D_5 - T_8}{1 + D_5\phi\epsilon_p}\frac{1-\theta_p\beta}{T_\pi} \tag{A.19}$$

$$a_{2g} = \frac{T_8}{1 + D_5\phi\epsilon_p}\frac{1-\theta_p\beta}{T_\pi} \tag{A.20}$$

$$a_\pi = \theta_p\frac{\beta}{(1-\theta_p)T_\pi} \tag{A.21}$$

A.3 Proof for Uniqueness of the equilibrium

Following Eggertsson (2011), the system of equations can be written as following forms:

$$E_t z_{t+1} = Az_t + ae_t \tag{A.22}$$

Where z_t is a 2 by 2 matrix of nonpredetermined endogenous state variables, e_t is a vector of exogenous distributions. The conditions for a Unique Bounded Solutions are as follows:

iff Case 1: $(a)det(A) > 1 \quad (b)det(A) - tr(A) > -1 \quad (c)det(A) + tr(A) > -1$.

Case 2: $(d)det(A) - tr(A) < -1 \quad (e)det(A) + tr(A) < -1$.

Either one of the case has to be satisfied to have unique equilibrium. Case 1 is often easier to be satisfied so that in this paper I only consider Case 1.

Let $z_t = [\pi_t \quad \hat{Y_t}]'$.

First Consider the case when the nominal interest rate is binding, express only the terms related to z_t, then the AS (when the past output deviation term is ignored) and AD equations are as follows:

$$\begin{aligned} AS: a_\pi E_t\pi_{t+1} &= \pi_t - a_{3y}E_t\hat{Y_{t+1}} - a_{1y}\hat{Y_t} \Rightarrow \\ E_t\pi_{t+1} &= a_\pi^{-1}\pi_t - a_\pi^{-1}a_{3y}E_t\hat{Y_{t+1}} - a_\pi^{-1}a_{1y}\hat{Y_t} \end{aligned} \tag{A.23}$$

$$AD: E_t\hat{Y_{t+1}} = \hat{Y_t} - \hat{\sigma}E_t\pi_{t+1} \tag{A.24}$$

Substitute the AD equation 2.55 into AS equation 2.54:

$$
\begin{aligned}
AS :\Rightarrow E_t\pi_{t+1} &= \frac{\pi_t}{a_\pi} - \frac{a_{3y}}{a_\pi}\hat{Y}_t + \frac{\hat{\sigma}a_{3y}}{a_\pi}E_t\pi_{t+1} - \frac{a_{1y}}{a_\pi}\hat{Y}_t \\
\Rightarrow (a_\pi - a_{3y}\hat{\sigma})E_t\pi_{t+1} &= \pi_t - (a_{3y} + a_{1y})\hat{Y}_t \\
\Rightarrow E_t\pi_{t+1} &= \frac{1}{a_\pi - a_{3y}\hat{\sigma}}\pi_t - \frac{a_{3y} + a_{1y}}{a_\pi - a_{3y}\hat{\sigma}}\hat{Y}_t
\end{aligned}
\tag{A.25}
$$

Substitute AS equation 2.54 into the AD equation 2.55:

$$
\begin{aligned}
AD : E_t\hat{Y_{t+1}} &= \hat{Y}_t - \hat{\sigma}(a_\pi^{-1}\pi_t - a_\pi^{-1}a_{3y}E_t\hat{Y_{t+1}} - a_\pi^{-1}a_{1y}\hat{Y}_t) \\
\Rightarrow E_t\hat{Y_{t+1}} &= \frac{\hat{\sigma}a_{1y} + a_\pi}{a_\pi - a_{3y}\hat{\sigma}}\hat{Y}_t - \frac{\hat{\sigma}}{a_\pi - a_{3y}\hat{\sigma}}\pi_t
\end{aligned}
\tag{A.26}
$$

At last I express equation 2.56 and 2.57 in the following form: $E_t z_{t+1} = Az_t$. Where the A matrix is the following:

$$
A \equiv \begin{bmatrix} \frac{1}{a_\pi - a_{3y}\hat{\sigma}} & -\frac{a_{3y}+a_{1y}}{a_\pi - a_{3y}\hat{\sigma}} \\ -\frac{\hat{\sigma}}{a_\pi - a_{3y}\hat{\sigma}} & \frac{\hat{\sigma}a_{1y}+a_\pi}{a_\pi - a_{3y}\hat{\sigma}} \end{bmatrix}
\tag{A.27}
$$

$tr(A) = \frac{1+\hat{\sigma}a_{1y}+a_\pi}{a_\pi - a_{3y}\hat{\sigma}}$ and $det(A) = \frac{\hat{\sigma}a_{1y}+a_\pi-\hat{\sigma}(a_{3y}+a_{1y})}{(a_\pi - a_{3y}\hat{\sigma})^2} = \frac{1}{a_\pi - a_{3y}\hat{\sigma}}$. Condition (a) and (c) are trivially satisfied, given that the trace and determinant are both positive. To satisfy condition (b), I need $det(A) - tr(A) = \frac{-a_\pi - \hat{\sigma}a_{1y}}{a_w - a_{3y}\hat{\sigma}} > -1 \equiv -a_\pi - \hat{\sigma}a_{1y} > -a_\pi + \hat{\sigma}a_{3y} \equiv a_{3y} < -a_{1y}$

Consider the case when the nominal interest rate is not binding. The same AS equation 2.54 applies, and the AD equation becomes:

$$
AD : E_t\hat{Y_{t+1}} = \hat{Y}_t - \hat{\sigma}E_t\pi_{t+1} + \hat{\sigma}\phi_\pi\pi_t + \hat{\sigma}\phi_y\hat{Y}_t
\tag{A.28}
$$

Substitute AD equation 2.59 into AS equation 2.54:

$$\begin{aligned} AS: a_\pi E_t\pi_{t+1} &= \pi_t - a_{3y}(\hat{Y}_t - \hat{\sigma}E_t\pi_{t+1} + \hat{\sigma}\phi_\pi\pi_t + \hat{\sigma}\phi_y\hat{Y}_t) - a_{1y}\hat{Y}_t \\ \Rightarrow (a_\pi - a_{3y}\hat{\sigma})E_t\pi_{t+1} &= -(a_{3y} + a_{3y}\hat{\sigma}\phi_y + a_{1y})\hat{Y}_t + (1 - a_{3y}\hat{\sigma}\phi_\pi)\pi_t \\ \Rightarrow E_t\pi_{t+1} &= -\frac{a_{3y} + a_{3y}\hat{\sigma}\phi_y + a_{1y}}{a_\pi - a_{3y}\hat{\sigma}}\hat{Y}_t + \frac{1 - a_{3y}\hat{\sigma}\phi_\pi}{a_\pi - a_{3y}\hat{\sigma}}\pi_t \end{aligned} \tag{A.29}$$

Substitute AS equation 2.54 into AD equation 2.59, I have the following:

$$\begin{aligned} AD: E_t\hat{Y_{t+1}} &= \hat{Y}_t - \hat{\sigma}(a_\pi^{-1}\pi_t - a_\pi^{-1}a_{3y}E_t\hat{Y_{t+1}} - a_\pi^{-1}a_{1y}\hat{Y}_t) + \hat{\sigma}\phi_\pi\pi_t + \hat{\sigma}\phi_y\hat{Y}_t \\ \Rightarrow (a_\pi - \hat{\sigma}a_{3y})E_t\hat{Y_{t+1}} &= [(1 + \hat{\sigma}\phi_y)a_\pi + \hat{\sigma}a_{1y}]\hat{Y}_t + \hat{\sigma}(\phi_\pi a_\pi - 1)\pi_t \\ \Rightarrow E_t\hat{Y_{t+1}} &= \frac{(1 + \hat{\sigma}\phi_y)a_\pi + \hat{\sigma}a_{1y}}{a_\pi - \hat{\sigma}a_{3y}}\hat{Y}_t + \frac{\hat{\sigma}(\phi_\pi a_\pi - 1)}{a_\pi - \hat{\sigma}a_{3y}}\pi_t \end{aligned} \tag{A.30}$$

Express equation 2.60 and 2.61 into the form $E_t z_{t+1} = Az_t$, then the A matrix is:

$$A \equiv \begin{bmatrix} \frac{1 - a_{3y}\hat{\sigma}\phi_\pi}{a_\pi - a_{3y}\hat{\sigma}} & -\frac{a_{3y} + a_{3y}\hat{\sigma}\phi_y + a_{1y}}{a_\pi - a_{3y}\hat{\sigma}} \\ \frac{\hat{\sigma}(\phi_\pi a_\pi - 1)}{a_\pi - \hat{\sigma}a_{3y}} & \frac{(1+\hat{\sigma}\phi_y)a_\pi + \hat{\sigma}a_{1y}}{a_\pi - \hat{\sigma}a_{3y}} \end{bmatrix} \tag{A.31}$$

The associated trace and determinant are: $tr(A) = \frac{1 - a_{3y}\hat{\sigma}\phi_\pi + (1+\hat{\sigma}\phi_y)a_\pi + \hat{\sigma}a_{1y}}{a_\pi - \hat{\sigma}a_{3y}}$ and $det(A) = \frac{(a_\pi + a_\pi\hat{\sigma}\phi_y + \hat{\sigma}a_{1y})(1 - a_{3y}\hat{\sigma}\phi_\pi) + (\hat{\sigma}\phi_\pi a_\pi - \hat{\sigma})(a_{3y} + a_{3y}\hat{\sigma}\phi_y + a_{1y})}{a_\pi - \hat{\sigma}a_{3y}}$
$= \frac{a_\pi + a_\pi\hat{\sigma}\phi_y - a_{3y}\hat{\sigma}\phi_\pi - a_\pi a_{3y}\hat{\sigma}\phi_\pi - a_\pi a_{3y}\hat{\sigma}^2\phi_y\phi_\pi - a_{3y}a_{1y}\hat{\sigma}^2\phi_\pi + \hat{\sigma}a_\pi a_{3y} + \hat{\sigma}^2\phi_\pi a_\pi a_{3y}\phi_y + \hat{\sigma}a_{1y}a_\pi\phi_\pi - \hat{\sigma}a_{3y} - \hat{\sigma}^2 a_{3y}\phi_y}{(a_\pi - \hat{\sigma}a_{3y})^2}$.

APPENDIX B

APPENDIX FOR THE SECTION THREE

B.1 Data Source

- Consumption: sum of personal consumption expenditures on non-durable goods (Table 1.1.5, Line 5) and services (Table 1.1.5, Line 6) divided by GDP deflator and by population

- Inflation: First difference of GDP deflator.

- Federal Funds rate: Effective federal funds rate from St. Louis FRED website and the shadow rate created by Wu and Xia (2015) for the zero lower bound periods 2009 Q3 to 2015 Q4.

- Investment: sum of Gross private domestic investment (Table 1.1.5, Line 7) and personal consumption expenditures on durable goods (Table 1.1.5, Line 4), divided by the GDP deflator and by population.

- Government spending: government consumption expenditures and gross investment (Table 1.1.5, Line 22) divided by the GDP deflator and population.

- Debt: Market value of federal debt held by public from the Dallas Fed website divided by the GDP deflator and by population.

- Capital and labor tax rate: see Zubairy (2014). FIT denotes federal income taxes (Table 3.2, Line 3), SIT denotes state and local income taxes (Table 3.3, Line 3), W denotes wages and salaries (Table 1.12, Line 3), PRI denotes propritor's income (Table 1.12, Line 9) and CI denotes capital income, which is sum of rental income (Table 1.12, Line 12), corporate profits (Table 1.12, Line 13) and net interest (Table 1.12, Line 18). CSI is total contributions to government social insurance (Table 3.1, Line 7), and EC denotes total compensation of employees (Table 1.12, Line 2). CT denotes corporate taxes (Table 3.1, Line 3) and PT denotes property taxes (Table 3.3: Line 9).

- Unemployment Benefits: Unemployment insurance (W825RC1, from Saint Louis FRED website) divided by unemployment population (labor force population LNS11000000 from BLS, times the unemployment rate) and the GDP deflator.

  Following Smets and Wouter (2007), GDP, consumption, investment, government spending, unemployment benefits and debt are defined as the difference of the natural log of the level variable times 100, plus the model implied growth rate, which is equal to the sample average growth rate of per capita real GDP.

- The effective federal funds rate is the empirical data, and observed inflation is the natural log of one plus the inflation rate. Wage tax rate, labor tax rate, labor force participation rate and unemployment rate are in natural log to match with the definition of corresponding endogenous variables in model.

B.2 Estimated equations in the model

The endogenous equations in this paper are shown as follows. Asset pricing kernel:

$$m_{t+1} = \beta E_t \frac{\psi_{t+1}}{\psi_t(1+g_{A,t+1})} \tag{B.1}$$

Consumption FOC:

$$\psi_t = (1-\omega)\tilde{c}_t^{-\chi} c_t - b(\frac{c_{t-1}}{1+g_{A,t}})^{\chi-1} \tag{B.2}$$

where $\tilde{c}_t$ is a composite consumption function given as $\tilde{c}_t = [(1-\omega)(c_t - b\frac{c_{t-1}}{(1+gA_t)})^{\chi} + \omega(c_t^H - b\frac{c_{t-1}^H}{(1+gA_t)})^{\chi}]^{\frac{1}{\chi}}$

Bond FOC:

$$1 = E_t m_{t+1} \frac{R_t}{\pi_{t+1}} + \frac{1}{\psi_t} lq_t \tag{B.3}$$

where lq_t is the liquidity shock.

Investment FOC:

$$1 = p_{k',t}(1 - \tilde{S}_t - \tilde{S}_t' \frac{i_t(1+g_{A,t})}{i_{t-1}}) + E_t m_{t+1}(1+g_{t+1}) p_{k',t+1} \tilde{S_{t+1}'} (i_{t+1}/i_t)^2 (1+g_{t+1}) \tag{B.4}$$

Capital FOC:

$$1 = E_t m_{t+1}(1+g_{A,t+1})(1-\delta_k + R_{k,t}) \tag{B.5}$$

Marginal product of capital:

$$R_{k,t} = \frac{\alpha}{\Xi} y_t \frac{p_{m,t}}{k_t} \tag{B.6}$$

Law of motion for physical capital:

$$k_{t+1} = \frac{(1-\delta_k)k_{t-1}}{(1+g_{A,t+1})} + (1-\tilde{S}_t) i_t MEI_t \tag{B.7}$$

where MEI_t is the marginal efficiency of investment shock.

R&D goods production function:

$$RD_t = \delta z_t^{\rho_{RD}} \tag{B.8}$$

The R&D value function:

$$\rho_{RD} z_t^{\rho_{RD}-1} \delta = E_t m_{t+1} Ome_t (1 + g_{A,t+1}) \tilde{V_{t+1}} \tag{B.9}$$

where Ome_t is the R&D innovation shock.

The recursive form of the value function:

$$\tilde{V_t} = \tau_t + (1 - z_t \eta_{ino,t})(1 + g_{A,t+1}) E_t m_{t+1} \tilde{V_{t+1}} \tag{B.10}$$

The current value of the R&D process:

$$\tau_t = (\Xi - 1) \frac{\alpha}{\Xi} p m_t y_t \tag{B.11}$$

The endogenous technology growth rate:

$$g_{A,t} = z_t (\gamma - 1) \eta_{ino,t} \tag{B.12}$$

Resource constraint:

$$y_t = c_t + i_t + RD_t + g_t \tag{B.13}$$

Home consumption function:

$$cH_t = \eta_{H,t} (1 - L_t) \tag{B.14}$$

where $\eta_{H,t}$ is the home productivity shock.

Pricing equations:

$$\frac{X_{1,t}}{X_{2,t}} = \Big(\frac{1 - \theta_p (\pi_{t-1}^{\iota_p})^{\frac{1}{\lambda_{p,t}}} (\pi_t)^{\frac{1}{\lambda_{p,t}}}}{1 - \theta_p}\Big)^{-\lambda_{p,t}} \tag{B.15}$$

$$X_{1,t} = (1+\lambda_{p,t})\lambda_t y_t p_t^m + \theta_p \beta (\pi_t^{\iota_p})^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \pi_{t+1}^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} X_{1,t+1} \tag{B.16}$$

$$X_{2,t} = \lambda_t y_t + \theta_p \beta (\pi_t^{\iota_p})^{-\frac{1}{\lambda_{p,t}}} \pi_{t+1}^{\frac{1}{\lambda_{p,t+1}}} X_{2,t+1} \tag{B.17}$$

The marginal value of hiring a worker for a firm:

$$J_t = vp_t - wp_t \tag{B.18}$$

The recursive form for the value function of hiring:

$$vp_t = pm_t + \rho E_t m_{t+1}(1+g_{A,t+1}) vp_{t+1} \tag{B.19}$$

The recursive form of the value function of wage payment:

$$wp_t = w_t(1-\tau_{w,t}) + \rho E_t m_{t+1}(1+g_{A,t+1}) wp_{t+1} \tag{B.20}$$

The Nash bargaining condition:

$$J_t = \frac{1-\eta}{\eta}(V_t - U_t) \tag{B.21}$$

The unemployment benefits value function:

$$\eta_{D,p,t} = D\eta_{D,t} + \rho E_t m_{t+1}(1+g_{A,t+1}) \eta_{D,p,t+1} \tag{B.22}$$

The value of the worker being matched with a firm:

$$V_t = wp_t + E_t m_{t+1}(1+g_{A,t+1})(\rho V_{t+1} + (1-\rho)s(f_{t+1}V_{t+1} + (1-f_{t+1})U_{t+1}) + (1-\rho)(1-s)(\Upsilon_{t+1} + N_{t+1})) \tag{B.23}$$

The value of being unemployed:

$$U_t = \eta_t^D + E_t m_{t+1}[s f_{t+1} V_{t+1} + s(1-f_{t+1})U_{t+1} + (1-s)(\Upsilon_{t+1} + N_{t+1})]. \tag{B.24}$$

The value of non-participation:

$$N_t = \lambda_t \eta_t^H + E_t m_{t+1}[e_{t+1}(f_{t+1}V_{t+1} + (1 - f_{t+1})U_{t+1} - \Upsilon_{t+1}) + (1 - e_{t+1})N_{t+1}]. \quad \text{(B.25)}$$

Job Finding rate:

$$f_t = x_t l_{t-1}/(L_t - \rho l_{t-1}) \quad \text{(B.26)}$$

The probability of leaving non-participation:

$$e_t = (L_t - sL_{t-1}) - \rho l_{t-1} - \rho \frac{l_{t-1}}{1 - l_{t-1}}. \quad \text{(B.27)}$$

Unemployment rate:

$$u_t = (L_t - l_t)/l_t. \quad \text{(B.28)}$$

Total vacancies:

$$vTot_t = v l_{t-1} \quad \text{(B.29)}$$

Law of motion of employment:

$$l_t = (\rho + x_t) l_{t-1}. \quad \text{(B.30)}$$

Matching function:

$$x_t l_{t-1} = \eta_{ME,t} \sigma_m (l_t - \rho l_{t-1})^{\sigma} vTot_t^{1-\sigma} \quad \text{(B.31)}$$

where ME_t is the matchimng efficiency shock.

Marginal utility of home consumption:

$$U_{CH,t} = \frac{\omega}{1 - \omega} \left(\frac{c_{H,t} - b\frac{c_{H,t-1}}{(1+gA_t)}}{c_t - b\frac{c_{t-1}}{1+gA_t}}\right)^{\chi - 1} \lambda_t \quad \text{(B.32)}$$

Monetary Policy rule, the Taylor rule:

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(R_{SS} + r_\pi(\pi_t - \pi_{SS}) + r_{\delta_y}/4(y_t - y_{t-1})) + \epsilon_{R,t} \quad \text{(B.33)}$$

where $\epsilon_{R,t}$ is the monetary policy shock.

Government budget constraint:

$$b_t R_t = b_{t-1}\pi_t(1 + gA_t) + g_t + tr - \tau_{w,t} w_t l_t + \eta_{D,t} D(L_t - l_t) \tag{B.34}$$

Government spending process:

$$g_t = \rho_g g_{t-1} + (1 - \rho_g) g_{SS} - \rho_{gb}(b_t - b_{t-1} - \rho_{gy}(y_t - y_{t-1})) + \epsilon_{g,t} \tag{B.35}$$

where $\epsilon_{g,t}$ is the government spending shock.

Wage tax process:

$$\tau_{w,t} = \rho_w \tau_{w,t-1} + (1 - \rho_w)\tau_{w,SS} + \rho_{w,b}(b_t - b_{t-1}) + \rho_{w,y}(y_t - y_{t-1}) + \epsilon_{w,t} \tag{B.36}$$

where $\epsilon_{w,t}$ is the labor tax shock.

Unemployment benefits process:

$$\eta_{D,t} = \rho_{n,D}\eta_{D,t-1} + (1 - \rho_{n,D})\eta_{D,SS} - \rho_{n,b}(b_t - b_{t-1}) - \rho_{D,y}(y_t - y_{t-1}) + \epsilon_{nD,t} \tag{B.37}$$

where $\epsilon_{nD,t}$ is the unemployment benefits shock. Government capital law of motion:

$$k_{G,t+1} = (1 - \delta_g)\frac{k_{G,t}}{1 + g_{A,t+1}} + g_t \tag{B.38}$$

Neutral technology shock:

$$\eta_{tech,t} = \rho_t \eta_{tech,t-1} + \epsilon_{tech,t} \tag{B.39}$$

where $\epsilon_{tech,t}$ is the neutral technology shock.

Home productivity process:

$$\eta_{H,t} = \rho_H \eta_{H,t-1} + (1 - \rho_H)\eta_{H,SS} + \epsilon_{H,t} \tag{B.40}$$

where $\epsilon_{H,t}$ is the home productivity shock.

The definition of composite consumption:

$$\tilde{c_t} = ((1-\omega)(c_t - b\frac{c_{t-1}}{1+gA_t})^{\chi} + \omega(c_{H,t} - b\frac{c_{H,t-1}}{1+g_{A,t}})^{\chi})^{1/\chi} \tag{B.41}$$

Marginal efficiency of investment process:

$$\eta_{MEI,t} = \rho_i \eta_{MEI,t-1} + \epsilon_{i,t} \tag{B.42}$$

where ϵ_i is the marginal efficiency of investment shock.

The matching efficiency process:

$$\eta_{ME,t} = \rho_{me} \eta_{ME,t-1} + \epsilon_{me,t} \tag{B.43}$$

where $\epsilon_{me,t}$ is the matching efficiency shock.

The liquidity preference shock:

$$\eta_{lq,t} = \rho_{lq} \eta_{lq,t-1} + \epsilon_{lq,t} \tag{B.44}$$

where $\epsilon_{lq,t}$ is the liquidity preference shock.

The mark up process:

$$\lambda_{p,t} = (1-\rho_{lp})\lambda_{p,SS} + \rho_{lp}\lambda_{p,t-1} + \epsilon_{lp,t} \tag{B.45}$$

where $\epsilon_{lp,t}$ is the mark up shock.

R&D innovation shock process:

$$\eta_{ino,t} = (1-\rho_{ino})\eta_{ino,t-1} + \epsilon_{ino,t} \tag{B.46}$$

where $\epsilon_{Ome,t}$ is the R&D innovation shock.

F.O.C. employment condition:

$$pl_t = w_t(1-\tau_{w,t}) - \eta_{D,t}D + E_t m_{t+1}(1+g_{A,t+1})pl_{t+1}\rho(1-f_{t+1}) \tag{B.47}$$

F.O.C. labor force participation condition:

$$0 = \eta_{D,t}D + pl_t f_t - \frac{U_{CH,t}}{\psi_t}(c_{H,t} + Fcal_t)/(1-L_t) - \frac{U_{CH,t}}{\psi_t}FcalL_t - E_t m_{t+1}(1+g_{A,t+1})\frac{U_{CH,t+1}}{\psi_{t+1}}Fcal^{`}_t \tag{B.48}$$

labor force participation adjustment cost equation:

$$Lcal_t = \frac{U_{CH,t}}{\psi_t}Fcal_t + E_t m_{t+1}(1+g_{A,t+1})\frac{U_{CH,t+1}}{\psi_{t+1}}Fcal^{`}_t \tag{B.49}$$

The observation equations:

$$dy_t = y_t - y_{t-1} + g_{A,t} \tag{B.50}$$

$$di_t = i_t - i_{t-1} + g_{A,t} \tag{B.51}$$

$$dc_t = c_t - c_{t-1} + g_{A,t} \tag{B.52}$$

$$dg_t = g_t - g_{t-1} + g_{A,t} \tag{B.53}$$

$$db_t = b_t - b_{t-1} + g_{A,t} \tag{B.54}$$

$$d\eta_{D,t} = \eta_{D,t} - \eta_{D,t-1} + g_{A,t} \tag{B.55}$$

$$yy_t = y_t(1+g_{A,t}) \tag{B.56}$$

$$cc_t = c_t(1+g_{A,t}) \tag{B.57}$$

B.3 Tables of Section Three

Table B.1: Nonestimated model parameters and calibrated variables

Parameters	Value	Description
δ_K	0.025	Depreciation rate of physical capital
δ_G	0.02	Depreciation rate of government spending capital
β	0.9968	Discount factor
ρ	0.9	Job Survival Probability
$(1-b)(1-\chi)^{-1}$	3	Elasticity of substitution market and home consumption
$100(\pi^A-1)$	2	Annual net inflation target
$400(1+g_A)$	1.6623	Annual GDP growth rate
Q	0.7	Vacancy filling rate
u	0.055	Unemployment rate
L	0.67	Labor force to population ratio
G/Y	0.2	Government consumption to output ratio
B/Y	0.5862	Debt to output ratio
α_G	0.05	Productive government spending share
$(\eta FSf)/y$	0.4945	labor search cost to output ratio
η	0.5	Nash bargaining share
z	0.036	creative destruction probability
ρ_{RD}	1.344	R&D elasticity
RD/y	0.0238	R&D cost to output ratio

Table B.2: Prior and Posterior of the estimated parameters

Parameters	Prior Mean	Prior Dist. type	Post. Mean	5% Value	95% Value	Post. Std.
ρ_R	0.75	Beta	0.9602	0.9536	0.9666	0.15
r_π	1.20	Gamma	1.1304	1.1235	1.1367	0.05
$r_{R\delta_y}$	0.20	Gamma	0.2769	0.2735	0.2815	0.05
b	0.66	Beta	0.7531	0.7457	0.7611	0.10
α	0.33	Beta	0.2951	0.2939	0.2963	0.01
$DSHARE$	0.318	Beta	0.3713	0.3680	0.3748	0.05
σ	0.50	Beta	0.6870	0.6711	0.7026	0.10
ρ_g	0.50	Beta	0.9935	0.9929	0.9941	0.20
ρ_w	0.50	Beta	0.9132	0.9020	0.9300	0.20
ρ_{nD}	0.50	Beta	0.4975	0.4893	0.5057	0.10
ρ_t	0.50	Beta	0.8530	0.8290	0.8775	0.20
ρ_i	0.50	Beta	0.9443	0.9379	0.9505	0.20
ρ_{nH}	0.50	Beta	0.7151	0.7054	0.7243	0.20
ρ_{lp}	0.50	Beta	0.2446	0.2387	0.2518	0.10
ρ_{me}	0.50	Beta	0.6883	0.6722	0.7061	0.10
ρ_{Ome}	0.50	Beta	0.0952	0.0854	0.1085	0.20
ρ_{lq}	0.50	Beta	0.9004	0.8897	0.9111	0.20
ρ_{gb}	0.50	Gamma	0.0254	0.0131	0.0390	0.25
ρ_{wb}	0.50	Gamma	0.3729	0.3548	0.3839	0.25
ρ_{nDb}	0.50	Gamma	0.4840	0.4673	0.5058	0.25
ρ_{gy}	0.50	Beta	0.6558	0.6450	0.6685	0.20
ρ_{wy}	0.50	Beta	0.7135	0.7033	0.7249	0.20
ρ_{nDy}	0.50	Beta	0.0757	0.0610	0.0907	0.20

Table B.3: Unconditional Variance Decomposition Table

Variables	$\epsilon_{R,t}$	$\epsilon_{g,t}$	$\epsilon_{w,t}$	$\epsilon_{nD,t}$	$\epsilon_{tech,t}$	$\epsilon_{nH,t}$	$\epsilon_{i,t}$	$\epsilon_{lp,t}$	$\epsilon_{me,t}$	$\epsilon_{ino,t}$	$\epsilon_{lq,t}$
y_t	2.44	0.82	0.00	0.00	0.21	0.13	94.76	0.41	0.05	0.16	1.03
c_t	10.04	1.55	0.00	0.00	0.14	0.38	82.21	1.14	0.07	0.22	4.25
yy_t	2.44	0.82	0.00	0.00	0.21	0.13	94.74	0.41	0.05	0.17	1.03
i_t	0.81	0.28	0.00	0.00	0.24	0.10	97.75	0.33	0.04	0.13	0.34
$g_{A,t}$	0.81	0.24	0.00	0.00	0.02	0.02	12.68	0.16	0.01	85.71	0.34
w_t	0.42	0.14	0.00	0.02	0.05	0.04	31.77	67.11	0.16	0.06	0.22
π_t	1.79	2.58	0.02	0.00	0.12	0.11	62.25	23.37	0.15	0.14	9.48
R_t	19.31	7.43	0.00	0.00	0.08	0.18	67.12	2.28	0.10	0.13	3.37
L_t	1.51	0.52	0.00	0.00	0.16	0.12	96.50	0.35	0.03	0.19	0.63
l_t	1.51	0.52	0.00	0.00	0.16	0.12	96.50	0.35	0.03	0.19	0.63
f_t	1.22	0.45	0.00	0.00	0.15	0.33	92.12	0.89	4.13	0.18	0.52
e_t	1.41	0.50	0.00	0.00	0.16	0.23	96.10	0.79	0.04	0.19	0.59
x_t	1.27	0.47	0.00	0.00	0.16	0.35	95.58	0.93	0.53	0.18	0.54
u_t	1.26	0.46	0.00	0.00	0.16	0.34	94.96	0.92	1.18	0.18	0.53

B.4 Figures of Section Three

Figure B.1: Historical Variance Decomposition for Undetrended Output

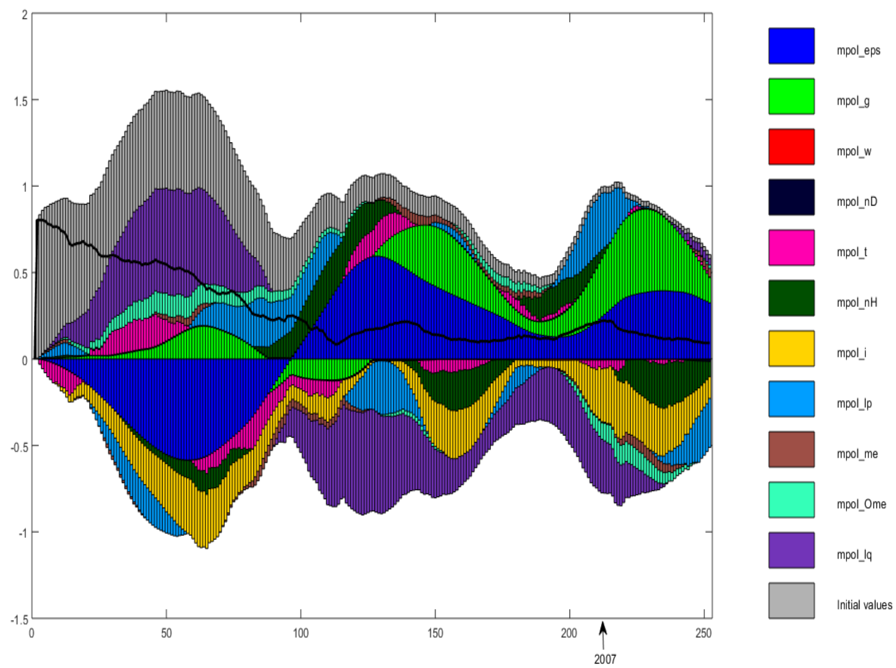


Figure B.2: Historical Variance Decomposition for detrended Output

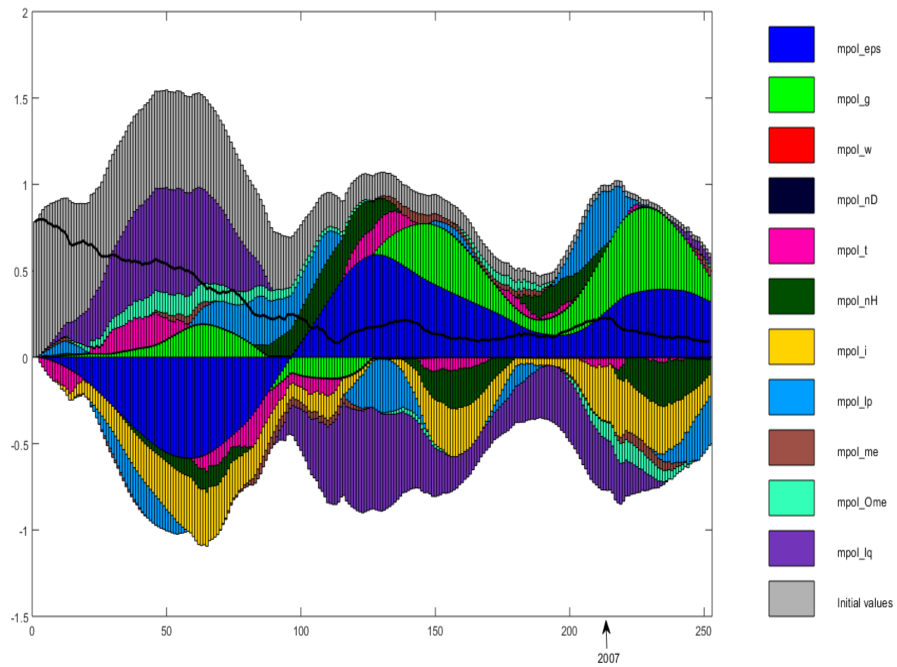


Figure B.3: Historical Variance Decomposition for Labor Force Participation rate

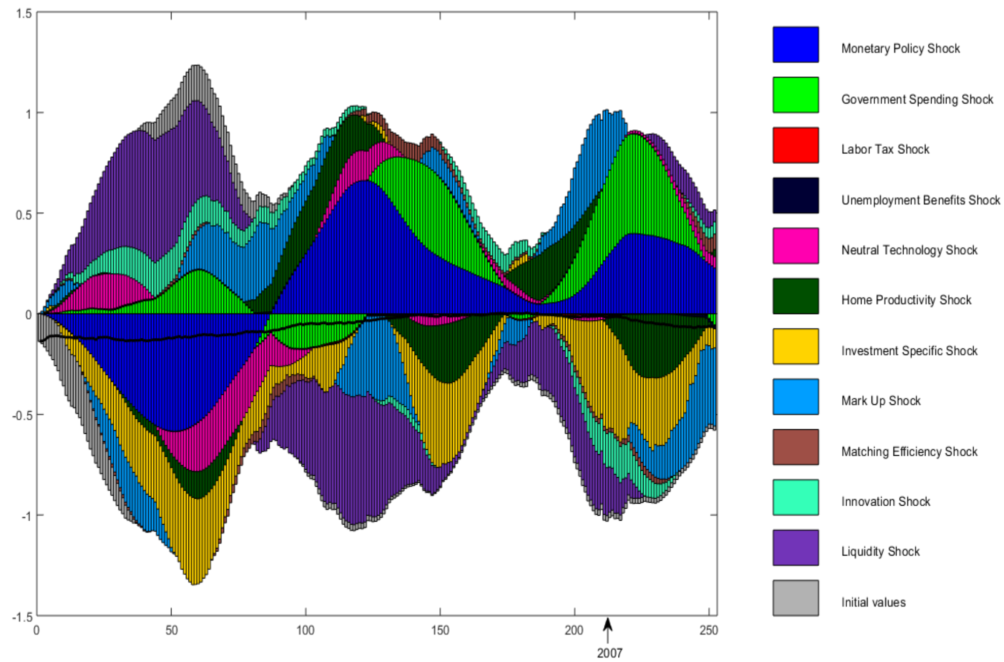


Figure B.4: Historical Variance Decomposition for Unemployment rate

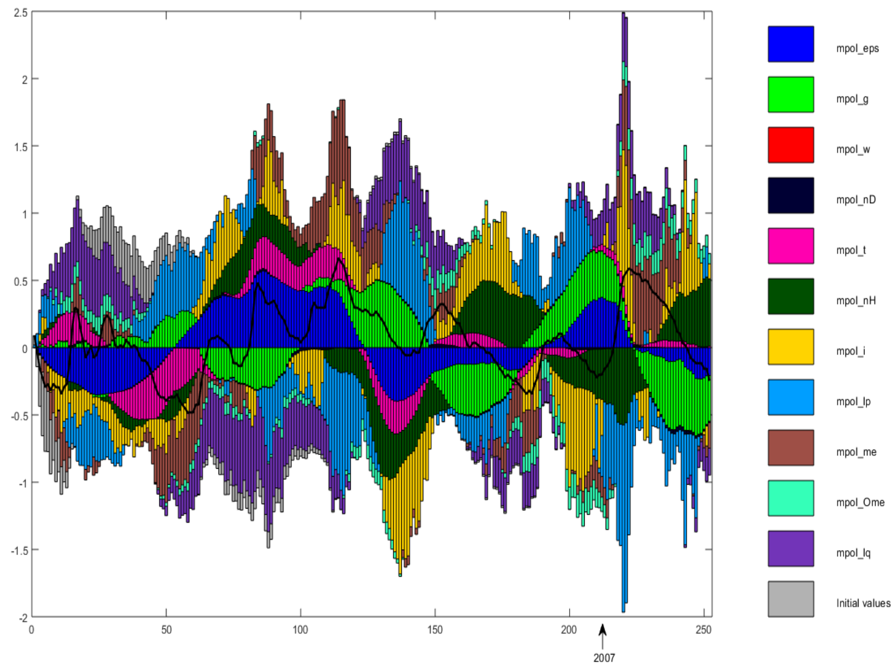


Figure B.5: Impulse Response Functions for Real Variables, Monetary Policy Shock

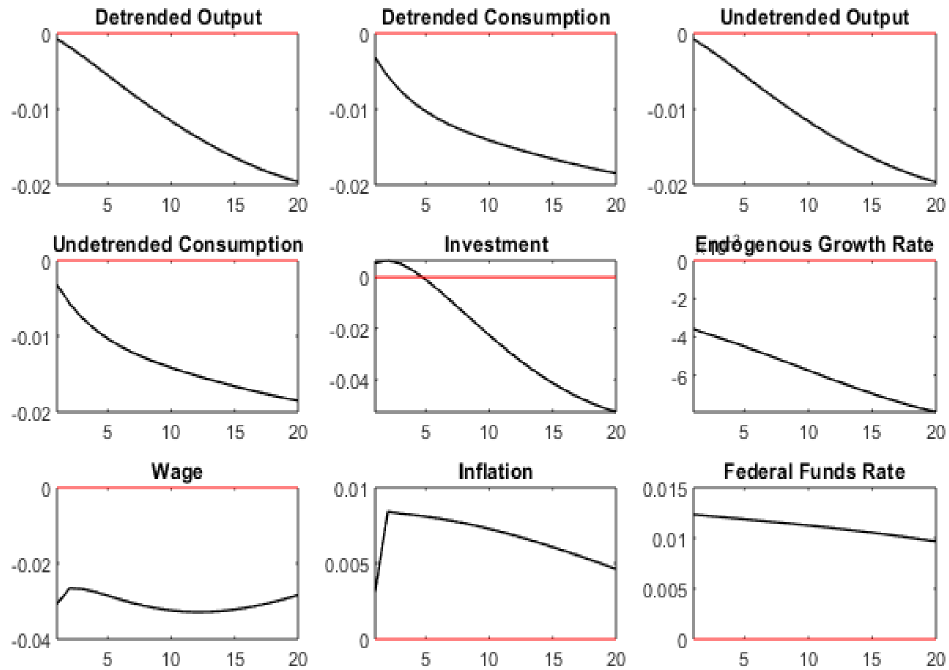


Figure B.6: Impulse Response Functions for Labor Variables, Monetary Policy Shock

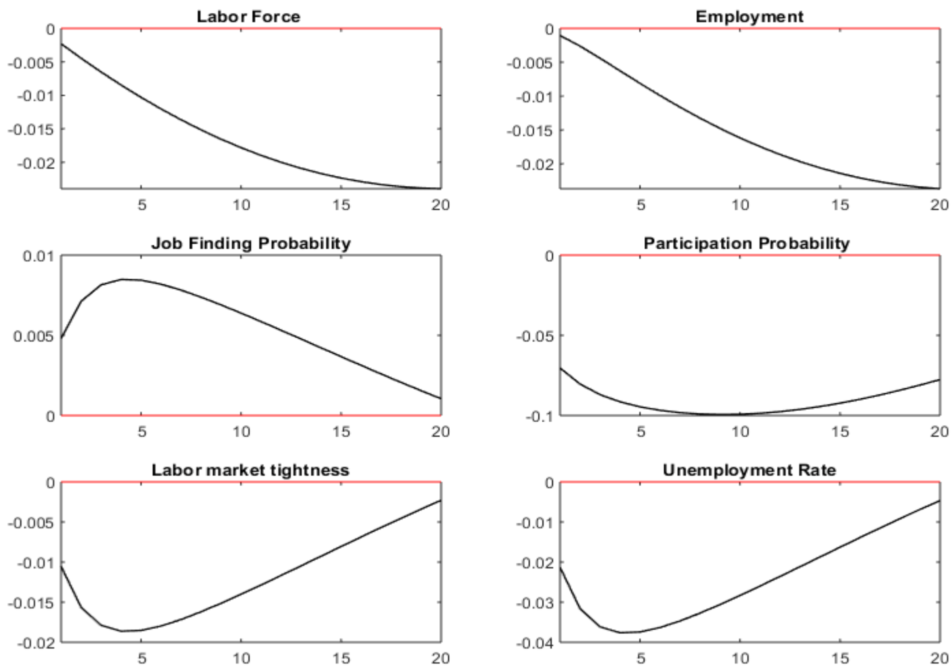


Figure B.7: Impulse Response Functions for Real Variables, Government Spending Shock

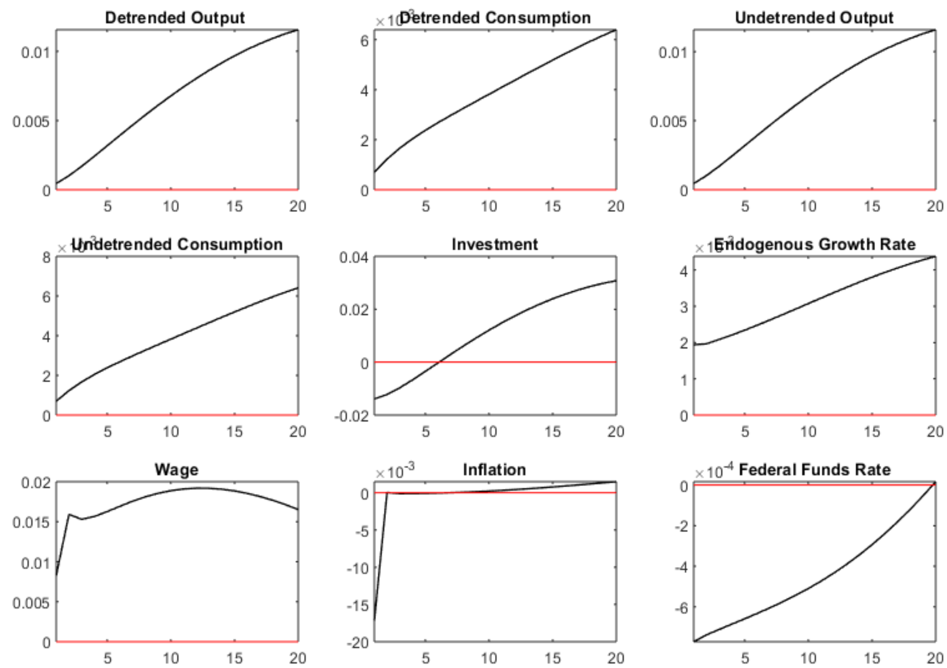


Figure B.8: Impulse Response Functions for Labor Variables, Government Spending Shock

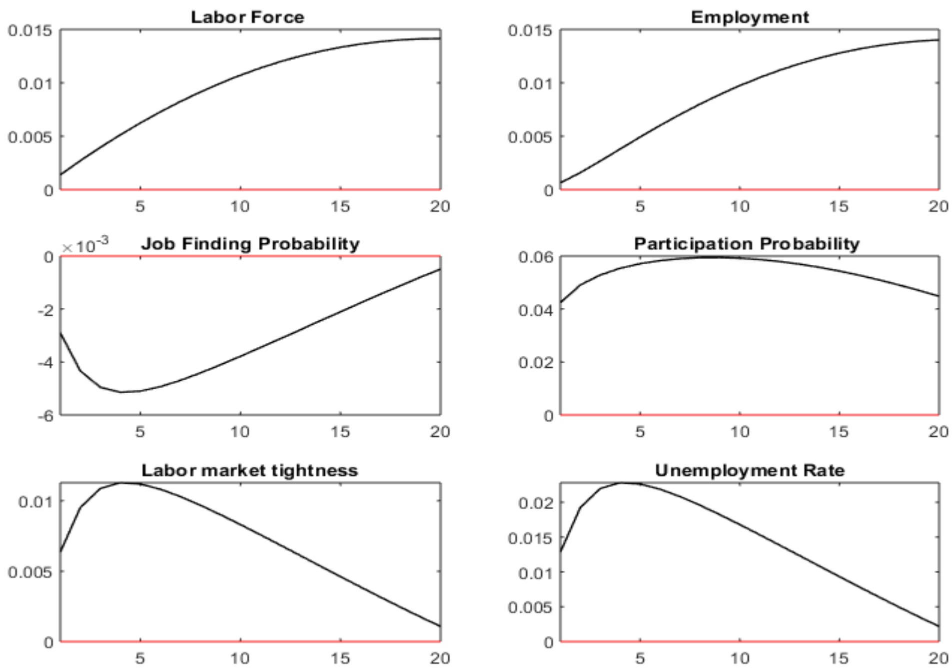


Figure B.9: Impulse Response Functions for Real Variables, Neutral Technology Shock

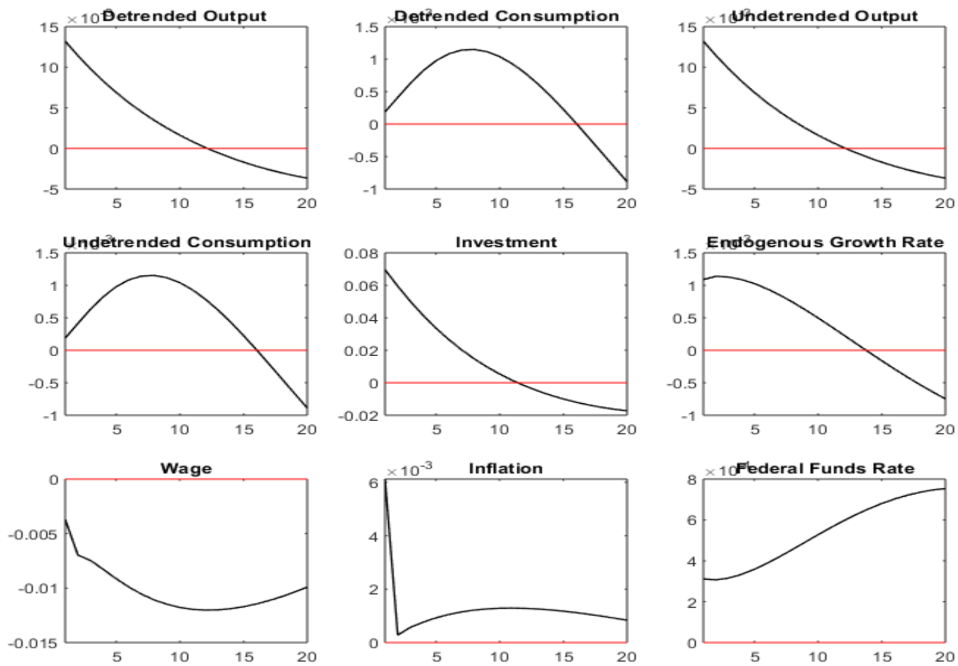


Figure B.10: Impulse Response Functions for Labor Variables, Neutral Technology Shock

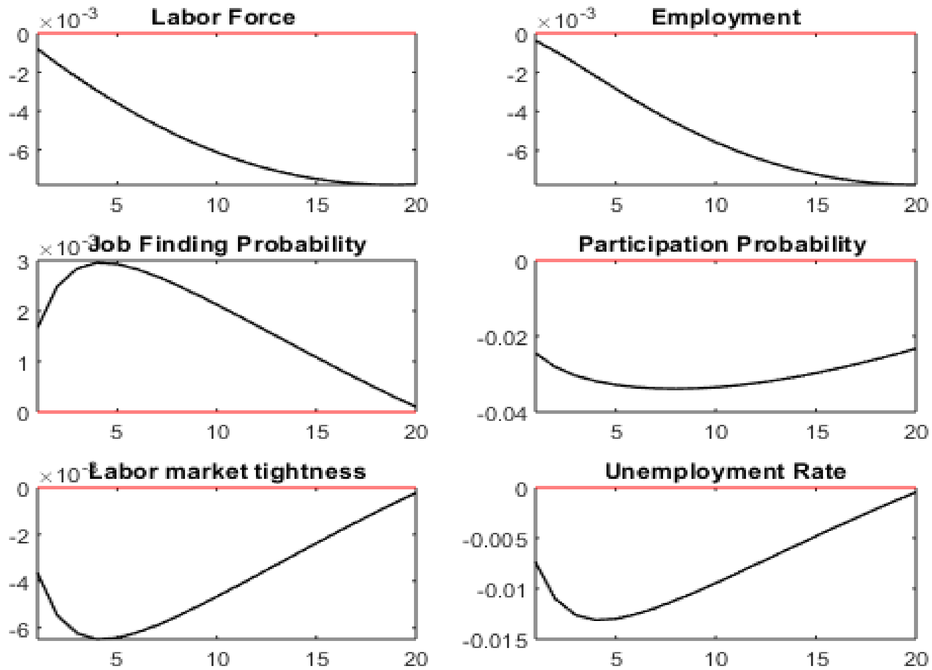


Figure B.11: Impulse Response Functions for Real Variables, Innovation Shock

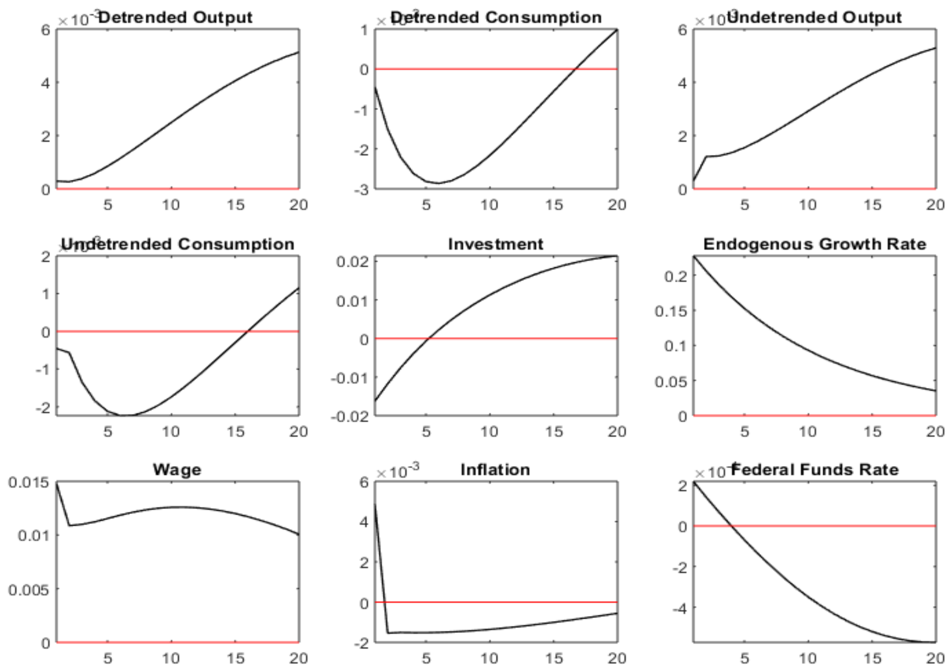
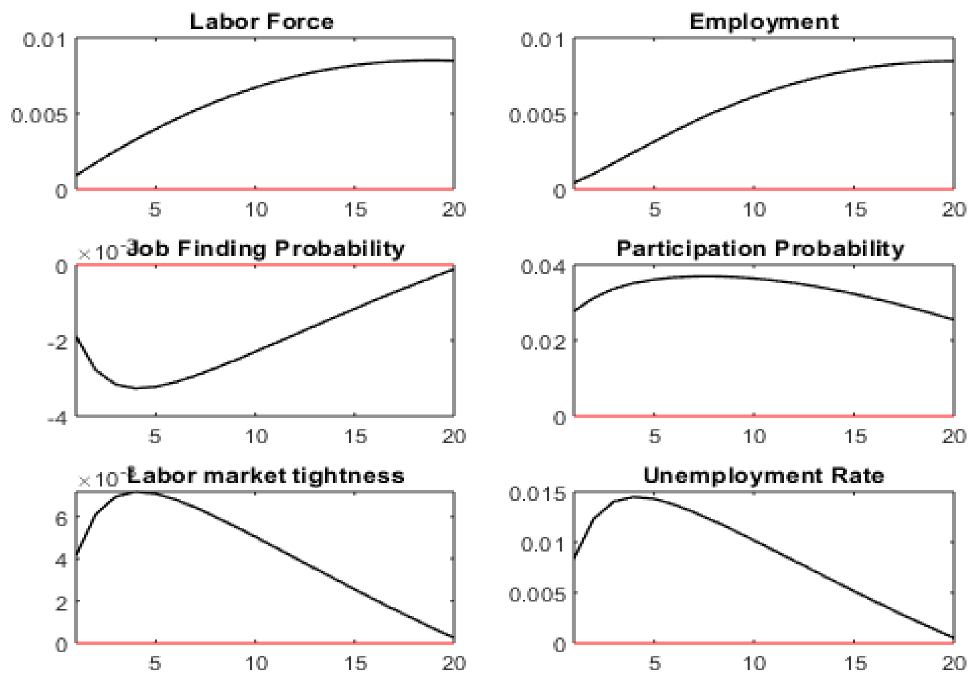


Figure B.12: Impulse Response Functions for Labor Variables, Innovation Shock



APPENDIX C

APPENDIX OF SECTION FOUR

C.1 Additional Tables of Section Four

Table C.1: Test Statistics, P Values-Stocks Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0005	0.0175	0.0118	0.0005	0.0185	0.0144
LR_{UC}	0.0000	0.0000	0.2675**	0.0000	0.0000	0.0075
LR_{IND}	0.965**	0.0018	0.0024	0.9650**	0.0032	0.2878**
LR_{C}	0.0000	0.0000	0.0055	0.0000	0.0000	0.0158*
DQ Test	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0043	0.0441	0.0408	0.0043	0.0393	0.0353
LR_{UC}	0.0000	0.0771**	0.0655**	0.0000	0.0011	0.0000
LR_{IND}	0.0668**	0.0005	0.0035	0.0668**	0.0172*	0.0499**
LR_{C}	0.0000	0.0005	0.0026	0.0000	0.0002	0.0000
DQ Test	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Testing Period: 2002-01-03 to 2018-07-25

Table C.2: Test Statistics, P Values-Stocks Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0008	0.0056	0.0056	0.0008	0.0064	0.0103
LR_{UC}	0.0000	0.0849**	0.0849**	0.0000	0.1651**	0.9035**
LR_{IND}	0.9682**	0.7795**	0.7795**	0.9682**	0.0382*	0.6022**
LR_{C}	0.0001	0.2179**	0.2179**	0.0001	0.0446*	0.8666**
DQ Test	0.0938**	0.5169**	0.4936**	0.0000	0.0145*	0.6953**
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0024	0.0278	0.0318	0.0024	0.0231	0.0318
LR_{UC}	0.0000	0.0000	0.0016	0.0000	0.0000	0.0016
LR_{IND}	0.9046**	0.0861**	0.1739**	0.9046**	0.1742**	0.5370**
LR_{C}	0.0000	0.0001	0.0027	0.0000	0.0000	0.0056
DQ Test	0.0000	0.0011	0.0064	0.0000	0.0004	0.0112*

Testing Period: 2002-01-03 to 2006-12-29

Table C.3: Test Statistics, P Values-Stocks Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0013	0.0357	0.0238	0.0026	0.0331	0.0212
LR_{UC}	0.0025	0.0000	0.0012	0.0158*	0.0000	0.0071
LR_{IND}	0.9589**	0.0139*	0.0690**	0.9179**	0.2539**	0.3432**
LR_C	0.0105*	0.0000	0.0001	0.0540**	0.0000	0.0172*
DQ Test	0.4546**	0.0000	0.0000	0.6506**	0.0000	0.0049
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0106	0.0754	0.0728	0.0106	0.0675	0.0489
LR_{UC}	0.0000	0.0027	0.0068	0.0000	0.0353*	0.9000**
LR_{IND}	0.6789**	0.1910**	0.3118**	0.6789**	0.3956**	0.1344**
LR_C	0.0000	0.0047	0.0155*	0.1475**	0.0760**	0.3236**
DQ Test	0.0003	0.0000	0.0000	0.0003	0.0000	0.3254**

Testing Period: 2007-01-04 to 2009-12-31

Table C.4: Test Statistics, P Values-Stocks Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0000	0.0181	0.0111	0.0026	0.0204	0.0144
LR_{UC}	NaN	0.0006	0.6009**	0.0000	0.0000	0.0546**
LR_{IND}	NaN	0.1968**	0.0277*	NaN	0.0719**	0.4679**
LR_{C}	NaN	0.0014	0.0774**	NaN	0.0000	0.1211**
DQ Test	NaN	0.0000	0.0000	NaN	0.0000	0.0000
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0032	0.0427	0.0408	0.0032	0.0390	0.0325
LR_{UC}	0.0000	0.1118**	0.0446*	0.0000	0.0150*	0.0001
LR_{IND}	0.0151*	0.0203*	0.0346*	0.0151*	0.1569**	0.2825**
LR_{C}	0.0000	0.0191*	0.0143*	0.0000	0.0190*	0.0002
DQ Test	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Testing Period: 2010-01-01 to 2018-07-25

Table C.5: Test Statistics, P Values-Bonds Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0	0.0162	0.0132	0	0.0212	0
LR_{UC}	NaN	0.0002	0.0468*	NaN	0.0000	NaN
LR_{IND}	NaN	0.9060**	0.7693**	NaN	0.0006	NaN
LR_{C}	NaN	0.0009	0.1328**	NaN	0.0000	NaN
DQ Test	NaN	0.0003	0.0644**	NaN	0.0000	NaN
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0	0.0450	0.0473	0	0.0428	0.0005
LR_{UC}	NaN	0.1256**	0.4182**	NaN	0.0284*	0.0000
LR_{IND}	NaN	0.0168*	0.0093	NaN	0.0157*	0.9654**
LR_{C}	NaN	0.0178*	0.0244*	NaN	0.0049	0.0000
DQ Test	NaN	0.0001	0.0001	NaN	0.0000	0.0000

Testing Period: 2002-01-07 to 2018-12-31

Table C.6: Test Statistics, P Values-Bonds Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0	0.0104	0.0080	0	0.0152	0
LR_{UC}	NaN	0.8787**	0.4682**	NaN	0.0839**	NaN
LR_{IND}	NaN	0.6006**	0.6875**	NaN	0.2916**	NaN
LR_C	NaN	0.8618**	0.7089**	NaN	0.1288**	NaN
DQ Test	NaN	0.9920**	0.9877**	NaN	0.1735**	NaN
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0	0.0385	0.0401	0	0.0377	0.0008
LR_{UC}	NaN	0.0532**	0.0983**	NaN	0.0380*	0.0000
LR_{IND}	NaN	0.4163**	0.1918**	NaN	0.3801**	0.9680**
LR_C	NaN	0.1108**	0.1088**	NaN	0.0790**	0.0000
DQ Test	NaN	0.2525**	0.4005**	NaN	0.4655**	0.0000

Testing Period: 2002-01-07 to 2006-12-31

Table C.7: Test Statistics, P Values-Bonds Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0	0.0332	0.0346	0	0.0492	0
LR_{UC}	NaN	0.0000	0.0000	NaN	0.0000	NaN
LR_{IND}	NaN	0.8534**	0.9146**	NaN	0.0081	NaN
LR_C	NaN	0.0000	0.0000	NaN	0.0000	NaN
DQ Test	NaN	0.0000	0.0000	NaN	0.0000	NaN
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0	0.0731	0.0745	0	0.0731	0.0013
LR_{UC}	NaN	0.0061	0.0039	NaN	0.0061	0.0000
LR_{IND}	NaN	0.0054	0.0072	NaN	0.0054	0.9588**
LR_C	NaN	0.0004	0.0004	NaN	0.0005	0.0000
DQ Test	NaN	0.0000	0.0000	NaN	0.0000	0.0000

Testing Period: 2007-01-01 to 2009-12-31

Table C.8: Test Statistics, P Values-Bonds Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0	0.0138	0.0089	0	0.0151	0
LR_{UC}	NaN	0.0881**	0.5907**	NaN	0.0233*	NaN
LR_{IND}	NaN	0.3519**	0.5491**	NaN	0.5412**	NaN
LR_C	NaN	0.1514**	0.7232**	NaN	0.0634**	NaN
DQ Test	NaN	0.0034	0.5750**	NaN	0.0006	NaN
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0	0.0391	0.0422	0	0.0356	0
LR_{UC}	NaN	0.0141*	0.0832**	NaN	0.0010	NaN
LR_{IND}	NaN	0.7996**	0.9946**	NaN	0.5842**	NaN
LR_C	NaN	0.0475*	0.2229**	NaN	0.0037	NaN
DQ Test	NaN	0.0001	0.0004	NaN	0.0003	NaN

Testing Period: 2010-01-01 to 2018-12-31

Table C.9: Test Statistics, P Values-Currencies Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0071	0.0196	0.0142	0.0071	0.0191	0.0158
LR_{UC}	0.0448*	0.0000	0.0101*	0.0448*	0.0000	0.0004
LR_{IND}	0.0179*	0.0066	0.0016	0.0179*	0.0000	0.4079**
LR_C	0.0081	0.0000	0.0002	0.0081	0.0000	0.0015
DQ Test	0.0000	0.0000	0.0000	0.0000	0.0000	0.0006
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0165	0.0522	0.0558	0.0165	0.0456	0.0499
LR_{UC}	0.0000	0.5080**	0.0901**	0.0000	0.1844**	0.9690**
LR_{IND}	0.0010	0.0000	0.0000	0.0008	0.0002	0.0261*
LR_C	0.0000	0.0001	0.0001	0.0000	0.0004	0.0840**
DQ Test	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

Testing Period: 2002-01-02 to 2018-11-02

Table C.10: Test Statistics, P Values-Currencies Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0047	0.0199	0.0103	0.0047	0.0223	0.0111
LR_{UC}	0.0383*	0.0019	0.9013**	0.0383*	0.0002	0.6884**
LR_{IND}	0.8103**	0.0973**	0.6020**	0.8103**	0.1524**	0.5742**
LR_C	0.1138**	0.0020	0.8662**	0.1138**	0.0003	0.7880**
DQ Test	0.5018**	0.0000	0.0294*	0.5215**	0.0000	0.8530**
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0175	0.0509	0.0541	0.0175	0.0485	0.0390
LR_{UC}	0.0000	0.8769**	0.5062**	0.0000	0.8149**	0.0636**
LR_{IND}	0.3983**	0.0545**	0.0361*	0.3983**	0.2529**	0.1657**
LR_C	0.0000	0.1555**	0.0893**	0.0000	0.5061**	0.0684**
DQ Test	0.0000	0.0028	0.0006	0.0000	0.0079	0.1103**

Testing Period: 2002-01-02 to 2006-12-31

Table C.11: Test Statistics, P Values-Currencies Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0237	0.0422	0.0277	0.0237	0.0356	0.0224
LR_{UC}	0.0012	0.0000	0.0001	0.0012	0.0000	0.0031
LR_{IND}	0.0686**	0.0494	0.0000	0.0686**	0.0138*	0.0538**
LR_C	0.0010	0.0000	0.0000	0.0010	0.0000	0.0019
DQ Test	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0449	0.0779	0.0805	0.0449	0.0633	0.0686
LR_{UC}	0.5139**	0.0011	0.0004	0.5139**	0.1035**	0.0252*
LR_{IND}	0.0169*	0.0013	0.0024	0.0169*	0.0023	0.2025**
LR_C	0.0467*	0.0000	0.0000	0.0467*	0.0025	0.0362*
DQ Test	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Testing Period: 2007-01-01 to 2009-12-31

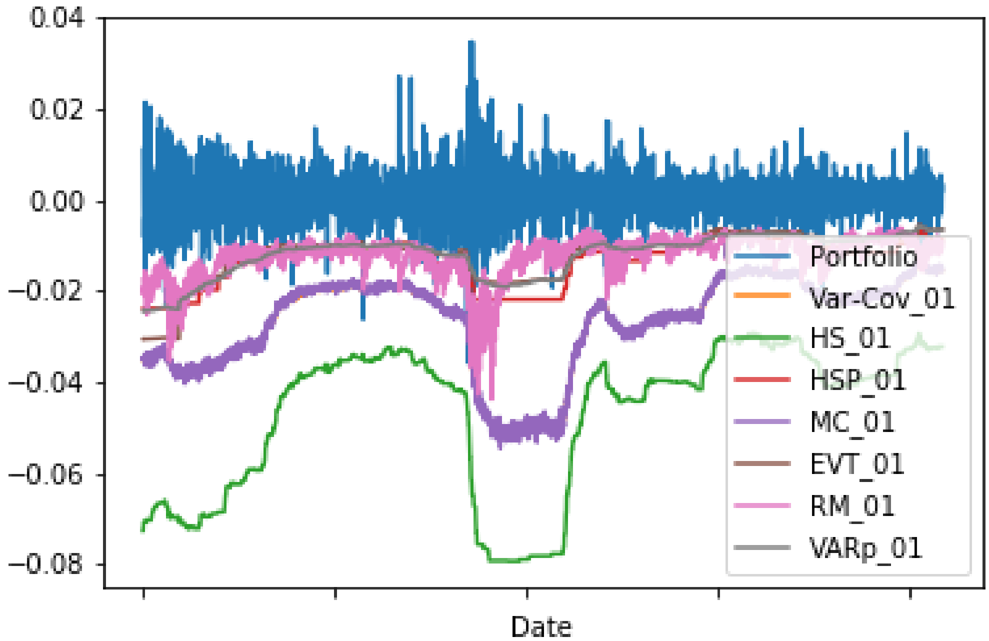
Table C.12: Test Statistics, P Values-Currencies Portfolio

1 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0027	0.0117	0.0117	0.0027	0.0117	0.0162
LR_{UC}	0.0000	0.4249**	0.4249**	0.0000	0.4249**	0.0067
LR_{IND}	0.8568**	0.4320**	0.3119**	0.8568**	0.4320**	0.2755**
LR_C	0.0002	0.5342**	0.4361**	0.0002	0.5342**	0.0140*
DQ Test	0.0616**	0.0093	0.0006	0.0612**	0.0944**	0.0086
5 Percent Confidence Interval						
	Var-Cov	Port Var	HS	Monte Carlo	EVT	RM
Hit Rate	0.0063	0.0442	0.0483	0.0059	0.0379	0.0496
LR_{UC}	0.0000	0.2035**	0.7096**	0.0000	0.0065	0.9377**
LR_{IND}	0.6731**	0.0959**	0.1053**	0.6953**	0.0522**	0.2816**
LR_C	0.0000	0.1114**	0.2513**	0.0000	0.0037	0.5584**
DQ Test	0.0000	0.0000	0.0002	0.0000	0.0000	0.2268**

Testing Period: 2010-01-04 to 2018-11-02

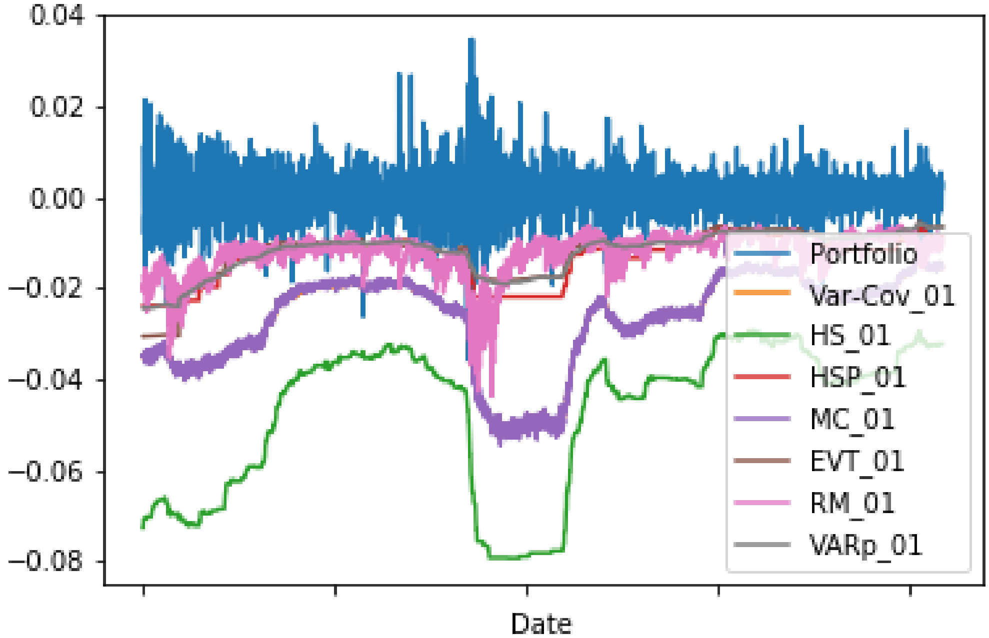
C.2 Figures of Section Four

Figure C.1: Forecasted Value-at-Risk values and true observations at one percent interval for stock portfolio



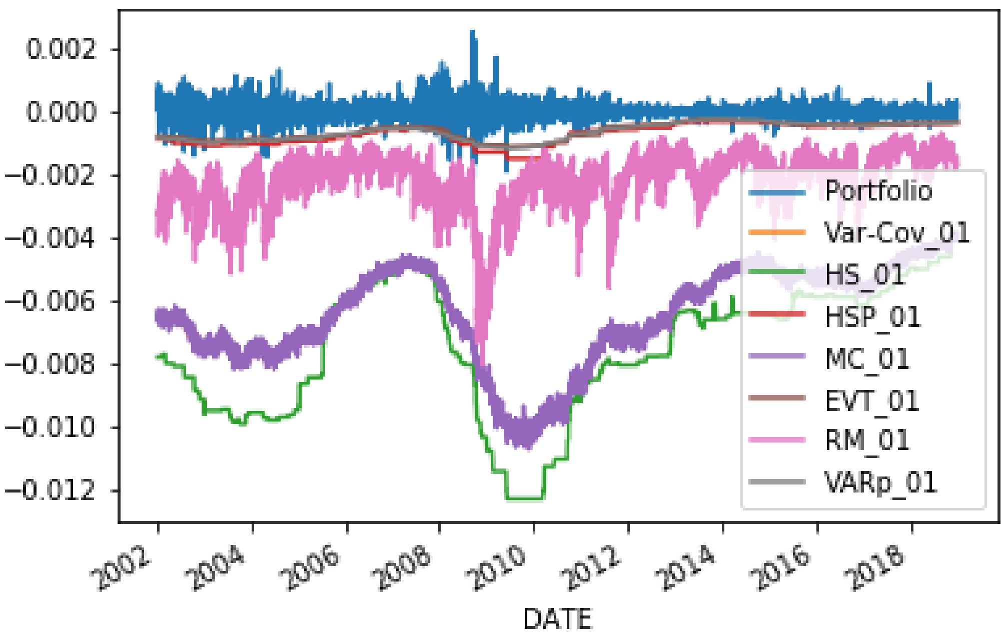
Portfolio stands for the realized observation of the portfolio, Var-Cov_01 stands for variance covariance method, HS_01 stands for historical simulation applied to each asset, HSP_01 stands for historical simulation applied to the whole portfolio, MC_01 stands for Monte Carlo method using multivariate normal distribution, EVT_01 stands for extreme value theorem method, RM_01 stands for Risk Metrics, VARp_01 stands for portfolio variance method.

Figure C.2: Forecasted Value-at-Risk values and true observations at five percent interval for stock portfolio



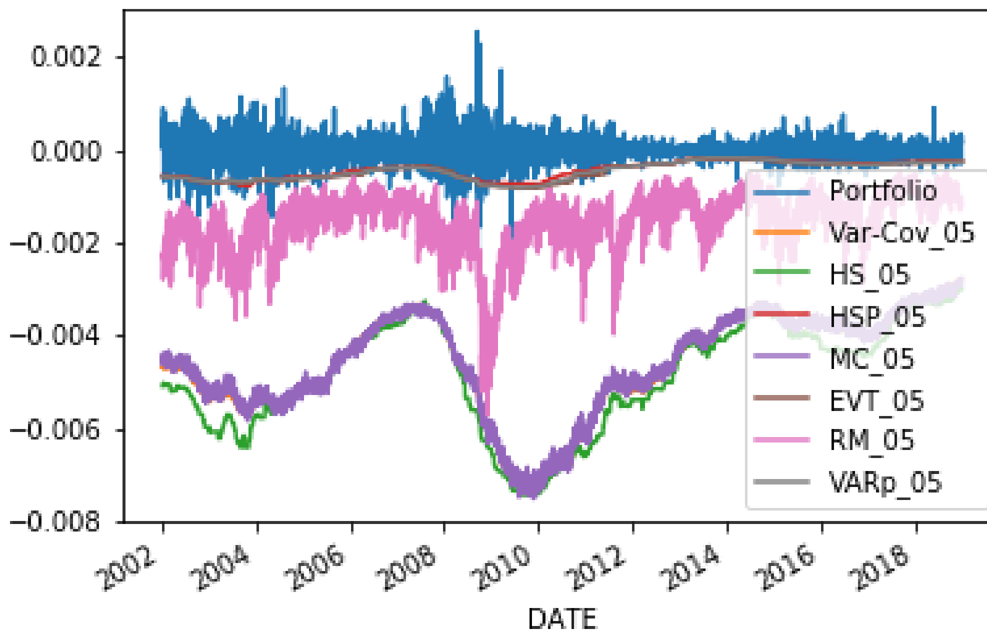
Portfolio stands for the realized observation of the portfolio, Var-Cov_05 stands for variance co-variance method, HS_05 stands for historical simulation applied to each asset, HSP_05 stands for historical simulation applied to the whole portfolio, MC_05 stands for Monte Carlo method using multivariate normal distribution, EVT_05 stands for extreme value theorem method, RM_05 stands for Risk Metrics, VARp_05 stands for portfolio variance method.

Figure C.3: Forecasted Value-at-Risk values and true observations at one percent interval for Bond portfolio



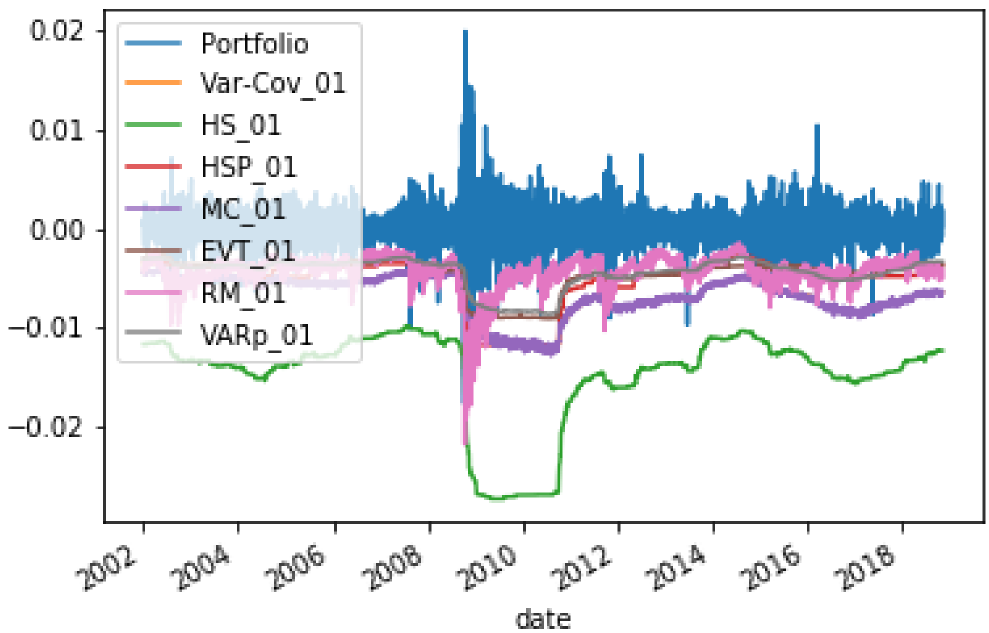
Portfolio stands for the realized observation of the portfolio, Var-Cov_01 stands for variance co-variance method, HS_01 stands for historical simulation applied to each asset, HSP_01 stands for historical simulation applied to the whole portfolio, MC_01 stands for Monte Carlo method using multivariate normal distribution, EVT_01 stands for extreme value theorem method, RM_01 stands for Risk Metrics, VARp_01 stands for portfolio variance method.

Figure C.4: Forecasted Value-at-Risk values and true observations at five percent interval for Bond portfolio



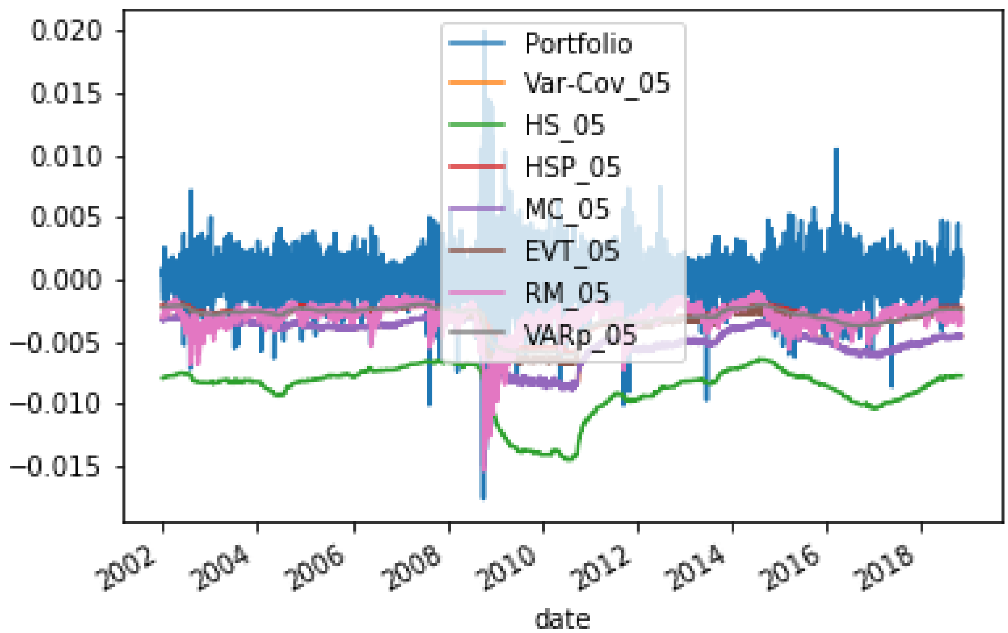
Portfolio stands for the realized observation of the portfolio, Var-Cov_05 stands for variance co-variance method, HS_05 stands for historical simulation applied to each asset, HSP_05 stands for historical simulation applied to the whole portfolio, MC_05 stands for Monte Carlo method using multivariate normal distribution, EVT_05 stands for extreme value theorem method, RM_05 stands for Risk Metrics, VARp_05 stands for portfolio variance method.

Figure C.5: Forecasted Value-at-Risk values and true observations at one percent interval for Currencies portfolio



Portfolio stands for the realized observation of the portfolio, Var-Cov_01 stands for variance covariance method, HS_01 stands for historical simulation applied to each asset, HSP_01 stands for historical simulation applied to the whole portfolio, MC_01 stands for Monte Carlo method using multivariate normal distribution, EVT_01 stands for extreme value theorem method, RM_01 stands for Risk Metrics, VARp_01 stands for portfolio variance method.

Figure C.6: Forecasted Value-at-Risk values and true observations at five percent interval for Currencies portfolio



Portfolio stands for the realized observation of the portfolio, Var-Cov_05 stands for variance covariance method, HS_05 stands for historical simulation applied to each asset, HSP_05 stands for historical simulation applied to the whole portfolio, MC_05 stands for Monte Carlo method using multivariate normal distribution, EVT_05 stands for extreme value theorem method, RM_05 stands for Risk Metrics, VARp_05 stands for portfolio variance method.