hep-ph/0703107
MIFP-07-07
March, 2007

Yukawa Couplings in a Model with Gauge, Higgs and Matter Unification

Ilia Gogoladze[a], Chin-Aik Lee[a], Yukihiro Mimura[b] and Qaisar Shafi[a]

[a] *Bartol Research Institute, Department of Physics & Astronomy, University of Delaware, Newark, DE 19716, USA*

[b] *Department of Physics, Texas A&M University, College Station, TX 77843-4242, USA*

Abstract

We discuss how unification of the gauge, Higgs and (three chiral family) matter superfields can be realized from the compactification of a six dimensional supersymmetric $SU(8)$ gauge theory over the orbifold $\mathbb{R}^4 \times T^2/\mathbb{Z}_3$. The bulk gauge interaction includes Yukawa interactions to generate masses for quarks and leptons after the electroweak symmetry is broken. The Yukawa matrices in this case turn out to be antisymmetric, and thus not phenomenologically viable. To overcome this we introduce brane fields which are vector-like under the standard model gauge symmetry, and so do not alter the number of chiral families. In such a setup, the observed fermion masses and mixings can be realized by taking into account suppression effects from the effective Wilson line couplings and large volume of the extra dimensions.

1 Introduction

The discovery of atmospheric and solar neutrino oscillations have yielded convincing evidence for new physics beyond the standard model (SM). Many possible extensions of the SM have been proposed, and one of the most compelling ideas is quark-lepton unification and that of grand unified theories (GUTs) [1, 2]. Combined with supersymmetry (SUSY), one obtains a rather attractive framework for physics beyond the standard model. Gauge couplings can be unified in the minimal SUSY standard model (MSSM), and the masses and mixings of quarks and leptons can be also explored in a (more or less) unified setup. However, the Higgs sector of these theories is usually problematic, leading to well known problems of fine-tuning, doublet-triplet splitting, dimension five nucleon decay, etc.

More recent discussion of SUSY GUTs often invoke one or more extra dimensions to circumvent some of these problems. After dimensional reduction, a vector/tensor field in some higher dimensional compactified space decomposes into a number of scalar, vector and tensor components in four dimensions (4D). The left- and right-handed Weyl fermions in 4D are unified in higher dimensional fermions. The idea of compactification has been applied to break symmetries via orbifold boundary conditions [3, 4, 5]. When applied to GUT models, the colored Higgs particles can be projected out, while the unprojected zero mode of the Higgs doublet remains light. Dimension five proton decay mediated by the colored Higgs fields is forbidden in the model [3]. Though the gauge symmetry is explicitly broken by the orbifold conditions, the gauge couplings can still unify provided that the brane localized gauge interactions are suppressed by the large volume of the extra dimension [4]. In such a framework, the idea of gauge-Higgs unification [6] was recently revived [7]. The scalar Higgs fields can be unified with the gauge fields in some higher dimensional vector field(s). Through the orbifold boundary condition, the higher dimensional gauge symmetry gets broken since the generators of the associated 4D gauge bosons are projected out. The broken generators for the extra dimensional components can have massless modes, and the Wilson line operator can be identified as the Higgs bosons breaking the symmetry that remains in 4D [8]. More precisely, via a Hosotani transformation, we can show that the resulting model is equivalent to one with a trivial Wilson line and non-Abelian orbifold projections breaking electroweak symmetry. The SUSY non-renormalization theorem protects the scalar from acquiring a large mass and which therefore survives at low energy. Interestingly, this idea is compatible with the extension of the SM to large gauge symmetries including GUTs.

An interesting consequence of gauge-Higgs unification is that Yukawa interactions can arise from the gauge interaction when fermions are also higher dimensional bulk fields [9, 10]. The 4D zero modes of fermions can be chiral due to orbifold projections, and the higher dimensional extension of the fermion kinetic term with covariant derivative can include Yukawa couplings, with some of the higher dimensional components of the gauge fields identified as Higgs fields. In the left-right symmetric realization of such a model [11], the matter representation for achieving gauge-Yukawa unification can be much simpler than that of the SM construction, and indeed, unification of gauge and Yukawa coupling constants can be realized [10]. In the models over a 5D $\mathcal{N}=1$ SUSY $S^1/\mathbb{Z}_2$ orbifold with bulk gauge symmetries such as $SO(11)$ and $SU(8)$, which break down to $SU(4)_c \times SU(2)_L \times SU(2)_R$ in 4D, matter fields are unified in hypermultiplets, and all three gauge couplings and third generation Yukawa couplings (top, bottom, tau and Dirac tau neutrino) can be unified. The top quark mass as well as the MSSM parameter $\tan\beta$, a ratio of vacuum expectation values (VEVs) for up- and down-type Higgs, are predicted. The prediction of the top quark mass is in good agreement with the experiment. Thus, unification of the gauge and Yukawa coupling constants can be an important signal of extra dimensions at ultra high energy scales [12].

In SUSY extensions, the matter fields can be unified in higher dimensional gauge multiplets [13, 14], especially if the model consists of $N=4$ vector multiplet in 4D language, which can arise from 6D $\mathcal{N}=(1,1)$ SUSY. Interestingly, three chiral families can be obtained in the case of a $T^2/\mathbb{Z}_3$ orbifold [13]. The three families originate from the three chiral supermultiplets in the $N=4$ gauge multiplet. Since three is the maximal number of chiral multiplet in 4D, this may explain family replication.

A hypermultiplet in the adjoint representation of the bulk gauge symmetry in 5D $\mathcal{N}=1$ SUSY $S^1/\mathbb{Z}_2$ orbifold model can be incorporated into the gauge multiplet in 6D $\mathcal{N}=(1,1)$ SUSY orbifold models. In Ref.[15], it is found that all the matter fields for one family, the Higgs doublets, as well as gauge fields of the SM can be unified in 6D $\mathcal{N}=(1,1)$ SUSY $SU(8)$ gauge multiplets with $T^2/\mathbb{Z}_6$ orbifold. The three gauge couplings and the third generation Yukawa couplings can also be unified in the model. Alternative gauge groups [16] and extensions including seven dimensional models [17, 18] have been discussed. Since no other bulk matter fields can be introduced, the model can explain why only the third family is heavy. A shortcoming of this type of model is that there is no unique way of specifying the discrete $\mathbb{Z}_6$ orbifold projections so that we get the matter and Higgs fields as the zero modes. The first and second families are treated as brane fields to cancel the brane localized gauge anomalies. The Yukawa couplings for the first and second families are suppressed by a large volume factor, but there is no good reason as to why the mass of the first family is hierarchically small. Thus, the masses and mixings are introduced by hand. If the gauge symmetry is extended to a group such the $SO(16)$, the two families can be included in the vector multiplet [16], though the discrete charge assignment is more complicated.

If we choose a $T^2/\mathbb{Z}_3$ orbifold for the $SU(8)$ model, on the other hand, the discrete charge assignment is simple and almost unique if $N = 1$ SUSY survives in 4D. In this case, the three chiral families, the Higgs fields as well as the gauge fields are naturally unified in one multiplet. However, the Yukawa matrix is antisymmetric, which stems from the fact that the chiral superfields in the gauge multiplet are in adjoint representations of the bulk gauge symmetry. As a consequence, after electroweak symmetry breaking, two families have degenerate masses, and the first family is massless. One may introduce brane localized interactions to break the mass degeneracy through cancellation. This looks not only unnatural, but is also inconsistent with the volume suppression of the brane interactions. Without the latter, gauge coupling unification can be adversely affected by brane localized couplings.

In this paper, we will construct a phenomenological viable three-family model from a $T^2/\mathbb{Z}_3$ orbifold construction based on $SU(8)$, such that the three chiral families, as well as the gauge and Higgs fields are all unified. We will introduce brane localized fields which are vector-like under the SM group, and which are needed to cancel the gauge anomalies originating from the additional $U(1)$ gauge symmetries. We will discuss how the second family masses can be suppressed due to the large volume of the extra dimensions. The mass of the first family can be further suppressed by a mechanism involving Wilson line operators.

This paper is organized as follows: In section 2, we will construct a 6D $\mathcal{N} = (1, 1)$ SUSY $T^2/\mathbb{Z}_3$ orbifold model with $SU(8)$ bulk gauge symmetry, which breaks down to $N = 1$ SUSY with $SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)^2$ at the 4D fixed points. The three chiral matter multiplets of the SM and the Higgs fields are obtained from the bulk gauge multiplet. The Yukawa and gauge interactions are unified. In section 3, we study how the fermion mass hierarchy can be realized. Section 4 explores fermion masses and mixings in more detail. Our conclusions are summarized in Section 5.

2 Gauge, Higgs and Three-Family Unification

In this section we will construct a model in which the 4D gauge, Higgs superfield and three families of matter superfield are unified in a 6D $\mathcal{N} = (1, 1)$ SUSY gauge (super)multiplet associated with the gauge group $G = SU(8)$. The gauge multiplet consists of a vector field, four real scalars, and both left- and right-handed Weyl fermions. The maximal R-symmetry in $\mathcal{N} = (1, 1)$ SUSY is $Sp(2)_L \times Sp(2)_R$. The vector field is a singlet under the R-symmetry and decomposes into a 4D vector field A_μ and two real scalar fields A_5 and A_6. The four real scalars in the gauge multiplet transform as $(\mathbf{2}, \mathbf{2})$ under the R-symmetry, while the left(right)-handed Weyl spinors transform as $(\mathbf{1}, \mathbf{2})$ $((\mathbf{2}, \mathbf{1}))$. From a 4D point of view, there are four left-handed Weyl spinors. All together, these fields are reorganized in a single $N = 4$ gauge multiplet, which consists of one $N=1$ vector superfield V and three chiral superfields Φ_i ($i=1,2,3$) in 4D. The scalar component of Φ_1 is $A_5 - iA_6$. [The formalism of 6D models is described in Ref.[19].]

The two extra dimensions are compactified over a flat $T^2/\mathbb{Z}_3$ orbifold. The orbifold transformation $\mathbf{R}$ is $z \to \omega z$, where $z = x_5 + ix_6$ and $\omega = e^{2\pi i/3}$. The transformation $\mathbf{R}$ can also act on the internal symmetry of the Lagrangian, which in our class of models is the product of $Sp(2)_L$, $Sp(2)_R$ and $Aut(G)$. This extension of $\mathbf{R}$ can break SUSY as well as the bulk gauge group G. Depending on the discrete charge assignment, the 4D $N=4$ SUSY can be broken down to $N=0$, 1 or 2.

If at least $N=1$ SUSY survives at a 4D fixed point, the orbifold conditions for the superfields V and Φ_i are

$$
\begin{aligned}
V(x^\mu, \bar{\omega}\bar{z}, \omega z) &= U V(x^\mu, \bar{z}, z) U^{-1}, &(1)\\
\Phi_1(x^\mu, \bar{\omega}\bar{z}, \omega z) &= \bar{\omega}\, U \Phi_1(x^\mu, \bar{z}, z) U^{-1}, &(2)\\
\Phi_2(x^\mu, \bar{\omega}\bar{z}, \omega z) &= \bar{\omega}^l\, U \Phi_2(x^\mu, \bar{z}, z) U^{-1}, &(3)\\
\Phi_3(x^\mu, \bar{\omega}\bar{z}, \omega z) &= \bar{\omega}^m\, U \Phi_3(x^\mu, \bar{z}, z) U^{-1}, &(4)
\end{aligned}
$$

where U is an $SU(8)$ matrix. Since there are higher dimensional versions of the gauge interaction term $\mathrm{tr}\, \Phi_1[\Phi_2, \Phi_3]$ in the Lagrangian, the condition $1+l+m=0$ (mod 3) needs to be satisfied. The only possible choices are $(l,m) = (1,1), (2,0), (0,2)$. For the case $(l,m) = (2,0)$, Φ_1 and Φ_2 form a hypermultiplet, and V and Φ_3 form an $N=2$ vector multiplet. In the same way, $N=2$ SUSY remains for $(l,m) = (0,2)$. Thus, $(l,m) = (1,1)$ is the unique choice to obtain $N=1$ SUSY at the 4D fixed points with the orbifold conditions on all three chiral superfields being the same. We will choose $l=m=1$ hereafter.

The $SU(8)$ matrix U is chosen to be

$$
U = \mathrm{diag}(1,1,1,1,\bar{\omega},\bar{\omega},\omega,\omega), \qquad (5)
$$

resulting in the breaking of the $SU(8)$ gauge symmetry to $G_{422} \times U(1)_A \times U(1)_B$ in 4D, where G_{422} is the gauge symmetry $SU(4)_c \times SU(2)_L \times SU(2)_R$ [1]. The $SU(8)$ adjoint representation $\mathbf{63}$ decomposes as

$$
\mathbf{63} \to \begin{pmatrix} (\mathbf{15},\mathbf{1},\mathbf{1})_{0,0} & (\mathbf{4},\mathbf{2},\mathbf{1})_{1,-1} & (\mathbf{4},\mathbf{1},\mathbf{2})_{1,1} \\ (\bar{\mathbf{4}},\mathbf{2},\mathbf{1})_{-1,1} & (\mathbf{1},\mathbf{3},\mathbf{1})_{0,0} & (\mathbf{1},\mathbf{2},\mathbf{2})_{0,2} \\ (\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{-1,-1} & (\mathbf{1},\mathbf{2},\mathbf{2})_{0,-2} & (\mathbf{1},\mathbf{1},\mathbf{3})_{0,0} \end{pmatrix} \oplus (\mathbf{1},\mathbf{1},\mathbf{1})_{0,0} \oplus (\mathbf{1},\mathbf{1},\mathbf{1})_{0,0}\,, \qquad (6)
$$

where the generators of $U(1)_A$ and $U(1)_B$ subgroups are chosen and normalized to be $\mathrm{diag}\,(1,1,1,1,-1,-1,-1,-1)/2$ and $\mathrm{diag}\,(0,0,0,0,1,1,-1,-1)$, respectively. Although the representations are vector-like under the subgroup, the zero modes of the superfields Φ_i are chiral after orbifold projection, and thus a chiral field theory in 4D is obtained. The zero modes correspond to three copies of $(\mathbf{4},\mathbf{2},\mathbf{1})_{1,-1}$ and $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{-1,-1}$ representations, and also three copies of $(\mathbf{1},\mathbf{2},\mathbf{2})_{0,2}$[1]. These can be interpreted as the $SU(2)_L$ ($SU(2)_R$) doublet matter chiral superfields Ψ_i (Ψ_i^c) for quarks and leptons, and the Higgs bidoublets H_i, respectively. As a result, the gauge and Higgs fields as well as the three families of SM matter fields plus three right-handed neutrinos are unified in one gauge multiplet.

There is only one pair of Higgs doublets in the MSSM. Indeed, if more than two pairs of doublets survive well below the unification scale, they can spoil gauge coupling unification. The brane interaction of H_i with the G_{422} singlets can make two of the bidoublets massive, so that only one linear combination remains light.

It is worth noting that obtaining the three chiral families does not depend on the details of the discrete charge assignments. Three copies of chiral fields are always obtained if $N = 1$ SUSY remains in 4D. The converse of this statement also holds.

Due to the $SU(8)$ bulk gauge unification, the $SU(4)_c$, $SU(2)_L$ and $SU(2)_R$ gauge couplings from a 4D point of view are unified at the cutoff M_* [4, 20],

$$g_c^2 = g_{2L}^2 = g_{2R}^2 = g^2 \equiv \frac{g_{6D}^2}{V} = \frac{\hat{g}_{6D}^2}{VM_*^2}, \tag{7}$$

where V is the volume of the extra dimensions. The 6D gauge coupling g_{6D} is a dimensionful parameter, which can be turned into a dimensionless coupling $\hat{g}_{6D}$ by employing the cutoff M_*. A brane localized gauge kinetic term can modify this unification, but this can be suppressed if VM_*^2 is sufficiently large.

Since the scalar component of Φ_1 is a higher dimensional gauge field, the bulk gauge interaction includes the term $\mathrm{tr}\,\Phi_1[\Phi_2,\Phi_3]$, which contains the Yukawa couplings,

$$g\,\epsilon_{ijk}\Psi_i\Psi_j^c H_k, \tag{8}$$

with conventional normalization of the gauge coupling, $\mathrm{tr}\,T^aT^b = 1/2\,\delta^{ab}$, the bulk Yukawa coupling constant is the same as the gauge coupling g.

Because the chiral superfields are in the adjoint representation, the bulk Yukawa coupling is antisymmetric. One family is naturally predicted to be massless, which, to a good approximation, is desirable as far as the first generation is concerned. However, the two non-zero mass eigenvalues of the fermions are degenerate, which is a terribly wrong prediction. Although a brane localized interaction, $y'_{ijk}\Psi_i\Psi_j^c H_k$, may be introduced to solve the problem, such interactions are suppressed by the volume factor. This suppression is needed, as previously mentioned,

[1]Note that the Higgs bidoublet is also chiral with respect to one of the $U(1)$ symmetries.

to preserve gauge coupling as well as gauge-Yukawa unification. In addition, the fermion mass hierarchy between the second and third families would then arise from fine-tuning, which is not attractive. In the next two sections we will show how phenomenologically viable fermion masses and mixings can be achieved in this class of models.

3 Fermion Mass Hierarchy

We have three chiral families $\Psi_i\,(\mathbf{4},\mathbf{2},\mathbf{1})_{1,-1}$, $\Psi_i^c\,(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{-1,-1}$ and $H_i\,(\mathbf{1},\mathbf{2},\mathbf{2})_{0,2}$ from the zero modes. There are gauge anomalies at each 4D fixed point [21] involving the $U(1)$ symmetries. In order to cancel these anomalies and retain three chiral families, we introduce brane fields which are vector-like under G_{422} but not with respect to the $U(1)$'s. To break the $U(1)$ symmetries, some G_{422} singlet fields with suitable $U(1)$ charges are also introduced.

Suppose that brane superfields $\bar{\Psi}_b^c\,(\mathbf{4},\mathbf{1},\mathbf{2})$, and $\Psi_b^c\,(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ are introduced with the following interaction:

$$\int d^6x\,\delta(x_5)\delta(x_6)\int d^2\theta\,\left(M\bar{\Psi}_b^c(\Psi_b^c+r_i\Psi_i^c)+y_{ik}\Psi_i\Psi_b^cH_k\right). \tag{9}$$

The mass scale M may be related to the $U(1)_{A,B}$ breaking scale. The fermion mass matrix (e.g. for up-type quark) is given by

$$\begin{pmatrix} u_1 & u_2 & u_3 & \bar{U}_b^c \end{pmatrix}\begin{pmatrix} 0 & a_3 & -a_2 & m_1 \\ -a_3 & 0 & a_1 & m_2 \\ a_2 & -a_1 & 0 & m_3 \\ r_1M & r_2M & r_3M & M \end{pmatrix}\begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ U_b^c \end{pmatrix}, \tag{10}$$

where $a_i = g\langle H_{iu}^0\rangle$ and $m_i = y_{ik}\langle H_{ku}^0\rangle$. If there is no other up-type Higgs field with a large VEV, we have $a \equiv (a_1^2+a_2^2+a_3^2)^{1/2} = gv_u$. We neglect the brane localized coupling $y'_{ijk}\Psi_i\Psi_j^cH_k$. Due to volume suppression, y_{ik} is suppressed, and thus $m_i \ll a$. For the same reason, the dimensionless coefficients $r_i \gg 1$. Without loss of generality, the a_2, a_3 and r_3 entries can be eliminated by the transformations $\Psi'_i = V_{ij}\Psi_j$ and $\Psi_i^{c\prime} = V_{ij}\Psi_j^c$, where V is a unitary matrix. Thus, we can write the up-type quark mass matrix as

$$M_u = \begin{pmatrix} 0 & 0 & 0 & m'_1 \\ 0 & 0 & a & m'_2 \\ 0 & -a & 0 & m'_3 \\ r'_1M & r'_2M & 0 & M \end{pmatrix}. \tag{11}$$

With $M \gg v_u$, the mass eigenvalues can be calculated to be

$$m_u^2 m_c^2 m_t^2 = \frac{a^4 r_1'^2 m_1'^2}{1+r^2}, \tag{12}$$

$$m_u^2 m_c^2 + m_u^2 m_t^2 + m_c^2 m_t^2 = a^2 \frac{(a + r_2' m_3')^2 + r^2 m_1'^2 + r_1'^2 (a^2 + m^2)}{1+r^2}, \tag{13}$$

$$m_u^2 + m_c^2 + m_t^2 = a^2 + \frac{(a + r_2' m_3')^2 + r_2'^2 (m_1'^2 + m_2'^2) + r_1'^2 (a^2 + m^2)}{1+r^2}, \tag{14}$$

where $r^2 = r_1^2 + r_2^2 + r_3^2$ and $m^2 = m_1^2 + m_2^2 + m_3^2$. Therefore, under the assumption of volume suppression ($a \gg m$ and $r \gg 1$), the quark masses are hierarchical if $r_1' \ll r$ is satisfied and one finds

$$m_t \simeq a, \quad m_c \simeq \frac{a}{r} + m_3', \quad m_u \simeq a \frac{r_1'}{r} \frac{m_1'}{m_c}. \tag{15}$$

For this case, the top, bottom, tau (and also Dirac tau neutrino) Yukawa couplings are unified at the GUT scale together with the gauge coupling:

$$g_c = g_{2L} = g_{2R} = y_t = y_b = y_\tau = y_{\nu_\tau}. \tag{16}$$

The scalar components of Ψ_1 and Ψ_1^c, as well as of H_1, are identified with the transverse components A_z of the gauge field. Because A_z can always be gauged away on the branes, it is not possible to introduce brane couplings to the zero modes of the first family. In the limit when there is no brane interactions for Ψ_1, Ψ_1^c and H_1, we obtain $a_2, a_3, m_1 \to 0$ and $r_1 \to 0$. The first generation is then massless, while the second and third family masses are hierarchical.

Since the first family is not truly massless we need a small correction to the brane coupling. There could exist a gauge invariant coupling involving a Wilson line closed path with a nontrivial winding number around the torus. This coupling is expected to be exponentially suppressed [22, 23], which could provide an explanation of the observed suppression of the first family mass terms [24].

4 Fermion Masses and Mixings

In the previous section we have seen how a fermion mass hierarchy can be realized through suppression of the Wilson line couplings and from volume suppression. In this section we discuss the Yukawa matrices in more detail. Since a left-right symmetry survives at 4D fixed points, the quark CKM mixings vanish if only one bidoublet is light, which is assumed for successful gauge coupling unification. Though it is an attractive feature at leading order, we have to break the proportional relationship between the up- and down-type quark mass matrices, as well as lepton Dirac mass matrices, to obtain realistic quark and lepton masses and mixings [25].

In our setup, the breaking of G_{422} down to the SM can happen by the VEVs of brane fields, which can break the proportional relationship of the fermion mass matrices. For example, suppose that there is a nonrenormalizable coupling, $\Sigma\Psi_i^c\bar{\Psi}_b^c S/M_*$, in addition to the coupling $\Psi_i^c\bar{\Psi}_b^c S$, where S is a singlet under G_{422} with appropriate $U(1)$ charges to make the coupling gauge-invariant and Σ is a triplet under $SU(2)_R$. Then, r_i in the fermion mass matrix such as Eq.(10) can be slightly different for up- and down-type quarks. There is enough freedom to fit fermion masses and mixings if we include this type of couplings with $SU(4)_c$ and $SU(2)_R$ adjoint fields.

Consider one such modification with $(r_3/r_2)_u = 1+\delta/2$ and $(r_3/r_2)_d = 1-\delta/2$ for example. Then, in the basis used in Eq.(11), the quark mass matrices can be written as

$$M_u = \begin{pmatrix} 0 & 0 & 0 & m_1^u \\ 0 & 0 & a_u & m_2^u \\ 0 & -a_u & 0 & m_3^u \\ \epsilon_u r_u M & r_u M & 0 & M \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & 0 & 0 & m_1^d \\ 0 & 0 & a_d & m_2^d \\ 0 & -a_d & 0 & m_3^d \\ \epsilon_d r_d M' & r_d M' & \delta r_d M' & M' \end{pmatrix}. \tag{17}$$

Integrating out the heavy vector-like quark fields, we obtain

$$M_u^{3\times3} = \begin{pmatrix} 0 & m_1^u & 0 \\ \epsilon_u a_u & \frac{a_u}{r_u} + m_3^u & 0 \\ 0 & m_2^u & a_u \end{pmatrix}, \quad M_d^{3\times3} = \begin{pmatrix} 0 & m_1^d & 0 \\ \epsilon_d a_d & \frac{a_d}{r_d} + m_3^d & \delta a_d \\ 0 & m_2^d - \delta\frac{a_d}{r_d} & a_d \end{pmatrix}. \tag{18}$$

The bulk field for the right-handed second family is switched into the bulk field Ψ_b^c in the large $r_{u,d}$ limit. In the expression for 3×3 mass matrices, the second and third rows are switched. It is easy to find that $V_{cb} \simeq \delta$, and one can also fit the relations $V_{ub} \sim V_{us}V_{cb}$ and $V_{cb} \sim m_s/m_b$. The (1,1) elements in the quark mass matrices are tiny since the effective Wilson line coupling is suppressed. This is useful to explain the empirical relation $V_{us} \sim \sqrt{m_d/m_s}$. The hierarchy within the up- and down-type quarks, e.g. $m_u/m_t \ll m_d/m_b$, is not explained in this model.

For the neutrino sector, the nonrenormalizable coupling $\Psi_i^c\Psi_j^c\bar{\chi}^c\bar{\chi}^c$ is needed to obtain heavy right-handed Majorana masses. The brane field $\bar{\chi}^c$ transforms as $(\mathbf{4},\mathbf{1},\mathbf{2})$ under G_{422}. The observed bilarge neutrino mixing is not automatically realized in the model, but there is enough freedom to fit the neutrino data. The large (essentially maximal) atmospheric neutrino mixing may be explained by the choice $r_2 \simeq r_3$. The model displays antisymmetry to leading order under the exchange of Φ_2 and Φ_3, which is related to the $Sp(2)_R$ symmetry in the bulk. As in the quark sector, due to the left-right gauge symmetry, the mixing angles from the neutrino Dirac sector are effectively small in the basis where the charged-lepton Yukawa matrix is diagonal. Namely, even if $r_2 \simeq r_3$, a large mixing in the charged-lepton Yukawa matrix is essentially canceled by the mixing in the Dirac neutrino Yukawa matrix. When considering type I seesaw, the observed large mixing needs to be generated by suitably choosing the right-handed Majorana mass matrix, irrespective of the symmetry. If we consider type II seesaw [26], on the other hand, there is no reason to cancel the large mixing in the charged-lepton Yukawa matrix and the exchange symmetry between Φ_2 and Φ_3 can be the origin of the large atmospheric neutrino mixing.

5 Conclusion and Discussion

We have studied a 6D $\mathcal{N} = (1, 1)$ SUSY model with $SU(8)$ bulk gauge symmetry compactified over a flat orbifold $T^2/\mathbb{Z}_3$. The $SU(8)$ symmetry is broken through orbifold compactification to $SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_A \times U(1)_B$. The three chiral families of the SM together with three right-handed neutrinos, as well as Higgs fields, are the zero modes. Thus, gauge, Higgs and matter fields of the SM are unified in a single gauge supermultiplet in higher dimension. However, this leads an antisymmetric Yukawa coupling matrix for the three chiral families. As a consequence, two of the fermion mass eigenvalues are degenerate, which is not acceptable. To solve this problem, one could consider the orbifold $T^2/\mathbb{Z}_6$ [15], so that the undesired eigenstate is projected out, only one family arises from the bulk fields, and the third family Yukawa couplings are unified with the gauge couplings. In this paper, on the other hand, we have constructed a realistic model employing brane localized fields which form vector-like pairs under the SM. Being vector-like, these fields do not alter the number of chiral families. In this setup, the fermion mass hierarchy can be realized as a result of suppression of the effective Wilson line couplings as well as the large volume of the extra dimensions. The third family Yukawa couplings maintain their asymptotic unification with the gauge couplings. Ignoring threshold effects at both the unification and weak scales yields $m_t = 178$ GeV and $\tan\beta = 51$ [12, 27].

While $SU(8)$ was employed to construct our model, we briefly comment on other choices for the bulk gauge group. We find that except for $SU(8)$, a complete set of three chiral quark and lepton families as well as Higgs fields is hard to achieve for a $T^2/\mathbb{Z}_3$ orbifold. In some cases, the SM particles are partially included in the bulk fields, while in some other cases, more than three chiral families are included in the bulk fields and a flavor gauge symmetry survives in 4D. In the latter case, it seems better to work with the $T^2/\mathbb{Z}_6$ orbifold. If the bulk fields only partially include the SM matter fields, the other remaining chiral fields are brane localized fields. In such partially unified models, only the top quark Yukawa coupling can be unified with the gauge couplings, and this may help explain the up-down hierarchy such as $m_u/m_t \ll m_d/m_b$.

Acknowledgments

This work is supported in part by the DOE Grant # DE-FG02-91ER40626 (I.G.), (C.L.) and (Q.S.).

References

[1] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).

[2] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[3] Y. Kawamura, Prog. Theor. Phys. **103**, 613 (2000) [hep-ph/9902423]; *ibid.* **105**, 999 (2001) [hep-ph/0012125]; *ibid.* **105**, 691 (2001) [hep-ph/0012352]; G. Altarelli and F. Feruglio, Phys. Lett. B **511**, 257 (2001) [hep-ph/0102301]; A. B. Kobakhidze, Phys. Lett. B **514**, 131 (2001) [hep-ph/0102323].

[4] L. J. Hall and Y. Nomura, Phys. Rev. D **64**, 055003 (2001) [hep-ph/0103125]; *ibid.* D **65**, 125012 (2002) [hep-ph/0111068]; *ibid.* D **66**, 075004 (2002) [hep-ph/0205067]; A. Hebecker and J. March-Russell, Nucl. Phys. B **613**, 3 (2001) [hep-ph/0106166]; T. Li, Phys. Lett. B **520**, 377 (2001) [hep-th/0107136]; Nucl. Phys. B **619**, 75 (2001) [hep-ph/0108120]; T. Asaka, W. Buchmuller and L. Covi, Phys. Lett. B **523**, 199 (2001) [hep-ph/0108021]; B. Kyae, C. A. Lee and Q. Shafi, Nucl. Phys. B **683**, 105 (2004) [hep-ph/0309205].

[5] Y. Mimura and S. Nandi, Phys. Lett. B **538**, 406 (2002) [hep-ph/0203126]; I. Gogoladze, Y. Mimura and S. Nandi, Phys. Lett. B **554**, 81 (2003) [hep-ph/0210320].

[6] N. S. Manton, Nucl. Phys. B **158**, 141 (1979); D. B. Fairlie, J. Phys. G **5**, L55 (1979); Phys. Lett. B **82**, 97 (1979); P. Forgacs and N. S. Manton, Commun. Math. Phys. **72**, 15 (1980); G. Chapline and R. Slansky, Nucl. Phys. B **209**, 461 (1982); S. Randjbar-Daemi, A. Salam and J. Strathdee, Nucl. Phys. B **214**, 491 (1983); N. V. Krasnikov, Phys. Lett. B **273**, 246 (1991); D. Kapetanakis and G. Zoupanos, Phys. Rept. **219**, 1 (1992); I. Antoniadis and K. Benakli, Phys. Lett. B **326**, 69 (1994) [hep-th/9310151]; H. Hatanaka, T. Inami and C. S. Lim, Mod. Phys. Lett. A **13**, 2601 (1998) [hep-th/9805067].

[7] See, for example, G. R. Dvali, S. Randjbar-Daemi and R. Tabbash, Phys. Rev. D **65**, 064021 (2002) [hep-ph/0102307]; N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **513**, 232 (2001) [hep-ph/0105239]; L. J. Hall, Y. Nomura and D. R. Smith, Nucl. Phys. B **639**, 307 (2002) [hep-ph/0107331]; I. Antoniadis, K. Benakli and M. Quiros, New J. Phys. **3**, 20 (2001) [hep-th/0108005]; K. Choi, N. Haba, K. S. Jeong, K. Okumura,

Y. Shimizu and M. Yamaguchi, JHEP **0402**, 037 (2004) [hep-ph/0312178]; C. A. Scrucca, M. Serone, L. Silvestrini and A. Wulzer, JHEP **0402**, 049 (2004) [hep-th/0312267]; N. Haba, Y. Hosotani, Y. Kawamura and T. Yamashita, Phys. Rev. D **70**, 015010 (2004) [hep-ph/0401183]; A. Aranda and J. L. Diaz-Cruz, Phys. Lett. B **633**, 591 (2006) [hep-ph/0510138]; N. Haba, S. Matsumoto, N. Okada and T. Yamashita, JHEP **0602**, 073 (2006) [hep-ph/0511046]; N. Maru and K. Takenaga, Phys. Lett. B **637**, 287 (2006) [hep-ph/0602149].

[8] Y. Hosotani, Phys. Lett. B **126**, 309 (1983); Phys. Lett. B **129**, 193 (1983); Phys. Rev. D **29**, 731 (1984); Annals Phys. **190**, 233 (1989).

[9] G. Burdman and Y. Nomura, Nucl. Phys. B **656**, 3 (2003) [hep-ph/0210257]; N. Haba and Y. Shimizu, Phys. Rev. D **67**, 095001 (2003) [hep-ph/0212166].

[10] I. Gogoladze, Y. Mimura and S. Nandi, Phys. Lett. B **560**, 204 (2003) [hep-ph/0301014].

[11] Q. Shafi and Z. Tavartkiladze, Phys. Rev. D **66**, 115002 (2002); N. Cosme and J. M. Frere, Phys. Rev. D **69**, 036003 (2004) [hep-ph/0303037].

[12] I. Gogoladze, Y. Mimura, S. Nandi and K. Tobe, Phys. Lett. B **575**, 66 (2003) [hep-ph/0307397].

[13] T. Watari and T. Yanagida, Phys. Lett. B **532**, 252 (2002) [hep-ph/0201086]; *ibid.* B **544**, 167 (2002) [hep-ph/0205090].

[14] T. Li, JHEP **0403**, 040 (2004) [hep-ph/0309199].

[15] I. Gogoladze, Y. Mimura and S. Nandi, Phys. Lett. B **562**, 307 (2003) [hep-ph/0302176].

[16] I. Gogoladze, Y. Mimura and S. Nandi, Phys. Rev. Lett. **91**, 141801 (2003) [hep-ph/0304118]; Phys. Rev. D **69**, 075006 (2004) [hep-ph/0311127].

[17] I. Gogoladze, T. j. Li, Y. Mimura and S. Nandi, Phys. Lett. B **622**, 320 (2005) [hep-ph/0501264]; Phys. Rev. D **72**, 055006 (2005) [hep-ph/0504082].

[18] I. Gogoladze, T. Li and Q. Shafi, Phys. Rev. D **73**, 066008 (2006) [hep-ph/0602040].

[19] N. Marcus, A. Sagnotti and W. Siegel, Nucl. Phys. B **224**, 159 (1983); N. Arkani-Hamed, T. Gregoire and J. Wacker, JHEP **0203**, 055 (2002) [hep-th/0101233].

[20] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B **436**, 55 (1998) [hep-ph/9803466]; Nucl. Phys. B **537**, 47 (1999) [hep-ph/9806292].

[21] C. A. Scrucca and M. Serone, Int. J. Mod. Phys. A **19**, 2579 (2004) [hep-th/0403163].

[22] C. Csaki, C. Grojean and H. Murayama, Phys. Rev. D **67**, 085012 (2003) [hep-ph/0210133].

[23] L. E. Ibanez, Phys. Lett. B **181**, 269 (1986).

[24] I. Gogoladze, C. A. Lee, Y. Mimura and Q. Shafi, in preparation.

[25] See, for example, Q. Shafi and Z. Tavartkiladze, Nucl. Phys. B **549**, 3 (1999) [hep-ph/9811282]; S. F. King, JHEP **0508**, 105 (2005) [hep-ph/0506297].

[26] G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**, 287 (1981); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).

[27] J. L. Chkareuli and I. G. Gogoladze, Phys. Rev. D **58**, 055011 (1998) [hep-ph/9803335].