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Non-Extremal Rotating Black Holes in Five-Dimensional Gauged Supergravity

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Abstract

Supersymmetric black holes in five-dimensional gauged supergravity must necessarily be rotating, and so in order to study the passage to black holes away from supersymmetry, it is of great interest to obtain non-extremal black holes that again have non-zero rotation. In this paper we find a simple framework for describing non-extremal rotating black holes in five-dimensional gauged supergravities. Using this framework, we are able to construct a new solution, describing the general single-charge solution of $\mathcal{N}=2$ gauged supergravity, with arbitrary values for the two rotation parameters. Previously-obtained solutions with two or three equal charges also assume a much simpler form in the new framework, as also does the general solution with three unequal charges in ungauged $\mathcal{N}=2$ supergravity. We discuss the thermodynamics and BPS limit of the new single-charge solutions, and we discuss the separability of the Hamilton-Jacobi and Klein-Gordan equations in these backgrounds.

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1 Introduction

With the developments in the understanding the AdS/CFT correspondence over the last few years, the investigation of solutions of gauged supergravities, and, especially, five-dimensional gauged supergravity, has acquired a new significance. In particular, it is of considerable interest to study five-dimensional solutions that provide a generalisation of the pure anti-de Sitter geometry whose dual boundary theory is an $\mathcal{N} = 4$ supersymmetric conformal field theory. Black holes provide a natural class of such generalisations. As was discussed in [1], by considering rotating asymptotically AdS black holes, one can make contact via the AdS/CFT correspondence with four-dimensional conformal field theories in a rotating Einstein universe.

The example considered in [1] was the purely gravitational Kerr-AdS black hole in five dimensions. This is a non-supersymmetric background, and so from the AdS/CFT viewpoint it does not enjoy the the added control and protection that would be exhibited in a BPS or near-BPS configuration. Supersymmetric rotating black holes have been found in five-dimensional gauged supergravities [2, 3, 4, 5, 6]. It is worth emphasising that there are no static black-hole solutions in five-dimensional gauged supergravity, and that rotation is essential in order to avoid the occurrence of naked singularities. Perhaps the greatest interest from the point of view of the dual CFT's arises if one can study AdS black hole backgrounds as one moves away from the supersymmetric situation, and this provides a further motivation for studying non-extremal black holes with rotation.

In the case of five-dimensional minimal gauged supergravity, the general solution, with mass parameter M, charge parameter q and the two independent rotation parameters a and b, was obtained in [5]. This generalised a previous result in [7], in which the two rotation parameters were set equal.

It would also be of interest to obtain the general non-extremal rotating solution in the case of five-dimensional $\mathcal{N} = 2$ supergravity coupled to matter. The most relevant case corresponds to $\mathcal{N} = 2$ supergravity coupled to two vector multiplets. This case, which therefore has three vector fields in total, gauging $U(1)^3$, can also be viewed as the truncation to the abelian subgroup of the maximal gauged $SO(6)$ supergravity. The solution for non-extremal rotating black holes with $a = b$ was obtained in [8]. A special case in which two of the three charges are set equal, with the third having a value related to these, was obtained for all $a \neq b$ in [4]. Also in the same paper, another special case, with $b = 0$ and only one charge, was constructed.

In the present paper, we extend this previous single-charge result by obtaining the general solution for a five-dimensional rotating black hole with arbitrary rotation parameters a and b, in the case that just one of the three $U(1)$ gauge fields carries a charge. In some sense this can be viewed as the most general "basic" solution of the $\mathcal{N}=2$ theory. Our approach to constructing this solution involves first recasting the metrics into a form that leads eventually to a rather simple presentation of the result. We also find that the same type of transformation, applied to previously-known cases, leads to rather simple expressions in those cases too.

The paper is organised as follows. In section 2 we introduce our general ansatz for the class of rotating black-hole metrics we shall consider. Then, we present our new results for the general single-charge rotating black holes. In section 3 we discuss the thermodynamics of the new solutions, followed in section 4 by the construction of the supersymmetric limit. In section 5, we examine the previously-known rotating black hole solutions in five-dimensional gauged supergravity, and show how these too fit elegantly within the formulation that we have adopted in this paper. In addition, we show that the general 3-charge solution in ungauged five-dimensional supergravity, which was constructed in [9], also has a very simple expression when written in this formalism. The paper ends with conclusions in section 6.

2 The General Single-Charge Rotating Black Hole

In this section, we construct the general solution for a single-charge non-extremal rotating black hole in five-dimensional $U(1)^3$ gauged $\mathcal{N}=2$ supergravity, characterised by the mass, charge, and two rotation parameters a and b.

The bosonic sector of the relevant $\mathcal{N}=2$ theory can be derived from the Lagrangian

$$e^{-1}\,\mathcal{L} = R - \tfrac{1}{2}\partial\vec{\varphi}^2 - \tfrac{1}{4}\sum_{i=1}^{3} X_i^{-2}\,(F^i)^2 + 4g^2\,\sum_{i=1}^{3} X_i^{-1} + \tfrac{1}{24}|\epsilon_{ijk}|\,\epsilon^{\mu\nu\rho\sigma\lambda}F^i_{\mu\nu}\,F^j_{\rho\sigma}\,A^k_\lambda\,, \qquad (1)$$

where $\vec{\varphi}=(\varphi_1,\varphi_2)$, and

$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1-\frac{1}{\sqrt{2}}\varphi_2}\,, \qquad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1+\frac{1}{\sqrt{2}}\varphi_2}\,, \qquad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}\,. \qquad (2)$$

All the solutions that we shall consider in this paper, comprising the new general single-charge rotating black holes, and also the previously-known solutions with two equal charges or three equal charges, as well as the general solutions in the ungauged theory with three unequal charges, can all be cast in a simple manner within the following formalism. We write the metrics as

$$ds_5^2 \;=\; (H_1H_2H_3)^{1/3}\,(x+y)\,d\hat{s}_5^2\,,$$

$$d\hat{s}_5^2 \;=\; -\Phi\,(dt+\mathcal{A})^2 + ds_4^2\,, \tag{3}$$

with the scalars and gauge potentials given by

$$\begin{aligned} X_i &= H_i^{-1}\,(H_1 H_2 H_3)^{1/3}\,,\\ A^1 &= \frac{2m}{x+y}\,H_1^{-1}\,\{s_1 c_1 dt + s_1 c_2 c_3[ab d\chi + (y-a^2-b^2)d\sigma] + c_1 s_2 s_3(ab d\sigma - y d\chi)\}\,, \end{aligned} \tag{4}$$

with A^2 and A^3 given by cyclically permuting the subscripts on the right-hand side. The functions H_i are given by

$$H_i = 1 + \frac{2m s_i^2}{x+y}\,, \tag{5}$$

and we are using the shorthand notation

$$s_i = \sinh\delta_i\,, \qquad c_i = \cosh\delta_i\,, \tag{6}$$

where δ_i are the charge parameters. The four-dimensional base metric in (3) takes the form

$$ds_4^2 = \Big(\frac{dx^2}{4X} + \frac{dy^2}{4Y}\Big) + \frac{U}{G}\Big(d\chi - \frac{Z}{U}\,d\sigma\Big)^2 + \frac{XY}{U}\,d\sigma^2\,, \tag{7}$$

where X is a function of x, Y is a function of y, and G, U and Z are functions of both x and y. The "Kaluza-Klein" 1-form $\mathcal{A}$ appearing in (3) lives purely in the four-dimensional base space, and takes the form

$$\mathcal{A} = f_1\,d\sigma + f_2\,d\chi\,. \tag{8}$$

The functions f_1 and f_2 depend only on x and y, as does Φ, which is given by

$$\Phi = \frac{G}{(x+y)^3\,H_1\,H_2\,H_3}\,. \tag{9}$$

The inverse of the metric $d\hat{s}_5^2$ is given by

$$\begin{aligned} \Big(\frac{\partial}{\partial\hat{s}_5}\Big)^2 &= -\frac{1}{\Phi}\Big(\frac{\partial}{\partial t}\Big)^2 + 4X\Big(\frac{\partial}{\partial x}\Big)^2 + 4Y\Big(\frac{\partial}{\partial y}\Big)^2 + \frac{G}{U}\Big(\frac{\partial}{\partial\chi} - f_2\,\frac{\partial}{\partial t}\Big)^2\\ &\quad + \frac{1}{UXY}\Big(U\frac{\partial}{\partial\sigma} + Z\,\frac{\partial}{\partial\chi} - (f_1\,U + f_2\,Z)\,\frac{\partial}{\partial t}\Big)^2\,. \end{aligned} \tag{10}$$

Since there is no solution-generating technique for deriving charged black holes from neutral black holes in gauged supergravity (unlike the situation in ungauged supergravity), there is really no way other than a combination of guesswork, followed by explicit verification, for obtaining the charged solutions. We were led to write the ansatz for the metric, gauge potentials and scalar fields in the manner we have presented above by considering all the previously-obtained examples. The specific results for the new general single-charged rotating black holes, which we shall present below, were obtained by making a detailed comparison of various known cases, transformed into the format of the ansatz above, and then making a conjecture for the form of the solution. Finally, we substituted this into the equations of motion following from (1), to verify that it was indeed a solution. In doing this, we made extensive use of the Mathematica algebraic computing language.

Our new results for the general single-charge rotating black hole in five-dimensional gauged $\mathcal{N}=2$ supergravity are as follows. Taking $\delta_2=\delta_3=0$, and writing $\delta_1=\delta$, we find

$$
\begin{aligned}
X &= (x+a^2)(x+b^2) - 2mx + g^2(x+a^2)(x+b^2)[x+2ms^2-(a^2+b^2)s^2+2absc]\,,\\
Y &= -(a^2-y)(b^2-y)[1-g^2(y+(a^2+b^2)s^2-2absc)]\,,\\
G &= (x+y)(x+y-2m)+g^2(x+y)^2\,(x-y+a^2+b^2)H\,,\\
U &= yX-xY+s^2W\,,\qquad Z=ab(X+Y)+scW\,,\\
W &= -2g^2m(a^2-y)(b^2-y)x+g^4(x+a^2)(x+b^2)(a^2-y)(b^2-y)(x+y+2ms^2)\,,\\
\Phi &= \frac{G}{(x+y)^3\,H}\,,\\
\mathcal{A} &= s\,(xd\chi+abd\sigma)+c\,[abd\chi-(x+a^2+b^2)d\sigma]\\
&\quad +\frac{1}{G}\Big[-s\,(x+y-2m)(xd\chi+abd\sigma)-c\,(x+y)[abd\chi-(x+a^2+b^2-2m)d\sigma]\\
&\qquad +g^2(x+a^2)(x+b^2)(x+y+2ms^2)(cd\sigma-sd\chi)\Big]\,. \qquad (11)
\end{aligned}
$$

The gauge potentials in (4) reduce to $A^2=A^3=0$ and

$$
A^1=\frac{2ms}{x+y+2ms^2}\,[cdt+abd\chi+(y-a^2-b^2)d\sigma]\,, \qquad (12)
$$

and the H_i functions are given by $H_2=H_3=1$ and

$$
H_1\equiv H=1+\frac{2ms^2}{x+y}\,. \qquad (13)
$$

The solution we have presented here has four non-trivial parameters, namely m, δ, a and b (with $s=\sinh\delta$, $c=\cosh\delta$), which characterise the mass, charge and two angular momenta respectively. In the next section, we shall derive the conserved charges associated with this solution, and also study the thermodynamic quantities.

3 Global Structure and Thermodynamics

The black hole solution we have constructed is of cohomogeneity 2, with the metric functions depending on the non-compact radial coordinate x, and the compact coordinate y which runs from $y=a^2$ to $y=b^2$. If we define $y=a^2\cos^2\theta+b^2\sin^2\theta$, then θ plays the role of the latitude coordinate of the round 3-sphere, viewed as a foliation of Clifford tori,

$$
d\Omega_3^2=d\theta^2+\cos^2\theta\,d\phi^2+\sin^2\theta\,d\psi^2\,. \qquad (14)
$$

The coordinates σ and χ can be related to two azimuthal $U(1)$ coordinates ϕ and ψ with canonical 2π periods via the redefinitions

$$\sigma = \frac{1}{a^2-b^2}\Big(\frac{a(\phi-\tilde a\, g^2\, t)}{\Xi_{\tilde a}} - \frac{b(\psi-\tilde b\, g^2\, t)}{\Xi_{\tilde b}}\Big)\,, \qquad \chi = \frac{1}{a^2-b^2}\Big(\frac{b(\phi-\tilde a\, g^2\, t)}{\Xi_{\tilde a}} - \frac{a(\psi-\tilde b\, g^2\, t)}{\Xi_{\tilde b}}\Big)\,, \tag{15}$$

where

$$\Xi_{\tilde a} = 1-\tilde a^2 g\,, \qquad \Xi_{\tilde b} = 1-\tilde b^2 g^2\,, \tag{16}$$

and we have defined "boosted" rotation parameters $\tilde a$ and $\tilde b$, given by

$$\tilde a = a\, c - b\, s\,, \qquad \tilde b = b\, c - a\, s\,. \tag{17}$$

The inclusion of t in the redefinitions ensures that ϕ and ψ define an asymptotically static frame.

The radial coordinate x runs to the asymptotically flat region as $x\to\infty$, and there is an outer horizon at $x=x_0$, the largest root of X. Straightforward calculations show that the entropy and temperature are given by

$$\begin{aligned} S &= \frac{\pi^2(x_0+a^2)(x_0+b^2)(c+s\,\tilde a\,\tilde b\, g^2)}{2\Xi_{\tilde a}\Xi_{\tilde b}\sqrt{x_0-g^2 s^2(x_0+a^2)(x_0+b^2)}}\,,\\ T &= \frac{\pi}{4S\,\Xi_{\tilde a}\Xi_{\tilde b}[x_0-g^2 s^2(x_0+a^2)(x_0+b^2)]}\Big(x_0^2-a^2b^2-g^4 s^2(x_0+a^2)^2(x_0+b^2)^2\\ &\quad +g^2[x_0^2(2x_0+a^2+b^2)-s(x_0^2-a^2b^2)(2a\,b\,c-s(a^2+b^2)]\Big)\,. \end{aligned} \tag{18}$$

The angular velocities of the horizon, measured with respect to the azimuthal coordinates ϕ and ψ of the asymptotically static frame, are given by

$$\Omega_\phi = \frac{a(c+s)(\Xi_{\tilde a}+g^2(x_0+a^2))}{(x_0+a^2)(1+s(c+s)(1+\tilde a\,\tilde b\, g^2)}\,, \qquad \Omega_\psi = \frac{b(c+s)(\Xi_{\tilde b}+g^2(x_0+b^2))}{(x_0+b^2)(1+s(c+s)(1+\tilde a\,\tilde b\, g^2)}\,. \tag{19}$$

The corresponding angular momenta, defined via Komar integrals $J=1/(16\pi)\int *dK$ where $K=\partial/\partial\phi$ or $\partial/\partial\psi$, are given by

$$J_\phi = \frac{\pi\, a\, m(c+s\,\tilde a\,\tilde b\, g^2)}{2\Xi_{\tilde a}^2\,\Xi_{\tilde b}}\,, \qquad J_\phi = \frac{\pi\, b\, m(c+s\,\tilde a\,\tilde b\, g^2)}{2\Xi_{\tilde a}\,\Xi_{\tilde b}^2}\,. \tag{20}$$

The electric potential and conserved electric charge are given by

$$\begin{aligned} \Phi &= \frac{s(c+s)(\Xi_{\tilde a}+g^2(x_0+a^2))}{1+s(c+s)(1+\tilde a\,\tilde b\, g^2)} = \frac{s(c+s)(\Xi_{\tilde b}+g^2(x_0+b^2))}{1+s(c+s)(1+\tilde a\,\tilde b\, g^2)}\,,\\ Q &= \frac{\pi m\, s(c+s\,\tilde a\,\tilde b\, g^2)}{2\Xi_{\tilde a}\,\Xi_{\tilde b}}\,. \end{aligned} \tag{21}$$

Note that we have the identity $\tilde{\Xi}_{\tilde{a}} + g^2(x_0 + a^2) = \tilde{\Xi}_{\tilde{b}} + g^2(x_0 + b^2)$, which is implied by the relation $a^2 - b^2 = \tilde{a}^2 - \tilde{b}^2$.

The mass (or energy) of the black hole can be easily obtained by integrating the first law of thermodynamics, $dE = T\,dS + \Omega_\phi\,dJ_\phi + \Omega_\psi\,dJ_\psi + \Phi\,dQ$; it is given by

$$E = \frac{\pi\,m}{4\Xi_{\tilde{a}}^2\,\Xi_{\tilde{b}}^2}\Big\{2\Xi_{\tilde{a}} + 2\Xi_{\tilde{b}} - \Xi_{\tilde{a}}\Xi_{\tilde{b}} + (\Xi_{\tilde{a}} + \Xi_{\tilde{b}})[4s\,c\,\sqrt{(1-\Xi_{\tilde{a}})(1-\Xi_{\tilde{b}})} + s^2(2-\Xi_{\tilde{a}})(2-\Xi_{\tilde{b}})]\Big\}\,, \tag{22}$$

4 The BPS limit

By considering the AdS superalgebra, as was discussed in [11], one can see that a BPS limit of the general non-extremal solution arises when

$$E + g\,J_\phi + g\,J_\psi + Q = 0\,. \tag{23}$$

(Equivalent BPS limits arise for other sign choices.) From our expressions in the previous section, we find that (23) implies

$$e^{2\delta} = 1 + \frac{2}{(\tilde{a} + \tilde{b})g}\,. \tag{24}$$

Expressed in terms of the original rotation parameters a and b, this BPS condition can be rewritten as

$$(a+b)g\,\sinh\delta = 1\,. \tag{25}$$

In this supersymmetric limit, where there exists a Killing spinor η, the Killing vector

$$\ell = \frac{\partial}{\partial t} - g\frac{\partial}{\partial\phi} - g\frac{\partial}{\partial\psi} \tag{26}$$

has a spinorial square root, in the sense that $\ell^\mu = i\,\bar\eta\,\gamma^\mu\eta$. It has norm given by

$$\ell^\mu\,\ell_\mu = -\frac{1}{H}\,. \tag{27}$$

As discussed in [2], the single-charge BPS rotating solution with a single angular momentum always has naked closed timelike curves, and the inclusion of the additional rotation parameter does not alter the feature. Thus there are no regular black hole solutions in the BPS limit.

5 Previously-Known Rotating Black Holes

In this section, we present the previously-known rotating black hole solutions of five-dimensional supergravity, using the formalism that we have introduced in section 2. These amount to three cases. The first is the case found in [4] with two charges set equal and the third related to this, in gauged $\mathcal{N}=2$ supergravity. The second case, obtained in [5], is where all three charges are equal in $\mathcal{N}=2$ gauged supergravity; this can be viewed also as the general solution in minimal gauged supergravity. The third case is the general solution in ungauged $\mathcal{N}=2$ supergravity, with three unequal charges, which was obtained in [9]. All three of these cases can be represented elegantly within the formulation of section 2, and thus to present them we need only specify the various functions and gauge potentials.

5.1 Two equal charges in gauged supergravity

In this solution, obtained in [4], we have $\delta_1=\delta_2=\delta$, with $\delta_3=0$. In the ungauged theory, this choice of charge parameters would imply that two of the three physical conserved charges were equal and non-vanishing, whilst the third vanished. As was shown in [4], in the case of the solution in gauged supergravity the third physical charge is actually non-vanishing too, with a value related to those of the other two. We find that in the formalism of section 2, this solution is given by

$$
\begin{aligned}
X &= (x+a^2)(x+b^2)-2mx+g^2(x+a^2+2ms^2)(x+b^2+2ms^2)x\,,\\
Y &= -(a^2-y)(b^2-y)(1-g^2\,y)\,,\\
G &= (x+y)(x+y-2m)+g^2(x+y)^2\,(x-y+a^2+b^2+2ms^2)H\,,\\
U &= yX-xY\,,\qquad Z=ab(X+Y)\,,\\
\Phi &= \frac{G}{(x+y)^3\,H^2}\,,\\
\mathcal{A} &= ab d\chi-(x+a^2+b^2+2ms^2)d\sigma\\
&\quad+\frac{1}{G}\Big[-(x+y+2ms^2)[ab d\chi-(x+a^2+b^2-2m)d\sigma]\\
&\qquad+g^2(x+a^2+2ms^2)(x+b^2+2ms^2)(x+y+2ms^2)d\sigma\Big]\,.
\end{aligned}
\tag{28}
$$

That the gauge potentials in (4) reduce to

$$
\begin{aligned}
A^1 &= A^2=\frac{2msc}{x+y+2ms^2}\,[dt+ab d\chi+(y-a^2-b^2)d\sigma]\,,\\
A^3 &= \frac{2ms^2}{x+y}\,(ab d\sigma-y d\chi)\,,
\end{aligned}
\tag{29}
$$

and the functions H_i reduce to $H_3 = 1$, and

$$H_1 = H_2 = H = 1 + \frac{2ms^2}{x+y}\,. \tag{30}$$

5.2 Three equal charges in gauged supergravity

This solution, obtained in [5], which can also be viewed as the general rotating black hole solution in five-dimensional minimal gauged supergravity, corresponds in the formalism of section 2 to taking $\delta_1 = \delta_2 = \delta_3 = \delta$. We find that it then takes the form

$$\begin{aligned}
X &= (x+a^2)(x+b^2) - 2mx \\
&\quad + g^2(x+a^2+2ms^2)(x+b^2+2ms^2)[x+2ms^2-(a^2+b^2)s^2+2absc]\,, \\
Y &= -(a^2-y)(b^2-y)[1-g^2(y+(a^2+b^2)s^2-2absc)]\,, \\
G &= (x+y)(x+y-2m) + g^2(x+y)^2\,(x-y+a^2+b^2+2ms^2)H^2\,, \\
U &= yX - xY + s^2 W\,, \qquad Z = ab(X+Y) + scW\,, \\
W &= -2g^2 m(a^2-y)(b^2-y)[x(c^2+s^2)+(a^2+b^2)s^2+2ms^4] \\
&\quad + g^4(x+a^2+2ms^2)(x+b^2+2ms^2)(a^2-y)(b^2-y)(x+y+2ms^2)\,, \\
\Phi &= \frac{G}{(x+y)^3\,H^3}\,, \\
\mathcal{A} &= s(xd\chi + abd\sigma) + c[abd\chi - (x+a^2+b^2+2ms^2)d\sigma] + 2ms^3\,d\chi \\
&\quad + \frac{H}{G}\Big[-s(x+y-2m)(xd\chi+abd\sigma) - c(x+y)[abd\chi-(x+a^2+b^2-2m)d\sigma] \\
&\quad + g^2(x+a^2+2ms^2)(x+b^2+2ms^2)(x+y+2ms^2)(cd\sigma - sd\chi)\Big]\,.
\end{aligned} \tag{31}$$

The gauge potentials in (4) reduce to

$$A^1 = A^2 = A^3 = \frac{2msc}{x+y+2ms^2}\,\{dt + c[abd\chi + (y-a^2-b^2)d\sigma] + s(abd\sigma - yd\chi)\}\,, \tag{32}$$

and the functions H_i are given by

$$H_1 = H_2 = H_3 = H \equiv 1 + \frac{2ms^2}{x+y}\,. \tag{33}$$

5.3 Three unequal charges in ungauged supergravity

This solution was first obtained in [9], by applying a solution-generating procedure to add charges to the neutral five-dimensional rotating black hole of Myers and Perry [10]. We find that in the formulation of section 2, it takes the simple form

$$X = (x+a^2)(x+b^2) - 2mx\,,$$

$$
\begin{aligned}
Y &= -(a^2-y)(b^2-y)\,,\\
G &= (x+y)(x+y-2m)\,,\\
U &= yX - xY\,,\qquad Z = ab(X+Y)\,,\\
\Phi &= \frac{G}{(x+y)^3\, H_1H_2H_3}\,,\\
\mathcal{A} &= \frac{2mc_1c_2c_3}{G}[(a^2+b^2-y)d\sigma - abd\chi] - \frac{2ms_1s_2s_3}{x+y}\,(abd\sigma - yd\chi)\,.
\end{aligned}
\tag{34}
$$

6 Conclusions

In this paper, we have constructed the general solution for a singly-charged rotating black hole in five-dimensional gauged supergravity. It can be viewed as a solution in the maximal $\mathcal{N}=8$ gauged theory, with the charge being carried in a single $U(1)$ subgroup of the $SO(6)$ gauge group. Equivalently, it can be viewed as a solution in $\mathcal{N}=2$ gauged supergravity coupled to two vector multiplets, with the charge carried in a single $U(1)$ factor of the $U(1)^3$ gauge group. The solution generalises a special case obtained in [4], in which only one of the two rotation parameters was non-vanishing.

In addition to obtaining the new solution, we have also found a simple framework within which all the currently-known five-dimensional non-extremal rotating black holes can be described. This framework involves writing the metric as a fibration over a four-dimensional base, as given in (3) and (7). The expressions for the new singly-charged solution appeared in section 2, and for the previously-obtained solutions with two or three equal charges in section 5. Also in section 5, we presented the strikingly-simple expressions in this framework for the ungauged rotating black holes with three unequal charges. One may hope that the rather simple forms of all these examples may persist in the most general case with three unequal charges in gauged supergravity, which is not yet known explicitly.

It is interesting to note that all the known five-dimensional rotating black hole metrics discussed in this paper have the feature that the Hamilton-Jacobi equation and the Klein-Gordon equation in these backgrounds exhibit separability. To be precise, the massless Hamilton-Jacobi and Klein-Gordon equations are separable in all the backgrounds, and additionally, the massive equations are also separable if the three charges are equal. The determinant of the metric is given by $\sqrt{-g_5} = \frac14 (H_1H_2H_3)^{1/3}\,(x+y)$, and hence

$$
\sqrt{-g_5}\,\Big(\frac{\partial}{\partial s_5}\Big)^2 = \tfrac14\Big(\frac{\partial}{\partial \hat s_5}\Big)^2\,. \tag{35}
$$

It is straightforward to check that the components of the hatted inverse metric, which can be read of from (10), are all of the form of a sum of a function of x and a function of y

provided the charges are equal. This immediately implies a manifest separability. In the case of unequal charges one has separability only if the mass is zero, implying that an overall function of x and y can be factored out. When the charges are equal, the Hamilton-Jacobi and Klein-Gordon equations are separable both in the massless and massive cases.

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