

Charged Kerr-de Sitter Black Holes in Five Dimensions

M. Cvetič^{*1}, H. Lü^{†‡3} and C.N. Pope^{†2}

**Department of Physics and Astronomy,
University of Pennsylvania, Philadelphia, PA 19104, USA*

*†George P. and Cynthia W. Mitchell Institute for Fundamental Physics,
Texas A&M University, College Station, TX 77843-4242, USA*

*‡Interdisciplinary Center for Theoretical Study,
University of Science & Technology of China, Hefei, Anhui 230026, China*

ABSTRACT

We construct a general class of non-extremal charged Kerr-de Sitter black holes in five dimensions, in which the two rotation parameters are set equal. There are three non-trivial parameters, namely the mass, charge and angular momentum. All previously-known cases, supersymmetric and non-supersymmetric, that have equal angular momenta are encompassed as special cases.

¹ Research supported in part by DOE grant DE-FG02-95ER40893, NSF grant INT03-24081, and the Fay R. and Eugene L. Langberg Chair.

² Research supported in part by DOE grant DE-FG03-95ER40917.

³ Research supported in part by grants from the Chinese Academy of Sciences and the grants from the NSF of China with Grant No: 10245001, 90303002.

Charged black holes with non-zero cosmological constant provide important gravitational backgrounds for testing the AdS/CFT correspondence [1, 2]. In particular, for charged black holes in the anti-de Sitter background, the black hole charge plays the role of the R-charge [3] in dual field theory. In addition, the thermodynamic stability as well as analogs of the Hawking-Page phase transition for such configurations shed light [3, 4, 5] on the phase structure of the strongly coupled dual field theory. First examples of non-extremal charged black holes in five dimensions, as solutions of a gauged supergravity theory, were obtained in [6]. (For generalizations to other dimensions and their higher dimensional origin as spinning branes see [7].)

An important generalisation of static charged black holes is to allow for the rotation. While the four-dimensional charged rotating black hole solutions with non-zero cosmological constant, the Kerr-Newman-de Sitter metrics, were found long ago [8], analogs of five-dimensional solutions have not been constructed yet. General five-dimensional rotating charged black holes with two non-equal rotation parameters in the zero cosmological constant background were obtained in [9] by employing generating techniques associated with the underlying non-compact duality symmetries. Five-dimensional uncharged rotating black holes with non-zero cosmological constant, the five-dimensional Kerr-de Sitter metrics, were obtained a few years ago in [10]. In addition, certain five-dimensional extremal charged rotating solutions with non-zero cosmological constant have been found [11, 12, 13], however, the non-extremal generalisations have not yet been obtained.

The purpose of this letter is to present the solutions for general charged rotating five-dimensional black holes with a cosmological constant, in the special case where the two rotation parameters are equal. To be precise, we consider solutions of the coupled Einstein-Maxwell system that arises as the bosonic sector of minimal gauged five-dimensional supergravity, described by the Lagrangian

$$\mathcal{L} = \sqrt{-g} \left(R - 12\lambda - \frac{1}{4}F^2 + \frac{1}{12\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda \right). \quad (1)$$

Specialisations of our solutions encompass all the previously-known cases mentioned above. The general case has three independent non-trivial parameters, comprising the mass, the charge and the angular momentum.

The solutions that we have obtained take the following form:

$$ds_5^2 = - \left(1 - \Sigma \lambda r^2 - \frac{2M}{r^2} + \frac{Q^2}{r^4} \right) dt^2 + \frac{dr^2}{W} + r^2 d\Omega_3^2 - \frac{J^2}{r^4} [Q^2 - 2(M+Q)r^2] (\sin^2 \theta d\phi + \cos^2 \theta d\psi)^2$$

$$-2J \left(\lambda \beta r^2 + \frac{2M+Q}{r^2} - \frac{Q^2}{r^4} \right) dt (\sin^2 \theta d\phi + \cos^2 \theta d\psi), \quad (2)$$

$$A = \frac{\sqrt{3}Q}{r^2} [dt - J (\sin^2 \theta d\phi + \cos^2 \theta d\psi)]. \quad (3)$$

where

$$W = 1 - \lambda r^2 - \left[2M + 2\lambda J^2 (M+Q) - 2\lambda J^2 (2M+Q) \beta + 2\lambda^2 J^4 (M+Q) \beta^2 \right] \frac{1}{r^2} \\ + \left[(\lambda \beta J^2 - 1)^2 Q^2 + J^2 (\lambda Q^2 + 2(M+Q)) \right] \frac{1}{r^4}, \quad (4)$$

$$\Sigma = 1 + \lambda \beta^2 J^2, \quad (5)$$

and

$$d\Omega_3^2 = d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2 \quad (6)$$

is the metric on the unit 3-sphere. There are three non-trivial parameters, which may be taken to be (M, J, Q) , which are related to the mass, angular momentum and charge. We shall remark further on the (trivial) parameter β below.

For some purposes it is convenient to rewrite the metric (2) in terms of left-invariant 1-forms σ_i on S^3 . Defining

$$\begin{aligned} \sigma_1 &= \cos \tilde{\psi} d\tilde{\theta} + \sin \tilde{\psi} \sin \tilde{\theta} d\tilde{\phi}, \\ \sigma_2 &= -\sin \tilde{\psi} d\tilde{\theta} + \cos \tilde{\psi} \sin \tilde{\theta} d\tilde{\phi}, \\ \sigma_3 &= d\tilde{\psi} + \cos \tilde{\theta} d\tilde{\phi}, \end{aligned} \quad (7)$$

where

$$\psi - \phi = \tilde{\phi}, \quad \psi + \phi = \tilde{\psi}, \quad \theta = \frac{1}{2}\tilde{\theta}, \quad (8)$$

then we have

$$\begin{aligned} d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2 &= \frac{1}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2), \\ \sin^2 \theta d\phi + \cos^2 \theta d\psi &= \frac{1}{2}\sigma_3. \end{aligned} \quad (9)$$

The metric (2) can be written as

$$ds^2 = -\frac{r^2 W dt^2}{4b^2} + \frac{dr^2}{W} + \frac{1}{4}r^2 (\sigma_1^2 + \sigma_2^2) + b^2 (\sigma_3 + f dt)^2, \quad (10)$$

where

$$\begin{aligned} b^2 &= \frac{1}{4}r^2 \left(1 - \frac{J^2 Q^2}{r^6} + \frac{2J^2 (M+Q)}{r^4} \right), \\ f &= -\frac{J}{2b^2} \left(\lambda \beta r^2 + \frac{2M+Q}{r^2} - \frac{Q^2}{r^4} \right) \equiv \frac{U}{2b^2}. \end{aligned} \quad (11)$$

The potential (3) becomes

$$A = \frac{\sqrt{3}Q}{r^2} (dt - \frac{1}{2}J \sigma_3). \quad (12)$$

Note that the function W , given in (5), can be written as

$$W = \frac{4b^2}{r^2} (f^2 b^2 - g_{00}) = \frac{1}{r^2} (U^2 - 4b^2 g_{00}), \quad (13)$$

where g_{00} is the coefficient of dt^2 in (2).

As we remarked earlier, the parameter β is trivial. Here, we demonstrate this in the case when $Q = 0$.¹ To demonstrate its triviality when $Q = 0$, the key observation is that one can perform the coordinate transformation

$$\tilde{\psi} \longrightarrow \psi + ct \quad (14)$$

in the metric (10), which therefore has the effect of shifting the function f by the additive constant c :

$$f \longrightarrow f + c. \quad (15)$$

This allows us to eliminate β as an independent parameter, when $Q = 0$. To see this, we introduce the constant k by

$$k \equiv \frac{\sqrt{1 + 4\lambda\beta J'^2} - 1}{2\lambda\beta J'^2}, \quad (16)$$

in terms of which we define new mass and angular momentum parameters via

$$M = M'/k^2, \quad J = k J'. \quad (17)$$

It is easy to see that the metric functions W , b and f are now expressed as

$$W = 1 - \lambda r^2 - \frac{2M'(1 + \lambda J'^2)}{r^2} + \frac{2M' J'^2}{r^4}, \quad (18)$$

$$b^2 = \frac{1}{4}r^2 \left(1 + \frac{2M' J'^2}{r^4}\right), \quad (19)$$

$$f = \frac{1 - \sqrt{1 + 4\lambda\beta J'^2}}{J'} - \frac{4M' J'}{r^4 + 2M' J'^2}. \quad (20)$$

Using the constant shift transformation (15) induced by the coordinate transformation (14), we see that the first term in (20) can be removed, and hence β no longer appears in the metric. This proves that β is a trivial parameter, in this situation when $Q = 0$. See [14] for a proof that β is trivial also when $Q \neq 0$. It is nonetheless useful to retain the redundant parameter β , since this provides a convenient way to consider various limits.

¹We are grateful to Gary Gibbons and Malcolm Perry for discussion that led to this conclusion. More recently, it has been shown in [14] that β is also trivial when $Q \neq 0$.

Reductions to Previously-known Solutions:

1. In the case where the charge Q vanishes, the solutions reduce to those of Hawking, Hunter and Taylor-Robinson [10], in the special case of equal angular momenta in the two orthogonal transverse 2-planes. To be precise, if we send $r^2 \rightarrow (r^2 + J^2)/\Sigma$, $M \rightarrow M/\Sigma$, $J \rightarrow a/\Sigma$ and choose $\beta = \Sigma$, then the metric (2) reduces to that given in [10], with their rotation parameters a and b set equal.
2. If instead we take the cosmological constant λ to vanish, and set

$$\beta = c+s, \quad Q = 2\mu s c, \quad M = \mu(c^2+s^2), \quad J = (c-s)\ell, \quad r^2 \rightarrow r^2 + \ell^2 + 2\mu s^2 \quad (21)$$

where $c \equiv \cosh \delta$ and $s \equiv \sinh \delta$, then the solutions (2) and (3) reduce to a special case of those found in [9], in which the two angular momenta are set equal, $\ell_1 = \ell_2 = \ell$, and the three charges of the more general Einstein-Maxwell-Dilaton considered there are set equal. As the parameter δ is increased from 0 to ∞ , with an accompanying inverse scaling of μ so that the charge Q and mass remain finite, the solutions interpolate between zero-charge Kerr black holes and supersymmetric extremal black holes.

- 3 One can obtain BPS metrics with $\lambda \neq 0$ by taking $Q = \pm M$. These encompass the previously-known BPS solutions. Specifically, we find:

$$\text{Klemm-Sabra [11]:} \quad Q = -M, \quad \beta = 0, \quad (22)$$

$$\text{Gutowski-Reall [13]:} \quad Q = M, \quad J = \frac{1}{2}\sqrt{-\lambda} M, \quad \beta = -\frac{2}{\lambda M}. \quad (23)$$

Note that the Gutowski-Reall case, for which $\Sigma \equiv 1 + \lambda\beta^2 J^2 = 0$, requires that the cosmological constant λ be negative. The constant M is denoted by r_0^2 in [11]. In the Klemm-Sabra case, the constants M and J are denoted by q and $-a/q$ in [11].

The Gutowski-Reall solution and the Klemm-Sabra solution are both supersymmetric, within the minimal $\mathcal{N} = 2$ supergravity whose bosonic sector is described by (1). We can obtain a more general class of supersymmetric solutions with $Q = M$, generalising [13], by imposing only one additional condition, namely that $\beta^2 J^2 = -1/\lambda$. (Although trivial as a parameter in general, here β effectively parameterises a family of inequivalent BPS limits.)

Global Analysis:

Horizons occur where the $r^2 W/(4b^2)$ prefactor of the dt^2 term in (10) vanishes. Closed time-like curves (CTCs) occur if $b^2 < 0$, and so for a regular black hole with no naked

CTCs, we require that $W(r)$ vanish at some positive value of r for which b^2 is still positive as r reduces from infinity (or from the cosmological horizon if $\lambda > 0$). It follows from (13) that naked CTCs can arise if, for example, $|Q| > M$ and $\Sigma > 0$. If $|Q|$ is sufficiently small in comparison to M , these regularity conditions will be satisfied for a range of values of the parameter β . Here we present a comprehensive study of the conditions on the four parameters for a regular black hole in which there are no naked singularities or CTCs.

The properties of the general solutions are largely determined by the functions W and b^2 . The solutions are invariant under $r \leftrightarrow -r$, with a curvature singularity at the fixed point $r = 0$. Thus without loss of generality we need consider only the region $r \geq 0$. Here we shall consider in detail only the solutions with a negative cosmological constant.

For solutions to be free of naked singularities, there should exist an event horizon at $r = r_+ > 0$, which is the largest root of W . For the solutions to be free of naked CTCs, we require that $b_+^2 \equiv b(r_+)^2 > 0$, and that b^2 remain positive for all $r > r_+$. The entropy and the temperature of the solution are then given by

$$T = \frac{rW'}{8\pi b} \Big|_{r=r_+}, \quad S = \pi^2 r^2 b \Big|_{r=r_+}. \quad (24)$$

For our solutions, r_+^2 is indeed the largest root of W when we have $W'(r_+) \geq 0$. Thus, for a well-defined regular black hole, we have the following conditions:

$$r_+ > 0, \quad W(r_+) = 0, \quad b_+^2 > 0, \quad W'(r_+) \geq 0. \quad (25)$$

For our solutions, the minima of $r^4 b^2$ occur at $r < r_+$, thus the condition $b_+^2 > 0$ also ensures that $b^2 > 0$ for all $r > r_+$. To simplify the analysis, it is convenient to define dimensionless quantities (m, q, j, α) by

$$M = m r_+^2, \quad Q = q r_+^2, \quad J = j r_+, \quad \lambda = -\frac{\alpha^2}{r_+^2}. \quad (26)$$

The global structure of the uncharged solutions, which reduce to those in [10] with $a = b$, is well established. In order to study the global structure of the general solutions with $Q \neq 0$, let us assume that there exists an event horizon at $r_+ > 0$, and then we have

$$\begin{aligned} W(r_+) = & -(2m + 2q - q^2)(1 + \alpha^2 \beta j^2)^2 + 2\alpha^2 q j^2 \beta + 2(1 + \alpha^2)(m + q) j^2 + \\ & + 1 + \alpha^2 + 2q - \alpha^2 j^2 q^2 = 0. \end{aligned} \quad (27)$$

This is a quadratic equation for β . The existence of a solution for a real value of β requires that the discriminant, which is given by

$$\Delta = \frac{16\alpha^4 b_+^2 (4b_+^2 - r_+^2)^2}{r_+^6 (2m + 2q - q^2)^2} \left(2(1 + \alpha^2)(m + q) - \alpha^2 q^2 \right), \quad (28)$$

be non-negative. Here b_+ is given by

$$b_+^2 = \frac{1}{4}r_+^2 (1 + j^2(2m + 2q - q^2)). \quad (29)$$

Thus we have

$$2(1 + \alpha^2)(m + q) - \alpha^2 q^2 \geq 0, \quad 1 + j^2(2m + 2q - q^2) > 0. \quad (30)$$

For r_+ to be the event horizon, we need further to require that r_+^2 be the largest root of W . Assuming this to be the case, the temperature and entropy are given by

$$\begin{aligned} S &= \frac{1}{2}\pi^2 r_+^3 \sqrt{1 + j^2(2m + 2q - q^2)}, \\ T &= \frac{\pi r_+^2}{4(2m + 2q - q^2)S} \left(-4j^2(m + q)^2 + 2(1 + 2\alpha^2)(m + q) \right. \\ &\quad \left. - (2 + 3\alpha^2)q^2 - 2(1 + \alpha^2 j^2 \beta)q^3 \right), \end{aligned} \quad (31)$$

where β is given by (27). The requirement that r_+ be the largest root of W provides a further constraint that the right hand side of the T above is non-negative, in addition to constraints given in (30).

Since W is a cubic polynomial in the variable r^2 , there might arise three positive roots for r^2 . In this case, W' for the smallest root would also be positive, seemingly satisfying the above conditions. To examine this, we denote the other two roots by \tilde{r}_\pm^2 , and we find that

$$r_+^2 - \tilde{r}_\pm^2 = \frac{(1 + 3\alpha^2)r_+^2}{2\alpha^2} \pm \frac{r_+ \sqrt{\pi(1 + 3\alpha^2)^2 r_+^2 - 16\alpha^2 T S}}{2\alpha^2 \sqrt{\pi}}. \quad (32)$$

If $TS > \pi(1 + 3\alpha^2)r_+^2/(16\alpha^2)$, then the function W has only one real root for r^2 , namely r_+^2 . If on the other hand, $0 \leq TS \leq \pi(1 + 3\alpha^2)r_+^2/(16\alpha^2)$, there can be two additional roots, but r_+^2 is the largest one. In particular, when $T = 0$, r_+ and \tilde{r}_- coincide, implying that W has a second-order zero, as one would expect.

Note that the temperature and entropy calculation above becomes singular when m and q are related by $m = \frac{1}{2}q^2 - q$. In this case, we have

$$b_+^2 = \frac{1}{4}r_+^2 > 0. \quad (33)$$

In fact it is easy to see that b^2 is positive-definite for $r \geq r_+$, implying the absence of naked CTCs. The existence of a horizon requires that

$$\beta = -\frac{1 + \alpha^2 + 2q + j^2 q^2}{2\alpha^2 j^2 q}. \quad (34)$$

The temperature and the entropy are given by

$$S = \frac{1}{2}\pi^2 r_+^3, \quad T = \frac{1}{8\pi r_+} \left(-j^4 q^4 - 2(3 - \alpha^2)j^2 q^2 - \alpha^4 + 6\alpha^2 + 3 \right). \quad (35)$$

Note that in this case, the entropy is independent of any dimensionless parameters. The requirement $W'(r_+) \geq 0$ implies that

$$-3 - 2\sqrt{3} + \alpha^2 \leq j^2 q^2 \leq -3 + 2\sqrt{3} + \alpha^2. \quad (36)$$

The case of extremality, $T = 0$, occurs at the equalities of the above. Note that as a function of $j^2 q^2$, the temperature lies in the range $0 \leq T \leq 3/(2\pi r_+)$.

In the above analysis, we started with the conditions (25), with β being arbitrary. For special solutions such as those in [10] ($\beta = 1$) and [13] ($\beta = 1/(\alpha j)$), it would become cumbersome to extract the conclusions from our final results. In these cases, it is more convenient to discuss directly the condition (25), with the specific β values substituted in.

The existence of Killing spinors in these solutions does not in general preclude the possibility of having naked CTCs. The $Q = -M$ solutions, which include the Klemm-Sabra specialisation when $\beta = 0$, have naked CTCs for all values of β . This can be easily seen from the fact that when $Q = -M$, we have

$$W = \frac{(r^2 - (1 - \lambda\beta J^2)Q)^2}{r^4} - 4\lambda b^2. \quad (37)$$

Thus, for negative cosmological constant λ , the horizon, $W = 0$, occurs for $b^2 < 0$, only.

For the other branch of the supersymmetric solution, where $M = Q$ and $\Sigma = 0$, we have $(-g_{00}) = (1 - Q/r^2)^2$, which is non-negative. It follows from (13) that the horizon at $W = 0$ occurs where $b^2 < 0$, unless $U = 0$ at $r^2 = Q$. This condition is uniquely satisfied by the Gutowski-Reall solution (23). The conditions for extremality ($T = 0$) and supersymmetry of our solutions do not necessarily coincide. The supersymmetric solutions we discussed above have two parameters Q and J , and these black holes in general have non-vanishing temperature. This result contradicts the assumption about the stability of configurations that is normally associated with supersymmetry. However, as we saw above, the supersymmetric black holes with non-vanishing temperature all have naked CTCs. The one that does not have naked CTCs, found in (23), indeed has zero temperature.

To conclude, we have obtained a general class of five-dimensional de Sitter charged rotating black hole solutions in which the two angular momenta are set equal. The solutions depend on three non-trivial parameters, namely the mass, the angular momentum and the charge. All previously-known cases, extremal and non-extremal, supersymmetric or non-supersymmetric, that have equal momenta are encompassed as special cases. We analysed

the conditions under which one obtains black holes with no naked singularities or closed time-like curves. We also found that the supersymmetry condition does not in general imply zero temperature. However, the supersymmetric solutions with non-vanishing temperature all have naked closed time-like curves. The only one [13] with no naked closed time-like curves indeed has zero temperature.

The general class of charged AdS₅ black holes presented in this paper provides a fertile ground to further study their thermodynamics, stability and phase transitions as well as having other implications for the AdS₅/CFT₄ correspondence. In particular, it would be interesting to study further the connection between thermodynamics and the closed time-like curves from the point of view of $D = 4$ super-conformal field theory.

Further generalizations of such rotating configurations to two non-equal rotation parameters as well as to three non-equal charges of five-dimensional gauged supergravity with $U(1)$ gauged isometry, are other important directions [15]. In addition, it would be interesting to generalise these solutions to charged Kerr-de Sitter solutions in higher dimensions, thus extending the general spinning charged black holes with zero cosmological constant [16] and the general Kerr-de Sitter solutions, obtained recently in [17], to charged black holes in de Sitter backgrounds.

Acknowledgment

We are grateful to Zhi-Wei Chong, Gary Gibbons and Malcolm Perry for useful discussions. We are grateful to the Relativity Group in DAMTP, Cambridge (M.C. and C.N.P.) and CERN theory Division (M.C.) and ICTS at USTC, Heifei (H.L.) for the hospitality.

Note Added

In an earlier version of this paper (the one published in Phys. Lett. **B598**, 273 (2004)), it was wrongly stated that the parameter β was non-trivial when $Q \neq 0$. This has recently been corrected by S.F. Ross and O. Madden [14].

References

- [1] J.M. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2**, 231 (1998), hep-th/9711200.

- [2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, Phys. Lett. **B428**, 105 (1998), hep-th/9802109.
- [3] M. Cvetič and S.S. Gubser, *Phases of R-charged black holes, spinning branes and strongly coupled gauge theories*, JHEP **9904**, 024 (1999), hep-th/9902195.
- [4] M. Cvetič and S.S. Gubser, *Thermodynamic stability and phases of general spinning branes*, JHEP **9907**, 010 (1999), hep-th/9903132.
- [5] A. Chamblin, R. Emparan, C.V. Johnson and R.C. Myers, *Charged AdS black holes and catastrophic holography*, Phys. Rev. **D60**, 064018 (1999), hep-th/9902170.
- [6] K. Behrndt, M. Cvetič and W.A. Sabra, *Non-extreme black holes of five dimensional $N = 2$ AdS supergravity*, Nucl. Phys. **B553**, 317 (1999), hep-th/9810227.
- [7] M. Cvetič, M.J. Duff, P. Hoxha, James T. Liu, H. Lü, J.X. Lu, R. Martinez-Acosta, C.N. Pope, H. Sati and T.A. Tran, *Embedding AdS black holes in ten and eleven dimensions*, Nucl. Phys. **B558**, 96 (1999), hep-th/9903214.
- [8] B. Carter, *Separability of the Killing Maxwell system underlying the generalized angular momentum constant in the Kerr-Newman black hole metrics*, J. Math. Phys. **28** (1987) 1535.
- [9] M. Cvetič and D. Youm, *General rotating five dimensional black holes of toroidally compactified heterotic string*, Nucl. Phys. **B476**, 118 (1996), hep-th/9603100.
- [10] S.W. Hawking, C.J. Hunter and M.M. Taylor-Robinson, *Rotation and the AdS/CFT correspondence*, Phys. Rev. **D59**, 064005 (1999), hep-th/9811056.
- [11] D. Klemm and W.A. Sabra, *Charged rotating black holes in 5d Einstein-Maxwell-(A)dS gravity*, Phys. Lett. **B503**, 147 (2001), hep-th/0010200.
- [12] D. Klemm and W.A. Sabra, *General (anti-)de Sitter black holes in five dimensions*, JHEP **0102**, 031 (2001), hep-th/0011016.
- [13] J.B. Gutowski and H.S. Reall, *Supersymmetric AdS(5) black holes*, JHEP **0402**, 006 (2004), hep-th/0401042.
- [14] O. Madden and S.F. Ross, *On uniqueness of charged Kerr-AdS black holes in five dimensions*, hep-th/0409193.

- [15] M. Cvetič, H. Lü and C.N. Pope, *Charged rotating black holes in five-dimensional $U(1)^3$ gauged $\mathcal{N} = 2$ supergravity*, hep-th/0407058, to appear in Phys. Rev **D**.
- [16] M. Cvetič and D. Youm, *Near-BPS-saturated rotating electrically charged black holes as string states*, Nucl. Phys. **B477**, 449 (1996), hep-th/9605051.
- [17] G.W. Gibbons, H. Lü, D.N. Page and C.N. Pope, *The general Kerr-de Sitter metrics in all dimensions*, hep-th/0404008, to appear in J. Geom. Phys.