KUL-TF-03-04 MIFP-03-02 ULB-TH/03-03
March 2003 **hep-th/0303211**

A Supersymmetric and Smooth Compactification of M-theory to AdS_5

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ABSTRACT

We obtain smooth M-theory solutions whose geometry is a warped product of AdS_5 and a compact internal space that can be viewed as an S^4 bundle over S^2. The bundle can be trivial or twisted, depending on the even or odd values of the two diagonal monopole charges. The solution preserves $\mathcal{N} = 2$ supersymmetry and is dual to an $\mathcal{N} = 1$ $D = 4$ superconformal field theory, providing a concrete framework to study the AdS_5/CFT_4 correspondence in M-theory. We construct analogous embeddings of AdS_4, AdS_3 and AdS_2 in massive type IIA, type IIB and M-theory, respectively. The internal spaces have generalized holonomy and can be viewed as S^n bundles over S^2 for $n = 4$, 5 and 7. Surprisingly, the dimensions of spaces with generalized holonomy includes $D = 9$. We also obtain a large class of solutions of $AdS \times H^2$.

[1] Research supported in part by the Federal Office for Scientific, Technical and Cultural Affairs through the Interuniversity Attraction Pole P5/27 and the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime.

[2] Research supported in part by DOE grant DE-FG03-95ER40917.

[3] Research supported in part by the Francqui Foundation (Belgium), the Actions de Recherche Concertées of the Direction de la Recherche Scientifique - Communauté Francaise de Belgique, IISN-Belgium (convention 4.4505.86) and by a "Pole d'Attraction Interuniversitaire."

1 Introduction

AdS_5 spacetime arises naturally in type IIB supergravity, which provides a non-trivial and relatively simple framework for examining the holographic principle *via* the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence [1, 2, 3]. The embedding of AdS_5 spacetime in eleven-dimensional supergravity has also been studied in the past. A smooth but non-supersymmetric compactification of eleven-dimensional supergravity to AdS_5 was obtained in [4] where the internal space is a Kähler manifold. More recently, an internal background of $\mathbb{CP}^2 \times T^2$ was found in [5]. Although the compactification is not supersymmetric at the level of supergravity, it was argued in [5] that it is fully supersymmetric at the level of M-theory, since it is T-dual to the $\mathrm{AdS}_5 \times S^5$ of type IIB theory. In the above two examples, the AdS spacetime and internal manifold are direct products without warp factors. Smooth but non-supersymmetric M-theory solutions have been constructed in [6], which are warped and twisted products of $\mathrm{AdS}_5 \times S^2$ or $\mathrm{AdS}_5 \times H^2$ with a squashed four-sphere.

In [7, 8], supersymmetric embeddings of AdS_5 in M-theory were found as warped geometries with a compact internal metric. This construction can be understood from that the fact that S^5 can be expressed as a foliation of S^3 and S^1. One can then T-dualize the $\mathrm{AdS}_5 \times S^5$ of type IIB theory on the $U(1)$ bundle of the S^3 and obtain a solution in M-theory [9]. However, there is a naked singularity in such a construction, since the $U(1)$ circle of the S^3 can shrink to zero. Supersymmetric and smooth embeddings of AdS_5 in M-theory were obtained in [10]. The eleven-dimensional metric is a warped product of AdS_5 with an internal metric that can be viewed as an S^4 bundle over H^2, a hyperbolic 2-plane. The construction can give rise to both $\mathcal{N} = 2$ and $\mathcal{N} = 4$ supersymmetry.

In this letter, we report a supersymmetric and smooth compactification of M-theory to AdS_5, with the internal space being an S^4 bundle over S^2. The construction is only possible for $\mathcal{N} = 2$ supersymmetry, and hence it gives rise to the minimum AdS_5 gauged supergravity coupled to matter. This solution provides a supergravity dual to $\mathcal{N} = 1$ $D = 4$ superconformal field theory. We also obtain supersymmetric and smooth compactifications of M-theory to AdS_2 and type IIB to AdS_3. The internal space is an S^p bundle over S^2, where $p = 7$ and 5, respectively. We also construct a supersymmetric compactification of massive IIA to AdS_4, which is singular.

2 $AdS_5 \times S^2$ in M-theory

We begin by considering the sector of $D = 7$ gauged supergravity with two diagonal $U(1)$ vector fields. The relevant Lagrangian is given by

$$\hat{e}^{-1}\mathcal{L}_7 = \hat{R} - \tfrac{1}{2}(\partial\phi_1)^2 - \tfrac{1}{2}(\partial\phi_2)^2 - \hat{V} - \tfrac{1}{4}\sum_{i=1}^{2} X_i^{-2}\,(\hat{F}^i_{(2)})^2\,, \tag{2.1}$$

where $X_i = e^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}$ with

$$\vec{a}_1 = (\sqrt{2}\,, \sqrt{\tfrac{2}{5}})\,, \qquad \vec{a}_2 = (-\sqrt{2}\,, \sqrt{\tfrac{2}{5}})\,. \tag{2.2}$$

The scalar potential $\hat{V}$ is given by [11]

$$\hat{V} = g^2\,(-4X_1\,X_2 - 2X_0\,X_1 - 2X_0\,X_2 + \tfrac{1}{2}X_0^2)\,, \tag{2.3}$$

where $X_0 = (X_1\,X_2)^{-2}$. The potential can be expressed in terms of the superpotential

$$\hat{W} = \frac{g}{\sqrt{2}}\,(X_0 + 2X_1 + 2X_2)\,. \tag{2.4}$$

We now consider a 3-brane ansatz

$$\begin{aligned} ds^2 &= e^{2u}\,dx^\mu\,dx_\mu + e^{2v}\,\lambda^{-2}\,d\Omega_2^2 + d\rho^2\,,\\ F^i_{(2)} &= \epsilon\,m_i\,\lambda^{-2}\,\Omega_{(2)}\,, \end{aligned} \tag{2.5}$$

where the constant ϵ takes the values 1, -1 and 0, if $d\Omega_2^2$ is the metric for a unit S^2, hyperbolic H^2 or 2-torus T^2. $\Omega_{(2)}$ is the corresponding volume form. The system admits the following first-order equations

$$\begin{aligned} \frac{d\vec{\phi}}{d\rho} &= \sqrt{2}\Big(-\frac{\epsilon}{2\sqrt{2}}\,(m_1\,\vec{a}_1\,X_1^{-1} + m_2\,\vec{a}_2\,X_2^{-1})\,e^{-2v} + \frac{d\hat{W}}{d\vec{\phi}}\Big)\,,\\ \frac{dv}{d\rho} &= -\frac{1}{5\sqrt{2}}\Big(2\sqrt{2}\,\epsilon\,(m_1\,X_1^{-1} + m_2\,X_2^{-1})\,e^{-2v} + \hat{W}\Big)\,,\\ \frac{du}{d\rho} &= \frac{1}{5\sqrt{2}}\Big(\frac{\epsilon}{\sqrt{2}}\,(m_1\,X_1^{-1} + m_2\,X_2^{-1})\,e^{-2v} - \hat{W}\Big)\,, \end{aligned} \tag{2.6}$$

provided that the constraint

$$\lambda^2 = (m_1 + m_2)\,g \tag{2.7}$$

is satisfied. This set of first-order equations were derived for the case of H^2 in [10] by studying the Killing spinor equations of $D = 7$ gauged supergravity. In [12], a different method was used to obtain them for H^2, S^2 and T^2. The equations of (2.6) were analysed in detail in [12]. Here, we report on only a subclass of solutions where $\vec{\phi}$ and v are constants. In this case, for $\epsilon = 0$, the solution is nothing but AdS$_7$ written in Poincaré coordinates.

For $\epsilon = \pm 1$, we find that the solution is given by

$$
\begin{aligned}
e^{\sqrt{2}\phi_1} &= \frac{m_2 - m_1 \pm \sqrt{m_2^2 + m_1^2 - m_1\, m_2}}{m_2}\,, \qquad e^{-\sqrt{\frac{5}{2}}\,\phi_2} = \tfrac{4}{3}\cosh(\phi_1/\sqrt{2})\,,\\
e^{-2v} &= -\frac{\epsilon\, g\, e^{-\frac{3}{\sqrt{10}}\phi_2}}{m_1\, e^{-\frac{\phi_1}{\sqrt{2}}} + m_2\, e^{\frac{\phi_1}{\sqrt{2}}}}\,, \qquad u = -\tfrac{1}{2} g\, e^{-\frac{4}{\sqrt{10}}\phi_2}\,\rho\,.
\end{aligned}
\tag{2.8}
$$

This solution is invariant under the simultaneous interchanges of $m_1 \leftrightarrow m_2$ and $\phi_1 \leftrightarrow -\phi_1$. The reality conditions of the solution constrain the constants m_i and g, as well as the choice of $\pm$ in the solution. Let us first consider the case $\epsilon = -1$, corresponding to $d\Omega_2^2$ as the metric of a unit (non-compact) hyperbolic 2-plane. In this case, the reality of the solution implies that $m_1\, m_2 \ge 0$. This includes the choice of $m_1 = 0$ (or $m_2 = 0$) and $m_1 = m_2$, which were discussed in [10]. The first case gives rise to $\mathcal{N} = 4$ supersymmetry in $D = 5$, whilst the second case gives rise to $\mathcal{N} = 2$ supersymmetry.

We are particularly interested in a compact internal space. Thus, we now turn to the choice of $\epsilon = +1$, corresponding to $d\Omega_2^2$ as the metric of S^2. In this case, the reality conditions for (2.8) imply that $m_1\, m_2 < 0$. The condition (2.7) implies further that $m_1 \ne -m_2$. Therefore, the AdS$_5 \times S^2$ solution can only have $\mathcal{N} = 2$ supersymmetry, but cannot arise from the pure $D = 7$ minimal gauged supergravity.

If we define a charge parameter $q = 2m_1/(m_1 + m_2)$, then the condition for having S^2 versus H^2 can be summarised as

$$
\begin{aligned}
q \in [0, 2] \quad &\Longrightarrow \quad H^2\,,\\
q \in (-\infty, 0) \text{ or } (2, \infty) \quad &\Longrightarrow \quad S^2\,.
\end{aligned}
\tag{2.9}
$$

It is straightforward to lift this solution to $D = 11$ by using the ansatz obtained in [11]. Since the solutions for general m_i are rather complicated to present, we only consider a representative example with $m_1 = 5g$ and $m_2 = -3g$. The M-theory metric is given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{1}{3}} \Big[ds_{\rm AdS_5}^2 + \frac{1}{g^2\, c} \Big\{ \frac{1}{4c} (d\theta^2 + \sin^2\theta\, d\varphi^2) \\ &\quad + \frac{1}{\Delta} \Big(\tfrac{1}{4}\, d\mu_0^2 + \tfrac{1}{5} (d\mu_1^2 + \mu_1^2\, (d\phi_1 - \tfrac{5}{2} \cos\theta\, d\varphi)^2) \\ &\quad + d\mu_2^2 + \mu_2^2\, (d\phi_2 + \tfrac{3}{2} \cos\theta\, d\varphi)^2 \Big) \Big\} \Big] \,, \end{aligned} \tag{2.10}$$

where $c = 10^{-2/5}$ and μ_i are spherical coordinates which satisfy $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$. The warp factor Δ is given by

$$\Delta = c\, (4\mu_0^2 + 5\mu_1^2 + \mu_2^2) > 0 \,. \tag{2.11}$$

The AdS_5 metric is given by

$$ds_{\rm AdS_5}^2 = e^{-\frac{2\rho}{R}}\, dx^\mu\, dx_\mu + d\rho^2 \,, \tag{2.12}$$

where the AdS radius is given by $R = \frac{1}{2c\, g}$. The 4-form field strength in $D = 11$ can also be obtained using the reduction ansatz in [11]. It is given by

$$\begin{aligned} {*F}_{(4)} &= -(2\, g)^{-1}\, (8\mu_0^2 + 15\mu_1^2 + 7\mu_2^2)\, \epsilon_{(5)} \wedge \sin\theta\, d\theta \wedge d\varphi \\ &\quad + g^{-1} \Big(\tfrac{1}{5} d(\mu_1^2) \wedge (d\phi_1 - \tfrac{5}{2} \cos\theta\, d\varphi) - 3 d(\mu_2^2) \wedge (d\phi_2 + \tfrac{3}{2} \cos\theta\, d\varphi) \Big) \wedge \epsilon_{(5)} \,. \end{aligned} \tag{2.13}$$

where $\epsilon_{(5)}$ is the volume form for the AdS_5 metric.

Thus, the internal space of the $D = 11$ metric can be viewed as an S^4 bundle over S^2, with two diagonal $U(1)$ bundles. In general, the internal metric can be labeled by $(q_1, q_2) = (\frac{2m_1}{m_1+m_2}, \frac{2m_2}{m_1+m2})$. In the specific example above, $(q_1, q_2) = (5, -3)$ and the solution is smooth everywhere. For general (m_1, m_2), the metric does not have a power-law singularity. However, it could have a conical orbifold singularity, which is absent only if (q_1, q_2) are integers. Since the q_i satisfy the constraint $q_1 + q_2 = 2$, it follows that they are either both even or both odd integers. In the even case, the bundle is topologically trivial, whilst it is twisted for the odd case.

3 $\mathrm{AdS}_4 \times S^2$ in massive type IIA

The scalar potential in gauged supergravity with two $U(1)$ isometries was obtained in [13]. From this, we deduce that the relevant Lagrangian involving the two $U(1)$

vector fields is given by

$$\hat{e}^{-1}\mathcal{L}_6 = \hat{R} - \tfrac{1}{2}(\partial\phi_1)^2 - \tfrac{1}{2}(\partial\phi_2)^2 - \hat{V} - \tfrac{1}{4}\sum_{i=1}^{2} X_i^{-2}\,(\hat{F}^i_{(2)})^2\,, \qquad (3.1)$$

where $X_i = e^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}$ with

$$\vec{a}_1 = (\sqrt{2}\,, \frac{1}{\sqrt{2}})\,, \qquad \vec{a}_2 = (-\sqrt{2}\,, \frac{1}{\sqrt{2}})\,. \qquad (3.2)$$

The scalar potential is given by

$$\hat{V} = \tfrac{4}{9}g^2\,(X_0^2 - 9X_1\,X_2 - 6X_0\,X_1 - 6X_0\,X_2)\,, \qquad (3.3)$$

where $X_0 = (X_1\,X_2)^{-3/2}$. As in the previous case, the scalar potential can be expressed in terms of the superpotential

$$\hat{W} = \frac{g}{\sqrt{2}}\,(\tfrac{4}{3}X_0 + 2X_1 + 2X_2)\,. \qquad (3.4)$$

We consider a membrane solution of the type given by (2.5). The system admits the following first-order equations

$$\begin{aligned}
\frac{d\vec{\phi}}{d\rho} &= \sqrt{2}\Big(-\frac{\epsilon}{2\sqrt{2}}\,(m_1\,\vec{a}_1\,X_1^{-1} + m_2\,\vec{a}_2\,X_2^{-1})\,e^{-2v} + \frac{d\hat{W}}{d\vec{\phi}}\Big)\,,\\
\frac{dv}{d\rho} &= -\frac{1}{4\sqrt{2}}\Big(\frac{3}{\sqrt{2}}\,\epsilon\,(m_1\,X_1^{-1} + m_2\,X_2^{-1})\,e^{-2v} + \hat{W}\Big)\,,\\
\frac{du}{d\rho} &= \frac{1}{4\sqrt{2}}\Big(\frac{\epsilon}{\sqrt{2}}\,(m_1\,X_1^{-1} + m_2\,X_2^{-1})\,e^{-2v} - \hat{W}\Big)\,, \qquad (3.5)
\end{aligned}$$

provided that the constraint $\lambda^2 = (m_1+m_2)g$ is satisfied. The solutions were analysed in detail in [12]. Here, we shall only consider the subset of solutions with constant scalars. For $\epsilon = 0$, one just reproduces the AdS$_6$ metric in Poincaré coordinates. For $\epsilon = \pm 1$, we have

$$\begin{aligned}
\mathrm{e}^{\sqrt{2}\phi_1} &= \frac{3}{2}\,\frac{m_2 - m_1 \pm \sqrt{(m_2 - m_1)^2 + \frac{4}{9}m_1\,m_2}}{m_2}\,, \qquad \mathrm{e}^{-\sqrt{2}\phi_2} = \frac{3}{2}\cosh(\frac{\phi_1}{\sqrt{2}})\,,\\
\mathrm{e}^{-2v} &= -\frac{4\epsilon\,g\,\mathrm{e}^{-\frac{\phi_2}{\sqrt{2}}}}{m_1\,\mathrm{e}^{-\frac{\phi_1}{\sqrt{2}}} + m_2\,\mathrm{e}^{\frac{\phi_1}{\sqrt{2}}}}\,, \qquad u = -\frac{g}{3}\mathrm{e}^{-\frac{3}{\sqrt{8}}\phi_2}\,\rho\,. \qquad (3.6)
\end{aligned}$$

As in the $D = 7$ result, we can define a charge parameter $q = \frac{2m_1}{m_1+m_2}$. We have H^2 or S^2 depending on the following condition:

$$\begin{aligned}
q \in [0,2] &\implies H^2\,,\\
q \in (-\infty,0) \text{ or } (2,\infty) &\implies S^2\,. \qquad (3.7)
\end{aligned}$$

When $q=0$ or $q=2$, the system has $\mathcal{N}=4$ supersymmetry. Otherwise, we have $\mathcal{N}=2$ supersymmetry.

Using the reduction ansatz in [13, 14], it is straightforward to lift the solution back to $D=10$, giving rise to a solution of massive type IIA supergravity. The metric is given by

$$\begin{aligned} ds_{10}^2 &= \mu_0^{\frac{1}{12}}\, X_0^{\frac{1}{8}}\, (X_1\, X_2)^{\frac{1}{4}}\, \Delta^{\frac{3}{8}} \Big[ds_6^2 + g^{-2}\,\Delta^{-1}\Big(X_0^{-1}\, d\mu_0^2 \\ &\quad + X_1^{-1}\, (d\mu_1^2 + \mu_1^2\, (d\varphi_1 + g\, A^2_{\scriptscriptstyle (1)})^2) + X_2^{-1}\, (d\mu_2^2 + \mu_2^2\, (d\varphi_2 + g\, A^1_{\scriptscriptstyle (2)})^2)\Big)\Big]\,, \end{aligned} \tag{3.8}$$

where $\Delta = \sum_{\alpha=0}^2 X_\alpha\, \mu_\alpha^2 > 0$ and $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$. Thus, the $D=10$ metric is a warped product of AdS_4 with an internal six-metric, which is an S^4 bundle over S^2 or H^2, depending on the charge parameter p, according to the rule (3.7).

As an example of a supersymmetric, though singular, compactification of AdS_4 from massive IIA, we can take $m_1 = 7g$ and $m_2 = -5g$, and a choice of negative sign in (3.6). This gives $X_0 = 6c$, $X_1 = 7c$ and $X_2 = c$, where $c = 6^{-1/4}\, 7^{-3/8}$. Also, $A^1_{\scriptscriptstyle (1)} = -\frac{7}{2g} \cos\theta\, d\varphi$ and $A^2_{\scriptscriptstyle (1)} = \frac{5}{2g} \cos\theta\, d\varphi$, and the radius of AdS_4 is given by $R = 1/(2c\, g)$.

4 $\mathrm{AdS}_3 \times S^2$ in type IIB

Let us now consider the $D=5$ minimal gauged supergravity coupled to two vector multiplets. The Lagrangian is given by

$$e^{-1}\mathcal{L}_5 = \hat{R} - \tfrac{1}{2}(\partial\phi_1)^2 - \tfrac{1}{2}(\partial\phi_2)^2 - \frac{1}{4}\sum_{i=1}^{3} X_i^{-2} (\hat{F}^i_{\scriptscriptstyle (2)})^2 - \hat{V} + e^{-1}\, \tfrac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda}\, \hat{F}^1_{\mu\nu}\, \hat{F}^2_{\rho\sigma}\, \hat{A}^3_\lambda\,, \tag{4.1}$$

where $X_i = e^{\frac{1}{2}\vec{a}_i\cdot\vec{\phi}}$ with

$$\vec{a}_1 = (\sqrt{2}\,, \frac{2}{\sqrt{6}})\,, \qquad \vec{a}_2 = (-\sqrt{2}\,, \frac{2}{\sqrt{6}})\,, \qquad \vec{a}_3 = (0, -\frac{4}{\sqrt{6}})\,. \tag{4.2}$$

The scalar potential is given by

$$\hat{V} = -4g^2 \sum_{i=1}^{3} X_i^{-1}\,. \tag{4.3}$$

The scalar potential $\hat{V}$ can also be expressed in terms of the superpotential

$$\hat{W} = \sqrt{2}\, g\, \sum_i X_i\,. \tag{4.4}$$

We find that the string solution of the type given by (2.5) admits the following first-order equations

$$\begin{aligned}\frac{d\vec\phi}{d\rho} &= \sqrt{2}\Big(-\frac{\epsilon}{2\sqrt2}(m_1\,\vec a_1\,X_1^{-1}+m_2\,\vec a_2\,X_2^{-1}+m_3\,\vec a_3\,X_3^{-1})\,e^{-2v}+\frac{d\hat W}{d\vec\phi}\Big)\,,\\ \frac{dv}{d\rho} &= -\frac{1}{3\sqrt2}\Big(\sqrt2\,\epsilon\,(m_1\,X_1^{-1}+m_2\,X_2^{-1}+m_3\,X_3^{-1})\,e^{-2v}+\hat W\Big)\,,\\ \frac{du}{d\rho} &= \frac{1}{3\sqrt2}\Big(\frac{\epsilon}{\sqrt2}(m_1\,X_1^{-1}+m_2\,X_2^{-1}+m_3\,X_3^{-1})\,e^{-2v}-\hat W\Big)\,,\end{aligned}\tag{4.5}$$

provided that the constraint $\lambda^2=(m_1{+}m_2{+}m_3)g$ is satisfied. The general solution for this system was analysed in [12]. Here, we consider only the solutions with constant scalar fields. For $\epsilon=0$, the solution is AdS_5 in Poincaré coordinates. For $\epsilon=\pm1$, fixed-point solutions exist only for non-vanishing m_i. The solution is given by

$$\begin{aligned}e^{\sqrt2\,\phi_1} &= \frac{m_1}{m_2}\Big(\frac{m_3+m_2-m_1}{m_3-m_2+m1}\Big)\,,\qquad e^{\sqrt6\,\phi_2}=\frac{m_1\,m_2\,(m_3^2-(m_1-m_2)^2)}{m_3^2\,(m_1+m_2-m_3)^2}\,,\\ e^{-2v} &= -\epsilon\,g\,\Big(\frac{(m_1+m_2-m_3)(m_3^2-(m_1-m_2)^2)}{m_1^2\,m_2^2\,m_3^2}\Big)^{\frac13}\,,\\ u &= -g\,e^{\frac{\phi_2}{\sqrt6}}\,\Big(\cosh(\phi_1/\sqrt2)+\tfrac12 e^{-\sqrt{\frac32}\,\phi_2}\Big)\,\rho\,.\end{aligned}\tag{4.6}$$

The reality condition of the solution implies that when three vectors with the magnitudes $|m_i|$ can form a triangle, $d\Omega_2^2$ should be the H^2 metric. On the other hand, when they cannot form a triangle, the metric should be that of S^2.[1] If any of the m_i vanish, there is no fixed-point solution, except when one vanishes with the remaining two equal [12].

Using the reduction ansatz obtained in [11], we can easily lift the solution back to $D=10$. Since the solution with general m_i is complicated to present, we consider a simpler case with $m_2=m_1$. The $D=10$ type IIB metric is

$$\begin{aligned}ds_{10}^2 &= \sqrt{\Delta}\Big\{ds^2_{\mathrm{AdS}_3}+\epsilon\,g^{-2}(\frac{m_1}{m_3-2m_1})^{1/3}\,(\tfrac12 q_1\,d\Omega_2^2+d\theta^2)\\ &\quad+g^{-2}\,\Delta^{-1}\Big[c^{-1/3}\,\cos^2\theta\,\Big(d\psi^2+\sin^2\psi\,(d\varphi_1+\tfrac12 q_1\,A_{(1)})^2\\ &\quad+\cos^2\psi\,(d\varphi_2+\tfrac12 q_1\,A_{(1)})^2\Big)+c^{2/3}\,\sin^2\theta\,(d\varphi_3+\tfrac12 q_3\,A_{(1)})^2\Big]\Big\}\,,\end{aligned}\tag{4.7}$$

where

$$c=\Big|\frac{m_1}{2m_1-m_3}\Big|\,,\qquad \Delta=c^{1/3}\,\cos^2\theta+c^{-2/3}\sin^2\theta>0\,,\qquad dA_{(1)}=\Omega_{(2)}\,,$$

[1] $\mathrm{AdS}_3\times S^2$ solutions were also recently found in [15] in a different construction.

$$ds^2_{\rm AdS_3} = e^{-\frac{2\rho}{R}}\,(-dt^2+dx^2)+d\rho^2\,,\qquad R=\Big|\frac{2m_1}{g\,(4m_1-m_3)\,c^{1/3}}\Big| \tag{4.8}$$

We have introduced the charge parameters $q_i = 2m_i/(m_1+m_2+m_3)$, and hence they satisfy the constraint $q_1+q_2+q_3=2$. In the above solution, if $|m_3|<2|m_1|$, we should have $\epsilon=-1$, corresponding to H^2; if $|m_3|>2|m_1|$, we should have $\epsilon=1$, corresponding to S^2. In general, the internal metric is an S^5 bundle over S^2 or H^2, depending the values of the q_i according to the above rules.

5 $\mathbf{AdS_2}\times S^2$ in M-theory

Let us now consider the $U(1)^4$ gauged $N=2$ supergravity in four dimensions. The Lagrangian is given by

$$e^{-1}\mathcal{L}_4 = \hat R - \tfrac12(\partial\phi_1)^2 - \tfrac12(\partial\phi_2)^2 - \tfrac12(\partial\phi_3)^2 - \frac14\sum_{i=1}^4 X_i^{-2}(\hat F^i_{(2)})^2 - \hat V\,, \tag{5.1}$$

where $X_i = e^{\frac12\vec a_i\cdot\vec\phi}$ with

$$\vec a_1=(1,1,1),\quad \vec a_2=(1,-1,-1),\quad \vec a_3=(-1,1,-1),\quad \vec a_4=(-1,-1,1)\,. \tag{5.2}$$

The scalar potential is given by

$$\hat V = -4g^2\sum_{i<j} X_i\,X_j\,, \tag{5.3}$$

which can also be expressed in terms of the superpotential

$$\hat W = \sqrt2\,g\,\sum_{i=1}^4 X_i\,. \tag{5.4}$$

The magnetic black hole solution of the type given by (2.5) admits the following first-order equations

$$\begin{aligned}
\frac{d\vec\phi}{d\rho} &= \sqrt2\Big(-\frac{\epsilon}{2\sqrt2}\sum_{i=1}^4 m_i\,\vec a_i\,X_i^{-1}\,e^{-2v}+\frac{d\hat W}{d\vec\phi}\Big)\,,\\
\frac{dv}{d\rho} &= -\frac{1}{2\sqrt2}\Big(\frac{\epsilon}{\sqrt2}\sum_{i=1}^4 m_i\,X_i^{-1}\,e^{-2v}+\hat W\Big)\,,\\
\frac{du}{d\rho} &= \frac{1}{2\sqrt2}\Big(\frac{\epsilon}{\sqrt2}\sum_{i=1}^4 m_i\,X_i^{-1}\,e^{-2v}-\hat W\Big)\,,
\end{aligned} \tag{5.5}$$

provided that the constraint $\lambda^2 = g\sum_{i=1}^4 m_i$ is satisfied. The general solution for this system was analysed in [12]. Here, we consider only solutions with constant scalar fields. For $\epsilon = 0$, the solution is AdS_4 in Poincaré coordinates. For $\epsilon = \pm 1$, we have not obtained the general solution for abitrary m_i, although we have found many examples of specific solutions. We do find a class of special solutions by setting $m_2 = m_3 = m_4$. This enables us to consistently set $\phi_1 = \phi_2 = \phi_3 \equiv \phi$. For this truncation, fixed-point solutions for $\epsilon = \pm 1$ are given by

$$\begin{aligned} e^{2\phi} &= \frac{3m_2 - m_1 \pm \sqrt{(m_1-m_2)(m_1-9m_2)}}{2m_2}\,, \qquad e^{-2v} = 4g\,\epsilon\,\frac{\sinh\phi}{m_1\,e^{-2\phi} - m_2}\,,\\ u &= -\tfrac12 g\Big(\frac{2(m_1\,e^{-\frac32\phi} + 3m_2\,e^{\frac12\phi})}{m_2 - m_1\,e^{-2\phi}}\sinh\phi + e^{\frac32\phi} + 3e^{-\frac12\phi}\Big)\,\rho\,. \end{aligned} \tag{5.6}$$

The reality condition of the solution implies that for $\epsilon = -1$, corresponding to H^2, we must have either $m_2 > 0$ and $0 < m_1 \le m_2$ or $m_2 < 0$ and $m_2 \le m_1 < -3m_2$. For $\epsilon = 1$, corresponding to S^2, we must have $m_2 \le 0$ and $m_1 > -3m_2$. The $AdS_2 \times H^2$ has also been found in [16].

Using the reduction ansatz obtained in [11], we can easily lift the solution back to $D = 11$ with the metric

$$\begin{aligned} ds_{11}^2 &= \Delta^{2/3}\Big\{ds^2_{AdS_2} + \frac{e^{2v}}{(m_1+3m_2)g}d\Omega_2^2\\ &\quad + \frac{1}{g^2\,\Delta}\Big[e^{-\frac32\phi}\Big(d\mu_1^2 + \mu_1^2(d\phi_1 + \frac{\epsilon\,m_1}{m_1+3m_2}A_{(1)})^2\Big)\\ &\quad + e^{\frac12\phi}\sum_{i=2}^4\Big(d\mu_i^2 + \mu_i^2(d\phi_i + \frac{\epsilon\,m_2}{m_1+3m_2}A_{(1)})^2\Big)\Big]\Big\}\,, \end{aligned} \tag{5.7}$$

where

$$\begin{aligned} \Delta &= (e^{\frac32\phi} - e^{-\frac12\phi})\mu_1^2 + e^{-\frac12\phi} > 0\,, \quad dA_{(1)} = \Omega_{(2)}\,, \quad ds^2_{AdS_2} = -e^{-\frac{2\rho}{R}}\,dt^2 + d\rho^2\,,\\ R &= \frac{2}{g}\Big[\frac{2(m_1\,e^{-\frac32\phi} + 3m_2\,e^{\frac12\phi})}{m_2 - m_1\,e^{-2\phi}}\sinh\phi + e^{\frac32\phi} + 3e^{-\frac12\phi}\Big]^{-1}\,. \end{aligned} \tag{5.8}$$

In general, the internal metric is an S^7 bundle over S^2 or H^2, depending the values of the m_i.

6 Conclusions

We have obtained a large class of supersymmetric embeddings of AdS spacetime in M-theory, as well as type IIB and massive type IIA theories. The internal spaces can be viewed as S^n bundles over S^2 or H^2. Similar solutions have been discussed in [17, 18, 19]. In particular, we have found a smooth embedding of AdS_5 in M-theory, with a compact internal space of an S^4 bundle over S^2. The bundle can be trivial or twisted, depending on the two diagonal monopole charges. The solution preserves $\mathcal{N}=2$ supersymmetry; it is a supergravity dual to an $\mathcal{N}=1$, $D=4$ superconformal field theory on the boundary of AdS_5. This provides a concrete framework to study $\mathrm{AdS}_5/\mathrm{CFT}_4$ from the point of view of M-theory.

The internal spaces of these embeddings may be regarded as concrete realisations of spaces with generalized holonomy groups advocated in [20], since they are not Ricci flat and involve a form field. An especially interesting example is the S^7 bundle over S^2 or H^2, which is nine-dimensional. While nine-dimensional Ricci-flat manifolds do not have an irreducible special holonomy group, our aforementioned solutions are explicit examples of nine-dimensional spaces which have generalized special holonomy.

The embeddings of AdS spacetimes in M-theory and string theories discussed in this paper all involve warp factors. We expect that there are many further examples of such solutions. It is of interest to explore them, both from the AdS/CFT perspective as well as for a more concrete understanding and classification of spaces with generalized special holonomy.

ACKNOWLEDGMENT

We are grateful to Gary Gibbons, Chris Pope and Ergin Sezgin for useful discussions.

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