## Three-Pion Hanbury-Brown-Twiss Correlations in Relativistic Heavy-Ion Collisions from the STAR Experiment

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Data from the first physics run at the Relativistic Heavy-Ion Collider at Brookhaven National Laboratory, $\mathrm{Au}+\mathrm{Au}$ collisions at $\sqrt{s_{N N}}=130 \mathrm{GeV}$, have been analyzed by the STAR Collaboration using three-pion correlations with charged pions to study whether pions are emitted independently at freezeout. We have made a high-statistics measurement of the three-pion correlation function and calculated the normalized three-particle correlator to obtain a quantitative measurement of the degree of chaoticity of the pion source. It is found that the degree of chaoticity seems to increase with increasing particle multiplicity.

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Two-pion Hanbury Brown and Twiss (HBT) interferometry in principle provides a means of extracting the space-time evolution of the pion source produced at kinematic freeze-out in relativistic heavy-ion collisions (1, 2). An underlying assumption of this method is that pions are produced from a completely chaotic source, i.e. a source in which the hadronized pions are created with random quantum particle production phases. In applications of two-pion HBT the validity of this assumption is usually tested by extracting the " $\lambda$-parameter" which in a simple picture is unity for a fully chaotic source and zero for a fully coherent source [1, 2]. However, this parameter
also depends on many other factors, such as contamination from other particles in the pion sample, unresolvable contributions from the decay of long-lived resonances and unstable particles ( $\omega, \eta, \eta^{\prime}, K^{0}, \Lambda$, etc.), and inaccurate Coulomb corrections (2).

A better determination of the source chaoticity is possible by using three-particle correlations. Normalizing the three-pion correlation function appropriately by the two-pion correlator, the effects from particle misidentification and decay contributions can be removed [3], thereby isolating possible coherence effects in the particle emission process. The resulting three-pion correlator
$r_{3}$ provides the means of extracting the degree of source chaoticity by examining its value in the limit of zero relative momentum. Recent measurements at the CERN SPS from experiments NA44 and WA98 have focused on extracting $r_{3}$ from three-pion correlations [4, 5]. While these studies have produced results which are consistent with a chaotic source for $\mathrm{Pb}+\mathrm{Pb}$ collisions $\left(\sqrt{s_{N N}}=17\right.$ GeV ), NA44 in particular has shown for $\mathrm{S}+\mathrm{Pb}$ collisions $\left(\sqrt{s_{N N}}=20 \mathrm{GeV}\right)$ a result which does not appear to be consistent with the chaotic assumption. All of these prior results suffer from low statistics which limits their significance. We present here using charged pions the first high-statistics heavy-ion study of three-pion correlations, resulting in the first accurate measurement of the degree of chaoticity in $\mathrm{Au}+\mathrm{Au}$ collisions at the Relativistic Heavy Ion Collider (RHIC). Note that a similar study has recently been carried out for CERN LEP $e^{+} e^{-}$ collisions [6] which reports a fully chaotic source.

The present three-pion correlation study by the STAR experiment at RHIC supplements the published two-pion correlation data from $\mathrm{Au}+\mathrm{Au}$ collisions at $\sqrt{s_{N N}}=130$ GeV 7]. A summary of the three-pion results will be presented for two multiplicity classes. By looking at collision classes with different multiplicities we can vary the impact parameter, and thus the number of initially colliding nucleons, and study the effect of the size of the colliding system on the source chaoticity. We discuss the method of normalization of the correlation function and its extrapolation to vanishing relative momentum in order to extract the source chaoticity; we estimate the various systematic uncertainties associated with these procedures.

Before presenting our experimental results, we first outline the formalism which guided our analysis (for details see Ref. [3, 8] and references therein). The measured observable is the normalized three-pion correlator:

$$
\begin{equation*}
r_{3}\left(Q_{3}\right)=\frac{\left(C_{3}\left(Q_{3}\right)-1\right)-\left(C_{2}\left(Q_{12}\right)-1\right)-\left(C_{2}\left(Q_{23}\right)-1\right)-\left(C_{2}\left(Q_{31}\right)-1\right)}{\sqrt{\left(C_{2}\left(Q_{12}\right)-1\right)\left(C_{2}\left(Q_{23}\right)-1\right)\left(C_{2}\left(Q_{31}\right)-1\right)}} \tag{1}
\end{equation*}
$$

Here $Q_{3}=\sqrt{Q_{12}^{2}+Q_{23}^{2}+Q_{31}^{2}}$ and $Q_{i j}=\sqrt{-\left(p_{i}-p_{j}\right)^{2}}$ are the standard invariant relative momenta [4, 5] which can be computed for each pion triplet from the three measured momenta $\left(\mathbf{p}_{1}, \mathbf{p}_{2}, \mathbf{p}_{3}\right) . C_{2}\left(p_{i}, p_{j}\right)=\frac{P_{2}\left(p_{i}, p_{j}\right)}{P_{1}\left(p_{i}\right) P_{1}\left(p_{j}\right)}=$ $C_{2}\left(Q_{i j}\right)$ and $C_{3}\left(p_{1}, p_{2}, p_{3}\right)=\frac{P_{3}\left(p_{1}, p_{2}, p_{3}\right)}{P_{1}\left(p_{1}\right) P_{1}\left(p_{2}\right) P_{1}\left(p_{3}\right)}=$ $C_{3}\left(Q_{3}\right)$, where $P$ represents the momentum probability distribution. In Ref. [3] the ratio $r_{3}$ is defined in terms of functions which depend on all 9 components of $\left(\mathbf{p}_{1}, \mathbf{p}_{2}, \mathbf{p}_{3}\right)$; however, limited statistics even in our highstatistics sample requires a projection of both the numerator and denominator onto a single momentum variable, $Q_{3}$.

For fully chaotic sources $r_{3} / 2$ approaches unity as all relative momenta (and thus $Q_{3}$ ) go to zero. If the source is partially coherent, a relationship can be established [3] between the limiting value of the three-pion correlator at $Q_{3}=0$ and $\varepsilon$, the fraction of pions which are emitted chaotically from the pion source $(0 \leq \varepsilon \leq 1)$ :

$$
\begin{equation*}
\frac{1}{2} r_{3}\left(Q_{3}=0\right)=\sqrt{\varepsilon} \frac{3-2 \varepsilon}{(2-\varepsilon)^{3 / 2}} \tag{2}
\end{equation*}
$$

$\varepsilon$ gives an upper limit on the value of the two-pion $\lambda$ parameter, which is sensitive to the fraction of coherent pairs in a sample, i.e. $\lambda=\varepsilon(2-\varepsilon)$ assuming no other effects on $\lambda$ such as long-lived resonances [2]. Eq. (2) is not affected by the projection onto a single relative momentum variable. To exploit it and extract the degree of chaoticity $\varepsilon$, the measured data for $r_{3}$ must, however, be extrapolated from finite $Q_{3}$ to $Q_{3}=0$.

Similar to the two-boson correlation function, the three-boson correlation function $C_{3}\left(Q_{3}\right)$ is calculated from the data by taking the ratio $\frac{A\left(Q_{3}\right)}{B\left(Q_{3}\right)}$ and normalizing it to unity at large $Q_{3}$. Here $A\left(Q_{3}\right)=\frac{d N}{d Q_{3}}$ is the threepion distribution as a function of the invariant three-pion relative momentum, integrated over the total momentum of the pion triplet as well as all other relative momentum components. It is obtained by taking three pions from a single event, calculating $Q_{3}$, and binning the results in a histogram. $B\left(Q_{3}\right)$ is the analogous mixed event distribution which is computed by taking a single pion from each of three separate events. Because of the zero in the denominator of the normalized three-pion correlator $r_{3}$ at large $Q_{i j}$, the particular method of normalization of $C_{2}$ and $C_{3}$ can have a strong effect on the calculation. The propagation of statistical errors through the $r_{3}$ functions, however, accounts for these effects completely. In fact, it is only with the very high statistics available from STAR that the calculation can be considered in the range $15<Q_{3}<120 \mathrm{MeV} / c$. This range is large enough to provide reliable extrapolation to $Q_{3}=0$.

Data for the present results are from about 300 K events taken during the $\sqrt{s_{N N}}=130 \mathrm{GeV} \mathrm{Au}+\mathrm{Au}$ run at STAR using the Time Projection Chamber (TPC) [9] as the primary tracking detector. In the discussion that follows, all phase space cuts and experimental corrections are similar to the two-pion HBT analysis 7]. A set of multiplicity classes was created by taking the $12 \%$ most central for the high-multiplicity class and the next


FIG. 1: Three-pion correlation function for central AuAu events using $\pi^{-}$triplets. Statistical and statistical+systematic errors are shown.
$20 \%$ most central for the mid-multiplicity class. For both multiplicity bins, tracks were constrained to have $p_{T}$ in the range $0.125<p_{T}<0.5 \mathrm{GeV} / c$, and pseudorapidity $|\eta|<1.0$. A vertex cut was also applied to events such that the vertex along the z-axis (beam direction) had to fall within $\pm 75 \mathrm{~cm}$ of the center of the detector. In the range $15<Q_{3}<120 \mathrm{MeV} / c$, approximately 150 million triplets were included in both the negative and positive pion studies.

The $C_{2}$ correlation function was corrected for Coulomb repulsion with a finite Gaussian source approximation, using an integration of Coulomb wave functions 10]. In calculating $C_{3}$, the correction was applied by taking the product of three two-pion corrections, obtained from the three possible pairs formed from each mixed-event triplet. This type of correction approximates the threebody Coulomb problem to first order 11, 12]. Other methods to more accurately estimate the true three-body Coulomb effect show a $5-10 \%$ smaller correction 13]. This difference was applied to the Coulomb correction factor in calculating $C_{3}$, and the resulting shifts in the $r_{3}$ function were found to be within systematic uncertainties. A separate study examined the effect of inappropriately applying the Coulomb correction to pions which come from long-lived resonances [14]. Using a rescattering model 15], the value of $r_{3}$ was found to increase by $10 \%$ when pairs and triplets of pions which contain pions from long-lived resonances were inappropriately Coulomb corrected. Effects of finite momentum resolution on $r_{3}$ were also studied using this model and were found to be insignificant. The $1 \sigma$ uncertainty in determining $Q_{3}$ is found to be about $10 \mathrm{MeV} / \mathrm{c}$.

Figure 1 shows the $C_{3}$ correlation function for negatively charged pions in the high-multiplicity bin. The shape of $C_{3}$ is mostly built up of products of two-pion correlations with the effect of true three-pion correlations


FIG. 2: Calculation of $r_{3}$ for (a) central and (b) mid-central $\pi^{-}$events, and (c) central and (d) mid-central $\pi^{+}$events. The fits shown use Eq. (33). Statistical and statistical+systematic uncertainties are shown.
being more subtle. At large $Q_{3}, C_{3}$ approaches unity and for an ideal pion source, i.e. $\lambda=1, C_{3}$ would approach 6 at $Q_{3}=0$ (this is not the present case since $\lambda<1$ ). A Gaussian parameterization is inadequate to describe this correlation function; this is consistent with results obtained in other experiments and a simulation 4, [5, 15]. In calculating $r_{3}$, the actual binned values of the correlation function for the various values of $Q_{3}$ are used instead of a fit 15]. In order to use Eq. (1), triplets are obtained that pass all of the momentum space and experimental cuts. $Q_{3}, Q_{12}, Q_{23}$ and $Q_{31}$ are calculated from the triplet and the three pairs that can be formed from the triplet. The values $C_{3}\left(Q_{3}\right), C_{2}\left(Q_{12}\right), C_{2}\left(Q_{23}\right)$ and $C_{2}\left(Q_{31}\right)$ are then computed from the binned actual two- and three-pion correlation functions. These values are then used to calculate $r_{3}$, which is then binned as a function of $Q_{3}$. The average for each bin is then calculated to obtain the final result. Systematic uncertainties are greatest at the low $Q_{3}$ end due to track merging effects and the uncertainty in the Coulomb correction. The parameters controlling these effects were modified $\pm 20 \%$ from the nominal values to obtain the overall systematic uncertainty in each bin.

The results for the two multiplicity bins are shown in Figure 2 for $\pi^{-}$and $\pi^{+}$, plotted as functions of $Q_{3}^{2}$. Plotting in this way is suggested by the theoretical analysis in [3] which shows that the leading relative momentum dependencies in the numerator and denominator of Eq. (11) are quadratic 16], allowing for a linear extrapolation of the results shown in Figure 2 to $Q_{3}=0$ by fitting them
to the form

$$
\begin{equation*}
r_{3}\left(Q_{3}\right) / 2=r_{3}(0) / 2-\alpha Q_{3}^{2} \tag{3}
\end{equation*}
$$

where $r_{3}(0) / 2$ and $\alpha$ are fit parameters. From Figure 2 it appears that the normalized three-pion correlator $r_{3}\left(Q_{3}\right)$ does indeed show a leading quadratic dependence for the smaller $Q_{3}^{2}$ values (Eq. (3) was fit to the range $0<Q_{3}<60 \mathrm{MeV} / \mathrm{c}$ ).

The resulting intercepts $r_{3}(0) / 2$ are shown in Figure 3 along with the results of WA98 and NA44. Error bars for STAR points are statistical+systematic. As mentioned earlier, the systematic error is computed by varying several parameters independently, including particle track cut parameters. The variation of the parameters is seen to produce, in general, asymmetric variations in the extracted intercepts. Intercepts from the quadratic fits as well as from quartic fits (i.e. adding a quartic term to Eq. (3) and fitting over the broader range $0<Q_{3}<120$ $\mathrm{MeV} / \mathrm{c}$ ) are shown for comparison, and are seen to agree within errors. The STAR $\pi^{+}$and $\pi^{-}$results are also seen to agree within error bars. NA44 reported a result close to unity for $\mathrm{Pb}-\mathrm{Pb}$ interactions, but a much lower result for $\mathrm{S}-\mathrm{Pb}$ [4], both with no clear $Q_{3}$ dependence. The $\mathrm{Pb}-\mathrm{Pb}$ result from WA98 is somewhat smaller than that from NA44, although they agree within error bars, and the $Q_{3}$-dependence in their result is similar to what is seen in STAR [5].

Figure 4 shows the results from the calculation of $\varepsilon$ for STAR's measurements, and for those from WA98 and NA44, plotted versus charged particle multiplicity per unit pseudorapidity, $d N / d \eta$. The calculation was done starting with the results of Figure 3 decreasing them by $10 \%$ to approximately take into account the overcorrection produced by Coulomb-correcting long-lived resonances (see earlier discussion) and using Eq. (2). It was assumed that the $10 \%$ correction also applies to the SPS data, and to be conservative, a $\pm 5 \%$ systematic uncertainty on the correction (i.e. $10 \% \pm 5 \%$ ) was included in all of the error bars shown. The plot shows an increasing trend in the STAR $\pi^{-}$and $\pi^{+}$results going from midcentral to central collisions. For the mid-central data, the results for $\varepsilon$ show a partially chaotic source, as seen in the SPS results. The central data appear to give a mostly chaotic pion source. Including the SPS measurements into the overall systematics, there appears to be, within the uncertainties shown, a systematic increase in $\varepsilon$ with increasing $d N / d \eta$, the smallest value being for SPS S-Pb collisions and the largest value for STAR central Au-Au collisions ( $d N / d \eta$ for charged particles at mid- $\eta$ for SPS $\mathrm{S}-\mathrm{Pb}$, SPS Pb-Pb, STAR mid-central, and STAR central are approximately 100 (scaled from S-S), 370, 280, and 510 , respectively 17, 18]). It is also found for the STAR results that the upper limit on the two-pion $\lambda$-parameter obtained from $\varepsilon$ using the relationship mentioned earlier is in the range $0.71-0.81$ for mid-central and $0.91-0.97$ for central events. The actual values for $\lambda$ extracted from


FIG. 3: $r_{3}\left(Q_{3}=0\right) / 2$ from STAR and two other experiments [4, 5]. For STAR, (a) central and (b) mid-central results are shown for $\pi^{-}$(circles) and $\pi^{+}$(squares) data. The other experiments use $\pi^{-}$data only. STAR results for fitting with both a quadratic and quartic functions are shown.

STAR $\pi^{-}-\pi^{-}$HBT measurements are $0.53 \pm 0.02$ for mid-central and $0.50 \pm 0.01$ for central events (the $\pi^{+}-\pi^{+}$ values agree with these within errors) 7]. The lower $\lambda$ values extracted from the two-pion experiment can be explained in terms of long-lived resonance effects, which nicely cancel out in a three-pion analysis 15.

In summary, we have presented three-pion HBT results for $\sqrt{s_{N N}}=130 \mathrm{GeV}$ data at STAR, and have shown that for the central multiplicity class the STAR data indicate a large degree of chaoticity in the source at freezeout, whereas for the mid-central class the source is less chaotic. Our $r_{3}$ results are close to those extracted in SPS $\mathrm{Pb}+\mathrm{Pb}$ collisions, but differ from the low value obtained in SPS $\mathrm{S}+\mathrm{Pb}$ collisions. The comparison between SPS and STAR results suggests a systematic dependence of the chaoticity on particle multiplicity. High statistics from STAR have allowed a normalized three-pion correlator calculation that extends to $120 \mathrm{MeV} / c$ in $Q_{3}$, and the dependence on this variable has been shown to be quadratic in nature for low $Q_{3}$. STAR's measured values provide increased confidence in the validity of standard HBT analyses based on the assumption of a chaotic source for central collisions at RHIC.

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FIG. 4: Chaotic fraction, calculated from Eq. (2), and plotted versus charged particle multiplicity per unit pseudorapidity for the same experiments as in Figure 3 The meanings of the symbols used in this figure are the same as in Figure 3

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