A relation between proton and neutron asymptotic normalization coefficients for light mirror nuclei and its relevance to nuclear astrophysics.

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We show how the charge symmetry of strong interactions can be used to relate the proton and neutron asymptotic normalization coefficients (ANCs) of the one nucleon overlap integrals for light mirror nuclei. This relation extends to the case of real proton decay where the mirror analog is a virtual neutron decay of a loosely bound state. In this case, a link is obtained between the proton width and the squared ANC of the mirror neutron state. The relation between mirror overlaps can be used to study astrophysically relevant proton capture reactions based on information obtained from transfer reactions with stable beams.

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The astrophysical S-factor associated with the peripheral proton capture reaction $B(p,\gamma)A$ at stellar energies is well known[1] to be related to the Asymptotic Normalization Coefficient (ANC) of the virtual decay $A \to B+p$. The same ANCs play a crucial role in other peripheral processes such as transfer reactions whose cross sections are significantly higher and therefore more easily measurable than those of the direct capture processes at astrophysically relevant energies [1]. The study of ANCs of astrophysical interest is a new and rapidly developing direction in modern experimental nuclear physics [2, 3]. However, in order to exploit these ideas to determine the ANCs for light proton-rich nuclei of importance to nuclear astrophysics the corresponding transfer reactions often require the use of weak radioactive beams which generally involves more difficult and less accurate experiments than are possible with stable beams. The higher intensities of stable beams means that they can be used at energies below the Coulomb barrier where the sensitivity to optical potentials, which are the main uncertainty of ANCs determined from transfer reactions, is minimised. We point out here that the ANC of the virtual neutron decay of the nucleus mirror to A, which may be susceptible to study with stable beams, is related in a model independent way by the charge symmetry of nuclear forces to the ANC of the corresponding proton decay of A. We propose to exploit this new insight to predict peripheral reaction cross sections in stars.

An asymptotic normalization coefficient (ANC) is one of the fundamental characteristics of the virtual decay of a nucleus into two clusters and is equivalent to the coupling constants in particle physics [4]. When multiplied by a trivial factor, it equals to the on-shell amplitude for the virtual decay into two clusters and it determines the large distance behaviour of the projection of the bound state wave function of the nucleus onto a binary channel.

In earlier work [5, 6], the ANCs for the one-nucleon virtual decays of the mirror pairs ^{8}B $-$ ^{8}Li and ^{12}N $-$ ^{12}B were studied in a microscopic approach. The calculated ANCs themselves depended strongly on the choice of the nucleon-nucleon (NN) force but the ratios of ANCs for mirror pairs were practically independent of the choice of the NN force. This observation is based so far entirely on the calculations using detailed models of nuclear structure. We now show that it follows naturally as a consequence of the charge symmetry of nuclear forces [27].

The ANC C_{lj} for the one-nucleon virtual decay $A \to B+N$ is defined via the tail of the overlap integral $I_{lj}(r)$ between the wave functions of nuclei A and $B = A-1$, where l is the orbital momentum and j is the total relative angular momentum between B and N. Asymptotically, this overlap behaves as

$$\sqrt{A}\, I_{lj}(r) \approx C_{lj} \frac{W_{-\eta,l+1/2}(2\kappa r)}{r}, \quad r \to \infty, \tag{1}$$

where $\kappa = (2\mu\epsilon/\hbar^2)^{1/2}$, ϵ is the one-nucleon separation energy, $\eta = Z_B Z_N e^2 \mu/\hbar^2\kappa$, μ is the reduced mass for the $B+N$ system and W is the Whittaker function. It follows from [4, 5, 7] that C_{lj} can be expressed in terms of the many-body wave functions of the nuclei A and B:

$$\begin{aligned} C_{lj} &= -\frac{2\mu\sqrt{A}}{\hbar^2} \\ &\times \langle [[\varphi_l(i\kappa r) Y_l(\hat{\boldsymbol{r}}) \otimes \chi_{\frac{1}{2}}]_j \otimes \Psi_{J_B}]_{J_A} || \hat{\mathcal{V}} || \Psi_{J_A} \rangle, \end{aligned} \tag{2}$$

where

$$\varphi_l(i\kappa r) = e^{-i\sigma_l} F_l(i\kappa r)/\kappa r, \tag{3}$$

F_l is the regular Coulomb wave function at imaginary momentum $i\kappa$, $\sigma_l = \arg\Gamma(l+1+i\eta)$, r is the distance between N and the center-of-mass of B and

$$\hat{\mathcal{V}} = \sum_{i=1}^{A-1} V_{NN}(|\boldsymbol{r}_i - \boldsymbol{r}_A|) + \Delta V_{coul} = \hat{\mathcal{V}}_N + \Delta V_{coul}, \tag{4}$$

$$\Delta V_{coul} = \sum_{i=1}^{A-1} \frac{e_i e_A}{|\boldsymbol{r}_i - \boldsymbol{r}_A|} - \frac{Z_B e_A e}{r}. \tag{5}$$

Here e_i (e_A) is the charge of the i-th (A-th) nucleon, Z_B is the charge of the residual nucleus B and V_{NN} is the

TABLE I: Ratio $\mathcal{R}$ of the proton and neutron squared ANC's for mirror overlap integrals calculated with GPT [9], Volkov V1 [10], Brink-Boeker B1 [11] and four versions of the M3Y effective NN potentials [12, 13]. M3Y(R), M3Y(HJ) and M3Y(P) were fitted to the oscillator G-matrix elements of the Reid, Hamada-Johnston and Paris NN potentials respectively and M3Y(E) was fitted to the oscillator G-matrix elements derived from the NN scattering data. Analytical estimates $\mathcal{R}_0$ are also shown.

Overlap	Mirror overlap	j	GPT	V1	B1	M3Y(E)	M3Y(R)	M3Y(P)	M3Y(HJ)	$\mathcal{R}_0$
$\langle^6Li\|^7Be\rangle$	$\langle^6Li\|^7Li\rangle$	1/2	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05
		3/2	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05
$\langle^7Be\|^8B\rangle$	$\langle^7Li\|^8Li\rangle$	1/2	1.18	1.17	1.18	1.20	1.17	1.17	1.19	1.12
		3/2	1.20	1.20	1.20	1.22	1.19	1.20	1.22	1.12
$\langle^{11}C\|^{12}N\rangle$	$\langle^{11}B\|^{12}B\rangle$	1/2	1.45	1.42	1.44	1.45	1.42	1.42	1.43	1.37
		3/2	1.45	1.44	1.46	1.46	1.42	1.43	1.46	1.37
$\langle^{12}C\|^{13}N\rangle$	$\langle^{12}C\|^{13}C\rangle$	1/2	1.26	1.24	1.24	1.26	1.23	1.24	1.25	1.19
$\langle^{14}N\|^{15}O\rangle$	$\langle^{14}N\|^{15}N\rangle$	1/2	1.53	1.50	1.51	1.53	1.51	1.51	1.52	1.48
		3/2	1.54	1.51	1.51	1.57	1.51	1.52	1.56	1.48
$\langle^{15}N\|^{16}O\rangle$	$\langle^{15}O\|^{16}O\rangle$	1/2	1.55	1.54	1.54	1.57	1.54	1.55	1.56	1.52

two-body nuclear NN potential. If the separated nucleon is a neutron, φ_l is replaced by the Bessel function $j_l(i\kappa r)$.

ANCs can be obtained from Eq. (2) using wave functions which model the structure of nuclear interior well, for example, from the oscillator shell model [28]. The incorrect behavior of these model wave functions at large distances plays a minor role because of the presence of the short range NN potential on the right hand side in Eq. (2) [7]. We have performed such calculations for several $0p$ nuclei, some of which are of astrophysical importance, with a range of NN potentials using fixed $0\hbar\omega$ wave functions obtained in [8],[29]. The oscillator radii was chosen to provide correct sizes for the nuclei considered. In these calculations mirror nuclei have exactly the same wave functions but, of course, the mirror ANCs are different because of different functions $\varphi_l(i\kappa r)$ involved. The $|C_{lj}|^2$ values change by a factor of two for different NN potential choice, but the ratio $\mathcal{R} = |C_p/C_n|^2$, where C_p and C_n are the proton and neutron ANCs for mirrors and hence may refer to different nuclei, changes by less than 4% for each mirror pair of overlaps (see Table 1 and [30]).

The observed effect has the following explanation. We first replace ΔV_{coul} by $V_{coul}(r) - Z_B e_A e/r$ where $V_{coul}(r)$ is the monopole Coulomb interaction of the A-th nucleon with the nucleus B. This ignores higher multipole components of ΔV_{coul}. Eq. (2) can then be replaced exactly by a formula in which ΔV_{coul} is removed from the matrix element and $\varphi_l(r)$ is replaced by $\varphi_l^{mod}(r)$. The latter is defined as the regular solution of the Schrödinger equation with the potential $V_{coul}(r)$ and which is normalized so that $\varphi_l(r) = \varphi_l^{mod}(r)$ outside the charge radius of B. Inside the charge radius, the potential $V_{coul}(r)$ varies little over the nuclear volume and can be replaced by a constant equal to the separation energies difference $\epsilon_n - \epsilon_p$. Hence, in the nuclear interior $r < R_N$, which is all that matters on the right-hand-side of Eq. (2), we can use

$$\varphi_l^{mod}(r) = \frac{F_l(i\kappa_p r_N)}{\kappa_p R_N j_l(i\kappa_n R_N)} j_l(i\kappa_n r), \quad r \le R_N, \qquad (6)$$

where $i\kappa_p$ and $i\kappa_n$ are determined by the proton and neutron separation energies ϵ_p and ϵ_n. Using Eq. (6) in the modified Eq. (2) and making the assumption that the difference between the wave functions for mirror pairs in the nuclear interior can be ignored, we find

$$\mathcal{R} \approx \mathcal{R}_0 = \left| \frac{F_l(i\kappa_p R_N)}{\kappa_p R_N \, j_l(i\kappa_n R_N)} \right|^2 . \qquad (7)$$

$\mathcal{R}_0$ depends on the NN force only implicitly through R_N.

The values of $\mathcal{R}_0$, presented in Table I, have been calculated for $R_N = 1.3 \cdot B^{1/3}$. They change by less than 2%, when R_N is varied from 2.5 to 4.5 fm in each case, and are smaller by less than 7% than the $\mathcal{R}$ values obtained from microscopic calculations. Eq. (7) correctly predicts the dependence of $\mathcal{R}$ on neutron and proton separation energies. The tendency of $\mathcal{R}_0$ to inderestimate $\mathcal{R}$ can be attributed to the contributions from the r^{-2} and r^{-3} multipoles of ΔV_{coul}. When these multipoles are excluded from the microscopic calculations, the $\mathcal{R}$ values decrease and become equal to $\mathcal{R}_0$ within the uncertainty in its definition.

In practice, overlap integrals for transfer reactions are frequently modelled as normalised single-particle wave functions times spectroscopic factors S, so that $C_{p(n)} = \sqrt{S_{p(n)}}\, b_{p(n)}$, where $b_{p(n)}$ is the single-particle proton (neutron) ANC. The derivation above shows that the result Eq.(7) is valid for $\mid b_p/b_n \mid^2$ if we assume that the single particle wave functions in the interior and the nuclear single particle potentials are the same for p and n. The ratio $\mathcal{R}_b = (b_p/b_n)^2$ is therefore expected to have only weak dependence on these potentials. We have verified this for a range of potentials chosen to simultaneouly reproduce fixed proton and neutron binding energies. The individual ANC's b_n and b_p vary by up to a factor of 2 but the ratio $\mathcal{R}_b$ is stable to within 3% with an average which agrees with Eq.(7)[31]. If we assume that the spectroscopic factors S_p and S_n are equal for mirror pairs then we have an alternative way of estimating $\mathcal{R}$. Note however that our derivation of Eq.(7) involves fewer assumptions than in this alternative approach and in fact

TABLE II: Squared ratio $(b_{max}/b_{min})^2$ of the maximal and minimal values of b, average ratio of squared ANC's $\langle\mathcal{R}_b\rangle$ analytical estimates $\mathcal{R}_0$ and experimental ratios $\mathcal{R}^{exp}$. Where several experimental values of ANC's are available, we take their average. Also shown are proton ϵ_p and neutron ϵ_n separation energies (in MeV), number of nodes n and orbital momentum l.

Overlap	ϵ_p	Mirror overlap	ϵ_n	nl	$\left(\frac{b_{max}}{b_{min}}\right)^2$	$\langle\mathcal{R}_b\rangle$	$\mathcal{R}_0$	$\mathcal{R}^{exp}$	Ref.for C_p^{exp}	Ref.for C_n^{exp}
$\langle ^7Be\|^8B\rangle$	0.137	$\langle ^7Li\|^8Li\rangle$	2.033	$0p$	1.23	1.01±0.01	1.12	1.08 ± 0.15	[17]	[18]
$\langle ^{11}C\|^{12}N\rangle$	0.601	$\langle ^{11}B\|^{12}B\rangle$	3.370	$0p$	1.67	1.30±0.02	1.37	1.28 ± 0.29	[15]	[16]
$\langle ^{14}N\|^{15}O(\frac{3}{2}{}^+_1)\rangle$	0.507	$\langle ^{14}N\|^{15}N(\frac{3}{2}{}^+_1)\rangle$	3.026	$1s$	1.68	3.62±0.03	4.09			
$\langle ^{15}N\|^{16}O\rangle$	12.128	$\langle ^{15}O\|^{16}O\rangle$	15.664	$0p$	2.55	1.55±0.02	1.52			
$\langle ^{16}O\|^{17}F(\frac{5}{2}{}^+_1)\rangle$	0.601	$\langle ^{16}O\|^{17}O(\frac{5}{2}{}^+_1)\rangle$	4.144	$0d$	2.15	1.21±0.03	1.21	1.33 ± 0.20	[20, 21, 22, 23]	[19]
$\langle ^{16}O\|^{17}F(\frac{1}{2}{}^+_1)\rangle$	0.106	$\langle ^{16}O\|^{17}O(\frac{1}{2}{}^+_1)\rangle$	3.273	$1s$	1.56	702±4	796			
$\langle ^{22}Mg\|^{23}Al\rangle$	0.123	$\langle ^{22}Ne\|^{23}Ne\rangle$	4.419	$0d$	1.50	2.67·10^4	2.61·10^4			
$\langle ^{26}Si\|^{27}P\rangle$	0.859	$\langle ^{26}Mg\|^{27}Mg\rangle$	6.443	$1s$	1.80	40.3±1.1	43.3			

does not appeal to the concept of spectroscopic factor at all. Our approach is therefore much more general and provides a basis for further refinement of the value of the ratio $\mathcal{R}$ predicted by theory. For the mirror pairs ^{8}B - ^{8}Li, ^{12}N - ^{12}B and ^{17}F - ^{17}O, where the experimental values of the proton C_p^{exp} and neutron C_n^{exp} ANC's are simultaneously available, both $\langle\mathcal{R}_b\rangle$ and $\mathcal{R}_0$ agree with $\mathcal{R}^{exp} = |C_p^{exp}/C_n^{exp}|^2$ within the error bars (see Table II).

Near the edge of stability, where neutron separation energies become very small, the corresponding mirror proton states manifest themselves as resonances. The width Γ_p of a narrow proton resonance is related to the ANC of the Gamow wave function for this resonance by the equation $\Gamma_p = \mu/\kappa_p|C_p|^2$ [14]. The ANC C_p can be calculated from Eqs. (2) and (3) using the regular Coulomb function $F_l(\kappa_p r)$ of a real argument [4]. Therefore, a link must exist between Γ_p and the ANC of mirror neutron bound states. The ratio $\mathcal{R}_\Gamma = \Gamma_p/|C_n|^2$ is then approximated by an equation similar to Eq. (7):

$$\mathcal{R}_\Gamma \approx \mathcal{R}_0^{res} = \frac{\kappa_p}{\mu}\left|\frac{F_l(\kappa_p R_N)}{\kappa_p R_N\, j_l(i\kappa_n R_N)}\right|^2 \tag{8}$$

Alternatively, $\mathcal{R}_\Gamma$ can be approximated by the single-particle ratio $\mathcal{R}_\Gamma^{s.p.} = \Gamma_p^{s.p.}/b_n^2$ if the spectroscopic factors and single-particle potential wells for mirror bound-unbound pairs are assumed equal. We have calculated $\mathcal{R}_\Gamma^{s.p.}$ for the ^{8}B(1$^+$), ^{12}N(2$^+$), ^{13}N($\frac{1}{2}^+$) and ^{13}N($\frac{5}{2}^+$) resonances using a set of two-body Woods-Saxon potentials which reproduce both the separation energy of the loosely-bound neutron and the position of the mirror proton resonance. In the case of $l \neq 0$, for different choice of the two-body potentials the ratios $\mathcal{R}_\Gamma^{s.p.}$ change by about 3% while $\Gamma_p^{s.p.}$ changes by up to a factor of 2 (see Table III). This is the same as in the case of bound mirror pairs of overlaps. However, for $l = 0$, where the centrifugal barrier is absent and non-resonant contributions are larger, the change in $\mathcal{R}_\Gamma^{s.p.}$ is larger and reaches 11%. The average value of $\mathcal{R}_\Gamma^{s.p.}$ agrees with $\mathcal{R}_0^{res}$ for the $l = 2$ resonance ^{13}N($\frac{5}{2}^+$) but is smaller than $\mathcal{R}_0^{res}$ by 16%, 20% and 37% for ^{8}B(1$^+$), ^{12}N(2$^+$) and ^{13}N($\frac{1}{2}^+$) respectively. The $\mathcal{R}_0^{res}$ values themselves are quite stable with respect to different choice of R_N except in the case of the $l = 0$ resonance ^{13}N($\frac{1}{2}^+$) where the uncertainty of $\mathcal{R}_0^{res}$ is 5%.

The proton widths of ^{8}B(1$^+$) ^{13}N($\frac{1}{2}^+$) and ^{13}N($\frac{5}{2}^+$) and neutron ANC's for their mirror states are known experimentally. The ratios $\mathcal{R}_\Gamma^{exp} = \Gamma_p^{exp}/|C_n^{exp}|^2$ for these states are shown in Table III. In all these cases, the single-particle approximation $\mathcal{R}_\Gamma \approx \mathcal{R}_\Gamma^{s.p.}$ is not confirmed. For ^{8}B(1$^+$), $\mathcal{R}_\Gamma^{exp}$ is larger than $\mathcal{R}_\Gamma^{s.p.}$ and agrees with $\mathcal{R}_0^{res}$, but for ^{13}N($\frac{1}{2}^+$) and ^{13}N($\frac{5}{2}^+$) $\mathcal{R}_\Gamma^{exp}$ is significantly lower than $\mathcal{R}_\Gamma^{s.p.}$ and $\mathcal{R}_0^{res}$. This result suggests that estimates based on the relation $\Gamma_p = S_p\Gamma_p^{s.p.}$ and the assumption $S_p = S_n$ can be unreliable.

The present work confirms the existence of a link between proton and neutron mirror ANC's both for bound-bound and bound-unbound mirror pairs. Therefore, neutron ANC's obtained with stable beams can be used to predict cross sections of low-energy direct and resonance proton capture reactions. Although more accurate theoretical ratios for $\mathcal{R}$ and $\mathcal{R}_\Gamma$ are required for these purposes, the estimates $\langle\mathcal{R}_b\rangle$, $\mathcal{R}_0$, $\mathcal{R}_\Gamma^{s.p.}$ and $\mathcal{R}_0^{res}$ of the present paper can already be used in some cases. In fact, the ratio $\mathcal{R}$ has already been used to predict the direct ^{11}C$(p,\gamma)^{12}$N capture cross sections in [6] and the results obtained there are in a good agreement with the predictions based on proton ANC's recently measured in [15]. Also, the astrophysical S-factor for the ^{7}Be$(p,\gamma)^8$B reaction has been calculated in [18] based on the $\langle\mathcal{R}_b\rangle$ estimate and experimentally measured neutron ANC in ^{8}Li. Another example is the proton width of the ^{12}N(2$^+$) resonance for which only an upper limit of 20 keV is available. Using the neutron ANC for the mirror ^{12}B(2$^+$) state from [16], we can predict that Γ_p is equal to 5.9±1.0 or 6.9 ± 1.2 keV for the $\mathcal{R}_\Gamma \approx \langle\mathcal{R}_\Gamma^{s.p.}\rangle$ and $\mathcal{R}_\Gamma \approx \mathcal{R}_0^{res}$ assumptions respectively. These values are less uncertain than the currently available experimental limit $\Gamma_p < 20$ keV.

Among other cases of astrophysical interest is the

TABLE III: The ratio $\Gamma_p^{max}/\Gamma_p^{min}$ of the maximal and minimal proton widths, average ratio of $\mathcal{R}_\Gamma^{s.p.}$, analytical estimates $\mathcal{R}_0^{res}$ and experimental ratios $\mathcal{R}_\Gamma^{exp}$. Where several experimental values of ANC's are available, we take their average.

Proton resonance	Bound mirror analog	l	$\frac{\Gamma_p^{max}}{\Gamma_p^{min}}$	$\langle \mathcal{R}_\Gamma^{s.p.} \rangle$	$\mathcal{R}_0^{res}$	$\mathcal{R}_\Gamma^{exp}$	Ref.for Γ_p^{exp}	Ref.for C_p^{exp}
^{8}B(1^+, 0.774)	^{8}Li(1^+, 0.980)	1	1.43	$(1.70 \pm 0.03) \cdot 10^{-3}$	$2.03 \cdot 10^{-3}$	$(2.29 \pm 0.40) \cdot 10^{-3}$	[24]	[18]
^{12}N(2^+, 0.960)	^{12}B(2^+, 0.953)	1	1.61	$(1.22 \pm 0.01) \cdot 10^{-5}$	$1.42 \cdot 10^{-5}$			
^{13}N($\frac{1}{2}^+$, 2.36)	^{13}C($\frac{1}{2}^+$, 3.09)	0	1.55	$(5.98 \pm 0.32) \cdot 10^{-5}$	$8.5 \cdot 10^{-5}$	$(4.57 \pm 0.57) \cdot 10^{-5}$	[25, 26]	[16]
^{13}N($\frac{5}{2}^+$, 3.55)	^{13}C($\frac{5}{2}^+$, 3.85)	2	2.01	$(1.37 \pm 0.03) \cdot 10^{-2}$	$1.42 \cdot 10^{-2}$	$(1.06 \pm 0.21) \cdot 10^{-2}$	[25]	[16]

astrophysical S-factor for the direct capture reaction ^{14}N$(p,\gamma)^{15}$O$(\frac{3}{2}{}_1^+)$, which is mainly responsible for the energy production in the CNO cycle. The $^{15}O(\frac{3}{2}{}_1^+)$ state is separated from the neighbouring $^{15}O(\frac{5}{2}{}_2^+)$ state by only 70 KeV, which influences the precision of measurements involving this state. The spacing between the mirror $^{15}N(\frac{5}{2}{}_2^+)$ and $^{15}N(\frac{3}{2}{}_1^+)$ states is larger and therefore the ANC for the $\langle ^{14}\text{N}|^{15}\text{N}(\frac{3}{2}{}_1^+)\rangle$ overlap integral can be determined using neutron transfer reactions to higher accuracy than the $^{15}O(\frac{3}{2}{}_1^+)$ ANC. Also, direct contributions to the cross sections of the ^{22}Mg$(p,\gamma)^{23}$Al and ^{26}Si$(p,\gamma)^{27}$P reactions, involving proton-rich radioactive nuclei, could be calculated through the mirror neutron ANC's which can be determined using stable targets ^{22}Ne and ^{26}Mg. These reaction are relevant to the nucleosynthesis in novae and are being intensively investigated.

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[27] We believe that charge symmetry rather than full charge independence is involved because mirror nuclei have the same number of n-p pairs.
[28] The oscillator shape of the single-particle wave functions makes the correct treatment of the center-of-mass possible, which is crucial for the nuclei considered here.
[29] The more consistent approach used in [5], in which the same NN potential is used both in Eq. (2) and in the shell-model Hamiltonians, leads to similar results for the ratio of the mirror ANCs.
[30] We have found a mistake in the computer code for proton ANC's used in Refs. [5, 6]. The corrected values are given in Table 1.
[31] The difference between the average $< \mathcal{R}_b >$ and $\mathcal{R}_0$ is largest for weakly bound states. We will return to the reason for this elsewhere.