Gravitational Forces in the Randall-Sundrum Model with a Scalar Stabilizing Field

R. Arnowitt and J. Dent

Center For Theoretical Physics, Department of Physics
Texas A&M University, College Station,TX 77843-4242, USA

Abstract

We consider the problem of gravitational forces between point particles on the branes in a five dimensional (5D) Randall-Sundrum model with two branes (at y_1 and y_2) and S^1/Z_2 symmetry of the fifth dimension. The matter on the branes is viewed as a perturbation on the vacuum metric and treated to linear order. In previous work [23] it was seen that the trace of the transverse part of the 4D metric on the TeV brane, $f^T(y_2)$, contributed a Newtonian potential enhanced by $e^{2\beta y_2} \cong 10^{32}$ and thus produced gross disagreement with experiment. In this work we include a scalar stabilizing field ϕ and solve the coupled Einstein and scalar equations to leading order for the case where ϕ_0^2/M_5^3 is small and the vacuum field $\phi_0(y)$ is a decreasing function of y. f^T then grows a mass factor $e^{-\mu r}$ where however, μ is suppressed from its natural value, $\mathcal{O}(M_{Pl})$, by an exponential factor $e^{-(1+\lambda_b)\beta y_2}$, $\lambda_b > 0$. Thus agreement with experiment depends on the interplay between the enhancing and decaying exponentials. Current data eliminates a significant part of the parameter space, and the Randall-Sundrum model will be sensitive to any improvements on the tests of the Newtonian force law at smaller distances.

1 Introduction

Higher dimensional models in particle physics with dimension D > 4 have been the subject of much theoretical investigation over the past two decades. Higher dimensional theory arises naturally in string/M-theory and is phenomenologically interesting as they offer the possibility of explaining fundamental features of nature that would not be possible in 4D theory. The simplest phenomenology of this type is the 5D Randall-Sundrum model (RS1)[1, 2] where the fifth dimension y is compactified with S^1/Z_2 symmetry so that one can think of space as bounded by two 4D orbifold planes (3-branes) at $y_1 = 0$ and $y_2 = \pi\rho$ with boundary conditions at y_1 and y_2 to enforce the S^1/Z_2 symmetry. With no matter on the branes, the 5D Einstein equations have a vacuum solution which preserves 4D Poincaire invariance on the branes

$$ds^2 = e^{-2A(y)}\eta_{ij}dx^i dx^j + dy^2 \tag{1}$$

where

$$A(y) = \beta|y| \ ; \ \ y_1 - \epsilon \le y \le y_2 - \epsilon \ ; \ \ \epsilon > 0 \tag{2}$$

and η_{ij} is the Lorentz metric. Thus if all basic masses are naturally of Planck size and the physical world lives on the y_2 brane, such a structure offers a new way of understanding the gauge hierarchy (without undue fine tuning) not available in 4D theory. For example, consider a scalar field χ on the y_2 brane which we may treat as a perturbation on the vacuum state. The action has the form

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}(g^{ij}\partial_i\chi\partial_j\chi + m^2\chi^2) \tag{3}$$

where we use the notation μ, ν = 0,1,2,3,5 and i, j = 0,1,2,3. Letting $\chi' = e^{-\beta y_2}\chi$, the theory then takes canonical form with a mass parameter

$$\bar{m} = e^{-\beta y_2} m \tag{4}$$

and the observed mass on the y_2 brane would be of TeV size if $e^{-\beta y_2} \simeq 10^{-16}$ i.e. $\beta y_2 \simeq 35$. Thus a Planck size mass travelling on the y_2 brane has its mass effectively supressed by the strong 5D gravitational forces (much as an electron traveling in a solid has its mass modified by the electric fields there). The question remains, however, as to whether the 5D theory will produce other additional phenomena that would violate known observations on the physical y_2 brane. Initial analysis examined whether the Friedmann-Robertson-Walker (FRW) cosmology on the y_2 brane could be achieved in this model [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. This was found to indeed be the case provided that in addition to gravity being in the 5D bulk, one must stabilize the vacuum metric, which is most easily accomplished by adding a scalar field in the bulk, $\phi(x^i, y)$[14]. Then both relativistic and non-relativistic matter could be accommodated in the cosmology[7, 8, 13], the distance between the branes being governed by the density of non-relativistic matter [13].

A second question that has been examined is whether the 5D theory correctly reproduces the known gravitational forces between particles. Here we treat the matter on the branes as a perturbation to the vacuum metric:

$$ds^2 = e^{-2\beta y}(\eta_{ij} + h_{ij})dx^i dx^j + h_{i5} dy dx^i + (1 + h_{55})dy^2 \tag{5}$$

There is also a large literature on this subject [15, 16, 17, 18, 19, 20, 21, 22, 23] Refs.[15-22] deal only with the gravitational forces on the $y_1 = 0$ "Planck" brane (where all the masses are of Planck size), and find normal Newtonian forces hold between particles on the Planck brane (along with negligibly small Kaluza-Klein corrections). In previous work [23], we have examined in addition the physically relevant forces on the y_2 "TeV" brane, and unlike other discussions make sure that the coordinate conditions chosen do not lead to bent branes (so that the S^1/Z_2 boundary conditions can be correctly imposed). To see what occurs for this case, it is convenient to make a 4D ADM decomposition of h_{ij} [24, 25]

$$h_{ij} = h^{TT}_{ij} + h^{T}_{ij} + h_{i,j} + h_{j,i} \tag{6}$$

where h^{TT}_{ij} is transverse and traceless ($\partial^i h^{TT}_{ij} = 0 = h^{i\,TT}_i$)and h^T_{ij} is transverse with a non-zero trace $f^T(\partial^i h^T_{ij} = 0, h^{i\,T}_i = f^T)$. One may write

$$h^T_{ij} = \frac{1}{3}\pi_{ij} f^T \ \ ; \ \ \pi_{ij} \equiv \eta_{ij} - O_{ij} \tag{7}$$

where

$$O_{ij} \equiv \frac{\partial_i \partial_j}{\Box^2} \tag{8}$$

(In the above and following, four dimesional indices are raised and lowered with the Lorentz metric η_{ij}.) What was found in [23] was that h^{TT}_{ij} gave rise to leading order to normal Newtonian forces bewteen particles on the Planck or TeV branes. However f^T gave a Newtonian contribution on the

TeV brane that was enhanced by a factor of $e^{2\beta y_2} \simeq 10^{32}$, thus producing a gross disagreement with experiment.

None of the analyses discussed above, [15, 16, 17, 18, 19, 20, 21, 22, 23], have included a scalar field $\phi_0(x^i, y)$ [14] to stabilize the vacuum metric. Such a scalar field might produce a mass for the f^T field, thus modifying it gravitational potential. In this paper we examine the effects of introducing such a stablizing contribution. It is not possible to solve the coupled Einstein scalar field equations in closed form, but an iterative solution can be obtained when ϕ_0^2/M_5^3 is small (where $\phi_0(y)$ is the vacuum solution and M_5 is the 5D Planck mass) and $\phi_0(y)$ is a decreasing function. Within this framework we find that f^T indeed grows a mass μ but the mass is exponentially suppressed. The f^T contribution then appears effectively massless over a distance $r \lesssim 1/\mu$ which can be anomalously large due to the exponential reduction of μ. Thus whether the theory is in agreement with current experimental tests of the Newtonian force law at small distances depends on the interplay of the amount of suppression of μ compared to the size of the enhancement factor $e^{2\beta y_2}$ of the amplitude of f^T, and in fact current experiment strongly constrains allowable scalar field models of this type.

In Sec. 2, we give the choice of coordinate conditions we use and write down the Einstein and scalar field equations. In Secs. 3 and 4 we state the expansion procedure we use to solve the equations, discuss the solutions of the Einstein equations and evaluate f^T at y_1 and y_2 (which is what is needed to calulate the effect of f^T on particles on the branes). Sec. 5 is devoted to the scalar field equations, and in Sec. 6 we calculate the leading effects to the fields for a case where a rigorous vacuum solution for $\phi_0(y)$ exists. In Sec.7 we calculate the Newtonian forces on particles on the branes. Conclusions are given in Sec.8. The Appendix shows that the results obtained in Sec.7 are valid for a general class of models where ϕ_0^2/M_5^3 is small and $\phi_0(y)$ is a decreasing function of y.

2 Coordinate Conditions

The action for our system has the form

$$\begin{aligned} S &= \int d^5x\sqrt{-g^5}[-\frac{1}{2}M_5^3 R - \Lambda] + \int d^5x\sqrt{-g^5}[-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)] \\ &+ \sum_\alpha \int d^5x\sqrt{-g^4}[\mathcal{L}_{m_\alpha} - V_\alpha(\phi)]\delta(y - y_\alpha) \end{aligned} \quad (9)$$

where M_5 is the 5D Planck mass, R is the 5D curvature scalar, $\mathcal{L}_{m_\alpha}$ is the Lagrangian for point particles on the y_α brane (α = 1,2), $\phi(x^\mu)$ is the scalar field that stabilizes the vacuum metric, $V(\phi)$ is the bulk potential, and V_α are the brane potentials. We write

$$\phi(x^\mu) = \phi_0(y) + \delta\phi(x^\mu) \tag{10}$$

where $\phi_0(y)$ is the vacuum solution and $\delta\phi$ is the perturbation due to matter on the branes. The vacuum equations read

$$4A'^2 - A'' = -\frac{2}{3M_3^5}[\Lambda + V(\phi_0)] - \frac{1}{3M_5^3}\sum_\alpha \delta(y - y_\alpha)V_\alpha(\phi_0) \tag{11}$$

$$4A'^2 - 4A'' = -\frac{2}{3M_3^5}[\Lambda + V(\phi_0)] - \frac{1}{M_5^3}\phi_0'^2 - \frac{4}{3M_5^3}\sum_\alpha \delta(y - y_\alpha)V_\alpha(\phi_0) \tag{12}$$

$$\phi_0'' - 4A'\phi_0' - V'(\phi_0) - \sum_\alpha \delta(y - y_\alpha)V_\alpha'(\phi_0) = 0 \tag{13}$$

where $A' \equiv dA(y)/dy$, $A'' \equiv d^2A/dy^2$, etc., and $V'(\phi_0) \equiv dV/d\phi_0$, etc.

The bulk and brane potentials are arbitrary except that they must be fine tuned to cancel the effects of the bulk cosmological constant Λ so that the net brane cosmological constant vanishes. Most of the analysis can be done without specifying V and V_α, making use of the field equations Eqs.(11-13). However, to estimate the size of effects, it is useful to have an explicit rigorous solution of Eqs.(11-13) and one has been given in [6]. Thus vacuum functions

$$A(y) = \beta y + \frac{1}{12}\frac{\phi_1^2}{M_5^3}e^{-2by} \tag{14}$$

$$\phi_0(y) = \phi_1 e^{-by} \tag{15}$$

are the solutions of Eqs.(10-12) for the choice

$$V(\phi_0(y)) = (\frac{1}{2}b^2 + 2b\beta)\phi_0^2 - \frac{b^2}{6M_5^3}\phi_0^4 \tag{16}$$

$$\begin{aligned} V_\alpha(\phi_0(y)) &= V_\alpha(\phi_0(y_\alpha)) + (-1)^\alpha 2b\phi_0(y_\alpha)(\phi_0(y) - \phi_0(y_\alpha)) \\ &+ \gamma_\alpha \frac{1}{2}(\phi_0(y) - \phi_0(y_\alpha))^2 \end{aligned} \tag{17}$$

where $\Lambda = -6M_5^3\beta^2$ (the fine tuning of the cosmological constant),

$$V_\alpha(\phi_0(y_\alpha)) = (-1)^{\alpha+1}[6M_5^3\beta - b\phi_0^2(y_\alpha)] \tag{18}$$

and γ_α are arbitrary constants. We see that the effect of the scalar field is to add a term to A(y) of size ϕ_0^2/M_5^3. Since naturalness implies that all masses should be of the same order and comparable to M_{Pl} we will assume $b \approx \beta$, and $b > 0$. The gauge hierarchy condition then requires $\beta y_2 \simeq 35$ [6] so that the addition to $A(y)$ is a rapidly decreasing quantity.

We assume that matter on the branes represent a perturbation to the vacuum state and so we solve the full field equations to first order in $h_{\mu\nu}(x^\alpha)$ and $\delta\phi(x^\alpha)$. We begin by reviewing the coordinate conditions we will use in the following analysis. The general transformation

$$x'^\mu + \xi^\mu = x^\mu \tag{19}$$

that preserves the S^1/Z_2 symmetry with no brane bending is constrained by

$$\xi^5(x^i, y_1) = 0 = \xi^5(x^i, y_2) \tag{20}$$

As discussed in [23], one may use these to set h_{5i} to zero, but in general it is not possible to have h_{55} vanish without introducing brane bending. We thus assume in the following that

$$h_{5i} = 0 \ ; \ \ i = 0, 1, 2, 3 \tag{21}$$

There still remains some gauge freedom. Thus under a general transformation preserving Eqs.(20, 21), the components of the metric transform to first order as [23]

$$\delta h_{55} = 2\xi^5_{,5} \tag{22}$$

$$\delta h_{ij}^{TT} = 0 \ ; \ \ \delta f^T = -6A'\xi^5 \tag{23}$$

$$\delta h_i^T = e^{2A}\xi_i^T \tag{24}$$

$$\delta(\Box^2 h^L) = 2e^{2A}\Box^2\xi^L + 2A'e^{-2A}(e^{2A}\xi^L)_{,5} \tag{25}$$

where we have decomposed h_i and ξ_i into transverse and longitudinal parts, e.g. $h_i = h_i^T + h_{,i}^L$ where $\partial^i h_i^T \equiv 0$. Eq.(21) requires in addition that

$$\xi_i^T = e^{-2A(y)} F_i^T(x^i) \tag{26}$$

$$(e^{2A}\xi_L)_{,5} = -e^{2A}\xi_5 \tag{27}$$

where $F_i^T(x^i)$ is an arbitrary function independent of y. The gauge change in $\delta\phi(x^i)$ is (to first order)

$$\delta[\delta\phi(x^i)] = \phi_0'(y)\xi^5(x^i) \tag{28}$$

We see that h_{ij}^{TT} (which contains the Kaluza-Klein modes) is gauge invariant, and Eq.(20) implies that f^T and $\delta\phi$ are invariant on the branes at y_1 and y_2. Eq.(22) shows explicitly that it is not possible to choose a gauge function ξ^5 obeying Eq.(20) that sets h_{55} to zero everywhere since integrating Eq.(22) to try to do this one has only one constant of integration to satisfy the two boundary conditions of Eq.(20). As discussed in [23], it is possible however to choose a ξ^5 that sets h_{55} to zero on each brane

$$h_{55}(x^i, y_\alpha) = 0 \ ; \ \ \alpha = 1, 2 \tag{29}$$

and we will use this gauge in some of the discussions below.

We conclude this section by recording the field equations and boundary conditions. The 5D Einstein equations read

$$R_{ij}^{TT}: \quad (\frac{1}{2}\partial_5^2 - 2A'\partial_5 + \frac{1}{2}e^{2A}\Box^2)h_{ij}^{TT} = -\frac{e^{2A}}{M_5^3}\sum_\alpha T_{ij}^{TT}(y_\alpha)\delta(y - y_\alpha) \tag{30}$$

$$R_{j5}: \quad \frac{1}{2}\partial_5\eta^{kl}(\partial_j h_{kl} - \partial_l h_{jk}) + \frac{3}{2}A'\partial_j h_{55} = -\frac{1}{M_5^3}(\partial_j\delta\phi)\phi_o' \tag{31}$$

$$\eta^{ij}R_{ij}: \quad (\frac{1}{2}\partial_5^2 - 4A'\partial_5)(\Box^2 h^L + f^T) + e^{2A}\Box^2 f^T + 2A'\partial_5 h_{55} + \frac{e^{2A}}{2}\Box^2 h_{55} + 4h_{55}(A'' - 4A'^2) = -\frac{8}{3M_5^3}V'(\phi_o)\delta\phi + \frac{1}{M_5^3}\sum_\alpha \delta(y - y_\alpha)(\frac{Te^{2A}}{3} - \frac{4}{3}V_\alpha'(\phi_o)\delta\phi + \frac{2}{3}h_{55}V_\alpha(\phi_o)) \tag{32}$$

$$R_{55}: \quad (\frac{1}{2}\partial_5^2 - A'\partial_5)(\Box^2 h^L + f^T) + \frac{e^{2A}}{2}\Box^2 h_{55} + 2A'\partial_5 h_{55} + h_{55}(4A'' - 4A'^2 - \frac{(\phi_o')^2}{M_5^3}) = -\frac{2}{3M_5^3}V'(\phi_o)\delta\phi - \frac{2\phi_o'\delta\phi'}{M_5^3} + \frac{1}{M_5^3}\sum_\alpha \delta(y - y_\alpha)(\frac{Te^{2A}}{3} - \frac{4}{3}V_\alpha'(\phi_o)\delta\phi + \frac{2}{3}h_{55}V_\alpha(\phi_o)) \tag{33}$$

$$\partial^i\partial^j R_{ij}: \quad (\frac{1}{2}\partial_5^2 - \frac{5}{2}A'\partial_5)(\Box^2 h^L) + \frac{1}{2}e^{2A}\Box^2 f^T - \frac{1}{2}A'\partial_5 f^T + \frac{1}{2}A'\partial_5 h_{55} + \frac{1}{2}e^{2A}\Box^2 h_{55} + h_{55}(A'' - 4A'^2) = -\frac{2}{3M_5^3}V'e^{-2A}\delta\phi + \tag{34}$$

$$\frac{1}{3M_5^3}\sum_\alpha \delta(y-y_\alpha)(Te^{2A}-V'_\alpha(\phi_o)\delta\phi+\frac{1}{2}h_{55}V_\alpha(\phi_o))$$

In the above $T \equiv \eta^{ij}T_{ij}$. The $\delta(y-y_\alpha)$ terms on the right hand side imply that the bulk solutions obey the boundary conditions

$$(-1)^{\alpha+1}[\partial_5(\Box^2 h^L + f^T) + 8A'h_{55}]\Big|_{y=y_\alpha} \tag{35}$$
$$= \frac{1}{3M_5^3}[Te^{2A} - 4V'_\alpha\delta\phi + 2h_{55}V'_\alpha]\Big|_{y=y_\alpha}$$

or equivalently using the vacuum equations

$$(-1)^{\alpha+1}[\partial_5(\Box^2 h^L - \frac{1}{3}f^T)]\Big|_{y=y_\alpha} = \frac{e^{2A}}{3M_5^3}T(y)\Big|_{y=y_\alpha} \tag{36}$$

Eqs. (30-34) represent a complete set of Einstein equations.

The $\delta\phi$ equation reads

$$e^{2A}\Box^2\delta\phi - 4A'\delta\phi' + \delta\phi'' - V''(\phi_o)\delta\phi + h_{55}V'(\phi_o) + \frac{1}{2}\partial_5(\Box^2 h^L + f^T)\phi'_o$$
$$-\frac{1}{2}\phi'_o\partial_5 h_{55} = \sum_\alpha \delta(y-y_\alpha)[\frac{1}{2}h_{55}V'_\alpha + V''_\alpha\delta\phi] \tag{37}$$

with boundary conditions

$$\delta\phi'(y_\alpha) = (-1)^{\alpha+1}\frac{1}{2}[\frac{1}{2}V'_\alpha h_{55} + V''_\alpha\delta\phi]\Big|_{y=y_\alpha} \tag{38}$$

3 R_{j5} Equation

We consider first the R_{j5} equation. Inserting in Eq.(6) and the orthogonal decomposition of h_i, Eq.(31) becomes

$$\frac{1}{2}\partial_5(\partial_j f^T - \Box^2 h_j^T) + \frac{3}{2}A'\partial_j h_{55} = -\frac{\phi'_0}{M_5^3}\partial_j\delta\phi \tag{39}$$

which can be decomposed into its transverse and longitudinal parts

$$\partial_5 f^T + 3A'h_{55} + 2\frac{\phi'_0}{M_5^3}\delta\phi = 0 \tag{40}$$
$$\partial_5 h_j^T = 0 \tag{41}$$

Eq.(41) implies that $h^T_j = h^T_j(x^i)$ is independent of y and one may use the remaining gauge freedom of Eqs.(24) and (26) to set h^T_j to zero,

$$h^T_j(x^i) = 0 \tag{42}$$

Each of the terms in Eq.(40) are gauge variant, and it is interesting to see how the gauge invariance of the sum arises. Thus using Eqs.(22),(23), and (28), the gauge change of the left hand side (lhs) of Eq.(40) is

$$\delta(lhs) = \partial_y(-6A'\xi^5) + 6A'\xi^5_{,5} + 2\frac{(\phi_0')^2}{M_5^3}\xi^5 \tag{43}$$

or

$$\delta(lhs) = (-6A'' + 2\frac{(\phi_0')^2}{M_5^3})\xi^5 \tag{44}$$

Using the vacuum metric equations, Eqs.(11,12) this reduces to

$$\delta(lhs) = -\frac{2}{M_5^3}\sum_\alpha \delta(y-y_\alpha)V_\alpha(\phi_0)\xi^5(x^i,y) \tag{45}$$

which vanishes as a consequence of Eq.(20). Thus the gauge invariance of Eq.(40) is directly related to the condition that there be no brane bending.

Eq.(40) allows us to eliminate h_{55} in terms of f^T and $\delta\phi$

$$h_{55} = -\frac{1}{3A'}\partial_5 f^T - \frac{2}{3M_5^3}\frac{\phi_0'}{A'}\delta\phi \tag{46}$$

a relation that holds thoughout the bulk. As mentioned in Sec.2, it is possible to choose a special gauge so that h_{55} vanishes on the branes, Eq.(29). In that case one has

$$\partial_5 f^T(x^i,y)\Big|_{y=y_\alpha} = -\frac{2}{M_5^3}(\phi_0'(y)\delta\phi(x^i,y))\Big|_{y=y_\alpha} \tag{47}$$

and one can eliminate $\partial_5 f^T$ in terms of $\delta\phi$ on the branes. [An alternate possibility is to choose a special gauge such that on the branes

$$h_{55}\Big|_{y=y_\alpha} = -\frac{2}{3M_5^3}(\frac{\phi_0'}{A'}\delta\phi)\Big|_{y=y_\alpha} \tag{48}$$

and then $\partial_5 f^T$ would vanish on the branes. However, in the following we will make use of Eq.(29)].

4 R_{55} and $\eta^{ij}R_{ij}$ Equations

Eqs.(32) and (33) give relations between f^T, h^L, and h_{55}. A convenient way to analyse these is to first consider the difference Eq.(33)-Eq.(32). Then the $\delta(y-y_\alpha)$ terms cancel and the resulting equation

$$3A'\partial_5(\Box^2 h^L + f^T) - e^{2A}\Box^2 f^T + h_{55}(12A'^2 - \frac{(\phi_0')^2}{M_5^3}) = \frac{2}{M_5^3}V'(\phi_0)\delta\phi - \frac{2\phi_0'}{M_5^3}\delta\phi' \quad (49)$$

is valid both in the bulk and on the branes. Eliminating h_{55} by Eq.(46) one has

$$\begin{aligned}\partial_5(\Box^2 h^L - \frac{1}{3}f^T) &= \frac{1}{3A'}e^{2A}\Box^2 f^T - \frac{(\phi_0')^2}{3M_5^3})\partial_5 f^T \\ &+\frac{2}{9A'M_5^3}\frac{\phi_0'}{A'}(12A'^2 - \frac{(\phi_0')^2}{M_5^3})\delta\phi \\ &+\frac{2}{3A'^2M_5^3}V'(\phi_0)\delta\phi - \frac{2\phi_0'}{3M_5^3A'})\delta\phi'\end{aligned} \quad (50)$$

One can integrate Eq.(50) to obtain h^L in terms of f^T and $\delta\phi$. The boundary conditions Eq.(36) involve precisely the same combination as the l.h.s. of Eq.(50), and since Eq.(50) has been seen to hold on the branes (with no $\delta(y-y_\alpha)$ singular terms), its solution can be inserted into Eq.(36). In the static approximation one has

$$T(y_\alpha) = -e^{2A(y_\alpha)}T_{00}(y_\alpha) \quad (51)$$

so that on the boundaries one has

$$[\frac{1}{3A'}e^{2A}\Box^2 f^T - \frac{(\phi_0')^2}{9A'^2M_5^3})\partial_5 f^T + \frac{2}{3A'M_5^3}(V' + \frac{\phi_0'}{3A'}(4A' - \frac{(\phi_0')^2}{3A'M_5^3})) - \frac{2\phi_0'}{3A'M_5^3}\delta\phi']\Big|_{y=y_\alpha} = \frac{(-1)^\alpha}{3M_5^3}e^{2A(y_\alpha)}T_{00}(y_\alpha) \quad (52)$$

Finally in the gauge choice of Eq.(47) this reduces to

$$\begin{aligned}\Box^2 f^T\Big|_{y=y_\alpha} = &\frac{(-1)^\alpha}{M_5^3}A'T_{00}(y_\alpha) - \\ &\frac{2}{M_5^3}[e^{-2A}\delta\phi(V'(\phi_0) + 4A'\phi_0' - \frac{\gamma_\alpha}{2}\phi_0')]\Big|_{y=y_\alpha}\end{aligned} \quad (53)$$

Note that f^T and $\delta\phi$ are gauge invariant on the branes so that Eq.(53) is a gauge invariant relation.

The quantity that governs the Newtonian potential is h_{00}, and in the static limit this is given on the branes by

$$h_{00}(x^i, y_\alpha) = h_{00}^{TT}(x^i, y_\alpha) - \frac{1}{3} f^T(x^i, y_\alpha) \qquad (54)$$

Eq.(53) determines the f^T contribution in terms of T_{00} and $\delta\phi$. The effect of the scalar stabilizing term is to add an additional term, the bracket of Eq.(53), and modify the A' factor in the first term, e.g. for the example of Eqs.(14) and (15)

$$A' = \beta - \frac{b}{6} \frac{\phi_0^2}{M_5^3} \qquad (55)$$

To examine the effects of these modifications to f^T we consider next the $\delta\phi$ field equation.

5 $\delta\phi$ Field Equation

The $\delta\phi$ field equation, Eq.(37), depends on both h_{55} and the combination $\partial_5(\Box^2 h^L + f^T)$. One may eliminate h_{55} using Eq.(46) and $\partial_5(\Box^2 h^L + f^T)$ by Eq.(50). One gets in this way a rather complicated equation involving only $\delta\phi$ and f^T. While f^T is determined on the branes by Eq.(53) (and is gauge invariant there), it is gauge variant in the bulk, as is $\delta\phi$. However from Eqs.(23) and (28), the combination

$$Q \equiv \delta\phi + \frac{1}{6} \frac{\phi_0'}{A'} f^T \qquad (56)$$

is gauge invariant in the bulk. Thus if we eliminate $\delta\phi$ in terms of Q, one will obtain an equation involving only Q, f^T, $\partial_5 f^T$, and $\partial_5^2 f^T$. However, the latter three are gauge variant in the bulk, and so gauge invariance implies that the coefficients of these three quantities must actually vanish leaving an equation involving only the gauge invariant quantity Q. A detailed and somewhat lengthy calculation shows that this is indeed the case and the equation for Q reduces to the following relatively simple form in the bulk

$$e^{2A}\Box^2 Q + Q'' - 4A'Q' - V''Q + \frac{4}{3M_5^3}(2(\phi_0')^2 + \frac{\phi_0'}{A'}V')Q - \frac{2(\phi_0')^4}{9A'^2 M_5^6} Q = 0 \qquad (57)$$

Eq.(57) thus gives us an uncoupled equation that determines Q in the bulk. One may limit the solution to the branes and impose the boundary conditions of Eq.(38). In terms of Q, this reads

$$[\delta Q' - \frac{1}{6}(\frac{\phi_0'}{A'})' - \frac{1}{6}\frac{\phi_0'}{A'}\partial_5 f^T]\Big|_{y=y_\alpha} = \frac{1}{2}(-1)^{\alpha+1}[\frac{1}{2}V_\alpha' h_{55} + V_\alpha'' Q - -\frac{1}{6}V_\alpha'' f^T]\Big|_{y=y_\alpha} \tag{58}$$

One needs

$$(\frac{\phi_0'}{A'})' = \frac{\phi_0''}{A'} - \frac{\phi_0'}{A'^2}A'' \tag{59}$$

and using Eqs.(11-13)

$$(\frac{\phi_0'}{A'})' = (4\phi_0' + \frac{V'}{A'} - \frac{1}{3M_5^3}\frac{(\phi_0')^3}{A'^2}) + \sum_\alpha \delta(y-y_\alpha)\frac{V_\alpha'}{A'} \tag{60}$$

We interpret the prescription of Eq.(58) to mean the limit from the bulk as $y \to y_\alpha$ and so the last term in Eq.(60) does not contribute to Eq.(58). One has then the boundary condition

$$[\delta Q' - \frac{1}{6}(4\phi_0' + \frac{V'}{A'})f^T - \frac{1}{6}\frac{\phi_0'}{A'}\partial_5 f^T]\Big|_{y=y_\alpha} = \frac{1}{2}(-1)^{\alpha+1}[[\frac{1}{2}V_\alpha' h_{55} + V_\alpha'' Q - \frac{1}{6}\frac{\phi_0'}{A'}V_\alpha'' f^T]\Big|_{y=y_\alpha} \tag{61}$$

It is convenient now to make use of the gauge condition Eq.(29) which also allows us to eliminate $\partial_5 f^T(x^i, y_\alpha)$ by Eq.(47) (from the R_{j5} field equation)

$$\partial_5 f^T\Big|_{y=y_\alpha} = -\frac{2}{M_5^3}[\phi_0'(Q - \frac{1}{6}\frac{\phi_0'}{A'}f^T)]\Big|_{y=y_\alpha} \tag{62}$$

The Q boundary condition then becomes

$$[Q' - \frac{1}{6}(4\phi_0' + \frac{V'}{A'})f^T + \frac{1}{3M_5^3}\frac{(\phi_0')^2}{A'}(Q - \frac{1}{6}\frac{\phi_0'}{A'}f^T)]\Big|_{y=y_\alpha} = \frac{1}{2}(-1)^{\alpha+1}[V_\alpha'' Q - -\frac{1}{6}\frac{\phi_0'}{A'}V_\alpha'' f^T]\Big|_{y=y_\alpha} \tag{63}$$

We can also eliminate $\delta\phi$ in terms of Q in Eq.(53) yielding

$$\Box^2 f^T\Big|_{y=y_\alpha} = \frac{(-1)^\alpha}{M_5^3}A' T_{00}(y_\alpha) + \frac{2}{M_5^3}[e^{-2A}(\frac{1}{2}(-1)^{\alpha+1}\phi_0' V_\alpha'' - V' - 4A'\phi_0')(Q - \frac{1}{6}\frac{\phi_0'}{A'}f^T)]\Big|_{y=y_\alpha} \tag{64}$$

6 Leading Order Solutions

As discussed above the Newtonian potential is obtained from the static approximation to $h_{00}(x^i, y_\alpha)$ on the branes, given in Eq.(54). $h_{00}^{TT}(x^i, y_\alpha)$ is to be obtained by solving Eq.(30) (which is similar to the result of [23] when no scalar field was present except for the modification of $A(y)$). $f^T(x^i, y_\alpha)$ on the branes is governed by the coupled equations Eqs.(63) and (64). Since Eq.(63) depends on Q', one cannot use it to eliminate Q in Eq.(64) (to obtain an equation depending only on f^T) and one must first solve Eq.(57) for Q in the bulk and then insert it into the boundary conditions Eqs.(63) and (64) and use those to determine $f^T(x^i, y_\alpha)$ on the branes. (Thus it is the boundary conditions on the branes that couple Q and f^T.) The Newtonian potential then arises from the $1/r$ part of $h_{00}(x^i, y_\alpha)$. Since both Eq.(30) and Eq.(57) are decoupled equations, the above analysis is in principle doable.

An analytic solution of the second order differential equations Eqs.(30) and (57) is not possible due to the fact they depend on the complicated functions $A(y)$ and $\phi_0(y)$. We note, however, that the corrections to $A(y)$ is proportional to ϕ_0^2/M_5^3. On the y_2 brane this contains a factor of $e^{-2\beta y_2} \approx 10^{-32}$ and is very small. If we assume also $\phi_1^2/M_5^3 \ll 1$, this correction is also small on the y_1 brane and one can consider an iteration scheme based on the smallness of ϕ_0^2/M_5^3. Thus to lowest order, Eq.(30) gives rise to the same gravitational potential as in [23] (where no scalar field was present)

$$V^{TT}(y_\alpha) = -\frac{4}{3}\frac{\beta}{8\pi M_5^3}\frac{1}{r}[\bar{m_\alpha}\bar{m_\alpha}' + \bar{m_1}\bar{m_2}] \tag{65}$$

where $\bar{m_\alpha} = e^{-\beta y_\alpha} m_\alpha$ are the observed masses on the y_α brane (and $m_\alpha \approx \mathcal{O}(M_{Pl})$). Higher order effects are presumably small since they are scaled by ϕ_0^2/M_5^3.

Eq.(57), which determines Q, shows a similar structure. Thus in the example of Eq.(16), V'' begins as a constant with $\mathcal{O}(\phi_0^2/M_5^3)$ corrections and the remaining terms have $\mathcal{O}(\phi_0^2/M_5^3)$, $\mathcal{O}(\phi_0^4/M_5^6)$, ... corrections. Thus to lowest order, in the bulk Q obeys the equation

$$e^{2\beta y}\Box^2 Q + Q'' - 4\beta Q' - \gamma^2 Q = 0 \tag{66}$$

where

$$\gamma^2 = [V''(\phi_0)]_{\phi_0=0} \;\; ; \;\; \beta = [A']_{\phi_0=0} \tag{67}$$

(and $\gamma^2 = b^2 + 4b\beta = \mathcal{O}(M_{Pl}^2)$ in the model of Eq.(16)). Again, higher order corrections should be of $\mathcal{O}(\phi_0^2/M_5^3)$ and be small. The boundary conditions Eq.(63) contain corrections to the zeroth order part of $\mathcal{O}(\phi_0' f^T)$ as well as $\mathcal{O}((\phi_0')^3/M_5^3)f^T)$, $\mathcal{O}(((\phi_0')^5/M_5^6)Q)$. If one neglects all these corrections, Eq.(66) represents free waves propagating in the bulk (discussed in some detail in [6]). They in general give no $1/r$ Newtonian contribution (without extreme fine tuning of parameters). Alternately, one might impose a Sommerfeld boundary condition that requires excitations in the bulk to arise from matter on the branes. To first order this will occur by including the $\mathcal{O}(\phi_0' f^T)$ term, since to zero'th order f^T is proportional to T_{00}. We thus take as the first order boundary condition

$$[Q' - \frac{1}{2}(-1)^{\alpha+1}\gamma_\alpha Q]\Big|_{y_\alpha} = -\frac{1}{6}[\frac{\gamma^2}{\beta b} - 4 + \frac{1}{2}(-1)^{\alpha+1}\frac{\gamma_\alpha}{\beta}]b\phi_0 f^T\Big|_{y_\alpha} \tag{68}$$

where

$$\gamma_\alpha \equiv V_\alpha''(\phi_0)\Big|_{\phi_0=0} \;\; ; \;\; b \equiv -\frac{\phi_0'}{\phi_0} \tag{69}$$

Turning to the second boundary equation Eq.(64) one might at first suggest that one could ignore the bracket on the r.h.s. as it is $\mathcal{O}((\phi_0^2/M_5^3)f^T)$, and include its effects in by iteration. However, to lowest order, the T_{00} term gives a Newtonian piece to $f^T \sim 1/r$, and if one inserts that into the bracket term, one sees that in the static limit ($\Box^2 \to \nabla^2$) the next approximation goes as $\nabla^{-2}(1/r)$ which is infrared divergent. Thus one must include the lowest order part of the bracket in the first approximation:

$$\begin{aligned}\Box^2 f^T\Big|_{y=y_\alpha} &= \frac{(-1)^\alpha}{M_5^3}\beta T_{00}(y_\alpha) + \\ &\frac{2}{M_5^3}[e^{-2\beta y}(4\beta b - \frac{1}{2}(-1)^{\alpha+1}\gamma_\alpha b - \gamma^2)\phi_0(Q + \frac{1}{6}\frac{b\phi_0}{\beta}f^T)]\Big|_{y=y_\alpha}\end{aligned} \tag{70}$$

We have kept the Q term on the r.h.s. of Eq.(70) as we will see below it is of size $\phi_0 f^T$ as a consequence of Eq.(68).

7 First Order Solutions for Q and f^T

To obtain the f^T part of the Newtonian potential to first order one must solve Eq.(66), insert it into the coupled boundary conditions Eqs.(68) and

(70), and then solve these equations for $f^T(y_\alpha)$. To solve Eq.(66) we let

$$Q(x^i, y) = \int d^4p e^{ipx} Q(p^i, y) \tag{71}$$

and set

$$Q(p, y) = e^{2\beta y} R(p, \xi) \;\; ; \;\; \xi(y) = \frac{m}{\beta} e^{\beta y} \tag{72}$$

where $m^2 = -p^2 = (p^0)^2 - \vec{p}^2$. (In Eq.(72) m/β is shorthand for $(m^2/\beta^2)^{1/2}$ where the branch cut is defined by being real and positive for $m^2 > 0$.) One finds that R obeys the Bessel equation and so the general solution to Eq.(66) is

$$Q(p, y) = e^{2\beta y}[A(p) J_\nu(\xi) + B(p) N_\nu(\xi)] \tag{73}$$

where $\nu = (4 + \gamma^2/\beta^2)^{1/2}$, and A and B are constants of integration to be determined by Eq.(68).

It is convenient to introduce the notation

$$\lambda_b = b/\beta \tag{74}$$

Since by hypothesis b and β are $\mathcal{O}(M_{Pl})$ we expect $\lambda_b = \mathcal{O}(1)$. We require $b > 0$ so that $\phi_0(y)$ is a decreasing function, and $\beta > 0$ to achieve the gauge hierarchy so that $\lambda_b > 0$. With this notation $\gamma^2/\beta^2 = \lambda_b^2 + 4\lambda_b$ in the model of Eq.(16) so that

$$\nu = 2 + \lambda_b > 2 \tag{75}$$

(The Appendix shows that the above actually represents the leading terms of a general model when $\phi_0^2(y_1)/M_5^3$ is small and $\phi_0(y)$ is a decreasing function.) Imposing the boundary conditions Eq.(68) determines A and B to be

$$\begin{aligned} A &= \frac{1}{D}[\xi_2 N_{\nu-1}(\xi_2) - (\nu - 2 - \lambda_2) N_\nu(\xi_2)][-\frac{(\lambda_b + \lambda_1)}{6\beta}\phi_0'(y_1) f^T(y_1)] \\ &- \frac{1}{D}[\xi_1 N_{\nu-1}(\xi_1) - (\nu - 2 + \lambda_1) N_\nu(\xi_1)][-\frac{e^{2\beta y_2}}{6\beta}(\lambda_b - \lambda_2)\phi_0'(y_2) f^T(y_2)] \end{aligned} \tag{76}$$

and

$$\begin{aligned} B &= -\frac{1}{D}[\xi_2 J_{\nu-1}(\xi_2) - (\nu - 2 - \lambda_2) J_\nu(\xi_2)][-\frac{(\lambda_b + \lambda_1)}{6\beta}\phi_0'(y_1) f^T(y_1)] \\ &+ \frac{1}{D}[\xi_1 J_{\nu-1}(\xi_1) - (\nu - 2 + \lambda_1) J_\nu(\xi_1)][-\frac{e^{2\beta y_2}}{6\beta}(\lambda_b - \lambda_2)\phi_0'(y_2) f^T(y_2)] \end{aligned} \tag{77}$$

where $\xi_{1,2} = \xi(y_{1,2})$ and D is given by

$$D = [\xi_1 J_{\nu-1}(\xi_1) - (\nu - 2 + \lambda_1)J_\nu(\xi_1)][\xi_2 N_{\nu-1}(\xi_2) - (\nu - 2 - \lambda_2)N_\nu(\xi_2)]$$
$$-[\xi_1 N_{\nu-1}(\xi_1) - (\nu - 2 + \lambda_1)N_\nu(\xi_1)][\xi_2 J_{\nu-1}(\xi_2) - (\nu - 2 - \lambda_2)J_\nu(\xi_2)] \quad (78)$$

The boundary condition Eq.(70) reduces to leading order in momentum space to

$$m^2 f^T(y_\alpha) = (-1)^\alpha \frac{\beta}{M_5^3} T_{00}(y_\alpha) \quad (79)$$
$$+2\frac{\beta b}{M_5^3}[(-1)^\alpha \lambda_\alpha - \lambda_b]\phi_0[(AJ_\nu(\xi) + BN_\nu(\xi)) + \frac{\lambda_b}{6}e^{-2\beta y}\phi_0 f^T(y)]\Big|_{y=y_\alpha}$$

where

$$\lambda_\alpha \equiv \frac{\gamma_\alpha}{2\beta} = \mathcal{O}(1) \quad (80)$$

To calculate the Newtonian contribution of $f^T(y_\alpha)$ we take the static limit ($m^2 = -p^2 \to -\vec{p}^2$) and take the low momentum limit of the right hand side (rhs) by limiting $\xi_{1,2} \to 0$. For ξ_2 this means we assume

$$\xi_2 = \frac{m}{\beta}e^{\beta y_2} \ll 1 \quad (81)$$

or we are considering momenta

$$p \leq \beta e^{-\beta y_2} \approx M_{Pl}10^{-16} \approx 1TeV \quad (82)$$

corresponding to distances $r \gtrsim 10^{-17}$cm. (The expansion for ξ_1 is valid for distances greater than the Planck length). Thus for all experimental tests of Newtonian forces, this expansion is valid. Keeping the leading terms we find for $f^T(y_2)$ that

$$AJ_\nu(\xi_2) + BN_\nu(\xi_2) \simeq \quad (83)$$
$$\frac{1}{D}[\frac{(\nu - 2 - \lambda_2)\lambda_b}{6\nu\pi}\phi_1(\lambda_b + \lambda_1)f^T(y_1)$$
$$-\frac{(\nu - 2 + \lambda_1)\lambda_b}{6\nu\pi}(\lambda_b - \lambda_2)\phi_1 f^T(y_2)$$
$$+\frac{(\nu + 2 + \lambda_2)\lambda_b}{6\nu\pi}\phi_1(\lambda_b + \lambda_1)f^T(y_1)$$
$$-\frac{(\nu + 2 - \lambda_1)\lambda_b}{6\nu\pi}(\lambda_b - \lambda_2)\phi_1 f^T(y_2)e^{-2\nu\beta y_2}]$$

where expanding $1/D$ gives

$$1/D \cong -\frac{\nu\pi e^{-\nu\beta y_2}}{(\nu+2+\lambda_2)(\nu-2+\lambda_1)}[1+\frac{(\nu+2-\lambda_1)(\nu-2-\lambda_2)}{(\nu-2+\lambda_1)(\nu+2+\lambda_2)}e^{-2\nu\beta y_2}] \quad (84)$$

Eq.(79) in the static limit then becomes to leading order (the second term of Eq.(84) is negligible)

$$-\vec{p}^2 f^T(y_2) = \frac{\beta}{M_5^3}T_{00}(y_2) + \alpha^2 e^{-(2+2\lambda_b)\beta y_2}[f^T(y_2) - f^T(y_1)] \quad (85)$$

where

$$\alpha^2 = \frac{2}{3}\frac{\phi_1^2}{M_5^3}\beta^2\lambda_b^2\frac{(2+\lambda_b)(\lambda_2-\lambda_b)}{4+\lambda_b+\lambda_2} \quad (86)$$

The $e^{-(2+2\lambda_b)\beta y_2}$ in Eq.(85) comes from the $e^{-\nu\beta y_2}$ of $1/D$ and the ϕ_0 factor in Eq.(70) (i.e. $\phi_0 = \phi_1 e^{-\lambda_b\beta y_2}$).

The analysis for $f^T(y_1)$ is more subtle. Here we find that the numerator of $AJ_\nu(\xi_1) + BN_\nu(\xi_1)$ terms of Q gives contributions of size $e^{-\nu\beta y_2}$ (from e.g. $N_\nu(\xi_2)J_\nu(\xi_1) \sim (\xi_1/\xi_2)^\nu$) and size $e^{\nu\beta y_2}$ (from $J_\nu(\xi_2)N_\nu(\xi_1)$). Multiplying by $1/D$ then gives terms of size $e^{-2\nu\beta y_2}$ and $\mathcal{O}(1)$. The $\mathcal{O}(1)$ term actually cancels with the $\mathcal{O}(1)$ term of $\phi_0 f^T(y_1)/6$ of Eq.(79), and so one must keep the second factor in the bracket of Eq.(84) to get a total result of size $e^{-2\nu\beta y_2}$ on the rhs:

$$-\vec{p}^2 f^T(y_1) = -\frac{\beta}{M_5^3}T_{00}(y_1) + \alpha^2 e^{-(4+2\lambda_b)\beta y_2}(f^T(y_2) - f^T(y_1)) \quad (87)$$

Note that λ_1 does not enter in these leading order results.

One can now easily solve Eqs.(85) and (87) to get in coordinate space the results

$$-\frac{1}{3}f^T(y_2) = \frac{1}{3}\frac{G_N\bar{m}_2}{r}e^{-\mu r}e^{2\beta y_2} - \frac{1}{3}\frac{G_N}{r}(m_1+\bar{m}_2)(1-e^{-\mu r}) \quad (88)$$

and

$$-\frac{1}{3}f^T(y_1) = -\frac{1}{3}\frac{G_N}{r}(m_1+\bar{m}_2) + \frac{1}{3}\frac{G_N}{r}\bar{m}_2 e^{-\mu r} - \frac{1}{3}\frac{G_N}{r}e^{-2\beta y_2}(m_1+\bar{m}_2)(1-e^{-\mu r}) \quad (89)$$

where the Newton constant is given by

$$G_N \equiv \frac{\beta}{8\pi M_5^3} \tag{90}$$

and

$$\mu^2 \equiv \alpha^2 e^{-(2+2\lambda_b)\beta y_2} \tag{91}$$

The requirement $\mu^2 \geq 0$ implies $\lambda_2 \geq \lambda_b$ or $\lambda_2 < -(4+\lambda_b)$.

In the limit $\phi_1 \to 0$ (no scalar field) Eqs.(88) and (89) reduce to the results of [23]. The presence of the scalar field does indeed grow a mass for the f^T field and so in the limit $\mu_2 r \gg 1$ the remaining $1/r$ piece in Eq.(88) precisely combines with the h_{00}^{TT} of Eq.(65) to give a total Newton potential with Newton constant of Eq.(90) on the TeV brane. (On the Planck brane the last term in Eq.(89) is negligible and one gets an additional factor of 5/3.) However, the factor $e^{2\beta y_2}$ in the first term of Eq.(88) remains, and the mass μ is suppressed by $e^{-(1+\lambda_b)\beta y_2}$. One may ask how large r has to get so that this anomalous behavior becomes negligible and Newtonian physics is reproduced on the TeV brane. As a measure of the effects seen here, we assume that the Newtonian force has been measured at the 1% level, so that the large dominant term in Eq.(88) implies

$$\frac{1}{3} e^{-\mu r} e^{2\beta y_2} < 10^{-2} \tag{92}$$

or

$$\mu r > 2\beta y_2 - ln(0.03) \tag{93}$$

Since we are assuming ϕ_1 is small and $\beta \simeq M_{Pl}$ we set $\phi_1^2/M_5^3 = 1/10$, $\beta = 1.22 \times 10^{19} GeV$ and $e^{2\beta y_2} = 10^{32}$. Eq.(93) then implies

$$f^{1/2} e^{-\lambda_b \beta y_2} r > 4.827 \times 10^{-15} cm \tag{94}$$

where

$$f = \frac{\lambda_b^2 (2+\lambda_b)(\lambda_2 - \lambda_b)}{4 + \lambda_b + \lambda_2} \tag{95}$$

Current gravitational force experiments have been done at a separation between masses as small as $10\mu m$[26]. Hence we require

$$f^{1/2} e^{-\lambda_b \beta y_2} > 4.827 \times 10^{-12} \tag{96}$$

Eq.(96) gives an exclusion contour in the $\lambda_2 - \lambda_b$ parameter space. For example for $\lambda_2 = 1$, one requires

$$\lambda_b < 0.67 \tag{97}$$

to avoid disagreement with experiment. The general excluded region is shown in Fig.1. Thus the absence of any deviation from the Newtonian force law already rules out a large amount of parameter space.

8 Conclusions

We have examined here the gravitational forces between point particles on the branes in the 5D Randall-Sundrum model with two branes and S^1/Z_2 symmetry (the RS1 model). In terms of the orthogonal decomposition of the 4D part of the metric of Eq.(5), the static Newtonian forces should arise from $h_{00} = h_{00}^{TT} - f^T/3$ on the branes $y = y_\alpha, \alpha = 1, 2$, where h_{00}^{TT} is the transverse traceless part of the metric (and also contains the Kaluza-Klein corrections) and f^T is the trace of the transverse part of the metric. In order to impose the S^1/Z_2 boundary conditions correctly, it is necessary that the coordinate conditions chosen do not produce brane bending. Thus we assume here only that $h_{5i}(x^i, y) = 0, i = 0, 1, 2, 3$. While it is not possible to have h_{55} vanish everywhere, one can assume it vanishes on the branes. $h_{ij}^{TT}(x^i, y)$ is gauge invariant with respect to the remaining gauge freedom, and f^T is gauge invariant on the branes (and plays the role of the radion).

Without a scalar field, the amplitude of $f^T(y_2)$ is enhanced by a factor $e^{2\beta y_2} \simeq 10^{32}$ making the theory in serious disagreement with experiment[23]. In this work we have included a scalar stabilizing field in the bulk $\phi(x^i, y) = \phi_0(y) + \delta\phi$, where $\phi_0(y)$ is the vacuum solution and $\delta\phi$ responds to matter on the branes. The presence of ϕ can allow $f^T(y_2)$ to grow a mass, suppressing it. To examine this possibility we considered the case where ϕ_0^2/M_5^3 was small and $\phi_0(y)$ is a decreasing function of y. Then one can obtain analytically the leading order corrections. One finds that $f^T(y_2)$ does indeed grow a factor $e^{-\mu r}$ but is still enhanced by the $e^{2\beta y_2}$ factor. Further, the mass μ of Eq.(91) is suppressed by the exponential factor $e^{-(1+\lambda_b)\beta y_2}$ where λ_b defined in Eq.(74) is positive. Thus whether the RS1 model is in agreement with current small distance measurements of the Newtonian force law depends upon a subtle interplay between the amplitude enhancement and the exponential suppression of the mass. Current data eliminates large parts of the parameter space. The remaining allowed region is shown in Fig.1.

The Randall-Sundrum 1 model shows interesting features not intuitively expected. Thus the fact that an exponential appears in the metric (a feature of the solution of the 5D vacuum Einstein equations) modifies the ideas of naturalness. While one would expect that the mass of f^T would scale by β, i.e. $\mu \sim \beta = \mathcal{O}(M_{Pl})$ (with perhaps a model dependent factor) the unexpected feature is the additional (model dependent) exponential factor, i.e. $\mu \sim \beta e^{-(1+\lambda_b)\beta y_2}$. Since exponentials vary rapidly, they radically change the 'natural' expectation of the size of μ. Such phenomena are intrinsic to the Randall-Sundrum model, since one is using exponentials to create an 'unnatural' solution of the gauge hierarchy problem. Further, the inverse of the very exponentials needed for the gauge hierarchy can enter from metric factors appearing in the denominator and do so in the amplitude of f^T i.e. $f^T \sim e^{2\beta y_2}$. It is thus remarkable that the theory can survive the experimental tests of the Newtonian force law. Improvements of these experiments at distances smaller than $10\mu m$ will therefore further test the model.

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A Appendix

In Sec.7 we considered a special solution for A and ϕ_0 of Eqs.(14-15). We show here that this actually represents the leading terms of the more general Eqs.(11-13) when $\phi_0(x^i, y_1)$ is small (i.e. $\phi_0^2(y_1)/M_5^3 \ll 1$) and ϕ_0 is a decreasing function of y.

For the situation considered, we can expand $A'(y)$ and $V(\phi_0)$ in a power series in ϕ_0^2

$$A'(y) = \beta + \mathcal{O}(\phi_0^2) + \ldots \tag{A.1}$$

$$V(\phi_0) = \frac{1}{2}\gamma^2\phi_0^2 + \mathcal{O}(\phi_0^4) + \ldots \tag{A.2}$$

where β and γ^2 are arbitrary constants. Eq.(13) in the bulk gives then

$$\phi_0'' - 4\beta\phi_0' - \gamma^2\phi_0 + \mathcal{O}(\phi_0^3) = 0 \tag{A.3}$$

To leading order then (since ϕ_0 is decreasing)

$$\phi_0 = \phi_1 e^{-by} + \ldots \quad ; \quad b > 0 \tag{A.4}$$

where

$$b^2 + 4\beta b - \gamma^2 = 0 \tag{A.5}$$

It is convenient to introduce the parameter

$$\lambda_b \equiv \frac{b}{\beta} \tag{A.6}$$

As discussed in [6], since $b > 0$, the gauge hierarchy requires $\beta > 0$ so that

$$\lambda_b > 0 \tag{A.7}$$

and Eq.(A.5) implies

$$\frac{\gamma^2}{\beta^2} = (\lambda_b^2 + 2)^2 - 4 \tag{A.8}$$

Eqs.(11) and (12) imply in the bulk that

$$3A'' = \frac{(\phi_0')^2}{M_5^3} \tag{A.9}$$

and inserting Eq.(A.4) (with the conventional boundary condition $A(0) = 0$) one finds

$$A = \beta y + \frac{\phi_1^2}{12 M_5^3} e^{-2by} + \ldots \tag{A.10}$$

showing that the special solutions Eqs.(14) and (15) are the leading terms in the more general case. The higher terms in Eqs.(A.4) and (A.10) are determined by the choice of bulk and brane potentials $V(\phi)$ and $V_\alpha(\phi)$.

As discussed in [6], it is still possible to achieve a solution of the gauge hierarchy when $b < 0$ and $\phi_0(y)$ is an increasing function of y. This situation is more complicated than the one treated in this paper since the terms $(\phi_0^2(y_1)/M_5^3)^2 Q$ for example in Eq.(57) might become large and dominate. Then an analytic solution as discussed here does not seem possible.

(A.11)

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