## Seesaw–constrained MSSM, Solution to the SUSY CP Problem and a Supersymmetric Explanation of $\epsilon'/\epsilon$

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## Abstract

We analyze CP violation in the supersymmetric standard model (MSSM) embedded minimally into a left-right symmetric gauge structure with the seesaw mechanism for neutrino masses. With the plausible assumption of universal scalar masses it is shown that CP violation in the hadron sector of the MSSM is described by a single phase residing in the supersymmetry breaking Lagrangian. This improves the CP properties of the MSSM by providing a natural solution to the SUSY CP problem. Furthermore,  $\epsilon'/\epsilon$  vanishes in this model above the seesaw scale; extrapolation to the weak scale then leads to a prediction in agreement with the NA31 and the recent KTeV observations. The electric dipole moment of the neutron is naturally suppressed, we estimate its magnitude to be about  $4 \times 10^{-29}$  ecm. Additional predictions include a tightly constrained super particle spectrum and vanishingly small CP asymmetries in the *B* meson system.

Recent evidences for neutrino oscillations imply that the standard model must be extended to accommodate small neutrino masses. An elegant model that provides an explanation of the small neutrino masses via the seesaw mechanism [1] is the left-right symmetric model of weak interaction. Two seemingly unrelated puzzles of the standard model, viz., the stability of the Higgs-boson mass and the question of the origin of the electroweak symmetry breaking, seem to require its supersymmetric extension – the MSSM [2] – for a proper resolution. It is well known that MSSM by itself is plagued with a number of problems which were not present in the standard model. In this Letter, we will be concerned with one such problem, viz., the emergence of a plethora of arbitrary CP phases that require severe fine-tuning to explain the smallness of observed CP violation in the Kaon system as well as the non-observation of the electric dipole moment of the neutron [3–5]. This is known as the SUSY CP problem. A promising approach towards its resolution would be to realize MSSM as a low energy limit of a theory where this as well as other problems of MSSM are cured. Here we explore supersymmetric left-right model with the seesaw mechanism for neutrino masses as a candidate theory.

It has been pointed out that in the supersymmetric version of the left-right model (SU-SYLR) [6,7] which embodies the seesaw mechanism (and therefore explains the small neutrino masses) the constraints of parity invariance provide a simple resolution of the SUSY CP problem [8]. Neutrino masses as suggested by SuperKamiokande require that the scale above which the SUSYLR model manifests itself is at least  $10^{12}$  GeV, if the seesaw mechanism is implemented via the renormalizable terms in the superpotential, or close to the canonical GUT scale of  $10^{16}$  GeV if one envisions the Planck scale suppressed non-renormalizable terms as the source of the seesaw. In either case the model could finally be embedded into an SO(10) grand unified theory. The basic assumption of this paper is that above the scale where the SUSYLR model is valid, the Lagrangian is invariant under the parity transformation and below the seesaw scale the theory is MSSM, with constraints on its parameters as required for its embedding into the SUSYLR model.

We will focus our attention on the flavor mixing and CP violation in the minimal versions of the high scale SUSYLR (or SO(10)) model. By minimal we mean that we must have only one multiplet that gives rise to fermion masses, i.e. one left-right bidoublet (10 in the case of SO(10)). Since this one multiplet contains both the  $H_u$  and  $H_d$  multiplets of the MSSM, it immediately leads to a proportionality between the up and the down Yukawa coupling matrices [9,10]. This is called up-down unification. The quark mixing angles all vanish at the tree-level, but they are induced by loop diagrams involving the exchange of supersymmetric particles. This considerably restricts the flavor and CP violating interactions in the model and makes it very predictive. Note that we have not assumed the existence of any extraneous discrete symmetries. This is one of our key starting points.

There is a second set of constraints on the model which follows from the existence of parity invariance of the Lagrangian prior to symmetry breaking [8]. Above the seesaw scale they imply that the familiar  $\mu$  and B parameters as well as the gluino mass terms are real. Furthermore, the Yukawa coupling matrices as well as the SUSY breaking A matrices (the trilinear terms that involves the squarks) are Hermitean. In fact, left-right symmetry implies that there is only one A matrix in the squark sector, which evolves to the two  $A_{u,d}$ matrices of low energy MSSM. It was noted in Ref. [8] that these parity-implied constraints solve the SUSY CP problem in the sense that the electric dipole moment of the neutron is comfortably consistent with the present experimental limits [11] without any need for fine-tuning. We thus see that the combination of up-down unification and the constraints of parity invariance considerably restricts flavor structure of both the quark and the squark sector of the model [9,10] and one might therefore expect that in addition to solving the SUSY CP problem, there are predictions by which the model can be tested.

In this paper we supplement the seesaw-constrained MSSM just described with the plausible assumption of universal scalar masses. We shall keep the trilinear scalar A terms arbitrary, subject of course to left-right symmetric constraints. In this theory, there is only one CP phase residing in the A term that characterizes all CP violating phenomena in the quark sector. It is thus on par with the conventional CKM model as far as the number of CP violating phases is concerned.

At tree level, the up and down Yukawa matrices in the model are proportional. This results in the vanishing of the CKM angles at tree level. Non-zero quark mixings arise only from one-loop corrections involving the elements of the A matrices. There is no other source of flavor mixing in the model. The  $A_{ij}$  are thereby fixed to a narrow range, resulting in a very predictive model. The value of the single CP phase is fixed by the requirement that the model reproduce the observed value of  $\epsilon_K$ , the indirect CP violating measure in the Kaon system. The resulting low energy theory is the MSSM, but without the SUSY CP problem and with its parameters restricted to a very narrow range.

An interesting prediction of the model is that, owing to the constraints of parity invariance,  $\epsilon'/\epsilon$  is vanishingly small above the seesaw scale  $v_R$ . However, as the theory is RGE evolved to the weak scale, manifest parity invariance disappears and a non-negligible value for  $\epsilon'/\epsilon$  emerges. We calculate this value and find it to be in good agreement with the recent KTeV [12] and previous NA31 [13] results.

A similar suppression also occurs for the electric dipole moment (edm) of the neutron and we find that its value at the weak scale is  $\sim 4 \times 10^{-29}$  ecm. This and the fact that up–down unification restricts the allowed parameter space of the MSSM considerably provide tests of the model.

Let us start by giving a brief derivation of the up-down unification relation in the SU-SYLR models. As is well known, the gauge group for this model is  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  with the standard assignment where  $Q, Q^c$  denote left-handed and right-handed quark doublets and  $\Phi$  denotes the (2,2,0) Higgs bi-doublet. The  $SU(2)_R \times U(1)_{B-L}$  symmetry could either be broken by B - L = 2 triplets – the left-handed triplet  $\Delta$  and the right-handed triplet  $\Delta^c$  (accompanied by  $\overline{\Delta}$  and  $\overline{\Delta}^c$  fields, their conjugates to cancel anomalies) or by B - L = 1 doublets  $\chi$  (left) and  $\chi^c$  (right) along with  $\overline{\chi}$  and  $\overline{\chi}^c$ . Let us write down the gauge invariant matter part of the superpotential involving these fields:

$$W = \mathbf{Y}_q Q^T \tau_2 \Phi \tau_2 Q^c + \mathbf{Y}_l L^T \tau_2 \Phi \tau_2 L^c + i (\mathbf{f} L^T \tau_2 \Delta L + \mathbf{f}_c L^{cT} \tau_2 \Delta^c L^c) .$$
(1)

Below the  $v_R$  scale, the  $H_{u,d}$  contained in the bi-doublet field will emerge as the MSSM doublets, but in general in arbitrary combinations with other doublet fields in the model. The single coupling matrix  $Y_q$  therefore describes the flavor mixing in the MSSM in both the up and the down sectors leading to the relations

$$Y_u = \gamma Y_d; \qquad Y_\ell = \gamma Y_{\nu^D} \tag{2}$$

which we call up-down unification. The parameter  $\gamma$  is unity if the multiplets  $H_u$  and  $H_d$  of MSSM are contained entirely in  $\Phi(2, 2, 0)$ , but  $\gamma$  can differ from one if additional doublets contribute to  $H_u$  and  $H_d$ . At first sight the first of the relations in Eq. (2) might appear phenomenologically disastrous since it leads to vanishing quark mixings and unacceptable quark mass relations. We showed in Ref. [9] that after including the one-loop corrections involving the exchange of supersymmetric particles to the relations in Eq. (1), there exists a large range of parameters (though not the entire range in the MSSM) where correct quark mixings as well as masses can be obtained. In Ref. [9], we explored the parameter space that allowed for arbitrary bilinear squark masses and mixings as well as arbitrary form for the supersymmetry breaking trilinear A matrix. We focused on a class of solutions for large  $\tan \beta \sim 35 - 40$ , corresponding to  $\gamma = 1$ , where all quark masses mixings and CP violating phenomena could be explained. The magnitude of  $\tan \beta$  can be reduced for  $\gamma \ge 1$ . In this paper, we focus on a predictive scenario where all flavor mixing arises from the trilinear Aterms. We use the small  $\tan \beta$  scenario ( $\gamma \gg 1$ ) to explain the observed CP violation while satisfying the FCNC constraints from the K meson system.

The model becomes much more predictive once the assumption of universal scalar masses is imposed on the theory. This implies that at the Planck scale, the only phase of the SU-SYLR theory resides in one of the three off-diagonal entries of the SUSY breaking trilinear coupling matrix A. To see how this comes about, note that parity symmetry of the Lagrangian imposed above the  $v_R$  scale implies that  $Arg(B) = Arg(\mu) = 0$  and  $Y_{q,l} = Y_{q,l}^{\dagger}$ and  $A_{q,l} = A_{q,l}^{\dagger}$ . A Hermitean  $3 \times 3$  matrix has only three independent phases. However, above the  $v_R$  scale, quark masses can be diagonalized and made real (this is because there is only one matrix  $\mathbf{Y}_q$  in Eq. (1).) It therefore follows that redefining the phases of any two quark superfields (in both the right and the left sector) we can make two of the three off diagonal  $A_{ij}$ 's real. Thus we are left with only one phase in the theory. Thus the tree level MSSM parameters above the  $v_R$  scale can be summarized as follows (we only discuss the quark-squark sector):

$$M_{u}^{(0)} = \gamma \tan \beta M_{d}^{(0)}$$
(3)  

$$M_{\tilde{Q}}^{2} = M_{\tilde{u}^{c}}^{2} = M_{\tilde{d}^{c}}^{2} = M_{H_{u,d}}^{2} = m_{0}^{2}\mathbf{I};$$
  

$$M_{\tilde{W}} = M_{\tilde{B}} = M_{\tilde{g}} = M_{1/2}$$
(4)  

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13}e^{i\delta_{13}^{p}} \\ A_{12} & A_{22} & A_{23} \\ A_{13}e^{i\delta_{13}^{p}} & A_{23} & A_{33} \end{pmatrix}.$$

The parameter  $\gamma$  is related to the mixing between the  $SU(2)_L$  doublets in  $\Phi$  and those in other multiplets in the theory such as  $\chi$  in the case of the non-renormalizable seesaw model or those in  $(2, 2, \pm 2)$  multiplets which may be included in the renormalizable seesaw model<sup>1</sup>. Note that all  $A_{ij}$  in Eq. (4) are real and further, we could have chosen to place the phase  $\delta_{13}^p$  at any off diagonal entry of A. However the final results are independent of this choice.

The first task before us is to compute the one-loop corrections to the quark mass matrices both in the up and the down sector and obtain the desired masses and mixings. The one-loop expressions for the mass corrections to the down type quarks are given in the Refs. [9,10,14]. The up sector corrections can be obtained by replacing  $(A_d, \lambda_d, v_d)$  by  $(A_u, \lambda_u, v_u)$ . In the down sector, there are three types of flavor contributions:  $M_d = v_d[Y_q(1 + c_1 \tan \beta) + c_2(A_d/M_{SUSY}) + c_3\delta_{33}]$ . Here the  $c_i$  are dimensionless loop factors. The  $c_1$  and  $c_2$  terms arise from the gluino graph, the  $c_3$  term which contributes significantly only to the *b*-quark mass is from the chargino graph.  $M_u$  is given by  $M_u = v_u[Y_q(1 + c_1/\tan \beta + c_2(A_u/M_{SUSY})]$ . Clearly there is a mismatch between  $M_u$  and  $M_d$ , which implies violation of proportionality and non-zero CKM angles.

<sup>&</sup>lt;sup>1</sup>The advantage of including the  $(2, 2, \pm 2)$  multiplets instead of the  $\chi$ -type doublets is that the former maintain the property of automatic R-parity conservation.

Although the scalar masses are assumed to be universal at the Planck scale, the non– diagonal nature of the A matrix will induce via the RGE off-diagonal elements in the up and down squark mass matrices. Since our calculation is going to be done at the SUSY scale of a few hundred GeV, we must extrapolate the parameters down from the Planck scale via the  $v_R$  scale down to  $M_{SUSY}$ . The RGE's for extrapolation below  $v_R$  are those of MSSM and are well known [15]. Between  $v_R \leq \mu \leq M_{P\ell}$ , we use the RGE corresponding to the SUSYLR model. We keep only the one-loop terms. It turns out that the main effect of running from the Planck scale to  $v_R$  in the squark sector is to split the third generation squarks slightly from the first two generations due to the large third generation Yukawa coupling. This effect is further amplified via the RGE in the process of running from  $v_R$  to the SUSY scale. We work in a basis where the tree-level Yukawa coupling matrix is diagonal. As a result, Yukawa matrices at  $M_{SUSY}$  also will be diagonal. However, the superpartner masses which start at the Planck scale as diagonal matrices acquire off-diagonal terms both at the  $v_R$  scale and at  $M_{SUSY}$ . At  $M_{SUSY}$ , the  $\hat{Q}$ ,  $\tilde{u^c}$  and  $\tilde{d^c}$  masses are no longer equal. The off-diagonal entries of these matrices become complex. Similarly, the A-matrix which was Hermitean at the  $v_R$  scale also loses its hermiticity. Using these extrapolated quantities, we compute the one-loop corrections to the up and down quark mass matrices and diagonalize them to fit the known quark mixings and masses. We find a range of input parameters at  $M_{P\ell}$  which leads to the correct quark masses and mixings. We then make sure that the chosen values of the  $A_{ij}$ 's do not lead to excessive flavor changing neutral current effects. We have used the constraints quoted in Ref. [16, 17].

Before presenting an explicit example of the parameter choice and the corresponding mass predictions, let us recall one of the main results of Ref. [9]. We showed that, to get the correct mixing angles, we need to have  $\delta_{12,LR} \simeq 4.4 \times 10^{-3} - 6.2 \times 10^{-3}$ ,  $\delta_{23,LR} \simeq$  $(.84-1.8) \times 10^{-2}$ , where the range comes from varying the parameter  $x \equiv M_{\tilde{g}}^2/m_{\tilde{q}}^2$ .  $\delta_{12,LR}$  is determined from the Cabibbo angle,  $\delta_{23,LR}$  from  $V_{cb}$ . (Here  $\delta_{ij}$  are the flavor violating squark mixing parameters.) On the other hand, from Ref. [16], we note that the upper bound on  $Im(\delta_{12,LR}) \leq (2-4) \times 10^{-4}$ . It is then clear from this that the phase  $\delta_{13}^p$  in the A matrix should be of order  $10^{-2}$ . Our detailed numerical analysis also seems to generate fits to all parameters only for a phase of this order of magnitude. Furthermore, we find that if we scale the squark masses, the A matrix and  $M_{\tilde{g}}$  by a common factor k, the quark mixings remain unchanged. This might lead one to suspect that the SUSY breaking parameter range is not limited in the theory. But since  $\epsilon_K$  in our model is coming entirely from the SUSY box graph and it scales like  $m_{\tilde{q}}^{-2}$ , the squarks cannot be too heavy. There is a scaling relation between the CP phase and the SUSY breaking parameters. The  $\epsilon'/\epsilon$  however scales differently. As a result, we are forced to a narrow range of the SUSY breaking masses.

To present a concrete example, we consider a case with  $\tan \beta = 3$ ,  $m_0 = 80$  GeV, and  $M_{1/2} = 180$  GeV. For the trilinear A matrix we choose:  $(A_{11}, A_{12}, A_{13}, A_{22}A_{23}, A_{33}) =$ (1.2, 1.8, -2.2, -12, 17, 50) GeV and  $\delta_{13}^p = 0.02$ . We determine  $\mu$  from the radiative breaking of the electroweak symmetry. Its magnitude in the above parameter space is  $\mu \simeq 290$  GeV. We choose the sign of  $\mu$  as preferred by  $b \to s\gamma$  decay. For the quark masses and mixings, we find:

$$V_{us} \simeq -0.21, V_{cb} \simeq 0.035, V_{ub} \simeq -0.0033, V_{td} \simeq -0.012, \text{ and } J \simeq 7 \times 10^{-7}.$$
 (5)  
 $M_d = (-0.0042, -0.059, 2.62) \text{ GeV}, M_u = (-0.0024, 0.61, 162) \text{ GeV}.$ 

The top mass (pole) is 172 GeV. The other masses at their respective mass scales (or at 1 GeV for u, d, s) can be obtained by mulityplying with the following QCD correction factors  $\eta$ :  $\eta_b=1.59$ ,  $\eta_c=2.1$ ,  $\eta_s=2.4$ ,  $\eta_d=2.4$  and  $\eta_u=2.4$ . The fit for the quark masses and mixings is quite satisfactory.

The squark mass matrices are  $6 \times 6$  with the  $3 \times 3$  submatrices denoted by  $M_{LL}^2, M_{RR}^2$  and  $M_{LR}^2$ . These submatrices need to be rotated by the matrices which are used to diagonalize the up and the down quark mass matrices. If the left-handed and the right-handed rotations for the quark masses are given by  $U_{l,i}$  and  $U_{r,i}$  where i's can be u or d, then we write the rotated down type squark submatrices (for example) as:  $U_{l,d}M_{LL}^2U_{l,d}^{\dagger}, U_{r,d}M_{RR}^2U_{r,d}^{\dagger}$  and  $U_{l,d}M_{LR}^2U_{r,d}^{\dagger}$ . We present only the rotated down squark mass matrices in this case since they are the only ones that enter in the discussion of CP violation in K-decays:

$$M_{LL}^{2} = \begin{pmatrix} 210442 & -1.8 - 198i & -3.5 - 398i \\ -1.8 + 198i & 209509 & -1594 - 0.86i \\ -3.5 + 398i & -1594 + 0.86i & 174085 \end{pmatrix};$$
(6)  
$$M_{RR}^{2} = \begin{pmatrix} 193030 & -2.4 - 263.6i & -5.7 - 651i \\ -2.4 + 263.6i & 191835 & -2583 - 1.4i \\ -5.7 + 651i & -2583 + 1.4i & 187244 \end{pmatrix};$$
$$M_{LR}^{2} = \begin{pmatrix} 180 + 7.2 \times 10^{-5}i & 0.23 + 86.2i & 8.85 + 979.8i \\ 0.23 - 86.3i & 2430 + 0.0062i & 3060 + 1.72i \\ 7.9 - 874.8i & 2718.3 - 1.52i & 3885.6 + 5.0 \times 10^{-4}i \end{pmatrix}.$$

We need these matrices to calculate flavor changing processes. We find that all the flavor changing constraints arising from the SUSY exchange are consistent with the bounds obtained in [16]. The six down type squark masses in this example are given by: (459, 458, 440, 439, 433 and 415) GeV and  $M_{\tilde{g}} = 501$  GeV. The parameter space of the model is quite constrained, the example above and its overall rescaling (discussed later) are the only solution we have found.

Let us now turn to CP violation in this model. Note that as mentioned before, the value of  $\epsilon_K$  is used to determine the input phase of the theory. It is clear from the CKM mixing matrix of the above example that the rephasing invariant J-parameter is of order ~  $7 \times 10^{-7}$ . If the CKM phase is to explain  $\epsilon_K$ , the value needed is  $J \sim 2 \times 10^{-5}$ . Thus  $\epsilon_K$  has a purely supersymmetric origin here. The dominant contribution is from gluino box graph involving the LL and RR terms in the squark mass matrix. These LL and RR terms also have their origin in the off-diagonal A terms through the RGE. All parameters in our model are then essentially fixed. In fact we have tried to vary them to see the effect. What we find is that fitting the quark mixings essentially implies that we must vary  $m_0$ ,  $m_{1/2}$  and A by a common factor k relative to the example just given. The value of  $\epsilon_K$  is sensitive to k since the CKM contribution to the real part of  $\Delta m_K$  is insensitive to it whereas the imaginary part of the matrix element which receives its dominant contribution from the supersymmetric box graphs scales like  $m_{\tilde{q}}^{-2}$ . Thus the only freedom allowed in our choice of squark and gaugino masses is whatever comes from the uncertainty in the hadronic matrix elements. Using the 12 elements of the LL and RR mass matrices of Eq. (6), we find that the experimental bound on  $\epsilon_K$  is nearly saturated. We use Ref. [17] to find the QCD corrected bound on  $\sqrt{\left[|Im(\delta_{12,LL}^d \delta_{12,RR}^d)|\right]}$  which is ~  $1.5 \times 10^{-4}$  for  $m_{\tilde{q}} \sim 460$  GeV and  $x \sim 1.2. \ (\delta^d_{12,LL} \equiv M^2_{12,LL}/m^2_{\tilde{a}})$ 

Let us now turn to our prediction for  $\epsilon'/\epsilon$ . As usual the dominant contributions here are from the penguin diagrams. An important point to note is that since all parameters are now fixed, we might have a conflict with the measured value of  $\epsilon'/\epsilon$ . The reason for this apprehension is that Masiero and Silvastrini [17] quote an upper limit on  $Im\delta_{12,LR} \simeq 10^{-5}$ , which is a factor of 10 smaller than the value required by us to fit  $\epsilon_K$ . This is where exact parity symmetry comes to the rescue. The point is that the CP-violating  $\Delta I = 1/2$ penguin Hamiltonian contributed by the exchange of squarks is proportional to the difference  $\delta_{12,LR} - \delta^*_{21,LR}$ , which vanishes above the  $v_R$  scale due to the constraints of parity symmetry mentioned above. However, the RGE running makes this difference nonzero by roughly  $\sim \frac{Y_{33}^2}{16\pi^2} ln \frac{v_R}{M_{SUSY}}$ , which is at the 10% level. This qualitative conclusion is borne out by our detailed numerical calculation. The diagram for the operator  $\bar{d}^{\alpha}_{L}\sigma^{\mu\nu}t^{A}_{\alpha\beta}s^{\beta}_{R}G^{A}_{\mu\nu}$  is formed by the squark line  $\tilde{d}_L - \tilde{b}_L - \tilde{s}_R$  in the loop formed by the squark and gluino lines. The magnitudes of the mixings are given  $\delta_{13,LL} (\equiv M_{LL_{13}}^2 / \tilde{m}_{\tilde{q}}^2)$  and  $\delta_{32,LR} (\equiv M_{LR_{32}}^2 / \tilde{m}_{\tilde{q}}^2)$ . Similarly we have the other diagram where the operator  $\bar{d}^{\alpha}_{R}\sigma^{\mu\nu}t^{A}_{\alpha\beta}s^{\beta}_{L}G^{A}_{\mu\nu}$  is formed by the squark line  $\tilde{d}_R - \tilde{b}_R - \tilde{s}_L$ . We have to subtract one diagram from the other to calculate  $\epsilon'/\epsilon$  and we predict  $\epsilon'/\epsilon \simeq 3 \times 10^{-3}$  for the above squark mass matrix and the lattice value for the hadronic matrix elements.

Let us now discuss the parameter space where we can have the right amount of CP violation in  $\epsilon'/\epsilon$ . One simple way is if we change  $m_0$ ,  $M_{1/2}$  and the matrix A in our example by a common factor of k, then  $V_{CKM}$  and the fermion masses will remain the same. However one needs to change the phase in order to fit  $\epsilon_K$  but  $\epsilon'/\epsilon$  might go out of the experimental range. For example if we use k = 2, which corresponds to  $m_{\tilde{q}} = 900$  GeV, we find that the phase  $\delta_{13}^p$  is near 0.1 to fit  $\epsilon_K$ , however our prediction of  $\epsilon'/\epsilon$  becomes a factor 2.5 smaller than what we had before. If k is decreased to 0.5 (corresponds to  $m_{\tilde{q}} = 230$  GeV), then the prediction of  $\epsilon'/\epsilon$  becomes a factor of 3 larger. The detailed predictions for the CP violating parameters  $\epsilon'/\epsilon$  along with the neutron edm  $d_n^e$  for various choices of k are given in Table I. We see from the Table that k somewhere between 0.7 to 2 is acceptable. This is the allowed spread in the squark mass parameters and the other SUSY breaking parameters i.e. squark and gluino masses somewhere between 300 GeV to a TeV. Note that the ratio of the gluino mass to the squark mass is essentially fixed, it is about 1.2 in all the fits. This prediction could serve as a crucial test of the model. We also find that  $\tan\beta$  cannot be increased beyond about 6. Larger  $tan\beta$  would require larger value of the off diagonal elements of A (to fit  $V_{cb}$ ,  $V_{us}$  etc). Through RGE, this would yield a SUSY contribution to  $\Delta m_K$  that is beyond the experimental limit. Note that the contribution to  $\Delta m_K$  will grow as  $(\tan \beta)^2$ , so there is really very little room for  $\tan \beta > 6$ .

Coming to the electric dipole moment of the neutron, we first note that, at the  $v_R$  scale, the flavor structure of the model is specified by the diagonal Yukawa matrices and Hermitean A matrices. The neutron edm would vanish in this limit, since it is given by the imaginary component of the (11) element of the A matrix. However once we extrapolate down to the weak scale, the situation changes and we get a nonvanishing, but quite consistent, edm for neutron.

Table 1. The predictions for  $\frac{\epsilon'}{\epsilon}$  and neutron edm for different values of k. k = 1 corresponds to  $m_0 = 80$  GeV and  $m_{1/2} = 180$ GeV.

k	$\frac{\epsilon'}{\epsilon}$	Neutron edm (ecm)
0.5	$1.0 \times 10^{-2}$	$5.0 \times 10^{-29}$
1.0	$3.1 \times 10^{-3}$	$5.2 \times 10^{-29}$
1.5	$2.4 \times 10^{-3}$	$6.0 \times 10^{-29}$
2.0	$1.2 \times 10^{-3}$	$7.0 \times 10^{-29}$

To calculate neutron edm we have considered 3 operators [4]:  $O_1 = -\frac{i}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 qF_{\mu\nu}$ ,  $O_2 = -\frac{i}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 T_a qG_{\mu\nu}^a$  and  $O_3 = \frac{1}{6}f_{abc}G_{\mu\rho}^aG_{\nu}^{b\rho}G_{\lambda\sigma}^c\epsilon^{\mu\nu\lambda\sigma}$ , where  $G_{yz}^x$  is the gluon field strength and  $f_{abc}$  are the Gell-mann coefficients. The effective Lagrangian with the Wilson coefficients is given by:  $L = \sum_{i=1}^{3} C_i(Q)O_i(Q)$ . We evaluate the  $C_i$ s at the weak scale and then we multiply by  $\eta_i$  in order to evaluate them at 1.18 GeV [5]. We use  $\eta_1 \sim 1.53$  and  $\eta_2 = \eta_3 = 3.4$ . Finally we use naive dimensional analysis [18] to calculate the quark edms ( $d_q = C_1(1.18) + \frac{e}{4\pi}C_2(1.18) + \frac{1.18e}{4\pi}C_3(1.18)$ ) and then use the quark models to calculate the neutron edm. We estimate  $d_n^e \simeq 10^{-28} - 10^{-29}$  ecm. A rough intuitive way to see this number is to note that the dominant contribution to  $d_n^e$  comes from the  $A_{11}$  term which is complex and use the naive estimate from the formula  $\frac{\alpha_s}{4\pi} \left(\frac{m_{\tilde{q}}}{m_{\tilde{q}}^2}\right) Im[\delta_{11,LR}]$ .

Before concluding, let us comment on other models with a supersymmetric origin of CP violation. One of the early models of this type is that of Ref. [19], where all CP violation is supposed to result from the phase of the gluino mass. Our model is different from that work since parity symmetry makes the gluino mass real above  $v_R$  and small at the weak scale. A related class of models with approximate CP invariance [20] generically leads to superweak CP violation which is now excluded. The second model is the recent phenomenological analysis of Ref. [21], where it is noted that a value of  $\delta_{12,LR} \simeq 10^{-5}$  would produce the right value for the  $\epsilon'/\epsilon$  satisfying all other constraints. Here we have constructed essentially a complete theory that on RGE extrapolation leads to these parameters. As a result, we have more specific predictions e.g. the  $d_n^e$  as well as the fact that there will be no measurable CP violation in the B-sector. It should be noted that while the constraint of parity by itself does not require the Wino mass  $M_2$  to be real, when embedded into a scheme such as SO(10) where  $M_2$  gets related to  $M_3$ ,  $M_2$  becomes real and there is no additional source of CP violation.

In conclusion, we have found that the requirement that MSSM be embedded into a supersymmetric left-right framework above a scale of  $10^{12}$  GeV to explain the small neutrino masses observed in the Super-Kamiokande experiment, imposes very stringent constraints on the parameters of the MSSM. First it predicts the value of  $\epsilon'/\epsilon$  in agreement with experiment and the edm of neutron comfortably consistent with the present upper limits. We also find the super-partner masses to be in a very narrow range with the gluino mass not much different from the squark masses, which can provide a test of the model. The parameter  $\tan \beta$  is not expected to exceed about 6. In the CP violation sector, a test of the model is the absence of significant CP violating effects in the B-sector. The standard KM contribution is negligible, so is the supersymmetric contribution (see Eq. (6)). These models can be readily unified into SO(10) models or other unification groups that contain the SUSY left-right model as a subgroup in them.

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