ESSAYS ON HUMAN CAPITAL, INEQUALITY, AND LABOR SUPPLY

A Dissertation

by

XIN LI

Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Chair of Committee,	Yuzhe Zhang
Committee Members,	Dennis W. Jansen
	Pedro Bento
	Quan Li
Head of Department,	Timothy J. Gronberg

May 2019

Major Subject: Economics

ABSTRACT

This dissertation is to investigate economic questions, especially questions about labor market, using methodology built upon modern economic theory and empirical study. The topics that I study in this dissertation include human capital, inequality, labor supply, and life-cycle wage growth. All models throughout the dissertation are variants of life-cycle human capital models that are based on the work by Ben-Porath. Ben-Porath human capital model is the canonical model in the labor market literature to explain life-cycle wage growth and human capital investment.

The dissertation presents and analyzes the key properties of the benchmark Ben-Porath model. It can be shown that the benchmark model can replicate the hump-shaped life-cycle income profile reasonably well but it fails to account for the fact that income inequality increases over the life-cycle in real life. To address this issue, I propose two possible extensions to the benchmark model that can solve the problem. The first way is to introduce ability heterogeneity among individuals and the second one is to introduce shocks to the human capital accumulation process. The reason why these two extensions can address the failure of the benchmark model will be discussed in detail. Since Ben-Porath model is the backbone model throughout the dissertation, detailed exposition of the benchmark model as well as its variants will help to understand the dissertation.

I build a life-cycle risky human capital model to study the sources of lifetime inequality faced by high school graduates. High school graduates face two career paths when they graduate: to enter college or to enter the labor market directly. Each path is featured by uncertainty (shocks) and the paper wants to investigate the relative importance of initial conditions and shocks in accounting for lifetime inequality. I find that differences in initial conditions as of a real-life age of 19 can only account for approximately 40 percent of the variation in realized lifetime earnings and lifetime utility. If we focus on the individuals who choose the college path, I find that the shocks in college periods play a more important role in determining the variation of inequality than do initial conditions and shocks in working periods. This fact indicates that the college investment is very risky.

I investigate the importance of labor supply in explaining gender differences in life-cycle wage growth, i.e, what fraction of gender differences in life-cycle wage growth can be attributed to different labor supply patterns? To answer this question, I build a life-cycle model in which agents make decisions about consumption, saving, labor supply and human capital investment. I find that the different labor supply patterns between genders can explain about 66 percent of the gender differences in life-cycle wage growth. Labor supply affects wage growth through two effects: experience effect and expectation effect. I find that experience effect can explain about 38 percent of the wage growth differences between genders at age 55 while expectation effect can explain 22 percent. The results indicate that labor supply plays a more important role in explaining gender wage differences than the findings in the previous literature and lends support to the importance of expectation in affecting people's human capital investment decisions.

DEDICATION

To my father, my mother, my sister, and my girlfriend.

ACKNOWLEDGMENTS

I would like to express my gratitude to my advisor, Prof. Yuzhe Zhang, for his continuous support throughout the course of my Ph.D. Next, I acknowledge my committee members, Prof. Dennis Jansen, Prof. Pedro Bento, and Prof. Quan Li for their valuable suggestions and comments on my research.

Last but not least, I am sincerely grateful to my loving parent, sister, and girlfriend who have given huge support to me throughout.

CONTRIBUTORS AND FUNDING SOURCES

Contributors

This work was supported by a dissertation committee consisting of Professor Yuzhe Zhang, Dennis Jansen, and Pedro Bento of the Department of Economics and Professor Quan Li of the Department of Political Science.

All work for the dissertation was completed independently by the student.

Funding Sources

Graduate study was supported by a scholarship from Texas A&M University.

TABLE OF CONTENTS

Page

LIST OF FIGURES

LIST OF TABLES

TABLE Page

1. INTRODUCTION: BEN-PORATH MODEL AND EXTENSIONS

1.1 Benchmark Ben-Porath Model

I present the mechanism of Ben-Porath model in a discrete time framework. While the original paper is based on continuous-time framework and the presentation is more transparent with continuous time, discrete time model is much easier to compute and interpret.

Suppose that all individuals live for J periods. In each period, each individual has 1 unit of time which can be divided into two activities, investing in human capital and working. I use i to denote the time devoted to human capital investment and $(1 - i)$ is the time used for working. New human capital can be produced if the agent devotes non-zero time ($i \neq 0$) to human capital investment. The human capital production function is as follows,

$$h_{j+1} = a(h_j i_j)^{\alpha} + (1 - \delta)h_j. \tag{1.1}$$

The above human capital production function has two inputs, human capital level in the previous period (h_j) and time (i_j). The parameter α measures the degree of diminishing marginal return of human capital investment. It is smaller than 1 and larger than 0. The parameter δ represents human capital depreciation rate. The typical empirical estimate for this parameter lies in the range of 0 and 0.1. The parameter a is a scale parameter. In the later exposition of this chapter as well as later chapters, parameter a will play a very important role and we interpret a as the ability of each individual. For now, we can interpret a as a fixed number. The human capital production function is strictly increasing and concave in time devoted to human capital investment (i).

Following the well-explained reasons in [1], I assume that the labor market is competitive and each worker bears the cost of human capital investment. That is, if the individual chooses not to denote any time to human capital investment ($i = 0$), the amount of output that the individual can produce would be h_j. However, if the worker chooses to spend positive time in human capital investment, he/she can not produce any output for the time spent in human capital. In other words, individuals will give up current earnings in order to augment their human capital level, which is beneficial to them in all later periods over the life-cycle. The price of human capital is normalized to one, by the above argument, we know that the earning for each individual in period j is

$$h_j(1 - i_j).$$

The model is a life-cycle model with a total of J periods. We can formulate the problem as an optimization problem of multiple periods. For ease of exposition, I assume that all individuals are risk neutral. In this case, the life-cycle utility maximization problem is equivalent to the life-cycle earnings maximization problem. I assume that all workers discount future earnings with a factor of β. The optimization problem can be formulated as

$$\max_{\{i_j\}_{j=1}^J} \sum_{j=1}^{J} h_j(1 - i_j) \qquad \text{subject to} \tag{1.2}$$

$$h_{j+1} = a(h_j i_j)^\alpha + (1 - \delta)h_j \tag{1.3}$$

$$0 \leq i \leq 1. \tag{1.4}$$

$$h_1 \text{ is given.} \tag{1.5}$$

We can reformulate the above dynamic optimization problem as a dynamic programming problem using Bellman's equation. In each period, the single state variable is human capital (h_j) and the single control variable is the time spent in human capital investment (i_j). I use V_j to denote the value function for period j. For all periods before the last one, the dynamic programming problem is

$$V_j(h_j) = \max_{i_j} h_j(1 - i_j) + \beta V_{j+1}(h_{j+1}) \qquad \text{subject to} \tag{1.6}$$

$$h_{j+1} = a(h_j i_j)^\alpha + (1 - \delta)h_j \tag{1.7}$$

$$0 \leq i_j \leq 1. \tag{1.8}$$

For the last period, the problem can be simplified to

$$v_J(h_J) = h_J(1 - i_J). \tag{1.9}$$

In the case of interior solution for i_j, the first order condition for i_j is

$$\begin{aligned} h_j &= \beta \frac{dV_{j+1}(h_{j+1})}{dh_{j+1}} \frac{dh_{j+1}}{di_j} \\ &= \beta \frac{dV_{j+1}(h_{j+1})}{dh_{j+1}} \left(a\alpha h_j^{\alpha} i_j^{\alpha-1} \right) \end{aligned} \tag{1.10}$$

The left-hand side of equation (1.10) represents the marginal cost of human capital investment. If the individual chooses to spend one more unit of time (or Δ, where Δ is a very small positive number) in human capital investment, the amount of earnings he/she would earn less equals h_j (or $h_j \cdot \Delta$). The right-hand side of equation (1.10) represents the marginal return of human capital investment. The increase in human capital level due to the human capital investment equals $\frac{dh_{j+1}}{di_j}$ (or $\frac{dh_{j+1}}{di_j} \cdot \Delta$). The associated increase in the value function because of the increase of human capital level equals $\frac{dV_{j+1}(h_{j+1})}{dh_{j+1}} \frac{dh_{j+1}}{di_j}$ (or $\frac{dV_{j+1}(h_{j+1})}{dh_{j+1}} \frac{dh_{j+1}}{di_j} \cdot \Delta$). The discounted increase in the value should be multiplied by β, which is exactly the right-hand side of equation (1.10). The first order condition states that, in the optimal, the marginal cost of human capital investment should equal the marginal return.

The envelope theorem for the problem is

$$\begin{aligned} \frac{dV_j(h_j)}{dh_j} &= (1 - i_j) + \beta \frac{dV_{j+1}(h_{j+1})}{dh_{j+1}} \frac{dh_{j+1}}{dh_j} \\ &= (1 - i_j) + \beta \frac{dV_{j+1}(h_{j+1})}{dh_{j+1}} \left(a\alpha h_j^{\alpha-1} i_j^{\alpha} + (1 - \delta) \right) \end{aligned} \tag{1.11}$$

The left-hand side of equation (1.11) represents the marginal increase in the value if the current human capital level (h_j) increases by one unit. The right-hand side decomposes the increase into two parts. The first part is due to the increase in current earnings, which equals $(1 - i_j)$. Besides, the increase in human capital level of period j can lead to increase in human capital of period

$j+1$. The second part measures the magnitude of the increase in discounted value because of the increase in human capital of period $j+1$.

By combing the first order condition (1.10) and envelope theorem (1.11), we can get the following closed form result, [1]

$$\frac{dV_j(h_j)}{dh_j} = \frac{1-[\beta(1-\delta)]^{J+1-j}}{1-\beta(1-\delta)} \tag{1.12}$$

Equation (1.12) states that as individuals age and get closer to the final period, the value of of human capital decreases. This makes sense since there fewer years for individuals to reap the benefit of human capital accumulation as they go through the life-cycle.

Another result that we can get is

$$\begin{aligned} h_j i_j &= \left(\beta a\alpha \frac{dV_{j+1}(h_{j+1})}{dh_{j+1}}\right)^{\frac{1}{1-\alpha}} \\ &= \left(\beta a\alpha \frac{1-[\beta(1-\delta)]^{J-j}}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} \end{aligned} \tag{1.13}$$

The left-hand side of equation (1.13) measures the foregone income for an individual who

---

[1]

Proof of (1.12).

$$\begin{aligned} \frac{dV_j(h_j)}{dh_j} &= (1-i_j) + \beta \frac{dV_{j+1}(h_{j+1})}{dh_{j+1}} \frac{dh_{j+1}}{dh_j} \\ &= (1-i_j) + \beta \frac{1}{\beta} h_j \frac{1}{a\alpha h_j^{\alpha} i_j^{\alpha-1}} \left(a\alpha h_j^{\alpha-1} i_j^{\alpha} + (1-\delta)\right) \\ &= (1-i_j) + \frac{1}{a\alpha}(h_j i_j)^{1-\alpha}\left(a\alpha h_j^{\alpha-1} i_j^{\alpha} + (1-\delta)\right) \\ &= (1-i_j) + i_j + \frac{1}{a\alpha}(1-\delta)(h_j i_j)^{1-\alpha} \\ &= 1 + \frac{1}{a\alpha}(1-\delta)(h_j i_j)^{1-\alpha} \\ &= 1 + \frac{1}{a\alpha}(1-\delta)\beta a\alpha \frac{dV_{j+1}(h_{j+1})}{dh_{j+1}} \\ &= 1 + \beta(1-\delta)\frac{dV_{j+1}(h_{j+1})}{dh_{j+1}} \end{aligned}$$

Since human capital after period J provides no value to the individual, the derivative $\frac{dV_{J+1}(h_{J+1})}{dh_{J+1}}$ should be zero. Therefore, $\frac{dV_J(h_J)}{dh_J}$ equals 1 and $\frac{dV_{J-1}(h_{J-1})}{dh_{J-1}}$ equals $1+\beta(1-\delta)$. Following this logic, it is trivial to show that (1.12) holds for any period j. $\square$

decides to spend i_j fraction of time in investing in human capital, so it measures the magnitude of human capital investment in terms of money. Equation (1.13) shows that the investment should decrease as individuals get closer to the last period. This result confirms the intuition. Another special result is that the value of investment, $h_j i_j$, only depends on β, a, α, and δ. Particularly, it does not rely on the human capital level of the individual. In other words, for two individuals that share the same discount factor β, scale parameter a, elasticity parameter α, and human capital depreciation rate δ but have different initial human capital level, we would expect that they would have exactly the same forgone income $h_j i_j$ over the whole life-cycle. This property is called Ben-Porath neutrality. This property holds since we assume that individuals are risk neutral and that there are no shocks over the life-cycle. Otherwise, this result might break down.

The benchmark Ben-Porath model can explain why individuals experience income growth over the life-cycle, especially rapid growth over the early stage of the life-cycle pretty well. The mechanism is as follows: in the early stage of the life-cycle, individuals devote a large fraction of time to accumulating human capital. This leads to a lower income in early periods and income will start increasing rapidly for two reasons. First, individuals will spend less time in human capital investment. Second, human capital level starts increasing due to human capital investment made in previous periods. In later stages of the life-cycle, the time spent in human capital investment is relatively small and this makes little change to the human capital level. We would expect relatively slow income change in these periods. Besides, in the last very few periods, individuals almost make no human capital investment. Income would probably fall due to the presence of human capital depreciation.

To get a better idea of how the model works, I present a version of the model by assigning specific values to model parameters. In this case, I assume that $J = 40$, $\alpha = 0.8$, $\beta = 0.96$, $a = 0.07$, $\delta = 0.005$, $h_1 = 2$. Figure 1.1 depicts how the time spent in human capital investment evolves over the life-cycle. As predicted by the theory, investment time falls as agents approach retirement. During the last 10 periods, it can be seen that the investment time is almost zero. Figure 1.2 shows the life-cycle profile of income. The model produces a hump-shaped age-profile

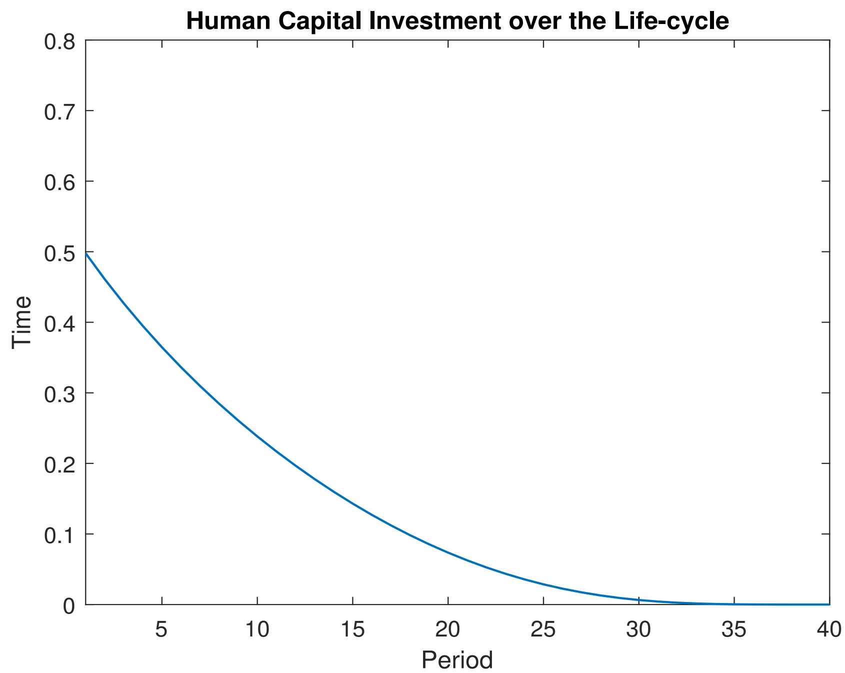

Figure 1.1: Time spent in human capital investment over the lifecycle: benchmark model

of income, which is consistent with empirical estimates of life-cycle income profile using real-world data.[2] We can see that income increases considerably during the first 15 periods and starts decreasing during the last 10 periods because of almost zero human capital investment and positive human capital depreciation.

1.2 The Limitation of the Benchmark Model

In the benchmark Ben-Porath model, it is assumed that different individuals could have different initial human capital but impose that they should be otherwise identical. In other words, they should have the same discount factor β, scale parameter a, elasticity parameter α, and human capital depreciation rate δ. In this subsection, I will present empirical evidence that will conflict with the predictions of the benchmark model and explain why the benchmark model fails in this regard. In the next subsection, I will show possible extensions of the benchmark model to make

[2] In later chapters, I will present how to estimate life-cycle income profile using PSID data, the main data I use throughout the essay.

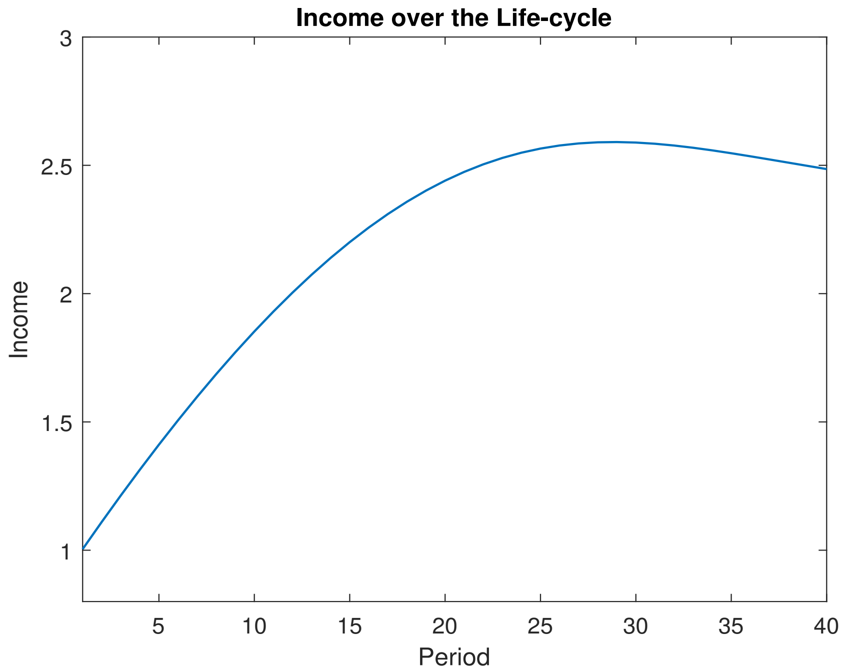

Figure 1.2: Income over the lifecycle: benchmark model

the model make more realistic predictions.

Equation (1.13), or the Ben-Porath neutrality, makes very strong predictions for the benchmark model. In particular, it means that individuals with higher human capital will spend less time in human capital accumulation compared to individuals with lower human capital. Therefore, the gap of human capital among individuals will shrink over the life-cycle even though there might exist large human capital differentials at the beginning of the life-cycle. Accordingly, the income gap among individuals will tend to fall over the life-cycle since income is tightly related to human capital under the Ben-Porath framework.

As I have already shown, the benchmark model does a good job of replicating the hump-shaped life-cycle income profile estimated using real-world data. However, due to the reasons mentioned in the above paragraph, the benchmark Ben-Porath will predict that the income inequality among individuals will decrease over the life-cycle, which is inconsistent with the empirical estimate. If we use the variance of log income to measure the dispersion of income, the empirical estimate of

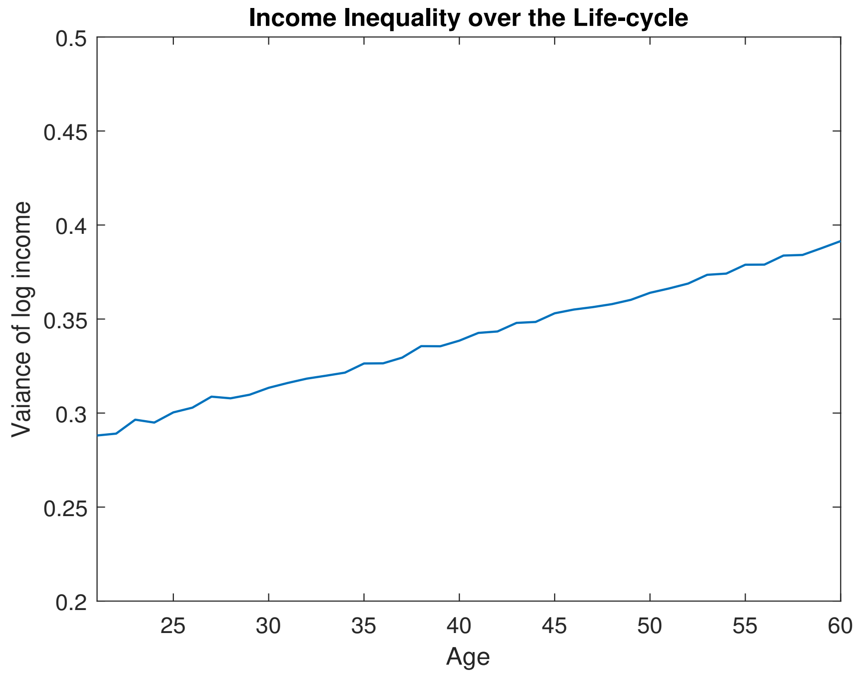

Figure 1.3: Variance of log income over the life-cycle: data

the life-cycle profile of variance of log income shows that income inequality increases over the life-cycle. Figure 1.3 shows the empirical estimate.

To get a better understanding of why income inequality decreases over the life-cycle, I simulate the life-cycle behaviors of 50000 individuals by using the parameter values used in the last subsection.[3] To introduce heterogeneity among individuals, I assume that the initial human capital is uniformly distributed with lower bound 2 and upper bound 4. In other words,

$$h_1 \sim U(2, 4).$$

The life-cycle profile of average time spent in human capital and average income are very similar to figure 1.1 and figure 1.2. What's new in this case is the life-cycle profile of income inequality since we introduce heterogeneity among individuals. Figure 1.4 depicts the life-cycle profile of variance of log income. As can be expected from the theoretical predictions of the

[3]To be specific, $J = 40$, $\alpha = 0.8$, $\beta = 0.96$, $a = 0.07$, $\delta = 0.005$

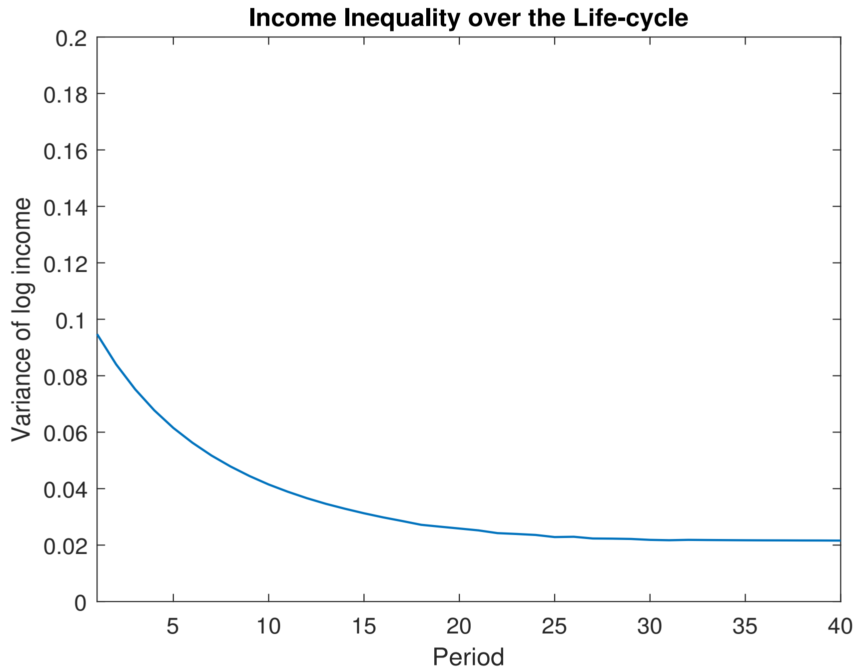

Figure 1.4: Variance of log income over the life-cycle: benchmark model

benchmark model, the fact that human capital and time spent in human capital investment are negatively related lead to the drop in income inequality over the life-cycle.

Based on the analysis above, we know that the benchmark Ben-Porath model fails to account for the data feature that income inequality over the life-cycle decreases. Some might wonder that if this model property is due to the fact that individuals are risk neutral, therefore we have got very strong predictions like equation (1.13). It turns out that this is not the case. If we assume that individuals are risk-averse and they solve lifetime utility maximization problems instead of lifetime earnings maximization problems, the model would still predict a decreasing trend of income inequality over the life-cycle. Individuals with higher human capital still spend less time inhuman capital accumulation compared to those with lower human capital.

In the next subsection, I will introduce several ways of extending the current framework in order to make the predictions of the model more realistic. The first way is to introduce ability differences among individuals. The second way is to introduce human capital accumulation shocks. Finally, we can combine both ability differentials and human capital shocks to account for the data feature. The key point, in this case, is how to distinguish the separate role played by each of the two channels.

1.3 Extensions of the Benchmark Model

1.3.1 Ability Heterogeneity

One way to address the drawbacks of the benchmark model is to introduce ability heterogeneity among individuals. For example, [2] introduce ability differences to study the relative importance of initial conditions versus shocks in accounting for lifetime inequality. In previous arguments, we interpret a as a scale parameter or a fixed number that is common to all individuals. In the current scenario, a can be interpreted as the ability of each individual. We can think of ability as the IQ of each individual that displays big dispersion among the population in real life. Besides, we typically assume that ability (or IQ) is fixed over the life-cycle. IQ cannot be changed but human capital can be accumulated. As we can see from equation (1.13), human capital investment is an increasing function of a. Therefore, individuals with higher ability tend to do more human capital investment and this will lead to higher human capital in later periods. The prerequisite to generate the increase of income inequality over the life-cycle is that the correlation between ability and initial human capital is positive. In this case, the initial gap of human capital among individuals will be enlarged since those with higher human capital (mostly with higher ability) will do more human capital investment over the life-cycle. Otherwise, with zero correlation or negative correlation between ability and initial human capital, those with lower initial human capital will eventually catch up with those with higher initial human capital since they spend more time in human capital accumulation. That will cause an unrealistic prediction of decreasing income inequality among individuals over the life-cycle, similarly to the prediction of the benchmark Ben-Porath model.

The argument in the above paragraph explains why the presence of heterogeneity in ability and positive correlation between ability and initial human capital can lead to an increase in income inequality. Now, I present numerical solutions of the model and simulate the behaviors of 50000

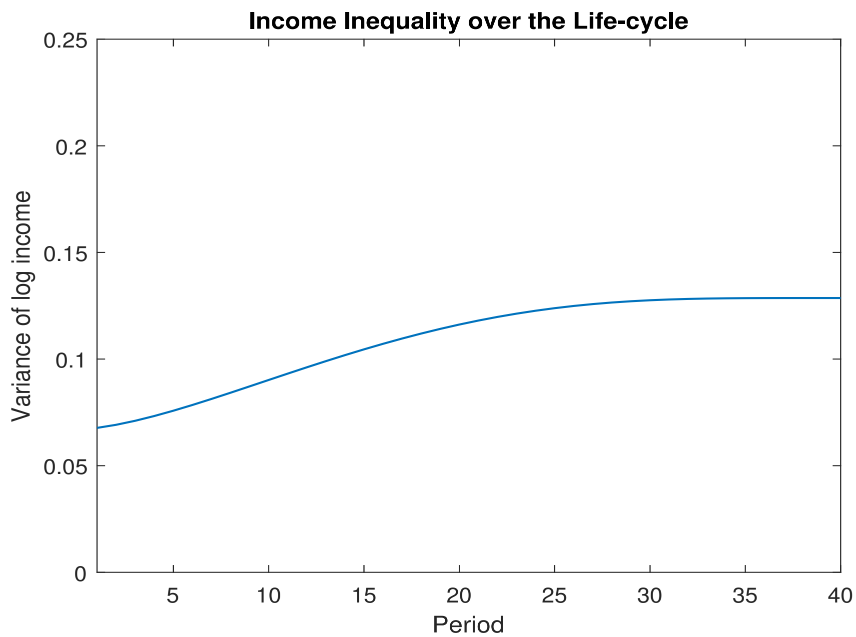

Figure 1.5: Variance of log income over the life-cycle: model with ability differences, $\rho = 0.8$

individuals for several specific parameter configurations of the model. For all scenarios, I assume that $J = 40$, $\alpha = 0.8$, $\beta = 0.96$, $\delta = 0.005$, $h1 \sim U(2, 6)$, $a \sim U(0.04, 0.08)$. I assume that initial human capital and ability have different Pearson correlation coefficients (ρ) between different scenarios.[4] I consider four cases: $\rho = 0.8$; $\rho = 0.5$; $\rho = 0.2$, $\rho = -0.1$. Not surprisingly, shapes of the life-cycle profile of average income and average time spent in human capital investment look very similar to what we have got in previous figures. However, income inequality shows hugely different evolution patterns over the life-cycle. Figure 1.5 to figure 1.8 depict life-cycle profiles of the variance of log income for the four cases.

It can be seen from figure 1.5 that when the correlation between initial human capital and ability is as high as 0.8, the income inequality increases over the whole life-cycle. This numerical version of the model is consistent with what I have argued before. When the correlation is positive but not that large, as in the case of figure 1.6 where the correlation coefficient is 0.5, the income inequality actually drops in the early stage of the life-cycle before it starts increasing. The income

[4]In practice, I use copulas to generate dependency between two uniformly distributed variables.

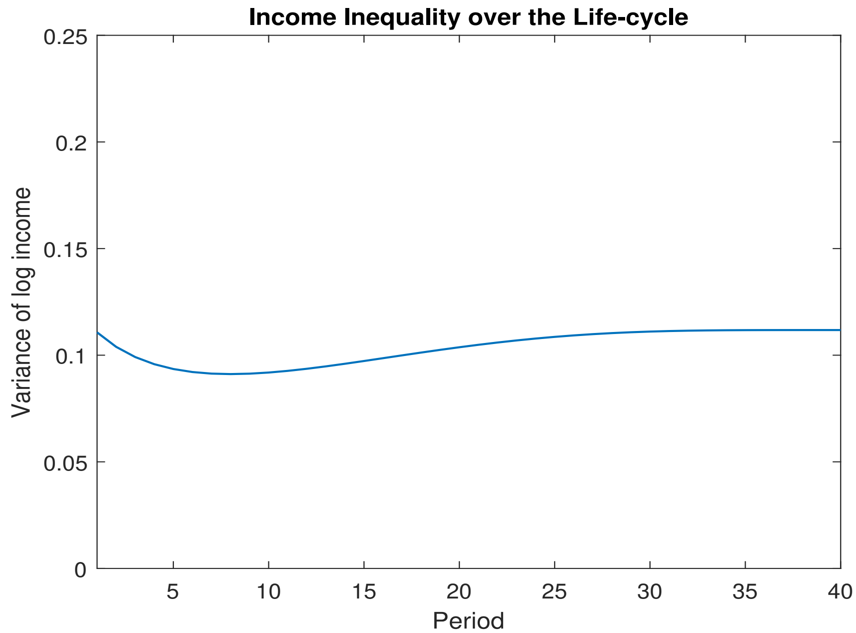

Figure 1.6: Variance of log income over the life-cycle: model with ability differences, $\rho = 0.5$

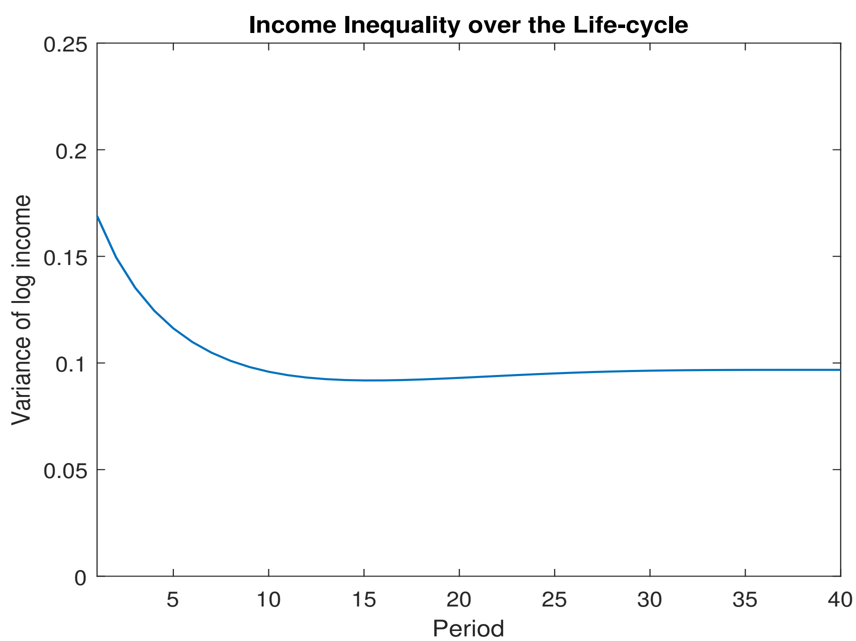

Figure 1.7: Variance of log income over the life-cycle: model with ability differences, $\rho = 0.2$

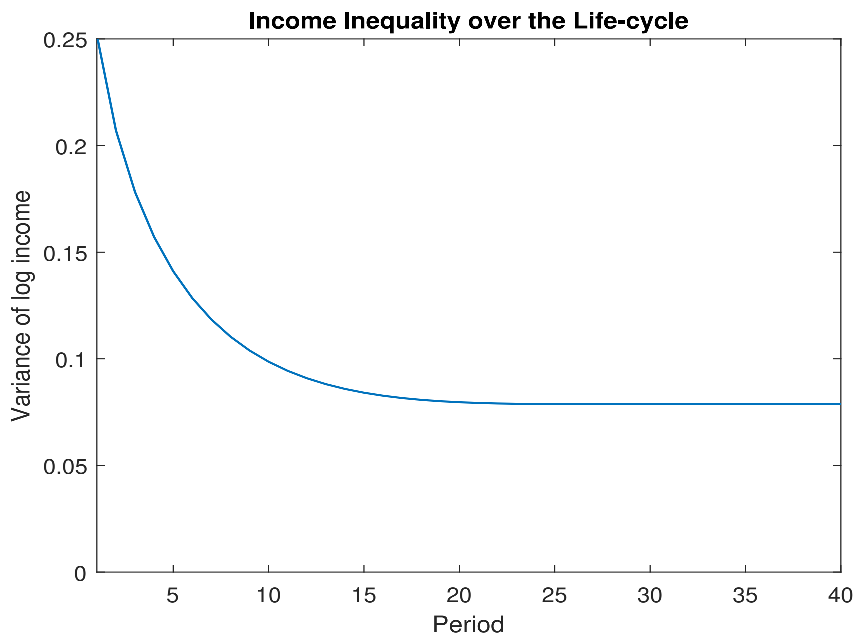

Figure 1.8: Variance of log income over the life-cycle: model with ability differences, ρ = -0.2

inequality over the life-cycle displays a U-shape. Figure 1.7 and 1.8 shows that when the correlation coefficient is a very small positive number or even negative, the life-cycle profile of income inequality displays a decreasing trend, a result that we should expect.

This subsection analyzes one possible extension to the benchmark Ben-Porath model that could possibly solve the failure of the benchmark model in accounting for one important feature of real-life data. I have shown, with several numerical simulations, that the presence of heterogeneity in ability and large enough positive correlation between ability and initial human capital can lead to an increase in income inequality over the life-cycle.

It is natural to ask whether a positive correlation between initial human capital and ability is a reasonable assumption in real-world. I have argued that IQ can be considered as the empirical counterpart of ability in the model. In terms of human capital, we can think of human capital as the amount of knowledge that each individual has acquired to make him/her productive in the labor market. The positive correlation between ability and initial human capital makes perfect sense since we would expect people with higher IQ to acquire more knowledge compared to those with lower IQ. In real life, we also observe that people with higher IQ tend to acquire new knowledge faster than those with lower IQ. This is also consistent with equation (1.1), which is the law of motion for human capital.

1.3.2 Human Capital Shocks

Another modification of the benchmark model that could possibly lead to increasing income inequality over the life-cycle is to add shocks to the human capital accumulation process. In the benchmark model, there is no uncertainty and human capital evolves from period to period in a deterministic way. The addition of shocks to the human capital accumulation process can lead to increasing income inequality because some individuals are lucky and receive positive shocks for many periods while some might be unlucky and receive negative shocks for a long time. Therefore, the presence of shocks will increase the dispersion of human capital levels among individuals over time and this will end up with higher income inequality in later stages of the life-cycle.

It is also straightforward that larger variance of shocks will contribute to steeper increasing income inequality trend over the life-cycle. The scenario of no shocks is the extreme case that the variance of shocks is zero. In this scenario, I have shown that the life-cycle profile of variance of log income actually depicts a decreasing trend. Therefore, as the variance of shocks gets larger, we would expect that the life-cycle profile of income inequality would rotate counter-clockwise and start to display an increasing trend at some point.

To better illustrate how the addition of human capital shocks would affect the life-cycle profile of income inequality, I present numerical simulations of the model with four different parameter configurations. The parameters that are shared across the four scenarios are as follows: $J = 40$, $\alpha = 0.8$, $\beta = 0.96$, $a = 0.05$, $\delta = 0.005$, $h1 \sim U(2, 4)$. I assume the following law of motion for human capital after the introduction of human capital shocks,

$$h_{j+1} = s\big(a(h_j i_j)^\alpha + (1 - \delta)h_j\big),$$

Where s denotes the human capital shock. I assume that shocks follow discrete uniform distribution and there are five possible values. The expectation of shocks is assumed to be one in all four cases. To be specific, I assume the following four different scenarios. Scenario 1: $s \in \{1, 1, 1, 1, 1\}$; scenario 2: $s \in \{0.96, 0.98, 1, 1.02, 1.04\}$; scenario 3: $s \in \{0.92, 0.96, 1, 1.04, 1.08\}$. scenario 4: $s \in \{0.8, 0.9, 1, 1.1, 1.2\}$. Scenario 1 actually corresponds to the extreme case that there is no uncertainty. The variance of shocks become larger from scenario 1 to scenario 4,. Figure 1.9 to figure 1.12 display the life-cycle profiles of the variance of log income for all four scenarios. As can be expected from theory, the life-cycle profile of income inequality shows a decreasing trend in the model configuration with no shocks and the profile starts to rotate counter-clockwise as the variance of shocks gets bigger.

As argued by Huggett et al. (2011), the wage dynamics generated by the above risky human capital model is consistent with the estimates of other wage models commonly used in the literature. For example, the above model can generate high persistence of wage dynamics even though the shocks are assumed to be independently and identically distributed.

The interpretation of shocks is also important. For example, if individuals take on-the-job training, a positive shock could be that the quality of the training program is better than expected. One thing to note is that the shocks apply to both existing human capital and newly acquired human capital. I have come up with one example that represents a positive shock in this case. In the 1980s and early 1990s, researchers studying neural network were often laughed by their peers since neural network was regarded as a dead direction at that time. Accordingly, the earnings of researchers in that field were not high. With the availability of more powerful computers and large volumes of data, neutral network has proved to outperform almost all other models and the researchers in this field have gotten a huge gain in terms of payoff. A negative shock in this case could be a loss of human capital due to some injuries that harm your health and productivity.

Human capital shocks add another dimension of heterogeneity to the benchmark model because shocks would lead to different history of luck for different individuals. These different luck experiences over the life-cycle would contribute to the increasing dispersion of human capital as well as income. In this spirit, the function of shocks is very similar to the productivity shocks extensively used in the incomplete market literature, as [3] and [4]. The literature using the incomplete market framework typically assumes infinitely-lived ex-ante identical agents. Agents differ in the ex-post since they receive different productivity shocks from period to period. These papers use the combined assumption of incomplete market and productivity shocks to investigate issues like wealth inequality. In this chapter, I have shown that, in a life-cycle framework, the introduction of human capital shocks can help to increase the income inequality over the life-cycle.

1.3.3 Ability Heterogeneity v.s. Shocks: Where Is the Separate Line?

In the previous two subsections, I have shown that there are two possible mechanisms to extend the benchmark Ben-Porath model in order to make more realistic predictions. The first option is to introduce ability heterogeneity and assume a positive correlation between ability and initial human capital. The second way is to add shocks to human capital accumulation process. A natural question to ask is then what's the separate line of these two types of forces. Specifically, to let the model's predictions be more consistent with empirical estimates, we could either assume the presence of heterogeneity and shut down the possibility of human capital shocks, or assume that there is no ability heterogeneity and shocks should account for all the increasing trend of income inequality over the life-cycle. I believe that either way bears its only merits and either mechanism should play an important role in real life. In other words, we should expect that the correct (in a relative sense) model should include both ability differences and human capital shocks.

If we build the more complicated version of the model by adding both ability heterogeneity and human capital shocks, we still need to provide a convincing answer to another question before we can take the model seriously and use it answer important economic questions. That is, how to quantify the magnitude of the correlation coefficient between initial human capital and ability (ρ), and the magnitude of the variance of human capital shocks (σ). Since both larger ρ and σ will lead to higher income inequality over the life-cycle, it is obvious that we can not distinguish the magnitude of ρ and σ simply from the life-cycle profile of income inequality. For example, it is totally possible that the parameter configuration of $\rho = 0.6$ and $\sigma = 0.6$ will produce very similar life-cycle profile of variance of log income as the case of $\rho = 0.8$ and $\sigma = 0.4$. Without a good

Figure 1.9: Variance of log income over the life-cycle: model without shocks

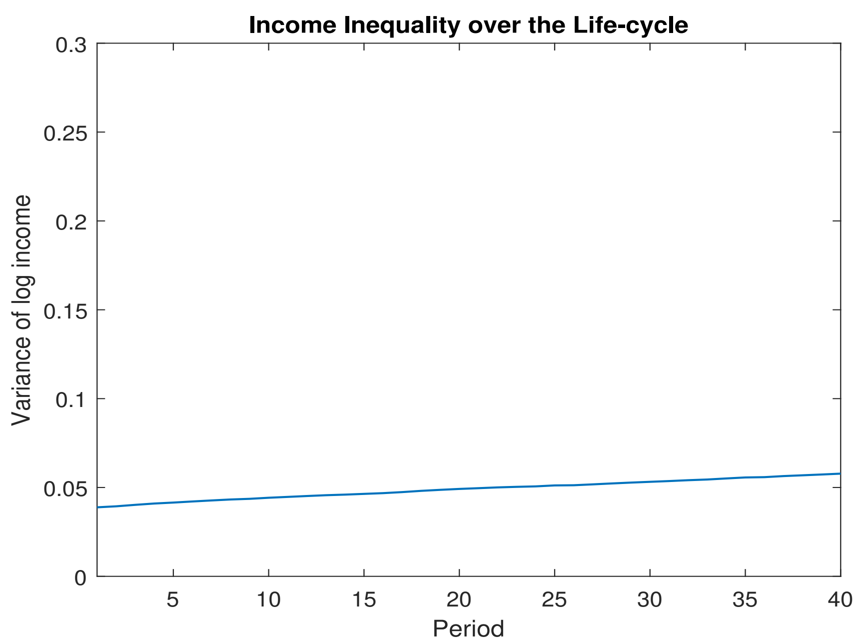

Figure 1.10: Variance of log income over the life-cycle: model with shocks, s = [0.96, 0.98, 1, 1.02, 1.04]

Figure 1.11: Variance of log income over the life-cycle: model with shocks, s = [0.92, 0.96, 1, 1.04, 1.08]

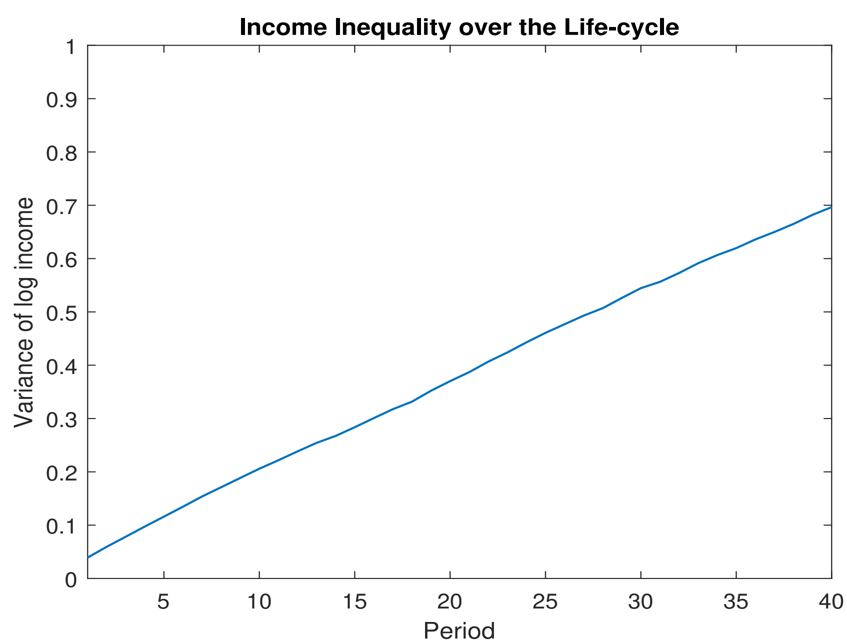

Figure 1.12: Variance of log income over the life-cycle: model with shocks, s = [0.8, 0.9, 1, 1.1, 1.2]

estimate of the correlation coefficient and variance of shocks, the model can not be used to answer important empirical questions and can only be of theoretical use.

In later chapters, I will illustrate how to address this problem in detail. Here, I present the general idea. The strategy was first proposed by [2]. In that paper, they build a life-cycle model to study the relative importance of initial conditions and shocks in accounting for lifetime inequality. The idea is that human capital shocks can be estimated using wage data in later stages of the life-cycle. As predicted by the model, in the last few years of the life-cycle, individuals almost spend no time in human capital investment. In this case, changes in an agent's wage solely come from human capital shocks. The magnitude of the variance of shocks can be estimated if we assume the distribution of shocks. In the next chapter, I will present in detail how to estimate the variance.

After we get the estimate of the variance, the correlation between initial human capital and ability will be determined by the residual between the empirical estimates and the predictions of the model with zero correlation. The procedure that I use is to calibrate the model to let the model's predictions match the empirical data moments as good as possible. Again, I will discuss the calibration strategy in later chapters when I build richer models to answer different empirical questions.

1.4 Summary

In this chapter, I first present and show relevant properties of the benchmark Ben-Porath model in section 1.1. It's shown that the benchmark model can produce the hump-shaped life-cycle profile of income pretty well. The model also implies that the time spent in human capital investment decreases as individuals approach retirement. However, the limitation of the benchmark model comes from the fact that it predicts individuals with higher initial human capital spend less time in investing in human capital than those with lower initial human capital. Therefore, the model predicts that income inequality decreases as individuals get older, which is opposite to the empirical life-cycle profile of income inequality estimated using real-life data. In section 1.3, I show that there are two ways to modify the benchmark model to address this problem. Subsection 1.3.1 introduces ability heterogeneity among individuals and subsection 1.3.2 presents the addition of human capital shocks to the benchmark model.

Ability differences among individuals, coupled with positive correlation between initial human capital and ability, can make individuals with higher initial human capital (most likely, also those higher ability) also devote more time to human capital investment. This will cause the dispersion of human capital to increase over the life-cycle, so is the income inequality.

In essence, the presence of human capital shocks adds another dimension of heterogeneity to the benchmark Ben-Porath model. In the benchmark model, individuals differ in initial human capital and this is the only dimension of heterogeneity among individuals. The addition of shocks will lead to different luck history experienced by individuals, which can interpreted as another dimension of heterogeneity, at least in the ex-post sense. These different luck history help to generate increasing income inequality over the life-cycle.

Subsection 1.3.3 put forward an important question that should be taken seriously before we can use different versions of the Ben-Porath model to address empirical questions. That is we should find a way to accurately quantify the correlation coefficient between initial human capital and ability as well as the variance of human capital shocks. The way to tackle this problem is to estimate the variance of shocks and calibrate the model to determine the correlation coefficient. We will see detailed presentation in later chapters.

2. COLLEGE CHOICE AND SOURCES OF LIFETIME INEQUALITY

In this chapter, I investigate the sources of lifetime inequality by enriching the Ben-Porath framework that I discussed in the last chapter. As as we all know, inequality is very common phenomenon in real life. Different people have different income, different consumption levels, different career paths and so on. Among these inequalities, some are due to the differences established early in life; for example, people with higher IQ tend to have higher lifetime earnings; people with better family background might have more chances to succeed. We attribute these inequalities to initial conditions. Some inequalities come from the differences in luck experienced in the following periods of life-cycle; two similar persons when they are young might end up with totally different career paths or two seemingly different people at age of 20 might have similar lifetime earnings. We attribute these inequalities to luck or shocks. So a natural question to ask would be: what's the relative importance of initial conditions versus shocks in accounting for the dispersion of lifetime inequality?

The choice between a college path and a no-college path is critical for every high school graduate. A college path is featured by two types of shocks, shocks in college periods and shocks in working periods, while a no-college path is only associated with the latter one. For any career path, we know lifetime inequality is partially due to different initial conditions and partly due to different shocks received in the lifetime. So some questions to ask would be: are the relative importance between initial conditions and shocks different for the two career paths? which part of riskiness, shocks received in college periods or shocks received in working periods, accounts for more of the variation in lifetime inequality for those who choose college paths?

It's critical to provide convincing answers to these questions. First and foremost, the relative riskiness of the two career paths can be contrasted. A career path in which shocks play a relatively more important role than initial conditions should indicate that this path is riskier than the other one. Consequently, the relative importance of policies directed at improving initial conditions (e.g., nutrition, education) against those directed at providing insurance for shocks (e.g., unemployment insurance) can be contrasted for the two career paths. Second, by investigating the relative importance of the two types of shocks for college students, we can know the effectiveness of the policies that insure students against college failure (e.g., income contingent loans).

To answer this question, I develop a quantitative life-cycle Ben-Porath human capital model that incorporating college periods and working periods which endogenizes the college and college dropout decisions. There are three dimensions of heterogeneity among individuals: initial human capital, ability and wealth. Shocks constitute an important part in the human capital accumulation process. Initial human capital will determine the income of individuals after they just graduate and enter the labor market. In the life-cycle income profile, initial human capital controls the intercept. Higher ability individuals can learn faster and accumulate more human capital over the lifetime, therefore ability can be viewed as rotating the income profile.

In the model, every individual belongs to one of the three educational groups: high school graduates, college dropouts and college graduates.[1] The model is calibrated to match the dynamics of income distribution over the life-cycle for each of the three educational groups. I present the empirical estimates of life-cycle profiles of mean earnings and measures of earnings inequality (Gini coefficient) for all three educational groups using income data of US males. The familiar hump-shaped average income profiles are produced and that income inequality increases with age over most of the working lifetime for all three groups.

As I have discussed in chapter 2, the model can produce hump-shaped average income profiles because individuals make human capital investment decisions over the whole life-cycle. In the early stages of the life-cycle, they spend less time in market work and devote more time to human capital accumulation, therefore the average income is lower. They accumulate more human capital and spend more time in market work as they age, therefore the average income increases over time. In the very last few years, income decrease because of human capital depreciation and almost zero time put into investing in human capital.

[1] In real world, we know there are individuals that do not belong to any of these three groups. But these individuals represent a very small fraction of the whole population. I don't consider the choices of these individuals to make the model parsimonious.

The reason why the model can produce increasing income inequality has been investigated in chapter 2 using relatively simple models and simulations. The two mechanisms that work together to account for this model feature are ability heterogeneity and human capital shocks. Ability differences among individuals as well as a positive correlation between initial human capital and ability will make individuals with higher initial human capital also take more time to accumulate human capital. Correspondingly, the dispersion of human capital, as well as income inequality, increases over the life-cycle. Human capital shocks can produce large dispersion of human capital even though the variance of initial human capital is small.

To identify the magnitude of the variance of shocks individuals receive in working periods, I follow the strategy that was proposed by [2]. In that paper, they utilized the fact that individuals almost spend no time in producing new human capital in the last few years of the life-cycle. This implies that, late in life, changes in income for these individuals are only due to the shocks they receive. The variance of the shocks can be estimated using the moments of changes in income for older males. After the variance of the shock is estimated, the joint distribution of initial human capital and ability can be calibrated to make the model match the life-cycle profiles of average income and dispersion of income for all three educational groups.

The relative importance of shocks and initial conditions in accounting for variation of lifetime inequality can be quantified after the calibration of the model. I find that shocks received over the college periods and shocks received over the working lifetime are more important than initial conditions as a source of lifetime inequality. In the benchmark calibration, as of age 19, initial conditions can only explain about 37.5 percent of the variation in lifetime earnings. For no-college path agents, we can see that initial conditions can account for approximately 40 percent of the variation in lifetime earnings and lifetime utility. For those who choose to attend college, we can see that initial conditions can only account for less than 30 percent of the variation in lifetime earnings and slightly more than 30 percent of the variation in lifetime utility. The shocks in college periods play a critically important role by the fact that these shocks can explain 41.6 percent of the variation in lifetime earnings and 38.4 percent of the variation in lifetime utility.

All previous results should be interpreted carefully. In the paper, the distribution of initial conditions at the age of 19 is taken as exogenously given.[2] However, we all know that the distribution of human capital in a later age is determined by the joint distribution of human capital and ability in an earlier age, say age 10. Therefore, the results I find that shocks seem to play a more important role than initial conditions in accounting for variation of lifetime inequality should only be interpreted as applying at the age of 19.

Most of the previous previous literature studying lifetime inequality is based on incomplete-markets model, where each worker's productivity and labor income evolve exogenously. For example, [5] analyze use such a model to investigate lifetime inequality. Similar models have also been widely used to study economic questions like wealth inequality and taxation.[3] [5] finds that slightly more than half of the variation in lifetime earnings is because of shocks received in the working periods. In the opposite, [10] builds and estimates a life-cycle career-choice model, which they find that initial conditions should play a more important role. They find that initial conditions at the age of 16 should explain about 90 percent of the variance in lifetime utility.

There exist some undesirable features with the incomplete-markets model with exogenous income. In the first place, all of the increase in income inequality over the life-cycle is attributed to human capital shocks. Therefore, the importance of human capital shocks may be overemphasized. In the second place, since income is exogenous in these papers, the model is silent on a lot of policy issues that may affect inequality through income. To this end, [2] develop a risky human capital model that features idiosyncratic shocks to human capital. Their finds are that, as of age 23, differences in initial conditions account for more than 60 percent of the variation in lifetime earnings and lifetime utility. Our paper is closely related to [2], but we model explicitly the career path decisions faced by high school graduates and the risky college periods. It is important to include college choice decisions when studying lifetime inequality for two reasons. On one hand, the college will help to increase human capital or initial conditions in general. On the other hand, college investment is very risky since the college dropout rate is very high in the US.

[2]To be precise, I back out the distribution of initial conditions by calibrating the model.

[3]See [6]; [7]; [8]; [9] and others.

The chapter is organized as follows. Section 3.3 illustrates the model. Section 3.2 provides the empirical estimates of life-cycle profiles of average earnings and earnings dispersion as well as the shock process. Section 2.3 discusses how to set model parameters. Section 3.5 analyzes the properties of the model. Section 2.5 provides quantitative answers to the research questions. Section 3.7 concludes.

2.1 Model

Time is discrete and runs from $t = 1, 2, ..., T$. The economy is populated with agents who live for J periods. Each period represents one calendar year. All agents enter the model as high school graduates at real life age 19, which corresponds to model age 1. At model age 1, the agents in the economy are heterogeneous in three dimensions: innate ability a, initial human capital h_1, and initial financial wealth k_1. Let $x_1 = (h_1, k_1, a)$ denote the initial conditions. We assume that the innate ability does not change over an agent's lifetime. An agent maximizes the expected lifetime utility, taken the triple (h_1, k_1, a) as given.

At model age 1, all agents will decide whether or not to go to college. Agents who choose to go to college will take the college path; those who choose no-college path will enter the labor market and start working immediately. Human capital can be accumulated both through college education and on the job training. In each period, all agents are endowed with one unit of time. In college periods, we assume that agents devote all their time to human capital accumulation and no job earnings are acquired. In working periods, the one unit of time can be spent either in working or in human capital accumulation.

The sources of risk to agents over the life-cycle are different for those who choose different career paths. For those who choose no-college path, the only risk comes from idiosyncratic shocks to their human capital in the working periods. For those who attend college, they receive idiosyncratic shocks to their human capital both in college periods and in working periods. In the next several subsections, we consider the two career paths and the individual's decision-making problems.

2.1.1 No-college Path

Individuals who choose to take this path will enter the labor market immediately after high school graduation at age 1. The maximization problem for the no-college path is stated below:

$$V_1^{NC}(h_1, k_1, a) = \max_{\{c_j, n_j, k_j\}_{j=1}^{J}} E_1\{\sum_{j=1}^{J} \beta^{j-1}(u(c_j) + u_{NC})\} \quad \text{subject to} \tag{2.1}$$

$$c_j + k_{j+1} = k_j(1 + r_j) + e_j - T_j(e_j), \ \forall j \text{ and } k_{J+1} = 0 \tag{2.2}$$

$$e_j = w_j h_j(1 - n_j), \text{ if } j < J_R \text{ and } e_j = 0 \text{ otherwise} \tag{2.3}$$

$$h_{j+1} = exp(z_j^w)(H(h_j, n_j, a) + h_j) \tag{2.4}$$

$$c_j \geq 0, \ k_j \geq -b, \ 0 \leq n_j \leq 1, \ \forall j \tag{2.5}$$

We use script NC to denote no-college path. In each period, individuals get utility both from consumption and from job characteristics associated with no-college path, u_{NC}, which is assumed to be same to all agents. We can think that u_{NC} contains all the factors that could affect an agent's utility besides consumption level, such as leisure, social recognition associated with no-college path. Equation (2.2) represents the budget constraint in each period. e_j denotes earnings in period j and $T_j(e_j)$ represents income taxation. Financial assets pay a real interest rate r_j in period j. Equation (2.3) says that earnings e_j, before retirement age J_R, equals the product of the unit price for human capital w_j, an agent's human capital h_j and the time fraction that is put into work $(1-n_j)$. Condition (2.5) is about nonnegative constraint on consumption, the exogenous borrowing constraint on financial wealth, and the constraint on time faction that can be used to accumulate human capital. Equation (2.4) is the law of motion for human capital: z_j^w is the shock to human capital and $H(h_j, n_j, a)$ is the human capital accumulation technology on the job. Following [11] we specify it as follows,

$$H(h_j, n_j, a) = a(h_j n_j)^{\alpha_{NC}},$$

where $\alpha_{NC} \in (0, 1)$, is the elasticity of human capital investment on the job for a no-college career path; it determines the degree of diminishing marginal returns of human capital investment. The human capital production depends on an agent's ability, human capital and the time fraction spent in human capital investment.

The value function, $V_j^{NC}(h_j, k_j, a)$ gives the maximum present value of lifetime utility at age j given state variables $x_j = (h_j, k_j, a)$. In recursive form, the objective function (2.1) can be written as follows,

$$V_j^{NC}(h_j, k_j, a) = \max_{\{c_j, n_j\}} \left((u(c_j) + u_{NC}) + \beta E_j V_{j+1}^{NC}(h_{j+1}, k_{j+1}, a) \right) \tag{2.6}$$

2.1.2 College Path

The agent can choose a college path as an alternative at age 1 and they will stay in college for at most fours years. Once they finish the college period, they will get a bachelor's degree and enter the labor market. At the beginning of each college period, they can also choose to drop out the college and join the labor market as a college dropout. Since agents have the option to drop out of college, I find that it is easier to formulate the maximization problem in recursive form.

First, we consider the value function for the college dropout path. At age 1, nobody can choose the college dropout path. The dropout path can only be pursued after a college path is chosen. In other words, only those who choose the college path have the option to drop out. In recursive form,

$$V_j^{CD}(h_j, k_j, a) = \max_{\{c_j, n_j\}} \left((u(c_j) + u_{CD}) + \beta E_j V_{j+1}^{CD}(h_{j+1}, k_{j+1}, a) \right), \quad j \geq 2 \tag{2.7}$$

I use script CD to denote college dropout path. The maximization problem for the college dropout path is the same as that for the no-college path with the only exception that u_{CD} replaces u_{NC} to capture the utility associated with job characteristics. So, the agents solve problem (2.7) subject to constraints (2.2) to (2.5).

Next, we consider the value function for the college path. From the first period in the working lifetime onwards, namely period 5 in the model, we have the following recursive formulation,

$$V_j^C(h_j,k_j,a) = \max_{\{c_j,n_j\}} \left((u(c_j)+u_C) + \beta E_j V_{j+1}^C(h_{j+1},k_{j+1},a)\right), \quad j \geq 5 \tag{2.8}$$

I use script C to denote college path. The agents solve (2.8) subject to constraints (2.2) to (2.5). Following the idea of [12], we make the assumption that the elasticity for human capital investment is different for college graduates and high school graduates. For college graduates, we let the elasticity be α_C, which belongs to $(0,1)$.

Now, we are ready to consider the value function for the college path in the college period. Again, in recursive form,

$$V_j^C(h_j,k_j,a) = \max_{\{c_j,n_j\}} \left((u(c_j)+u_C) + \beta E_j V_{j+1}^{EC}(h_{j+1},k_{j+1},a)\right), \quad 1 \leq j \leq 4 \tag{2.9}$$

$$V_{j+1}^{EC}(h_{j+1},k_{j+1},a) = \max\left(V_{j+1}^C(h_{j+1},k_{j+1},a), V_{j+1}^{CD}(h_{j+1},k_{j+1},a)\right) \tag{2.10}$$

$$c_j + k_{j+1} = k_j(1+r_j) - m \tag{2.11}$$

$$h_{j+1} = exp(z_j^c)(H^C(h_j,a)+h_j) \tag{2.12}$$

$$c_j \geq 0,\ k_j \geq -b. \tag{2.13}$$

Equation (2.10) captures the fact that college students have the option to drop out in the college period. (2.11) is the budget constraint and m is the education cost, which is the same for all agents. (2.12) is the law of motion for human capital in college period. $H^C(h_j,a)$ is the human capital production function in the college, we specify it as, $H^C(h_j,a) = ah_j^\eta$, where $\eta \in (0,1)$ is the elasticity parameter. z_j^c is the idiosyncratic shock to human capital in college periods.

2.1.3 The Agent's Decisions

At age 1, agents make the decision of whether or not to attend college. A college path is pursued if and only if

$$V_1^C(h_1,k_1,a) \geq V_1^{NC}(h_1,k_1,a), \tag{2.14}$$

Agents who choose the college path will reconsider their career paths at the beginning of period $2, 3, 4$. If they find that the discounted lifetime utility associated with a dropout path is higher than that if they continue the college, they will drop out. For example, if at the beginning of period 3, someone finds that

$$V_3^{CD}(h_3, k_3, a) > V_3^{C}(h_3, k_3, a),$$

he/she will drop out college with two years' experience in college and enter the labor market as a college dropout.

Now, we analyze the trade-offs between the two career paths. The cost of attending college comes in two parts: a goods cost and a time cost since agents can not get any earnings for at most 4 periods. The benefit of a college path is that you can take all your time to accumulate human capital, which, in expectation, will lead to a higher human capital stock when you graduate. Since earnings in working lifetime are directly influenced by human capital, that means you can earn more later in life. Also, the elasticity of human capital investment on the job differs between college graduates and high school graduates (college dropouts have the same elasticity as high school graduates). That means if you can manage to graduate in college, you will get a different (possibly higher) production elasticity in later working periods.

2.1.4 Model Specifications

The utility function is of CRRA form, $u(c) = c^{1-\sigma}/(1-\sigma)$.

The initial condition $x_1 = (h_1, k_1, a)$ is of multivariate log-normal distribution. In the benchmark case, we consider the case where initial financial wealth is set to zero. In this case, $x_1 = (h_1, a)$ and it is bivariate log-normally distributed; $x_1 \sim LN(\mu_x, \Sigma)$.

We assume that the idiosyncratic shocks to human capital are independent and identically distributed both in the college periods and in the working lifetime and follow a normal distribution. $z_j^w \sim N(\mu_w, \sigma_w^2)$ and $z_j^c \sim N(\mu_c, \sigma_c^2)$.

Following [2], we let the tax system T_j includes a social security and an income tax. Before retirement, $T_j(e_j) = T_j^{ss}(e_j) + T_j^{inc}(e_j)$. We assume the social security and income tax both have flat tax rates : $T_j^{ss}(e_j) = \tau_{ss} \times e_j$ and $T_j^{inc}(e_j) = \tau_{inc} \times e_j$. After retirement, the social security pays a common amount which equals 40 percent of average economy-wide income in the last year of working periods.

The unit price for human capital grows exogenously at a constant rate g, $w_j = (1+g)^{j-1}$. The interest rate is set to be constant in each period, $r_j = r$.

2.2 Empirical Analysis

The data I use is the Panel Study of Income Dynamics(PSID) 1969-2013 family files. The PSID is a longitudinal study of a sample of US individuals (men, women, and children) and the family units in which they reside. Data is annual from 1968 to 1997, and biennial since then.

I use data to address three issues. First, I use data to calculate two college statistics: college enrollment rate and college dropout rate. Second, I estimate how average earnings and measures of earnings inequality (Gini coefficient is what I use) evolve over the life-cycle for each cohort. Third, I estimate the properties of the shocks associated with human capital accumulation from the data.

2.2.1 College Statistics

The two college statistics are calculated in the following way: the college enrollment rate is the ratio between the number of observations that attend college and the number of total observations; the college dropout rate is the ratio between the number of observations that get a bachelor's degree and the number of observations that attend college.

The estimated college enrollment rate and college dropout rate are 49.2% and 49.1%, respectively. As we can see from the statistics, the college dropout rate is nearly half in the United States, which might indicate that college investment is very risky.

2.2.2 Life-cycle Profiles

The life-cycle profiles of earnings dynamics are estimated for all three education levels: high school graduates, college dropouts and college graduates. Each educational group is classified in the following way: first, the observations with 12 years of education are treated as high school graduates; second, the observations that enter college, end up with 13-15 years of education and no bachelor's degrees are obtained are treated as college dropouts; third, the observations that end up with 16 years of education or above are treated as college graduates.

I estimate the life-cycle profiles for mean earnings and earnings Gini coefficient from age 23 to 60. Only the earnings data for males who are the household head is used. I use a centered 7-year bin to estimate relevant earnings statistics at a specific age. For males over age 30, I require that they work at least work 520 hours per year and earn at least 9000 dollars (in 2010 dollars). For males age 30 and below, I require that they work at least 260hours per year and earn at least 6000 dollars (in 2010 dollars).

Following [2], I use the following two-step method to estimate the life-cycle profiles of earnings dynamics. First, the statistic of interest at age bin j and at year t is calculated and labeled as $stat_{j,t}$. For example, for earnings Gini coefficient, I set $stat_{j,t} = G_{j,t}$, where $G_{j,t}$ is the Gini coefficient of real earnings of all males in some education group in the age bin centered at age j and at year t.[4] Second, Several factors, including cohort effects, age effects and time effects can generate the earnings statistics. The age effects (or life-cycle profiles) are what we are really interested in. The following econometric model is employed to estimate the age effects,

$$stat_{j,t} = \alpha_c^{stat} + \beta_j^{stat} + \gamma_t^{stat} + \epsilon_{j,t}^{stat},$$

where α_c^{stat}, β_j^{stat} and γ_t^{stat} are cohort, age and time effects respectively and $\epsilon_{j,t}^{stat}$ is the error term. We have got a perfect co-linear problem here since birth cohort c equals exactly time t minus age j ($c = t - j$). In order to estimate the age effect, some further restrictions need to made here.

Two alternative ways to solve the co-linear problem are provided in the literature.[5] The cohort effects view, which shuts down the time effects, says that γ_t^{stat} should be set to be 0 for all t. The time effects view, which shuts down the cohort effects, argues that α_c^{stat} should be set to be 0 for all c. [7] argues that time effects view is more appropriate for many applications, so I will mainly

[4]Consumer Price Index is used to convert nominal earnings to real earnings.
[5]See [13], [7], and [2] among others.

present the estimation results of the time effects view in the main text. Ordinary least squares estimation method is employed to estimate the coefficients.

In Figure 2.1, I graph the life-cycle profiles of mean earnings for the three educational categories. Figure 2.1 generates the familiar hump-shaped age profiles for mean earnings. However, the age profiles have different shapes for different educational groups. For high school graduates, the earnings ratio between age 50 and age 23 is approximately 1.7; for college graduates, the statistic is approximately 2.6. Even though the earnings gap between college graduates and high school graduates is not very large at age 23, the gap is getting larger when they age. The age profile for college dropouts lies in between college graduates and high school graduates and it is much closer to the age profile of high school graduates.

Figure 2.2 graphs the age effects of earnings Gini coefficient. We can see that the Gini coefficients increase with age for all three educational groups. College graduates have the largest Gini coefficient and the most obvious increase trend in the life-cycle compared to the other two groups.

2.2.3 Human Capital Shocks

To estimate the mean and variance of human capital shocks, I employ the strategy posed by [2]. The earnings equation implies that an agent's wage rate equals the product of the unit price of human capital and an agent's human capital when no time is spent in human capital investment. The model also implies that individuals spend almost no time in human capital investment in the last few years of the working lifetime. This occurs because the value of making new human capital investment decreases as the individual approaches retirement. Accordingly, we can utilize the changes in the agent's wage rate over the last few working years to estimate the mean and variance of human capital shocks.

I assume that an individual spends zero time in human capital investment between age j through $j+n$. Then we know $e_j = w_j h_j$ and $h_{j+1} = exp(z_j^w) h_j$. We have the following relationships,

$$e_{j+n} = w_{j+n} h_{j+n} = w_{j+n} exp(z_{j+n-1}^w) h_{j+n-1} = w_{j+n} \prod_{t=0}^{n-1} exp(z_{j+t}^w) h_j \tag{2.15}$$

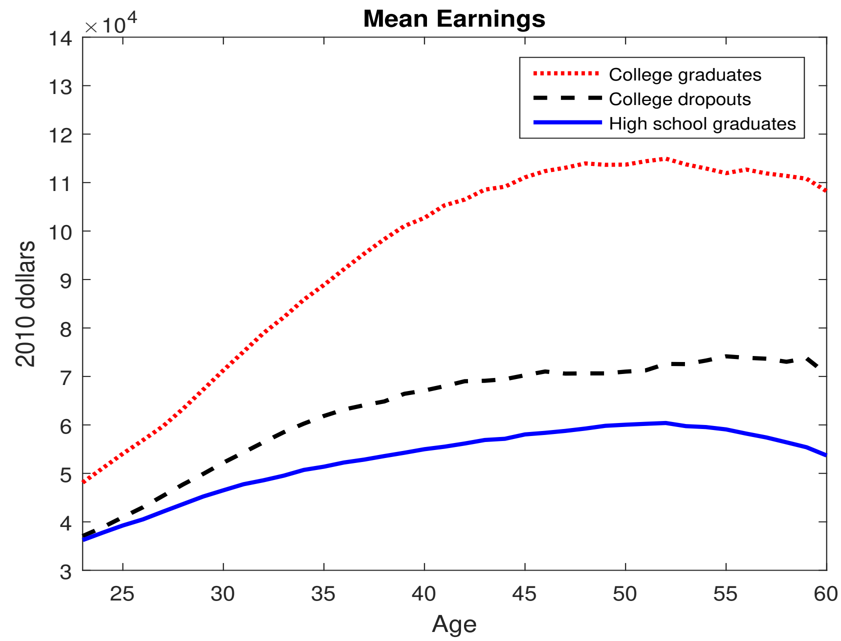

Figure 2.1: Mean earnings by age: data

Figure 2.2: Earnings Gini coefficient by age: data

Take log to the above equation, we have that

$$\tilde{e}_{j+n} = log(e_{j+n}) = \tilde{w}_{j+n} + \sum_{t=0}^{n-1} z^w_{j+t} + \tilde{h}_j, \tag{2.16}$$

where $\tilde{x} = log(x)$. I use $y_{j,n}$ to denote the log-earnings-difference, then we have

$$y_{j,n} = \tilde{e}_{j+n} - \tilde{e}_j + \epsilon_{j+n} - \epsilon_j = \tilde{w}_{j+n} - \tilde{w}_j + \sum_{t=0}^{n-1} z^w_{j+t} + \epsilon_{j+n} - \epsilon_j, \tag{2.17}$$

where ϵ_j is the measurement error in period j.

I use equation (2.17) and impose the assumption that the measurement errors ϵ_j are independent and identically distributed over time and people. Let $Var(\epsilon_j) = \sigma^2_\epsilon$. Then, we have the following moment condition,

$$Var(y_{j,n}) = n\sigma^2_w + 2\sigma^2_\epsilon \tag{2.18}$$

In practice, we focus on older workers since these workers, to a great extent, satisfy the condition that no human capital investment is made. We follow male workers between age 50 and 60 for four years and thus calculate three differences (i.e. $y_{j,n}$ for $n = 1, 2, 3$). To do the estimation, all cross-sectional variances across panel years are used. Two-step Generalized Method of Moments estimation method is used to estimate the variance.

The estimation results are summarized in Table 2.1. There is no big difference in the standard deviation of the human capital process among three educational groups. This is equal to say that a one standard deviation shock change the human capital by 15% ($e^{0.138} \approx 1.15$). The ratio of mean earnings between adjacent model periods equals $(1 + g)e^{\mu_w + \frac{1}{2}\sigma^2_w}$ when agents make no human capital investments. $\hat{\mu}_w$ is calculated for each educational group to accommodate the fact that mean earnings depreciates 2% per year in the late working lifetime.

Parameter	high school graduates	college dropouts	college graduates
$\hat{\sigma}_w$	0.136	0.138	0.142
$\hat{\mu}_w$	-0.0308	-0.0311	-0.0317

Table 2.1: Estimation of human capital shocks

2.3 Calibration

This section calibrates the model. All model parameters are divided into two groups. The first group of model parameters is set using parameters found in the literature or common knowledge. This step is done before we solve the model. The rest model parameters are set so that the predictions of the model match the age profiles of meaning earnings and earnings Gini coefficient, as well as the college enrollment rate and college dropout rate.

For the parameters that are set before solving the model, the parameter values and their sources are summarized in Table 2.2. We assume that the agents live 57 periods in the model, which corresponds to a real-life age of 75. All individuals begin to receive retirement benefits at age $J_R = 43$. The exogenous borrowing limit is chosen to be 120, which is equivalent to say that all agents can borrow as much as $300k$ 2010 dollars after normalization. The education cost of 12 corresponds to $30k$ 2010 dollars. The discount factor is set to be 0.98. The value of the risk aversion parameter is set to $\sigma = 2$, which is in the range of the estimates by [14]. Following Huggett et. al. (2011), the interest rate r and the social security tax rate τ_{ss} is set to be 0.042 and 0.106 respectively. The growth rate of human capital price is 0.0014 based on Huggett et. al. (2006). The income tax rate is the estimate of the average tax rate for 40-60% quantiles of the U.S. citizens, based on [15].

The strategy of setting the remaining model parameters is to set the model in line with the life-cycle profiles of mean earnings and earnings Gini Coefficient as well as the two college relevant statistics. The parameters to be determined are those governing the distribution of initial conditions $x_1 \sim LN(\mu_x, \Sigma)$, the two elasticities of the human capital technology on the job, α_{NC} and α_C for non-college graduates and college graduates, the elasticity of the human capital production

Parameter	Symbol	Value	Source
Life-cycle periods	J	57	
Retirement age	J_R	43	
Borrowing limit	b	120	
Education cost	m	12	
Discount factor	β	0.98	
Risk parameter	σ	2	[14]
Growth rate for h.c. price	g	0.0014	[16]
Interest rate	r	0.042	[2]
Income tax rate	τ_{inc}	0.070	[15]
Social Security	τ_{ss}	0.106	[2]

Table 2.2: Calibration: parameters chosen without solving the model

Parameter	μ_a	μ_h	σ_a	σ_h	ρ	α_{NC}	α_C	η	σ_c	u_{CD}	u_C
Value	-1.38	3.01	0.10	0.32	0.52	0.68	0.74	0.44	0.23	0.008	0.027

Table 2.3: Calibration: parameters chosen by solving the model

function in the college, η, the standard deviation of human capital shock in college, σ_c, and two utility terms u_{CD} and u_C.[6] Table 2.3 displays the calibration results of these parameters.

2.4 Properties of the Model

In this section, I discuss a number of properties of the model.

The life-cycle profiles of mean earnings, earnings Gini coefficient generated by the model are displayed in Figure 2.3 and Figure 2.4. The model produces the familiar hump-shaped life-cycle profile of mean earnings. Individuals spend more time in producing new human capital in the early stages of the life-cycle than at later ages. Therefore, individuals accumulate human capital and average earnings display a rapid increasing trend in the early stages of the life-cycle. In the later stages of the life-cycle, average earnings drop for all three groups because human capital depreciates and individuals spend almost no time in producing new human capital anymore.

[6] the utility term for no-college path u_{NC} is normalized to 0.

Figure 2.5 graphs the life-cycle profiles of the average time devoted to human capital production and the average human capital levels. Panel A shows that more than 30 percent of the time is devoted to human capital investment at age 23 but less than 3 percent of time is spent after age 55 for college students. This trend is very similar for all three educational groups except that college dropouts and high school graduates tend to spend less time in investing human capital over the life-cycle compared to college students. The results confirm the validity of the empirical strategy to estimate human capital shocks. Panel B shows that the life-cycle profile of average human capital is also hump-shaped. As we can see from the graph, the human capital profiles for high school graduates and college dropouts are almost the same and extremely flat over the life-cycle. The human capital for college graduates increase before age 47 and then starts to fall.

From table 2.3, we can see that the correlation coefficient between initial human capital and ability is 0.52, which is positive and consistent with what I have argued in chapter 2. It is necessary for the model to have a positive correlation between initial human capital and ability to produce increasing earnings inequality over the life-cycle even in the presence of human capital shocks.

2.5 Discussion

Now, we are ready to use the well calibrated model to answer the research questions that are posed in the introduction.

The variance of all relevant variables are decomposed into variation due to initial conditions versus variation due to shocks. I consider both lifetime utility and lifetime earnings following [2].[7] A random variable is the summation of its conditional mean and the deviation from its conditional mean. Since conditional mean and the deviation from conditional mean are orthogonal to each other, the variance of the random variable equals the sum of the variance of the conditional mean and the variance around the conditional mean. A statistical equation can be written as,

$$Var(Y) = Var(E[Y|X]) + E[Var(Y|X)] \tag{2.19}$$

[7] Lifetime earnings and lifetime utility are defined as $\sum_{j=1}^{J_R} e_j/(1+r)^{j-1}$ and $\sum_{j=1}^{J} \beta^{i-1} u(c_j)$

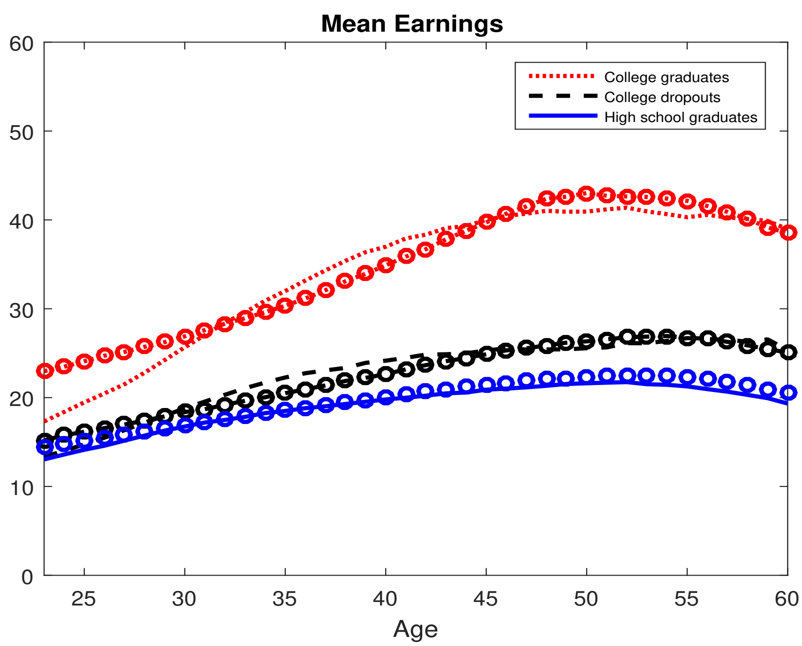

Figure 2.3: Mean earnings by age: data and model.

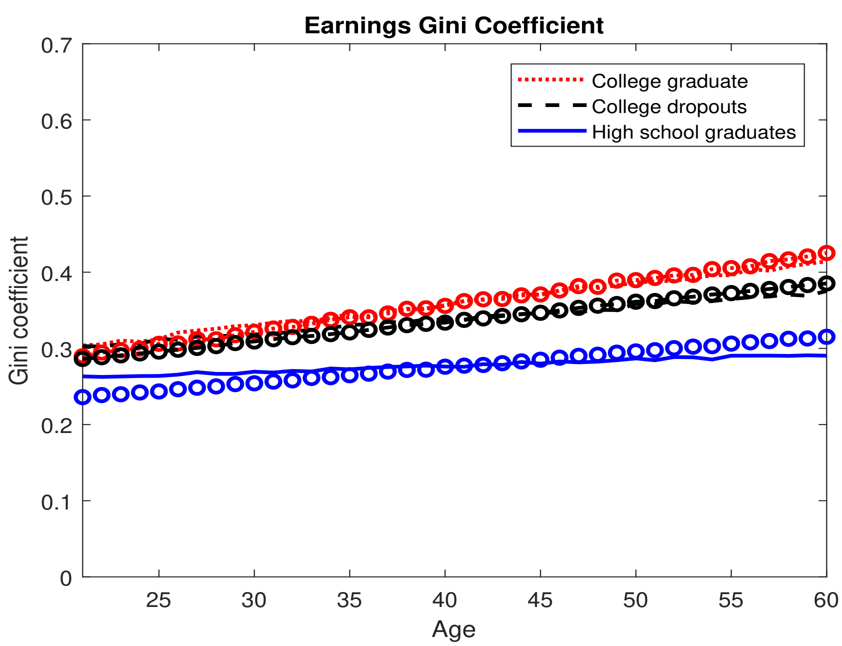

Figure 2.4: Earnings Gini coefficient by age: data and model

I do the variance decomposition for lifetime earnings and lifetime utility and calculate the ratio $Var(E[Y|X])/Var(Y)$. The ratio is the fraction of lifetime inequality that is due to initial conditions. Table 2.4 presents the decomposition results. I analyze lifetime inequality as of age 19. In the model, I find that 37.5 percent of the variation in lifetime earnings and 41.4 percent of the variation in lifetime utility can be attributed to initial conditions. The findings indicate that initial conditions are not as important as shocks in accounting for the variations in lifetime inequality.

Next, I focus on the individuals that choose different career paths. For those who choose no-college path, they only have shocks in the working lifetime. For those individuals who choose to attend college, they face two types of shocks: shocks in college periods and shocks in working lifetime. For those individuals, I do the variance decomposition by two steps. First, I eliminate the risk in the college periods and calculate the corresponding variance terms. Second, I eliminate the risk in working periods and calculate the corresponding variance terms. By these two steps, I can decompose the variation in lifetime inequality to three parts for college-path agents: initial conditions, shocks in college periods and shocks in working lifetime.

Table 2.5 and Table 2.6 present the results of this decomposition. For no-college path agents, we can see that initial conditions can account for approximately 40 percent of the variation in lifetime earnings and lifetime utility. For those who choose to attend college, we can see that initial conditions can only account for less than 30 percent of the variation in lifetime earnings and slightly more than 30 percent of the variation in lifetime utility. The shocks in college periods play a very much important role by the fact that these shocks can explain 41.6 percent of the variation in lifetime earnings and 38.4 percent of the variation in lifetime utility. These statistics indicate that college investment is really risky.

2.6 Conclusions

This paper analyzes the sources of lifetime inequality. I find that differences in initial conditions as of a real-life age of 19 can only account for approximately 40 percent of the variation in realized lifetime earnings and lifetime utility. If we focus on the individuals who choose the college path, we find that the shocks in college periods play a more important role in determining the variation of

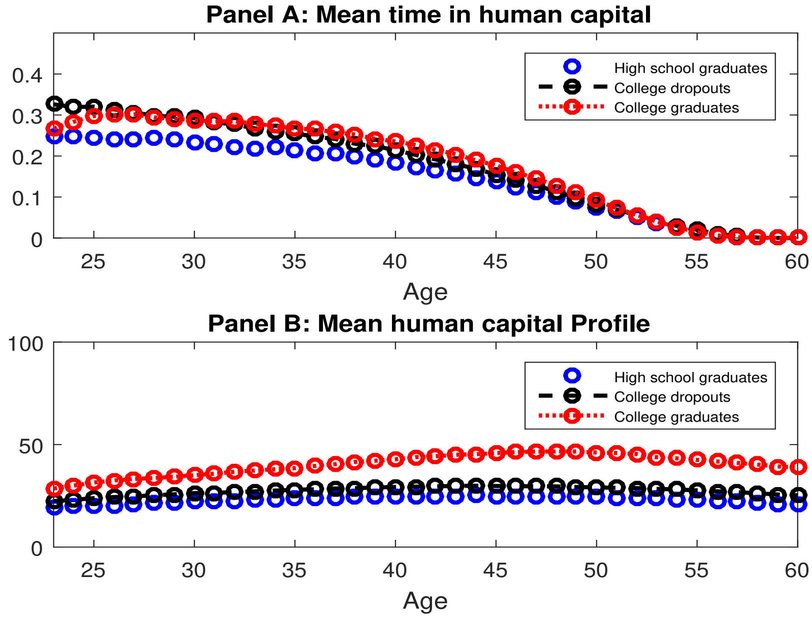

Figure 2.5: Properties of human capital by age

Statistic	Benchmark Model
Fraction of Variance in Lifetime Earnings Due to Initial Conditions	37.5
Fraction of Variance in Lifetime Earnings Due to Shocks	62.5
Fraction of Variance in Lifetime Utility Due to Initial Conditions	41.4
Fraction of Variance in Lifetime Utility Due to Shocks	58.6

Table 2.4: Sources of lifetime inequality

Statistic	Benchmark Model
Fraction of Variance in Lifetime Earnings Due to Initial Conditions	38.6
Fraction of Variance in Lifetime Earnings Due to Working Shocks	61.4
Fraction of Variance in Lifetime Utility Due to Initial Conditions	40.2
Fraction of Variance in Lifetime Utility to Working Shocks	59.8

Table 2.5: Sources of lifetime inequality: no-college path

Statistic	Benchmark Model
Fraction of Variance in Lifetime Earnings Due to Initial Conditions	26.1
Fraction of Variance in Lifetime Earnings Due to College Shocks	41.6
Fraction of Variance in Lifetime Earnings Due to Working Shocks	32.3
Fraction of Variance in Lifetime Utility Due to Initial Conditions	30.7
Fraction of Variance in Lifetime Utility Due to College Shocks	38.4
Fraction of Variance in Lifetime Utility to Working Shocks	30.9

Table 2.6: Sources of lifetime inequality: college path

inequality than do initial conditions and shocks in working periods. This fact indicates that college investment is very risky.

The conclusions only apply to the specific model at the age of 19. Results might change if we choose a different model or consider a different starting age. However, the framework in this chapter should help to motivate other research work which investigates the forces that affect individuals' decisions before the age of 19.

3. HUMAN CAPITAL, LABOR SUPPLY, AND LIFE-CYCLE WAGE GROWTH DIFFERENTIALS BETWEEN GENDERS

3.1 Introduction

A salient feature of the US labor market is that the average wage of male workers is much higher than that of females workers and that males have higher wage growth over the life-cycle than females. Another feature is that males have much higher labor force participation rates than females over the life-cycle. Human capital theory is the dominant framework to explain wage growth and gender differences in wage growth over the life-cycle. According to the theory, labor supply can affect wage growth through two effects. The first effect is the experience effect. It is natural to assume that individuals accumulate no human capital if they choose to stay out of the labor market. Instead, their human capital level should decrease due to depreciation. Therefore, the higher labor force participation rates of male workers help them accumulate more human capital and get higher wages than female workers. The second effect is the expectation effect. Females usually expect lower future labor force attachment, which leads to smaller investment in human capital than males since human capital investment is costly. So gender differences in future work expectation are a critical determinant of existing male-female differences in life-cycle wage growth.

The main objective of this paper is to provide a quantitative assessment of the importance of labor supply in explaining gender differences in life-cycle wage growth, i.e, what fraction of gender differences in life-cycle wage growth can be attributed to different labor supply patterns? What is the separate role played by each of the two effects through which labor supply impact wage growth in accounting for the gender differences? A convincing answer to these questions is of significant importance. First, and most simply, an answer serves to contrast the potential importance of different factors that can cause gender wage differences. For example, labor market discrimination has been widely regarded as another important factor that can lead to lower wage of female workers.

By quantifying the importance of the “labor supply" factor, we can draw an indirect conclusion about the importance of the “labor market discrimination" factor. Consequently, we can know the relative effectiveness of the public policies that directed at eliminating labor market discrimination. Second, a decomposition of the relative importance of the experience effect and expectation effect can depict a clear and transparent picture of how labor supply affects wage dynamics. It also addresses the question of whether expectation is vital in affecting individuals’ human capital accumulation. Third, a useful framework for answering these questions should also be central in the analysis of a wide range of topics in labor economics and public policies.

I build a life-cycle model which features endogenous labor supply, endogenous human capital investment and consumption-saving decision. Males and females differ in the sense that females face higher average labor market participation cost than males and this leads to lower labor market participation. This is the only source of heterogeneity between genders. Males and females are the same in all other respects. Most importantly, I assume that they have the same human capital production technology. Within each gender category, I assume that individuals differ in initial human capital, learning ability, and labor market participation cost. Initial human capital can be thought of determining the intercept of an agent’s wage profile, whereas learning ability acts to rotate this profile. I also assume that human capital accumulation is subject to idiosyncratic shocks in each period, therefore human capital and labor earnings are risky.

To answer the research questions, I implement two calibration procedures: “benchmark calibration" and “alternative calibration”. In the benchmark calibration, the model is calibrated so that the predictions of the model match the wage and labor supply dynamics for male workers over the life cycle. In this calibration, the parameters that determine the labor supply patterns over the life-cycle, which I call the “labor supply parameters” and the parameters that govern how human capital is accumulated over the life-cycle, which I call the “the human capital parameters” are jointly determined. As can be shown in later sections, the model does a good job of matching the data targets.

In the alternative calibration, I calibrate the model to match the labor supply dynamics for female workers by choosing labor supply parameters while holding the human capital parameters calibrated in the benchmark calibration fixed. By design, these two calibration procedures can totally isolate the impact of labor supply on wage growth. The result of this quantitative exercise shows that different labor supply patterns between genders can explain about 66% of the gender differences in life-cycle wage growth. The literature typically finds this number to be between one third and one half. My result suggests that labor supply actually plays a more important role in determining gender wage gaps compared to the findings in the literature. This is one novel finding of the paper. The second research question involves the decomposition of the two effects, namely the experience effect and the expectation effect. The decomposition results show that experience effect alone can explain about 38 percentage points while the expectation effect alone can explain about 22 percentage points. The rest 6 percentage points might be due to the interaction effect between experience and expectation. To my best knowledge, this is the first paper to provide a quantitative assessment of the importance of the two effects. Particularly, the paper shows that the expectation effect alone can contribute to more than 20 percentage points of the gender differences in life-cycle wage growth. In other words, the expectation effect plays a non-negligible role in affecting life-cycle wage dynamics even though experience effect account for more of the gender wage growth differentials. This is the second important finding of the paper.

Related Literature A large body of literature tries to explain why male workers have higher wage and higher wage growth than female workers based on the fact that males and females have different labor supply patterns. The literature is split into two strands. The first strand mainly focuses on the experience effect. The paper in this direction include [17], [18], [19], [20], [21], [22] and so on. By using different model specifications, these papers find that the experience effect could account for approximately one third to one-half of the wage differences between genders. In [18], they employ a rich work history model to characterize the presence of interpersonal differences in the timing of work experience and differences in the total amount of experience accumulated. They find that 12% of the male-female wage gap is due to differences in the timing of work experience. [19] finds that labor supply can explain about one half of the gender gap in early-career wage growth. In [17], the authors use Panel Study of Income Dynamics (PSID) data to show that changes in accumulated experience can account for a large share of the decline in gender wage gaps.

The second strand studies the possible effect of expectation about future labor supply on gender wage differences. This includes [23], [24], [25], [26], [27] and so on. [23] is the first paper to bring about the idea that gender differences in future work expectations might serve to explain the wage gap between genders. They argue that optimal investment in human capital depends on prospective utilization of the capital being accumulated. The fact that female workers are more likely to detach from the labor market than male workers reduces their incentive to invest in human capital. They find that expected career interruptions indeed have a negative influence on the human capital investment of females. [24] finds that women's planned labor force interruptions lead to reduced human capital investment and lower wages. [25] finds that preference for future labor force attachment is significantly related to rates of post-school human capital accumulation among young white working women. Another finding is that young women may underestimate their future labor force attachment, which implies that women may underinvest in human capital.

All the papers listed above are based on regression analysis instead of a theoretical decision-making framework. In [26], they build a theoretical decision-making model to analyze the joint determination of females' earnings and labor force participation over the life cycle. They find that the interruptions affect the profitability of the investment in human capital, which in turn determines earnings. However, they do not provide any quantitative results. Another paper that set up a theoretical model is [27], they build a life-cycle model to study the possible effect of fertility decisions on gender differences in early-career wage growth. They show that compared to males, females tend to take less effort to do human capital investment when they expect less labor force detachment because of childbearing possibilities. However, they focus on the early part of the life-cycle instead of the whole life-cycle. The expectation about the labor supply over the entire life-cycle affects the human capital investment decision today, therefore a model incorporating the whole lifetime periods will be more appropriate if the expectation effect is one of our main interests.

The rest of the paper is organized as follows. Section 2 documents the wage dynamics and labor supply dynamics for both males and females over the life-cycle. Section 3 presents the model. Section 4 introduces the benchmark calibration of the model. Section 5 investigates the properties and shows the fit of the model. Section 6 introduces the alternative calibration, conducts several quantitative experiments and answer the two research questions. Section 7 concludes.

3.2 Empirical Analysis

The data that I use is the Panel Study of Income Dynamics (PSID) 1969-2015 family files. The PSID is a longitudinal study of a representative sample of US individuals (men, women, and children) as well as the family units in which they reside. The survey started from 1968 when about 5000 families were first interviewed. The survey gathered information about these families and all their descendants from then on. Data is annual from 1968 to 1997, and biennial since then.

To focus on as homogeneous a group as possible, we only use data of white male and white female high school graduates. This can eliminate race differences as well as educational attainment differences among individuals. Because labor supply patterns for both males and females, particularly for females, have been constantly evolving over the last 50 years, I choose to focus on the 1946 -1954 cohort for the empirical analysis. The cohort effect can be eliminated by focusing on this particular cohort. Another reason is that, for the 1946-1954 cohort, we can have a complete history of the workers' labor force participation and wage over the entire life-cycle. The individuals born in 1946 just turned to 21 years old when the survey started and those born in 1954 were 60 years old when they were interviewed in 2015. This is of critical importance for two reasons. First, data covering the entire working periods enables us to extract the life-cycle wage profiles and life-cycle labor force participation profiles for both males and females. This will provide high quality data moments for me to do the calibration. Second, since expectation about future labor supply over the entire life-cycle affects the human capital investment decision today, the life-cycle data is important to correctly model the expectation effect.

The empirical data moments that I choose to estimate are life-cycle profiles of average hourly wage, life-cycle profiles of the dispersion of hourly wage, life-cycle profiles of labor force participate rates and the distribution of the total number of working years by quintiles. To be specific, the life-cycle profiles are estimated from age 21 to 60. To calculate the labor force participation rates, I treat those who have worked less than 520 hours in one year as not participating in the labor market in that year and those who have worked at least 520 hours as participating in the labor market. The restriction on minimum hours amounts to a quarter of full-time work hours. The hourly wage is constructed as the yearly earnings divided by the total number of hours worked in that year. Note that the sample only includes those who participated in the labor market when I construct the hourly wage. To calculate wage and labor supply statistics at a specific age and year, I use a five-year age bin.

I adopt a two-step methodology to estimate all relevant life-cycle profiles. First, I calculate the statistic of interest for both males and females in age bin j at time t and label this as $stat_{j,t}$. For example, I set $stat_{j,t} = w_{j,t}$ for average hourly wage. Here $w_{j,t}$ is real mean hourly wage of all relevant observations in the age bin centered at age j in year t.[1] For labor force participation rate, I set $stat_{j,t} = p_{j,t}$, where $p_{j,t}$ is the labor force participation rate of all relevant observations in the age bin centered at age j in year t. Second, the statistic is viewed as being generated by several factors, including age effect and time effect. The age effect is what we are really interested in.[2] I posit an econometric model governing the evolution of the statistic as indicated below.

$$stat_{j,t} = \beta_j^{stat} + \gamma_t^{stat} + \epsilon_{j,t}^{stat},$$

where β_j^{stat} and γ_t^{stat} represent the coefficients in front of the age dummies and time dummies and $\epsilon_{j,t}^{stat}$ is the error term. Ordinary least squares is implemented to estimate the coefficients. The life-cycle profile is constructed using β_j^{stat}.

In Figure 3.1, I graph the life-cycle profiles of labor force participation rates for males and females. It generates the familiar hump-shaped age profile of labor force participation rates for

[1] I use the Consumer Price Index to convert nominal earnings to real earnings.

[2] In the paper, age effect is equivalent to age profile or life-cycle profile. I will use these three terms interchangeably in later illustration

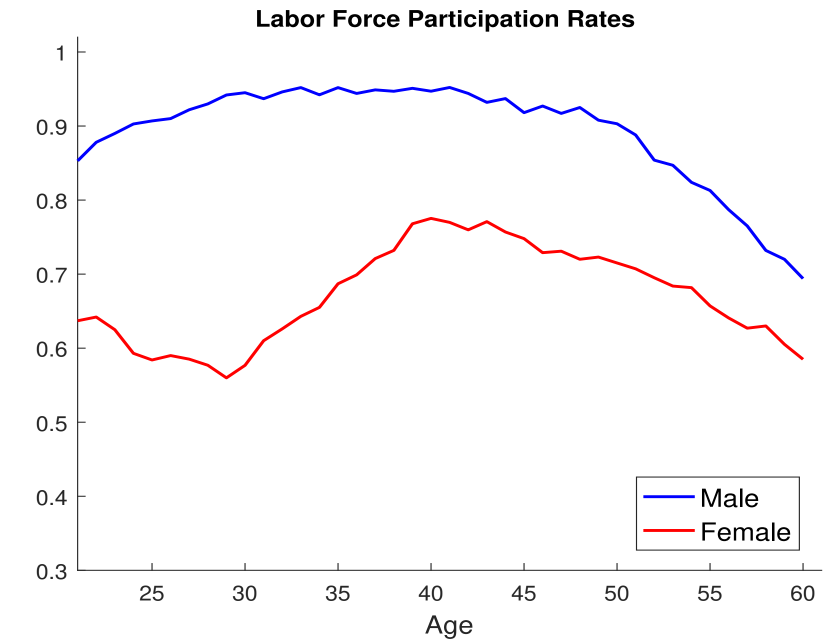

Figure 3.1: Life-cycle profile of labor force participation rates

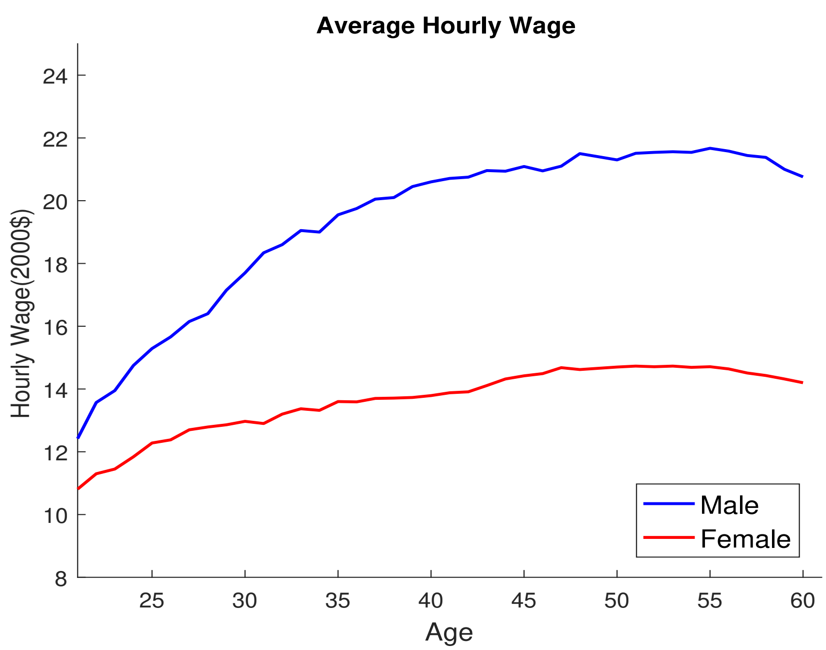

Figure 3.2: Life-cycle profile of average hourly wages

males. The labor force participation rate is increasing at the early stage of life-cycle and starts to decrease around age 40. The age profile for females is more complicated than that for males. The labor force participation rate is decreasing between age 21 to 30. The main reason for this declining trend might be that females tend to make fertility decisions in this period and many of them choose to detach from the labor market. Females choose to come back to the labor market after they give birth, which gives rise to the increasing trend of labor force participation rates between age 30 and 40. After 40, the participation rate starts to decline again. One prominent feature of the data is that there is a considerable gap in labor force participation rates between males and females over the life-cycle. Males have higher participation rates than females at every age of the life-cycle. The biggest gap occurs around age 30 and is approximately 40 percentage points. The smallest gap is still around 10 percentage points and occurs around the retirement age. The average labor force participation rate is around 88% for males and 67% for females over the entire life-cycle.

Table 3.1 presents the distribution statistics of labor market participation history for both males and females. For each individual in the sample, I have the complete history of labor market activities.[3] The total number of years that the individual has worked for the first ten years (age 21 to 30), first twenty years (age 21 to 40) and the whole life-cycle (age 21 to 60) can be calculated accordingly. I report the average number of total working years for each of the five quintiles of the corresponding distribution. It is clear that labor supply differences across gender are substantial. Males tend to work more in each of the five quintiles of the corresponding distribution. This is in accord with the evidence that males have higher labor force participation rates than females over the life-cycle. The statistics in Table 3.1 serves to grasp the distributional pattern of working history for males and females. The table will also be used in the later benchmark calibration. A good fit of the model's predictions to the data moments displayed in Table 3.1 adds our confidence in the validity of the model.

[3] Note that the data is is annual from 1968 to 1997, and biennial since then. This means that the data is missing for every two years since year 1997. I use the following criterion to judge whether the individual has worked at some age, say age j: if the individual worked at both age $j-1$ and age $j+1$, the individual is treated as working at age j; if the individual neither worked at age $j-1$ nor at age $j+1$, the individual is treated as not working at age j; if the individual is observed working only in one of the two years, the participation status is determined randomly (50 % for each possibility)

	Distribution of total years of working by quintiles					
	Age 21-30		Age 21-40		Age 21-60	
	Male	Female	Male	Female	Male	Female
Q1	6.5	2.3	15.3	6.7	28.9	14.8
Q2	8.7	4.5	17.4	10.2	32.4	20.4
Q3	9.6	6.4	18.7	13.6	35.6	27.9
Q4	10	7.2	19.6	15.3	38.1	32.2
Q5	10	9.3	20	18.8	40	37.9

Table 3.1: Distribution of total years of working by quintiles

The life-cycle profiles for the average hourly wage are reported in Figure 3.2. The familiar hump-shaped age profiles of wage are produced both for males and females. A salient feature of the labor market is that the average hourly wage of males is substantially higher than the average hourly wage of females. Also, male workers have higher wage growth than female workers over the life-cycle. The gender gap in wage is about 1.5 dollars at age 21 and the gap is as high as 7 dollars at age 55, when the wage for both males and females tend to reach the prime. While the wage gap in gender is not very large at the beginning of the life-cycle, the gap is getting larger when they age. The wage ratio between age 55 and age 21 is 1.75 for males and 1.36 for females.

Table 3.2 reports the life-cycle wage dynamics for both males and females. All statistics are reported every five years. Column (a) and column (c) show the average hourly wage for males and females respectively. Column (b) and column (d) represent the wage growth from age 21 for males and females respectively. Since this paper makes no attempt to explain the gender gap in wage at the beginning of the life-cycle, I focus on the wage growth differentials between genders. So the last column (e) calculates the wage growth differences. We can see that male workers have a 9.25 dollar wage growth from age 21 to 55 while the number for female workers is only 3.90. The difference of life-cycle wage growth between genders is 5.35 dollars. This number is what this paper wants to explain: that is, how much of the gender difference in life-cycle wage growth can be explained by different labor supply patterns?

Age	Male		Female		
	Wage	ΔWage from 21	Wage	ΔWage from 21	Δ Wage Growth
	(a)	(b)	(c)	(d)	(e)=(b)-(d)
21	12.42	NA	10.81	NA	NA
25	15.29	2.87	12.28	1.47	1.43
30	17.70	5.28	12.97	2.16	3.12
35	19.55	7.13	13.60	2.79	4.34
40	20.61	8.19	13.79	2.98	5.21
45	21.09	8.67	14.42	3.61	5.06
50	21.34	8.92	14.70	3.89	5.03
55	21.67	**9.25**	14.71	**3.90**	**5.35**
60	20.76	8.34	14.20	3.39	4.95

Table 3.2: Life-cycle wage dynamics for male and female

3.3 Model

3.3.1 Model Set-up

I build a life-cycle model in which individuals make decisions about consumption, saving, labor supply, and human capital investment. Time is discrete and runs from $t = 1, 2, ..., T$. The economy is populated with individuals who live from period $j = 1$ to period $j = J$. Each period represents one calendar year.[4] All individuals enter the model as high school graduates at age 21, which corresponds to model age 1. At model age 1, individuals in the economy are heterogeneous in four dimensions: initial human capital h_1, initial financial wealth k_1, learning ability a and working type w, Let $x_1 = (h_1, k_1, a, w)$ denote the initial conditions. Individuals are able to accumulate human capital using a Ben-Porath(1967) style technology if they choose to participate in the labor market. The cost associated with labor market participation depends on each individual's working type and age. We assume that the working type and ability do not change over an individual's lifetime. An agent maximizes the expected lifetime utility, taken the vector of state variables (h_1, k_1, a, w) as given.

I model the extensive margin of labor supply. I assume that the utility function in period j is as

[4] I use “period" and “age" interchangeably in the paper.

follows,

$$u_j(c_j, n_j, s_j) = \frac{c_j^{1-\theta}}{1-\theta} - n_j s_j \tag{3.1}$$

Here c_j is the consumption level and the binary variable $n_j \in \{0, 1\}$ indicates whether the agent chooses to work. The labor market participation cost for each individual is denoted as s_j. I assume that the labor market participation cost consists of two components: fixed cost and stochastic cost. The fixed cost depends on the individual's working type and I allow the fixed cost to vary systematically across age. I assume that the fixed cost for each agent is of the following parametric form,

$$w_j = w \cdot (1 + \gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + \gamma_4 j^4) \tag{3.2}$$

Here γ_1, γ_2, γ_3, γ_4 are four parameters that jointly determine the fixed cost for each individual at each age over the life-cycle. It is also clear that the fixed cost depends on the working type of each individual.

The parametric form for the labor market participation cost s_j is as follows,

$$s_j = \exp(w_j + \sigma_\epsilon \epsilon_j) \tag{3.3}$$

where ϵ_j is an independent and identically-distributed standard normal random variable. Therefore s_j follows a log-normal distribution. The location and scale parameters are the fixed cost w_j and the standard deviation of the stochastic cost σ_ϵ respectively.

At each period, each individual decides either to participate in the labor market or not. If the individual chooses to participate in the labor market, he/she chooses how much time, i_j, to invest in human capital accumulation. I assume that all individuals are endowed with one unit of time for each period, so they use the rest, $1 - i_j$, to perform market work. The individual acquires labor income from market work. The human capital production function is of Ben-Porath(1967) type,

$$h_{j+1} = \exp(z_j)((1-\delta)h_j + a(h_j i_j)^\alpha) \tag{3.4}$$

where a is the learning ability and h_j is the human capital level at period j. δ is the human capital depreciation rate. z_j is an idiosyncratic shock to human capital accumulation. $\alpha \in (0,1)$, is the curvature parameter for the human capital production technology; it determines the degree of diminishing marginal returns of human capital investment.

If an individual chooses not to participate in the labor market, he/she invests no time in human capital production($i_j = 0$) and human capital depreciates at the constant rate δ.

I assume that human capital is general and the labor market is perfectly competitive. As a consequence, workers will bear the cost of human capital investment and the wage rate will be adjusted downwards by the fraction of time invested in human capital accumulation. The rental rate of human capital is normalized to one so that the wage rate for each unit of effective labor supply is the human capital level h_j. Therefore the pre-tax labor wage income for an agent is $h_j(1 - i_j)$.

The maximization problem for each agent can be formulated as the following recursive form,

$$V_j(h_j, k_j, a, w; \epsilon_j) = \max_{c_j, n_j, i_j} \{ \frac{c_j^{1-\theta}}{1-\theta} - n_j s_j + \beta E_j V_{j+1}(h_{j+1}, k_{j+1}, a, w; \epsilon_{j+1}) \} \quad \text{s.t.} \tag{3.5}$$

$$c_j + k_{j+1} = k_j(1+r) + e_j - T(e_j) + \tau_j,\ k_{J+1} = 0 \tag{3.6}$$

$$e_j = h_j(1 - i_j)n_j,\ \text{if } j < J_R \text{ and } e_j = 0 \text{ otherwise} \tag{3.7}$$

$$h_{j+1} = \exp(z_j)((1-\delta)h_j + a(h_j i_j)^\alpha n_j) \tag{3.8}$$

$$s_j = \exp(w_j + \sigma_\epsilon \epsilon_j) \tag{3.9}$$

$$w_j = w(1 + \gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + \gamma_4 j^4) \tag{3.10}$$

$$\tau_j = max\{0, \underline{c} - [(1+r)k_j + e_j - T(e_j)]\} \tag{3.11}$$

$$c_j \geq 0,\ k_j \geq 0,\ n_j \in \{0,1\},\ 0 \leq i_j \leq 1 \tag{3.12}$$

Equation (3.6) indicates the budget constraint at age j. k_j stands for financial wealth and r is the risk free real interest rate. $T(e_j)$ is the tax function imposed on the labor income. Government transfers, τ_j, provide a consumption floor $\underline{c}$ as in [28]. I assume that individuals leave no bequest so that they run out of the financial wealth during the last period of their lives. This is why we impose the restriction that $k_{J+1} = 0$. Condition (3.12) refers to the nonnegative constraint on consumption, the exogenous borrowing constraint on financial wealth and the constraint on time faction that can be used to accumulate human capital. Equations (3.7) to (3.10) have been directly specified in the previous argument and I just skip the illustration of these equations here. Equation (3.11) refers to how the government transfers are determined. If the total available financial resource, $[(1+r)k_j + e_j - T(e_j)]$, is smaller than the consumption floor, the government will provide transfers.

3.3.2 Solutions to the Model

In the model, we have two shocks that affect the decisions of each individual: the stochastic component of labor market participation cost, ϵ_j, and the human capital production shock, z_j. The timing within a period in our model is as follows: the participation cost shock ϵ_j is realized at the start of each period. Given the value of this shock and of the vector of state variables, $x_j = (h_j, k_j, a, w)$, the individual makes decisions about consumption, saving, labor supply and human capital investment. After these decisions are made, the human capital production shock z_j is realized. This human capital production shock, z_j, together with the decisions about labor force participation and human capital investment, will determine the human capital level in the next period.

Let $x_j = (h_j, k_j, a, w)$ denote the vector of state variables. Equation (3.5) can be rewritten as follows,

$$V_j(x_j;\epsilon_j) = \max_{c_j, n_j, i_j} \{u_j(c_j, n_j, s_j) + \beta E_j V_{j+1}(x_{j+1};\epsilon_{j+1})\}$$

The expectation on the right hand side is over the human capital production shock z_j and the labor market participation cost shock in period $j+1$.

I solve the agent's maximization problem in two steps. The first step is to solve the optimal decisions conditional on the labor supply decision and then the labor supply decision is determined by comparing the value functions associated with different labor supply decisions.

The optimal consumption $\tilde{c}_j^1(x_j)$, human capital investment, $\tilde{i}_j^1(x_j)$ decisions conditional on working ($n_j = 1$) depend only on x_j and can be acquired from

$$\{\tilde{c}_j^1(x_j), \tilde{i}_j^1(x_j)\} = \underset{\{c_j, i_j\}}{\text{argmax}}\{\frac{c_j^{1-\theta}}{1-\theta} + \beta E_j V_{j+1}(x_{j+1}; \epsilon_{j+1} | n_j = 1)\} \tag{3.13}$$

and the conditional value function apart from labor market participation cost is

$$\tilde{V}_j^1(x_j) = \frac{\tilde{c}_j^1(x_j)^{1-\theta}}{1-\theta} + \beta E_j V_{j+1}(x_{j+1}; \epsilon_{j+1} | \tilde{c}_j^1(x_j), \tilde{i}_j^1(x_j), n_j = 1)\} \tag{3.14}$$

Conditional on not participating in the labor market ($n_j = 0$), the optimal consumption decisions can be calculated as follows,

$$\{\tilde{c}_j^0(x_j)\} = \underset{\{c_j\}}{\text{argmax}}\{\frac{c_j^{1-\theta}}{1-\theta} + \beta E_j V_{j+1}(x_{j+1}; \epsilon_{j+1} | n_j = 0, i_j = 0)\} \tag{3.15}$$

The conditional value function function is

$$\tilde{V}_j^0(x_j) = \frac{\tilde{c}_j^0(x_j)^{1-\theta}}{1-\theta} + \beta E_j V_{j+1}(x_{j+1}; \epsilon_{j+1} | \tilde{c}_j^0(x_j), n_j = 1, i_j = 0)\} \tag{3.16}$$

Since I assume that ϵ_j follows an independent and identically-distributed standard normal distribution, the conditional policy functions and the conditional value functions in equations (3.13) to (3.16) do not depend on ϵ_j.

At each period, the agent decides whether to work or not. He/she chooses to participate in the labor market if

$$\tilde{V}_j^1(x_j) - s_j \geq \tilde{V}_j^0(x_j) \tag{3.17}$$

This implies that there exists a threshold value

$$s_j^*(x_j) = \tilde{V}_j^1(x_j) - \tilde{V}_j^0(x_j) \tag{3.18}$$

such that

$$n_j = \begin{cases} 1, & \text{if } s_j \leq s_j^*(x_j) \\ 0, & \text{if } s_j > s_j^*(x_j) \end{cases} \tag{3.19}$$

From equation (3.3), we know $\ln(s_j^*(x_j)) = w_j + \sigma_\epsilon \epsilon_j^*(x_j)$, so we can get the threshold value of ϵ_j as

$$\epsilon_j^*(x_j) = \frac{1}{\sigma_\epsilon}\{\ln(s_j^*(x_j)) - w_j\} \tag{3.20}$$

Then the agent's labor supply decision is determined by

$$n_j = \begin{cases} 1, & \text{if } \epsilon_j \leq \epsilon_j^*(x_j) \\ 0, & \text{if } \epsilon_j > \epsilon_j^*(x_j) \end{cases} \tag{3.21}$$

Given the vector of state variables, $x_j = (h_j, k_j, a, w)$, the agent chooses to participate in the labor market with probability $\Phi(\epsilon_j^*(x_j))$ and chooses not to participate in the labor market with probability $1 - \Phi(\epsilon_j^*(x_j))$, where $\Phi(\cdot)$ is the cumulative density function for standard normal distribution.

Since s_j is log-normally distributed with location parameter w_j and scale parameter σ_ϵ, we know

$$E(s_j | s_j \leq s_j^*(x_j)) = \exp(w_j + \frac{\sigma_\epsilon^2}{2}) \frac{\Phi(\epsilon_j^*(x_j) - \sigma_\epsilon)}{\Phi(\epsilon_j^*(x_j))} \tag{3.22}$$

Therefore

$$\begin{aligned} E[V_{j+1}(x_{j+1}; \epsilon_{j+1}) | x_{j+1}] =& \Phi(\epsilon_{j+1}^*(x_{j+1})) \Big\{ \tilde{V}_{j+1}^1(x_{j+1}) - \exp(w_{j+1} + \frac{\sigma_\epsilon^2}{2}) \frac{\Phi(\epsilon_{j+1}^*(x_{j+1}) - \sigma_\epsilon)}{\Phi(\epsilon_{j+1}^*(x_{j+1}))} \Big\} \\ & + (1 - \Phi(\epsilon_{j+1}^*(x_{j+1}))) \tilde{V}_{j+1}^0(x_{j+1}) \end{aligned} \tag{3.23}$$

By the law of iterated expectation, we know that

$$E_j V_{j+1}(x_{j+1};\epsilon_{j+1}) = E_j\left\{E[V_{j+1}(x_{j+1};\epsilon_{j+1})|x_{j+1}]\right\} \tag{3.24}$$

The expectation is over the distribution of the human capital production shock, z_j, which I assume follows an independent and identically distributed normal distribution.

$$z_j \sim \mathcal{N}(-\frac{\sigma_z^2}{2}, \sigma_z^2) \tag{3.25}$$

where σ_z^2 is the variance of z_j and we know the expectation of $\exp(z_j)$ is one.

The human capital level in the first period is assumed to be log-normally distributed, $h_1 \sim \mathcal{LN}(\mu_h, \sigma_h^2)$. The working type is assumed to be normally distributed, $w \sim \mathcal{N}(\mu_w, \sigma_w^2)$. In the benchmark case, we consider the case where initial financial wealth are set to zero.

Following [2], I let the tax system T includes a social security and an income tax. Before retirement, I assume that the income tax is progressive: $T^{inc}(e_j) = 1 - \eta_1 \tilde{e}^{-\eta_2}$, where $\tilde{e} = \frac{e_j}{\bar{e}}$ and $\bar{e}$ is the mean before-tax income in the economy. After retirement, the social security pays a common retirement transfer which is set to be 40 percent of mean economy-wide earnings in the last period of an agent's working periods.

3.4 Benchmark Calibration

In this section, I discuss the benchmark calibration of the model. I ask the model to account for key features of the dynamics of labor force participation and wage distribution for males. This calibration process serves as the benchmark calibration. The model parameters are set so that the predictions of the model match the age profiles of labor force participation rate, average hourly wage, dispersion of hourly wage as well as the distribution of labor force participation history for males.

The model parameters consist of two subsets of parameters. The parameters in the first subset are those that discipline how human capital is accumulated over the lifetime. I call this subset of parameters "human capital parameters". The human capital parameters include the mean and the standard deviation for the distribution of initial human capital, the mean and the standard deviation for the distribution of learning ability level, the correlation between initial human capital and learning ability, the curvature parameter in the human capital production function, the human capital depreciation rate and the standard deviation of the human capital shock. The second subset of model parameters are those that determine how individuals make labor supply decisions over the life-cycle. I call this subset of parameters “labor supply parameters". The labor supply parameters include the mean and the standard deviation for the distribution of working type, the standard deviation of the shock to labor force participation cost and the coefficients in the polynomial which determines how the fixed working cost evolve over the lifetime.

Table 3.3 reports the calibration results for the model parameters. The correlation between the initial human capital and learning ability is 0.64. This positive correlation is critical in accounting for the increasing wage dispersion over the life-cycle, which is well illustrated in [2]. The curvature parameter in the human capital production function is 0.58 in our model. This parameter has been estimated in the literature. The estimates, surveyed by [14], lie in the range 0.5 to about 0.9. Our result in accordance with the literature. The human capital depreciation rate is set to 0.022, which is also in the range of estimates in the literature. Another point to note is that γ_3 and γ_4 are both zeros. This is because the age profile of labor force participation for male workers is hump-shaped and relatively simple. Therefore, a quadratic function of age is rich enough to represent the fixed cost.

3.5 Properties of the Model

In this section, I report the ability of the model to match the data targets produced using males data and documented in section 3.2. I also provide and analyze a number of other properties of the model. To be specific, I analyze how the return and cost of labor supply evolve over the life-cycle.

3.5.1 Fit of the Model

The life-cycle profiles of labor force participation rate, average hourly wage, and dispersion of wage produced by the benchmark model are displayed in Figure 3.3 to 3.5. The corresponding

Human Capital Parameters		Value
Initial human capital distribution: mean	μ_h	2.59
Initial human capital distribution: s.d.	σ_h	0.28
Initial ability distribution: mean	μ_a	-1.22
Initial ability distribution: s.d.	σ_a	0.104
Correlation	ρ	0.64
Curvature of human capital production	α	0.58
Human capital depreciation	δ	0.022
Human capital shock: s.d.	σ_z	0.115
Labor Supply Parameters		
Working type: mean	μ_w	-1.604
Working type: s.d.	σ_w	0.236
Participation cost shock: s.d	σ_ϵ	0.387
Coefficient	γ_1	0.022
Coefficient	γ_2	$-5.48e^{-04}$
Coefficient	γ_3	0
Coefficient	γ_4	0

Table 3.3: Parameters chosen to match the data moments based on males data

estimates using PSID data are also displayed as a comparison. The model succeeds in generating the hump-shaped life-cycle wage profile. The mechanism is that individuals devote much more time to human capital accumulation at the early stage of the life-cycle than at later ages, which lead to a net accumulation of human capital at early ages. Therefore, the average wage increases with age as human capital level increases with age. As individuals approach retirement, they reduce the investment in human capital since the number of working periods that the individuals can reap the benefit of human capital investment falls. The wage starts to fall around age 55 and this is the direct consequence of human capital depreciation and barely any human capital investment in the last 5 years of the life-cycle.

The model also generates the hump-shaped age profile of labor force participation rate and the match between the model and the data is pretty good. Males tend to increase their labor force attachment in the early part of the life-cycle and decrease labor supply towards the end of the life-cycle. I will postpone the detailed analysis of the mechanism behind this model feature to the next

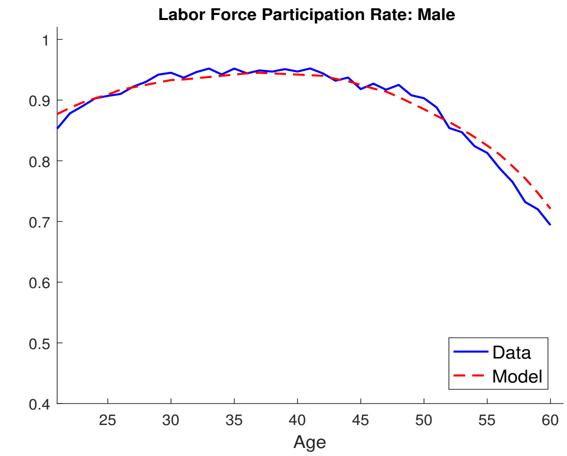

Figure 3.3: Fit of model: labor force participation rate for males

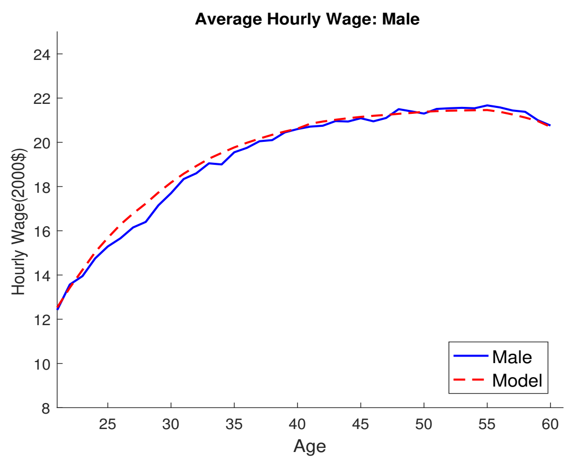

Figure 3.4: Fit of model: average hourly wage for males

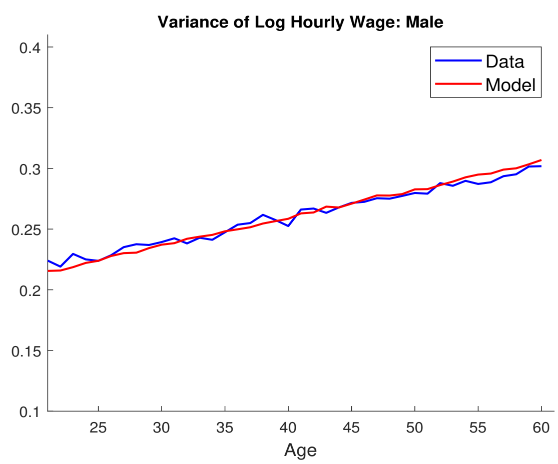

Figure 3.5: Fit of model: variance of log hourly wage for males

	Distribution of total years of working: Male					
	Age 21-30		Age 21-40		Age 21-60	
	Data	Model	Data	Model	Data	Model
Q1	6.5	6.1	15.3	14.8	28.9	27.5
Q2	8.7	8.9	17.4	17.7	32.4	32.8
Q3	9.6	9.7	18.7	19.0	35.6	36.1
Q4	10	9.9	19.6	19.3	38.1	38.3
Q5	10	10	20	20	40	39.9

Table 3.4: Fit of model: distribution of total years of working for males

subsection where I illustrate how the return and cost of labor supply evolve over the life-cycle.

The variance of log wage is to used to indicate the dispersion of wage. The main advantage of using variance of log wage is that it is scale invariant. The model replicates the age profile of wage dispersion over the lifetime estimated using the PSID data. This increasing dispersion mainly comes from two sources of model features. First, the heterogeneity in learning ability and the positive correlation between initial human capital and learning ability ensures that individuals with higher initial human capital also tend to have higher learning ability. These individuals tend to invest more in human capital over the life-cycle and this leads to higher wage dispersion over time. Without the heterogeneity in learning ability, the dispersion of wage would decrease with age as human capital would converge because of the concavity of the human capital production function. The second reason is that we impose the existence of idiosyncratic shocks to human capital investment. The shocks associated with human capital accumulation will cause the wage to disperse over the life-cycle.

The model also does an excellent job of replicating the pattern of the distribution of working history for males, as shown in Table 3.4. The discrepancy between the model's predictions and the data is very small, which indicates that our model has the ability to account for the main features observed in the data.

3.5.2 Labor Supply over the Life-cycle

As is shown in Figure 3.3, the age profile of labor force participation rate for male workers is hump-shaped. Male workers tend to increase labor supply between age 21 and 40 and reduce labor supply afterward. From the model set-up, we know that at each period, individuals compare the return (or benefit) to the cost of labor supply and make optimal labor supply decisions accordingly. From equation (3.18), we know that $\tilde{V}_j^1(x_j) - \tilde{V}_j^0(x_j)$ is the return of labor supply given the state variables and s_j is the labor participation cost facing each individual. Each individual will choose to participate in the labor market as long as the return of labor supply exceeds the cost of labor supply.

Now, we analyze how the return of labor supply evolves over the life-cycle for a typical worker. The return of labor supply is determined by three effects: substitution effect, income effect, and human capital effect. First, as the wage increases over the life-cycle, the substitution effect induces labor supply to increase. The substitution effect increases over most part of the life-cycle and decreases a little bit at late life-cycle periods due to the fact that the age profile of wage is hump shaped. Second, individuals accumulate financial wealth as they age and the income effect reduces the incentive of working when individuals become wealthier. So the income effect decreases over the lifetime. Third, the last effect that affects the return of labor supply is the human capital effect. This effect arises since current labor supply can possibly lead to human capital accumulation and higher future human capital level. In other words, current labor supply can not only bring wage income but also increase the ability of earning money in future periods. The human capital effect also decreases over the life-cycle since the number of periods over which individuals can obtain the benefit of human capital accumulation falls as individuals age. The total return of labor supply over the life-cycle depends on the three effects mentioned above. The profile for this total effect also depends on the parameters of the model. One exercise that I do is to impose constant labor force participation cost[5] over the life-cycle on the calibrated model economy. The counterfactual

[5]This amounts to setting γ_1, γ_2, γ_3, γ_4 to zero and keeping other parameters unchanged in our calibrated model economy.

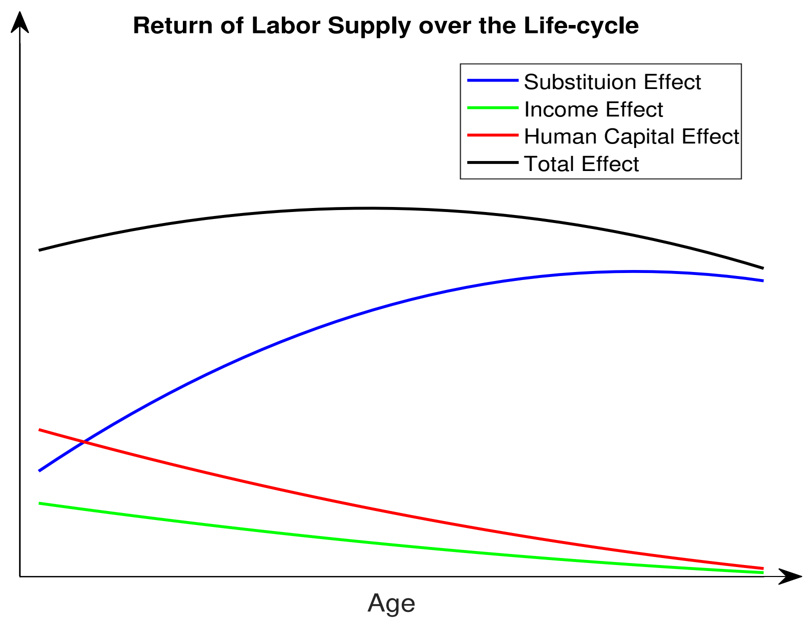

Figure 3.6: Return of labor supply over the life-cycle

life-cycle profile of labor force participation rates is still hump shaped although it is slightly fatter than what we get in Figure 3.3. This means that the total return of labor supply in our model economy is hump shaped. In Figure 3.6, the three effects and the corresponding trends of variation over the life-cycle are plotted.

The cost of the labor supply in each period depends on the parametric form and the corresponding parameters of the fixed cost. The parameters that govern how the fixed cost of labor supply evolves over the life-cycle are γ_1, γ_2, γ_3 and γ_4. From Table 3.3, we know that, for males, fixed cost is a binomial function of age. I plot the age profile of mean cost of labor supply for a typical worker in Figure 3.7.[6] It is U-shaped, which means that male workers have relatively larger labor force participation cost at young and old ages while having smaller cost at middle ages. Note that the curve is rather flat and the cost of labor supply over the life-cycle does not change much. In other words, if I impose the restriction that the fixed part of the labor supply cost remains constant

[6] By saying a typical worker, I mean that the value of the working type for the worker is the mean value of the distribution of the working type. In our calibrated model, that is to set $w = -1.664$. The mean cost of labor supply at age j is calculated as $\exp(w_j + \frac{1}{2}\sigma_\epsilon^2)$, where w_j is calculated according to equation (3.2).

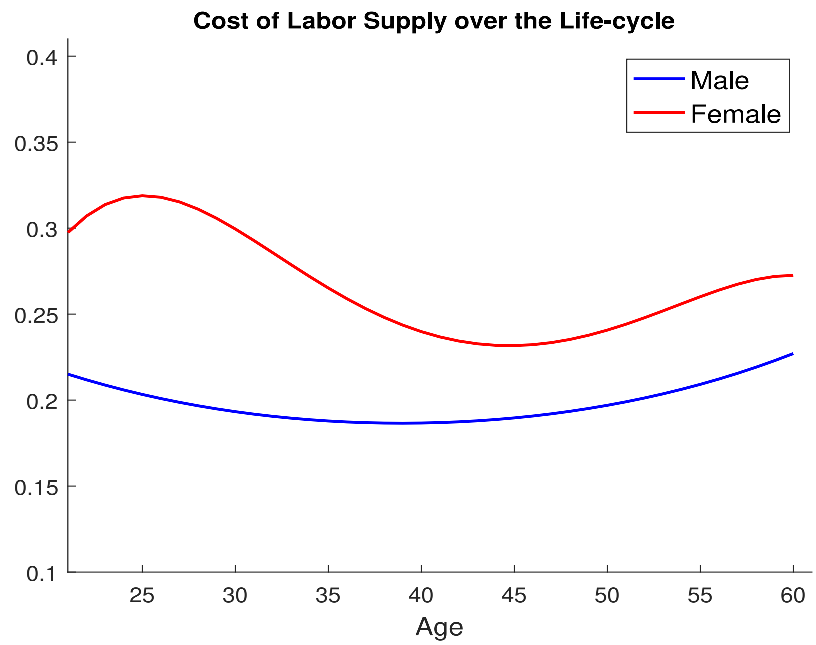

Figure 3.7: Cost of labor supply over the life-cycle

over the life-cycle, I can still produce the hump-shaped labor force participation profile over the lifetime, as I have mentioned in the last paragraph. For females, the pattern of the labor supply cost is more complicated, I will postpone the analysis to later sections.

3.6 Alternative Calibration and Quantitative Experiments

In section 3.5, I show that the model economy can replicate the data features pretty well when the model is calibrated using males' data. The calibration strategy is to take the males' data as the benchmark and pin down two sets of model parameters by matching the predictions of the model to the data targets produced using males' data. The two sets of parameters, as I describe in section 3.4, are "human capital parameters" and "labor supply parameters". In this section, I discuss another calibration procedure, which I call the "alternative calibration" to answer relevant research questions. The detailed illustration of the alternative calibration will constitute the first subsection. In the second subsection, I conduct two quantitative exercises to decompose the two effects by which labor supply affect life-cycle wage growth and study their separate contributions

Labor Supply Parameters		Value
Working type: mean	μ_w	-1.248
Working type: s.d.	σ_w	0.272
Participation cost shock: s.d	σ_ϵ	0.303
Coefficient	γ_1	0.045
Coefficient	γ_2	$-5.97e^{-3}$
Coefficient	γ_3	$2.11e^{-4}$
Coefficient	γ_4	$-2.27e^{-6}$

Table 3.5: Parameters chosen to match the data moments based on females' data.

to the gender differences in life-cycle wage growth.

3.6.1 Alternative Calibration

As stated in the introduction, the main goal of this paper is to provide a quantitative assessment of the importance of labor supply in explaining gender differences in life-cycle wage growth, i.e, what fraction of gender differences in life-cycle wage growth can be attributed to different labor supply patterns? To answer this question, I recalibrate the model to match the life-cycle profiles of labor force participation rate and the distribution of working history for females, holding human capital parameters unchanged while choosing different labor supply parameters. The counterfactual life-cycle wage profile would indicate how the wage for male workers would evolve over the lifetime if we impose the restriction that male workers have the same labor supply pattern as females. I eliminate the possible effects of different human capital technologies and different human capital depreciation rates on wage growth differences by holding the human capital parameters constant when I do the recalibration exercise. Therefore, the differences in life-cycle wage growth between the benchmark calibration and this counterfactual calibration, which I call "alternative calibration", are solely due to the fact that individuals in the benchmark calibration have the same labor supply pattern as males while individuals in alternative calibration have the same labor supply pattern as females.

Table 3.5 reports the labor supply parameters in the alternative calibration. One obvious feature

	Distribution of participation history: Female					
	Age 21-30		Age 21-40		Age 21-60	
	Data	Model	Data	Model	Data	Model
Q1	2.3	2.5	6.7	6.8	14.8	14.6
Q2	4.5	4.2	10.2	9.9	20.4	19.5
Q3	6.4	6.4	13.6	13.1	27.9	27.4
Q4	7.2	7.1	15.3	15.3	32.2	32.0
Q5	9.3	9.0	18.8	18.3	37.9	37.4

Table 3.6: Fit of model: distribution of working history for females

is that the mean value of the distribution of working type is larger than that in the benchmark calibration. This makes sense since higher labor supply cost should be introduced in order to induce lower labor force participation rate. Another feature is that the dependence of fixed cost on age is more complex. This can be seen from Figure 3.7 where the cost of labor supply for females depends on age in a more complicated way. The cost increases in the first 5-10 years, then it starts to decrease until age 40. After that, it increases again until the end of the working periods. This is consistent with the estimated age profile of labor force participation for females.

Figure 3.8 and Table 3.6 shows that the model has the ability to replicate the age profile of labor force participation rate and the distribution of working history for females. This indicates that individuals in the alternative calibration behave the same as females in terms of labor supply. The good match between the predictions of the model to the data targets can be viewed as a good signal for subsequent quantitative exercises.

Table 3.7 shows the quantitative results produced by the benchmark calibration and the alternative calibration. With the quantitative results in hand, we are in a good position to answer the questions posed in the introduction. Column (a) and column (b) reports how many dollars the average wage increases from age 21 for males and females respectively. Column (c) reports the genders differences in the wage growth. We can see that male workers have a 9.25 dollar wage growth from age 21 to 55 while the number for female workers is only 3.90. The difference between genders is 5.35 dollars. Column (d) and column (e) report the wage increases from age

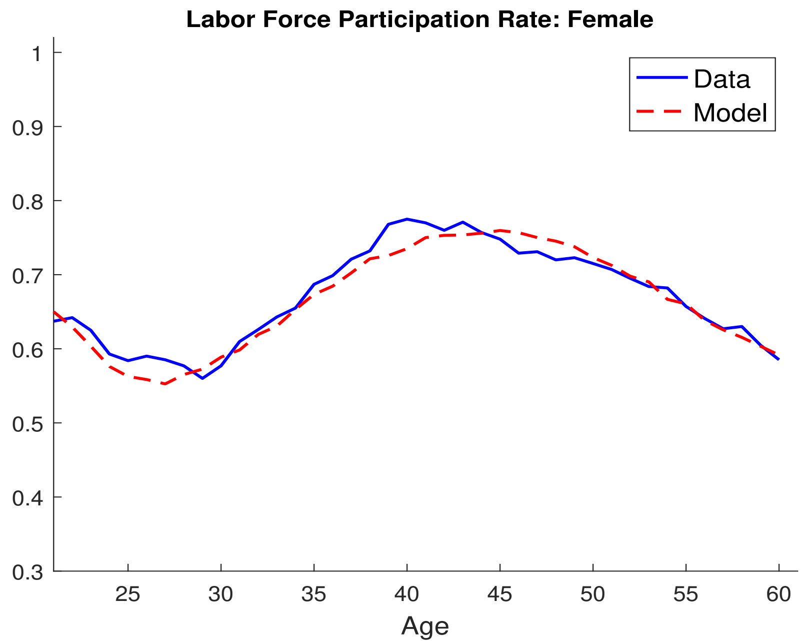

Figure 3.8: Fit of model: labor force participation rates for females

Age	Wage Growth: Data		Δ	Wage Growth: Model		Δ	Frac. exp.
	Male	Female		Benchmark	Alternative		
	(a)	(b)	(c)=(b)-(a)	(d)	(e)	(f)=(d)-(e)	(g)=(f)/(c)
25	2.87	1.47	1.43	3.15	2.13	1.02	71.3%
30	5.28	2.16	3.12	5.65	3.62	2.03	65.1%
35	7.13	2.79	4.34	7.24	4.55	2.69	62.0%
40	8.19	2.98	5.21	8.09	4.85	3.24	62.2%
45	8.67	3.61	5.06	8.62	5.19	3.43	67.7%
50	8.92	3.89	5.03	8.85	5.32	3.53	70.2%
55	9.25	3.90	**5.35**	8.93	5.37	**3.56**	**66.5%**
60	8.34	3.39	4.95	8.17	4.86	3.31	66.8%

Table 3.7: Gender differences in life-cycle wage growth: model vs data

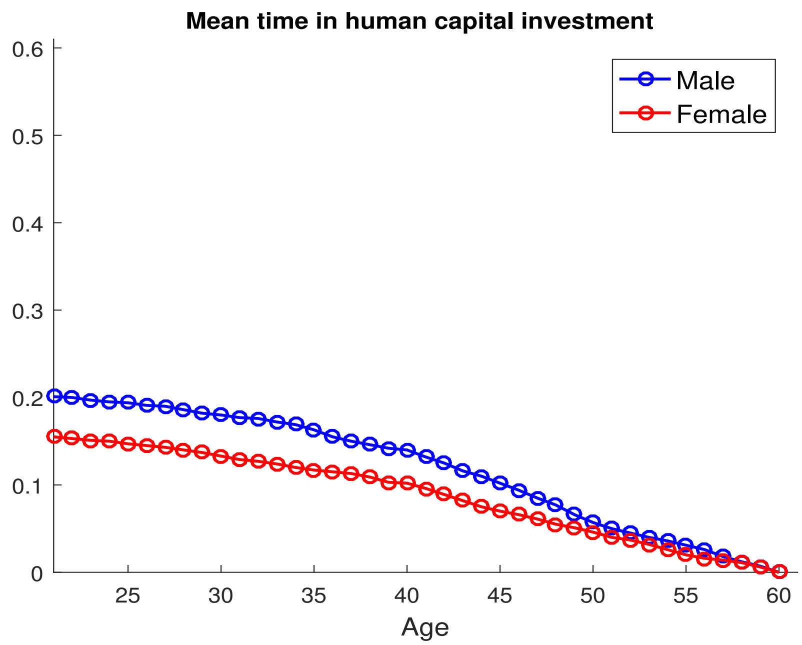

Figure 3.9: Mean time in human capital investment

21 predicted the benchmark calibration and the alternative calibration. The benchmark calibration and the alternative calibration predict 8.93 and 5.37 dollars increase in wage from age 21 to 55 separately. The difference between the two calibrations is 3.56 dollars. Therefore, this exercise implies that the different labor supply patterns between genders can explain about $\frac{3.56}{5.35} \approx 66.5\%$ of the gender differences in life-cycle wage growth. This means most of the gender differences in life-cycle wage growth can be attributed to different labor supply behaviors between genders. The result in the paper is larger than the findings in the literature, which provides an estimate in the range of one third to one half. Later, I will show that it is the inclusion of the expectation effect that plays a critical role.

In Figure 3.9, I report the average time spent in human capital investment over the lifetime. we can see that males spend more time in human capital investment than females over the life-cycle. Males have more incentive to invest in human capital than females because males on average expect higher labor force attachment than females. More investment in human capital accumulation leads to higher human capital level, which increases the ability to earn in the future. This is the mechanism through which expectation effect has an impact on life-cycle wage growth dynamics.

3.6.2 Decomposition of Experience Effect and Expectation Effect

In the previous quantitative experiment, both the experience effect and expectation effect play a role in explaining the gender differences in life-cycle wage growth. We know these two effects together can explain about 66 percent of the life-cycle wage growth differences between genders. In this subsection, I quantify the separate role played by each of these two effects in accounting for the gender differences. I conduct two decompositions. First, I simulate the lifetime histories of 50000 individuals. Note that here the policy function for labor force participation derived in alternative calibration and the policy function for human capital investment derived in the benchmark calibration are used. Since the distribution of working type is different between the two calibration strategies, I use a one-to-one correspondence when I match the optimal policy functions. To be specific, for any given value of working type, say $w = w_1$ drawn from alternative calibration, the corresponding working type for the benchmark calibration should be $w = w_2$ such that w_1 has the same percentile in the distribution drawn from alternative calibration as w_2 in the distribution drawn from the benchmark calibration. The life-cycle profile of wage is generated using the lifetime histories of the 50000 simulated individuals. The experiment separates the role of experience effect from that of the expectation effect since the individuals in the benchmark calibration and the current experiment only differs in experience while investing in human capital in the same way. Similarly, to separate the role of expectation effect, I simulate the lifetime histories of 50000 individuals using the policy function for labor force participation derived in the benchmark calibration and the policy function for human capital investment derived in alternative calibration. Again, the previous one-to-one correspondence rule is used when conducting the simulation. Then the counterfactual life-cycle profile is generated accordingly.

The quantitative results for the two decompositions are displayed in Table 3.8. Column (d) and column (e) list the gender differences in life-cycle wage growth explained by the experience effect and expectation effect respectively. We can see that experience effect alone can generate 2.03 dollars gender differences in wage growth at age 55 while expectation effect alone can generate 1.16 dollars gender differences. Column (e) and column (g) calculate the fraction of gender differences in wage growth over the life-cycle that can be attributed to experience effect and expectation effect separately. Experience effect can explain about 37.9 percent of the wage growth differences between genders at age 55 while expectation effect can explain 21.7 percent. Since the two effects together can explain 66.5 percent, I conjecture that the rest 6.9 percent might be due to the interaction effect between experience and expectation.

The results show that expectation effect is of critical importance in explaining the gender differences in life-cycle wage growth. That the paper finds different labor supply patterns can explain about 66% of the life-cycle wage growth differences, compared to the previous estimates in the range of one third to one half, is mainly due to the fact that the paper incorporates expectation effect directly in the model and quantify the expectation effect seriously. The results show the importance of expectation in affecting people's human capital decisions over the life-cycle.

However, one thing to note here is that the decomposition strategy used in this chapter is not standard. I admit that there are some possible problems with this method. First, by using this method, it's assumed that agents will make irrational decisions when they choose their behavior. For example, when I isolate the expectation effect, the policy function of labor force participation in the alternative calibration and the policy function of human capital investment in the benchmark calibration are used. Nobody will behave in this way in reality. Second, the two effects in the paper are not orthogonal to each other. If the two effects are totally orthogonal, we would expect that the summation of the two effects equals the total effect. Since the total effect is 66.5% and the two effects are about 37.9% and 21.7% respectively, a residual of about 7% is produced. Therefore, we should treat the results with a grain of salt. The results should be useful in providing some

	Gender differences in life-cycle wage growth						
Age	Data (a)	Two effects (b))	Frac. exp. (c)=(b)/(a)	Exper. effect (d)	Frac. exp. (e)=(d)/(a)	Expec. effect (f)	Frac. exp. (g)=(f)/(a)
25	1.43	1.02	71.3%	0.74	51.7%	0.18	12.6%
30	3.12	2.03	65.1%	1.31	42.0%	0.44	14.1%
35	4.34	2.69	62.0%	1.65	38.0%	0.82	18.8%
40	5.21	3.24	62.2%	1.87	35.9%	1.06	20.3%
45	5.06	3.43	67.7%	1.96	38.7%	1.11	19.7%
50	5.03	3.53	70.2%	2.01	39.9%	1.15	22.8%
55	**5.35**	**3.56**	**66.5%**	**2.03**	**37.9%**	**1.16**	**21.7%**
60	4.95	3.31	66.8%	1.86	37.5%	1.10	22.2%

Table 3.8: Gender differences in life-cycle wage growth: model vs data, decomposition results

benchmark results and motivating other better ways of addressing this problem.

3.7 Conclusions

This paper tries to link two salient features observed in the US labor market. The first one is that male workers have both higher average wage and higher life-cycle wage growth than female workers over the life cycle. The second feature is that male workers also have higher labor force participation rate than female workers. From human capital theory, labor supply could affect wage growth through two effects. The first effect is the experience effect, which says that individuals tend to accumulate less human capital and achieve slower wage growth if they spend more time out of the labor market. The second effect is the expectation effect, which says that individuals might have lower incentive to invest in human capital today if they expect lower labor market participation in the future. Based on the observed facts and human capital theory, the main objective of the paper is to provide a quantitative assessment of the importance of labor supply in explaining gender differences in life-cycle wage growth. To be specific, I ask two research questions: 1. What fraction of gender differences in life-cycle wage growth can be attributed to different labor supply patterns? 2. What's the separate role played by each of the two effects?

To answer the two questions, I build a life-cycle model in which agents make decisions about consumption, saving, labor supply, and human capital investment. Female workers are different than male workers because they face higher average labor market participation cost and this leads to lower labor market participation. Two calibration procedures: “benchmark calibration" and “alternative calibration” are introduced. In the benchmark calibration, the model is calibrated so that the predictions of the model match the wage and labor supply dynamics for male workers over the life cycle. As I have already shown in the paper, the model does a good job of matching the data targets.

In the alternative calibration, I calibrate the model to match the labor supply dynamics for female workers by choosing labor supply parameters while holding the human capital parameters calibrated in the benchmark calibration fixed. By design, these two calibration procedures can totally isolate the impact of labor supply on wage growth. The result of this quantitative exercise shows that different labor supply patterns between genders can explain about 66% of the gender differences in life-cycle wage growth. The literature typically finds this number to be between one third and one half. The result suggests that labor supply actually plays a more important role in determining gender wage gaps compared to the findings in the literature. This is the first novel finding of the paper. The decomposition results show that experience effect alone can explain about 38 percentage points while expectation effect alone can explain about 22 percentage points. The rest 6 percentage points might be due to the interaction effect between experience and expectation. To my best knowledge, this is the first paper to provide a quantitative assessment of the importance of the two effects. Particularly, the paper shows that expectation effect alone can contribute to more than 20 percentage points of the gender differences in life-cycle wage growth. In other words, expectation effect plays a non-negligible role in affecting life-cycle wage dynamics even though experience effect is the primary driving force. This is the second important finding of the paper.

4. SUMMARY AND CONCLUSIONS

In this dissertation, I investigate economic questions related to human capital, inequality, and labor supply by building models based on the Ben-Porath human capital model.

In chapter 1, I present and analyze the key properties of the benchmark Ben-Porath model. The benchmark model can produce the hump-shaped life-cycle income profile reasonably well but it fails to account for the fact that income inequality increases over the life-cycle in real life. Then, I show that there are two ways to address this issue. The first way is to introduce ability heterogeneity among individuals and the second one is to introduce shocks to the human capital accumulation process.

In chapter 2, I build a life-cycle risky human capital model to study the sources of lifetime inequality faced by high school graduates. High school graduates face two career paths when they graduate: to enter college or to enter the labor market directly. Each path is featured by uncertainty (shocks) and the chapter wants to investigate the relative importance of initial conditions and shocks in accounting for lifetime inequality. I find that differences in initial conditions as of a real-life age of 19 can only account for approximately 40 percent of the variation in realized lifetime earnings and lifetime utility. If we focus on the individuals who choose the college path, I find that the shocks in college periods play a more important role in determining the variation of inequality than do initial conditions and shocks in working periods. This fact indicates that the college investment is very risky.

In chapter 3, I investigate the importance of labor supply in explaining gender differences in life-cycle wage growth, i.e, what fraction of gender differences in life-cycle wage growth can be attributed to different labor supply patterns? To answer this question, I build a life-cycle model in which agents make decisions about consumption, saving, labor supply and human capital investment. I find that the different labor supply patterns between genders can explain about 66 percent of the gender differences in life-cycle wage growth. Labor supply affects wage growth through two effects: experience effect and expectation effect. I find that experience effect can explain about 38 percent of the wage growth differences between genders at age 55 while expectation effect can explain 22 percent. The results indicate that labor supply plays a more important role in explaining gender wage differences than the findings in the previous literature and lends support to the importance of expectation in affecting people's human capital investment decisions.

REFERENCES

[1] G. S. Becker, "Human capital theory," *Columbia, New York*, vol. 1964, 1964.

[2] M. Huggett, G. Ventura, and A. Yaron, "Sources of lifetime inequality," *American Economic Review*, vol. 101, no. 7, pp. 2923–54, 2011.

[3] S. R. Aiyagari, "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, vol. 109, no. 3, pp. 659–684, 1994.

[4] M. Huggett, "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of economic Dynamics and Control*, vol. 17, no. 5-6, pp. 953–969, 1993.

[5] K. Storesletten, C. I. Telmer, and A. Yaron, "Consumption and risk sharing over the life cycle," *Journal of Monetary Economics*, vol. 51, pp. 609–633, 2004.

[6] M. Huggett, "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, vol. 38, pp. 469–494, 1996.

[7] J. Heathcote, K. Storesletten, and G. L. Violante, "Two views of inequality over the life cycle," *Journal of the European Economic Association: Papers and Proceedings*, pp. 765–775, 2005.

[8] D. Krueger and F. Perri, "Does income inequaltiy lead to consumption inequality," *Review of Economic Studies*, vol. 73, pp. 163–193, 2006.

[9] F. Guvenen, "Learning your earning: Are labor income shocks really very persistent," *American Economic Review*, vol. 97, pp. 687–712, 2007.

[10] M. Keane and K. Wolpin, "The career decisions of young man," *Journal of Political Economy*, vol. 105, pp. 473–522, 1997.

[11] Y. Ben-Porath, "The production of human capital and the life cycle of earnings," *Journal of political economy*, vol. 75, no. 4, Part 1, pp. 352–365, 1967.

[12] J. J. Heckman, L. Lochner, and C. Taber, "Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of labor earnings with heterogeneous agents," *Review of Economic Dynamics*, vol. 1, pp. 1–58, 1998.

[13] A. Deaton and C. Paxson, "Intertemporal choice and inequality," *Journal of Political Economy*, vol. 102, pp. 437–467, 1994.

[14] M. Browning, L. P. Hansen, and J. J. Heckman, "Micro data and general equilibrium models," *Handbook of Macroeconomics, ed. J.B. Taylor and M. Woodford, (Elsevier Science B.V.)*, vol. 1, pp. 543–633, 1999.

[15] N. Guner, R. Kaygusuz, and G. Ventura, "Income taxation of u.s. households: Facts and parametric estimates," *Review of Economic Dynamics*, vol. 17, pp. 559–581, 2014.

[16] M. Huggett, G. Ventura, and A. Yaron, "Human capital and earnings distribution dynamics," *Journal of Monetary Economics*, vol. 53, pp. 265–290, 2006.

[17] F. D. Blau and L. M. Kahn, "Swimming upstream: Trends in the gender wage differential in the 1980s," *Journal of labor Economics*, vol. 15, no. 1, Part 1, pp. 1–42, 1997.

[18] A. Light and M. Ureta, "Early-career work experience and gender wage differentials," *Journal of labor economics*, vol. 13, no. 1, pp. 121–154, 1995.

[19] A. Manning and J. Swaffield, "The gender gap in early-career wage growth," *The Economic Journal*, vol. 118, no. 530, pp. 983–1024, 2008.

[20] R. K. Filer, "The usefulness of predicted values for prior work experience in analyzing labor market outcomes for women," *Journal of Human Resources*, pp. 519–537, 1993.

[21] M.-K. Kim and S. W. Polachek, "Panel estimates of male-female earnings functions," *Journal of Human Resources*, pp. 406–428, 1994.

[22] A. J. Wellington, "Changes in the male/female wage gap, 1976-85," *Journal of Human resources*, pp. 383–411, 1993.

[23] J. Mincer and S. Polachek, "Family investments in human capital: Earnings of women," *Journal of political Economy*, vol. 82, no. 2, Part 2, pp. S76–S108, 1974.

[24] R. Gronau, "Sex-related wage differentials and women's interrupted labor careers-the chicken or the egg," *Journal of labor Economics*, vol. 6, no. 3, pp. 277–301, 1988.

[25] S. H. Sandell and D. Shapiro, "Work expectations, human capital accumulation, and the wages of young women," *Journal of Human Resources*, pp. 335–353, 1980.

[26] Y. Weiss and R. Gronau, "Expected interruptions in labour force participation and sex-related differences in earnings growth," *The Review of Economic Studies*, vol. 48, no. 4, pp. 607–619, 1981.

[27] A. Erosa, L. Fuster, and D. Restuccia, "A quantitative theory of the gender gap in wages," *European Economic Review*, vol. 85, pp. 165–187, 2016.

[28] R. G. Hubbard, J. Skinner, and S. P. Zeldes, "Precautionary saving and social insurance," *The Journal of Political Economy*, vol. 103(2), pp. 360–399, 1995.