Pion nucleon coupling constant, Goldberger-Treiman discrepancy and πN σ term

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Abstract

We start by studying the Goldberger-Treiman discrepancy (GTd) $\Delta = (2.259 \pm 0.591)\%$. Then we look at the πN σ term, with the dimensionless ratio $\sigma_N/2m_N = 3.35\%$. Finally we return to predicting (via the quark model) the πN coupling constant, with GTd $\Delta \to 0$ as $m_q \to m_N/3$.

Given the recent new value of the πNN coupling constant [1]

$$g^2_{\pi NN}/4\pi = 13.80 \pm 0.12 \quad \text{or} \quad g_{\pi NN} = 13.169 \pm 0.057, \tag{1}$$

along with the observed axial current coupling [2]

$$g_A = 1.267 \pm 0.004, \tag{2}$$

combined with the measured pion decay constant [2]

$$f_\pi = (92.42 \pm 0.26)\text{MeV}, \tag{3}$$

the Goldberger-Treiman discrepancy (GTd) is

$$\Delta = 1 - \frac{m_N g_A}{f_\pi g_{\pi NN}} = (2.259 \pm 0.591)\%. \tag{4}$$

Here we have used the mean nucleon mass m_N= 938.9 MeV and have computed the overall mean square error.

To verify this GTd in Eq.(4), we employ the constituent quark loop with imaginary part [3]

$$\text{Im} f_\pi(q^2) = \frac{3g_{\pi qq}}{2}\frac{4\hat{m}}{8\pi}\left(1 - \frac{4\hat{m}^2}{q^2}\right)^{1/2}\Theta(q^2 - 4\hat{m}^2). \tag{5}$$

This follows from unitarity with the inclusion of a factor of 3 from colour. Following ref. [3] using the quark level Goldberger-Treiman relation $f_\pi g_{\pi qq} = \hat{m}$, the GTd to fourth order in q'^2 predicts via a Taylor series

$$\frac{f_\pi(q^2) - f_\pi(0)}{f_\pi(0)} = \frac{q^2}{\pi^2}\int_{4\hat{m}^2}^{\infty}\frac{dq'^2}{q'^4}\frac{3g^2_{\pi qq}}{4\pi}\left(1 - \frac{4\hat{m}^2}{q'^2}\right)^{1/2}\left(1 + \frac{q^2}{q'^2} + ...\right). \tag{6}$$

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On the pion mass shell $q^2 = m_\pi^2$, the integration in Eq.(6) can be evaluated analytically and gives a GTd

$$\bar{\Delta} = \frac{f_\pi(m_\pi^2)}{f_\pi(0)} - 1 = \frac{m_\pi^2}{8\pi^2 f_\pi^2}\left(1 + \frac{m_\pi^2}{10\hat{m}^2}\right) \approx 2.946\%. \tag{7}$$

The first term on the rhs is independent of $\hat{m}$, while in the small second term we take $\hat{m} = m_N/3$. This then leads to a net 2.946% correction in Eq.(7).

Since the physical GT relation becomes exact ($f_\pi g_{\pi NN} = m_N g_A$) when $m_\pi \to 0$ for a conserved axial current, it should not be surprising that the measured GTd in Eq.(4) of $(2.259 \pm 0.591)\%$ is within 1.16 standard deviations from the dispersion-theoretical G$\bar{\text{T}}$d $\bar{\Delta} = 2.946\%$ in Eq.(7). Appreciate that g_A is measured at $q^2 = 0$ while f_π is measured at $q^2 = m_\pi^2$ but $f_\pi(0)$ is inferred at $q^2 = 0$ via Eq.(7).

Just as the chiral-breaking $SU(2)$ GTd is 2–3%, the $SU(2) \times SU(2)$ πN σ term of 63 MeV corresponds to a dimensionless ratio of about 3%:

$$\frac{\sigma_N}{2m_N} = \frac{63 \text{ MeV}}{2 \times 938.9 \text{ MeV}} \approx 3.35\%. \tag{8}$$

Alternatively the chiral-limiting (CL) nucleon mass is related to the πN σ term as [4]

$$m_N^2 = (m_N^{CL})^2 + m_N \sigma_N, \quad \text{or with} \;\; \sigma_N = 63 \text{ MeV}, \tag{9}$$

$$\frac{m_N}{m_N^{CL}} - 1 = 3.53\%, \quad \text{with} \;\; m_N^{CL} = 906.85 \text{ MeV}. \tag{10}$$

Note the many 3% CL relations in Eqs. (4),(7),(8),(10) above. Now we justify the σ term $\sigma_N = 63$ MeV.

The explicit $SU(2) \times SU(2)$ chiral-breaking σ term is the sum of the perturbative GMOR [5] or quenched APE [6] part

$$\sigma_N^{GMOR} = (m_\Xi + m_\Sigma - 2m_N)\frac{m_\pi^2}{m_K^2 - m_\pi^2} = 26 \text{ MeV}, \tag{11}$$

$$\sigma_N^{APE} = (24.5 \pm 2) \text{ MeV}, \tag{12}$$

plus the nonperturbative linear σ model (LσM) nonquenched part [7] due to σ tadpoles for the chiral-broken m_π^2 and σ_N, with ratio predicting

$$\sigma_N^{L\sigma M} = \left(\frac{m_\pi}{m_\sigma}\right)^2 m_N \approx 40 \text{ MeV} \tag{13}$$

for $m_\sigma \approx 665$ MeV [8], a model-independent and parameter-free relation. Specifically, Eq.(13) stems from semi-strong LσM tadpole graphs generating σ_N and m_π^2. Their ratio cancels out the $< \sigma|H_{ss}|0 >$ factor. The LσM couplings $2g_{\sigma\pi\pi} = m_\sigma^2/f_\pi$ and $f_\pi g_{\sigma NN} = m_N$ then give $\sigma_N^{L\sigma M} = (m_\pi/m_\sigma)^2 m_N$ as found in Eq.(13). Since the $\sigma(600)$ has been observed [2], with a broad width, but the central model-independent value [8] is known to be 665 MeV, the chiral LσM mass ratio in Eq.(13) is expected to be quite accurate - while being free of model-dependent parameters. The authors of [9] find the σ meson between 400

MeV and 900 MeV, with the average mass 650 MeV near 665 MeV from [8]. Then the sum of (11,12) plus (13) is

$$\sigma_N = \sigma_N^{GMOR,APE} + \sigma_N^{L\sigma M} \approx (25+40)\text{ MeV} = 65\text{ MeV}. \tag{14}$$

Rather than add the perturbative plus nonperturbative parts as in Eq.(14), one can instead work in the infinite momentum frame (IMF) requiring squared masses [10] and only one term [11]

$$\sigma_N^{IMF} = \frac{m_\Xi^2 + m_\Sigma^2 - 2m_N^2}{2m_N}\left(\frac{m_\pi^2}{m_K^2 - m_\pi^2}\right) = 63\text{ MeV}. \tag{15}$$

Note that Eqs.(14) and (15) are both very near the observed value [12] (65 ± 5) MeV.

With hindsight, we can also deduce the πN σ term via PCAC (partially conserved axial current) at the Cheng-Dashen (CD) point [13] with background isospin-even πN amplitude

$$\bar{F}^+(\nu = 0, t = 2m_\pi^2) = \sigma_N/f_\pi^2 + O(m_\pi^4). \tag{16}$$

At this CD point, a recent Karlsruhe data analysis by G. Höhler [12] finds

$$\bar{F}^+(0, 2m_\pi^2) = \sigma_N/f_\pi^2 + 0.002m_\pi^{-1} = 1.02m_\pi^{-1}, \tag{17}$$

implying $\sigma_N = 63$ MeV for $f_\pi = 93$ MeV, $m_\pi = 139.57$ MeV.

We can unify the earlier parts of this paper by first inferring from Eq.(7) the chiral limit (CL) pion decay constant

$$f_\pi^{CL} = f_\pi/1.02946 \approx 89.775\text{ MeV} \tag{18}$$

using Eq.(7) and the observed [2] $f_\pi = (92.42 \pm 0.26)$ MeV. Then the quark-level GTr using the meson-quark coupling $g = 2\pi/\sqrt{3}$ [14] predicts the nonstrange quark mass in the CL as

$$\hat{m}^{CL} = f_\pi^{CL} g = 325.67\text{ MeV}, \tag{19}$$

close to the expected $\hat{m}^{CL} = m_N/3 \approx 313$ MeV. This in turn predicts the scalar σ mass in the CL as [7, 15]

$$m_\sigma^{CL} = 2\hat{m}^{CL} = 651.34\text{ MeV} \tag{20}$$

and then the on-shell LσM σ mass is

$$m_\sigma^2 - m_\pi^2 = (m_\sigma^{CL})^2 \approx (651.34\text{ MeV})^2 \quad \text{or} \quad m_\sigma \approx 665.76\text{ MeV}, \tag{21}$$

almost exactly the model-independent σ mass found in ref. [8], also predicting $\sigma_N^{L\sigma M}$ in Eq.(13).

In this letter we have linked the GT discrepancy Eqs.(4),(7) and the πN σ term Eqs.(14),(15) with the LσM values Eqs.(18)-(21). The predicted LσM value of $g_{\pi NN}$ is

$$g_{\pi NN} = N_c g g_A = 3(2\pi/\sqrt{3})1.267 = 13.79, \tag{22}$$

near the observed value in Eq.(1) with meson-quark coupling g. Substituting Eq.(22) into the GTd (Eq.(4)) in turn predicts in the quark model

$$\Delta = 1 - \frac{m_N}{3m_q} \to 0 \quad \text{as} \quad m_q \to m_N/3. \tag{23}$$

However meson-baryon couplings for pseudoscalars (P), axial-vectors (A) and $SU(6)$-symmetric states are known [16] to obey

$$(d/f)_P \approx 2.0, \quad (d/f)_A \approx 1.74, \quad (d/f)_{SU(6)} = 1.50, \tag{24}$$

where the scales of d, f characterize the symmetric, antisymmetric $SU(3)$ structure constants. Note that the ratio remains the same:

$$\frac{(d/f)_A}{(d/f)_P} = \frac{1.74}{2.0} = 0.87, \quad \frac{(d/f)_{SU(6)}}{(d/f)_A} = \frac{1.50}{1.74} \approx 0.86. \tag{25}$$

Thus to predict the quark-based πNN coupling constant we weight Eq.(22) by the scale factor of Eq.(25) in order to account for the $SU(6)$ quark content of g_A:

$$g_{\pi NN} = 3 \times 2\pi/\sqrt{3} \times 1.267 \times 0.87 = 12.00 \tag{26}$$

and this predicted coupling constant is near 13.169 from ref. [1], or 13.145 from ref. [17], or nearer still to 13.054 from ref. [18]. One could alter this 0.87 reduction of g_A in Eq.(26) by using the quark-based factor 3/5=0.6, where the $SU(6)$ factor for g_A of 5/3 becomes inverted for quarks as suggested in [19]. In any case the predicted πNN coupling lies between 12.00 and 13.79 in Eqs.(26),(22), midway near the recent data in Eq.(1).

In summary, as $m_\pi \to 0$, $\partial A_\pi \to 0$, the quark-level GT relation requires the observed $2-3\%$ GTd and 3% σ term ratio to predict $g_{\pi NN}$, with $\Delta, \bar{\Delta} \to 0$ as $m_q \to m_N/3$. We have computed the πN σ term in many different ways to find approximately $\sigma_N = 63$ MeV.

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