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Scattering of String Monopoles [†]

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In the low-velocity limit, multi-soliton solutions trace out geodesics in the static solution manifold with distance defined by a metric on moduli space. For the recently constructed multimonopole solutions of heterotic string theory, we obtain a flat metric to leading order in the impact parameter. This result agrees with the trivial scattering predicted by a test monopole calculation.

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1. Introduction

In recent work[1] an exact multimonopole solution of heterotic string theory was presented. This solution was obtained via a modification of the 't Hooft ansatz[2–6] for the Yang-Mills instanton. An analogous solution in Yang-Mills field theory saturates a Bogomoln'yi[7] bound and possesses the topology and far field limit of a multimonopole configuration, but has divergent action near each source. In the string solution, however, the divergences from the Yang-Mills sector are precisely cancelled by those from the gravity sector, so that the action is finite and easily computed[1]. In this letter, we study the dynamics of the string monopoles and find that, unlike BPS[8,7] monopoles, the string monopoles scatter trivially to leading order in the impact parameter.

We study the scattering of two string monopoles by two methods. The first approach computes the Manton metric on moduli space, which defines distance on the static solution manifold. We first invert the $O(\beta)$ time-dependent constraint equations and replace the solution into the action. The resultant kinetic action defines the metric on moduli space. A flat metric is obtained to leading order in the impact parameter, a result which implies trivial scattering between string monopoles.

An independent calculation of the dynamic force on a test string monopole moving in the background of a source string monopole yields a zero dynamic force to lowest order in the velocity, again implying trivial scattering. This computation thus confirms the flat metric result.

2. Manton Metric for String Monopoles

The bosonic fields for the exact self-dual multimonopole solution of heterotic string theory with zero background fermi fields are given by[1]

$$
\begin{aligned}
g_{\mu\nu} &= e^{2\phi}\delta_{\mu\nu}, \quad g_{ab} = \eta_{ab}, \\
H_{\mu\nu\lambda} &= \pm\epsilon_{\mu\nu\lambda\sigma}\partial^\sigma\phi, \\
e^{2\phi} &= e^{2\phi_0}f, \\
A_\mu &= i\overline{\Sigma}_{\mu\nu}\partial_\nu \ln f,
\end{aligned}
\tag{2.1}
$$

where $\mu, \nu, \lambda, \sigma = 1, 2, 3, 4$, $a, b = 0, 5, 6, 7, 8, 9$, $\overline{\Sigma}_{\mu\nu} = \overline{\eta}^{i\mu\nu}(\sigma^i/2)$ for $i = 1, 2, 3$ (σ^i, $i = 1, 2, 3$ are the 2×2 Pauli matrices) where

$$
\begin{aligned}
\overline{\eta}^{i\mu\nu} = -\overline{\eta}^{i\nu\mu} &= \epsilon^{i\mu\nu}, \qquad & \mu, \nu = 1, 2, 3, \\
&= -\delta^{i\mu}, \qquad & \nu = 4
\end{aligned}
\tag{2.2}
$$

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and where

$$f = 1 + \sum_{n=1}^{N} \frac{m_n}{|\vec{x} - \vec{a}_n|}, \qquad (2.3)$$

where m_n is the charge and $\vec{a}_n$ the location in the three-space (123) of the nth monopole. The anti-self-dual solution is similar, with the δ-term in (2.2) changing sign. This solution was shown to have multimonopole structure[1] in the three-space, each source having topological charge $Q = 1$ and magnetic charge $m = 1/g$, where g is the YM coupling constant.

If we make the identification $\Phi \equiv A_4$, then the gauge and Higgs fields may be simply written in terms of the dilaton as

$$\Phi^a = -\frac{2}{g} \delta^{ia} \partial_i \phi,$$

$$A_k^a = -\frac{2}{g} \epsilon^{akj} \partial_j \phi \qquad (2.4)$$

for the self-dual solution. For the anti-self-dual solution, the Higgs field simply changes sign. A toroidal compactification along the lines of [9] can be adopted, so that we consider the dynamics of our solution in the four-dimensional spacetime (0123). As usual, the existence of a static multi-soliton solution depends on the "zero force" condition.

Owing to the exactness condition $A_\mu = \Omega_{\pm\mu}$[10,11,12] (where $\Omega_{\pm\mu}$ is the generalized connection defined in [1]), the higher order in α' terms drop out from the action, and the static multimonopole mass can be computed from the tree-level action[13,1]

$$S = -\frac{1}{2\kappa^2} \left[\int dt \left(\int d^3x \sqrt{g} e^{-2\phi} \left(R + 4(\nabla\phi)^2 - \frac{H^2}{12} \right) + 2 \int_{\partial M} \left(e^{-2\phi} K - K_0 \right) \right) \right], \quad (2.5)$$

where we have added a Gibbons-Hawking surface term (GHST) to cancel the double derivative terms in the action[14,15,16,17,13]. ∂M is the surface boundary and K and K_0 are the traces of the fundamental form of the boundary surface embedded in the metric g and the Minkowskian metric η respectively. The addition of a surface term does not, of course, affect the equations of motion. The multimonopole mass is given by[1]

$$M_T = \frac{8\pi}{\kappa^2} \sum_{n=1}^{N} m_n, \qquad (2.6)$$

where $m_n = 1/g$ for $n = 1, 2...N$.

We wish to study dynamics of the string monopoles. Manton's prescription[18] for the study of soliton scattering may be summarized as follows. We first invert the constraint

equations of the system. The resultant time dependent field configuration does not in general satisfy the full time dependent field equations, but provides an initial data point for the fields and their time derivatives. Another way of saying this is that the initial motion is tangent to the set of exact static solutions. The kinetic action obtained by replacing the solution to the constraints into the action defines a metric on the parameter space of static solutions. This metric defines geodesic motion on the moduli space[18].

A calculation of the metric on moduli space for the scattering of BPS monopoles and a description of its geodesics was worked out by Atiyah and Hitchin[19]. Several interesting properties of monopole scattering were found, such as the conversion of monopoles into dyons and the right angle scattering of two monopoles on a direct collision course[19,20]. The configuration space is found to be a four-dimensional manifold M_2 with a self-dual Einstein metric.

In this section, we adapt Manton's prescription to study the dynamics of heterotic string monopoles. A similar procedure was followed in [13] for the Manton scattering of heterotic instantons. Indeed, many of the formal computations carry over from the instanton computation. For the monopoles, however, the divergences plagueing the instanton calculation are absent, thus rendering our task far simpler. In both cases, we follow essentially the same steps that Manton outlined for monopole scattering, but take into account the peculiar nature of the string effective action. Since we work in the low-velocity limit, our kinematic analysis is nonrelativistic.

We first solve the constraint equations for the soliton solutions. These equations are simply the $(0j)$ components of the equations of motion (see [21,13])

$$R_{0j} - \frac{1}{4}H_{0j}^2 + 2\nabla_0\nabla_j\phi = 0,$$
$$-\frac{1}{2}\nabla_k H^k{}_{0j} + H_{0j}{}^k\partial_k\phi = 0.$$

(2.7)

Note that we use the tree-level equations of motion, as the higher order corrections in α' automatically vanish. We wish to find an $O(\beta)$ solution to the above equations which represents a quasi-static version of (2.1) (i.e. a solution of the form (2.1) but with time dependent $\vec{a}_i$). In other words, we would like to give each source an arbitrary transverse velocity $\vec{\beta}_n$ in the (123) subspace of the four-dimensional transverse space and see what corrections to the fields are required by the constraints. The vector $\vec{a}_n$ representing the position of source n in the three-space (123) is given by

$$\vec{a}_n(t) = \vec{A}_n + \vec{\beta}_n t,$$

(2.8)

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where $\vec{A}_n$ is the initial position of the nth source. Note that at $t = 0$ we have an exact static multi-soliton solution. Our solution to the constraints will adjust our quasi-static approximation so that the initial motion in the parameter space is tangent to the initial exact solution at $t = 0$.

The $O(\beta)$ solution to the constraints is given by

$$e^{2\phi(\vec{x},t)} = 1 + \sum_{n=1}^{N} \frac{m_n}{|\vec{x} - \vec{a}_n(t)|},$$

$$g_{00} = -1, \qquad g^{00} = -1, \qquad g_{ij} = e^{2\phi}\delta_{ij}, \qquad g^{ij} = e^{-2\phi}\delta_{ij},$$

$$g_{0i} = -\sum_{n=1}^{N} \frac{m_n\vec{\beta}_n \cdot \hat{x}_i}{|\vec{x} - \vec{a}_n(t)|}, \qquad g^{0i} = e^{-2\phi}g_{0i}, \tag{2.9}$$

$$H_{ijk} = \epsilon_{ijkm}\partial_m e^{2\phi},$$

$$H_{0ij} = \epsilon_{ijkm}\partial_m g_{0k} = \epsilon_{ijkm}\partial_k \sum_{n=1}^{N} \frac{m_n\vec{\beta}_n \cdot \hat{x}_m}{|\vec{x} - \vec{a}_n(t)|},$$

where $i, j, k, m = 1, 2, 3, 4$, the $\vec{a}_n(t)$ are given by (2.8) and we use a flat space ϵ-tensor. Note that g_{00}, g_{ij} and H_{ijk} are unaffected to order β. Also note that we can interpret the solitons as either line sources in the four-dimensional space (1234) or point sources in the three-dimensional subspace (123).

The kinetic Lagrangian is obtained by replacing the expressions for the fields in (2.9) into (2.5). Since (2.9) is a solution to order β, the leading order terms in the action (after the quasi-static part) are of order β^2. In the volume term of the action, $O(\beta)$ terms in the solution give $O(\beta^2)$ terms in the kinetic action. As explained in [13], the contribution of the GHST to the kinetic action can be written in the form $m_s\beta^2/2$ for each source, and the contributions of the sources can be simply added. The GHST does not therefore play an important role in the dynamics of the string monopoles, but merely serves to give the correct total mass. Collecting all $O(\beta^2)$ terms in S_V we get the following kinetic Lagrangian density for the volume term:

$$\mathcal{L}_{kin} = -\frac{1}{2\kappa^2}\left(4\dot{\phi}\vec{M} \cdot \vec{\nabla}\phi - e^{-2\phi}\partial_i M_j \partial_i M_j - e^{-2\phi}M_k\partial_j\phi\left(\partial_j M_k - \partial_k M_j\right) \right.$$
$$\left. + 4M^2 e^{-2\phi}(\vec{\nabla}\phi)^2 + 2\partial_t^2 e^{2\phi} - 4\partial_t(\vec{M} \cdot \vec{\nabla}\phi) - 4\vec{\nabla} \cdot (\dot{\phi}\vec{M}) \right), \tag{2.10}$$

where $\vec{M} \equiv -\sum_{n=1}^{N} \frac{m_n\vec{\beta}_n}{|\vec{x}-\vec{a}_n(t)|}$. Henceforth let $\vec{X}_n \equiv \vec{x} - \vec{a}_n(t)$. The last three terms in (2.10) are time-surface or space-surface terms which vanish when integrated. Note that

the kinetic Lagrangian has the same form as in [13]. The contributions of the GHST are again simply flat kinetic terms.

In contrast to the instanton case, the kinetic Lagrangian $L_{kin} = \int d^3x \mathcal{L}_{kin}$ for monopole scattering converges everywhere. This can be seen simply by studying the limiting behaviour of L_{kin} near each source. For a single source at $r = 0$ with magnetic charge m and velocity β, we collect the logarithmically divergent pieces and find that they cancel:

$$\frac{m\beta^2}{2} \int r^2 dr d\theta \sin\theta d\phi \left(-\frac{1}{r^3} + \frac{3\cos^2\theta}{r^3} \right) = 0. \tag{2.11}$$

So unlike the instanton case, in which we were compelled to extract information from the convergent interaction terms, in this case we can use the self-terms directly.

We now specialize to the case of two heterotic monopoles of magnetic charge $m_1 = m_2 = m = 1/g$ and velocities $\vec{\beta}_1$ and $\vec{\beta}_2$. Let the monopoles be located at $\vec{a}_1$ and $\vec{a}_2$. Our moduli space consists of the configuration space of the relative separation vector $\vec{a} \equiv \vec{a}_2 - \vec{a}_1$. The most general kinetic Lagrangian can be written as

$$\begin{aligned} L_{kin} =& h(a)(\vec{\beta}_1 \cdot \vec{\beta}_1 + \vec{\beta}_2 \cdot \vec{\beta}_2) + p(a)\left((\vec{\beta}_1 \cdot \hat{a})^2 + (\vec{\beta}_2 \cdot \hat{a})^2 \right) \\ &+ 2f(a)\vec{\beta}_1 \cdot \vec{\beta}_2 + 2g(a)(\vec{\beta}_1 \cdot \hat{a})(\vec{\beta}_2 \cdot \hat{a}). \end{aligned} \tag{2.12}$$

Now suppose $\vec{\beta}_1 = \vec{\beta}_2 = \vec{\beta}$, so that (2.12) reduces to

$$L_{kin} = (2h + 2f)\beta^2 + (2p + 2g)(\vec{\beta} \cdot \hat{a})^2. \tag{2.13}$$

This configuration, however, represents the boosted solution of the two-static soliton solution. The kinetic energy should therefore be simply

$$L_{kin} = \frac{M_T}{2}\beta^2, \tag{2.14}$$

where $M_T = M_1 + M_2 = 2M = 16\pi m/\kappa^2$ is the total mass of the two soliton solution. It then follows that the anisotropic part of (2.13) vanishes and we have

$$\begin{aligned} g + p &= 0, \\ 2(h + f) &= \frac{M_T}{2}. \end{aligned} \tag{2.15}$$

It is therefore sufficient to compute h and p. This can be done by setting $\vec{\beta}_1 = \vec{\beta}$ and $\vec{\beta}_2 = 0$. The kinetic Lagrangian then reduces to

$$L_{kin} = h(a)\beta^2 + p(a)(\vec{\beta} \cdot \hat{a})^2. \tag{2.16}$$

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Suppose for simplicity also that $\vec{a}_1 = 0$ and $\vec{a}_2 = \vec{a}$ at $t = 0$. The Lagrangian density of the volume term in this case is given by

$$\mathcal{L}_{kin} = \frac{-1}{2\kappa^2} \left(\frac{3m^3 e^{-4\phi}}{2r^4} (\vec{\beta} \cdot \vec{x}) \left[\frac{\vec{\beta} \cdot \vec{x}}{r^3} + \frac{\vec{\beta} \cdot (\vec{x} - \vec{a})}{|\vec{x} - \vec{a}|^3} \right] - \frac{e^{-2\phi} m^2 \beta^2}{r^4} \right.$$
$$\left. - \frac{e^{-4\phi} m^3 \beta^2}{2r^4} \left(\frac{1}{r} + \frac{\vec{x} \cdot (\vec{x} - \vec{a})}{|\vec{x} - \vec{a}|^3} \right) + \frac{e^{-6\phi} m^4 \beta^2}{r^2} \left(\frac{1}{r^4} + \frac{1}{|\vec{x} - \vec{a}|^4} + \frac{2\vec{x} \cdot (\vec{x} - \vec{a})}{r^3 |\vec{x} - \vec{a}|^3} \right) \right).$$
$$(2.17)$$

The GHST contribution to the kinetic Lagrangian can be simply added after integration and will not affect the analysis below.

The integration of the kinetic Lagrangian density in (2.17) over three-space yields the kinetic Lagrangian from which the metric on moduli space can be read off. For large a, the nontrivial leading order behaviour of the components of the metric, and hence for the functions $h(a)$ and $p(a)$, is generically of order $1/a$. In fact, for Manton scattering of YM monopoles, the leading order scattering angle is $2/b$[22], where b is the impact parameter. In this paper, we restrict our computation to the leading order metric in moduli space. A tedious but straightforward collection of $1/a$ terms in the Lagrangian yields

$$\frac{-1}{2\kappa^2} \frac{1}{a} \int d^3x \left[-\frac{3m^4 e^{-6\phi_1}}{r^7} (\vec{\beta} \cdot \vec{x})^2 + \frac{m^3 e^{-4\phi_1}}{r^4} \beta^2 + \frac{m^4 e^{-6\phi_1}}{r^5} \beta^2 - \frac{3m^5 e^{-8\phi_1}}{r^6} \beta^2 \right], \quad (2.18)$$

where $e^{2\phi_1} \equiv 1 + m/r$. The first and third terms clearly cancel after integration over three-space. The second and fourth terms are spherically symmetric. A simple integration yields

$$\int_0^\infty r^2 dr \left(\frac{e^{-4\phi_1}}{r^4} - \frac{3m^2 e^{-8\phi_1}}{r^6} \right) = \int_0^\infty \frac{dr}{(r+m)^2} - 3m^2 \int_0^\infty \frac{dr}{(r+m)^4} = 0. \quad (2.19)$$

The $1/a$ terms therefore cancel, and the leading order metric on moduli space is flat. This implies that the leading order scattering is trivial. In other words, there is no deviation from the initial trajectories to leading order in the impact parameter.

The above result is rather surprising and suggests that, in addition to the static force, the leading order dynamic force also vanishes. For pure YM monopoles, this is certainly not the case. For the string monopoles, however, the dynamic YM force is precisely cancelled by the dynamic gravity sector force. In the next section, we adopt a different approach to the computation of the dynamic force in order to confirm the flat metric result.

3. Test Monopole Calculation

We now employ the test-soliton approach of [23,24] to compute the dynamic force exerted on a test string monopole moving in the background of a source string monopole. Again only the massless fields in the gravitational sector come in to play at tree-level. Since the monopoles have fivebrane structure, we adopt the fivebrane action of Duff and Lu[25,26]

$$
S_{\sigma_5} = - T_6 \int d^6 \xi \left(\frac{1}{2} \sqrt{-\gamma} \gamma^{mn} \partial_m X^M \partial_n X^N g_{MN} e^{-\phi/6} - 2\sqrt{-\gamma} \right.
$$
$$
\left. + \frac{1}{6!} \epsilon^{mnpqrs} \partial_m X^M \partial_n X^N \partial_p X^P \partial_q X^Q \partial_r X^R \partial_s X^S A_{MNPQRS} \right),
$$
(3.1)

where $m, n, p, q, r, s = 0, 5, 6, 7, 8, 9$ are fivebrane indices and $M, N, P, Q, R, S = 0, 1, ...9$ are spacetime indices (transverse indices are denoted by $i, j = 1, 2, 3, 4$). γ_{mn} is a $5 + 1$-dimensional worldsheet metric, g_{MN} is the canonical spacetime metric and A_{MNPQRS} is the antisymmetric six-form potential whose curl $K = dA$ is dual to the antisymmetric field strength $H_{\alpha\beta\gamma}$.

The multimonopole solution written in this frame is given by

$$
ds^2 = e^{2A} \eta_{mn} dx^m dx^n + e^{2B} \delta_{ij} dx^i dx^j,
$$
$$
A_{056789} = -e^C,
$$
(3.2)

where all other components of A_{MNPQRS} are set to zero and the dilaton ϕ and the scalar functions A, B and C are given by

$$
A = -\frac{(\phi - \phi_0)}{4},
$$
$$
B = \frac{3(\phi - \phi_0)}{4},
$$
$$
C = -2\phi + \frac{3\phi_0}{2},
$$
(3.3)

where ϕ_0 is the value of the dilaton field at infinity and

$$
e^{2\phi} = e^{2\phi_0} \left(1 + \sum_{n=1}^{N} \frac{m_n}{|\vec{x} - \vec{a}_n|} \right),
$$
(3.4)

where $\vec{x}$ and $\vec{a}_n$ are again vectors in the three-dimensional subspace (123) of the transverse space (1234).

The Lagrangian for a test monopole moving in a background of identical static source monopoles is given by substituting (3.2) in (3.1) and then eliminating the worldbrane metric. The result is

$$\mathcal{L}_6 = -T_6 \left[\sqrt{-\det(e^{-2\phi/3+\phi_0/2}\eta_{mn} + e^{4\phi/3-3\phi_0/2}\partial_m X^M \partial_n X_M)} - e^{-2\phi+3\phi_0/2} \right]. \quad (3.5)$$

Since the test-monopole moves only in the (123) subspace of the transverse space (there is no motion along or field dependence on the direction x_4), (3.5) reduces in the low-velocity limit to

$$\mathcal{L}_6 \simeq -T_6 \left[e^{-2\phi+3\phi_0/2} \left(1 - \tfrac{1}{2} e^{2(\phi-\phi_0)}(\dot{X}^i)^2 \right) - e^{-2\phi+3\phi_0/2} \right]$$
$$= \frac{T_6}{2} e^{-\phi_0/2}(\dot{X}^i)^2 , \quad (3.6)$$

where $i = 1, 2, 3$. Again both the static force and the nontrivial $O(v^2)$ velocity-dependent force vanish. Hence this result also predicts trivial scattering, in direct agreement with the flat Manton metric calculation.

4. Conclusion

In [1], an exact multimonopole solution of heterotic string theory was presented. An analogous solution in YM field theory was found to have divergent action near each source. In the string theory solution, however, the divergences from the Yang-Mills sector are exactly cancelled by divergences in the gravity sector. The cancellation between the gauge and gravitational sectors also influences the dynamics of the string monopoles. In this paper, we found from both a Manton metric on moduli space calculation and a test string monopole calculation that the leading order dynamic force between two string monopoles vanishes. This result implies trivial scattering between string monopoles to leading order in the impact parameter in the low-velocity limit.

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