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# THE PRICE OF DERIVING THE STANDARD MODEL FROM STRING

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## ABSTRACT

It is well known that precision LEP data on $\sin^2\theta_W$ and $\alpha_3$ are consistent with supersymmetric grand unification at an energy scale almost two orders of magnitude below that calculated in string models. We examine whether one can raise the unification scale, whilst retaining the successful predictions for $\sin^2\theta_W$ and $\alpha_3$, by adding to the Standard Model non-exotic multiplets of particles. We find that one must introduce extra (3,2) representations of $SU(3) \times SU(2)$. If there is just one such vector-like (3,2) representation, it must weigh less than about $10^{13}$ GeV. Moreover, there must be at least one other type of light matter representation. We exhibit a "twisted" version of the flipped $SU(5)$ model derived from string that can accommodate such light representations.

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The chasm between the Standard Model and string theory still yawns. On the one hand, the Standard Model emerges unscathed from successive skirmishes with experiment, its parameters progressively better determined. On the other hand, the structure of string theory is being progressively better understood, although the rules for selecting the correct string vacuum are still obscure. This has not prevented optimists from exploring phenomenologically-promising string models in the hopes of unveiling general features that might provide a generic signature of string theory and (who knows?) maybe even happening on the string vacuum in which we live.

A potential bridge across the chasm is provided by the renormalization group, which enables one to extrapolate reliably up to higher mass scales Standard Model parameters such as gauge couplings [1] or particle masses [2]. These extrapolations can be made very precisely once the light particle content is specified. Using the precision values of the $SU(3) \times SU(2) \times U(1)_Y$ gauge couplings determined at LEP and elsewhere, parametrize d by $\alpha_{em}$, $\sin^2\theta_W$ and $\alpha_3$, one can test the feasibility of grand unification at some large mass scale $m_{GUT}$ [3]-[5]. It is well known that such a grand unification is not attainable unless one invokes low-energy supersymmetry. Using just the particles of the minimal supersymmetric extension of the Standard Model, the observed values of $\alpha_{em}$, $\sin^2\theta_W$ and $\alpha_3$ are consistent with [3]-[5] $m_{GUT} \sim$ (1 to 2) $\times 10^{16}$ GeV.

While impressively high, this range of $m_{GUT}$ is almost two orders of magnitude below the unification scale $m_{SU}$ calculated in typical string models, which is about $10^{18}$ GeV [6]. One way of restating this discrepancy is the following: using the observed value of $\alpha_{em}$ and the calculated value of $m_{SU}$, and assuming there is no other low-energy ($\ll m_{Pl}$) physics beyond that in the minimal supersymmetric extension of the Standard Model, one can calculate from first principles the values of **both** $\sin^2\theta_W$ **and** $\alpha_3(m_{Z^0})$. Using $\alpha_{em}(m_{Z^0}) = 1/128$ and $m_{SU} = 10^{18}$ GeV, we find $\sin^2\theta_W \simeq 0.217$ and $\alpha_3(m_{Z^0}) \simeq 0.21$, to be compared with the experimental values $\sin^2\theta_W \simeq 0.233$ and $\alpha_3(m_{Z^0}) \simeq 0.11$. So near and yet so far!

It does not appear to be possible to bridge this gap by invoking uncertainties in the low-energy inputs, threshold effects, or uncertainties in the large extrapolation due to higher-loop effects, Yukawa couplings, or whatever [7]. The natural way to reconcile $m_{SU}$ with the calculated value of $m_{GUT}$ is to include additional light supermultiplets beyond those in the Standard Model. *A priori*, one might imagine that it must be quite easy to find combinations of light supermultiplets that increase $m_{GUT}$ to $m_{SU}$ without changing $\alpha_{em}$, $\sin^2\theta_W$ or $\alpha_3$. In fact, we find that it is not so easy.

We consider the addition of light supermultiplets belonging to the same non-exotic representations of $SU(3) \times SU(2) \times U(1)_Y$ as in the Standard Model, namely $Q \equiv (\underline{3}, \underline{2}, \frac{1}{6})$, $U^c \equiv (\underline{\bar{3}}, \underline{1}, -\frac{2}{3})$, $D^c \equiv (\underline{\bar{3}}, \underline{1}, \frac{1}{3})$, $L \equiv (\underline{1}, \underline{2}, -\frac{1}{2})$ and $E^c \equiv (\underline{1}, \underline{1}, -1)$. We find that the price of increasing $m_{GUT}$ to $m_{SU}$ is the inclusion of at least one vector-like $Q$ supermultiplet. If there is just one such state, it must weigh less than about $10^{13}$ GeV. Moreover, there must be at least one other type of light representation: $D^c$, $U^c$ or $E^c$.

These results are simply derived using the well-known [8] renormalization group equations for the Standard Model gauge couplings in the presence of possible additional matter supermultiplets, which yield

$$\frac{\pi}{6}\left(\frac{3}{5\alpha_{em}(m_{Z^0})}-\frac{8}{5\alpha_3(m_{Z^0})}\right)+\frac{1}{20}\Big[L_Q+L_{D^c}-L_L-L_{E^c}\Big]-0.21=\ln\left(\frac{m_{GUT}}{m_Z}\right) \quad (1)$$

and

$$\sin^2\theta_W(m_{Z^0})=\frac{1}{5}+\frac{7\alpha_{em}(m_{Z^0})}{15\alpha_3(m_{Z^0})}+\frac{\alpha_{em}(m_{Z^0})}{20\pi}\Big[7L_Q-3L_{D^c}-5L_{U^c}+3L_L-2L_{E^c}\Big]+0.0029 \quad (2)$$

where $\alpha_3(m_{Z^0})$ and $\sin^2\theta_W(m_{Z^0})$ are defined in the $\overline{\rm MS}$ scheme, $L_R \equiv \sum_i \ln(m_{GUT}/m_{R_i})$, where the sum runs over all supermultiplets $i$ in the representation $R$, and the numbers represent the small corrections to the one-loop results that are introduced by two-loop diagrams. In writing (1) and (2) we have neglected small corrections due to the sparticle thresholds that are treated in detail in Ref. [7], but are unimportant for this discussion.

We now ask what combinations of additional matter supermultiplets can reconcile $m_{GUT}$ (1) with the value of the unification scale calculated in a large class of string theories: $m_{SU} \simeq 10^{18}$ GeV, while retaining the prediction (2) of $\sin^2\theta_W(m_{Z^0})$, which appears to agree perfectly with LEP and other precision data. Conditions (1) and (2) then become

$$L_Q+L_{D^c}-L_L-L_{E^c}\simeq 82 \quad (3)$$

and

$$7L_Q-3L_{D^c}+3L_L-2L_{E^c}-5L_{U^c}\simeq -14 \quad (4)$$

Since these are just two linear equations for five unknowns, they have a wide range of solutions. Nevertheless, when we recall that all the $L_R \geq 0$, we find that some interesting information can be gleaned from Eqs. (3) and (4) [1].

The combination $3\times(3)+(4)$ tells us that

$$2L_Q-L_{U^c}-L_{E^c}\simeq 46 \quad (5)$$

and hence that *at least one* $Q=(\underline{3},\underline{2},\frac{1}{6})$ *representation of* $SU(3)\times SU(2)\times U(1)_Y$ *must weigh much less than* $m_{GUT}=m_{SU}$. Any such additional supermultiplets should be vector-like ($V$) if they weigh much more than $m_{Z^0}$. If we postulate just one such light vector-like supermultiplet $Q_V$, Eq. (5) tells us that $\ln(m_{GUT}/m_{Q_V})=\frac{1}{2}L_Q \gtrsim 11$, and hence $m_{Q_V}\lesssim 10^{13}$ GeV.

This is not the end of the story, since Eq. (4) then tells us that *at least one other supermultiplet must also weigh much less than* $m_{GUT}$. If we assume $L_{U^c}=L_{E^c}=0$ so as not to strengthen the upper bound (5) on $m_{Q_V}$, we need to postulate one or more light $D^c$ supermultiplets with $L_{D^c}=\frac{7}{3}(L_Q+2)$. If we postulate just one such vector-like supermultiplet $D^c_V$, we infer that $m_{D^c_V}\lesssim 2\times 10^5$ GeV. Alternatively, we could enforce the cancellation imposed by Eq. (4) by postulating that $L_{U^c}\neq 0, L_{D^c}=L_{E^c}=0$, in which case we find $m_{Q_V}\lesssim m_W$ and $L_{U^c}\gtrsim\frac{7}{5}(L_Q+2)$, so that more than one vector-like

[1] The theoretical error on the right-hand side of Eq. (3) is probably $\mathcal{O}(20)$, which will not affect our qualitative conclusions. The experimental error on the right-hand side of Eq. (4) is $\mathcal{O}(10)$, which is also negligible for our purposes.

$U^c$ multiplet is needed. In the case $L_{E^c} \neq 0$, $L_{D^c} = L_{U^c} = 0$ we find again that $m_{Q_V} \leq m_W$ and $L_{E^c} \gtrsim \frac{7}{2}(L_Q + 2)$, so that more than one vector-like $E^c$ multiplet is needed.

We conclude that embedding the Standard Model gauge group directly in a string model requires the addition of at least one extra $Q = (\underline{3}, \underline{2}, \frac{1}{6})$ representation weighing much less than $m_{GUT}$, and at least one other type of light representation: $D^c, U^c$ or $E^c$.

These conclusions are trivial deductions from well-known renormalization group equations. Nevertheless, we find them interesting in view of the incessant search for a "smoking gun" betraying string. Such extra light matter representations might be avoidable in some string unification scenarios invoking a larger field-theoretical gauge group at some energy below the Planck scale, but this would just be another realization of the deeper message of our paper. If string theory is correct, there must be *some* new physics below the Planck scale. It is not correct to assert that string theory only has implications for physics at Planckian energies. LEP and other precise low-energy data already tell us that string physics cannot be postponed until the Greek Kalends.

The alert reader will recognize that the revamped version of the flipped $SU(5)$ model derived from string in Ref. [9] does not contain any candidates for extra light $Q$ fields. While not wishing to prejudice the judgement of experiment on this model, which should await a more complete exploration of its possible pattern of v.e.v.'s and intermediate-mass thresholds, we conclude this paper by presenting a small variation of the revamped flipped $SU(5)$ model that offers the possibility of light $Q$ fields.

For the sake of simplicity, we choose to construct a variation of the model in which the three conventional generations of quarks and leptons are unchanged, and only the GUT Higgs and singlet sectors are modified. The revamped version [9] of the flipped $SU(5)$ string model is generated by a set of eight basis vectors of boundary conditions for the world-sheet fermions. Only one of them, called $\alpha$, treats the fermions in complex pairs transformed by phases $\pm i$, and is used to obtain unitary gauge group factors. The structure of the revamped model can be visualized via an intermediate stage in which the other seven basis elements together with $2\alpha$ define a model with observable $SO(10)$, hidden $SO(16)$ and extra $U(1)^6$ factors. At this stage, the model has six chiral $SO(10)$ families in $\underline{16}$ representations produced in pairs by the $b_{1,2,3}$ sectors, and two extra $\underline{16} + \overline{\underline{16}}$ pairs from the $b_4$ and $b_5$ sectors. In addition, there are eight $\underline{10}$'s: six from the Neveu-Schwarz sector and two from $b_4 + b_5$, as well as a number of singlets. The revamped model was then obtained by applying the vector $\alpha$, which breaks $SO(10)$ to $SU(5) \times U(1)$, $SO(16)$ to $SO(10) \times SO(6)$, and $U(1)^6$ to $U(1)^4$, à la Wilson line. The associated $\mathbf{Z}_2$ projection also removes half of the chiral families, while keeping $\underline{10} + (\underline{5} + \underline{1})$ and $\overline{\underline{10}} + (\bar{\underline{5}} + \underline{1})$ from the Higgs sectors $b_4$ and $b_5$. Here we discuss a modification of this last step, which consists of keeping $\underline{10} + \overline{\underline{10}}$ states from each of $b_{4,5}$ and thus producing an extra vector-like $Q = (\underline{3}, \underline{2}, \frac{1}{6})$ state, in addition to that "eaten" by the Higgs mechanism which breaks the $SU(5) \times U(1)$ GUT group.

This is achieved by modifying the vector $\alpha$:

$$\alpha: \quad \text{periodic for} \quad \bar{y}^{4,6}, y^6, w^6 \rightarrow \alpha': \quad \text{periodic for} \quad \bar{y}^1, \bar{w}^5, y^5, w^5 \tag{6}$$

with the other fermions retaining the same boundary conditions they had under $\alpha$. At the same time, we change some of the generalized GSO projection coefficients:

$$c\binom{\alpha}{b_{1,2,3,4}} = -1 = -c\binom{\alpha}{b_5} \to c\binom{\alpha'}{b_1} = 1 = -c\binom{\alpha'}{b_{2,3,4,5}} \tag{7}$$

As a result of these minor changes, there are two extra $U(1)$ factors in the gauge group, and the observable-sector matter fields become:

$$F_{1_{\frac{1}{2},0,0}}^{-\frac{1}{2},0,0} + (\bar{f}_1 + 1_1^c)^{\frac{1}{2},0,0}_{\frac{1}{2},0,0} \quad \text{and cyclic permutations of upper and lower indices} \tag{8}$$

$$F_{4_{-\frac{1}{2},0,0}} + F_{5_{0,-\frac{1}{2},0}} \quad \text{and c.c.} \tag{9}$$

$$h_{45_{\frac{1}{2},\frac{1}{2},0}},\ \left(h_{1_{1,0,0}} + \text{cyclic permutations}\right) \quad \text{and c.c.} \tag{10}$$

$$\phi_{\omega_{\frac{1}{2},-\frac{1}{2},0}},\ \phi_{y_{\frac{1}{2},-\frac{1}{2},0}},\ \phi_{3_{-\frac{1}{2},-\frac{1}{2},1}},\ \Phi_{12_{-1,1,0}},\ \Phi_{23_{0,-1,1}},\ \Phi_{13_{-1,0,1}}, \quad \text{and c.c.} \tag{11}$$

$$\Phi_{1,2,3} \tag{12}$$

where any $U(1)$ charge not shown explicitly vanishes. The hidden-sector matter fields become:

$$\underline{10} \text{ of } SO(10): \quad T_{1_{0,\frac{1}{2},\frac{1}{2}}}^{-\frac{1}{2},0,0} + \text{cyclic permutations of upper and lower indices} \tag{13}$$

$$T_{4_{0,\frac{1}{2},-\frac{1}{2}}} + T_{5_{\frac{1}{2},0,-\frac{1}{2}}}, \text{ and c.c.} \tag{14}$$

$$\underline{6} \text{ of } SU(4): \quad \Delta_{1_{0,\frac{1}{2},\frac{1}{2}}}^{\frac{1}{2},0,0} + \text{cyclic permutations of upper and lower indices} \tag{15}$$

and the half integer-charged states

$$\underline{4} \text{ of } SU(4): \quad X_{1_{\frac{1}{4},-\frac{1}{4},-\frac{1}{4}}}^{0,-\frac{1}{2},-\frac{1}{2}} + \text{cyclic permutations of upper and lower indices} \tag{16}$$

$$\underline{4} \text{ of } SU(4): \quad \bar{X}_{1_{-\frac{1}{4},\frac{1}{4},\frac{1}{4}}}^{0,-\frac{1}{2},-\frac{1}{2}} + \text{cyclic permutations of upper and lower indices} \tag{17}$$

The renormalizable trilinear part of the superpotential is:

$$\begin{aligned} W &= g\sqrt{2}\Big[F_1\bar{f}_1\bar{h}_1 + F_2\bar{f}_2\bar{h}_2 + F_3\bar{f}_3\bar{h}_3 \\ &+ F_4F_4h_1 + F_5F_5h_2 + \bar{F}_4\bar{F}_4\bar{h}_1 + \bar{F}_5\bar{F}_5\bar{h}_2 + F_4F_5h_{45} \\ &+ \frac{1}{\sqrt{2}}\left(F_4\bar{F}_5\phi_y + \bar{F}_4\bar{F}_5\bar{\phi}_\omega\right) + \frac{1}{2}\left(F_4\bar{F}_4\Phi_1 + F_5\bar{F}_5\Phi_2\right) \\ &+ h_1\bar{h}_2\Phi_{12} + h_1\bar{h}_3\Phi_{13} + h_2\bar{h}_3\Phi_{23} + \bar{h}_1h_2\bar{\Phi}_{12} + \bar{h}_1h_3\bar{\Phi}_{13} + \bar{h}_2h_3\bar{\Phi}_{23} \\ &+ h_{45}\bar{h}_3\phi_3 + \bar{h}_{45}h_3\bar{\phi}_3 \\ &+ \frac{1}{2}\Phi_3(h_{45}\bar{h}_{45} + \phi_3\bar{\phi}_3 + \phi_y\bar{\phi}_y + \phi_w\bar{\phi}_w) \\ &+ \Phi_{12}\bar{\Phi}_{13}\Phi_{23} + \bar{\Phi}_{12}\Phi_{13}\bar{\Phi}_{23} + \frac{1}{2}\Phi_{12}(\phi_y^2 + \phi_w^2) + \frac{1}{2}\bar{\Phi}_{12}(\bar{\phi}_y^2 + \bar{\phi}_w^2) \\ &+ \frac{1}{2}\left(\Phi_1T_4\bar{T}_4 + \Phi_2T_5\bar{T}_5\right) + \frac{1}{\sqrt{2}}\left(T_4\bar{T}_5\phi_w + \bar{T}_4T_5\bar{\phi}_y\right)\Big] \end{aligned} \tag{18}$$

The anomalous combination of $U(1)$ factors is

$$U(1)_A \equiv (Y_1 + Y_2 + Y_3)_{lower} - \frac{1}{2}(Y_1 + Y_2 + Y_3)_{upper} : Tr\ Q_A = 90$$

Comparing the above spectrum with that in the original version [9] of the revamped flipped $SU(5)$ model, we see that the $F_5+\bar{F}_5$ states in (9) replace $\underline{5}+\underline{\bar{5}}+\underline{1}+\underline{\bar{1}}$ states. The $F_5+\bar{F}_5$ representations contain $Q \equiv (\underline{3}, \underline{2}, \frac{1}{6})$ representations of $SU(3) \times SU(2) \times U(1)_Y$ that could be light, and hence able to pay the price (5) of string unification [2], together with candidates for light $D^c$ states. There are fewer singlet and hidden states than in the original version of the revamped model. The Yukawa couplings (18) offer candidate origins for the $t$, $b$ and $\tau$ masses, although they also offer the possibility that the $b$ and $\tau$ masses arise from higher-order non-renormalizable superpotential terms. It is easy to see that the three upper $U(1)$ $D$-terms can be cancelled by v.e.v.'s for the fields $\Delta_{1,2,3}$ that break $SU(4) \rightarrow SO(5)$, which still confines fractionally-charged hidden-sector states. The anomalous $U(1)$ $D$-term can be cancelled by v.e.v's for the tenplates $F_{4,5}$ of $SU(5)$ giving a GUT symmetry breaking scale $m_{GUT}$ close to $m_{SU}$, as suggested in Ref. [6]. However in this case, the $U(1)_{flipped}$ $D$-term cannot be cancelled and the model does not have a $D$-flat vacuum.

A detailed exploration of the phenomenological aspects of this new version of the revamped flipped $SU(5)$ model would take us beyond the scope of this paper, and within our present understanding only a $D-$ and $F-$flat vacuum could have small enough supersymmetry breaking. However, its existence does show that the price (5) of string unification is not exorbitative, this being just one example of a flipped $SU(5)$ model able to pay the price.

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[2] Note that, as argued in Refs. [6, 10], we expect the string unification scale $m_{SU}$ calculated for this version of flipped $SU(5)$ to be very similar to that calculated for the original revamped model.

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