Spin Diffusion in 2D XY Ferromagnet with Dipolar Interaction

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Abstract

In the ordered phase of 2D XY ferromagnet the dipole force induce a strong interaction between spin-waves in the long wavelength limit. This interaction leads to the transformation of the spin-wave excitation into a new soft-mode excitation in an intermediate range of wavelengths limited in magnitude and direction; and into an anomalous anisotropic diffusion mode excitation at long wavelengths. The dissipation of a spin-wave at short wavelengths is found to be highly anisotropic.

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The conventional condensed matter theory deals with elementary excitations and their interactions. The excitations such as electrons, phonons, spin-waves etc. have a propagating, wave-like nature. The momentum $\mathbf{p}$ and the energy ω of a single excitation are related by the dispersion relation (spectrum): $\omega = \epsilon(\mathbf{p})$. A weak interaction changes slightly the spectrum and leads to a finite life-time of the excitations [1]. Effect of a strong interaction is not so universal. Migdal [2] has shown that a strong electron-phonon interaction renormalizes substantially electron velocities and, if it exceeds a critical value, destroys the electron quasi-excitations. Recently a strong interaction of electrons in 2 dimensions with gauge fields has been shown to result in non-Fermi-liquid behavior [3].

In the long wavelength (hydrodynamic) limit quasi-excitations turn into classical modes, such as sound or spin-wave. Not only the propagating waves, but also particle, heat and spin diffusion can be considered as hydrodynamic modes. The interaction between these modes has been shown to be substantial in the critical region [4]; in the dynamics of liquid crystals [5], [6]; and in the dynamics of a Charge Density Wave interacting with impurities [7]. In all these systems the interaction leads to a drastic reconstruction of the dispersion relation.

In this Letter we solve the experimentally feasible magnetic model, in which a strong interaction between spin-waves leads to the replacement of the propagating spin-wave by a diffusion mode and to appearance of a new soft-mode in a range of momentum. This model is the two-dimensional XY ferromagnet with the dipolar interactions between spins.

A spin diffusion mode appears naturally in the paramagnetic phase and in the vicinity of the Curie point [8]. We consider a low temperature ordered phase, where no diffusion is expected but rather a propagating and weakly dissipating spin-wave mode. In a 3D ferromagnet the exchange interaction between spin-waves vanishes in the long wavelength limit [9]. The dipole force generates three spin-wave processes [8, 10] and violates the total spin conservation law. This interaction is dominant in the dissipation of a spin-wave via decaying it into the two spin-waves or via merging it with other spin-wave. But dissipation is weak.

In a 2D XY ferromagnet at low temperatures the dipolar interaction is relevant in the long wavelength limit, even despite of the low density of spin-waves. It was shown by one of the authors [11] that dipolar force induces an anomalous anisotropic scaling of spin-spin correlations in the ordered phase.

In this article we find an analogous dynamical scaling.

The dipolar force stabilizes the ferromagnetic long-range order [12], suppressing strong XY thermal fluctuation. Therefore, we represent the unit vector field of magnetization $\mathbf{S}$ by the two spin-wave fields – in-plane $\phi(\mathbf{x},t)$ and out-of-plane $\pi(\mathbf{x},t) = S^z$:

$$\mathbf{S} = \left(-\sqrt{1-\pi^2}\sin\phi; \sqrt{1-\pi^2}\cos\phi; \pi\right) \tag{1}$$

where both π and ϕ are small. The field $\pi(t,\mathbf{x})$ is canonically conjugated to the field $\phi(t,\mathbf{x})$.

The Hamiltonian of a 2D XY ferromagnet contains three terms: the exchange and the anisotropy energies:

$$H_{EA} = \sum_{\omega\,\mathbf{p}} (J\mathbf{p}^2|\phi_{\omega\mathbf{p}}|^2 + \lambda|\pi_{\omega\mathbf{p}}|^2)/2,$$

as well as the dipolar force term (see e.g. [11]):

$$H = H_{EA} + g\sum_{\omega\,\mathbf{p}} \left| p_x\phi_{\omega\mathbf{p}} + p_y\left(\phi^2/2\right)_{\omega\mathbf{p}}\right|^2 /2|\mathbf{p}|, \tag{2}$$

with the corresponding couplings being – the exchange constant J, the anisotropy λ and the dipole constant g [13]; The Hamiltonian (2) is written in terms of the Fourier transformed fields – $\phi_{\omega\mathbf{p}}$ and $\pi_{\omega\mathbf{p}}$, the abbreviation - $(\phi^2/2)_{\mathbf{p}\omega}$ - denotes the Fourier transform of $\phi^2(\mathbf{x},t)/2$. The anisotropy is assumed to be weak: $\lambda \ll J$. Therefore the spin $\mathbf{S}$ averaged over scales larger than $\sqrt{J/\lambda}$ turns into the plane. The bare spin-wave spectrum extracted from the quadratic part of the Hamiltonian (2) reads:

$$\epsilon^2(\mathbf{p}) = c^2\left(p^2 + p_0 p_x^2/p\right), \tag{3}$$

where $p = |\mathbf{p}|$, the characteristic wave-vector $p_0 = g/J$ and the spin-wave velocity $c = \sqrt{J\lambda}$. In the region $p_0 \ll p \ll \sqrt{\lambda/J}$ spin-waves have the linear, acoustic-like spectrum. We call this spherical shell of momentum the $\mathcal{A}$-shell.

The non-linear part of the Hamiltonian (2) H_{int} contains the three-leg and four-leg vertices. In particular the three-leg vertex is:

$$f(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) = \sum_{i=1}^{3} p_{ix}p_{iy}/\mid \mathbf{p}_i \mid \tag{4}$$

Since the sum of momenta entering the vertex is zero it depends on two momenta. Further we denote it as $f(\mathbf{p}_1, \mathbf{p}_2)$.

The dynamical equation associated with the Hamiltonian (2) reads:

$$\left(\frac{\omega^2}{\lambda} - \frac{\epsilon^2(\mathbf{p})}{\lambda} + i\frac{\omega}{\Gamma_0}\right)\phi_{\omega\mathbf{p}} = \frac{\delta H_{int}}{\delta\phi_{-\omega-\mathbf{p}}} + \eta_{\omega\mathbf{p}}. \tag{5}$$

The thermal noise $\eta_{\omega\mathbf{p}}$ and the bare dissipation coefficient Γ_0^{-1} generate the stochastic dynamics in (5). The noise-noise corelation function is related to the bare dissipation coefficient: $\langle|\eta_{\omega\mathbf{p}}|^2\rangle = 2T/\Gamma_0$. It vanishes at $T = 0$. We do not consider external sources of dissipation, spin-waves dissipate due to interaction generated by dipole force. Hence, we set $\Gamma_0^{-1} = +0$, and look for the generated dissipation: $\Gamma^{-1}(\omega, \mathbf{p})$.

The Green function $G(\omega, \mathbf{p})$ is defined as the linear response of the magnet to an external magnetic field with the same frequency and wave-vector. The l.-h.-side of Eq.5 represents the inverse of the bare Green function $\lambda^{-1}G_0^{-1}(\omega, \mathbf{p})$. According to the Fluctuation-Dissipation Theorem, the imaginary part of the Green function multiplied by the factor T/ω is the spin-spin correlation function $D(\omega, \mathbf{p})$. For convenience we use the dissipation function defined as: $b(\omega, \mathbf{p}) = \lambda/\Gamma(\omega, \mathbf{p})$. The self-energy term $\Sigma(\omega, \mathbf{p})$ equals $G_0^{-1}(\omega, \mathbf{p}) - G^{-1}(\omega, \mathbf{p})$ by definition. We notify the real and the imaginary part of the self-energy term as: $\Sigma = a^2(\omega, \mathbf{p}) - i\omega b(\omega, \mathbf{p})$. Thus, the Green function reads:

$$G^{-1}(\omega, \mathbf{p}) = \omega^2 - \epsilon^2(\mathbf{p}) - a^2(\omega, \mathbf{p}) + i\omega b(\omega, \mathbf{p}), \tag{6}$$

whereas the spin-spin correlation function reads:

$$D(\omega, \mathbf{p}) = \frac{b(\omega, \mathbf{p})}{[\omega^2 - \epsilon^2(\mathbf{p}) - a^2(\omega, \mathbf{p})]^2 + \omega^2 b^2(\omega, \mathbf{p})} \tag{7}$$

(we refer the factor T to vertices). We use the reduced temperature $t = T/4\pi J$ as a small and $L = \log(\sqrt{J\lambda}/g)$ as a large parameters (the latter means large $\mathcal{A}$-shell).

We employ the standard, so-called Janssen-De-Dominicis Functional method [14] to account for the non-linear terms in the stochastic Langevin equation (5). This method generates a diagrammatic expansion in powers of the bare vertices, associated with H_{int}. The main contribution to the self-energy is given by the one-loop diagram shown in Fig.1a, 1b. Our theory is valid only if the temperature is small:

$$t \log(\sqrt{J\lambda}/g) = tL \ll 1.$$

Under this condition the two-loop corrections are small and the diagram Fig.1b contributes to a negligible change of the spectrum (3) [15]. Such neglecting of the two-loop diagrams (vertex correction) was a major assumption in the so-called mode-coupling methods [8]. Later we proove this assumption. Thus, the Dyson equation for our problem is as follows:

$$\Sigma(\Omega, \mathbf{q}) = 2g^2\lambda^3 T \int \frac{d^2\mathbf{p}}{(2\pi)^2} \int \frac{d\omega}{2\pi} f^2(\mathbf{p}, \mathbf{q}) \\ D(\omega, \mathbf{p}) G(\omega + \Omega, \mathbf{p} + \mathbf{q}), \tag{8}$$

The functions $b(\omega, \mathbf{p})$ and $a(\omega, \mathbf{p})$ are even in both arguments [1]. The imaginary part of the self-energy is odd in frequency Ω. Hence, the equation for the dissipation function reads:

$$b(\Omega, \mathbf{q}) = g^2\lambda^3 T \int \frac{d^2\mathbf{p}}{(2\pi)^2} \int \frac{d\omega}{2\pi} f^2(\mathbf{p}, \mathbf{q}) \\ D(\omega, \mathbf{p}) D(\omega + \Omega, \mathbf{p} + \mathbf{q}). \tag{9}$$

The integrand in (9) is positive. Thus, the main contribution to $b(\Omega, \mathbf{q})$ comes from the region, where poles of the two D-functions coincide. The function $D(\omega, \mathbf{p})$ has poles at: $\omega \approx \pm\epsilon(\mathbf{p})$, in the $\mathcal{A}$-shell [16]. Following the terminology of the field theory we call the surface $\omega^2 = \epsilon^2(\mathbf{p})$ the mass-shell. The dissipation in the $\mathcal{A}$-shell is small and the D-function can be represented as a sum of δ-functions:

$$D(\omega, \mathbf{p}) \approx \sum_{\pm} \frac{\pi}{2\epsilon^2(\mathbf{p})} \delta(\Delta\omega_\pm), \tag{10}$$

where $\Delta\omega_\pm = \omega \pm \epsilon(\mathbf{p})$ measures the deviation from the mass-shell. After integrating ω out from the Eq.9 with the D-functions from (10) we recover the Fermi Golden Rule for the probability of the spin-wave decay and scattering processes.

Looking for the long wavelength quasi-excitations we need the self-energy at very small momenta $q \ll p_0$, which we denote as Σ_0. We expect quasi-excitations to be soft: $\Omega \ll cq$, and we restrict the wave-vector $\mathbf{q}$ of quasi-excitation to be directed along the magnetization: $|q_x| \ll q$. The essential contribution to the integral in Eq.8 comes from the internal momentum p being in the $\mathcal{A}$-shell and the internal frequency $\omega = \epsilon(\mathbf{p})$. Taking the integral over ω, with the D-function from Eq.(10), we find [16]:

$$\Sigma_0 = \frac{c^2 p_0^2 t}{4\pi} \int \frac{c^4 p^3 dp}{\epsilon^4(\mathbf{p})} \frac{\Omega \sin^2(2\psi) d\psi}{\Omega - cq\cos\psi + ib_1}, \tag{11}$$

where b_1 is the dissipation function $b(\omega, \mathbf{p})$ of a spin-wave inside the $\mathcal{A}$-shell and ψ is the direction of the internal spin-wave: $\sin\psi = p_x/p$. Note, that we added a Ω-independent contribution from the diagram Fig.1.b to the self-energy 11. $\mathrm{Re}\Sigma_0$ vanishes in the static limit $\Omega = 0$.

If $cq \gg b_1$ we make the integral over ψ in Eq.11 to find:

$$\Sigma_0(\chi) = c^2 p_0^2 t L \cos^2\chi \exp(-2i\chi), \tag{12}$$

where χ, defined by the equation $\cos\chi = \Omega/cq$, measures the deviation from the mass shell. If q is so small that $cq \ll b_1$ the Eq.11 implies the q-independent dissipation constant:

$$b_0 = c^2 p_0^2 t L \int \frac{d\psi}{4\pi} \frac{\sin^2(2\psi)}{b_1(\psi)}. \tag{13}$$

In this calculation we have used the fact that the dissipation of a spin-wave in the $\mathcal{A}$-shell b_1 depends only on the angle ψ between the direction of magnetization and the spin-wave wave-vector $\mathbf{p}$, which we prove below.

Anyway, we need $b_1(\psi)$. An unusual feature of our theory is that the dissipation process in the $\mathcal{A}$-shell is mediated by an off-mass-shell, virtual spin-wave. Indeed, the Eq.3 does not allow for decay or merging processes. Alternatively, the dissipation of a spin-wave in the $\mathcal{A}$-shell propagating in the direction specified with the angle ψ ($\sin\psi = q_x/q$) is mediated by an internal virtual spin-wave in (9), with a momentum $p \ll p_0 \ll q$ and a frequency $\omega < cp$. The integration over ω with one of the D-functions in (9) taken in the form (10), leads to a following equation:

$$b_1 = \frac{c^2 p_0^2 t f^2(\mathbf{0}, \mathbf{q})}{8\pi q^2} \int d^2\mathbf{p} D(\epsilon(\mathbf{p}+\mathbf{q}) - \epsilon(\mathbf{q}), \mathbf{p}) \tag{14}$$

Since $\omega = \epsilon(\mathbf{p}+\mathbf{q}) - \epsilon(\mathbf{q})$ we conclude that $\omega = cp\cos\Phi$, where $\Phi = \psi - \varphi$ is the angle between the vectors $\mathbf{q}$ and $\mathbf{p}$. Invoking the definition of the angle

χ for virtual spin-wave, we find that $\chi = \Phi$. Now, we look onto Eq.14 in more details:

$$b_1(\psi) = \frac{c^2 p_0^2 t}{2\pi} \mathrm{Im} \int \frac{\sin^2\psi \cos\psi \; dp d\varphi}{p^2 \sin^2\psi + p_0 p \varphi^2 + \Sigma_0(\psi)/c^2}. \tag{15}$$

Note, that $\varphi^2 \sim p/p_0 \ll 1$ in (15). Thus, $\chi = \psi$. In other words, the dissipation of a short wavelength spin-wave propagating in the direction ψ is determined by the scattering on the long wavelength virtual spin-wave, which lies on the specific distance off the mass-shell: $\omega/cp = \cos\psi$. The integration over p in (15) is confined towards the crossover region: $p \sim p_c = p_0\sqrt{tL}$.

Substituting $\Sigma_0(\psi)$ from Eq.12 into Eq.15 we find the anisotropic dissipation of a spin-wave mode in the $\mathcal{A}$-shell:

$$b_1(\psi) = \beta_1 t^{3/4} c p_0 \frac{\sin^{3/2}(2\psi)\sin(\psi/2)}{L^{1/4}\cos\psi}, \tag{16}$$

where the direction of the spin-wave is limited to the fundamental quadrant: $0 < \psi < \pi/2$. We found: $\beta_1 = \Gamma^2(1/4)/4\sqrt{2\pi} \approx 1.31$.

Let us return to very low momenta $p \ll b_1/c$. Plugging Eq.16 into Eq.13 one finds: $b_0 = \beta_0 c p_0 t^{1/4} L^{5/4}$, where $\beta_0 \approx 1.24$ was found numerically. The condition $cp_{DM} \sim b_1$ defines the crossover wave-vector: $p_{DM} \sim p_0 t^{3/4}/L^{1/4}$, between the self-energies Eq.12 and Eq.13.

The dissipation functions (12,13 and 16) conclude the self-consistent solution of the Dyson equation (8, 9).

We verify that the two-loop correction (see Fig.1c) is negligible. Note, that the main contribution to the diagram Fig.1c comes, if the two internal momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ are restricted to the $\mathcal{A}$-shell. Inside the $\mathcal{A}$-shell the Green and the D-functions live on the mass-shell. But, the prohibition of the three spin-wave processes leaves the Green function $G(\omega_1 - \omega_2, \mathbf{p_1} - \mathbf{p_2})$ to be off the mass-shell. The Green function off the mass-shell is small in temperature. A simple counting shows, then, that the two-loop dissipation function is $b_0' = b_0(1 + t^{1/4}O(t))$. The function $b_1'(\psi, q) - b_1(\psi)$, which represents the two-loop corrections for b_1, is small in $t^{1/4}$, and is also small in the ratio p_0/q.

Having the explicit expression for the self-energy we can analyse the dispersion relation – $\omega^2 = \epsilon^2(\mathbf{p}) + \Sigma(\omega, \mathbf{p})$ – in the range of small ω and p. New results are expected for the region $p < p_c = p_0\sqrt{tL}$ in which Σ_0 becomes comparable with $\epsilon^2(\mathbf{p})$. In a range of momentum $p_{DM} \ll p \ll p_c$ and angles $\psi \ll \sqrt{p_0 tL/p}$ we find a new propagating soft mode with the dispersion:

$$\omega = cp(p^2 + p_0 p\psi^2)^{1/2}/p_0\sqrt{tL} \tag{17}$$

The dissipation of the soft mode grows to the boundary of the region and becomes of the order of its energy at $\psi \sim \sqrt{p_0 tL/p}$ or $p \sim p_{DM}$. There is no soft mode beyond the indicated range. The spin-wave mode persists at $p > p_0\sqrt{tL}$. In a range $p \ll p_{DM}$ and small angles a new diffusion mode occurs with the dispersion:

$$\omega = -i\epsilon^2(\mathbf{p})t^{-1/4}L^{5/4}/\beta_0 c p_0 \tag{18}$$

The angular range of the diffusion mode increases with decreasing p and captures the entire circle at $p < p_0 tL$.

At even smaller wavelengths $p < p_A \ll p_{DM}$ the interaction between diffusion modes should be taken into account (p_A is the anomalous diffusion set up wave-vector [17]). The problem can be solved by the renormalization group method [17]. The growing interaction, although leaving invariant the diffusive nature of the spin propagation, changes the dispersion. For the propagation along the spontaneous magnetization it is: $\omega \propto -ip^{47/27}$, whereas for the propagation in the perpendicular direction it is: $\omega \propto -ip^{47/36}$. This is a dynamic analog of the non-Gaussian fixed point found in [11].

In conclusion, we discuss the experimental feasibility of the above considered effects in the epitaxial magnetic films. The main difficulty is that all of them are confined to rather long wavelengths. Therefore even weak in-plane anisotropy can suppress them. One need to use substrates with the six-fold symmetry axis. The six-fold anisotropy is much weaker than the tetragonal one. Moreover, it was predicted in [18] that there exists a temperature interval in which the hexagonal anisotropy vanishes at large distances. A proper substrate is, for example, a (111) face of Cu, Au etc. Unfortunately the epitaxial growth of ferromagnetic films on this faces has not been realized so far. In a recent work [19] a growth of a ultrathin Ru/C(1000) film has been reported. It is the in-plane ferromagnet. We hope that further sophistication of the experimental technics will allow to observe the predicted dynamical behavior.

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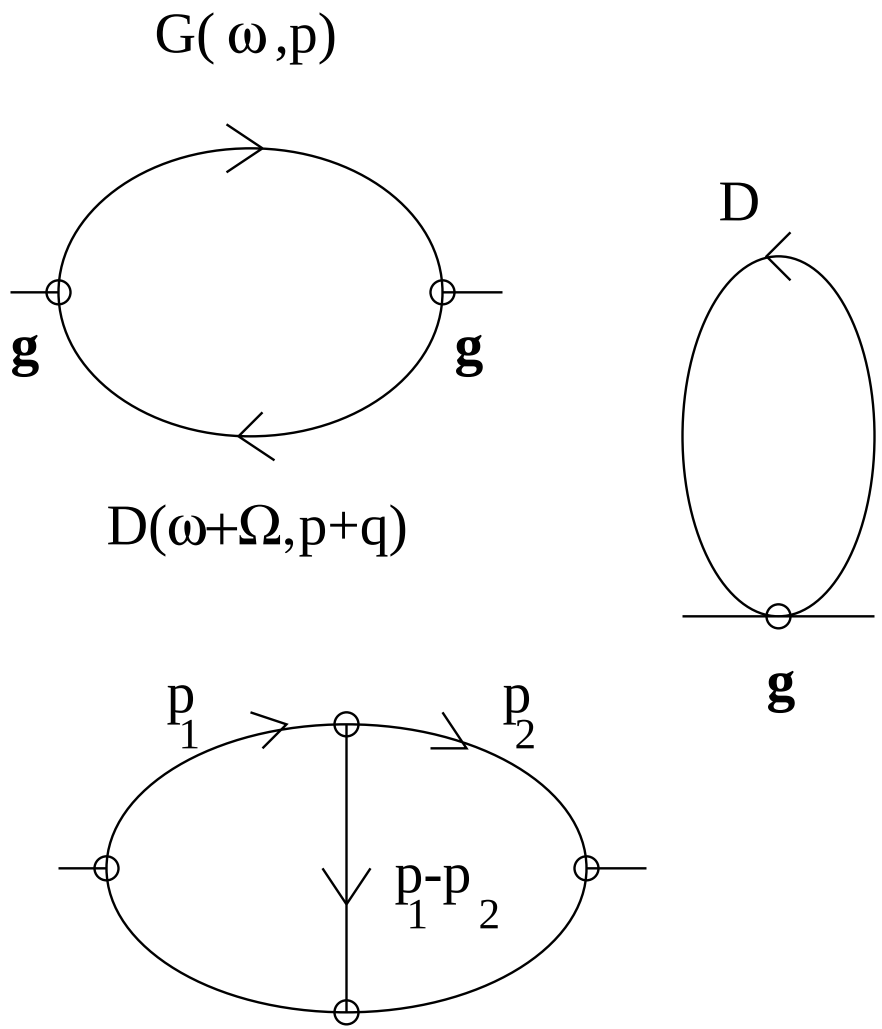
Figure Caption

- FIG.1. a) Main one-loop self-energy diagram; b) One-loop four-leg vertex diagram; c) Two-loop self-energy diagram. Momenta of internal lines are indicated.


G(ω ,p)
g
g
D(ω+Ω,p+q)
D
g
p
1
p
2
p-p
1
2