

Proton decay and $\mu \rightarrow e + \gamma$ Connection in a Renormalizable SO(10) GUT for Neutrinos

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Abstract

Supersymmetric SO(10) grand unified models with renormalizable Yukawa couplings involving **10**, **126** and **120** Higgs fields have been shown to give a very economical theory for understanding quark-lepton flavor in a unified framework. In previous papers, we showed how nucleon decay can be suppressed in these models without invoking cancellation, by choice of Yukawa flavor texture within a type II seesaw framework for neutrinos that explains all mixings and masses including the recently observed “large” θ_{13} . In this follow-up paper, we extend our earlier work to the case of type I seesaw and show that the recently measured “large” θ_{13} can be accommodated in this case while suppressing proton decay. We then point out that the two cases (type I and II) lead to different testable predictions for $B(\mu \rightarrow e + \gamma)$ and $B(\tau \rightarrow \mu(e) + \gamma)$ as well as different flavor final states in nucleon decay. In particular, we find that for the type I seesaw case, $B(\tau \rightarrow \mu + \gamma)$ can be observable while at the same time suppressing $B(\mu \rightarrow e + \gamma)$, whereas in the type II seesaw case, $B(\tau \rightarrow \mu + \gamma)$ is always suppressed whereas $B(\mu \rightarrow e + \gamma)$ is observable.

1 Introduction

The last decade has seen incredible experimental progress in the field of neutrino physics. Since the discovery of the oscillations of atmospheric neutrinos in 1998 followed by the confirmation of the solar neutrino oscillations discovered by Ray Davis in mid-sixties, many parameters describing the neutrino masses and mixings have been measured in various solar, atmospheric and accelerator and reactor experiments. The most recent such measurement is that of the remaining mixing parameter θ_{13} by reactor as well as accelerator experiments [1, 2, 3, 4, 5] which has created a lot of excitement in the theory community due to its implications for physics behind the neutrino masses. Attention has now turned to measuring the other missing pieces of informations: Dirac versus Majorana nature of neutrinos, the CP violating phase δ in the lepton sector as well as the mass hierarchy of neutrinos. The global analysis of the parameter fit [6] seem to have some preference for CP phase $\sim \pi$ although it is far too early to take this seriously. The “large” value of the 13 mixing angle (around $\sin \theta_{13} \simeq 0.15$) however, has raised the hope that the CP phase can be measured in near future.

What have we learnt about the physics behind the neutrino masses from these observations? The first challenge is to understand why the neutrino masses are so small and the second is to see if the same framework that explains the small neutrino mass also simultaneously explains the observed mixing pattern. A popular paradigm for this seems to be the so-called seesaw mechanism [7] where one extends the standard model by adding heavy Majorana right-handed neutrinos (known as the type I seesaw) or heavy SM Higgs triplets (known as the type II seesaw) [8]. Generic models based on seesaw are quite successful in achieving the first goal i.e. small neutrino masses. It is however more challenging to get an understanding of the mixings without further assumptions. The two approaches which have been used extensively are: (i) leptonic symmetries and (ii) grand unified theories. The closeness of the leptonic mixing parameters to group theory-like numbers (e.g. $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}$) have been the major driving force behind the symmetry approach. Most models with symmetries however predicted the third mixing angle $\theta_{13} = 0$, which have now been disproved by experiments as noted. While there are still a number of leptonic symmetry scenarios which can lead to large value of θ_{13} [9], the fact that

$$\sin \theta_{13} \sim \sqrt{\frac{\Delta m_{12}^2}{\Delta m_{23}^2}} \sim \theta_{Cabibbo} \quad (1)$$

may be suggestive of a “big” picture that unifies quarks and leptons such as those based on grand unified theories. In particular, if normal neutrino hierarchy is established, that would imply a similarity between the quark and the lepton sector, that would be consistent with expectations from grand unified theories (GUTs) and would further motivate the GUT approach to all

flavors.

One class of grand unified theories that have been extensively explored and appear quite successful in providing a unified description of quark-lepton flavor i.e. giving observed values of the neutrino mixings and masses, while being consistent with observed quark masses and CKM mixings is the one based on the group $SO(10)$ with or without supersymmetry and using **126** fields to break the $B - L$ subgroup of $SO(10)$. This class of models uses only renormalizable Yukawa interactions involving a **10** and **126** [10, 11] with/without an additional **120** [12, 13] to explain both the quark and lepton flavor puzzle. The renormalizability property restricts the fermion Yukawa couplings to have only a small number of parameters (without the need for adhoc symmetries) describing both the quark and the lepton sectors simultaneously so that the models become quite predictive. Indeed these models led to the prediction not only of the relation between neutrino mass ratios and Cabibbo angle in Eq.(1) but also the value of θ_{13} several years prior to its measurement [14] without the need for **120** field.

One of the key predictions of the supersymmetric (SUSY) GUT models is the enhancement of proton decay rate due to the presence of new supersymmetric contributions from color triplet Higgsino exchange. These contributions are known to severely constrain the nature of these theories and clearly if the above $SO(10)$ neutrino models are to be taken seriously, their consistency with current lower limits on proton lifetimes must be examined for both the decay modes $p \rightarrow e^+\pi^0$ where the key input is the value of the unification scale and the characteristic SUSY mode $p \rightarrow K^+\bar{\nu}$, where the key input is the nature Higgs fields that give mass to fermions. For the case of minimal $SO(10)$ models with **10**, **126**, with type II seesaw for neutrino masses, this consistency requires cancellation between parameters describing the Higgsino mixings and masses [15]. Since there are many proton decay modes that have been constrained by experiments [16] and all parameters of the model except for three Higgsino mixings are already determined, the fact that there is a consistent picture, is highly nontrivial. However this works only for low $\tan\beta$ regime of the MSSM parameter space. It was subsequently pointed out [17, 18] that once the **120** is included, the situation improved quite a bit within the type II seesaw framework i.e. by choice of an appropriate flavor structure for the different Yukawa coupling matrices, one can not only give a simultaneous fit to fermion masses but also suppress proton decay to the desired level, for both small and large $\tan\beta$, without need to invoke cancellation. It was shown that in the model with **120**, one can obtain θ_{13} as large as 0.15, which is in accord with the recent measurements.

An important question remained as to whether the idea of suppressing proton decay by choice of flavor structure works in models with **120** when type I seesaw is picked for understanding neutrino masses. The second question is: is there a way to distinguish between the

type I and type II seesaw models of this kind. Clearly, there are two obvious avenues to explore in this connection. One can look at the flavor structure of the proton decay final states and secondly the predictions for rare lepton flavor violating decays of type $B(\mu \rightarrow e + \gamma)$ and $B(\tau \rightarrow \mu(e) + \gamma)$. We investigate both these questions in this paper.

The reason to suspect that lepton flavor violation can probe different models is that in SUSY seesaw models all leptons are accompanied by their bosonic partners, the sleptons and their mixings can lead to large flavor violating effects at low energies. To prevent excessive flavor violating effects, one generally assumes that at some high scale, all slepton masses are equal. However as we extrapolate the theory down to the weak scale, slepton mixing are generated by the large neutrino mixings hidden in the Dirac Yukawa couplings of right-handed neutrinos [19]. In generic type I seesaw models without additional inputs, these rates can be suppressed by simply changing the seesaw scale since these mixings are proportional to products of Y_ν matrix elements and if the seesaw scale is lowered, seesaw formulae for neutrino masses demand that Y_ν become smaller in absolute magnitude thereby reducing the $\mu \rightarrow e + \gamma$ etc. rates. The same thing also happens in SU(5) GUTs, where the size of the Dirac neutrino Yukawa coupling is a free parameter because the right-handed neutrino is a gauge singlet and no definite prediction is possible.

However, in predictive GUT models such as the minimal renormalizable SO(10), coupling unification and fermion mass fits determine all Yukawa couplings of the theory and in particular, the Yukawa couplings of the RH neutrinos. Furthermore the values of the RH neutrino masses are also predicted making the lepton flavor violation predictions more definite. There are also separate contributions to LFV amplitudes coming from type II seesaw Yukawa couplings [20], which are not generally considered. Another contribution to LFV in all GUT models comes from charged fermion-quark-color triplet Higgs ($e^c u^c H_T$) coupling [21] which is independent of the seesaw mechanism; it is also predicted in these models since fermion mass fits also determine those couplings. Moreover, since the flavor structure of the Yukawa matrices is related to the lepton flavor violations (LFV), and the proton decay amplitudes within a SUSY SO(10) framework, these two predictions are in principle connected. We investigate this question in this paper for a specific realistic SO(10) model. We note that although some early studies of LFV in SO(10) models have been carried out in Ref.[22], where assumptions are made about expected relations between neutrino Yukawa matrix Y_ν and Y_u , in realistic models there may be significant deviations from these relations. We find this to be true in models we are considering where in addition to new forms for the **10** couplings, there are important $\overline{\mathbf{126}}$ Yukawa contributions to flavor mixing.

The new results of this paper are: (i) we present a detailed analysis of the necessary texture

to obtain “large” θ_{13} and suppressed proton decay for type I seesaw case and in particular discuss how the relation of 13 mixing and the ratio of the squared mass differences, Eq.(1) emerges: (ii) we show that in the case of type II seesaw mechanism, zero value of 13 neutrino mixing is disfavored if we want to suppress proton decay via flavor structure; (iii) we also show that the measured value of 13 neutrino mixing prefers $\delta_{\text{PMNS}} \sim \pi$ for the CP phase in the neutrino oscillation parameters and (iv) we compare the predictions of LFV for both type I and II seesaw and show how the required flavor structures in type I and type II seesaw cases can provide different predictions for LFV decays: in particular, we find that for the type I seesaw case, $B(\tau \rightarrow \mu + \gamma)$ can be observable while at the same time suppressing $B(\mu \rightarrow e + \gamma)$, whereas in the type II seesaw case, $B(\tau \rightarrow \mu + \gamma)$ is always suppressed.

This paper is organized as follows: in section II, we discuss the Yukawa matrices in SO(10) model; in section III, we discuss the suppression of nucleon decay amplitudes; in section IV, we discuss the LFV constraints; in section V, we discuss the flavor structures and predictions for type II seesaw scenario; in section VI we discuss the flavor structures and predictions for type I seesaw scenario; in section VII we discuss the predictions for lepton flavor violation in type I and II seesaw scenarios; in section VIII we discuss the predictions for nucleon decay in type I and II seesaw scenarios and we conclude in section IX.

2 Yukawa matrices for fermions in renormalizable SO(10)

The Yukawa terms in the superpotential of the renormalizable SO(10) model involve the couplings of **16**-dimensional matter spinors ψ_i with Higgs fields belonging to **10** (denoted by H) and $\overline{\Delta}$, and D , representing the $\overline{\mathbf{126}}$, and **120** dimensional representations, respectively and is given by:

$$W_Y = \frac{1}{2} \bar{h}_{ij} \psi_i \psi_j H + \frac{1}{2} \bar{f}_{ij} \psi_i \psi_j \overline{\Delta} + \frac{1}{2} \bar{h}'_{ij} \psi_i \psi_j D. \quad (2)$$

This equation holds at the GUT scale. In order to write the effective Yukawa couplings below the GUT scale, we extract the effective $H_{u,d}$ fields of MSSM which are linear combinations of the Higgs doublet fields not only in $H, \overline{\Delta}, D$ fields but also in other fields e.g. Δ and **210** fields used to break the GUT symmetry while maintaining supersymmetry down to the weak scale. We assume that the SO(10) symmetry is broken down to the standard model gauge symmetry by the vacuum expectation values of $\Delta(\mathbf{126}) + \overline{\Delta}(\overline{\mathbf{126}})$ and the **210** Higgs field. Our conclusions below are independent of this i.e. the GUT symmetry could have been broken down by other fields such **54**, **45** etc as long as they do not contribute to fermion masses. We also assume that only one pair of the linear combinations of the Higgs doublets remains massless (or more precisely, weak scale Higgsino mass, which is much smaller than the GUT scale) to break the

electroweak symmetry. Using this, we can write the Yukawa matrices that give rise to the fermion masses as those given by the linear combination of the original \bar{h} , \bar{f} and \bar{h}' couplings:

$$Y_u = h + r_2 f + r_3 h', \quad (3)$$

$$Y_d = r_1(h + f + h'), \quad (4)$$

$$Y_e = r_1(h - 3f + c_e h'), \quad (5)$$

$$Y_\nu = h - 3r_2 f + c_\nu h', \quad (6)$$

where r_i and c_e, c_ν are the functions of Higgs mixings, and h, f and h' matrices are the original couplings multiplied by Higgs mixings. More details about these equations can be found in Ref.[17].

The $\overline{\mathbf{126}}$ Higgs Yukawa coupling includes both left- and right-handed Majorana neutrino couplings:

$$\psi\psi\overline{\Delta} \supset \ell\ell\overline{\Delta}_L + \bar{\nu}\bar{\nu}\overline{\Delta}_R, \quad (7)$$

where $\overline{\Delta}_L$ denotes a $SU(2)_L$ triplet in the $\overline{\mathbf{126}}$ representation. As a result, in the case of triplet-part dominant type II seesaw neutrino mass [8], the neutrino mass is (approximately) proportional to the coupling matrix f . It is clear from the above equation that there is an intimate connection between the lepton and the quark sector since the same Yukawa coupling matrix f appears both in the charged fermion sector as well as the neutrino sector due to the seesaw mechanism. We assume CP conservation prior to symmetry breaking so that h and f are real symmetric matrices and we can choose a basis so that h is diagonal. The matrix h' is imaginary and anti-symmetric so that the total number of coupling parameters in the theory (prior to any assumption about nucleon decay) is twelve. Combined with the other parameters for the model i.e. (adding in $r_i, c_{e,\nu}$) and the v_{BL} which gives the overall scale of neutrino masses) the total number of parameters is eighteen. This is smaller than the number of observables i.e. 13 in the charged fermion sector and six in the neutrino sector excluding the Majorana phases. So the model is predictive. When combined with the flavor ansatz for suppressing nucleon decay mentioned above, the number becomes fifteen and the predictive power increases as we saw in Ref.[17].

3 Suppression of nucleon decay amplitude

In this section, we review the flavor suppression mechanism for nucleon decay in this model proposed in [17]. The dominant proton decay in SUSY GUTs arise from the exchange of color triplet higgsinos which are part of the $\mathbf{10}$, $\overline{\mathbf{126}}$ and $\mathbf{120}$ fields. The effective dimension five

operator induced by them can be written as

$$-W_5 = \frac{1}{2}C_L^{ijkl}q_kq_lq_i\ell_j + C_R^{ijkl}e_k^c u_l^c u_i^c d_j^c. \quad (8)$$

where the coefficients $C_{L,R}$ are functions of h , f and h' couplings:

$$C_L^{ijkl} = \sum_a \frac{1}{M_{T_a}}(X_{a1}\bar{h} + X_{a4}\bar{f} + \sqrt{2}X_{a3}\bar{h}')_{ij}(Y_{a1}\bar{h} + Y_{a5}\bar{f})_{kl}, \quad (9)$$

$$C_R^{ijkl} = \sum_a \frac{1}{M_{T_a}}(X_{a1}\bar{h} - X_{a4}\bar{f} + \sqrt{2}X_{a2}\bar{h}')_{ij}(Y_{a1}\bar{h} - (Y_{a5} - \sqrt{2}Y_{a6})\bar{f} + \sqrt{2}(Y_{a3} - Y_{a2})\bar{h}')_{kl}, \quad (10)$$

where M_{T_a} are masses of colored Higgs triplets, and X and Y are the colored Higgs mixings (Details are given in Ref.[17]).

As is noted in the introduction in order to suppress the nucleon decay amplitudes, one has to consider

1. Choice of SUSY breaking parameter (heavy squarks being preferable).
2. The (lightest) colored Higgs triplet is heavy and/or the colored Higgs mixings are small.¹
3. Special structure of Yukawa matrices.

All the above requirements may work in tandem to satisfy the current experimental constraints on proton lifetime without invoking cancellation. We will concentrate on the item 3 in this paper, and consider what kind of flavor structure is favorable to suppress the nucleon decay operators, while at the same time giving correct prediction for the fermion masses and mixings.

The important features that arise in the discussion of suppressing nucleon decay operators by a flavor structure are as follows [17, 23]:

1. In the current setup, there are multiple pairs of Higgs fields, and thus, there is freedom to cancel the decay amplitude. However, the required cancellations in both C_L and C_R operators are large, especially for large $\tan\beta$. Sometimes, C_R operators are ignored since they are suppressed by Higgsino dressing rather than gaugino dressing. It is true that the contribution from C_R to the nucleon decay amplitude is rather suppressed compared to C_L . However, in the case that the 1st generation masses are obtain by a choice of h and f coupling and $h_{11}, f_{11} \sim y_d$ (down quark Yukawa coupling), the C_R operators are far from being small. Indeed, even in the minimal-type of SU(5) model, when only one pair of Higgs fields couple to fermions and the dimension five operators are roughly in the form

¹We investigate what kind of SO(10) breaking vacua is preferable to obtain heavy colored Higgs in the current scheme in Ref.[18].

of $Y_d Y_u$, and leads to trouble due to the fact that both C_L and C_R cannot be suppressed simultaneously from the flavor structure [17] unless the colored Higgs spectrum is extended [18]. The SO(10) model (in which the cancellation is compatible with quark/lepton mass hierarchy) provides the rough structure of proton decay operators given by $Y_d Y_u$, and the size of C_L and C_R operators are much larger than the current experimental bound. We also stress that in SO(10) models, the Higgs triplets in $\mathbf{10}$ and $\overline{\mathbf{126}}$ have opposite D -parity and therefore appear with opposite signs in the $C_{L,R}$ expressions as in Eqs.(9) and (10). Therefore, the cancellation (between h and f) is unnatural. On top of this, such cancellation need to happen for each decay mode.

2. Since the cancellation between h and f is not naturally realizable, it is preferable that h coupling structure is similar to the up-type quark mass hierarchy, and the down-type quark and charged lepton mass hierarchy is generated by f and h' matrices. Even in this case, the $h_{ij} h_{kl}$ contribution is comparable to the current experimental bound even if $h_{11} \sim y_u$ and $h_{22} \sim y_c$, where y_u and y_c are up and charm quark Yukawa couplings. Therefore to avoid unnatural cancellation which has to be implemented for each decay mode, the simplest choice appears to be to have $h_{11} \ll y_u$ and $h_{22} \ll y_c$. This implies that the h coupling has no role in generating the 1st and 2nd generation masses and only 3rd generation masses are generated by h ; in other words, h coupling can be chosen to be a rank 1 matrix to leading order.
3. If the h coupling matrix is rank 1, and 1st and 2nd generation masses are generated from f and/or h' couplings, thus avoiding the need for cancellation between h and f . We however still need suppression of the contributions from f and h' . The simplest choice seems to be that the 1st generation masses and Cabibbo mixing are generated by h'_{12} , and f coupling does not contribute to them. Since the h' matrix is anti-symmetric, it does not contribute to the kl part of the C_L operator Eq.(9). In fact, since the down quark Yukawa coupling is much too large to satisfy the current experimental bounds for the the nucleon decays, the question of how to generate the first generation masses and Cabibbo angle becomes an important one and this is the reason way we adopt a $\mathbf{120}$ Higgs representation instead of an extra $\mathbf{10}$ Higgs field.
4. The size of f_{1i} components are also important for a natural suppression of the proton decay amplitude. If the f coupling does not contribute to the down-quark and electron masses, and Cabibbo mixing angle, f_{11} and f_{12} can be taken small. In fact, this choice of f and h' coupling matrices can be simply consistent to the empirical relations: $\sin \theta_C \simeq \sqrt{m_d/m_s}$, and $m_d m_s m_b \simeq m_e m_\mu m_\tau$. Under the assumption that the h coupling is dominantly large

and rank one and h' and f are the correction needed to generate the masses of the first and second generation quarks and leptons, respectively, the quark mixings V_{cb} and V_{ub} are generated to be small due to the left-right symmetry.

5. The last piece in this discussion involves the magnitude of the 13 element of f coupling. In the case of type II seesaw (triplet part dominant), a sizable f_{13} is needed to obtain a proper solar mixing and neutrino mass ratio. In this case, one still needs a cancellation in the nucleon decay amplitudes. It is important to study whether the f_{13} coupling can be made to be small. The size of f_{13} is important not only for the nucleon decay suppression but also for the lepton flavor violation as we will see next section.

4 Lepton Flavor Violation

We now turn to the discussion of lepton flavor violating rare decays in this class of models. It is well known that the SUSY GUTs predict the lepton flavor violating rare decays due to the possibility of large flavor mixings in the superpartner sector. In order to avoid too much FCNCs induced by them, the flavor universality of the SUSY breaking mass parameters is often invoked. Even so, the loop correction via the heavy GUT particles can generate flavor violation. Though there can be new flavor changing effects in quark sector in SUSY GUTs, the recent LHC result imply no large effects [24, 25]. We, therefore, concentrate on the flavor violation in the lepton sector.

The typical sources of LFV in SUSY SO(10) are as follows:

1. $e^c u^c H_C$ coupling (which is $\mathbf{10}_m \cdot \mathbf{10}_m \cdot \mathbf{H}_5$ coupling in SU(5)).
2. Dirac neutrino coupling ($Y_\nu \ell \nu^c H_u$).
3. Majorana neutrino coupling ($f_\nu \ell \ell \bar{\Delta}_L$).

In SO(10) model, the Dirac neutrino Yukawa coupling Y_ν is the effective combination of $\mathbf{10}$, $\overline{\mathbf{126}}$ and $\mathbf{120}$ doublet coupling with appropriate mixing ratios (as in Eq. 6) whereas the Majorana neutrino coupling f_ν is unified to the $\overline{\mathbf{126}}$ coupling (up to a Clebsch-Gordon coefficient).

One can express the RGE-induced off-diagonal elements of SUSY breaking masses in terms of the Y_ν and f_ν Yukawa couplings as follows:

$$\Delta M_{ij}^2 \simeq -\frac{3m_0^2 + A_0^2}{16\pi^2} \sum_k (Y_{\nu,ik}^* Y_{\nu,kj}) \ln \frac{M_U^2}{M_{R_k}^2} - \frac{9m_0^2 + 3A_0^2}{16\pi^2} \sum_k (f_{\nu,ik}^* f_{\nu,kj}) \ln \frac{M_U^2}{M_\Delta^2}, \quad (11)$$

The initial flavor universality is assumed and $M_{ij}^2 = m_0^2 \mathbf{1}$, and A_0 stands for an universal trilinear scalar coupling.

In general, the off-diagonal elements by these couplings can be parametrized as

$$\Delta M_{ij}^2 \sim -\kappa m_0^2 U \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^\dagger, \quad (12)$$

where the unitary matrix U is just for the parameterization in general. The mixing angles in U depends on the source of flavor violation. Once the source of the flavor violation is specified (with some assumptions), numerical quantities in the parameterization can be related to the observed values. For example, if the source only comes from the Dirac neutrino Yukawa coupling, $Y_\nu \equiv U_L Y_\nu^{\text{diag}} U_R^T$ (in the basis where charged-lepton mass matrix and the right-handed Majorana mass matrix are diagonal), we obtain $U = U_L^*$, $\kappa = (3 + m_0^2/A_0^2)/(8\pi^2)(Y_\nu^{\text{diag}})_{33}^2 \ln M_U/M_{R_3}$. The quantities k_i are specified by the hierarchy of the eigenvalues of the Yukawa couplings (with RGE corrections). If $U_R = \mathbf{1}$ is supposed, the unitary matrix U_L^* corresponds to the neutrino mixing matrix (up to renormalization group evolution). In general, the unitary matrix U_L is different from the neutrino mixing matrix. However, it is often assumed that the large neutrino mixing angles originates from U_L . In the SO(10) model, the angles in U in the parameterization is different from the neutrino mixing angles, and those are computed from the expression of Y_ν . In the triplet part dominant type II seesaw (i.e. type I seesaw part is negligible), the unitary matrix U corresponds to the neutrino mixing matrix if the f_ν coupling is the dominant source of the flavor violation.

The most stringent constraint is provided by the experimental upper limit on the branching ratio of $\mu \rightarrow e\gamma$ [26]:

$$\text{Br}(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}. \quad (13)$$

To suppress the $\mu \rightarrow e\gamma$ amplitude in SUSY models, the 12 and 13 elements of the slepton mass matrices, given by the equation below, have to be small:

$$M_{12}^2 = -\kappa m_0^2 \left(\frac{1}{2} k_2 \sin 2\theta_{12} \cos \theta_{23} + e^{i\delta} \sin \theta_{13} \sin \theta_{23} \right) e^{i(\beta-\alpha)}, \quad (14)$$

$$M_{13}^2 = -\kappa m_0^2 \left(\frac{1}{2} k_2 \sin 2\theta_{12} \sin \theta_{23} - e^{i\delta} \sin \theta_{13} \cos \theta_{23} \right) e^{i\beta}, \quad (15)$$

where θ_{ij} denotes mixing angles in the matrix U and α, β, δ are phases in U .

The left-handed $\mu \rightarrow e\gamma$ amplitude can be cancelled by a choice of k_2 and mixing angles. However, such cancellation cannot happen for both $\mu \rightarrow e\gamma$ and μ - e conversion simultaneously. As a consequence, the suppression of $\mu \rightarrow e\gamma$ and μ - e conversion requires (at least) one of the followings:

1. small κ (which means that the relevant Yukawa couplings are small),

2. both 12 and 13 mixings in U are small,
3. $k_i \ll 1$ (which means that the eigenvalues of Yukawa matrix is hierarchical) and 13 mixing is small.

If κ is small, $\tau \rightarrow \mu(e)\gamma$ is also small. In the other cases ((2) and (3) above), $\tau \rightarrow \mu\gamma$ can be sizable, staying just below the current experimental bound while satisfying the bounds from $\mu \rightarrow e\gamma$ and μ - e conversion if 23 mixing angle in U of the source is large. It is therefore important to sort out the models to see if $\tau \rightarrow \mu(e)\gamma$ can be observed just below the current bounds, with a potential for $\mu \rightarrow e\gamma$ to be observed near future. We will study the patterns of LFV in our SO(10) model using type I and type II seesaw models once we fit the neutrino masses and suppress proton decay.

5 Predictions for flavor structures in type II seesaw

We discuss the suitable choice of flavor structure in lepton sector, and how to accommodate two large neutrino mixing angles(for atmospheric and solar neutrino oscillations) and one (relatively) small 13 neutrino mixing angle naturally in type II seesaw in SO(10) GUT [23].

Let us first describe the flavor structure in a general case. We start from a basis where neutrino mass matrix is diagonal. In this basis, the neutrino mixing matrix is equal to the diagonalization matrix of charged lepton mass matrix M_e .

$$U_{\text{PMNS}}^* M_e M_e^\dagger U_{\text{PMNS}}^T = (M_e^{\text{diag}})^2. \quad (16)$$

Since the muon mass is much smaller than the tau lepton mass, one can decompose M_e as

$$M_e = M_e^0 + \delta M_e, \quad (17)$$

where M_e^0 is a rank 1 matrix and generate tau mass, and elements of δM_e is smaller than M_e^0 . The rank 1 matrix is written in general as

$$M_e^0 = \begin{pmatrix} c \\ b \\ a \end{pmatrix} \begin{pmatrix} c & b & a \end{pmatrix}. \quad (18)$$

In general we can make the rank 1 matrix symmetric by rotating the right-handed lepton fields, and therefore, we write it in the symmetric form. In that basis, δM_e is not necessarily a symmetric matrix. The rank 1 matrix can be diagonalized by two angles (or one can say one of the three angles is unphysical if $\delta M_e = 0$). A unitary matrix to diagonalize the rank 1 matrix

can be written as

$$U_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_a & -\sin \theta_a \\ 0 & \sin \theta_a & \cos \theta_a \end{pmatrix} \begin{pmatrix} \cos \theta_s & -\sin \theta_s & 0 \\ \sin \theta_s & \cos \theta_s & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (19)$$

$$= \begin{pmatrix} \cos \theta_s & -\sin \theta_s & 0 \\ \cos \theta_a \sin \theta_s & \cos \theta_a \cos \theta_s & -\sin \theta_a \\ \sin \theta_a \sin \theta_s & \sin \theta_a \cos \theta_s & \cos \theta_a \end{pmatrix}, \quad (20)$$

where

$$\tan \theta_s = \frac{c}{b}, \quad \tan \theta_a = \frac{\sqrt{b^2 + c^2}}{a}. \quad (21)$$

We define a basis rotated by U_0 . (We attach “hat” to distinguish from the original one.)

$$\hat{M}_e = \hat{M}_e^0 + \delta \hat{M}_e, \quad (22)$$

where $\hat{M}_e^0 = \text{diag}(0, 0, m_3)$ ($m_3 = a^2 + b^2 + c^2$), and $\delta \hat{M}_e = U_0(\delta M_e)U_0^T$. Define the diagonalization matrix of \hat{M}_e as V_e . Then the mixing unitary matrix is written as

$$U_{\text{PMNS}} = V_e U_0. \quad (23)$$

The 13 element of U_{PMNS} is

$$U_{e3} = -(V_e)_{12} \sin \theta_a + (V_e)_{13} \cos \theta_a, \quad (24)$$

From our assumption $m_3 \gg (\delta M_e)_{ij}$, we expect $(V_e)_{13}$ and $(V_e)_{23}$ to be small. The angle θ_a is almost same as the 23 mixing (for atmospheric neutrino oscillations) if $(V_e)_{23}$ is tiny. The solar angle is modified from θ_s by $(V_e)_{12}$. Naively, if $(V_e)_{13}$ can be negligible, the 13 mixing is $(V_e)_{12}/\sin \theta_{\text{atm}}$. At this stage, this is just a parametrization of 13 mixing. But it is very useful to work using the unification picture. Interestingly, the experimental measurements of the 13 mixing are consistent with $(V_e)_{12} = V_{us}$, which is the Cabibbo angle in the quark sector.

We note that U_{e3} is exactly equal to zero in the limit where only 33 element of δM_e is non-zero (in this limit, the rank of M_e is 2, and electron is massless). In general, θ_a and θ_s are large. If $(\delta M_e)_{33}$ dominantly generate muon mass, the 13 mixing angle is naturally small compared to the others, and the naive size of the 13 mixing is expected to be electron/muon mass ratio.

Generically, there is no reason why $(\delta M_e)_{33}$ is dominant compared to the other elements and $(V_e)_{12}$ is small (in the “hat” basis). In the triplet-dominant type II seesaw, however, such situation is quite natural because the correction of the charged lepton mass matrix and the neutrino mass matrix can be unified (up to factor) to the $\overline{\mathbf{126}}$ Higgs coupling f . Besides, the $\overline{\mathbf{126}}$ coupling is good to generate the strange quark and muon masses for the Clebsh-Gordon

coefficient. We emphasize that the situation is consistent with the suppression of nucleon decay amplitudes by a flavor structure. As we have explained, we are studying the situation where the fermion masses are dominantly given by rank 1 matrix (h), which gives third generation charged-fermion masses, and the f and h' coupling matrices give the first and second generation masses, and fermion mixings. Under this assumption, qualitative structure of the fermion masses and mixings can be easily reproduced in the case of triplet-part dominant type II seesaw. In addition to the natural realization of two large and one small neutrino mixings, the quark mixings are small under this assumption due to left-right symmetry in SO(10).

Let us illustrate the feature of the flavor structure in triplet-dominant type II seesaw. We choose a basis where f matrix is diagonal, and parametrize

$$h = h_{33} \begin{pmatrix} c^2 & bc & ac \\ bc & b^2 & ab \\ ac & ab & a^2 \end{pmatrix}, \quad f = f_{33} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (25)$$

In this basis, the neutrino mass matrix is (nearly) diagonal neglecting the type I seesaw term (which is assumed to be suppressed by heavy right-handed neutrinos). The neutrino mixing matrix is the diagonalization matrix of Y_e . For simplicity, the h' contribution is neglected in this illustration. Then, in the limits of $\lambda_1, \lambda_2 \rightarrow 0$ and $f_{33} \ll h_{33}$, one obtains

$$\tan^2 \theta_{\text{atm}} = \frac{b^2 + c^2}{a^2}, \quad \tan \theta_{\text{sol}} = \frac{c}{b}, \quad \sin \theta_{13} = 0. \quad (26)$$

The limit $\lambda_1, \lambda_2 \rightarrow 0$ corresponds to the massless electron limit and two zero eigenvalues of neutrinos. Therefore, it is naturally realized that two solar and atmospheric mixings are large, and small $\sin \theta_{13}$. The size of the 13 mixing is expected to arise from the electron/muon mass ratio, as well as the ratio of mass squared differences, $\Delta m_{12}^2/\Delta m_{23}^2$. In order to fit the electron mass without fine-tuning, the h'_{12} component is useful. If the fine-tuning to fit the electron mass is absent², the 13 neutrino mixing can be directly related to the electron/muon mass ratio, and $\Delta m_{12}^2/\Delta m_{23}^2$.

The first possibility is that the 13 neutrino mixing only depends on electron/muon mass ratio. This can be constructed to make the $\overline{\mathbf{126}}$ coupling matrix tri-bimaximal form by using a discrete flavor symmetry [23]. In this case, the $\Delta m_{12}^2/\Delta m_{23}^2$ dependence can be dropped, and the predicted 13 mixing is

$$\sin \theta_{13} \sim \sqrt{\frac{m_e}{m_\mu}} \sin \theta_{\text{atm}} \sim \frac{1}{3} V_{us} \sin \theta_{\text{atm}} \simeq 0.05. \quad (27)$$

² If one allows a cancellation to fit the electron mass, the prediction is lost, and the 13 mixing can have O(1) factor ambiguity. However, if one fit all the fermion masses and mixings without $\mathbf{120}$ Higgs coupling (only $\mathbf{10}$ and $\overline{\mathbf{126}}$ couple to fermions [10, 27]), there is predictivity of 13 mixing due to the reduction of parameters [11]. In this case, the electron mass is obtained by a cancellation. The prediction of the 13 mixing is related to the Cabibbo mixing angle (roughly $V_{us}/\sqrt{2}$), and the prediction can agree with the recent measured value.

This does not match the measured 13 mixing angle. We note that the tri-bimaximal form of $\overline{\mathbf{126}}$ is not preferable for the nucleon decay suppression, and we discard this possibility even without using the θ_{13} mixing.

The second possibility is that the 13 mixing angle is related to both the electron/muon mass ratio and $\Delta m_{12}^2/\Delta m_{23}^2$. The $\Delta m_{12}^2/\Delta m_{23}^2$ dependence of the 13 mixing angle is related to the f coupling structure, which is related to the proton decay suppression, which we will see later. Indeed, in the rank 1 structure, the size of f_{11} and f_{13} are important for both proton decay and LFV. The small f_{11} component is favorable to suppress proton decay amplitude, and it allows prediction of the 13 neutrino mixing angle.

Let us study how the 13 neutrino mixing angle is numerically restricted in the rank 1 h Yukawa structure:

$$h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_3 \end{pmatrix}, \quad f = \begin{pmatrix} u & 0 & x \\ 0 & y & z \\ x & z & w \end{pmatrix}, \quad h' = \begin{pmatrix} 0 & c_1 & -c_2 \\ -c_1 & 0 & c_3 \\ c_2 & -c_3 & 0 \end{pmatrix}. \quad (28)$$

Note that we can parametrize $f_{12} = f_{21} = 0$ without loss of generality, by diagonalizing 1st-2nd block of the matrix f . As we have noted, $u \rightarrow 0$ is preferable to suppress proton decay amplitude and to obtain the empirical relation $V_{us} \simeq \sqrt{m_d/m_s}$. Roughly speaking, the 3rd generation masses are given by h_3 , and the 2nd generation masses are given by f_{22} with r_2 and Clebsch-Gordon coefficient. The h'_{12} elements gives the first generation masses as well as the Cabibbo mixing angle. The absence of f_{11} provides natural understanding of the so-called Georgi-Jarskog relation (without a cancellation): $m_e m_\mu m_\tau \simeq m_d m_s m_b$ by a choice of $|c_e| \simeq 1$. Other parameters can be chosen to be consistent with the other quark mixings and atmospheric and solar mixing angles.

From $f_{12} = 0$, we obtain a simple relation among the eigenvalues of f and mixing angles:

$$\frac{f_1}{f_3} = u \sec^2 \theta_{13}^\nu - e^{-2i\delta} \tan^2 \theta_{13}^\nu + e^{-i\delta} (1 - u e^{2i\delta}) \sec \theta_{13}^\nu \tan \theta_{13}^\nu \tan \theta_{12}^\nu \tan \theta_{23}^\nu, \quad (29)$$

$$\frac{f_2}{f_3} = u \sec^2 \theta_{13}^\nu - e^{-2i\delta} \tan^2 \theta_{13}^\nu - e^{-i\delta} (1 - u e^{2i\delta}) \sec \theta_{13}^\nu \tan \theta_{13}^\nu \cot \theta_{12}^\nu \tan \theta_{23}^\nu, \quad (30)$$

where

$$f = U_\nu^T \begin{pmatrix} f_1 & & \\ & f_2 & \\ & & f_3 \end{pmatrix} U_\nu, \quad (31)$$

$$U_\nu = \begin{pmatrix} \cos \theta_{12}^\nu & -\sin \theta_{12}^\nu & 0 \\ \sin \theta_{12}^\nu & \cos \theta_{12}^\nu & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta_{13}^\nu & 0 & -e^{i\delta} \sin \theta_{13}^\nu \\ 0 & 1 & 0 \\ e^{-i\delta} \sin \theta_{13}^\nu & 0 & \cos \theta_{13}^\nu \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23}^\nu & -\sin \theta_{23}^\nu \\ 0 & \sin \theta_{23}^\nu & \cos \theta_{23}^\nu \end{pmatrix}. \quad (32)$$

In triplet-part dominant type II seesaw (and in normal hierarchy), we have

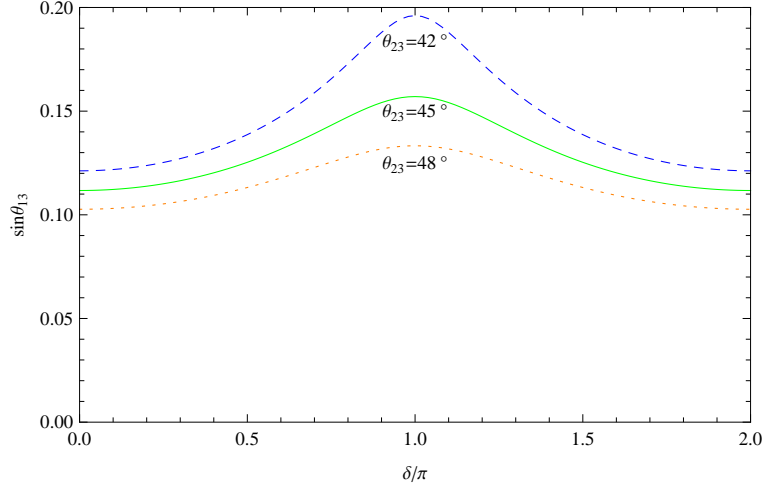


Figure 1: Magnitude of $\sin \theta_{13}$ depending on δ/π (and also θ_{23} , each choice of which gives one line), which is restricted in a narrow band.

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} = \frac{|f_2|^2 - |f_1|^2}{|f_3|^2 - |f_2|^2}. \quad (33)$$

In the limit $u \rightarrow 0$, we obtain

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} = \frac{4 \sin^2 \theta_{13}^\nu \csc 2\theta_{12}^\nu \tan \theta_{23}^\nu (\sin \theta_{13}^\nu \cos \delta + \cot 2\theta_{12}^\nu \tan \theta_{23}^\nu)}{1 - \sin^2 \theta_{13}^\nu (2 + \cot^2 \theta_{12}^\nu \tan^2 \theta_{23}^\nu) - 2 \sin^3 \theta_{13}^\nu \cos \delta \cot \theta_{12}^\nu \tan \theta_{23}^\nu}. \quad (34)$$

As a result, θ_{13} is bounded from below by $\Delta m_{12}^2/\Delta m_{23}^2$. Even if $u \neq 0$, θ_{13} has to be bounded because we obtain $f_1/f_3 \rightarrow u$, $f_2/f_3 \rightarrow u$ in the limit of $\theta_{13} \rightarrow 0$. In figure 1, we plot the 13 mixing angle as a function of the CP phase δ . Since the 13 mixing angle is sensitive to the deviation from the 45 degree of 23 mixing, we vary the 23 mixing angles. It is not sensitive to the 12 mixing angle. It is interesting to note that $\delta \sim \pi$ is preferable if the 13 mixing is larger value (depending on θ_{23}).

The 13 neutrino mixing angle can be modified by the diagonalization matrix of charged lepton masses, but the size correction is less than 0.05 radians³. In conclusion, tiny $\theta_{13} (< 0.05)$ is not allowed for the rank-1 structure with type II seesaw, irrespective of the detail of fitting of fermion masses and mixings. This is in contrast with the (nearly) tri-bimaximal model, where $\theta_{13} \simeq 1/3 V_{us} \sin \theta_{23} \sim 0.05$.

The suppression is 11,12 elements of f coupling reduces the ff contribution of the nucleon decay amplitudes drastically. Because the f matrix is generated the large neutrino mixings and the neutrino masses directly in the triplet dominant type II seesaw, the 13 element of f

³ The correction depends on relative phase freedom. The maximal correction (~ 0.05 radians) occurs when the relative phase is 0 or π .

cannot be zero. Therefore, it can still contribute to the nucleon decay amplitudes and we need a cancellation which can be achieved by choosing the colored Higgs mixings to suppress them.

As we have noted, the LFV can be also generated from the f coupling if the doublet Higgs mixing angle is small and the original \bar{f} coupling in the superpotential is large since (at least) $SU(2)_L$ triplet is lighter than the GUT scale. In that case, however, $\mu \rightarrow e\gamma$ is generated and the doublet Higgs mixing at GUT scale is bounded. If the doublet Higgs mixing is $O(1)$, and $\bar{f} \sim f$, the size of $\text{Br}(\mu \rightarrow e\gamma)$ is the same order of the current experimental bound. Since the size of 13 element is predicted in the type II seesaw model, the $\text{Br}(\tau \rightarrow \mu\gamma)$ can be predicted, and it is about $O(10^{-10})$, which is below the sensitivity at the LHCb. In the type II seesaw neutrino, it is not possible to enhance $\text{Br}(\tau \rightarrow \mu\gamma)$ to be of the order of the current experimental bound while satisfying the bound of $\mu \rightarrow e\gamma$ and μ - e conversion.

6 Type I seesaw

In the case of triplet-part dominant type II seesaw, the simple formula of the mixing angles are obtained, and the two large and one small neutrino mixing angles can be easily realized. On the other hand, for the type I seesaw case, the situation is more complicated since the seesaw formula includes an inverse of the f matrix. In this section, we investigate how the fermion masses and mixings are reproduced in type I seesaw with suppressed nucleon decays.

Because the Dirac neutrino Yukawa coupling is given as

$$Y_\nu = h - 3r_2 f + c_\nu h', \quad (35)$$

the type I seesaw neutrino mass matrix is proportional to

$$Y_\nu f^{-1} Y_\nu^T = (h + c_\nu h') f^{-1} (h - c_\nu h') - 6r_2 h + 9r_2^2 f. \quad (36)$$

Because the up-type quarks are more hierarchical than down-type quarks, r_2 is small in the current scheme, and the last two terms can be negligible. We, therefore, concentrate on the first term $N \equiv (h + c_\nu h') f^{-1} (h - c_\nu h')$.

We denote h and h' matrices as

$$h = \begin{pmatrix} h_1 & & \\ & h_2 & \\ & & h_3 \end{pmatrix}, \quad h' = \begin{pmatrix} 0 & c_1 & -c_2 \\ -c_1 & 0 & c_3 \\ c_2 & -c_3 & 0 \end{pmatrix}. \quad (37)$$

We study whether a hierarchical f matrix required by charged fermion fits and also proton decay suppression is consistent with the profile of observed neutrino mixings. For this purpose, we first obtain the f coupling by solving $N = (h + c_\nu h') f^{-1} (h - c_\nu h')$. Denoting

$$N = U \text{diag.}(n_1, n_2, n_3) U^T, \quad (38)$$

we obtain

$$f = (h - c_\nu h') U^* \text{diag.} \left(\frac{1}{n_1}, \frac{1}{n_2}, \frac{1}{n_3} \right) U^\dagger (h + c_\nu h'). \quad (39)$$

Defining \mathbf{x}_i as

$$\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{pmatrix} = U^\dagger (h + c_\nu h'), \quad (40)$$

we obtain

$$f = \frac{1}{n_1} \mathbf{x}_1^T \mathbf{x}_1 + \frac{1}{n_2} \mathbf{x}_2^T \mathbf{x}_2 + \frac{1}{n_3} \mathbf{x}_3^T \mathbf{x}_3. \quad (41)$$

The neutrino mass matrix \mathcal{M}_ν is expressed as

$$\mathcal{M}_\nu = -N \frac{v_u^2}{v_R}, \quad (42)$$

where v_u is a VEV of up-type Higgs field, and v_R is a VEV of **126** Higgs field which breaks SO(10) down to SU(5). (More precisely, this f coupling is original \bar{f} coupling as a convention). As we have noted, we are neglecting terms from $9r_2^2 f - 6r_2 h$, because each component of N is O(100) as we will see. The unitary matrix U is the neutrino mixing matrix (up to the diagonalization matrix of charged lepton mass matrix).

One can express f matrix using a general form of U . To capture the essence of the discussion, we first use a tri-bimaximal form for U , and later on add the corrections due to non-zero θ_{13} . Using the tri-bimaximal form

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (43)$$

we obtain

$$\mathbf{x}_1 = \frac{1}{\sqrt{6}} (c_\nu (c_1 - c_2) + 2h_1, c_\nu (2c_1 + c_3) - h_2, -c_\nu (2c_2 + c_3) - h_3), \quad (44)$$

$$\mathbf{x}_2 = \frac{1}{\sqrt{3}} (c_\nu (-c_1 + c_2) + h_1, c_\nu (c_1 - c_3) + h_2, c_\nu (-c_2 + c_3) + h_3), \quad (45)$$

$$\mathbf{x}_3 = \frac{1}{\sqrt{2}} (c_\nu (c_1 + c_2), -c_\nu c_3 + h_2, -c_\nu c_3 + h_3). \quad (46)$$

Our purpose is to find a solution where h_1, h_2, f_{1i} are suppressed. The solution can be found by assuming

$$c_1 - c_2 = 0 \quad (n_1, n_2 \ll n_3, \text{ normal hierarchy}), \quad (47)$$

$$c_1 + c_2 = 0 \quad (n_1, n_2 \gg n_3, \text{ inverted hierarchy}). \quad (48)$$

In the case of normal hierarchy, we need $v_R/n_3 \sim 10^{14}$ GeV, and therefore, we expect $n_3 \approx 100 - 1000$. Assuming $n_1 \ll n_2, n_3$, for the case of normal hierarchy, we have $n_2/n_3 \simeq \sqrt{\Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2} \sim 0.15$. By a choice of $c_1 = c_2$, the f matrix is obtained (assuming $h_1, h_2 \ll c_1, c_2 \ll c_3 \ll h_3$ to express in short) as

$$f \sim \frac{1}{6n_1} \begin{pmatrix} 4c_\nu^2 c_1^2 \frac{6n_1}{2n_3} & -2c_\nu^2 c_1 c_3 \frac{6n_1}{2n_3} & 2c_\nu c_1 h_3 \frac{6n_1}{2n_3} \\ -2c_\nu^2 c_1 c_3 \frac{6n_1}{2n_3} & c_\nu^2 c_3^2 & c_\nu c_3 h_3 \\ 2c_\nu c_1 h_3 \frac{6n_1}{2n_3} & c_\nu c_3 h_3 & h_3^2 \end{pmatrix}. \quad (49)$$

We emphasize that the 11,12,13 elements of f are suppressed by n_1/n_3 , which is precisely what is needed to suppress nucleon decay amplitudes.

In the case of inverted hierarchy, we need $|m_1| \simeq |m_2|$, and $|m_1| - |m_2| = \Delta m_{\text{solar}}^2/(2m_1)$. If $m_3 \ll m_1$, $m_1 \simeq \sqrt{\Delta m_{\text{atm}}^2}$. So, $n_1 \approx 100 - 1000$ in this case. Assuming $n_3 \ll n_1, n_2$, we can obtain that 11,12,13 elements of f are suppressed by n_3/n_1 similar to the normal hierarchy case. For example, the 13 element of f for the choice of $c_2 = -c_1$ and $h_1 = 0$ is

$$f_{13} = \frac{2c_\nu c_1 (-h_3 - c_\nu c_3 + 2c_\nu c_2)}{6n_1} + \frac{-2c_\nu c_1 (h_3 + c_\nu c_1 + c_\nu c_3)}{3n_2}, \quad (50)$$

which is equal to $-c_\nu c_1 h_3/n_1$ (for $n_1 \simeq n_2$), $c_\nu c_1 h_3/3n_1$ (for $n_1 \simeq -n_2$), for example.

In both cases, the f_{1i} elements can be suppressed naturally, and the suppression of f_{1i} is related to the hierarchy between the VEV of **126** and $v_u^2/m_\nu \sim 10^{14}$ GeV.

We note on the effect of modification from the tri-bimaximal case : ($U_{13} \neq 0$). In the inverted hierarchy case, the condition is just changed to $c_2 \cos \theta_{23} + c_1 \sin \theta_{23} = 0$ (as long as $h_1 = 0$). In the normal hierarchy case, we require $(\mathbf{x}_1)_1 = 0$ to make $f_{1i} \rightarrow 0$. In that case, $(\mathbf{x}_3)_1 \sim c_1$ similarly to the tri-bimaximal case. In the tri-bimaximal case, $(\mathbf{x}_2)_1 = 0$ is satisfied (for $h_1 = 0$). In the case of $\theta_{13} \neq 0$, $(\mathbf{x}_2)_1 \simeq c_1 \sin \theta_{13}/(\sin \theta_{12} \sin \theta_{23})$. Therefore, the f_{1i} elements are suppressed by $U_{13}^2 n_2/n_3$ and n_1/n_3 . Because of the approximate relation $n_2/n_3 \simeq \sqrt{\Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2} \sim \theta_{13}$, the size of the f_{1i} elements is not far different from the case of $\theta_{13} = 0$.

We solved the f matrix which can reproduce the mixing angles in U by Eq.(39). As we have seen, for the solution with suppressed f_{1i} , c_1 and c_2 has to be related ($|c_1| = |c_2|$ for the tri-bimaximal mixing). If we put the mixing angles in U , the condition is given as $\tan \theta_{23} \simeq |c_1/c_2|$. Inversely speaking, if we start from a f matrix with suppressed f_{1i} to suppress nucleon decay naturally, the large atmospheric mixing angle implies $|c_1/c_2| \simeq 1$. Interestingly, it naively implies a ‘‘post-diction’’ for quark mixing : $V_{ub} \sim V_{us} m_s/m_b$.

We note on the fitting of the charged fermion mass and mixing. The size of $c_\nu c_3$ has to be $O(0.1)$ as long as the muon mass comes from f_{22} . To obtain the proper value of V_{cb} , we need a cancellation between f and h' . It can also modify the atmospheric mixing from

the diagonalization of charged-lepton unless $c_e \sim +1$ (For example, at the SU(5)-like vacua, $c_e \sim -1$).

Apart from the detail fit, we have showed that the suppression of f_{1i} and $h_{1,2}$ is possible to reproduce the neutrino mixings in type I seesaw. Although the detail of the charged fermion masses and mixings may depend on the threshold corrections (both GUT scale and weak scale) or any other possible higher order effects, the suppression of f_{1i} can generate an interesting feature of the type I seesaw. The f_{1i} elements is suppressed by n_1/n_3 , as we have described. In this case, the f coupling can be a source of $\tau \rightarrow \mu\gamma$ without enhancing $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ if one of the SM decomposed representations in the **126** Higgs fields is lighter than the unification scale [18]. The feature of this solution is obtained because the f_{1i} components can be tiny to realize the neutrino masses, which is suitable to suppress proton decay amplitude. This is the main difference between type I and type II seesaw. In the case of triplet-part-dominant type II seesaw, it is impossible to enhance $\tau \rightarrow \mu\gamma$ around its current experimental bound after satisfying the bounds of $\mu \rightarrow e\gamma$ and μ - e conversion because a size of f_{13} element is needed to generate the proper neutrino oscillation data. In both cases, $\mu \rightarrow e\gamma$ can be generated just below the current experimental bound via the Dirac neutrino Yukawa coupling, or left-handed Majorana neutrino couplings. If $\tau \rightarrow \mu\gamma$ is discovered soon, the structure in type I seesaw with suppressed nucleon decay is preferred.

7 Predictions of $\mu \rightarrow e\gamma$ decay

In this section, we show the predictions of the branching ratio of $\mu \rightarrow e\gamma$ decay in the SO(10) model with proton decay suppression.

As is expressed in Eq.(12), the size of the off-diagonal elements is specified by κ , and the numerical quantity of κ is specified by the size of the coupling matrices and the SUSY breaking parameters. For example, if the source is the Dirac neutrino Yukawa coupling, the numerical quantity is roughly estimated as

$$\kappa m_0^2 \simeq \frac{1}{8\pi^2} (Y_{\nu 3}^{\text{diag}})^2 (3m_0^2 + A_0^2) \ln \frac{M_*}{M_{R3}}, \quad (51)$$

where $Y_{\nu 3}^{\text{diag}}$ is the 3rd eigenvalue of the Dirac Yukawa coupling matrix, M_* is a cutoff scale, M_{R3} is the 3rd right-hand Majorana mass, m_0 is a SUSY breaking universal scalar mass, and A_0 is a universal scalar trilinear coupling.

In Fig.2, we plot the branching ratio of $\mu \rightarrow e\gamma$ decay for various scenarios. We choose the unified gaugino mass to be $M_{1/2} = 700$ GeV (corresponding to the experimental lower limit of $m_{\tilde{g}} \simeq 1.7$ TeV for gluino mass) to satisfy the recent LHC experimental bounds, and $\tan \beta = 50$.

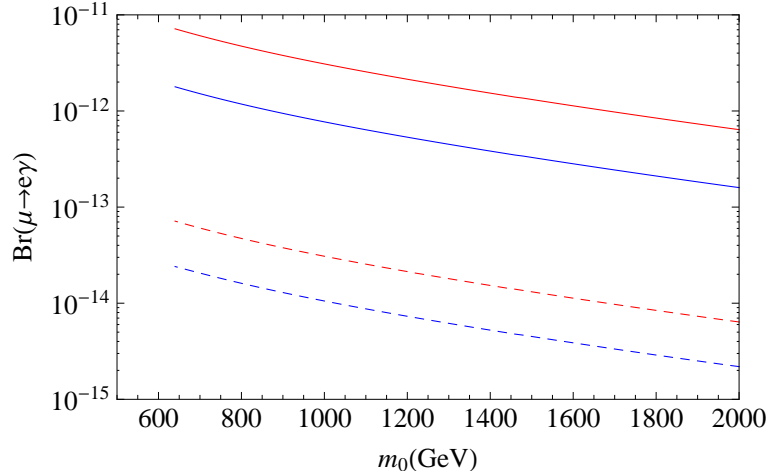


Figure 2: Branching ratio of $\mu \rightarrow e\gamma$ decay vs. the universal scalar mass m_0 . We choose $M_{1/2} = 700$ GeV, and $\tan\beta = 50$. The universal trilinear scalar coupling is chosen to satisfy $m_h = 125$ GeV. The current experimental bound is $\text{Br}(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}$ [26]. It is expected that the decay can be observed if the branching ratio is larger than 10^{-13} at the MEG experiment. The detail of the plots are given in the text.

In the plot, the universal trilinear coupling A_0 is chosen to be a value to make the lightest Higgs mass to be $m_h = 125$ GeV [28]. For example, for the universal scalar mass $m_0 = 1000$ GeV, we obtain $A_0 = -2000$ GeV (In our sign convention, the positive value of $M_{1/2}$ takes the trilinear scalar couplings to the negative direction by RGEs. Namely, for a boundary condition $A_0 = 0$, the scalar trilinear couplings are negative at the weak scale in the convention). We note that the flavor violation is enhanced because a large magnitude of A_0 is needed to obtain the Higgs mass. For larger m_0 , the left-right stop mixing becomes smaller (for fixed A_0). Therefore, in order to obtain $m_h = 125$ GeV for a larger value m_0 , a larger magnitude of A_0 is needed, and the flavor violation is more enhanced. As a result, the behavior of the plots are different from the simple dependence of the slepton masses. Due to a large magnitude of A_0 and a large left-right mixing, one of the stau masses becomes tachyonic for a small value of m_0 . As a result, m_0 has to be larger than 650 GeV for the plot. For the given boundary conditions, squark masses are heavier than 1.5 TeV.

We choose the following four scenarios to plot:

1. (Red lines)

The flavor violating source is the neutrino Dirac Yukawa coupling, and the unitary matrix U in Eq.(12) is the same as the neutrino mixing matrix. Namely, the Dirac neutrino mass matrix can be written as

$$Y_\nu = U Y_\nu^{\text{diag}}, \quad (52)$$

in the basis where the right-handed neutrino mass matrix is diagonal. We choose $M_{R3} = 10^{13}$ GeV (red solid line) and 3×10^{12} GeV (red dashed line).

2. (Blue solid line)

The flavor violating source is the Majorana coupling of the left-handed lepton doublets and $SU(2)_L$ triplets for the type II seesaw, and the triplet part provides the dominant contribution of the light neutrino masses (Then, the unitary matrix U is same as the neutrino mixing matrix). As we have explained, the coupling is the $\overline{\mathbf{126}}$ Higgs coupling in the $SO(10)$ model, and the second generation fermion masses are generated by this coupling in the current setup for the proton decay suppression. There is a free parameter for the $\mathbf{10}$ and $\overline{\mathbf{126}}$ Higgs mixing. We choose $\bar{f}_3^{\text{diag}} = 0.1$ for the original $\overline{\mathbf{126}}$ Higgs coupling without multiplying the Higgs mixing. The numerical quantity κ is proportional to \bar{f}_3^{diag} , and the branching ratio is roughly proportional to κ^2 . Therefore, for $\bar{f}_3^{\text{diag}} < 0.05$, the branching ratio is smaller than 10^{-13} . Because f_{33} provides the second generation fermions mass, and f_{23} generates the quark mixing V_{cb} , one can expect that the branching ratio can be more than 10^{-13} in this scenario.

3. (Blue dashed line)

The flavor violating source is the Dirac neutrino Yukawa coupling, Eq.(6), in the $SO(10)$ models. Contrary to the $SU(5)$ models, the Dirac neutrino Yukawa coupling is restricted in the current scenarios of the $SO(10)$ models. There is one free parameter c_ν in the Dirac neutrino Yukawa coupling. We choose $c_\nu = 2$ (which is the value of the $SU(5)$ -like vacuum, which is preferable since one obtains $c_e = -1$ and the Georgi-Jarskog relation can be naturally obtained). In order to observe the $\mu \rightarrow e\gamma$ decay at the MEG experiment [26], one needs a larger value of c_ν . The branching ratio is roughly proportional to c_ν^2 .

In addition to the above sources, there can be a flavor violating source in the $e^c u^c H_C$ coupling, and the colored Higgs loop can induce the off-diagonal elements in the right-handed charged slepton mass matrix, and can generate LFV via neutralino loop diagram. The contribution is calculated to be less than 10^{-15} for the branching ratio using the boundary condition of the SUSY particle spectrum.

In the above calculations, we assume the gaugino mass unification and the universality of the scalar masses. However, if there is a SUSY breaking contribution from the anomaly mediation, the gaugino mass unification can be relaxed even in the GUT modes. In that case, the gluino mass bounds from the LHC experiments does not necessarily restrict the Wino and Bino masses, and the $\text{Br}(\mu \rightarrow e\gamma)$ can be enhanced.

In summary, in the SU(5) GUTs with type I seesaw, the branching ratio of the $\mu \rightarrow e\gamma$ decay can be as large as the current experimental bounds, and its magnitude depends on the right-handed neutrino Majorana mass. In the type II seesaw SO(10) model with proton decay suppression, it is expected that the $\mu \rightarrow e\gamma$ is observed at the MEG experiments. In the type I seesaw SO(10), on the other hand, the branching ratio is smaller than 10^{-13} (for a natural size of c_ν) and $\mu \rightarrow e\gamma$ decay may be difficult to be observed.

As is explained, the $\tau \rightarrow \mu\gamma$ decay width cannot be large enough to be observed near future in type II seesaw SO(10) (and type I seesaw SU(5)) models, satisfying the experimental bounds of $\mu \rightarrow e\gamma$ and μ - e conversion. In type I seesaw SO(10) with proton decay suppression, the $\tau \rightarrow \mu\gamma$ decay width can become large by the $\overline{126}$ Higgs coupling without enhancing $\mu \rightarrow e\gamma$. The $\tau \rightarrow e\gamma$ decay is not enhanced in both type I and II seesaw scenarios. These features are important to distinguish the GUT models and the vacua of the GUT symmetry breaking.

8 Partial lifetime of nucleon in type I and type II

The hierarchical structure of the Yukawa coupling matrix is needed to suppress proton decay. As we have studied, natural suppression of the proton decay amplitude is possible in the type I seesaw relating to the hierarchy between the GUT scale and the typical seesaw scale $\sim 10^{14}$ GeV. In the type I seesaw, the structure is really simple and it allows predictions for the partial decay amplitude. which we investigate in this section.

We denote the coupling matrices in the basis where h is diagonal:

$$h = \begin{pmatrix} h_1 & & \\ & h_2 & \\ & & h_3 \end{pmatrix}, \quad f = \begin{pmatrix} u & v & x \\ v & y & z \\ x & z & w \end{pmatrix}, \quad h' = \begin{pmatrix} 0 & c_1 & -c_2 \\ -c_1 & 0 & c_3 \\ c_2 & -c_3 & 0 \end{pmatrix}. \quad (53)$$

As we have explained, we assume that the h coupling is rank 1 (namely, h_1 and h_2 are irrelevant to fit fermion masses). Then, we can choose $v = 0$ without loss of generality. In that basis, $u \rightarrow 0$ is important to suppress proton decay amplitude. In type II seesaw, x cannot be small to fit the large solar neutrino mixing. In type I seesaw, on the other hand, x is small and the smallness of x gives a predictivity to the decay amplitudes. In the following, we take $h_1, h_2, u, v \rightarrow 0$, but we keep x to describe the difference between type I and II seesaw.

The left-handed proton decay amplitudes (from the $LLLL$ dimension-five operator C_L) from the chargino dressing diagram can be written as

$$A_L(p \rightarrow K\bar{\nu}_\tau) \simeq -\beta_p g_2^2 (xp_1 - c_2 p_5) y \cos \theta_C \sin \theta_C X_{p \rightarrow K\bar{\nu}}, \quad (54)$$

$$A_L(p \rightarrow K\bar{\nu}_\mu) \simeq -\beta_p g_2^2 c_1 p_5 y \cos \theta_C \sin \theta_C X_{p \rightarrow K\bar{\nu}}, \quad (55)$$

where β_p is a hadron matrix element of proton, θ_C is the Cabibbo angle, p_1 and p_5 are the coefficients from the colored Higgs mixing for ff and $h'f$ contributions, respectively:

$$x_{L1} = X_{a4} \frac{1}{M_a} Y_{a5} = (M_T^{-1})_{54}, \quad (56)$$

$$x_{L5} = \sqrt{2} X_{a3} \frac{1}{M_a} Y_{a5} = \sqrt{2} (M_T^{-1})_{53}. \quad (57)$$

Here, X and Y are the diagonalization unitary matrix of the colored Higgs mass matrix M_T and we used the same notation in Ref.[17]. The factor $X_{p \rightarrow K\bar{\nu}}$ includes a loop function and chargino mixing angles θ_u, θ_v :

$$X_{p \rightarrow K\bar{\nu}} = (A_1 - A_2)(\cos^2 \theta_u H_{ue}^1 + \sin^2 \theta_u H_{ue}^2) + (A_1 + A_2)(\cos \theta_u \cos \theta_v H_{ud}^1 + \sin \theta_u \sin \theta_v H_{ud}^2), \quad (58)$$

$$A_1 = 1 + \frac{m_p}{m_{B'}} (F + \frac{1}{3}D), \quad A_2 = \frac{m_p}{m_{B'}} \frac{2}{3}D, \quad (59)$$

where $F \simeq 0.48$ and $D \simeq 0.76$ are coupling constants for interaction between the baryons and mesons, m_p is the proton mass, and $m_{B'}$ is an averaged baryon mass $m_{B'} \approx m_\Sigma \approx m_\Lambda$. The loop function is defined as

$$H_{ue}^\alpha = \frac{1}{m_{\chi_\alpha}} H \left(\frac{m_u^2}{m_{\tilde{\chi}_\alpha}^2}, \frac{m_e^2}{m_{\tilde{\chi}_\alpha}^2} \right), \quad (60)$$

where

$$H(x, y) = \frac{1}{x - y} \left(\frac{x \ln x}{x - 1} - \frac{y \ln y}{y - 1} \right), \quad (61)$$

H_{ud} is defined similarly by replacing the slepton mass into down-type squark mass, and m_{χ_α} is an eigenvalues of chargino masses. We assume that SUSY breaking squark, slepton mass matrices are proportional to identity matrix for simplicity. We neglect the subleading contribution from V_{cb} and V_{ub} . The $p \rightarrow K\bar{\nu}_e$ is suppressed by a factor $\sim \theta_C$, but can be generated due to the mixing between electron and muon in the basis where h is diagonal.

We find that $A_L(p \rightarrow K\bar{\nu}_\tau) \propto xp_1 - c_2 p_5$ and $A_L(p \rightarrow K\bar{\nu}_\mu) \propto c_1 p_5$. Therefore, if $x \rightarrow 0$ (as in type I seesaw), the left-handed amplitude of $p \rightarrow K\bar{\nu}$ can be suppressed by choosing a small p_5 . The smallness of p_5 is related to a vacuum selection of SO(10) breaking vacua. In type II, on the other hand, x cannot be small, and one needs to choose small values both p_1 and p_5 to suppress proton decay. If there are numbers of parameter in the colored Higgs mass matrices, the suppression is possible. However, it is not related to the vacuum selection.

Since in the case of $x \rightarrow 0$ in type I seesaw case the cancellation between the Yukawa couplings for different Higgs representations (ff and $h'f$ for example) is not required, the ratio of partial lifetime is predictable. In fact, the decay amplitudes of to anti-muon are obtained as

$$A_L(p \rightarrow \pi^0 \mu^+) \simeq \beta_p g_2^2 c_1 p_5 y \sin \theta_C \sin \theta_{uc} A_3 X_{p \rightarrow \mu+X}, \quad (62)$$

$$A_L(p \rightarrow K^0 \mu^+) \simeq -\beta_p g_2^2 c_1 p_5 y \cos \theta_C \sin \theta_{uc} A_4 X_{p \rightarrow \mu+X}, \quad (63)$$

where

$$X_{p \rightarrow \mu + X} = \cos \theta_u \cos \theta_v (H_{d\nu}^1 + H_{ue}^1) + \sin \theta_u \sin \theta_v (H_{d\nu}^2 + H_{ue}^2), \quad (64)$$

and

$$A_3 = \frac{1}{\sqrt{2}}(1 + F + D), \quad A_4 = 1 + \frac{m_p}{m_{B'}}(F - D). \quad (65)$$

The mixing angle θ_{uc} is an angle between u and c quarks in the basis and therefore, $\theta_{uc} \sim \sqrt{m_u/m_c}$. Therefore, we obtain the ratio of partial decay width in type I seesaw as

$$\frac{\Gamma(p \rightarrow K\mu)}{\Gamma(p \rightarrow K\bar{\nu})} \simeq \frac{|c_1|^2}{|c_1|^2 + |c_2|^2} \frac{\sin^2 \theta_{uc}}{\sin^2 \theta_C} \left| \frac{A_4 X_{p \rightarrow \mu + X}}{X_{p \rightarrow K\bar{\nu}}} \right|^2. \quad (66)$$

The maximal atmospheric mixing in the type I seesaw requires $|c_1| \simeq |c_2|$, and the ratio is predictive (up to the sfermion mass spectrum). In the case of type II seesaw, there is an additional parameter ($x - c_2 p_5/p_1$), and the ratio cannot be predicted. We comment that

$$\frac{\Gamma(p \rightarrow \pi^0 \mu^+)}{\Gamma(p \rightarrow K^0 \mu^+)} \simeq \sin^2 \theta_C \frac{A_3^2}{A_4^2} \quad (67)$$

is obtained for both type I and II since only $h'f$ contribution is dominated. For the decay to the third generation lepton (because tau lepton is heavier than proton, the decay to the ν_τ is the only case), ff piece also contributes if $f_{13} = x$ is not suppressed. Therefore, the ratio of partial decay width to neutrino and anti-lepton is not predictable in the type II, while it is predictable for type I seesaw. The same situation occurs for $n \rightarrow \pi\bar{\nu}$ decay.

In the above expressions, we neglect the contribution from the right-handed dimension-five proton decay operator C_R . They can contribute to the decays into $\bar{\nu}_\tau$, and can alter the prediction of the ratio of partial decay widths. The C_R contribution (for $h_1, h_2, u, v, x \rightarrow 0$) can be obtained as

$$A_R(p \rightarrow K\bar{\nu}_\tau) = \alpha_p y_c y_\tau (A_1 c_1 c_2 x_{R8} \cos \theta_C + A_2 (c_1 c_3 p_{R8} - c_2 y p_{R7} + c_1 z p_{R5}) \sin \theta_C) \\ \times (H_{ue}^2 \cos \theta_u \cos \theta_v + H_{ue}^1 \sin \theta_u \sin \theta_v), \quad (68)$$

where α_p is a hadron matrix element, and

$$p_{R5} = -\sqrt{2} X_{a2} \frac{1}{M_a} (Y_{a5} - \sqrt{2} Y_{a6}) = -\sqrt{2} (M_T^{-1})_{52} + 2 (M_T^{-1})_{62}, \quad (69)$$

$$p_{R7} = -\sqrt{2} X_{a4} \frac{1}{M_a} (Y_{a3} - Y_{a2}) = -\sqrt{2} ((M_T^{-1})_{34} - (M_T^{-1})_{24}), \quad (70)$$

$$p_{R8} = 2 X_{a2} \frac{1}{M_a} (Y_{a3} - Y_{a2}) = (M_T^{-1})_{32} - (M_T^{-1})_{22}. \quad (71)$$

For a large $\tan \beta \sim 50$, since c_i and y, z have to be large to fit the down-type quark Yukawa couplings, C_R contribution can make the decay width comparable to the current experimental

bounds, and the simple relation shown previously can be disturbed by the second term (A_2 contribution). If the Higgsino mass is much heavier than wino mass, the C_R contribution is suppressed rather than C_L contribution. We note that if we do not assume $h_1, h_2, u, v \rightarrow 0$, the magnitudes of C_R contribution can exceed the experimental bounds even for $\tan \beta \sim 5$ and we should not neglect the C_R contribution.

9 Conclusion

We study the Yukawa texture in a renormalizable SUSY SO(10) GUT model for neutrino masses that gives a “large” θ_{13} while at the time suppressing nucleon decay without invoking cancellation between coupling parameters. We consider cases with both type I and type II seesaw separately. In the type II seesaw scenario, we find that $\theta_{13} > 0.05$ radians irrespective of the detail of fitting of fermion masses and mixings (i.e. independent of the number of parameters) and that the measured value of 13 mixing angle prefers $\delta \sim \pi$ for the CP violating phase in the neutrino oscillations. We then study the predictions of these scenarios for lepton flavor violation. The branching ratio of $\mu \rightarrow e\gamma$ can be as large as the current experimental bound, and should be observed very soon. The $\text{Br}(\tau \rightarrow \mu\gamma)$ is predictable, and is about $O(10^{-10})$ in the type I seesaw case, if the bounds of $\mu \rightarrow e\gamma$ and μ - e conversion are satisfied.

In type I seesaw scenario, observed θ_{13} can be accommodated naturally. We find that 1-2 and 1-3 flavor violation from the FCNC source is suppressed compared to the type II scenario if the proton decay is suppressed naturally. This gives a larger value of $\text{Br}(\tau \rightarrow \mu\gamma)$ while $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ are suppressed. If the $\tau \rightarrow \mu\gamma$ decay is observed soon then definitely it will point towards a type I scenario in this kind of SO(10) model and the prediction for $\text{Br}(\mu \rightarrow e\gamma)$ can be checked for confirmation. Detailed numerical study of the model for fermion mass fits as well as lepton flavor violation is currently under way. The ratio of the partial decay width of proton to $K\mu$ and $K\bar{\nu}$ is predicted (up to uncertainties from sfermion masses) in this scenario.

The next generation of the baryon number violating nucleon decay experiments at Hyper-Kamiokande along with the information about the low energy SUSY states from the LHC may be used to distinguish between the different scenarios.

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