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Domain Walls with Localised Gravity and Domain-Wall/QFT Correspondence

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ABSTRACT

We review general domain-wall solutions supported by a delta-function source, together with a single pure exponential scalar potential in supergravity. These scalar potentials arise from a sphere reduction in M-theory or string theory. There are several examples of flat (BPS) domain walls that lead to a localisation of gravity on the brane, and for these we obtain the form of the corrections to Newtonian gravity. These solutions are lifted back on certain internal spheres to $D = 11$ and $D = 10$ as M-branes and D-branes. We find that the domain walls that can trap gravity yield M-branes or Dp -branes that have a natural decoupling limit, *i.e.* $p \leq 5$, with the delta-function source providing an ultra-violet cut-off in a dual quantum field theory. This suggests that the localisation of gravity can generally be realised within a Domain-wall/QFT correspondence, with the delta-function domain-wall source providing a cut-off from the space-time boundary for these domain-wall solutions. We also discuss the form of the one-loop corrections to the graviton propagator from the boundary QFT that would reproduce the corrections to the Newtonian gravity on the domain wall.

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1 Introduction

The most intriguing feature of the Randall-Sundrum II scenario [1] is the observation that a five-dimensional theory with a non-compact fifth direction can nevertheless describe four-dimensional gravity on a domain wall. In the original formulation [1], the configuration is a flat infinitely thin Z_2 symmetric domain wall with with an asymptotically AdS space-time.¹ Such a domain wall can be viewed as a solution of five-dimensional (AdS₅) supergravity in the bulk on either side of the 3-brane (tending to the AdS₅ horizon far from the brane), but it requires a delta-function source on the 3-brane. The origin of this singular source lies outside five-dimensional supergravity, and within a field-theoretic discussion, it is put by hand. Smooth supergravity solutions that exhibit the same feature of localising gravity on the brane remain elusive, and indeed no-go theorems have been established that demonstrate their absence, subject to certain sets of assumptions [7, 8, 9, 10, 11, 12, 13]. (For related work see also [14, 15, 16].)

In another development, it was suggested that there could be a complementarity between the localisation of gravity in the Randall-Sundrum scenario and the AdS/CFT correspondence [14, 17, 18, 19, 20, 21]. In the Randall-Sundrum scenario the introduction of the singular domain-wall (3-brane) source in the AdS space-time, before the boundary of AdS is reached, can be viewed within the AdS/CFT correspondence as the introduction of an ultra-violet cut-off in the boundary conformal field theory, thus introducing gravity on the domain wall [17]. An important test of this complementarity is to see whether the leading-order corrections to Newtonian gravity on the 3-brane in the Randall-Sundrum picture are in agreement with the predictions from the corrections to the graviton propagator induced by one-loop SCFT effects on the boundary [17, 18, 19]. The matching between the two approaches was recently demonstrated in [22].

It has been proposed that the conjectured AdS/CFT correspondence [23, 24, 25] can be extended to a more general class of theories, where a bulk supergravity admitting a domain-wall solution (describing a dilatonic vacuum) is dual to a quantum field theory on the boundary of the solution at infinity [26, 27, 28]; this is referred to as the Domain-wall/QFT correspondence. It is therefore natural to conjecture that under suitable circumstances there could be a complementarity between this Domain-wall/QFT correspondence and a

¹Smooth flat domain-wall configurations of this type were first found in $D = 4$ [2] as BPS configurations interpolating between supersymmetric isolated extrema in $N = 1$ supergravity theory. Subsequent generalisations led to the study of the global and local space-time structures of non-supersymmetric thin-wall configurations (bent domain-walls) [3] as well as dilatonic domain walls [4, 5] and were reviewed in [6].

localisation of gravity on the domain wall itself. In this paper we search for evidence that might support this conjecture and show that within this complementarity framework the localisation of gravity takes place precisely when a natural decoupling limit within the Domain-wall/QFT correspondence can be established.

To do this, we begin by studying a large class of BPS domain-wall solutions in supergravity theories, namely those that involve a single scalar field that has a pure single-exponential potential. These bulk Lagrangians are supplemented by a singular domain wall source. In section 2 we begin with a review of these solutions, including their space-time structure and the matching across the domain-wall source. By studying the equations governing linearised fluctuations of the graviton, we then study the conditions for the localisation of gravity on the wall. We also obtain the leading-order corrections to Newtonian gravity in those cases where the localisation occurs.

In section 3 we study the origins of pure single-exponential scalar potentials in supergravities, coming from reductions of M-theory or string theory. They can arise, for example, from generalised Scherk-Schwarz reductions on Ricci-flat internal spaces, or from reductions on internal Einstein spaces (such as spheres). We find that the Ricci-flat reductions never give rise to gravity-trapping domain walls, but that under appropriate circumstances the scalar potentials coming from spherical reductions can give domain walls that do trap gravity. We classify the higher dimensional origins of these solutions, and the circumstances under which the gravity trapping solutions can appear in the lower-dimensional theory.

We find that the examples where gravity-trapping occurs can all be lifted back to become the near-horizon regions of M-branes or Dp -branes with $p \leq 5$. These are precisely the branes for which a natural gravity-decoupling limit exists, which is a *sine qua non* for the possibility of establishing a Domain-wall/QFT correspondence. For the Dp -branes with $p \geq 6$, on the other hand, it is known that there is no natural decoupling limit [29, 30], and correspondingly, we find that in these cases there is no associated domain wall that can trap gravity. Thus we obtain strong evidence that the localisation of gravity on the domain wall, and the existence of a Domain-wall/QFT correspondence, go hand in hand. We explore the complementarity further in section 4, by considering the one-loop corrections to the graviton propagator from the Yang-Mills fields on the boundary. Concluding remarks are contained in section 5. In an appendix, we obtain the exact analytic expression for the Green function for the operator describing linearised gravity fluctuations, for one specific example of a gravity-trapping domain wall.

2 Supergravity domain walls and localisation of gravity

2.1 Supergravity domain-wall solutions

Our starting point is a d -dimensional action of the form

$$S = \int_{\Sigma_d} \sqrt{-g} d^d x (R - \frac{1}{2}(\partial\phi)^2 - 2\Lambda e^{b\phi}) + \int_{\Sigma_{d-1}} d^{d-1}x \mathcal{L}_{\text{source}}, \quad (1)$$

where the bulk contribution is viewed as arising from a supergravity theory whose higher dimensional origin, as an effective theory of sphere reduced M-theory or string theory, we shall discuss in the subsequent section. To this bulk action we added by hand a delta-function source for the domain wall; this delta function will provide a cut-off for the boundary of the bulk solution. For the case of the AdS bulk solution, this source provides a cut-off from the AdS boundary (in horospherical coordinates) and thus within the AdS/CFT correspondence provides the ultra-violet cut-off for the dual CFT description of the brane-world scenario [14, 17, 18, 19, 20, 21, 22].

Single-exponential scalar potentials in fact occur rather frequently in such circumstances, as we shall discuss in detail in the next section.² It is useful to parameterise the constant b that determines the strength of the coupling in terms of a constant Δ as follows:

$$b^2 = \Delta + \frac{2(d-1)}{d-2}. \quad (2)$$

The advantage of this parameterisation is that the value of Δ is preserved under toroidal dimensional reduction [31]. Note that a pure cosmological term, corresponding to $b = 0$, has

$$\Delta = \Delta_{\text{AdS}} \equiv -2 - \frac{2}{d-2} < -2. \quad (3)$$

The reality of b requires that Δ satisfy

$$\Delta \geq \Delta_{\text{AdS}}. \quad (4)$$

We shall review domain-wall solutions³ in the conformally-flat frame

$$ds^2 = e^{2A} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \quad (5)$$

where A and the bulk scalar field ϕ is a function only of z .

²One could refer to such bulk Lagrangians as “vacuum” Lagrangians where one has not excited any other massless scalar fields; see, for example, [28]. Situations involving additional fields can also be considered.

³These solutions were first considered [4] as BPS solutions in $d = 4$. Generalisations to D dimensions in the context of localisation of gravity were studied in [32, 33].

In principle these solutions can be obtained as solutions to the first order (BPS) differential equations (c.f. [4] for the $d = 4$ case), with the Israel matching conditions [34] across the singular domain wall source relating the tension ($\mathcal{T} = \int_{\Sigma_{d-1}} d^{d-1}x \mathcal{L}_{\text{source}}$) of the wall to the bulk cosmological constant parameters. For the sake of simplicity we choose to have the *same* bulk Lagrangian (1) on either side ($z > 0$ and $z < 0$) of the wall and thus the solution is Z_2 symmetric.⁴ In addition we shall focus on the positive tension walls ($\mathcal{T} > 0$) and only briefly discuss the negative tension branes. (The latter turn out not to be of interest for localisation of gravity.)

For $\Delta \neq -2$ the equations of motion following from (1) admit domain-wall solutions, given by

$$\begin{aligned} ds^2 &= H^{\frac{4}{(d-2)(\Delta+2)}} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \\ e^\phi &= H^{-\frac{2b}{\Delta+2}}, \quad H = 1 + k|z|, \end{aligned} \tag{6}$$

where

$$k^2 = \frac{(\Delta + 2)^2 \Lambda}{\Delta}. \tag{7}$$

In order to have a real solution, it is necessary that Λ and Δ have the same sign. As we shall discuss in the next section, this is indeed always the case in supergravity theories. The constant k can then be chosen to be either the positive or the negative root of (7). In the former case the z coordinate runs from $-\infty$ to $+\infty$ while in the latter the range of the transverse coordinate is finite. The choice of the sign of k has to be correlated with the matching condition across the singular domain wall source.

If $\Delta = -2$, the solution is instead given by

$$\begin{aligned} ds^2 &= e^{-\frac{2k}{d-2}|z|} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \\ \phi &= \frac{\sqrt{2}k}{\sqrt{d-2}}|z|, \end{aligned} \tag{8}$$

where k is now given by

$$k^2 = -2\Lambda(d-2), \tag{9}$$

which is real for negative Λ .

The matching conditions across the singular domain wall source imply that the energy density (tension) of the wall is related to the values of the cosmological constant parameters on either side of the wall. (For a detailed discussion pertinent to our situation,

⁴ The choice of a different bulk Lagrangian on either side ($z > 0$ or $z < 0$) of the wall, for example with a different choice of Λ and/or b on either side of the wall, would in turn allow one to construct Z_2 asymmetric walls.

see, e.g., Refs. [5, 6].) For the domain wall source associated with a typical solitonic kink the matching conditions is of the form

$$\sigma = \mathcal{T} = 2(A'_{z=0-} - A'_{z=0+}), \quad (10)$$

where the prime denotes a derivative with respect to z . This leads to

$$\begin{aligned} \Delta \neq -2: \quad \mathcal{T} &= -8 \operatorname{sign}[k(\Delta + 2)] \sqrt{\frac{\Lambda}{\Delta}}, \\ \Delta = -2: \quad \mathcal{T} &= \frac{8k}{d-2}. \end{aligned} \quad (11)$$

Thus positive-tension domain-wall solutions exist for $\Delta \leq -2$ with $k > 0$ and for $\Delta > -2$ with $k < 0$. Conversely, negative-tension domain walls arise for $\Delta \leq -2$ with $k < 0$ and for $\Delta > -2$ with $k > 0$. From now on, whenever we discuss domain walls with $\Delta \leq -2$, the lower bound (4) is to be understood.

The Riemann curvature of the metric (5) is of the form $A'' e^{-2A}$ or $(A')^2 e^{-2A}$. For the domain-wall solutions given in (6), these functions are of the form $H^{-2 - \frac{4}{(d-2)(\Delta+2)}}$. It follows that for the positive tension solutions with $\{\Delta_{\text{AdS}} < \Delta \leq -2, k > 0\}$ and $\{\Delta > -2, k < 0\}$, there are curvature singularities at $z = \pm\infty$ and $z = \pm\frac{1}{|k|}$, respectively⁵. Thus the positive-tension solutions have a null singularity, coinciding with the horizon, for $\Delta \leq -2$, and have naked singularities for $\Delta > -2$. (Of course the AdS example with $b = 0$ is non-singular at $z = \pm\infty$, corresponding to the AdS Cauchy horizon.)

The negative-tension solutions with $\{\Delta_{\text{AdS}} < \Delta \leq -2, k < 0\}$ have naked singularities at $z = \pm\frac{1}{|k|}$, while those with $\{\Delta > -2, k > 0\}$ are geodesically complete with $z = \pm\infty$ corresponding to the boundary of space-time.

2.2 Localisation of gravity on domain walls

Our focus is to identify within the framework a class of Z_2 symmetric domain-wall solutions without naked singularities that can localise gravity. A necessary condition for localised gravity on the brane at $z = 0$ is that the conformal scale factor e^{2A} in (5) should vanish at large $|z|$ and that the delta-function source have a positive tension, thus providing a trapping volcano-like effective potential at $z = 0$. For the solutions described above, this happens only for the $\Delta \leq -2$ and $k > 0$ cases, *i.e.* Z_2 -symmetric domain walls with positive tension, whose space-time geometry has at most null singularities at $z = \pm\infty$. To see the localisation of gravity in detail, one can examine the equation for small gravitational fluctuations on

⁵For special values of Δ , such as $\Delta = +1$ in $d = 4$, the Ricci scalar R may be finite but then $R_{\mu\nu}R^{\mu\nu}$ is infinite [4].

the brane. The fluctuations of the d -dimensional graviton, in the conformal frame, satisfy the equation of a minimally-coupled scalar field in the gravitational background, namely $\partial_M (\sqrt{-g} g^{MN} \partial_N \Phi) = 0$. (See, *e.g.*, [35, 36].) Following [1], we consider the Ansatz

$$\Phi = \phi(z) e^{i p \cdot x} = e^{-\frac{1}{2}(d-2)A} \psi(z) e^{i p \cdot x}, \quad (12)$$

where the A -dependent rescaling function is chosen so that ψ satisfies a Schrödinger-type equation,

$$-\frac{1}{2}\psi'' + U\psi = -\frac{1}{2}p^2\psi, \quad (13)$$

where the Schrödinger potential is given by

$$\begin{aligned} \Delta \neq -2: \quad U &= -\frac{(\Delta+1)k^2}{2(\Delta+2)^2 H(z)^2} + \frac{k}{\Delta+2} \delta(z), \\ \Delta = -2: \quad U &= \frac{1}{8}k^2 - \frac{1}{2}k \delta(z). \end{aligned} \quad (14)$$

The potential for $\Delta \neq -2$ was obtained in [32, 33].

It is evident from these expressions for U that there will be a zero-mass bound state if $\Delta \leq -2$, since then the delta function has a negative coefficient, and the “bulk” term is non-negative for all z . (Had we chosen the constant k to be negative, the solutions would have had naked singularities, and hence will not be considered.) In fact the potential is volcanic for $\Delta < -2$, whilst for $\Delta = -2$ it is a raised constant potential, again with a negative delta function. The massless wave-function is given by

$$\begin{aligned} \Delta < -2: \quad \psi &= e^{\frac{1}{2}(d-2)A} = H^{\frac{1}{\Delta+2}}, \\ \Delta = -2: \quad \psi &= e^{\frac{1}{2}(d-2)A} = e^{-\frac{1}{2}k|z|}. \end{aligned} \quad (15)$$

The trapping of gravity requires [36] that the wave-function be normalisable, *i.e.* $\int |\psi|^2 dz < \infty$, in order to obtain a finite leading-order contribution to the gravitational field on the brane. This condition is satisfied, for the domain walls without naked singularities that we are considering in this paper, if $-4 < \Delta \leq -2$. In view of the bound (4) resulting from the requirement that the constant b be real, we therefore have the criterion, for $d \geq 4$, that

$$-2 - \frac{-2}{d-2} \equiv \Delta_{\text{AdS}} \leq \Delta \leq -2. \quad (16)$$

The pure AdS case $b = 0$ leads to a trapping of gravity [1], and indeed we see from (3) that the value of Δ in this case lies strictly within the bound (16), for $d \geq 4$. When $d = 3$, we have $\Delta_{\text{AdS}_3} = -4$, and the norm of the wave-function is logarithmically divergent. This may be attributed to the degeneracy of two-dimensional pure gravity. In $d \geq 4$ the bound

(4) on Δ that is needed for reality of the exponential potential ensures that the $\Delta = -4$ limit in (16) is never attained.

It is worth noting that for $\Delta < -2$, including the AdS case, there is no mass gap in the spectrum (although, of course, the wavefunctions with small mass are delocalised away from the brane). On the other hand, when $\Delta = -2$ there is a distinct mass gap, with the continuum wavefunctions having $m^2 = -p^2 \geq \frac{1}{4}k^2$. Generally, for Δ approaching -2 from below, the effect of the localisation of gravity becomes more pronounced.

For Δ lying outside the range (16), there will be no trapping of gravity on the brane (for the examples with no naked singularities that we are considering here). In such cases one would have to resort to the more traditional Kaluza-Klein approach of compactifying the z coordinate. This can be done by introducing a second parallel brane [37, 38, 39].

In the next section we shall investigate the various exponential potentials of the form (1) that can arise in supergravities, and in particular, their associated values of Δ .

2.3 Corrections to Newtonian gravity

It is of interest to see how the leading-order Newtonian gravitational potential in the brane is modified for the various five-dimensional domain walls that we have found. In section 2, the massless bound-states (15) were found. Here, we shall obtain the wavefunctions describing massive gravity fluctuations also, and use these results in order to estimate the corrections to the leading-order Newtonian result.

The Schrödinger equation (13) can be solved exactly for the exponential scalar potentials that we are considering here. After imposing the boundary conditions at $z = 0$, we find that the Z_2 -symmetric massive wavefunctions are given by

$$\begin{aligned} \Delta < -2: \quad \psi_m &= c_m H^{1/2} \left[Y_{\nu-1} \left(\frac{m}{k} \right) J_{\nu} \left(\frac{m}{k} H \right) - J_{\nu-1} \left(\frac{m}{k} \right) Y_{\nu} \left(\frac{m}{k} H \right) \right], \\ c_m &= \sqrt{\frac{m}{\pi}} \left[Y_{\nu-1} \left(\frac{m}{k} \right)^2 + J_{\nu-1} \left(\frac{m}{k} \right)^2 \right]^{-1/2}, \\ \nu &= \frac{\Delta}{2(\Delta + 2)}, \end{aligned} \tag{17}$$

$$\begin{aligned} \Delta = -2: \quad \psi_m &= c_m \left(k \sin q|z| - 2q \cos q|z| \right), \\ c_m &= \frac{1}{2m\sqrt{\pi}q}, \\ q &= \sqrt{m^2 - \frac{1}{4}k^2}. \end{aligned} \tag{18}$$

Note that in the latter case, $\Delta = -2$, there is a mass gap and so we must have $m^2 \geq \frac{1}{4}k^2$ for the massive wavefunctions.

The corrections to the Newtonian gravitational potential between masses M_1 and M_2 can be estimated as follows [35]:

$$U(r) \sim \frac{G_4 M_1 M_2}{r} + \frac{G_5 M_1 M_2}{r} \int_{m_0^2}^{\infty} d(m^2) \psi_m(0)^2 e^{-mr}, \quad (19)$$

where m_0 is the lowest mass for the non-bound states. (This will be taken to be zero, except in the case where there is a mass gap.) The four-dimensional and five-dimensional Newton constants are related by $G_4 = k G_5$. For $\Delta < -2$ we therefore find

$$U(r) \sim \frac{G_4 M_1 M_2}{r} \left(1 + \frac{c}{(kr)^{2\nu-2}} + \dots \right), \quad (20)$$

where c is some constant of order 1. The two cases that are relevant to our discussion in this paper are $\Delta = -\frac{8}{3}$, giving $\nu = 2$, and $\Delta = -\frac{12}{5}$, giving $\nu = 3$. The former is the standard AdS Randall-Sundrum II scenario, with the well-known $1/r^3$ correction to the Newtonian potential; the latter gives instead a $1/r^5$ correction at leading order.

For $\Delta = -2$, we find

$$U(r) \sim \frac{G_4 M_1 M_2}{r} \left(1 + \frac{2e^{-\frac{1}{2}kr}}{\pi k} \int_0^{\infty} dy \frac{(y(y+k))^{1/2}}{y + \frac{1}{2}k} e^{-yr} \right), \quad (21)$$

from which we see that the leading-order corrections are of the form ⁶

$$U(r) \sim \frac{G_4 M_1 M_2}{r} \left[1 + \frac{2e^{-\frac{1}{2}kr}}{\sqrt{\pi}} \left(\frac{1}{(kr)^{3/2}} - \frac{9}{4(kr)^{5/2}} + \frac{345}{32(kr)^{7/2}} + \dots \right) \right]. \quad (22)$$

This is a Yukawa-like modification to Newtonian gravity. The essential $e^{-kr/2}$ factor reflects that we are dealing with a situation where there is a mass gap $\frac{1}{2}k$ separating the zero-energy bound state and the massive continuum. The formula (19) is consistent with the exact Green function we obtain in Appendix. The determination of the precise constant coefficients involves subtleties concerning the imposition of gauge conditions on the metric perturbations [40, 19, 41]. (The corrections to Newtonian gravity in a different domain wall whose spectrum has a mass gap, associated with a D3-brane distributed over a disc, was discussed in [35].)

3 Higher-dimensional origin of the scalar potentials

⁶We thank Konstadinos Sfetsos for pointing out an error in the normalisation factor in (18) in an earlier version of this paper, which has resolved a previous discrepancy with the Green function result obtained in the Appendix.

3.1 Exponential scalar potentials from Scherk-Schwarz reductions

The largest dimension in which a supergravity has a scalar potential is $D = 10$, in the massive type IIA theory. This has $\Delta = 4$. One way to obtain a scalar potential in a lower dimension is by performing a Scherk-Schwarz reduction of a supergravity without any scalar potential, in which an n -form field strength is taken to be proportional to some harmonic n -form on a Ricci-flat internal manifold. The first example of this kind was the reduction of type IIB supergravity on S^1 , where the “1-form field strength” $d\chi$ was taken to be $m dy$, where y is the tenth coordinate. In fact it was shown [42] that this reduction of type IIB is T-dual to the massive type IIA theory. Scherk-Schwarz reductions on tori were extensively studied in [43], and a complete classification of their exponential scalar potentials was given in [44]. The basic value of Δ that arises for any individual exponential term is always $\Delta = 4$. If N exponentials are combined, eliminating some scalar fields using the equations of motion, one is left with an exponential with $\Delta = 4/N$, where $2 \leq N \leq 8$ (the possible values of N depend on the dimension D [44]).⁷

In some cases the compactifying tori can be replaced by certain other Ricci-flat manifolds, such as K3 or a Calabi-Yau or Joyce manifold. This was discussed extensively in [45]. One of the examples is the Scherk-Schwarz reduction of M-theory on a 6-dimensional Calabi-Yau manifold Y , with the 4-form field strength residing in the 4'th cohomology class of Y . The resulting scalar potential, with $\Delta = 4/3$, was used to construct a domain-wall solution in $d = 5$ [38] that attempts to provide a field-theoretic realisation of the Hořava-Witten [37] construction. In a limit where the Calabi-Yau manifold becomes an orbifolded 6-torus, the domain wall solution can be viewed as an intersection of three 5-branes [45].

In all the examples of Scherk-Schwarz reduction on Ricci-flat internal manifolds, the value of Δ is positive and in addition $\Lambda > 0$, (unless the higher-dimensional theory already has a scalar potential with negative Δ .) As we discussed in the previous section, a single domain wall supported by such an exponential potential cannot trap gravity. It is then necessary to resort to the traditional Kaluza-Klein mechanism where the extra dimension is compact. One way to compactify the extra dimension is to take the coordinate z to lie on the interval S^1/Z_2 [38], with a domain wall at each endpoint.

⁷In fact all known exponential scalar couplings in ungauged supergravities have $\Delta = 4/N$. Interestingly, this provides strong supporting evidence for the belief that $D = 11$ supergravity cannot have a cosmological term, since from (3) this would have $\Delta = -20/9$. Since this value would be preserved under toroidal dimensional reduction [31], it would also imply the existence of lower-dimensional supergravities with this peculiar value of Δ .

3.2 Exponential scalar potentials from sphere reductions

Here we show that values of Δ that are less than -2 can commonly arise from sphere reductions of M-theory or string theory. Well-known examples are the supergravities with pure cosmological constants ($b = 0$) that arise from the S^4 and S^7 reductions of M-theory, and the S^5 reduction of type IIB supergravity. In this section we shall consider a general Lagrangian in D dimensions given by

$$\hat{\mathcal{L}} = \hat{e} \hat{R} - \frac{1}{2} \hat{e} (\partial\phi_1)^2 - \frac{1}{2n!} \hat{e} e^{a\phi_1} \hat{F}_{(n)}^2, \quad (23)$$

where in supergravity theories the constant a is parameterised by

$$a^2 = \frac{4}{N} - \frac{2(n-1)(D-n-1)}{D-2}, \quad (24)$$

and N is an integer. The case $N = 1$ can arise for all n -forms in supergravity. In particular, in $D = 10$ or $D = 11$ all the field strengths have $N = 1$ [46], and in fact all the individual field strengths in all maximal supergravities have $N = 1$. The case $N = 2$ can arise for 2-forms in $D \leq 9$, and 3-forms in $D \leq 6$, in non-maximal supergravities. $N = 3$ can arise for 2-forms in $D \leq 5$, and $N = 4$ for 2-forms in $D \leq 4$.

We now perform the following consistent reduction on S^n , using the Ansatz⁸

$$\begin{aligned} d\hat{s}_D^2 &= e^{-2\alpha\phi_2} ds_d^2 + g^{-2} e^{\frac{2(d-2)}{n}\alpha\phi_2} d\Omega_n^2, \\ \hat{F}_{(n)} &= m g^{-n} \Omega_n, \end{aligned} \quad (25)$$

where $d\Omega_n^2$ is the metric on the unit n -sphere, Ω_n is its volume form, and

$$\alpha = -\sqrt{\frac{n}{2(d-2)(D-2)}}. \quad (26)$$

This gives the following Lagrangian in $d = D - n$ dimensions [47]:

$$e^{-1} \mathcal{L} = R - \frac{1}{2} (\partial\phi_1)^2 - \frac{1}{2} (\partial\phi_2)^2 - \frac{1}{2} m^2 e^{a\phi_1 - 2(d-1)\alpha\phi_2} + n(n-1) g^2 e^{-\frac{2(D-2)}{n}\phi_2}. \quad (27)$$

Calculating the values of Δ for the two exponential terms, we find

$$\Delta_m = \frac{4}{N}, \quad \Delta_g = -2 + \frac{2}{n} \quad (28)$$

respectively. Note that, comparing with (1), the sign of Λ is indeed the same as that of Δ for each term, so that $\Delta \Lambda$ is non-negative for each term. One can use either of the

⁸Note that here we are always giving a magnetic charge to the n -form field strength. The case where the charge is instead electric can be handled within this framework by dualising the n -form to a $(D-n)$ -form. The case of the self-dual 5-form in type IIB supergravity was discussed in [47], and the final result is of the same form as (27). Also, in this section we consider $n \geq 2$, since the $n = 1$ case was the topic of section 3.1.

exponential terms by itself to construct a domain wall of the kind discussed in section 2, since one can turn off either of the parameters m and g , and then, if necessary, rotate the dilatons ϕ_1 and ϕ_2 to give a single dilatonic scalar in the remaining exponential, with the orthogonal (free) dilaton set to zero. As we discussed in section 2 there can be no localised gravity in either case, since the inequality (16) is not satisfied.

If instead m and g are both taken to be non-vanishing, we can still eliminate one of the two dilatonic scalars. The two exponentials then coalesce into one, with a different value of Δ . To see this, we make an orthonormal transformation to new scalars (ϕ, φ) , defined by

$$\phi_1 = -\phi \cos \beta - \varphi \sin \beta, \quad \phi_2 = \phi \sin \beta - \varphi \cos \beta, \quad (29)$$

with

$$\tan \beta = -\sqrt{\frac{a^2 n (D-2)}{2(d-2)(n-1)}}. \quad (30)$$

The Lagrangian (27) now becomes

$$e^{-1} \mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\varphi)^2 - V, \quad (31)$$

where

$$V \equiv e^{b\phi} \left(\frac{1}{2} m^2 e^{c_1 \varphi} - n(n-1) g^2 e^{c_2 \varphi} \right), \quad (32)$$

and

$$c_1 = \sqrt{\frac{8n}{N[2n - (n-1)N]}}, \quad c_2 = (n-1) \sqrt{\frac{2N}{n[2n - (n-1)N]}},$$

$$b^2 = -\frac{4(n-1)}{2n - (n-1)N} + \frac{2(d-1)}{d-2}. \quad (33)$$

For $N = 1$ and $N = 2$, the constants c_1 and c_2 are real for all n . If $N \geq 3$, the degree n of the n -form must be less than $N/(N-2)$.

In this section we shall consider just a 1-scalar solution of the form discussed in section 2. To do this, we first solve the φ equation by taking $\varphi = 0$, which therefore implies $c_1 m^2 = n(n-1) c_2 g^2$. We are then left with the potential

$$V = \frac{2(n-1)^2 g^2}{\Delta} e^{b\phi}, \quad (34)$$

where Δ characterises the strength of the dilaton coupling as in (2), and, from (33), is given by

$$\Delta = -\frac{4(n-1)}{2n - (n-1)N}. \quad (35)$$

This expression was also given in [32], where its implications for higher-dimensional origin of gravity trapping domain walls were discussed.

Comparing with (1), we see that the quantity $\Delta \Lambda$ is indeed always positive, as we stated in the previous section.

For the relevant values of N , we therefore find that Δ is as follows:

$$\begin{aligned}
N = 1, \quad D \leq 11: \quad \Delta &= -4 + \frac{8}{n+1}, \\
N = 2, \quad D \leq 9: \quad \Delta &= 2 - 2n, \\
N = 3, \quad D \leq 5: \quad \Delta &= 4 + \frac{8}{n-3}, \\
N = 4, \quad D = 4: \quad \Delta &= 2 + \frac{2}{n-2}.
\end{aligned} \tag{36}$$

In order to satisfy the inequality (16) that ensures the trapping of gravity on the brane, we can therefore have $N = 1$ with $n \geq 3$, or $N = 2$ with $n = 2$.

Note that the disallowed value of $(N, n) = (1, 2)$ corresponds to a reduction in which a Kaluza-Klein 2-form field strength is taken to be proportional to the volume form of an internal S^2 . The higher-dimensional theory could therefore itself be viewed as an S^1 reduction of pure gravity. Thus the above analysis shows that it is not possible to construct a gravity-trapping domain wall purely within an Einstein gravity theory (with no cosmological constant).

For a concrete example that is not without physical interest, we can enumerate the various ways of obtaining five-dimensional theories that are capable of trapping gravity on a 3-brane. They correspond to reducing an appropriate ordinary massless supergravity in $D = 10, 9, 8$ or 7 on S^5, S^4, S^3 or S^2 , respectively. Thus they can all be viewed as coming, for example, from reductions of type IIB supergravity, as follows:

	S^5	$S^1 \times S^4$	$T^2 \times S^3$	$T^3 \times S^2$
N	1	1	1	2
Δ	$-\frac{8}{3}$	$-\frac{12}{5}$	-2	-2

Table 1: The sphere reductions that give trapped gravity on a 3-brane in $D = 5$, lifted to type IIB.

In the next section, we discuss the higher-dimensional interpretations of these solutions, both in the type IIB and the type IIA and M-theory pictures.

3.3 Lifting of the domain walls to higher dimensions

The various domain-wall solutions of the previous section, supported by the potential V given in (34) can be lifted back on the n -sphere to D dimensions where they become the

near-horizon limits of $(D - n - 2)$ -branes. (See also [32].) To see this, consider the standard isotropic $(D - n - 2)$ -brane in D dimensions with metric

$$\begin{aligned} ds_D^2 &= \hat{H}^{-\frac{(n-1)N}{D-2}} \eta_{\mu\nu} dx^\mu dx^\nu + \hat{H}^{\frac{(d-1)N}{D-2}} (dr^2 + r^2 d\Omega_n^2), \\ e^{-\frac{2}{N}a\phi} &= \hat{H} \quad \hat{H} = 1 + \frac{Q}{r^{n-1}}. \end{aligned} \quad (37)$$

If we drop the “1” in the harmonic function \hat{H} , and make the coordinate redefinition

$$r = \left(1 + k|z|\right)^{-\frac{2}{(n-1)N-2}}, \quad (38)$$

then after performing the reduction of the D -dimensional $(D - n - 2)$ -brane metric on the n -sphere, using the reduction Ansatz given in (25), and making appropriate constant rescalings, we obtain precisely the d -dimensional domain-wall solution (6). Note that there is an exceptional case when $(n - 1)N = 2$, corresponding to $\Delta = -2$, for which we shall have instead

$$r = e^{-\frac{k}{n-1}|z|}, \quad (39)$$

reproducing (8).

For the cases of principal interest to us, with $\Delta \leq -2$, the regions where $z \rightarrow \pm\infty$ correspond to $r \rightarrow 0$. Thus in these cases the horizons of the domain-wall solution in the lower dimension d (*i.e.* $z \rightarrow \pm\infty$, far from the brane at $z = 0$) map into the horizon of the brane in the higher dimension $D = d + n$. By contrast, in the cases with $\Delta > -2$ the relation between r and z is of the form $r \sim (1 + k|z|)^c$ where c is a positive constant, and there is no region in the lower-dimensional solution that maps to the horizon of the higher-dimensional brane.

As an example, the domain-wall solutions listed in Table 1 have oxidation endpoints as follows:

Δ	$-\frac{8}{3}$	$-\frac{12}{5}$	-2	-2
Oxidation	D3	M5 (or D4)	D5	M5 \perp M5
Endpoint	on S^5	on $T^2 \times S^4$ (or $S^1 \times S^4$)	on $T^2 \times S^3$	on $T^4 \times S^2$ or K3 \times S^2

Table 2: Ten or eleven-dimensional origins of the $\Delta \leq -2$ five-dimensional domain walls.

The case $\Delta = -\frac{8}{3}$ is nothing but the well-known $\text{AdS}_5 \times S^5$ solution of type IIB supergravity, with no scalar field. The scalar potential for the case $\Delta = -\frac{12}{5}$ was obtained from the S^4 reduction of the type IIA theory with a non-vanishing 4-form field strength [48, 49], followed by an S^1 reduction. The potential for the first of the $\Delta = -2$ cases was obtained

from the S^3 reduction of ten-dimensional supergravity with a non-vanishing 3-form field strength [48, 50, 49], followed by a reduction on T^2 . The scalar potential corresponding to the second $\Delta = -2$ case arises from a new source. It is interesting that the D5-brane reduced on $T^2 \times S^3$ and the M5/M5-brane intersection reduced on $T^4 \times S^2$ (or $K3 \times S^2$) give rise to the same scalar potential. This may suggest some duality relation between the quantum field theory living on the world volume the D5-brane wrapped on T^2 and that of the M5/M5-brane wrapped on T^4 or K3.

In the previous subsection, we saw that the domain walls that can trap gravity are those with $N = 1, n \geq 3$ and $N = 2, n = 2$. All the M-branes in $D = 11$ and Dp-branes in $D = 10$ have $N = 1$. Clearly M-branes lead to domain walls in $D = 4$ and $D = 7$ that can trap gravity since the internal sphere dimension n is greater than 3. For Dp-branes, the corresponding lower-dimensional domain wall can trap gravity only for $p \leq 5$. In section 4, we shall show this may be related to the fact that a natural decoupling limit exists only for Dp-branes with $p \leq 5$, but not for $p \geq 6$.

3.4 Two-scalar domain-wall solutions

In this section we shall consider the domain-wall solutions for the two-scalar potentials obtained in section 3.2, with the second scalar φ no longer set to zero.⁹ We saw in section 3.3 that the single-scalar solutions where $\varphi = 0$ had the interpretation, after lifting back to $d + n$ dimensions on S^n , of being the near-horizon limits of isotropic $(d - 2)$ -branes. In fact we could have reversed the process, and derived the lower-dimensional domain walls by reducing these near-horizon limits on S^n . We shall use the analogous inverse procedure now in order to obtain the lower-dimensional two-scalar domain-wall solutions.

To do this, we begin by considering the $(D - n - 2)$ -brane solution (37) in D dimensions. This time, however, we shall retain the constant “1” in the harmonic function \hat{H} . Reducing the metric on S^n , following the Ansatz (25), we obtain the d -dimensional solution

$$\begin{aligned} ds_d^2 &= \hat{H}^{\frac{N}{d-2}} r^{\frac{2n}{d-2}} \eta_{\mu\nu} dx^\mu dx^\nu + \hat{H}^{\frac{N(d-1)}{d-2}} r^{\frac{2n}{d-2}} dr^2, \\ e^{-\frac{2}{Na} \phi_1} &= \hat{H}, \quad e^{\frac{2(d-2)\alpha}{n} \phi_2} = \hat{H}^{\frac{(d-1)N}{D-2}} r^2. \end{aligned} \quad (40)$$

⁹In the case of $b = 0$, the scalar potential would comprise only the massive breathing mode. The associated domain-wall solution using this breathing mode was obtained in [47]. In [51, 52] it was used in order to obtain a gravity-trapping model analogous to Randall-Sundrum II (with one 3-brane), exploiting the fact that the breathing mode is massive, and so it has a scalar potential with a minimum, rather than a maximum. (The inclusion of a delta-function source is still necessary.) The use of the breathing mode was then explored in the context of the Randall-Sundrum I scenario (with two 3-branes [39]) in [53, 54, 55].

The metric can be recast into a conformal frame

$$ds_d^2 = \hat{H}^{\frac{N}{d-2}} r^{\frac{2n}{d-2}} (\eta_{\mu\nu} dx^\mu dx^\nu + dy^2), \quad (41)$$

by introducing a new coordinate y , related to r by $dy = \hat{H}^{N/2} dr$, which implies

$$y = r {}_2F_1 \left[\frac{1}{1-n}, -\frac{1}{2}N; 1 + \frac{1}{1-n}; -\frac{Q}{r^{n-1}} \right]. \quad (42)$$

As r tends to infinity, y becomes proportional to r and therefore tends to infinity. As r approaches the horizon at $r = 0$, the coordinate y tends to $-\infty$. Thus we can introduce an appropriate cut-off by changing coordinate from y to z , defined by

$$y = -c(1 + k|z|), \quad (43)$$

where c and k are positive constants. By doing this we have introduced a domain-wall, with a standard delta-function curvature singularity, at $z = 0$, which corresponds to some value of r outside the $(D - n - 2)$ -brane's horizon at $r = 0$. As z tends to $\pm\infty$ the coordinate r tends to zero, and so the horizons on either side of the singular wall are mapped into the horizon of the $(D - n - 1)$ -brane. This is the desired two-scalar solution of the Lagrangian that we obtained in section 3.2. Clearly, it approaches the same form as the previous 1-scalar solution as z goes to $\pm\infty$, since in these regions r becomes small enough that the “1” in the harmonic function \hat{H} becomes negligible. It follows that the characteristic structures of the associated Schrödinger potentials for the two-scalar solutions will be essentially the same as for the corresponding 1-scalar solutions discussed in section 2.

4 Domain-wall/QFT correspondence

4.1 Decoupling limit and the localisation of gravity

Although only the D3-brane has a near-horizon limit of $\text{AdS}_5 \times S^5$, which leads to the conjecture of the $\text{AdS}_5/\text{CFT}_4$ correspondence, one expects that the world volumes of other D-branes in ten dimensions should also describe certain quantum field theories. It was observed in [26] that for any Dp -brane in ten dimensions there exists a dual frame where the metric becomes a direct product of $\text{AdS}_{10-n} \times S^n$, for $n \neq 3$, or $M_7 \times S^3$, where M_7 is seven-dimensional Minkowski space-time. Since dimensional reduction of a Dp -brane on S^{8-p} gives rise to a domain wall in general, this leads to the conjecture of a Domain-wall/QFT correspondence [26]. In [28], ellipsoidal distributions of Dp -branes were obtained, and it was shown these solutions can be consistently reduced on the internal transverse

spheres to give rise to multi-scalar domain walls, generalising the results of the Coulomb branch of the AdS/CFT correspondence [56, 57, 35, 58, 59, 60]. Here we shall investigate the Domain-wall/QFT correspondence in the context of the relation between the gravity-decoupling limit of a D-brane, the trapping of gravity on the associated lower-dimensional domain wall.

As we discussed in the previous sections, the Lagrangian (23) admits a $(D - n - 2)$ -brane solution, given in (37). There exists a dual frame $ds_{\text{dual}}^2 = e^{-a\phi/(n-1)} ds_{\text{Einst}}^2$, in which the near-horizon region of the dual frame metric becomes

$$ds_{\text{dual}}^2 \sim r^{(n-1)N-2} dx^\mu dx_\mu + \frac{dr^2}{r^2} + d\Omega_n^2. \quad (44)$$

Thus when $(n - 1)N = 2$, corresponding to $\Delta = -2$, the metric is $M_d \times S^n$; otherwise it is $\text{AdS}_d \times S^n$.

In order to make sense of the Domain-wall/QFT correspondence, it is necessary to examine whether there exists a natural decoupling limit in which the gravitational constant can be set to zero, so that the higher-dimensional p -brane limits to the domain wall. To do so, one needs to make a coordinate rescaling

$$r = u (\ell_p)^\gamma, \quad (45)$$

and then send the Planck length ℓ_p to zero while keeping u fixed. In this limit, the constant “1” in the harmonic function \hat{H} can be dropped if $\gamma > 1$, since the charge Q has to scale as ℓ_p^{n-1} . The natural decoupling limit exists if the metric can now be expressed as $ds_{\text{dual}}^2 = \ell_p^2 d\tilde{s}^2$ where $d\tilde{s}^2$ is independent of ℓ_p . For the solutions given by (37), we find that this can be achieved only when the following condition is satisfied:

$$(n - 1)N = \frac{2\gamma}{\gamma - 1}. \quad (46)$$

Thus for $N = 1$, which applies for all the D-branes in ten dimensions, the requirement that $\gamma > 1$ implies $n \geq 3$. (The $n = 3$ case requires $\gamma = \infty$.) In other words, the decoupling limit exists naturally only for Dp -branes with $p \leq 5$, which have $n \geq 3$. This condition for the existence of a decoupling limit coincides precisely with the requirement that gravity be localised on the brane as discussed in section 2.

This coincidence may not be surprising. It was shown that the Randall Sundrum II wall with the AdS_5 bulk geometry can be interpreted within an $\text{AdS}_5/\text{CFT}_4$ correspondence as a singular domain wall source that cuts off the boundary of AdS_5 [14, 17, 18, 19, 20, 21, 22]. The delta-function source in turn provides an ultra-violet cut-off in the dual CFT_4 . In the case of dilatonic domain walls, discussed in section 2, it should likewise be possible to

view the domain walls source for the gravity trapping solutions as an ultra-violet cut-off for the dual QFT within the Domain-wall/QFT correspondence. Thus the criterion to localise gravity could have been expected to coincide with the criterion for the decoupling of gravity in the complementary picture of Domain-wall/QFT correspondence.

The absence of a natural decoupling limit of Dp -branes with $p \geq 6$ is also related to the difficulties arising in M(atric) theory on a torus T^n with $n \geq 6$ [29, 30]. We showed in this paper that the corresponding domain walls do not have localised gravity, unlike the cases with $p \leq 5$. The difficulty may also be related to the fact that the U-duality group becomes exceptional for $D \leq 5$, which requires the dualisation of $(D - 1)$ -forms to enlarge the scalar coset [61].

4.2 One-loop corrections from the QFT

In the context of AdS/CFT, an interesting observation was recently made [22], to the effect that if one treats the Randall-Sundrum II picture as a UV cut-off applied to the AdS/CFT system, then the leading-order corrections to localised gravity on the brane are in exact agreement with the corrections to the graviton propagator induced by one-loop contributions from the super Yang-Mills fields on the boundary. One might expect, therefore, that it should be possible to see an analogous complementarity in the conjectured Domain-wall/QFT equivalence. The technology for performing such a detailed comparison is not yet sufficiently developed. However, we can determine what the general structure of the one-loop QFT corrections would have to be if the conjectured correspondence were to hold.

In section 2.3 we determined the leading corrections to Newtonian gravity at large distances r . It was observed in [22] that the $1/r^3$ correction in the Randall-Sundrum II case ($\Delta = -\frac{8}{3}$) was exactly reproduced by the corrections to the graviton propagator coming from the effect of closed super Yang-Mills loops, which, in momentum space, takes the general form

$$\Pi(p) \sim a \log \frac{p^2}{\mu^2} + b. \quad (47)$$

For the case $\Delta = -\frac{12}{5}$, we would instead need a propagator correction of the form

$$\Pi(p) \sim a p^2 \log \frac{p^2}{\mu^2} + \dots, \quad (48)$$

in order to yield, after a Fourier transformation, the required $1/r^5$ leading-order correction to the Newtonian potential.

For the $\Delta = -2$ examples, we saw in section 2.3 that the leading-order correction to Newtonian gravity is of the Yukawa-like form $e^{-kr/2} r^{-5/2}$. We find that in this case the needed corrections to the graviton propagator coming from one-loop effects in the boundary QFT would be of the form

$$\Pi(p) \sim \frac{1}{\left((p^2 + \frac{1}{4}k^2)^{\frac{1}{2}} + \frac{1}{2}k\right)^{\frac{1}{2}}}. \quad (49)$$

5 Conclusions

The primary goal of this paper was to study the localisation of gravity for a more general class of domain walls, and to provide a complementary description of these phenomena within the Domain-Wall/QFT correspondence.

For this purpose we studied thin, flat (BPS) domain walls in d dimensions for which the asymptotic geometry is a vacuum with a running dilaton. (The examples with asymptotic AdS space-times are special cases within this class of solutions.) The domain walls arise as solutions of a d -dimensional bulk Lagrangian with a scalar field whose potential is a pure single exponential $e^{b\phi}$, and where a singular domain-wall source is introduced. In particular, we reviewed the space-time structures of these solutions with the focus on those where the domain-wall source has positive tension with no naked singularities, and where the bulk Lagrangian determining the space-time on the two sides of the wall is the same. Thus the domain-wall geometry is Z_2 symmetric. (The domain wall with asymptotic AdS [1] is a particular example within this class of solutions.) In order to have a trapping of gravity on the wall, the tension must be positive, and in addition the coupling constant b in the scalar exponential potential must lie within certain bounds.

The exponential potentials responsible for such gravity-trapping domain-wall solutions can arise from certain sphere reductions in M-theory or string theory, whilst a generalised Scherk-Schwarz reduction on Ricci-flat internal space, which also gives a pure exponential potential, does not yield a value for b in the range necessary for the trapping of gravity. Specifically, we classified such examples of sphere compactifications and focused on the explicit examples that yield effective theories in $d = 5$.

The gravity-trapping domain-wall solutions can be lifted back on these internal spheres to $D = 11$ or $D = 10$. Their bulk geometries turn out to be the near-horizon regions of M-branes or Dp -branes (with $p \leq 5$), with the domain wall itself located at some distance outside the horizon, and the two horizons of the domain-wall solution corresponding to the horizon of the M-brane or Dp -brane. It is intriguing that these are precisely the near-horizon

geometries of the branes for which a natural gravity-decoupling limit exists, and so these are the examples for which a Domain-wall/QFT correspondence can be established. Conversely, the Dp -branes with $p \geq 6$, which do not have a natural decoupling limit, are associated with lower-domain walls that cannot trap gravity. This provides strong evidence to suggest that the localisation of gravity on the domain wall and the existence of a Domain-wall/QFT correspondence are closely related.

Note if we remove the delta-function source for the gravity-trapping domain walls, then the metric from one side runs from the null singularity at $-\infty$ to a (non-singular) boundary at some finite z . Thus the introduction of the singular delta-function source, before the boundary of this space-time is reached, can now be viewed within the Domain-wall/QFT correspondence as the introduction of an ultra-violet cut-off in the boundary field theory, thus generalising the complementarity between the Randall-Sundrum II scenario and the AdS/CFT correspondence [14, 17, 18, 19, 20, 21, 22].

An important test of this complementarity is to see if the leading-order corrections to Newtonian gravity on the dilatonic domain-walls are in agreement with the predictions from the corrections to the graviton propagator induced by one-loop QFT effects, which again generalises the studies within the AdS/CFT correspondence. Within this framework we analysed the linearised equation describing gravity fluctuations, and obtained the spectrum. In one example, where the spectrum has a mass gap, we obtained the exact analytic form for the associated Green function. In all the examples where gravity is trapped on the domain wall we obtained the leading-order corrections to Newtonian gravity, and from these we deduced the necessary forms of the one-loop boundary-QFT contributions to the graviton propagator. It would be interesting to study this in more detail, by comparing with detailed one-loop results from the boundary QFT, to obtain further tests of the proposed complementarity.

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A Exact Green function for $\Delta = -2$ gravity perturbations

The Green function satisfies the equation

$$-\frac{d^2 G(x, z; x', z')}{dz^2} - \square_4 G(x, z; x', z') + 2U G(x, z; x', z') = \delta^5(x, z; x', z'), \quad (50)$$

where for $\Delta = -2$ the Schrödinger potential U is given by the second line in (14). We saw earlier that the massless bound state wave-function u and the massive continuum wavefunctions u_q are given by

$$u(z) = e^{-\frac{1}{2}k|z|}, \quad u_q(z) = k \sin q|z| - 2q \cos q|z|, \quad (51)$$

where u_q corresponds to mass $m = \sqrt{q^2 + \frac{1}{4}k^2}$. The retarded Green function is then given by

$$G(x, z; x', z')_R = \int \frac{d^4 p}{(2\pi)^4} e^{ip \cdot (x-x')} \left\{ \frac{k u(z) u(z')}{2(\bar{p}^2 - (\omega - i\epsilon)^2)} + \int_0^\infty dq \frac{u_q(z) u_q(z')}{\pi(4q^2 + k^2) [\bar{p}^2 - (\omega - i\epsilon)^2 + q^2 + \frac{1}{4}k^2]} \right\}. \quad (52)$$

which can be evaluated explicitly. The static Green function is given by integrating over time, yielding $G(\vec{x}, z; \vec{x}', z') = \int_{-\infty}^\infty dt' G(x, z; x', z')$.

Taking $z = z' = 0$, we eventually obtain the following expression for the static Green function $G(\vec{x}, \vec{x}') = G(\vec{x}, 0; \vec{x}', 0)$:

$$\begin{aligned} G(\vec{x}, \vec{x}') &= \frac{k}{8\pi R} + \frac{e^{-\frac{1}{2}kR}}{4\pi^2 R} \int_0^\infty dy \frac{(y(y+k))^{1/2}}{y + \frac{1}{2}k} e^{-yR}, \\ &= \frac{k}{8\pi R} + \frac{k}{8\pi^2 R} K_1(\frac{1}{2}kR) \\ &\quad + \frac{k^2}{32\pi} \left(K_0(\frac{1}{2}kR) L_{-1}(\frac{1}{2}kR) + K_1(\frac{1}{2}kR) L_0(\frac{1}{2}kR) \right) - \frac{k}{16\pi R}, \end{aligned} \quad (53)$$

where $R = |\vec{x} - \vec{x}'|$, and $L_n(x)$ is the modified Struve function. This is a special case of the generalised hypergeometric function ${}_pF_q$ for $p = 1, q = 2$:

$$L_{-1}(x) = \frac{2}{\pi} {}_1F_2[1; \frac{1}{2}, \frac{3}{2}; \frac{1}{4}x^2], \quad L_0(x) = \frac{2x}{\pi} {}_1F_2[1; \frac{3}{2}, \frac{3}{2}; \frac{1}{4}x^2]. \quad (54)$$

Note that the first term in (53) is the usual one, which comes from the massless mode $u(z)$; all the remaining terms come from the massive modes $u_q(z)$. The form of this result, and in particular the integral associated with the massive modes, is identical to that obtained in section 2.3. Although the final term in (53) is of the same form as the leading-order result from $u(z)$, it is appropriate to keep it distinct since it is really just cancelling an equal

and opposite term that resides in the products of Bessel and Struve functions in a large- R expansion.

At large R , we therefore have

$$G(\vec{x}; \vec{x}') = \frac{k}{8\pi R} + \frac{k^2 e^{-\frac{1}{2}kR}}{4\pi^{3/2}} \left[\frac{1}{(kR)^{5/2}} - \frac{9}{4(kR)^{7/2}} + \frac{345}{32(kR)^{9/2}} + \dots \right]. \quad (55)$$

This is in precise agreement with the correction to the Newtonian potential discussed in section 2.3.

At small R , we find

$$G(\vec{x}; \vec{x}') = \frac{1}{4\pi^2 R^2} + \frac{k}{16\pi R} - \frac{k^2}{32\pi^2} \log(\frac{1}{2}kR) + \dots. \quad (56)$$

Thus, as expected, the gravity becomes effectively five-dimensional at small length-scales.

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