

Loop-corrected Entropy of Near-extremal Dilatonic p -branes

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ABSTRACT

It has recently been shown that for certain classical non-dilatonic p -branes, the entropy and temperature satisfy the ideal-gas relation $S \sim T^p$ in the near-extremal regime. We extend these results to cases where the dilaton is non-vanishing, but nevertheless remains finite on the horizon in the extremal limit, showing that the ideal-gas relation is again satisfied. At the classical level, however, this relation does break down if the dilaton diverges on the horizon. We argue that such a divergence indicates the breakdown of the validity of the classical approximation, and that by taking string and worldsheet loop corrections into account, the validity of the entropy/temperature relation may be extended to include these cases. This opens up the possibility of giving a microscopic interpretation of the entropy for all near-extremal p -branes.

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Semi-classical investigations have revealed many intriguing thermodynamic properties of black holes [1, 2, 3]. They have also raised many questions, related to the issues of predictability and information loss in the quantum evolution of the system. The resolution of these questions is expected to require a more complete understanding of the quantum theory of gravity. It is widely believed that string theory will supply this missing link. Accordingly, in recent studies, various authors have applied the semi-classical methods to the calculation of thermodynamic quantities for black hole and higher extended-object solitons in string theory or M-theory, and uncovered a striking relation to the microscopic description of entropy in terms of the counting of string states [4–17]. A crucial ingredient in establishing this correspondence was that the dilatonic scalar fields of the p -brane soliton remain finite on the horizon in the extremal limit, in order that the tree-level approximation be trustworthy. Isotropic extremal p -brane solutions of this type are very limited. There are in total six cases, namely the elementary membrane [18] and solitonic 5-brane [19] in $D = 11$, the self-dual 3-brane in $D = 10$ [20, 21], the dyonic string in $D = 6$ [22], and black holes in $D = 5$ and $D = 4$ with three and four independent participating field strengths respectively [23], as well as a special dyonic black hole in $D = 4$, which involves two participating field strengths, each of which carries both electric and magnetic charges [24]. In other words, in each case the number N of non-vanishing charges compatible with having some preserved supersymmetry must be maximal, namely $N = 1, 1, 2, 3, 4$ and 4 in $D = 11, 10, 6, 5, 4$ and 4 respectively. The general non-extremal generalisations of these solitons can be found in [25].

In $D = 11$, there is no dilaton field, and the microscopic interpretation of the entropy of the membrane and 5-brane was presented in [17]. The dilaton field decouples from the self-dual 3-brane in $D = 10$, and the microscopic discussion of the entropy was presented in [10]. In the remaining three cases, the dilatonic scalar fields do not decouple for generic values of the charges, but they do remain finite at the horizon in the extremal limit. When all the charges are equal, the dilatonic scalar fields then decouple; the resulting solutions were referred to as “non-dilatonic” p -branes in [17]. In this case, the dyonic string becomes the self-dual string in $D = 6$, and the black holes become Reissner-Nordström black holes in $D = 5$ and $D = 4$ respectively. (The dilaton field also decouples for the special dyonic black hole in $D = 4$, when the electric and the magnetic charges are equal.) In fact it is sometimes the case that multiply-charged black holes [26] may be regarded as bound states at threshold of singly-charged black holes [27, 28, 29, 30]. All the above solitons are regarded as “regular-dilaton” p -branes. (In contrast, “dilatonic” solutions in general refer to p -branes

whose dilatonic scalar fields do not decouple for any non-vanishing charges; moreover, these scalar fields diverge at the horizon in the extremal limit.) The $D = 5$ black hole solution is stainless [31], but can be oxidised to a boosted dyonic string in $D = 6$ dimensions. The entropy per unit p -volume is preserved under the oxidisation, and was shown to be precisely equal to that of the microscopic counting of the corresponding D-string states in the near-extremal regime [4]. There are two regular black holes in $D = 4$. One of them involves 4 field strengths, and reduces to the Reissner-Nordström black hole when all the charges are equal. The analogous analysis has also been carried out for this black hole [14, 13]. The other involves only two field strengths, each with electric and magnetic charges. This solution was referred to as the dyonic black hole of the second type in [24]. We shall extend the previous entropy analysis to include this case.

The first five cases above are the only p -brane solitons whose entropies have so far been given a microscopic interpretation. Furthermore, it has been shown that in the non-dilatonic cases, the entropy and temperature satisfy the ideal-gas relation $S \sim T^p$ [17], which is in concordance with the microscopic D-brane picture. We shall show that this discussion can be extended to the regular dilatonic cases. There are, however, many more p -brane solitons in the string or M-theory, in addition to the six cases discussed above. At the classical level, the ideal-gas entropy/temperature relation breaks down in all these other cases. This can be attributed to the fact that for all these solitons the dilaton, as well as the curvature and field strengths, diverges on the horizon in the extremal limit. These divergences indicate a breakdown of validity of the classical approximation, and it can be argued that the inclusion of string and worldsheet loop corrections will remove such singularities. We present a simple explicit example that illustrates how this happens. Then, we study the relation between entropy and temperature in the presence of loop corrections, and show that the ideal-gas entropy/temperature relation may be recovered. This may open up the possibility of giving a microscopic interpretation of the entropy for all near-extremal p -branes.

Let us begin with a discussion of the thermodynamic properties of classical p -branes. The relevant part of the bosonic Lagrangian is the tree-level approximation of the string effective action, namely

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{2n!} \sum_{\alpha=1}^N e^{-\vec{a}_\alpha \cdot \vec{\phi}} F_\alpha^2, \quad (1)$$

where $\vec{\phi} = (\phi_1, \dots, \phi_N)$, and F_α is a set of N n 'th-rank antisymmetric tensor field strengths, which give rise to a p -brane with world volume dimension $d = n - 1$ if they carry electric charges, or with $d = D - n - 1$ if they carry magnetic charges. The ‘‘dilaton vectors’’ \vec{a}_α are

constant vectors characteristic of the supergravity theory arising as the low energy limit of the string or the M-theory. Note that the Lagrangian (1) can be embedded into the string or the M-theory if the dot products $M_{\alpha\beta} = \vec{a}_\alpha \cdot \vec{a}_\beta$ satisfy [23]

$$M_{\alpha\beta} = 4\delta_{\alpha\beta} - \frac{2d\tilde{d}}{2(D-2)}, \quad (2)$$

where $\tilde{d} = D - d - 2$.

The metric of the black p -branes with N non-vanishing charges is given by [25]

$$ds^2 = e^{2A}(-e^{2f}dt^2 + dx^i dx^i) + e^{2B}(e^{-2f}dr^2 + r^2 d\Omega^2), \quad (3)$$

$$e^{2A} = \prod_{\alpha=1}^N \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu_\alpha\right)^{-\frac{\tilde{d}}{D-2}}, \quad e^{2B} = \prod_{\alpha=1}^N \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu_\alpha\right)^{\frac{d}{D-2}}, \quad (4)$$

where the coordinates (t, x^i) parameterise the d -dimensional world-volume of the p -brane, and the remaining coordinates of the D dimensional spacetime are r and the coordinates on the $(D - d - 1)$ -dimensional unit sphere, whose metric is $d\Omega^2$. The function f has a completely universal form [25]:

$$e^{2f} = 1 - \frac{k}{r^{\tilde{d}}}. \quad (5)$$

The dilatonic scalar fields $\varphi_\alpha = \vec{a}_\alpha \cdot \vec{\phi}$ are given by

$$e^{-\frac{1}{2}\epsilon\varphi_\alpha} = \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu_\alpha\right) \prod_{\beta=1}^N \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu_\beta\right)^{-\frac{d\tilde{d}}{2(D-2)}}, \quad (6)$$

where $\epsilon = 1$ for elementary solutions and $\epsilon = -1$ for solitonic solutions. The metric (4) has an outer horizon at $r = r_+ \equiv k^{1/\tilde{d}}$ and a curvature singularity at $r = r_- \equiv 0$. In general the curvature remains singular at the origin in the extremal limit $r_+ \rightarrow r_- = 0$. The mass per unit p -volume and the charges for each field strength are given by

$$m = k(\tilde{d} + 1) + k\tilde{d} \sum_{\alpha=1}^N \sinh^2 \mu_\alpha, \quad \lambda_\alpha = \frac{1}{2}\tilde{d}k \sinh 2\mu_\alpha. \quad (7)$$

The extremal limit corresponds to sending $k \rightarrow 0$ and $\mu_\alpha \rightarrow \infty$ while keeping the charges λ_α fixed. When all the charges are equal, the solutions reduce to single-scalar p -branes, described by the Lagrangian

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!}e^{-a\phi}F^2, \quad (8)$$

where the constant a can be parameterised by $a^2 = \Delta - 2d\tilde{d}/(D-2)$, with a given by $a^2 = (\sum_{\alpha,\beta}(M^{-1})_{\alpha\beta})^{-1}$ [24], implying that $\Delta = N/4$. In this case, the functions A and B in the metric are given by [25]

$$e^{2A} = \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu\right)^{-\frac{4\tilde{d}}{\Delta(D-2)}}, \quad e^{2B} = \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu\right)^{\frac{4d}{\Delta(D-2)}}. \quad (9)$$

Note that for choices of the dot products $M_{\alpha\beta}$ other than those given by (2), the equations of motion following from (1) can be cast into the form of Toda-like equations [32], which have not yet been solved. However their single-scalar truncations are solvable, yielding solutions of the same form (9), with values of Δ other than $4/N$. These solutions are not supersymmetric in the extremal limit.

The temperature and the entropy per unit p -volume for the p -brane metric (4) are given by [25]

$$T = \frac{\tilde{d}}{4\pi r_+} \prod_{\alpha=1}^N (\cosh \mu_\alpha)^{-1}, \quad S = \frac{1}{4} r_+^{\tilde{d}+1} \omega_{\tilde{d}+1} \prod_{\alpha=1}^N \cosh \mu_\alpha, \quad (10)$$

where $\omega_{\tilde{d}+1} = 2\pi^{\tilde{d}/2+1}/(\frac{1}{2}\tilde{d})!$ is the volume of the unit $(\tilde{d}+1)$ -sphere. For brevity we shall often refer to S , which is the entropy per unit p -volume, simply as the entropy. In the near-extremal regime, *i.e.* $k \ll \lambda_\alpha$ for all α ,¹ the temperature and the entropy are therefore related by

$$S \approx \gamma \left(\prod_{\alpha=1}^N \lambda_\alpha \right)^{\frac{\tilde{d}}{N\tilde{d}-2}} T^{\frac{2(\tilde{d}+1)-N\tilde{d}}{N\tilde{d}-2}}, \quad (11)$$

where $\gamma = \tilde{d}\omega_{\tilde{d}+1}(16\pi^2\tilde{d}^{-N-2})^{\tilde{d}/(N\tilde{d}-2)}/(16\pi)$. When the number N of charges in the p -brane solution satisfies

$$N = 2(D-2)/(d\tilde{d}), \quad (12)$$

we find that the entropy/temperature relation becomes

$$S \approx \gamma \left(\prod_{\alpha=1}^N \lambda_\alpha \right)^{\frac{d}{2}} T^{d-1} \sim T^p, \quad (13)$$

which has precisely the natural massless ideal-gas scaling, predicted by D-brane considerations since open strings on a Dirichlet p -brane can be viewed as an ideal gas of massless objects in a p -dimensional space. Note that if the charge parameters λ_α are all equal, in which case, the dilatonic scalars decouple, the ideal-gas entropy/temperature relation (13) reduces to the one found in [17]. It is interesting that this entropy/temperature relation holds even when the charges are not equal, in which case the dilatonic scalar fields do not decouple, although they do remain finite at the horizon in the extremal limit. Note that the temperature goes to zero in the extremal limit for all p -brane solitons satisfying the condition (12). On the other hand, such p -brane solitons have vanishing entropy in the extremal

¹The term ‘‘near-extremal regime’’ is used rather loosely here since in fact we are requiring the stronger condition that k be much smaller than each of the individual charges, rather than merely the sum of all the charges, so that we can approximate $\sinh 2\mu_\alpha$ in (7) by $\frac{1}{2}e^{2\mu_\alpha}$.

limit only for $p \geq 1$. Note also that when the condition (12) is satisfied, the entropy of the near-extremal p -branes can also be expressed as

$$S \sim (\delta m^2)^{\frac{d-1}{d}}, \quad (14)$$

which is consistent with the asymptotic density of states of p -branes [33]. Thus another approach that can yield a microscopic interpretation of the entropy is by counting the states of a fundamental p -brane.

It follows from (6) that if (12) is satisfied, the dilatonic scalar fields φ_α are finite at the horizon in the extremal limit. Moreover, we have $\sum_\alpha \varphi_\alpha = 0$, and hence there are a total of $(N - 1)$ non-vanishing scalar fields. It is worth remarking that the curvature and the field strengths are also finite at the horizon in the extremal limit when the condition (12) is satisfied. There are precisely five cases where N satisfies (12). When $N = 1$, this corresponds to the elementary membrane or the solitonic 5-brane in $D = 11$ where there is no dilaton field, or to the self-dual 3-brane in $D = 10$, where the dilaton decouples. The $N = 2$ case corresponds to the dyonic string in $D = 6$, with one dilaton field. The $N = 3$ and 4 cases correspond to the black holes in $D = 5$ and $D = 4$ with 2 and 3 dilatonic scalar fields respectively. In the latter three cases, the dilatonic scalar fields all decouple when the charges λ_α are all set equal.

For other values of N , (or, in the case of single-scalar solutions, other values of Δ) the entropy/temperature relation $S \sim T^{d-1}$ in the near-extremal regime breaks down. This breakdown of the relation can be expected since in these cases, the physical quantities such as dilatonic scalar fields, the field strengths and the curvature diverge at the horizon in the extremal limit. This indicates that for dilatonic p -branes, the tree-level approximation is not sufficient. In fact, as we shall argue next, the divergence of these physical quantities at the horizon in the extremal limit may merely be an artifact of the tree-level approximation to the effective action of the string. If we include also loop effects, the dilaton field, as well as the field strengths and the curvature, may be finite at the horizon, and the entropy/temperature relation of the resulting p -branes will satisfy precisely the natural massless ideal-gas scaling. In other words, these loop corrections have the effect of turning the tree-level singular-dilaton p -branes into regular-dilaton p -branes.

The inclusion of string and worldsheet loop corrections is in general very complicated; they will include higher powers of curvature and field strengths, and different exponents in the effective dilaton couplings [34]. Later, we shall argue that subject to rather general assumptions about the effect of higher-loop corrections, the ideal-gas entropy/temperature relation will be restored. First, let us consider a simple example which illustrates the

phenomenon, namely the heterotic string in $D = 6$, obtained by compactification of the $D = 10$ heterotic string on $K3$. In particular let us consider black hole solutions whose charges are carried by the Yang-Mills fields. The string loop-corrections in the effective Lagrangian have been calculated [35], and the relevant part is given by

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!}g(\phi)F^2, \quad (15)$$

where

$$g(\phi) = ve^{-\phi/\sqrt{2}} + \tilde{v}e^{\phi/\sqrt{2}}, \quad (16)$$

and v and \tilde{v} are constants. The first term in (16) is the classical contribution, and its coefficient v is always positive since it is essentially the Kac-Moody level [36, 37, 38]. The constant \tilde{v} can sometimes be negative. In this case, the kinetic energy of the gauge field becomes negative in the strong-coupling regime, indicating a phase-transition [38, 39, 40]. We shall restrict our attention in this paper to the cases where \tilde{v} is positive and hence the phase transition does not occur. The function $g(\phi)$ satisfies

$$\left. \frac{\partial g(\phi)}{\partial \phi} \right|_{\phi=\phi_0} = 0 \quad (17)$$

for $e^{\sqrt{2}\phi_0} = v/\tilde{v}$. This property, which is obviously not satisfied by the tree-level term alone, will play an important role in constructing regular-dilaton p -branes.

Given the standard p -brane ansatz for the field strength, black hole solutions for the loop-corrected Lagrangian (15) can be easily constructed. In fact, we can generalise the discussion to Lagrangians of the form (15) where F represents a field strength of arbitrary degree with some dilaton coupling of the form $g(\phi) = e^{-a\phi} + \dots$, where the first term is the classical contribution. Let us suppose that, as in the explicit example of the $D = 6$ string discussed above, the function $g(\phi)$ satisfies (17) for some ϕ_0 . It is then easy to see that there is a p -brane solution where the dilaton is the constant $\phi = \phi_0$, and the metric is given by

$$\begin{aligned} ds^2 &= e^{2A}(-e^{2f}dt^2 + dx^i dx^i) + e^{2B}(e^{-2f}dr^2 + r^2 d\Omega^2), \\ e^{-dA} &= 1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu, \quad e^{\tilde{d}B} = 1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu, \end{aligned} \quad (18)$$

with f again given by (5). The mass per unit volume and the charge are given by

$$m = \frac{2(D-2)k}{d} \sinh^2 \mu + k(\tilde{d} + 1), \quad \lambda = k \sqrt{\frac{(D-2)\tilde{d}}{2dg(\phi_0)}} \sinh 2\mu. \quad (19)$$

As for the previous black p -branes, the metric (18) has an outer horizon at $r_+ = k^{1/\tilde{d}}$, and a curvature singularity at $r = 0$. The temperature and the entropy per unit p -volume are

given by

$$T = \frac{1}{4\pi r_+} (\cosh \mu)^{-\frac{2(D-2)}{d\tilde{d}}}, \quad S = \frac{1}{4} r_+^{\tilde{d}+1} \omega_{\tilde{d}+1} (\cosh \mu)^{\frac{2(D-2)}{d\tilde{d}}}, \quad (20)$$

The extremal limit $r_+ \rightarrow 0$ is achieved by sending $k \rightarrow 0$ and $\mu \rightarrow \infty$ while keeping the charge λ fixed. In the near-extremal regime, the functions A and B behave as

$$e^A \sim r_+^{\tilde{d}/d}, \quad e^B \sim \frac{1}{r_+} \quad (21)$$

at the horizon. All the physical observables, including the field strength, are finite in this limit. In particular the curvature at $r = r_+$, which in the tree-level approximation was divergent as $r_+ \rightarrow 0$, is now, owing to the inclusion of loop corrections, regular in the extremal limit. It follows from (20) that the ideal-gas entropy/temperature relation (13) is restored. Note that in this case the entropy and energy δm also satisfy the relation (14).

The explicit $D = 6$ example above was rather exceptional, in that the 2-index field strength that we used for constructing the black-hole solution was derived from the Yang-Mills sector of the theory, rather than from the fields of the supergravity multiplet. Thus although a black-hole solution with an extremal limit can be constructed, it will not preserve any supersymmetry in this limit. Furthermore, the form (15) of the loop corrections is especially simple here, with no powers of F higher than the quadratic order arising. Owing to this simplicity, we were able to find an exact solution, and to show how the string-loop corrections gave rise to modifications that led again to the ideal-gas form of the entropy/temperature relation that was previously seen only for the regular-dilaton p -branes. We now turn to a general discussion of the expected form of the quantum-corrected p -brane solutions, to show that under rather broad assumptions, the quantum corrections can be expected to restore the ideal-gas entropy/temperature relation.

It has often been argued that the higher-order quantum corrections to the string effective action can be expected to smooth out the singularities that may arise in solutions of the purely classical limit. In particular, it is reasonable to expect that the quantum corrections will make all the physical quantities in the p -brane solutions finite at the outer horizon. In the example above, this phenomenon was indeed observed; in the string-loop corrected theory, the dilaton remained finite on the horizon (in fact, it became constant everywhere), and furthermore all invariant quantities built from the fields, such the curvature scalar and the square of the field strength, $F_{MN}F^{MN}$, remained finite on the horizon. In other examples, it may be that worldsheet loop corrections, rather than string loops, ensure finiteness of the physical quantities in the extremal limit. This is reminiscent of the stretched-horizon approach [28].

To proceed in general, let us make the assumption that the effect of loop corrections on *any* elementary or solitonic p -brane solution will be to render all physical invariant quantities finite at the horizon in the extremal limit. Thus we may consider such solutions that still have the general isotropic form (3), with the usual form of isotropic n -index field-strength configuration,

$$F = \lambda * \epsilon, \quad \text{or} \quad F = \lambda \epsilon, \quad (22)$$

where ϵ is the volume form on the unit $(D - d - 1)$ -sphere. Owing to the loop corrections however, the precise form of the solution will no longer be given by (4). If we calculate the square of the field strength, using (22) and (3), we find that it is proportional to $(r e^B)^{-2n}$, and thus to be finite on the horizon in the extremal limit, we must have

$$r_+ e^{B(r_+)} \longrightarrow c, \quad (23)$$

where c is some constant. Furthermore, we find that finiteness of the curvature scalar for the metric (3) at $r = r_+$ again requires (23) in the extremal limit, together with some further regularity condition for the function A . The entropy per unit p -volume, and the temperature, are given by [25]

$$S = \frac{1}{4} r_+^{\tilde{d}+1} \omega_{\tilde{d}+1} e^{(\tilde{d}+1)B(r_+)+(d-1)A(r_+)}, \quad (24)$$

$$T = \frac{\tilde{d}}{4\pi r_+} e^{A(r_+)-B(r_+)}. \quad (25)$$

Substituting (23) into these expressions, we therefore find that the entropy and temperature for the loop-corrected p -brane will take the form

$$S = \frac{1}{4} c^{D-2} \omega_{\tilde{d}+1} \left(\frac{4\pi}{\tilde{d}}\right)^{d-1} T^{d-1} \quad (26)$$

in the near-extremal regime. Thus we see that the assumption of regularity of physical fields on the horizon even in the extremal limit implies that the entropy/temperature relation will always take the ideal-gas form. Note that for the loop corrected p -branes satisfying the above ideal-gas relation, the explicit form of the function A is unimportant.

To summarise, we have studied the relation between entropy and temperature for near-extremal p -branes. It was recently shown [17] that in classical p -brane solitons where the dilaton is either absent (as in $D = 11$) or constant, the entropy is proportional to T^p . This relation is the one that is found for an ideal gas of massless particles in p spatial dimensions, which is in line with expectations from D-brane state-counting arguments. We have extended this result to include cases where the dilaton does not decouple, provided that it is regular on the horizon in the extremal limit. The entropy/temperature relation

breaks down in cases where the dilaton is singular on the horizon in the extremal limit. In fact the majority of classical p -brane solitons exhibit such singularities, and singularities in the curvature and field strengths, indicating a breakdown of the tree-level approximation. It can be argued that the inclusion of loop corrections or non-perturbative effects will remove these divergences. We have demonstrated that this assumption is sufficient to recover the ideal gas entropy/temperature relation for all the near-extremal p -branes, and hence may remove the previous obstacles to providing a microscopic interpretation of the entropy by the counting of states.

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