Quantum Zeno and Anti-Zeno Effect without Rotating Wave Approximation

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Abstract

The effect of the anti-rotating terms on the short-time evolution and the quantum Zeno (QZE) and anti-Zeno (AQZE) effects is studied for a two-level system coupled to a bosonic environment. A unitary transformation and perturbation theory are used to obtain the electron self-energy, energy shift and the enhanced QZE or the AQZE, simultaneously. The calculated Zeno time depends on the atomic transition frequency sensitively. When the atomic transition frequency is smaller than the central frequency of the spectrum of boson environment, the Zeno time is prolonged and the anti-rotating terms enhance the QZE; when it is larger than that the Zeno time is reduced and the anti-rotating terms enhance the AQZE.

PACS numbers: 03.65.Xp; 42.50.Ct; 03.65.Yz

The quantum Zeno effect (QZE) and anti-Zeno effects (AQZE) have been widely discussed for decades theoretically[1] and recently experimentally[2]. In an unstable quantum two-level (multi-level) system, frequently measurement can reduce or accelerate the decay processes [1, 3, 4, 5, 6, 7]. The survival probability at an excited state of a quantum system interacting with an environment is a decaying function of time. Frequent measurements at extremely short time interval may slow down the decay process, because the decay of the excited state is almost zero at the beginning of the decay process[1, 3], which is known as the QZE. It was also found that if the measurement time interval is short, but not extremely short, the decay of the excited state could be accelerated[3], which is known as the AQZE. Let $P(\tau)$ denote the survival probability (after a short time interval τ) at the initial state, which can be written as $P(\tau) = \exp(-\gamma(\tau)\tau)$. After N time measurements at equal τ, the survival probability reads $P^N(\tau) = \exp[-\gamma(\tau)N\tau] = \exp[-\gamma(\tau)t]$ with $\gamma(\tau)$ the effective decay rate. If $N = 1$, $P(t) = \exp(-\gamma(t)t)$, which goes to $P(t) \to \exp(-\gamma_0 t)$ for large enough t, where γ_0 is the decay rate under the Weisskopf-Wigner approximation. We will have the QZE if $\gamma(\tau) < \gamma_0$, and the AQZE if $\gamma(\tau) > \gamma_0$.

It is well known that the whole spectrum of the environment at off-resonance (with the transition frequency) is important for the QZE and AQZE. If the atomic transition frequency is located not at the maximum of the spectrum, we can have the QZE and AQZE depending on the measurement time interval. In the previous studies on QZE and AQZE [1, 3, 4, 5, 6, 7], the rotating wave approximation (RWA) is used. However, this approximation raises the question what the influence of the anti-rotating terms on the QZE and AQZE is, as they might have the same order contribution as the spectrum components off-resonant with the atomic transition, especially for the QZE where measurement time interval is extremely short[1, 3, 4]. Therefore we ask ourselves, "what is the role of anti-rotating terms on the QZE and AQZE?"

The model to describe an unstable quantum system is the following spin-boson model with the Hamiltonian[8, 9],

$$H = \frac{1}{2}\omega_0\sigma_z + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger + b_k)\sigma_x, \tag{1}$$

$b_k^\dagger$ (b_k) is the creation (annihilation) operator of boson mode with frequency ω_k, σ_x and σ_z are Pauli matrices describing the two-level system. ω_0 is the transition frequency between the up $|+\rangle$ and down state $|-\rangle$: $\sigma_z|\pm\rangle = \pm|\pm\rangle$. g_k is the coupling between the two-level system and the environment, which can be characterized by the interacting spectrum[3, 4, 8]: $G(\omega) = \frac{1}{4}\sum_k g_k^2 \delta(\omega - \omega_k)$. This model Hamiltonian is used for a large number of different physical and chemical processes, such as the atom-field interaction and the QZE in quantum optics[3, 4, 9], coupled quantum dots on a solid state substrate[8, 10], and the macroscopic quantum coherence experiment in SQUID's[11, 12].

The Hamiltonian (1) cannot be solved exactly and usually the RWA is used[3, 4, 9] for which the Hamiltonian is taken to be

$$H_{RWA} = \frac{1}{2}\omega_0\sigma_z + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger \sigma_- + b_k\sigma_+), \tag{2}$$

where $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$. H_{RWA} can be solved in the so-called one-boson sector with the initial state $|\psi(0)\rangle = |+\rangle|\{0_k\}\rangle$, where $|\{0_k\}\rangle$ is the vacuum state for every k. The survival amplitude of finding the system still in $|\psi(0)\rangle$ at $\tau > 0$ is[3, 4]

$$x_{RWA}(\tau) = \frac{1}{2\pi i}\int_B \frac{e^{p\tau}dp}{p + i\omega_0 + \frac{1}{4}\sum_k \frac{g_k^2}{p+i\omega_k}}, \tag{3}$$

where B is the so-called Bromwich path. The survival probability in the initial state is $P_{RWA}(\tau) = |x_{RWA}(\tau)|^2$ and the effective decay rate $\gamma_{RWA}(\tau)$ for a short interval τ can be calculated as follows[3],

$$\gamma_{RWA}(\tau) = 2\pi\int_0^\infty d\omega G(\omega)F(\omega - \omega_0), \tag{4}$$

$$F(\omega - \omega_0) = 2\sin^2\left[\frac{\omega - \omega_0}{2}\tau\right]/\pi\tau(\omega - \omega_0)^2. \tag{5}$$

Since $F(\omega - \omega_0) \to \delta(\omega - \omega_0)$ (the Dirac $\delta-$function) for large enough t, we have the decay rate $\gamma_0 = 2\pi G(\omega_0)$ in the Weisskopf-Wigner approximation. Eqs.(3) and (4) are main results of the RWA.

When the anti-rotating terms are included, the above method is no longer valid. Here we present an analytical approach, based on unitary transformation and perturbation theory to calculate the survival amplitude and the effective decay rate for

Hamiltonian (1) in order to clarify the impact of the anti-rotating terms on the short time evolution and on the QZE and AQZE. In the following the interacting spectrum of the environment is assumed as,

$$G(\omega) = \frac{\frac{1}{2}\alpha\omega\Omega^4}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}, \tag{6}$$

where coupling strength, α, is a dimensionless constant and Ω is the center of gravity of the spectrum[11, 12], and Γ is the width of the spectrum. We will show that the off-resonance ratio ω_0/Ω plays an important role and the effect of the anti-rotating terms must be taken into account especially for the off-resonant case $\omega_0/\Omega \ll (1-\Gamma/\Omega)$. We note that when $\Gamma^2 \ll \Omega^2$, $G(\omega)$ is mainly a sharp Lorentzian-type peak similar to the case of resonant Rabi oscillation[3, 11, 12]. Throughout this paper we set $\hbar = 1$.

We treat the anti-rotating terms by a unitary transformation[13]: $H' = \exp(S)H\exp(-S)$ with

$$S = \sum_k \frac{g_k}{2\omega_k}\xi_k(b_k^\dagger - b_k)\sigma_x. \tag{7}$$

Here we introduce in S a k-dependent function ξ_k and its form will be determined later. The transformation can be carried out, and the result is $H' = H_0' + H_1' + H_2'$,

$$H_0' = \frac{1}{2}\eta\omega_0\sigma_z + \sum_k \omega_k b_k^\dagger b_k - \sum_k \frac{g_k^2}{4\omega_k}\xi_k(2-\xi_k), \tag{8}$$

$$H_1' = \frac{1}{2}\sum_k g_k(1-\xi_k)(b_k^\dagger + b_k)\sigma_x - \frac{1}{2}\eta\omega_0 i\sigma_y \sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k), \tag{9}$$

$$H_2' = \frac{1}{2}\omega_0\sigma_z\left(\cosh\{\sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k)\} - \eta\right)$$
$$-\frac{1}{2}\omega_0 i\sigma_y\left(\sinh\{\sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k)\} - \eta\sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k)\right) \tag{10}$$

where

$$\eta = \exp[-\sum_k \frac{g_k^2}{2\omega_k^2}\xi_k^2]. \tag{11}$$

H_0' is the unperturbed part of H' and, obviously, it can be solved exactly because in which the two-level system and the bosons are decoupled. The eigenstate of H_0' is a direct product: $|\pm\rangle|\{n_k\}\rangle$, where $|\{n_k\}\rangle$ is the eigenstate of bosons with n_k bosons for mode k. In particular, the ground state of H'_0 is $|g_0\rangle = |-\rangle|\{0_k\}\rangle$.

H'_1 and H'_2 depend on g_k and are small, which are treated as perturbation. Because of the definition of η in Eq.(11), H'_2 contains the terms of two-boson and multi-boson non-diagonal transitions and its contribution to physical results is $(g_k^2)^2$ and higher. So, H'_2 can be omitted and we approximate $H' \approx H'_0 + H'_1$. H'_1 contains the terms of single-boson transition and we chose ξ_k as

$$\xi_k = \frac{\omega_k}{\omega_k + \eta\omega_0}, \tag{12}$$

so that H'_1 is of the form

$$H'_1 = \eta\omega_0 \sum_k \frac{g_k}{\omega_k}\xi_k \left[b_k^\dagger\sigma_- + b_k\sigma_+\right], \tag{13}$$

It is easily to check that $H'_1|g_0\rangle = 0$. We note that the transformed Hamiltonian H' is of a form similar to H_{RWA} but ω_0 and $g_k/2$ in (2) are replaced by $\eta\omega_0$ and $\eta\omega_0 g_k\xi_k/\omega_k$, respectively.

As $H'_1|g_0\rangle = 0$, we have $H'|g_0\rangle \approx (H'_0 + H'_1)|g_0\rangle = E_g|g_0\rangle$ with the ground state energy $E_g = -\frac{1}{2}\eta\omega_0 - \sum_k \frac{g_k^2}{4\omega_k}\xi_k(2-\xi_k)$, which is lower than the ground state energy $E_g^{RWA} = -\frac{1}{2}\omega_0$ in the RWA . It can be seen that the third term in H'_0 (or the second term in E_g) does not depend on the atomic transition frequency; it is the self-energy of the free electron due to the vacuum fluctuations. In previous treatment[9] the self-energy usually needs to be calculated separately.

The transition frequency (originally ω_0) is modified to $\omega_a = \eta\omega_0$ with the modification factor η due to the anti-rotating terms. Figure 1 shows η as a function of α for various values of ω_0. The horizontal dotted line is $\eta = 1$ in the RWA. We note that $(\eta-1)\omega_0$ is an energy shift of the two-level system resulted from the anti-rotating terms. By using this unitary transformation Eq.(7), we can obtain the self-energy and the energy shift simultaneously.

Since $|g_0\rangle$ is the ground state of H', the ground state of the original H is $\exp(-S)|g_0\rangle$. Then, the survival amplitude of finding the system in the initial state is $x(\tau) = \langle+|\langle\{0_k\}|\exp(-iH'\tau)|+\rangle|\{0_k\}\rangle$. Since H' is of a form similar to H_{RWA}, $x(\tau)$ can be calculated in the so-called one-boson sector in the same way as $x_{RWA}(\tau)$ was,

$$x(\tau) = \frac{1}{2\pi i}\int_B \frac{e^{p\tau}dp}{p + i\eta\omega_0 + \sum_k \frac{V_k^2}{p+i\omega_k}}, \tag{14}$$

where $V_k = \eta\omega_0 g_k \xi_k/\omega_k$. The survival probability in the initial state is $P(\tau) = |x(\tau)|^2$ and the effective decay rate $\gamma(\tau)$ for a short interval τ is

$$\gamma(\tau) = 2\pi \int_0^\infty d\omega G'(\omega) F(\omega - \eta\omega_0), \tag{15}$$

where $G'(\omega) = 4G(\omega)(\eta\omega_0)^2/(\omega+\eta\omega_0)^2 = G(\omega)f(\omega)$. The spectrum is modulated by the factor $f(\omega) = (2\omega_a)^2/(\omega+\omega_a)^2$. The physics of this factor is clear. It is proportional to $1/(\omega+\omega_a)^2$ because it comes from the anti-rotating terms. It is equal to 1 for $\omega = \omega_a$, because the decay rate at large enough time is proportional to $G(\omega_a)$. Please note that $f(\omega) > 1$ (or < 1) and $G'(\omega) >$(or $<$)$G(\omega)$ for $\omega <$(or $>$)ω_a.

For the short-time limit the survival probability is quadratic in τ: $P(\tau) = 1 - \tau^2/\tau_Z^2$ for $\tau \to 0$, which is explicitly different from the exponential decay law[3, 4, 6]. The quantity τ_Z is referred to as "Zeno time"[4, 6] and can be calculated by using Eq.(15),

$$\tau_Z = \left(\frac{d}{d\tau}\gamma(\tau)\right)^{-1/2}\Bigg|_{\tau\to 0} = \left(\int_0^\infty d\omega G'(\omega)\right)^{-1/2}, \tag{16}$$

while the Zeno time in the RWA is approximately $\tau_Z^{RWA} = (\int_0^\infty d\omega G(\omega))^{-1/2}$. One can check that τ_Z^{RWA} appears to be independent of ω_0 because the integrand $G(\omega)$ does not depend on ω_0. This should not be physically correct since the short-time evolution $P(\tau) = 1 - \tau^2/\tau_Z^2$ should depend on where the transition frequency ω_0 is located in the interacting spectrum. Our τ_Z without RWA does depend on ω_0, since $G'(\omega)$ is a function of ω_0. When the atomic transition frequency is smaller than the central frequency of the spectrum the Zeno time is prolonged; when it is larger the Zeno time is reduced, as shown in Fig.2. Also the energy shift is dependent on the location of ω_0 in the spectrum. The smaller the ratio ω_0/Ω or the larger the α (stronger interaction), the larger the energy shift will be (see Fig. 1).

In Fig.3, $\gamma(\tau)$ is plotted for ω_0 located in the low frequency part of the spectrum, $\omega = 0.2\Omega$. The dashed line is the result of RWA, and one can see that for extremely short time ($\gamma_0\tau < 0.01$) RWA predicts the QZE but for a short time ($\gamma_0\tau > 0.01$) there is a possibility of AQZE. However, by taking into account the anti-rotating terms, we only have the QZE and no AQZE. This is a general conclusion of our calculation for the off-resonant spectrum with $\omega_0/\Omega \ll (1-\Gamma/\Omega)$, which is different from that of Ref.[3].

Kofman and Kurizki[3] concluded that when ω_0 is significantly detuned from the maximum of $G(\omega)$ at Ω (so that $G(\omega_0) \ll G(\Omega)$), the effective decay rate $\gamma(\tau)$ grows with decreasing τ and leads to the AQZE of decay acceleration by frequent measurements. The reason can be understood by checking $G(\omega)$ and $G'(\omega)$ in Eqs.(4) and (15). The modification factor $f(\omega)$ due to the anti-rotating terms on the spectrum $G(\omega)$ is $f(\omega) < 1$ for $\omega > \omega_a$. When ω_a is much smaller (or smaller) than the spectrum center frequency Ω, the spectrum $G'(\omega)$ is greatly flattened (or flattened) compared with $G(\omega)$, see the inset of Fig.3 (or Fig. 4). The dephasing function[3] $F(\omega-\omega_0)$ is mainly a single-peak function with peak at ω_0 and width $\sim 1/\tau$. Since the integrand in Eq.(4) is $G(\omega)F(\omega-\omega_0)$, when ω_0 is far below the maximum of $G(\omega)$, one can check that $\gamma_{RWA}(\tau)$ grows with decreasing τ (AQZE) because $F(\omega-\omega_0)$ is then probing more of the rising part of $G(\omega)$. Our result is different from Ref.[3] because the integrand in Eq.(15) is $G'(\omega)F(\omega-\omega_a)$ and $\gamma(\tau)$ decreases with decreasing τ (QZE) since $F(\omega-\omega_a)$ already covers the main part of $G'(\omega)$.

In Fig.4 we plot $\gamma(\tau)$ for $\omega_0 = 0.5\Omega$. Here RWA predicts anti-QZE for $\gamma_0\tau > 0.09$, but our result predicts very weak anti-QZE for the region $0.24 < \gamma_0\tau < 0.58$, because the factor $f(\omega)$ coming from the anti-rotating terms slightly flattens the spectrum (see the inset of Fig.4).

For the resonant case, $\omega_a \sim \Omega$, our calculations with anti-rotating terms are nearly the same as those of RWA, because the factor $f(\omega)$ come from the anti-rotating terms is almost equal to 1 around the frequency $\omega = \Omega \sim \omega_a$, and changes the spectrum very little. This can be understood easily since the RWA is a good approximation for the resonant case.

Furthermore, for ω_0 larger than Ω, we have $f(\omega) > 1$ for $\omega < \omega_0$, that is to say, the main part of the spectrum enhanced by the anti-rotating terms. In Fig.5 we plot $\gamma(\tau)$ for $\omega_a = 1.5\Omega$, the region for AQZE is wider than predicted by RWA, because the anti-rotating terms raise the peak of $G'(\omega)$; see the inset of Fig.5.

In summary: The impact of the anti-rotating terms on the short-time evolution and the quantum Zeno and anti-Zeno effects is studied for a two-level system coupled to a bosonic environment. We present an analytical approach, based on a unitary transformation and a perturbation method. With this method, we can simultaneously obtain the electron self-energy, energy shift and the enhancement of the quantum Zeno or the anti-Zeno effects. The effective decay rate is calculated. The Zeno time depends on the atomic transition frequency sensitively. When the atomic transition frequency is smaller than the central frequency of the spectrum the Zeno time is prolonged and the anti-rotating terms enhance the QZE; when it is larger than that the Zeno time is reduced and the anti-rotating terms enhance the AQZE.

Acknowledgement

Part of the work of H.Z. was done when he visited Department of Physics, Baptist University of Hong Kong. H.Z. was supported partly by the National Natural Science Foundation of China (Grants No.90503007 and No.10734020). S.Y.Z. was supported by HKUST/CA/06-07/02 and FRG of HKBU.

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Figure Captions

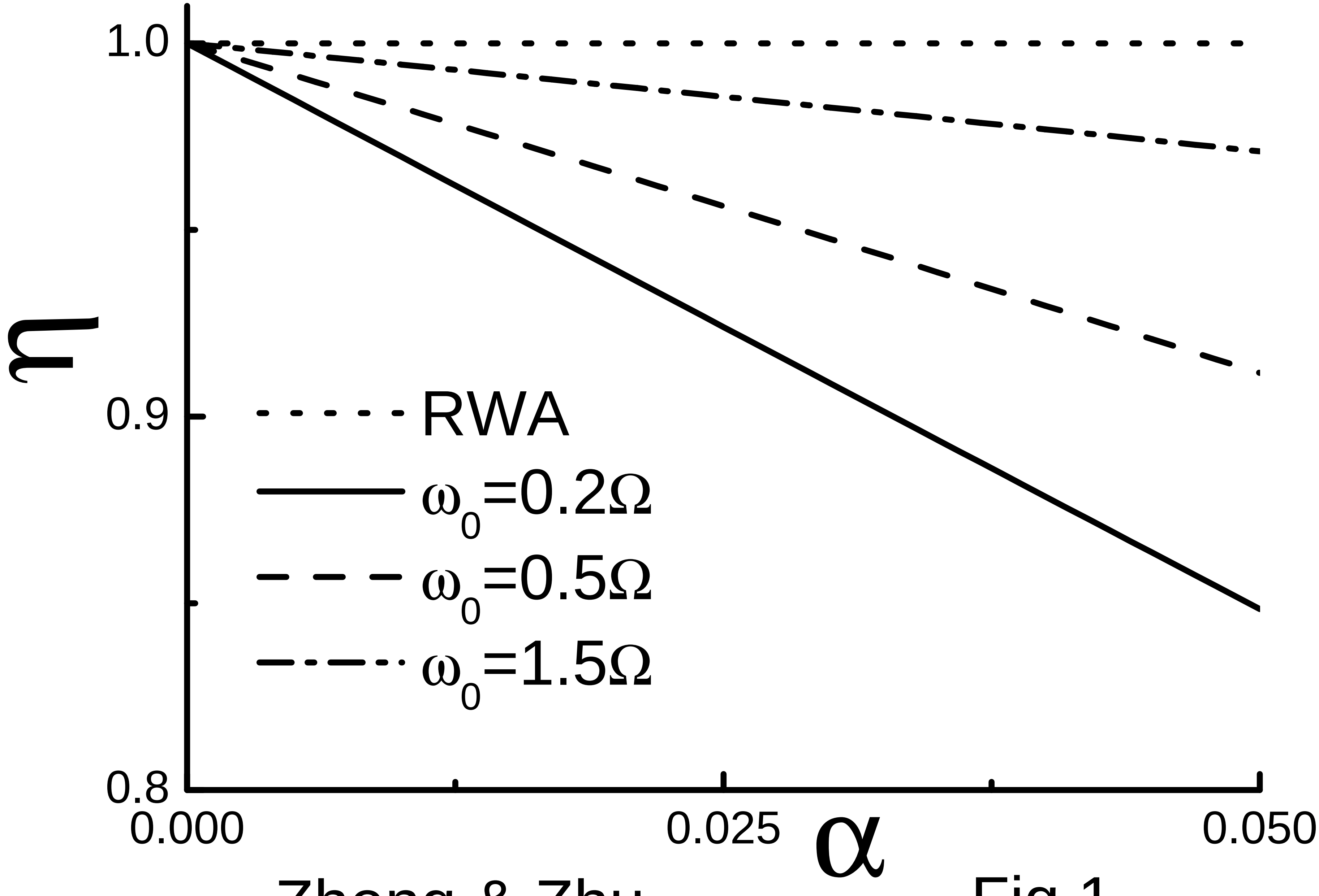
Fig.1 The renormalization factor η as a function of the coupling α.

Fig.2 The Zeno time as a function of the transition frequency ω_0.

Fig.3 Decay rate $\gamma(\tau)$ for $\alpha = 0.02$, $\omega_0 = 0.2\Omega$ and $\Gamma = 0.4\Omega$. The dashed line is the result of RWA. Inset: $G(\omega)$ (dotted line) and $G'(\omega)$ (solid line).

Fig.4 Decay rate $\gamma(\tau)$ for the below resonant spectrum with $\alpha = 0.02$, $\omega_0 = 0.5\Omega$ and $\Gamma = 0.4\Omega$. The dashed lines is the result of RWA. Inset: $G(\omega)$ (dotted line) and $G'(\omega)$ (solid line)

Fig.5 Decay rate $\gamma(\tau)$ for the above resonant spectrum with $\alpha = 0.02$, $\omega_0 = 1.5\Omega$ and $\Gamma = 0.4\Omega$. The dashed line is the result of RWA. Inset: $G(\omega)$ (dotted line) and $G'(\omega)$ (solid line)



Zheng & Zhu Fig.1

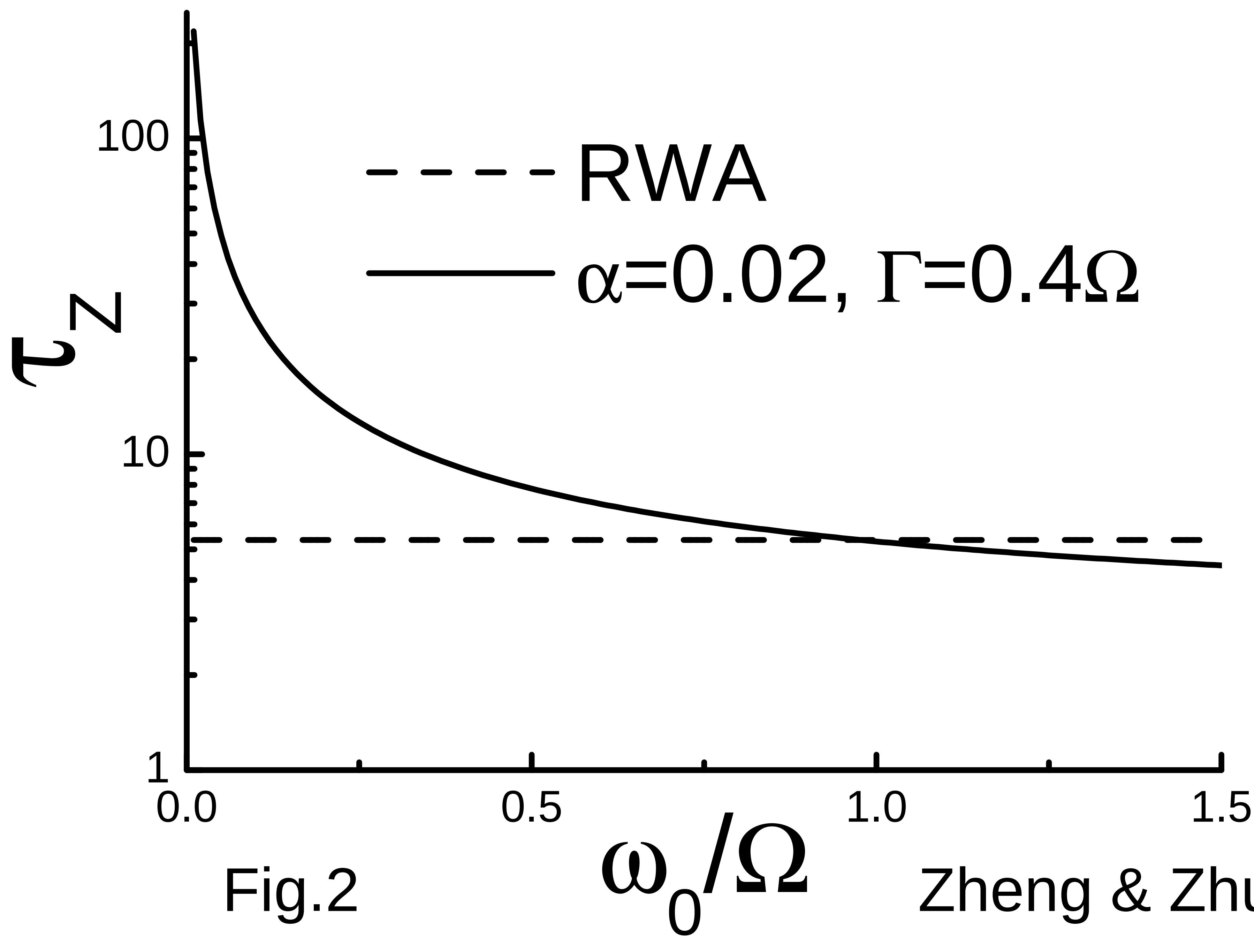


Fig.2 Zheng & Zhu

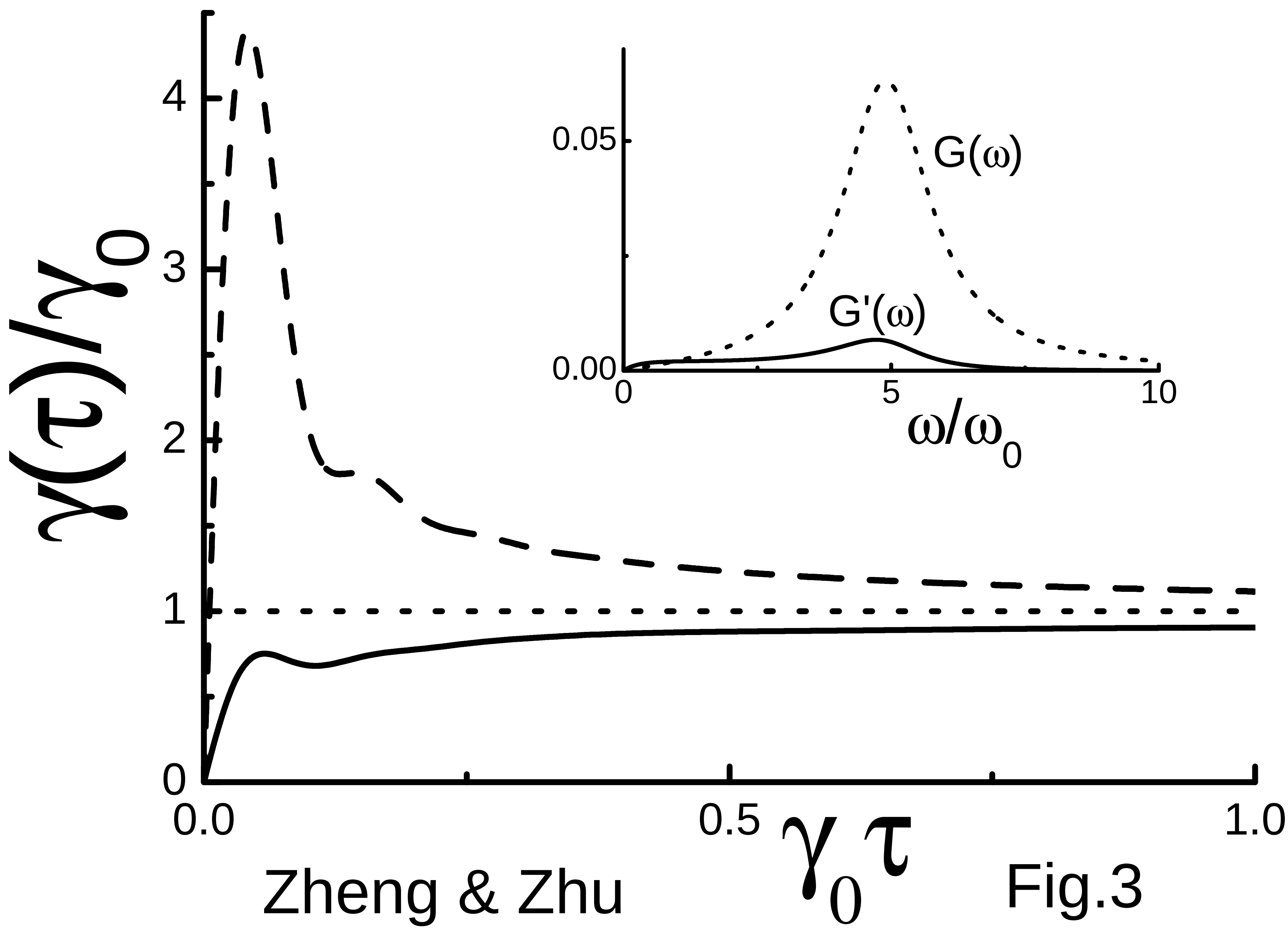


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Fig.3

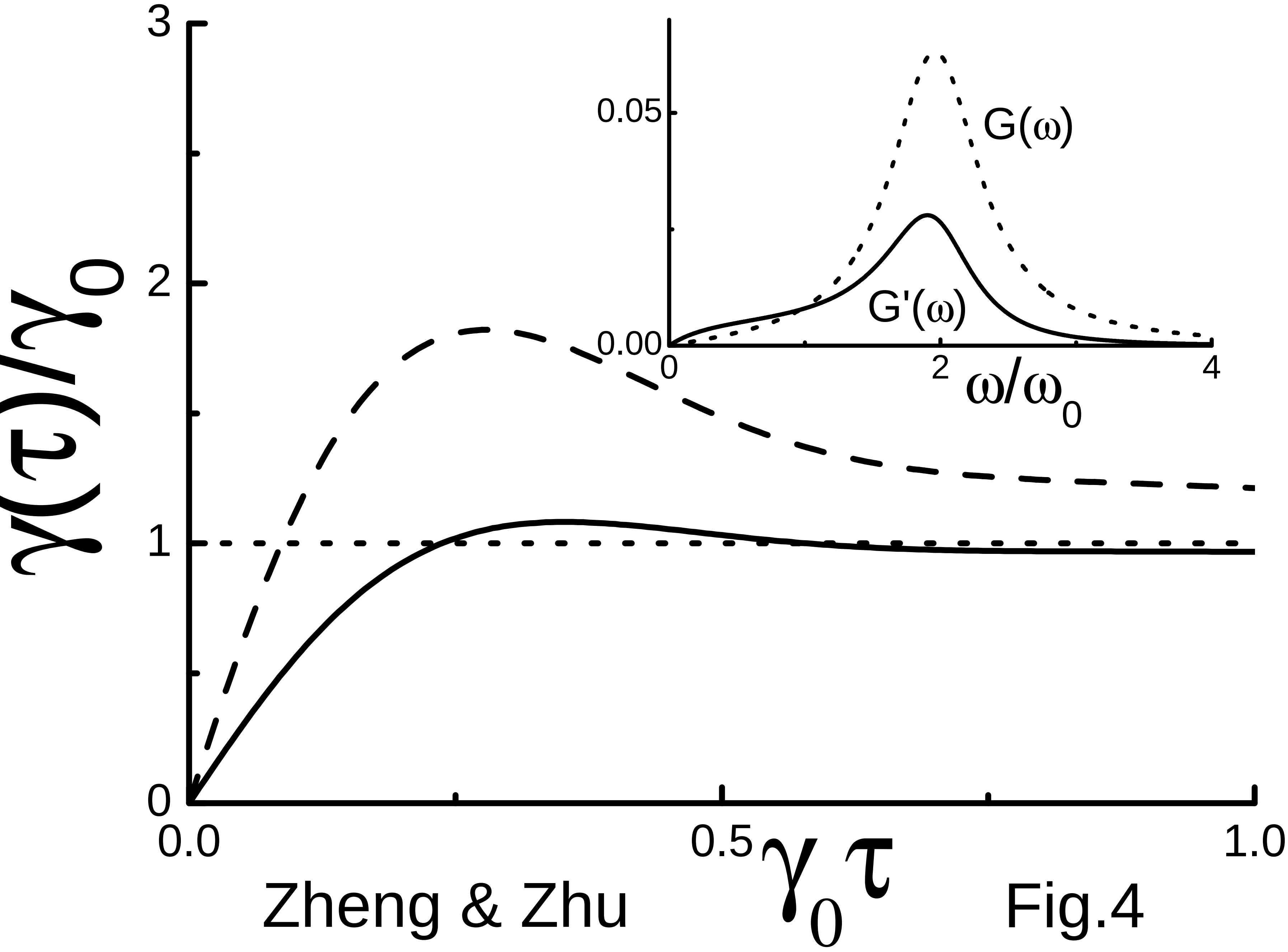

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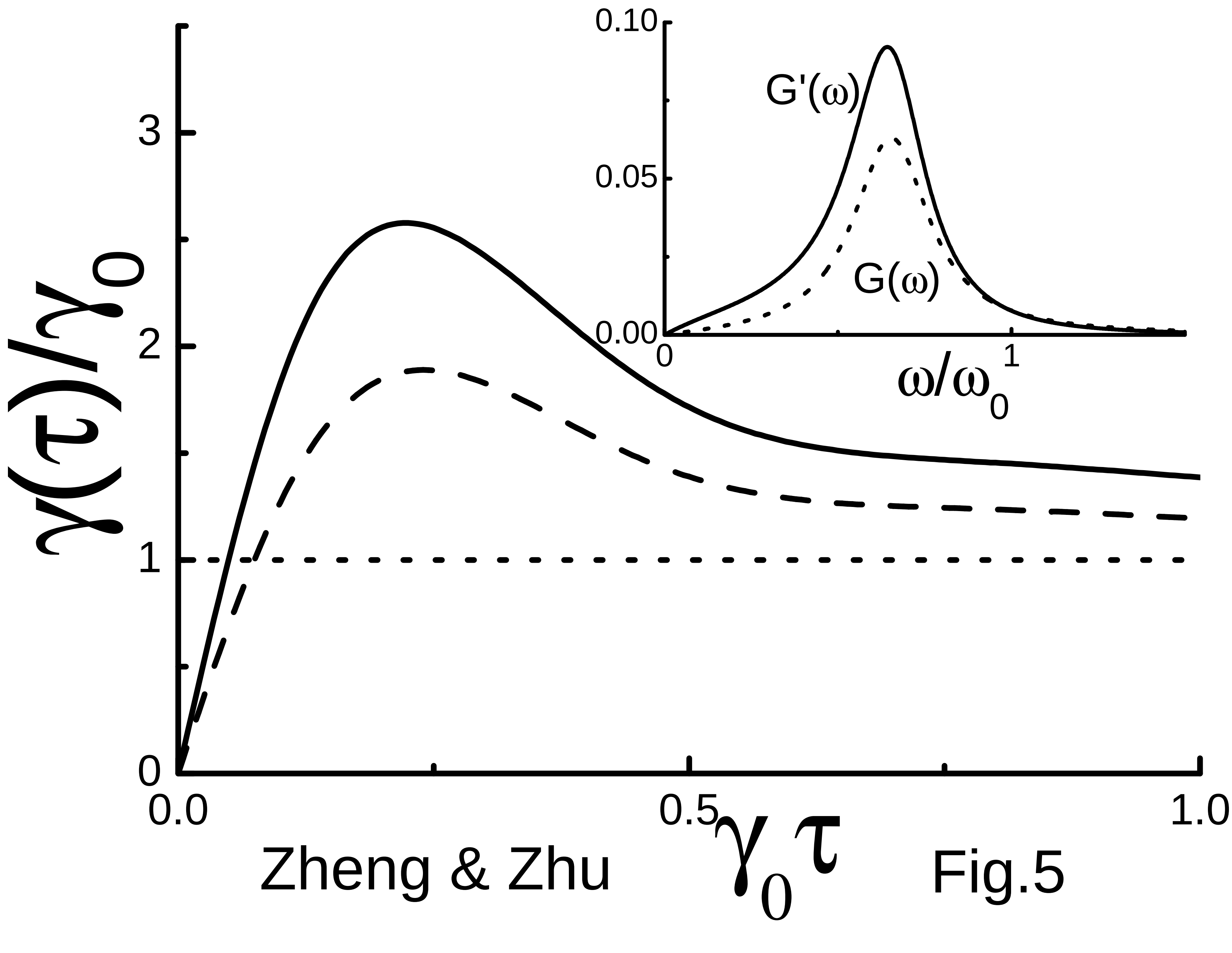
Fig.4



Zheng & Zhu Fig.5