

Optically Induced Spin Hall Effect in Atoms

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We propose an optical means to realize a spin hall effect (SHE) in neutral atomic system by coupling the internal spin states of atoms to radiation. The interaction between the external optical fields and the atoms creates effective magnetic fields that act in opposite directions on “electrically” neutral atoms with opposite spin polarizations. This effect leads to a Landau level structure for each spin orientation in direct analogy with the familiar SHE in semiconductors. The conservation and topological properties of the spin current, and the creation of a pure spin current are discussed.

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Information devices based on spin states of particles require a lot less power consumption than equivalent charge based devices [1]. To implement practical spin-based logical operations, a basic underlying theory, i.e. spin hall effect (SHE) has been widely studied for the creation of spin currents in semiconductors [2, 3, 4, 5]. Nearly all current publications on SHE involve some form of spin-orbit coupling, including the interaction between charged particles in semiconductors and external electric field. The physics of SHE in semiconductors is: in the presence of spin-orbit coupling, the applied electric field leads to a transverse motion (perpendicular to the electric field), with spin-up and spin-down carriers moving oppositely to each other, creating a transverse spin current. However, spin current can also be generated by interacting optical fields with charged particles in semiconductors [6, 7], even in absence of spin-orbit coupling [8].

In this letter, we show how SHE can be induced by optical fields in neutral atomic system. Quantum states of atoms can be manipulated by coupling their internal degrees of freedom (atomic spin states) to radiation, making it possible to control atomic spin propagation through optical methods. We consider here an ensemble of cold Fermi atoms interacting with two external light fields (Fig. 1). The ground ($|g_{\pm}, \pm\frac{1}{2}\rangle$) and excited ($|e_{\pm}, \pm\frac{1}{2}\rangle$) states are hyperfine angular momentum states (atomic spins) with their total angular momenta $F_g = F_e = 1/2$. The transitions from $|g_{-}, -\frac{1}{2}\rangle$ to $|e_{+}, \frac{1}{2}\rangle$ and from $|g_{+}, \frac{1}{2}\rangle$ to $|e_{-}, -\frac{1}{2}\rangle$ are coupled respectively by a σ_+ light with the Rabi-frequency $\Omega_2 = \Omega_{20} \exp(i(\mathbf{k}_2 \cdot \mathbf{r} + l_2 \vartheta))$ and by a σ_- light with the Rabi-frequency $\Omega_1 = \Omega_{10} \exp(i(\mathbf{k}_1 \cdot \mathbf{r} + l_1 \vartheta))$, where $\mathbf{k}_{1,2} = k_{1,2} \hat{\mathbf{e}}_z$ are the wave-vectors and $\vartheta = \tan^{-1}(y/x)$. l_1 and l_2 indicate that σ_+ and σ_- photons are assumed to have the orbital angular momentum $\hbar l_1$ and

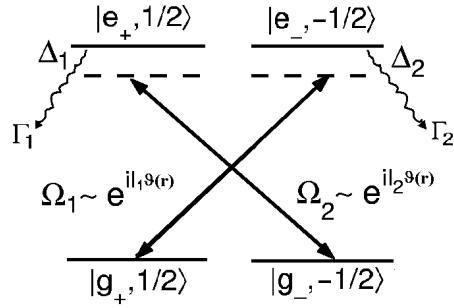


FIG. 1: Fermi atoms with four-level internal hyperfine spin states interacting with two light fields. This can be experimentally realized with alkali atoms, such as ${}^6\text{Li}$ atoms ($2^2S_{1/2}(F=1/2) \longleftrightarrow 2^2P_{1/2}(F'=1/2)$) [9].

$\hbar l_2$ along the $+z$ direction, respectively [10]. For simplicity, we replace the notations $|\alpha_{\pm}, \pm\frac{1}{2}\rangle$ by $|\alpha_{\pm}\rangle$ ($\alpha = e, g$). The \mathbf{r} -representation atomic wave function is denoted by $\Phi_{\alpha}(\mathbf{r}, t)$. It is helpful to introduce the slowly-varying amplitudes of atomic wave-functions by (setting $\omega_{g\pm} = 0$): $\Psi_{g\pm} = \Phi_{g\pm}$, $\Psi_{e+} = \Phi_{e+}(\mathbf{r}, t) e^{-i(\mathbf{k}_2 \cdot \mathbf{r} - (\omega_{e+} - \Delta_1)t)}$, $\Psi_{e-} = \Phi_{e-}(\mathbf{r}, t) e^{-i(\mathbf{k}_1 \cdot \mathbf{r} - (\omega_{e-} - \Delta_2)t)}$, where $\hbar\omega_{\alpha}$ is the energy of the state $|\alpha\rangle$, $\Delta_{1,2}$ are corresponding detunings. The total Hamiltonian $H = H_0 + H_1 + H_2$ of the system reads:

$$\begin{aligned} H_0 &= \sum_{\alpha=e\pm g\pm} \int d^3r \Psi_{\alpha}^* \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right) \Psi_{\alpha}, \\ H_1 &= \hbar \Delta_1 \int d^3r \Psi_{e+}^* S_{e+e+} \Psi_{e+} \\ &\quad + \hbar \int d^3r (\Psi_{e+}^* \Omega_{10} e^{il_1 \vartheta} S_{1+} \Psi_{g-} + h.a.), \quad (1) \\ H_2 &= \hbar \Delta_2 \int d^3r \Psi_{e-}^* S_{e-e-} \Psi_{e-} \\ &\quad + \hbar \int d^3r (\Psi_{e-}^* \Omega_{20} e^{il_2 \vartheta} S_{2+} \Psi_{g+} + h.a.), \end{aligned}$$

where the atomic operators $S_{e\pm e\pm} = |e_{\pm}\rangle\langle e_{\pm}|$, $S_{1+} = |e_+\rangle\langle g_-|$, $S_{2+} = |e_-\rangle\langle g_+|$, $S_{1+}^\dagger = S_{1-}$, $S_{2+}^\dagger = S_{2-}$, and

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$V(\mathbf{r})$ is an external trap potential. The collisions (s-wave scattering) between cold Fermi atoms are negligible. The interaction part of the Hamiltonian can be diagonalized with a local unitary transformation: $\tilde{H}_I = U(\mathbf{r})H_IU^\dagger(\mathbf{r}) = U(\mathbf{r})(H_1 + H_2)U^\dagger(\mathbf{r})$ where

$$U(\mathbf{r}) = \begin{bmatrix} U_1 & 0 \\ 0 & U_2 \end{bmatrix} \quad (2)$$

with

$$U_j = \begin{bmatrix} \cos\theta_j & \sin\theta_j e^{il_j\vartheta(\mathbf{r})} \\ -\sin\theta_j e^{-il_j\vartheta(\mathbf{r})} & \cos\theta_j \end{bmatrix}, \quad j = 1, 2.$$

Under this transformation the four eigenstates of interaction Hamiltonian can be obtained as $[\lvert\psi_+\rangle, \lvert\psi_-\rangle]^T = U_1[\lvert e_+\rangle, \lvert g_-\rangle]^T$ and $[\lvert\phi_+\rangle, \lvert\phi_-\rangle]^T = U_2[\lvert e_-\rangle, \lvert g_+\rangle]^T$. The mixing angles θ_j are defined by $\tan\theta_{1,2} = E_{\psi,\phi}^-/\Omega_{1,2}$, where the eigenvalues of $\lvert\psi_\pm\rangle$ and $\lvert\phi_\pm\rangle$ can be calculated by: $E_{\psi,\phi}^\pm = (\Delta_{1,2} \pm \sqrt{\Delta_1^2 + 4|\Omega_{1,2}|^2})/2$.

In order to suppress the spontaneous emission of the excited states, we consider the large detuning case, i.e. $\Delta_j^2 \gg \Omega_{j0}^2$. By calculating all values up to the order of Ω_{j0}^2/Δ_j^2 , one can verify that $\tan^2\theta_j = \Omega_{j0}^2/(\Delta_j^2 - \Omega_{j0}^2)$, and $E_{\psi,\phi}^+ \gg E_{\psi,\phi}^-$. We then invoke the adiabatic condition so that the population of the higher levels $\lvert\psi_+\rangle$ and $\lvert\phi_+\rangle$ is adiabatically eliminated. Moreover, the total system is confined to the ground eigenstates

$$\lvert\Psi\rangle = \cos\gamma\lvert S_\downarrow\rangle + \sin\gamma\lvert S_\uparrow\rangle, \quad (3)$$

where to facilitate further discussions, we have put the effective spin states [8]: $\lvert S_\downarrow\rangle = \lvert\psi_-\rangle$, $\lvert S_\uparrow\rangle = \lvert\phi_-\rangle$ with their z -component effective spin polarizations $S_z^\uparrow \approx \hbar/2$ and $S_z^\downarrow \approx -\hbar/2$. The parameter γ describes the probability of an atom in states $\lvert S_\downarrow\rangle$ and $\lvert S_\uparrow\rangle$, determined through the initial condition. One can also see the population of excited states $\lvert e_\pm\rangle$ is very small, therefore the atomic decay can be neglected in the present situation.

Under adiabatic condition, the local transformation $U(\mathbf{r})$ leads to a diagonalized $SU(2)$ gauge potential: $(e/c)\mathbf{A}_\alpha = -i\hbar\langle S_\alpha \nabla |S_\alpha\rangle$, ($\alpha = \downarrow, \uparrow$). Accordingly, the effective (scalar) trap potentials read $V_\alpha(\mathbf{r}) = V(\mathbf{r}) - \hbar|\Delta_j|^{-1}\Omega_{j0}^2 - (2m)^{-1}|\hbar\langle S_\alpha \nabla |S_\alpha\rangle|^2$ with $j = 1$ (for $\alpha = \downarrow$) and $j = 2$ (for $\alpha = \uparrow$). Specially we have $\mathbf{A}_\downarrow = -\mathbf{A}_\uparrow = \hbar c e^{-1} \frac{\Omega_0^2}{\Delta^2} (x\hat{e}_y - y\hat{e}_x)/\rho^2$ and $V_{eff}(\mathbf{r}) = V_{\uparrow,\downarrow}(\mathbf{r}) = V(\mathbf{r}) - \hbar\Omega_0^2/\Delta - \hbar^2 l^2 \Omega_0^4/(2m\Delta^4\rho^2)$ with $\rho = \sqrt{x^2 + y^2}$, when we choose $\Delta_1 = \Delta_2 = \Delta$, $\Omega_{10} = \Omega_{20} = \Omega_0$ and $l_1 = -l_2 = l$, i.e. the angular momenta of the two light fields are opposite in direction. This result is intrinsically interesting: by coupling the atomic spin states to radiation, we find the atomic system can be described as an ensemble of charged particles with opposite spins experiencing magnetic fields in opposite directions but subject to the same electric field. With the help of gauge and trap potentials,

we rewrite the Hamiltonian effectively as

$$H = \int d^3r \Psi_{s\downarrow}^* \left[\frac{1}{2m} (\hbar\partial_k + i\frac{e}{c} A_k)^2 \right] \Psi_{s\downarrow} + \int d^3r \Psi_{s\uparrow}^* \left[\frac{1}{2m} (\hbar\partial_k - i\frac{e}{c} A_k)^2 \right] \Psi_{s\uparrow} + \int d^3r (V_\downarrow |\Psi_{s\downarrow}|^2 + V_\uparrow |\Psi_{s\uparrow}|^2), \quad (4)$$

where $\Psi_{s\downarrow}(\mathbf{r}) = \cos\gamma\langle\mathbf{r}|S_\downarrow\rangle$ and $\Psi_{s\uparrow}(\mathbf{r}) = \sin\gamma\langle\mathbf{r}|S_\uparrow\rangle$ are spin wave functions in \mathbf{r} -representation.

Before calculating the spin current, we study the properties of spin currents in our model. We first consider a conservation law. The general spin density in the present system is calculated using $\vec{S}(\mathbf{r}, t) = \Psi_{s\downarrow}^* \vec{S}_\downarrow \Psi_{s\downarrow} + \Psi_{s\uparrow}^* \vec{S}_\uparrow \Psi_{s\uparrow}$ with $\vec{S}_{\uparrow\downarrow} = (S_x^{\uparrow\downarrow}, S_y^{\uparrow\downarrow}, S_z^{\uparrow\downarrow})$. Moreover, the spin current density $\vec{J}_k(\mathbf{r}, t) = (J_k^{sx}, J_k^{sy}, J_k^{sz})$ is defined by

$$\vec{J}_k = -\frac{i\hbar}{m} \vec{S}_\downarrow (\Psi_{s\downarrow}^* D_{1k} \Psi_{s\downarrow} - \Psi_{s\downarrow} D_{1k}^* \Psi_{s\downarrow}^*) - \frac{i\hbar}{m} \vec{S}_\uparrow (\Psi_{s\uparrow}^* D_{2k} \Psi_{s\uparrow} - \Psi_{s\uparrow} D_{2k}^* \Psi_{s\uparrow}^*), \quad (5)$$

where $D_{1k} = \partial_k + i\frac{e}{c} A_k$ and $D_{2k} = \partial_k - i\frac{e}{c} A_k$ are the covariant derivative operators. By a straightforward calculation, we verify the following continuity equation

$$\partial_t \vec{S}(\mathbf{r}, t) + \partial_k \vec{J}_k = A_k \sigma_z \hat{\mathbf{e}}_z \times \vec{J}_k(\mathbf{r}, t), \quad (6)$$

where σ_z is the usual Pauli matrix. It is easy to see that the right hand side of Eq. (6) equals zero for the s_z -component spin current. Thus the spin current J_k^{sz} is conserved. But there is no spin-orbit coupling in the present atomic system. Thus, it is easily verified that the orbit-angular momentum current is also conserved. These results underlines the conservation law for s_z -component of total angular momentum current of atoms. Secondly, it is interesting that the two spin functions in the effective Hamiltonian (4) are not independent: there is a nontrivial effective coupling mediated by an effective electromagnetic field. This implies that the spin current in our model could have a nontrivial hidden topology. We next discuss the topological properties of J_k^{sz} . We note that $\Psi_{s_\alpha} = n_a^{1/2} \zeta_\alpha$, where the complex $\zeta_\alpha = |\zeta_\alpha| e^{-i\varphi_\alpha}$ with $|\zeta_\downarrow|^2 + |\zeta_\uparrow|^2 = 1$, and n_a is the total atomic density which is assumed to be constant in our case. The s_z -component of eq. (5) can be recast as

$$J_k^{sz} = -i \frac{n_a \hbar^2}{2m} (\zeta_\downarrow \partial_k \zeta_\downarrow^* - \zeta_\downarrow^* \partial_k \zeta_\downarrow + \zeta_\uparrow^* \partial_k \zeta_\uparrow - \zeta_\uparrow \partial_k \zeta_\uparrow^*) - \frac{n_a \hbar^2 e}{mc} A_k. \quad (7)$$

Furthermore, we introduce the unit vector field, $\vec{\lambda} = (\bar{\zeta}, \vec{\sigma}\zeta)$, where $\zeta = (\zeta_\downarrow, \zeta_\uparrow^*)^T$ and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. It then follows that $\lambda_1 = \zeta_\downarrow^* \zeta_\uparrow^* + \zeta_\downarrow \zeta_\uparrow$, $\lambda_2 = i(\zeta_\downarrow \zeta_\uparrow - \zeta_\downarrow^* \zeta_\uparrow^*)$ and $\lambda_3 = |\zeta_\downarrow|^2 - |\zeta_\uparrow|^2$. Using these definitions we have

$$(\nabla \times \mathbf{J}^{sz})_m = -\frac{n_a \hbar^2 e}{mc} B_m - \frac{n_a \hbar^2}{2m} \epsilon_{mkl} \vec{\lambda} \cdot (\partial_k \vec{\lambda} \times \partial_l \vec{\lambda}). \quad (8)$$

The contribution $\vec{\lambda} \cdot (\partial_k \vec{\lambda} \times \partial_l \vec{\lambda})$ provides a topological term in the spin current induced by optical fields. Note each value of $\vec{\lambda}$ represents a point in the two-dimensional sphere S^2 . The variation of $\vec{\lambda}$ depends on the relative density of the two spin components $\gamma(\mathbf{r})$ and the sum of the phase spreadings $\varphi_{\uparrow}(\mathbf{r}) + \varphi_{\downarrow}(\mathbf{r})$. It is easily verified that $\vec{\lambda}$ can cover the entire surface S^2 , when the parameter distributions in the interaction region satisfy: $\varphi_1 + \varphi_2 : 0 \rightarrow 2n_1\pi$ and $\gamma : 0 \rightarrow n_2\pi/2$ with integers $n_1, n_2 \geq 1$. We then obtain a map degree between the unit vector field $\vec{\lambda}$ and the spatial vectors $F : \vec{\lambda} \rightarrow \mathbf{r}/r$ so that the closed-surface integral

$$\oint_s (\nabla \times \mathbf{J}^{s_z}) \cdot d\mathbf{S} \sim \oint_s ds_m \epsilon_{mkl} \frac{\mathbf{r}}{r} \cdot (\partial_k \frac{\mathbf{r}}{r} \times \partial_l \frac{\mathbf{r}}{r}) \sim 4\pi n,$$

where n is the winding number. In physics, such mapping corresponds to the formation of a local inhomogeneity in the densities of the spin-up and spin-down atoms. The Hamiltonian (4) can also be regarded as a system of *two-flavor* oppositely charged particles interacting with *one* external effective magnetic field. Such model has a wide range of applications to, e.g. two-band superconductivity [11], etc. It is interesting to note that, under certain conditions liquid metallic hydrogen, as an example, might allow for the coexistence of superconductivity with both electronic and protonic Cooper pairs [12]. Faddeev et al. have also discovered a series of nontrivial topological properties in such systems [13]. Based on this technique, we have developed an optical way to realize *two-flavor* artificially charged system, which could provide a deeper understanding of the essential physical mechanisms in cold atomic systems.

For practical application, an important goal is to create a pure spin current injection, where the massive current is zero. To this end, we propose using a columnar spreading light fields that [10] $\Omega_0(\mathbf{r}) = f\rho$ with $f > 0$. Furthermore, we set the x - y harmonic trap $V(\mathbf{r}) = \frac{1}{2}m\omega_{\perp}^2(\vec{p} + x_0\hat{\mathbf{e}}_x)^2$ centered at $\vec{p} = -x_0\hat{\mathbf{e}}_x$, where the frequency is tuned to $\omega_{\perp}^2 = (1 + \frac{\hbar l^2 f^2}{2m\Delta^3})\frac{2\hbar f^2}{m\Delta}$. It should be emphasized that the (positive) potential $V(\mathbf{r})$ can typically be obtained with a blue-detuned dipole trap method [14] for instance. The uniform magnetic and electric fields corresponding to the gauge and scalar potentials are

$$\mathbf{B}_{\downarrow}(-\mathbf{B}_{\uparrow}) = \frac{\hbar c}{e} \frac{f^2}{\Delta^2} \hat{\mathbf{e}}_z, \mathbf{E} = -(1 + \frac{\hbar l^2 f^2}{4m\Delta^3}) \frac{2\hbar f^2 x_0}{e\Delta} \hat{\mathbf{e}}_x, \quad (9)$$

along the z ($-z$) and x directions respectively. Atoms in different spin states $|S_{\alpha}\rangle$ experience the opposite magnetic fields \mathbf{B}_{α} but the same electric field \mathbf{E} . This leads to a Landau level structure for each spin orientation. To calculate the spin currents explicitly, one needs to obtain the eigenstates of the present system. However, the present forms of gauge and scalar potentials in the Hamiltonian (4) cannot be diagonalized easily; consequently, we need to resort to perturbation theory to calculate the spin currents. The Hamiltonian for a single atom in state $|S_{\alpha}\rangle$

can be written as: $H^{\alpha} = H_0^{\alpha} + H'$ ($\alpha = \downarrow, \uparrow$), where the perturbation part $H' = -eEx$ and

$$H_0^{\alpha} = \frac{\hbar^2 e B}{2mc} (R_{\alpha}^2 + P_{\alpha}^2) + \frac{1}{2m} p_z^2 \quad (10)$$

with $R_{\uparrow\downarrow} = (\frac{c}{eB})^{1/2}(p_x - \frac{e}{c}A_x^{\uparrow\downarrow})$, $P_{\uparrow\downarrow} = (\frac{c}{eB})^{1/2}(p_y - \frac{e}{c}A_y^{\uparrow\downarrow})$ and $B = |\mathbf{B}_{\uparrow,\downarrow}|$. One can verify that $[R_{\alpha}, P_{\beta}] = i\hbar\delta_{\alpha\beta}$, so the eigenfunction $\mu_{nk}^{\alpha}(R)$ of H_0^{α} is Hermite polynomial with the Landau level $E_{nk,\alpha} = (n+1/2)\hbar\omega + \hbar k^2/2m$ and $\omega = eB/mc$. For the weak field case, the spin/massive current carried by an atom can be calculated perturbatively to the first order correction on the state $|\mu_{nk}^{\alpha}\rangle$

$$(j_{sz,m}^y)_{nk,\alpha} = \langle \mu_{nk}^{\alpha} | j_{sz,m}^y | \mu_{nk}^{\alpha} \rangle + \\ + \left(\sum_{n'\alpha'} \frac{\langle \mu_{n'k}^{\alpha'} | H' | \mu_{nk}^{\alpha} \rangle \langle \mu_{nk}^{\alpha} | j_{sz,m}^y | \mu_{n'k}^{\alpha'} \rangle}{E_{n,\alpha} - E_{n',\alpha'}} \right. \\ \left. + h.a. \right), \quad (11)$$

where the spin current operator is $(j_{sz}^y)_{\alpha} = \frac{\hbar}{2}(S_z^{\alpha}v_y^{\alpha} + v_y^{\alpha}S_z^{\alpha})$ with $v_y^{\alpha} = [y, H^{\alpha}]/ih = (eB/m^2c)^{1/2}P_{\alpha}$ and the massive current operator $j_m = mv_y$. It is easy to see that $\langle \mu_{nk}^{\alpha} | j_{sz,m}^y | \mu_{n'k}^{\alpha'} \rangle = 0$ when $n' \neq n \pm 1$, thus only the terms with $n' = n \pm 1$ contribute in the above equation.

If the spatial spreads of the atoms in x and z directions are L_x and L_z , respectively, the average current density for the total system is given by $J_{sz,m}^y = \frac{1}{L_x L_z} \int dE ((j_{sz,m}^{(0)})_{nk,\alpha} + (j_{sz,m}^{(1)})_{nk,\alpha}) f(E)$ with $f(E)$ as the Fermi distribution function. For ${}^6\text{Li}$ atoms we may consider the initial condition that $\sin^2 \gamma = \cos^2 \gamma = 1/2$, i.e. the atoms have the equal probability in state $|S_{\downarrow}\rangle$ and $|S_{\uparrow}\rangle$, as in the usual optical trap [15]. Rewriting the perturbation part as $H' = i\hbar E(\frac{ec}{B})^{1/2} \frac{\partial}{\partial R}$ and substituting this result into Eq (11) and finally we get

$$J_{sz}^y = n_a (\hbar + \frac{\hbar^2 l^2 f^2}{4m\Delta^3}) \frac{\Delta x_0}{l}, \quad (12)$$

whereas the massive current J_m is zero. In fact, under present interaction of external effective electric and magnetic fields, the atoms with opposite velocities in y direction have opposite spin polarizations (see Fig.2). Thus the massive current vanishes whereas a pure spin current is obtained. This result allows us to create conserved spin currents without using atomic beams.

To observe SHE in present cold atomic system, a spin-sensitive measurement [16] can be used. For example, the technique of atom chip [17] can be used to implement the spin current described in the present model. The created spin current can lead to spin accumulations on opposite sides along y direction of the atom chip. Experimentally, one could detect such spatially separated spin polarizations using magnetic resonance force microscopy (MRFM) [18] for instance. Another means to detect the spin accumulation can also be achieved with fluorescence. For ${}^6\text{Li}$ atoms, one may perform resonant Raman

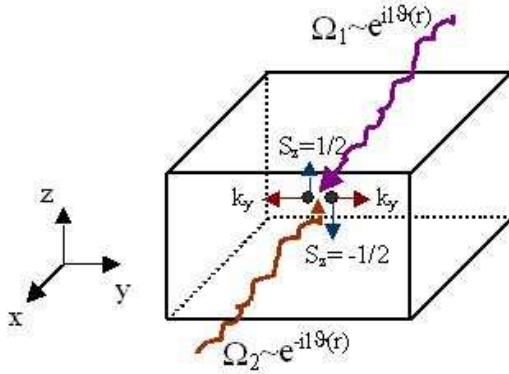


FIG. 2: (Color online) Under the interaction of effective electric and magnetic fields induced by light fields, atoms in state $|S_\downarrow\rangle$ and $|S_\uparrow\rangle$ have opposite momenta in y direction.

transitions from the accumulated spin state $S_z = -1/2$ to $2^2P_{1/2}(F = 3/2, m = -3/2)$ or from $S_z = 1/2$ to $2^2P_{1/2}(F = 3/2, m = 3/2)$ (D_1 line) and then observe the fluorescence. As long as the spin accumulations are separated to say larger than 5–10 microns, the two separated images can be resolved in an experiment.

One should also discuss the adiabatic condition employed in above calculations. Atomic motion may lead to transitions between ground eigenstates and excited ones, e.g. the element of transition between $|\psi_+\rangle$ and $|\psi_-\rangle$ can be calculated by $\tau_\pm = |\langle\psi_+|\partial_t|\psi_-\rangle| = |\bar{v} \cdot \nabla\theta_1(\mathbf{r}) + \frac{1}{2}l_1 \sin 2\theta_1 \bar{v} \cdot \nabla\vartheta(\mathbf{r})|$ where \bar{v} is the average resulting velocity in spin currents. The adiabatic condition requires $\tau_\pm \ll |E_\psi^+ - E_\psi^-|$. For a numerical evaluation one typically set the parameters $\Delta \sim 10^8 \text{s}^{-1}$, $l < 10^4$, $f = 5 \times 10^{10} (\text{s} \cdot \text{m})^{-1}$, $x_0 \approx 2.0 \mu\text{m}$. We then find the velocity $\bar{v} < 1.0 \text{ m/s}$ and $\tau_\pm / |E_\psi^+ - E_\psi^-| \sim 10^{-3} \ll 1$, which guarantees the validity of the adiabatic condition. In the case of ${}^6\text{Li}$ atomic system, the x - y trap potential $V(\mathbf{r})$ can be achieved through D_2 transition (from $2^2S_{1/2}$ to $2^2P_{3/2}$) with a blue detuning [9, 14]. With former parameters and by tuning the trap frequency to $\omega_\perp \approx 728.5 \text{ Hz}$, one achieves the uniform magnetic and electric fields in Eq. (9). Furthermore, if we employ an atomic system with the atomic density $n_a \approx 1.0 \times 10^{10} \text{ cm}^{-3}$, one can find the spin current $J_{s_z}^y \approx 1.322 \times 10^{-5} \text{ eV/cm}^2$. On the other hand, we note that for all practical purposes, the light fields have a finite cross section S_{xy} , which may have a boundary effect on the calculation of spin currents. For the cold Fermi atomic gas, the spatial scale of the interaction region is about 0.1 mm [15], then such boundary effect can be neglected since $lf^2S_{xy}/\Delta^2 \sim 10^2 \gg 1$. This means the effective magnetic flux induced by the light fields can support a sufficiently large degeneracy at each Landau level.

In conclusion we proposed an optical means to realize a new type of spin hall effect in the neutral atomic sys-

tem. The spin current created in this way is conserved and could possess interesting topological properties. The present atomic system is equivalent to a *two-flavor* artificially charged system, providing a direct analogy between the dynamics of electrons in solid systems, e.g. two-band superconductors [11, 12], and the behavior of cold atoms in optical potentials. The effect also provides understanding of the basic physical mechanisms of SHE with a wide range of applications in cold atomic systems.

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