Tightly Localized Stationary Pulses in Multi-Level Atomic System

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We show the pulse matching phenomenon can be obtained in the general multi-level system with electromagnetically induced transparency (EIT). For this we find a novel way to create tightly localized stationary pulses by using counter-propagating pump fields. The present process is a spatial compression of excitation so that it allows us to shape and further intensify the localized stationary pulses, without using standing waves of pump fields or spatially modulated pump fields.

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Recently, an important progress in electromagnetically induced transparency (EIT) \cite{1, 2, 3, 4, 5, 6, 7} is that experimental realization for the coherent control of stationary pulses was achieved by using standing waves of pump fields in the three-level system \cite{8, 9}. The creation of stationary pulses can enhance the nonlinear couplings between photons or collective excitations corresponding to stored photons, both of which are useful for deterministic logic operations. The key point in the creation of stationary pulses is expressed by the pulse matching phenomenon between the forward (FW) and backward (BW) propagating probe fields \cite{8, 9, 10}. For a three-level system, the technique to generate tightly localized stationary pulses involves the use of standing waves of pump fields with a frequency-comb or spatially modulated pump fields \cite{9}. However, such tight localization cannot be applied directly to applications in quantum nonlinear optics.

On the other hand, coherent manipulation of probe lights has been studied in the four-level double $\Lambda$-type system \cite{11, 12} and also in the general multi-level atomic system that interacts with many probe and pump fields \cite{13, 14}. It has also been shown in Ref. \cite{13} that, one can convert different probe lights into each other by manipulating the external pump fields based on such general EIT method, indicating a sort of pulse matching phenomenon between all applied probe fields. This observation also motivates us to probe into a new technique of creating localized stationary pulses based on multi-level atomic system. In this rapid communication, we shall show the tightly localized stationary pulses can be obtained through a spatial compression of excitation the general multi-level EIT system.

We consider the quasi-one dimensional system shown

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{(color online) (a) General $m$-level atomic system coupled to $m-2$ quantized probe and classical pump fields which propagate in $+z$ or $-z$ directions. (b) No. 1 to No. $m-3$ pump/probe pulses propagate in the $+z$ direction, while No. $m-2$ pump/probe pulse propagates in the $-z$ direction.}
\end{figure}

in Fig. 1(a) for an ensemble of $m$-level atoms interacting with $m-2$ quantized probe fields which couple the transitions from the ground state $|b\rangle$ to excited state $|e_\sigma\rangle$ ($1 \leq \sigma \leq m-2$) with coupling constants $g_\sigma$, and $m-2$ classical pump fields which couple the transitions from the state $|c\rangle$ to excited ones $|e_\sigma\rangle$ with Rabi-frequencies $\Omega_\sigma(z,t)$. All probe and pump fields are co-propagating in the $+z$ or $-z$ direction (Fig. 1(b)), and

\begin{equation}
E_\sigma(z,t) = \sqrt{\frac{\hbar \nu_\sigma}{2 e_0 V}} \hat{\xi}_\sigma(z,t) e^{i(k_{p\sigma}z-\nu_\sigma t)},
\end{equation}

\begin{equation}
\Omega_\sigma(z,t) = \Omega_{\sigma 0} e^{i(k_{c\sigma}z-\omega_\sigma t)},
\end{equation}

where $\sigma = 1, 2, ..., m-2$, $\hat{\xi}_\sigma$ and $\Omega_{\sigma 0}$ are slowly-varying amplitudes, $k_{p\sigma}$ and $k_{c\sigma}$, respectively $z$-component wave vectors of probe and pump fields, can be positive or negative. For $k_{p\sigma} > 0$ and $k_{c\sigma} > 0$ ($k_{p\sigma} < 0$ and $k_{c\sigma} < 0$), it means the $\sigma$th pair of probe and pump fields propagate in

\begin{itemize}
\item $+z$ direction.
\item $-z$ direction.
\end{itemize}
the +z (−z) direction. We consider all transitions to be at resonance. Under the rotating-wave approximation, the interaction Hamiltonian can be written as:

\[ \hat{\mathcal{V}} = - \int \frac{dz}{L} (\hbar N \sum_{\sigma=1}^{m-2} g_{\sigma} \hat{\sigma}_{c=\sigma}(z,t) \hat{\mathcal{E}}_{\sigma}(z,t) + h.c.), \]

(2)

where \( N \) is the total atom number, \( L \) is the length of the medium in the \( z \) direction, and the continuous atomic variables \( \hat{\sigma}_{\mu\nu}(z,t) = \sum_{\gamma \in N_z} \hat{\sigma}_{\mu\nu}^\gamma(t) \) are defined by a collection of \( N_z \gg 1 \) atoms in a very small length length interval \( \Delta z \). \( \hat{\sigma}_{c=\sigma}(z,t) = |e_{\sigma}^c \rangle \langle b^c| e^{-i(k_{\mu\nu} z - \omega_{\mu\nu} t)} \) and \( \hat{\sigma}_{e=\sigma}(z,t) = |e_{\sigma}^e \rangle \langle c^e| e^{-i(k_{\mu\nu} z - \omega_{\mu\nu} t)} \) are the slowly-varying parts of the \( \gamma \)th atomic flip operator. We note that an essential difference between our model and the three-level system is that for the case of multi-frequency optical pulses, the dark-state polaritons (DSPs) in the general multi-level EIT system is first obtained in [13, 14], where the single-mode probe pulses are considered. Accordingly, the dark- and bright-state polaritons (BSPs) in the present general multi-level system can be defined by:

\[ \Psi(z, t) = \cos \theta \prod_{j=1}^{m-3} \cos \phi_j \hat{E}_1 \]
\[ + \cos \theta \sum_{l=2}^{m-2} \sin \phi_{l-1} \prod_{j=l}^{m-3} \cos \phi_j \hat{E}_l \]
\[ - \sin \theta(t) \sqrt{N} \tilde{\sigma}_{bc}(z,t), \]

(8)

\[ \Phi(z, t) = \sin \theta \prod_{j=1}^{m-3} \cos \phi_j \hat{E}_1 \]
\[ + \sin \theta \sum_{l=2}^{m-2} \sin \phi_{l-1} \prod_{j=l}^{m-3} \cos \phi_j \hat{E}_l \]
\[ + \cos \theta(t) \sqrt{N} \tilde{\sigma}_{bc}(z,t), \]

(9)

which are superpositions of the atomic coherence and the \( m - 2 \) probe fields. The mixing angles \( \theta \) and \( \phi_j \) in the new quantum fields are defined through

\[ \tan \theta = \frac{g_1 g_2 \cdots g_{m-2} \sqrt{N}}{\left[ \sum_{j=1}^{m-2} (\Omega_{\sigma_0}^2 \prod_{l=1,l \neq l}^{m-2} g_{l}^2) \right]^{1/2}} \]

and

\[ \tan \phi_j = \frac{\prod_{l=1}^{j-1} g_l \Omega_{l+1,0}}{\left[ \sum_{l=1}^{j} (\Omega_{\sigma_0}^2 \prod_{l=1,l \neq l}^{j} g_{l}^2) \right]^{1/2}}. \]

Using the definitions above, one can transform the equations of motion for the probe fields and the atomic variables into the new field variables. With the low-excitation approximation and neglecting the nonlinear effects we find that the DSP field satisfies
(\partial_t + c \cos^2 \theta \cos \alpha_{m-2} \partial_z) \hat{\Psi} = -\hat{\theta} \hat{\Phi} + \sum_{j=1}^{m-2} \hat{\phi}_j \cos \theta \hat{s}_j - \frac{c}{2} \sin 2\theta \cos \alpha_{m-2} \partial_z \hat{\Phi}, + 
+ c \cos \theta \sum_{j=1}^{m-2} \prod_{l=j}^{m-3} \cos \phi_l \sin 2\phi_{l-1} \left( \frac{1}{2e} \frac{\nu_j}{k_{pj}} + \frac{\cos \alpha_{j-1}}{2} \right) \partial_z \hat{s}_j, \quad (10)

where we have defined

\cos \alpha_\sigma = \sum_{j=1}^{m-1} \frac{E_j}{g_j} \prod_{l=1, l \neq j}^{m-1} g_l^2, \quad \sigma = 1, 2, ..., m - 3

and \( \hat{s}_j = \partial_{\phi_j} \hat{\Psi} / \cos \theta \). It then follows that

\( \hat{s}_1 = \prod_{j=2}^{m-3} \cos \phi_j (- \sin \phi_1 \mathcal{E}_1 + \cos \phi_1 \mathcal{E}_2), \quad \hat{s}_2 = \prod_{j=3}^{m-3} \cos \phi_j (- \sin \phi_2 (\cos \phi_1 \mathcal{E}_1 + \sin \phi_1 \mathcal{E}_2) + \cos \phi_2 \mathcal{E}_3), \)

and generally

\( \hat{s}_k = \prod_{j=k+1}^{m-3} \cos \phi_j \hat{j}_k, \quad k = 1, 2, ..., m - 3, \quad (11) \)

\[ \Phi = \frac{\Gamma}{\sqrt{N}} \left( \sum_{j=1}^{m-2} \left( \frac{\Omega_j / \Omega_0}{g_j} \right)^2 \right)^{1/2} \cos \theta \frac{\partial_t (\sin \theta \Psi - \cos \theta \Phi)}{\Omega_0} \partial_t (\sin \theta \Psi - \cos \theta \Phi) + \cos \theta (g_1 \Omega_0 \sin \phi_1 \hat{s}_1 - \sum_{l=2}^{m-3} g_l \Omega_0 \cos \phi_{l-1} \hat{s}_{l-1}). \quad (13) \]

By comparing Eqs. (10) and (13) with the corresponding DSP and BSP fields in the three-level system, one can see a key difference is the appearance of \( \hat{s}_j(z, t) \) from the probe fields in our model. The adiabatic condition in the present case can be fulfilled only if \( \hat{s}_j(z, t) = 0 \) for all \( j \). However, the input probe pulses are generally independent of each other so that the fields \( \hat{s}_j \) need not be zero. To study the dynamics of the DSP field, we should therefore investigate first the pulse matching between all the probe fields needed for adiabatic condition. Bearing

these ideas in mind, we next explore the evolution of a set of normal fields by introducing

\( \hat{G}_{j,j+1} = -\sin \phi_{j,j+1} \hat{E}_j(z, t) + \cos \phi_{j,j+1} \hat{E}_{j+1}(z, t), \quad (14) \)

where \( j = 1, 2, ..., m - 3 \) and \( \tan \phi_{j,j+1} = g_j \Omega_{j+1} / g_{j+1} \Omega_j \). From the Eq. (13) and together with the results of \( \sigma_{bc} \) and \( \sigma_{bc} \) one can verify that the field \( \hat{G}_{j,j+1} \) satisfies the equation

\[ (\partial_t - c \cos^2 \beta \cos 2\phi_{j,j+1} \partial_z) \hat{G}_{j,j+1} = - \frac{g_j^2 \Omega_{j+1}^2 + g_{j+1}^2 \Omega_j^2}{\Gamma} \cos \beta \hat{\Omega}_{j+1} - \frac{1}{2} g_j g_{j+1} \sqrt{N} \sin 2\beta \partial_t \hat{E}_{j,j+1} + c \cos^2 \beta \sin 2\phi_{j,j+1} \partial_z \hat{E}_{j,j+1} + F(\hat{\epsilon}_\sigma, \sigma \neq j, j + 1) \quad (15) \]

with

\[ \tan^2 \beta = \frac{N \Omega_j^2 \Omega_{j+1}}{g_j^2 \Omega_{j+1}^2 + g_{j+1}^2 \Omega_j^2} \left( \frac{g_j^2 - g_{j+1}^2}{2} \right)^2 \Omega_0^{-4}, \]

and \( \hat{E}_{j,j+1} = \cos \phi_{j,j+1} \hat{E}_j(z, t) + \sin \phi_{j,j+1} \hat{E}_{j+1}(z, t) \). \( F(\hat{\epsilon}_\sigma) \) includes no \( \hat{E}_j \) or \( \hat{E}_{j+1} \). The first term in the right hand side of Eq. (15) reveals a very large ab-
results in Fig. 2 show how tight localization of stationary pulses can be readily obtained when the multi-level system is applied. In contrast, for a three-level system, a frequency-comb is used to create a localized pulse, filtering the off-resonant input probe pulses and retaining only the resonant one for creation 3. Generally, the total number of probe photons created by a frequency-comb in a three-level system is much less than in the current model; ii) The present technique can be freely controlled. For example, based on our results, we see that the pulse matching in the present case is between all of the probe pulses with different frequencies, say, $\Phi = \prod_{j=1}^{m} \tan \phi_{j+1} \Phi_j (l \geq 1, \sigma \leq m - 2)$. Thus, in principle, one can use just one pump field to achieve the stationary pulse by adjusting its intensity to match those of the other pump fields; iii) It requires no standing waves in the pump fields or spatially modulated pump fields to create localized stationary pulses.

Experimentally, the simplest multi-level system is an ensemble of four-level double Λ-type $^{87}$Rb atoms. The schematic diagram for experimental realization is shown in Fig.3. All the atoms first are trapped in state $|b\rangle$ ($5^2S_{1/2}$) and only the $\pm z$ directional propagation pump fields ($\Omega_1$ and $\Omega_2$) are applied to couple the transitions from $|c\rangle$ ($5^2S_{1/2}$) to $|e_1\rangle$ ($5^2P_{1/2}(F = 1)$) and $|e_2\rangle$ ($5^2P_{3/2}(F = 1)$) respectively. We then input the probe pulses ($\Phi_j$) and allow the system to achieve adiabatic condition. Finally, by adjusting $\Omega_1$ or $\Omega_2$ so that $g_1 \Omega_2 = g_2 \Omega_1$, we can create the stationary pulses for probe fields $\Phi_j (z, t) = \cos \theta \cos \phi \Phi_j, \Phi_j (z, t) = \cos \theta \sin \phi \Phi_j$, where $\Phi_j$ is determined by the Eq. 8 with $m = 4$. It can be ex-
FIG. 3: (color online) (a)(b) Schematic of experimental realization of stationary pulses with four-level double-A-type $^{87}$Rb atoms coupled to two single-mode quantized and two classical pump fields that propagate in $+z$ and $-z$ directions, respectively.

Expected that when the level number $m$ becomes larger, the more tightly localized stationary pulses can be created. According to the numerical results in Fig. 2, the effect becomes substantial when $m \geq 5$.

In conclusion, we have shown the tightly localized stationary pulses can be obtained in the general multi-level EIT system. We have examined the dynamics of DSPs in detail and found that, all the applied probe pulses with different frequencies contribute to the stationary pulses. The present process is therefore a spatial compression of excitation, which may be able to enhance further non-linear optical couplings and will have interesting applications in quantum nonlinear optics [17]. In particular, our technique may open up a novel way towards the spatial compression of many probe photons with small losses. According to the results in [13], if initially input is a non-classical probe pulse, e.g. a quantum superposition of coherent states, we may also generate entangled stationary pulses within our model.

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