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Elements of $\mathcal{F}$-ast Proton Decay

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Abstract

Gauge coupling unification in the Minimal Supersymmetric Standard Model (MSSM) strongly suggests the existence of a Grand Unified Theory (GUT), which could be probed by the observation of proton decay. Proton lifetime in the $p \to (e|\mu)^{+}\pi^0$ dimension six mode is proportional in the fourth power to the GUT mass scale, and inversely proportional in the fourth power to the GUT coupling. Flipped $SU(5)$ with strict MSSM field content is known to survive existing null detection limits for proton decay approaching 10^{34} years, and indeed, the lifetime predicted by prior studies can be so long that successful detection is not currently plausible. We provide an updated dictionary of solutions for the relevant flipped unification parameters with generic β-function coefficients, significantly upgrading the level of detail with which second order effects are treated, and correcting subtle published errors.

Recently studied classes of $\mathcal{F}$-theory derived GUT models postulate additional vector-like multiplets at the TeV scale which modify the renormalization group to yield a substantial increase in the $SU(3)_C \times SU(2)_L$ unified coupling. In conjunction with the naturally depressed $\mathcal{F}$-lipped $SU(5)$ partial unification mass M_{32}, the $\mathcal{F}$-resh analysis undertaken predicts comparatively $\mathcal{F}$-ast proton decay which only narrowly evades existing detection limits, and likely falls within the observable range of proposed next generation detectors such as DUSEL and Hyper-Kamiokande. The TeV-scale vector multiplets are themselves suitable for cross correlation by the Large Hadron Collider. Their presence moreover magnifies the gap between the dual mass scales of Flipped $SU(5)$, allowing for an elongated second stage renormalization, pushing the $\mathcal{F}$-inal grand unification to the doorstep of the reduced Planck mass.

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1. $\mathcal{F}$-undamentals

1.1. Introduction

The major problem in the Standard Model (SM) is that of the gauge hierarchy, *i.e.*, the Higgs boson mass square is not stable against quantum corrections and has quadratic divergences. Supersymmetry (SUSY) provides a natural solution to gauge hierarchy problem due to the opposing spin statistics for bosons and fermions. In the Minimal Supersymmetric Standard Model (MSSM) with R parity, under which the SM particles are even while their supersymmetric partners are odd, gauge coupling unification can be achieved [1–4], the lightest supersymmetric particle (LSP), typically the neutralino, can be a cold dark matter candidate [5–7], and the precision electroweak constraints can be satisfied. Also, the electroweak gauge symmetry can be broken radiatively due to the large top quark Yukawa coupling [8], and smallness of the neutrino masses can be explained via the seesaw mechanism [9–11]. In particular, gauge coupling unification strongly suggests the existence of a Grand Unified Theory (GUT), which can further explain charge quantization. If this scenario is realized in nature, there is a further unique and necessary implication which is of great and pressing interest.

As the lightest baryon, the proton, having no viable decay products within the Standard Model, seems to enjoy a life without age or end. It has long been suspected however, that protons are not actually forever [12]. Instability of the proton is indeed a ubiquitous signature of Grand Unification, the merger of fundamental forces necessarily linking quarks to leptons, and providing a narrow channel $p \to (e|\mu)^{+}\pi^{0}$ of dimension six decay via heavy gauge boson exchange. Following the direct verification of coupling unification, neutrino masses, and the third quark generation, and convincing secondary evidence for supersymmetry and the Higgs mechanism, proton decay is the last great unresolved prediction of the particle physics generation.

We will consider the problem of proton decay within both the Georgi-Glashow $SU(5)$ [13] GUT and also the Flipped $SU(5)$ [14–18] variation, both as pure particle theory and supplemented by string theoretic considerations from free fermionic constructions [19, 20] and Cumrun Vafa's F-theory [21–28]. Flipped $SU(5)$ with strict MSSM field content is known to survive existing null detection limits for proton decay approaching 10^{34} years [17, 29–31], while Standard $SU(5)$, via dimension five decay, could be in trouble [30, 32]. Flipped $SU(5)$, in which the 'ignoble' dimension five operators are automatically suppressed, is moreover phenomenologically preferred by the authors for a host of additional benefits [18] which include natural doublet-triplet splitting, avoidance of unwieldy adjoint Higgs multiplets, a mechanism of baryogenesis, consistency with precision electroweak data, a more inclusive accommodation of right-handed neutrinos, and an improved two-step seesaw mechanism [11] for generation of their eV scale mass.

It is an amusing game we have seen played out with the proton lifetime, in that the first success of a sufficiently suppressed decay rate can turn next into a 'failure' to predict results which are potentially observable. Of course, nature can do as she wishes. But there remains a comic absurdity to the notion that we could be right on the cusp of a detection at 10^{35-36} years, at just the scale where the macroscopic size of the experiment becomes unviable. The enormity of Avogadro's number has run most of the relay and our engineering limitations have let him down at the finish line!

A proper treatment of the problem necessarily touches upon an astonishingly broad set of interconnected topics. As input, we call on the precision measurements of couplings at the Z boson mass boundary, and numerically evaluated proton wavefunction matrix elements from lattice Quantum Chromodynamics. Updates to the rate of coupling renormalization from the detailed mass spectrum of light supersymmetric particles play a decisive role in the determination of the unified mass scale, on which proton lifetime is very strongly dependent. The preferred position within the supersymmetric parameter space is in turn constrained by the ability to realize electrically neutral cold dark matter and electroweak symmetry breaking, cosmological limits on dark matter relic density, the anomalous magnetic moment of the muon, and search limits for the Higgs boson. Detection and direct measurement of the light superpartners themselves constitutes one of the key reasons of entry for the Large Hadron Collider (LHC).

The subject of proton decay is highly differentially sensitive, in both preferred mode and predicted rate of decay, to the unified representation. The lifetime in the dimension six decay mode is proportional in the fourth power to the GUT mass scale, and inversely proportional in the fourth power to the GUT coupling. This extraordinary sensitivity to the unification point enhances even subtle distinctions between competing scenarios, and argues for great care and precision in the evaluation of the renormalization group running and of any model dependent peculiarities.

Flipped $SU(5)$ could be unique among proposed GUT scenarios for its simultaneously consistency with existing proton decay limits and all low energy phenomenology. It is a key feature of Flipped $SU(5)$, or more properly $SU(5) \times U(1)_{\mathrm{X}}$, that the $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}}$ gauge couplings g_3 and g_2 first merge into the common value $g_5(M_{32}) \equiv g_{32}$ at an intermediate mass M_{32}, proximal to the usual GUT scale and prior to a point of later full unification with the hypercharge-like remixed coupling g_{X}. In fact, it is exactly this additional compensatory freedom in the position of M_{32} which has allowed the flipped scenario to match the precision electro-weak measurements of LEP without unreasonable invocation of heavy threshold effects.

The current study represents, in our opinion, the most careful, comprehensive and accurate survey to date of proton lifetime predictions in the context of Flipped $SU(5)$, correcting certain mistakes of prior analysis, and significantly reducing the extent to which simplifying approximations produce errors in the calculation of critical second order effects. The results obtained are both surprising and noteworthy, yielding where one might expect only inconsequential variation within the noise, a highly significant downward shift of the central predicted lifetime. The net motion of more than a full order of magnitude toward the region of experimental detectability is of clear relevance to the great experimental proton decay searches, both ongoing (Super Kamiokande) and proposed (DUSEL, Hyper Kamiokande, LAGUNA).

Superstring phenomenology may be considered as a natural partner to particle physics, providing theoretical impetus and an ultimate connection to energies approaching the Planck scale. It is a generic prediction of string model building that extra heavy multiplets beyond the MSSM may exist. We have focused here on extensions to GUT phenomenology from the formulation called F-theory, which is highly favorable in its own right for the decoupling of gravitational physics, allowing the construction of local models. Such models naturally yield very specific predictions for new particle representations near the TeV scale, only a small number of such configurations successfully avoiding a Landau pole in a physically reasonable manner. The presence of such fields, themselves potentially verifiable at the LHC, has direct bearing on gauge renormalization and proton lifetime. Specifically, The modifications to the renormalization group from these fields yield a significant increase in α_{32}, which enhances the rate of proton decay.

It is the reduced scale M_{32} of Flipped $SU(5)$ relative to its Standard counterpart which is directly relevant to dimension six decay predictions, also acting to shorten the proton lifetime. Traditional analysis with MSSM field content has tended to indicate a rather mild splitting [29, 30], with $M_{32}/M_{32}^{\mathrm{max}}$ in the neighborhood of one or two parts out of three. However, inclusion here of extra matter at the TeV scale has the effect of making the splitting substantially more dramatic, due precisely to the enhanced gap between the $SU(5)$ and $U(1)_X$ couplings, realizing an 'extreme' Flipped $SU(5)$ scenario. Although the location of M_{32} itself experiences no great modification, and has actually been seen to increase mildly, the true grand unification point tends to push farther upward by two orders of magnitude or more, tantalizingly close to the string scale. The F-theoretic field content is in this sense just what the doctor ordered for Flipped $SU(5)$, tangibly justifying the expected separation between the GUT and String/Planck scales.

1.2. Outline of Topics

Continuing in Section (1), *$\mathcal{F}$-undamentals* , we proceed first with a general review of proton decay considerations, including a survey of existing experimental decay limits, and a look forward to the realistic proposals for next generation experiments. We also provide a broad background on the available proton decay channels, including some discussion of the fundamental amplitudes, and focusing in particular on the differential decay signatures of Standard vs. Flipped $SU(5)$ grand unification. Simple numerically parameterized formulae for the dimension six $p \to (e|\mu)^+\pi^0$ decay are provided for each scenario, including the dominant dependencies on the unification mass scale and coupling.

Also provided in Section (1) is a detailed presentation of the appropriate manner in which to analytically correct for the step-wise entrance of new particles into the gauge renormalization as their respective mass thresholds are crossed. The distinction in treatment between light and heavy thresholds is made clear. The section concludes with a detailed breakdown of the β-function coefficients for each gauge coupling, for each MSSM field, as is required for implementation of the previously described threshold factors. For handy reference, we also tabulate the β-coefficients for the field groupings which are typically consolidated in the output files of the standard particle physics computational analysis libraries.

Section (2), *$\mathcal{F}$-lipped $SU(5)$* , broadly reviews the phenomenology of our preferred GUT framework, Flipped $SU(5)\times U(1)_X$, including a discussion of field content and quantum numbers, neutrino masses, doublet-triplet splitting, compliance with precision low energy measurements, and the related implications for proton decay.

Section (3), *$\mathcal{F}$-theory* , consist of an introduction to Cumrun Vafa's F-theory, and its related model building techniques. In particular, we focus on the possible extensions to the TeV scale field content which F-theory can naturally provide. It is these fields which are directly responsible for magnifying the coupling separation at M_{32}, allowing for an elongated second stage renormalization, pushing grand unification to the doorstep of the reduced Planck mass. An immediate consequence of the enhanced $SU(5)$ gauge coupling is substantial speeding of the dimension six proton decay. We also supplement the previously published descriptions of possible heavy vector content in Flipped $SU(5)$ GUT models with the corresponding explicit constructions which extend Georgi-Glashow $SU(5)$.

Section (4), *$\mathcal{F}$-resh Analysis* , documents the majority of the original efforts undertaken in this study. Beginning with the renormalization group equations (RGEs) of the MSSM, we systematically establish a dictionary of solutions for the unification parameters of Standard and Flipped $SU(5)$, including threshold and second loop effects individually for each of the three couplings. We both distinguish and relate the two model classes, while carefully identifying and isolating the set of dependent variables in use at any moment, demonstrating how lack of caution in this regard has sometimes in the past led to a subtle pollution of resulting formulae.

Subsequently presented in Section (4), is a detailed approximate solution of the second loop contributions to the renormalization group. While applications are readily available for the numerical evaluation of second order effects, they do not generally enforce the flipped unification paradigm which we favor, and they moreover suffer from the lack of visual transparency which is endemic to any processed solution. The methods presented offer either an advanced launching position from which to initiate a numerical analysis, or a full, albeit more greatly simplified, closed form solution. Both approaches are parameterized to quickly and efficiently interface with the main results on gauge unification, with the formalism of the second loop corrections placed on equal footing with the companion correction factors from the thresholds. The updated treatment of these two factors is found to speed the baseline predictions for proton decay even without inclusion of the exotic F-theory field content.

Section (4) concludes with an elaboration on the methods used for evaluation of the GUT scale (heavy) mass thresholds. We reproduce an established formula for the single parameter approximation of the consequences of heavy thresholds to Standard $SU(5)$, and demonstrate that the historical cross-application of this formula to Flipped $SU(5)$ is inappropriate, proposing a more satisfactory alternative. We also make a correction to a previously published formula for the shift in the unification scale M_{32} which accompanies the activation of heavy thresholds in Flipped $SU(5)$.

Section (5), *$\mathcal{F}$-ast Proton Decay* , contains the principle results of the present study, in the form of description, tables and figures. We summarize the updates to data, field content, and methodology which have culminated in the present reductions in the predicted proton lifetime, distinguishing between the portion of the overall change which is to be attributed to each modification. Applying the techniques established in the prior section, we go on to undertake a thorough survey of the possible heavy threshold contributions to both Flipped and Standard $SU(5)$. Our study supports the previously advertised conclusion that heavy thresholds cannot save Standard $SU(5)$ from overly rapid dimension five proton decay, and makes the new suggestion that heavy effects in Flipped $SU(5)$ act only to suppress, and never to enhance, dimension six proton decay.

Section (6), *$\mathcal{F}$-inale* , opens with the final stage of Flipped $SU(5)$ grand unification, namely the second stage running of α_5 and α_X. We observe that the 'weakness' of the two step unification becomes here a great strength, without which the rationale for the splitting of the traditional single GUT scale from the Planck mass becomes more strained. The F-theory fields play a decisive role in this picture by stretching the natural Flipped $SU(5)$ scale separation across the required two-plus orders of magnitude.

We close the final section with a brief summary and conclusions, emphasizing the cohesive plot line that the cumulative effect of Flipped $SU(5)$ grand unification in the F-theory model building context, applying the freshly detailed methods of analysis presented herein, is surprisingly fast proton decay.

In an appendix, we offer a brief review of del Pezzo surfaces.

1.3. Proton Lifetime Considerations

Let us first briefly review the existing and proposed proton decay experiments. Despite the incredible contribution of knowledge made by Super-Kamiokande to the physics of neutrino oscillations, this is in fact not even the principal task of that experiment. Faced with the incomprehensibly long time scales on which proton decay is expected to be manifest, some 23 − 26 orders of magnitude older than the universe itself, we can only hope to observe this process within some reasonably finite interval by leveraging Avogadro's number to our benefit and watching some very large number of nuclei simultaneously. Indeed, that is precisely the role of the extravagant size allowed to this detector. This 50-kiloton (kt) water Čerenkov detector has set the current lower bounds of 8.2×10^{33} and 6.6×10^{33} years at the 90% confidence level for the partial lifetimes in the $p \to e^+ \pi^0$ and $p \to \mu^+ \pi^0$ modes [33].

Hyper-Kamiokande is a proposed 1-Megaton detector, about 20 times larger volumetrically than Super-K [34], which we can expect to explore partial lifetimes up to a level near 2×10^{35} years for $p \to (e|\mu)^{+}\pi^{0}$ across a decade long run. The proposal for the DUSEL experiment [35–37] features both water Čerenkov and liquid Argon (which is around five times more sensitive per kilogram to $p \to K^{+}\overline{\nu}_{\mu}$ than water) detectors, in the neighborhood of 500 and 100 kt respectively, with the stated goal of probing partial lifetimes into the order of 10^{35} years for both the neutral pion and K^{+} channels.

For the $p \to K^{+}\overline{\nu}$ partial lifetime as mediated by the triplet Higgsinos of $SU(5)$, a lower bound of 6.7×10^{32} years has been established at the 90% confidence level. This is a so-called dimension-five decay, summing the mass level of the two-boson, two-fermion effective vertex *à la* Fermi. The upper bound on its rate translates directly to a minimal mass for the color-triplet Higgs of around 10^{17} [GeV] [32]. Conversely though, compatibility of a strict unification with the precision LEP measurements of SM parameters at M_Z places an upper limit on this mass of order 10^{15} [GeV].

Flipped $SU(5)$ evades this incongruity by means of the 'missing-partner mechanism' [16] which naturally splits the heavy triplets $H_{\mathbf{3}}$ within the five of Higgs (h) away from the light electroweak components $H_{\mathbf{2}}$. Specifically, since the flipped **10** now contains a neutral element it is possible to allow vacuum expectation values for the breaking of $SU(5)$ to arise within a Higgs decaplet H from this representation. The GUT superpotential elements HHh and $\overline{H}\overline{H}\overline{h}$ then provide for the mass terms $\langle \nu^{c}_{H} \rangle d^{c}_{H} H_{\mathbf{3}}$ (and conjugate), while $H_{\mathbf{2}}$ is left light, having no partner in H with which to make a neutral pairing. So then is adroitly bypassed all insinuation of a hand-built term $Mh_{\mathbf{5}}h_{\bar{\mathbf{5}}}$ to finely tune against the putative adjoint GUT Higgs $h_{\mathbf{5}}h_{\bar{\mathbf{5}}}\Sigma_{\mathbf{24}}$ for fulfillment of this same goal. With that term goes also the undesirable triplet mixing, the dangerously fast proton decay channel and the fatal limits[1]. on the mass of $H_{\mathbf{3}}$ As for Standard $SU(5)$ however, this is just another splash of cold water from our friends at Super-K. And as we have mentioned, they have no shortage of cold water.

This diversion put aside, the dimension six decay $p \to (e|\mu)^{+}\pi^{0}$ may now regain our attention [17, 29, 30]. With aid of the SUSY extension, neither theory is under any fear from the current lower bound for this mode. That is not to say though that interesting differences do not exist between the pictures. In Standard $SU(5)$ there are two effective operators which contribute in sum to this rate. The first vertex arises from the term $\mathbf{10}\,\bar{\mathbf{5}}\,\mathbf{10}^{*}\,\bar{\mathbf{5}}^{*}$ and the second from $\mathbf{10}\,\mathbf{10}\,\mathbf{10}^{*}\,\mathbf{10}^{*}$ with a relative strength of $(1 + |V_{ud}|^{2})^{2}$. However, in Flipped $SU(5)$, e^{c}_{L} no longer resides within the **10**, so the positronic channel makes use of only e^{c}_{R} decays utilizing the operator which contains the representation $\bar{\mathbf{5}}$. Taking the central value of .9738(5) for the Cabibbo quark-mixing phase V_{ud} leads to a suppression of the total rate by a factor of about five after dividing out the correction $(1 + (1 + |V_{ud}|^{2})^{2})$. In opposition to this effect is a tendency toward more rapid decay due to dependence on the intermediate partial unification scale M_{32} rather than the traditional GUT value. In fact, we will see that this second distinction generally overwhelms the first, leading on balance to a net shorter prediction of the proton lifetime in Flipped $SU(5)$.

The effective dimension six operator for proton decay [38, 39] in Flipped $SU(5)$ is given following, suppressing the hermitian conjugate terms, where g_5 is the $SU(3)_C \times SU(2)_L$ unified gauge coupling, θ_c is the Cabibbo angle, and u, d, s are the up, down and strange quarks,

[1]The $d = 5$ mode is not *entirely* abandoned, as there does remain the supersymmetric term $\mu h\bar{h}$ for suppression of electroweak axions. It is slower though by a ratio $(\frac{\mu}{M_{H_3}})^{2}$, where $\mu \sim$ TeV.

respectively.

$$\bar{\mathcal{L}}_{\Delta B \neq 0} = \frac{g_5^2}{2M_{32}^2}\Big[(\epsilon^{ijk}\bar{d}_k^c e^{2i\eta_{11}}\gamma^\mu P_L d_j)(u_i\gamma_\mu P_L \nu_L) + (\epsilon^{ijk}(\bar{d}_k^c e^{2i\eta_{11}}\cos\theta_c + \bar{s}_k^c e^{2i\eta_{21}}\sin\theta_c)\gamma^\mu P_L u_j)(u_i\gamma_\mu P_L \ell_L)\Big] \quad (1)$$

The decay amplitude is proportional to the overall normalization of the proton wave function at the origin. The relevant matrix elements α and β are defined as:

$$\begin{aligned} \langle 0|\epsilon_{ijk}(u^i d^j)_R u_L^k|p(\mathbf{k})\rangle &\equiv \alpha\, \mathrm{u}_L(\mathbf{k}) \\ \langle 0|\epsilon_{ijk}(u^i d^j)_L u_L^k|p(\mathbf{k})\rangle &\equiv \beta\, \mathrm{u}_L(\mathbf{k}) \end{aligned} \quad (2)$$

The reduced matrix elements α and β have been calculated in a lattice approach [40], with central values $\alpha = \beta = 0.015$ [GeV^3] reported. Quoted errors are below 10%, corresponding to an uncertainty of less than 20% in the proton partial lifetime, negligible compared to other uncertainties present in our calculation.

Following the results of [32, 41] and references therein, we present a numerically parameterized expression for the desired lifetime in both Standard and Flipped $SU(5)$, with coefficients appropriate to the flipped specialization already absorbed in the latter case.

$$\tau^{\mathrm{SU(5)}}_{p\to(e|\mu)^+\pi^0} = 0.8 \times \left(\frac{M_{32}^{\max}}{10^{16}\ [\mathrm{GeV}]}\right)^4 \times \left(\frac{0.0412}{\alpha_5^{\max}}\right)^2 \times 10^{35}\ [\mathrm{Y}] \quad (3a)$$

$$\tau^{\mathcal{F}\text{-SU(5)}}_{p\to(e|\mu)^+\pi^0} = 3.8 \times \left(\frac{M_{32}}{10^{16}\ [\mathrm{GeV}]}\right)^4 \times \left(\frac{0.0412}{\alpha_5}\right)^2 \times 10^{35}\ [\mathrm{Y}] \quad (3b)$$

The proton lifetime scales as a fourth power of the $SU(5)$ unification scale M_{32}, and inversely, again in the fourth power, to the coupling $g_{32} \equiv \sqrt{4\pi\alpha_5}$ evaluated at that scale. This extreme sensitivity argues for great care in the selection and study of a unification scenario. Lower bounds can only ever exclude a model, never truly supporting any one competing suggestion. The real goal of course is to constrain this number from both directions. Assisted by the shorter net Flipped $SU(5)$ lifetime, the increased reach of next-generation experiments offers the tantalizing prospect of probing the most relevant parameter space.

We close this section with a survey of some characteristic predictions which have been made for Flipped $SU(5)$ proton decay based on the baryon-number violating effective potential of Eq. (1). Unknown parameters in this expression are the the CP-violating phases $\eta_{11,21}$ and lepton flavor eigenstates ν_L and ℓ_L, related to the mass diagonal mixtures as:

$$\nu_L = \nu_F U_\nu \qquad ; \qquad \ell_L = \ell_F U_\ell \quad (4)$$

These mixing matrices $U_{(\nu|\ell)}$ take on added currency in the age of neutrino oscillations. Having seen there evidence for near-maximal mixing, it seems reasonable to suspect that at least some $(e|\mu)$ entries are also $\mathcal{O}(1)$ in U_ℓ. From this point it will indeed be assumed that $|U_{\ell_{11,12}}|^2$ are $\mathcal{O}(1)$, thus avoiding further large numerical suppressions of both the $p \to (e|\mu)^+\pi^0$ rates[2]. No

[2]Note that there is no corresponding suppression of the $p \to \bar{\nu}\pi^+$ and $n \to \bar{\nu}\pi^0$ modes, since all neutrino flavors are summed over.

more can be said though regarding the ratio of $p \to e^+X$ and $p \to \mu^+X$ decays, and as such it would be good that any next-generation detector be equally adept at the exposure of either mode. Throughout this report we thus diligently emphasize the electronic-muonic product ambiguity as an essential Flipped $SU(5)$ characteristic.

Despite the discussed points of ignorance, it can still be robustly stated that [17]:

$$\begin{aligned}
\Gamma(p \to e^+\pi^o) &= \tfrac{\cos^2\theta_c}{2}|U_{\ell_{11}}|^2\Gamma(p \to \bar{\nu}\pi^+) = \cos^2\theta_c|U_{\ell_{11}}|^2\Gamma(n \to \bar{\nu}\pi^o) \\
\Gamma(n \to e^+\pi^-) &= 2\Gamma(p \to e^+\pi^o) \quad ; \quad \Gamma(n \to \mu^+\pi^-) = 2\Gamma(p \to \mu^+\pi^o) \\
\Gamma(p \to \mu^+\pi^o) &= \tfrac{\cos^2\theta_c}{2}|U_{\ell_{12}}|^2\Gamma(p \to \bar{\nu}\pi^+) = \cos^2\theta_c|U_{\ell_{12}}|^2\Gamma(n \to \bar{\nu}\pi^o)
\end{aligned} \tag{5}$$

We note [17, 41] that the Flipped $SU(5)$ predictions for decay ratios involving strange particles, neutrinos and charged leptons differ substantially from those of conventional $SU(5)$. Comparison of such characteristic signals then constitutes a potentially powerful tool for establishing mixing patterns and differentiating between GUT proposals.

1.4. Multiple Threshold Renormalization

To determine the effects of embedding TeV scale vector multiplets within a GUT scenario, it is useful to start by studying generally, and from first principles, the way in which renormalization kinks at intermediate thresholds influence the relationship between coupling strengths at the bounding scales.

The transition between any two such intermediate scales (containing no threshold transition) is given by the standard one-loop renormalization group equation[3] (cf. Figure 1).

$$\frac{2\pi}{\alpha_{i-1}} - \frac{2\pi}{\alpha_i} = b_i\,(\ln M_i - \ln M_{i-1}) \equiv \Delta_i \tag{6}$$

Summing over all steps, the total shift in the coupling is obtained. To facilitate comparison to the case with no intermediate thresholds, we will define a factor ξ_N to encapsulate all dependence on internal masses M_i and β-coefficients b_i.

$$\sum_{i=1}^{N} \Delta_i = \frac{2\pi}{\alpha_0} - \frac{2\pi}{\alpha_N} = b_N \ln\frac{M_N}{M_0} - \xi_N \tag{7}$$

$$\begin{aligned}
(-)\,\xi_N \equiv\; & b_N \ln M_0 - b_1 \ln M_0 + b_1 \ln M_1 - b_2 \ln M_1 \\
& + b_2 \ln M_2 - b_3 \ln M_2 + b_3 \ln M_3 + \cdots - b_N \ln M_{N-1} \\
=\; & \ln\left[\left(\frac{M_1}{M_0}\right)^{b_1}\left(\frac{M_2}{M_1}\right)^{b_2}\left(\frac{M_3}{M_2}\right)^{b_3} \times\cdots\times \left(\frac{M_0}{M_{N-1}}\right)^{b_N}\right]
\end{aligned} \tag{8}$$

[3]The index i here labels the threshold *segment* of the running being considered. It does not label the group to which α belongs.

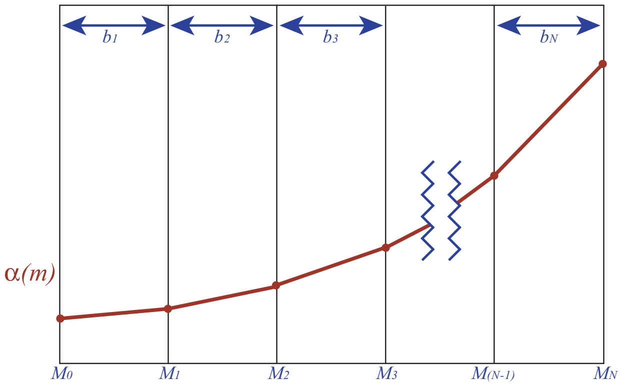

Figure 1: Kinking is depicted at multiple threshold points labeled $(0 \ldots N)$ in the extended running of some coupling α. The naming convention shown is used in related equations and calculations.

For concreteness in evaluation of Eq. (8), we will will specialize to the example $(N = 3)$.

$$\begin{aligned}
\xi_3 &= (-)\ \ln\left[\left(\frac{M_1}{M_0}\right)^{b_1}\left(\frac{M_2}{M_1}\right)^{b_2}\left(\frac{M_0}{M_2}\right)^{b_3}\right] \\
&= (-)\ \ln\left[\left(\frac{M_1}{M_0}\right)^{(b_1-b_2)}\left(\frac{M_2}{M_0}\right)^{b_2}\left(\frac{M_0}{M_2}\right)^{b_3}\right] \\
&= \ln\left[\left(\frac{M_1}{M_0}\right)^{(b_2-b_1)}\left(\frac{M_2}{M_0}\right)^{(b_3-b_2)}\right]
\end{aligned} \tag{9}$$

This compact form readily regeneralizes to any choice of N. Picking back up from Eq. (8), we group together repeated occurrences of each M_i, separating out the special case M_0. This term is then reabsorbed into the sum over intermediate masses by use of the identity $(b_N - b_1) = \sum_{i=1}^{N-1}(b_{i+1} - b_i)$. The notation $\delta b_i \equiv (b_{i+1} - b_i)$ is adopted.

$$\xi_N = -\ln M_0^{(b_N-b_1)} + \sum_{i=1}^{N-1} \ln M_i^{(b_{i+1}-b_i)} = \sum_{i=1}^{N-1} \delta b_i \ln\left(\frac{M_i}{M_0}\right) \tag{10}$$

The final result for running which crosses internal threshold points is then:

$$\left\{\frac{2\pi}{\alpha_0} - \frac{2\pi}{\alpha_N}\right\} + \left\{\xi_N \equiv \sum_{\text{internal}} \delta b \ln\left(\frac{M}{M_0}\right)\right\} = b_N \ln\frac{M_N}{M_0} \tag{11}$$

This expression has the appealing feature that ordering of the kinks at each M_i is fully decoupled, depending only on the *change* induced in the β-coefficient at each given mass.

The underlying symmetry between the bounding energy scales suggests that it should likewise be possible to achieve an analogous formula which references each threshold against the heavy mass M_N. Of course, our general low energy perspective on the unification mass as an output prediction of the experimentally determined electroweak parameters would seem to preclude this view. However, it is exactly the form which would be useful for consideration of the ultra heavy threshold terms which arise from masses in direct associated proximity to the unification scale. Guided by this observation, we may attempt to split our perspective by defining an intermediate index I, $(1 \leq I \leq N)$, such that the coefficient b_I prevails after action of what is considered to be the final light threshold, and until action of the subsequent, and by definition initial heavy, threshold. The principal analysis then refactors as follows, starting from Eqs. (7,10).

$$\begin{aligned}\frac{2\pi}{\alpha_0} - \frac{2\pi}{\alpha_N} &= b_N \ln \frac{M_N}{M_0} + \ln M_0^{(b_N - b_1)} - \sum_{i=1}^{N-1} \ln M_i^{(b_{i+1} - b_i)} \\ &= b_I \ln \frac{M_N}{M_0} + \ln M_0^{(b_I - b_1)} + \ln M_N^{(b_N - b_I)} - \sum_{i=1}^{N-1} \ln M_i^{(b_{i+1} - b_i)} \\ &= b_I \ln \frac{M_N}{M_0} - \sum_{i=1}^{I-1} (b_{i+1} - b_i) \times \ln \left(\frac{M_i}{M_0} \right) - \sum_{i=I}^{N-1} (b_{i+1} - b_i) \times \ln \left(\frac{M_i}{M_N} \right)\end{aligned} \tag{12}$$

We have then the following alternate prescription of Eq. (11) for the efficient distinction between light and heavy thresholds. It will be particularly useful for the work of subsequent sections that *only* the better known light fields contribute to the main β-coefficient b_I.

$$\left\{ \frac{2\pi}{\alpha_0} - \frac{2\pi}{\alpha_N} \right\} + \left\{ \xi_N \equiv \sum_{\text{light}} \delta b \ln \left(\frac{M}{M_0} \right) - \sum_{\text{heavy}} \delta b \ln \left(\frac{M_N}{M} \right) \right\} = b_I \ln \frac{M_N}{M_0} \tag{13}$$

As a note in passing, we remark that prior study [42] suggests the requisite conversion factor between the minimal subtraction and dimensional regularization renormalization schemes to be a comparatively insignificant contribution.

1.5. Computation of MSSM Thresholds

We review [43] in this section the determination of Standard Model quantum numbers and β-function coefficients. The Standard Model (plus right-handed neutrinos) electric- and hypercharge strengths for the sixteen matter fields of each generation are given in Table (1).

	$\begin{pmatrix} u \\ d \end{pmatrix}_L \otimes 3$	$u_R^c \otimes 3$	$d_R^c \otimes 3$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	e_R^c	ν_R^c
$\{\mathrm{Q}, \mathrm{Y}'\}$	$\left\{ \begin{pmatrix} +2/3 \\ -1/3 \end{pmatrix}, +\frac{1}{6} \right\}$	$\{-\frac{2}{3}, -\frac{2}{3}\}$	$\{+\frac{1}{3}, +\frac{1}{3}\}$	$\left\{ \begin{pmatrix} 0 \\ -1 \end{pmatrix}, -\frac{1}{2} \right\}$	$\{+1, +1\}$	$\{0, 0\}$

Table 1: Standard Model (plus right-handed neutrino) field content and quantum numbers.

The diagonal eigenvalues of T_3 onto each doublet of $SU(2)_L$ are $\pm 1/2$ for up/down elements respectively, and the hypercharge Y' is assigned so as to satisfy the defining relation $\mathrm{Q} = \mathrm{T}_3 + \mathrm{Y}'$. However, in a unification scenario, $U(1)_\mathrm{Y}$ and $SU(2)_L$ must have a common normalization, meaning that $\sum(\mathrm{T}_3)^2 = \sum(\mathrm{Y})^2$ for each family. There are four doublets, times two fields each, times $((\pm 1/2)^2 = 1/4)$, for a total sum over T_3 of 2. For hypercharge Y':

$$3 \times (+1/6)^2 \times 2 \; + \; 3 \times (-2/3)^2 \; + \; 3 \times (+1/3)^2 \; + \; (-1/2)^2 \times 2 \; + \; 1 \; = \; 10/3 \tag{14}$$

This necessitates the scaling $\mathrm{Y} = \sqrt{5/3}\,\mathrm{Y}'$, where absence of the prime denotes the GUT normalized hypercharge[4]. Specifically, the physical product $g'_{\mathrm{Y}'} \times \mathrm{Y}' \equiv g_{\mathrm{Y}} \times \mathrm{Y}$ of coupling times assigned charge must be invariant, so to *reduce* an overly large sum of numerical hypercharge counts, we must *increase* the coupling by the same proportionality.

The β-function coefficients have renormalization effects from self interactions of the gauge bosons, fermion loops, and scalar loops.

$$b_i = \frac{-11}{3}\,C_2(G_i) + \sum_f \frac{4}{3}\frac{d(R)C_2(R)}{d(G_i)} + \sum_\Phi \frac{1}{6} \tag{15}$$

C_2 is the second Casimir invariant of the representation, and d is the dimension of the representation. For $SU(N \geq 2)$, $C_2(G_i) = N$, and for $U(1)$, which lacks self interactions, $C_2 = 0$. Matter fields of the Standard Model are exclusively housed in fundamental representations, for which the dimension $d(R)$ is just N. The dimension $d(G_i)$ of the $SU(N)$ adjoint is $N^2 - 1$, the number of generators. The second Casimir eigenvalue of the fundamental is $C_2(R) = \frac{N^2-1}{2N}$. The fermion loop thus reduces to $\sum_f \frac{4}{3} \times \frac{1}{2}$ for a L/R vector pair with $N \geq 2$. For the $U(1)$ special case however, it becomes $\sum_f \frac{4}{3} \times \frac{1}{2} \times (\mathrm{Y})^2$, leaving just a factor of $4/3$ per family after making use of the previously summed normalization. For $SU(2)_L$, there is an overall factor of $1/2$ since only one chirality is active. There are four fundamental doublets, for a net of $4/3 \times 1/2 \times 1/2 \times 4 = 4/3$ per generation. For color $SU(3)$, there are two vector triplets per family, for a total again of $4/3 \times 1/2 \times 2 = 4/3$. There is a single scalar doublet Higgs in the Standard Model, and thus a single term in the sum for $U(1)$ and $SU(2)$ of the scalar Φ variety. As before, the normalization on Y' must be included, for a total of $1/6 \times 3/5 = 1/10$. All together, with totals for $N_G = 3$ generations and $N_H = 1$ Higgs doublet:

$$\begin{array}{ccccccccc} b_{\mathrm{Y}} & = & 0 & + & \frac{4}{3}N_G & + & \frac{1}{10}N_H & \Rightarrow & \frac{41}{10} \\ b_2 & = & \frac{-22}{3} & + & \frac{4}{3}N_G & + & \frac{1}{6}N_H & \Rightarrow & -\frac{19}{6} \\ b_3 & = & -11 & + & \frac{4}{3}N_G & + & 0 & \Rightarrow & -7 \end{array} \tag{16}$$

In the MSSM, there is an extra Higgs doublet, required to avoid complex conjugation in the holomorphic superpotential while providing both up- and down-type quark masses[5]. Each of the $8 + 3 + 1 = 12$ gauge bosons, and each of the 16 matter fields also receive a superpartner shifted in spin by a half increment. The 'gauginos' are fermionic, spin-1/2, as are the 'Higgsinos'. The matter fields shift down to spin-0. There are no new spacetime vectors, which would be necessity be gauged, thus further enlarging the spectrum and moreover creating a representational mismatch between the partners. For $SU(3)$, the only new spin-1/2 content are the adjoint of gluinos, $\tilde{g}$, for which $d(R) = d(G_i) = N^2 - 1$. This leaves just $C_2(R)$, which, for the adjoint, equals N. However, there is also a factor of $\frac{1}{2}$, similar to that seen previously in the $SU(2)_L$ case. One set of gluons/gluinos service all generations, so there is no N_G type factor. The sum is over a single 8-plet: $4/3 \times (N = 3) \times 1/2 = 2$. On the scalar side, there are generational sets of $\begin{pmatrix}\tilde{u}\\ \tilde{d}\end{pmatrix} \otimes 3$, $\tilde{u}^c \otimes 3$, and $\tilde{d}^c \otimes 3$. This represents four 3-plets of complex scalars for each family, each

[4]The defining relation $\tan\theta_{\mathrm{W}} \equiv g'_{\mathrm{Y}'}/g_{\mathrm{L}}$ for the Weinberg angle references the original primed hypercharge coupling.
[5]Or alternatively to avoid a chiral anomaly for the superpartner $\tilde{H}$.

contributing a factor of $1/6$ to each b_i, for a total of $1/6 \times 4 \times N_G = 2/3N_G$ The net shift in b_3 is $\delta b_3 = 2 + 2/3N_G$.

Focusing on $SU(2)$, the wino triplet contributes $4/3 \times (N = 2) \times 1/2 = 4/3$. *Each* of the Higgs doublets adds $1/6$, and their fermionic partners are in for $4/3 \times 1/2 \times N_H \times 1/2$, with appearance again of the halving term. The total Higgs portion of b_2 is $N_H \times (1/6 + 1/3) = N_H/2$, which replaces the prior $N_H/6$ factor. The generational scalar $SU(2)$ contributions come from the super partners of the left handed quarks and leptons, $\begin{pmatrix}\tilde{u}\\ \tilde{d}\end{pmatrix} \otimes 3$, and $\begin{pmatrix}\tilde{\nu}\\ \tilde{e}\end{pmatrix}$. The effect is tallied for four complex doublets at a rate of $1/6$ per family, for a total $2/3N_G$. The net shift is $\delta b_2 = 4/3 + 2/3N_G + (1/2 - 1/6)N_H$.

Finally, we consider the normalized hypercharge Y. The W's carry Q, but are hypercharge neutral. The $U(1)_{\mathrm{Y}}$ generator B^0 does not self-interact. The sole source for new spin-1/2 contributions is thus the Higgsinos. H_u and $\tilde{H}_u$ carry $\mathrm{Y}' = -1/2$, while the value is $+1/2$ for $(H_d, \tilde{H}_d)$. The sum of Y'^2 over two Higgsino doublets is thus $4 \times (1/2)^2 = 1$, giving a normalized hypercharge of $\sum(\mathrm{Y})^2 = 3/5$, and a β-function contribution of $\sum 4/3 \times 1/2 \times (\mathrm{Y})^2 = 2/5$. This is for $N_H = 2$, and we may more generally write the term as $N_H/5$. The original scalar H_d and the new H_u contribute $1/6 \times 3/5 = 1/10$ each, or $N_H/10$ together. The combined Higgs sector contribution to b_{Y} is thus $\frac{3}{10}N_H$. Finally, we have the scalar matter partners for each generation. Note that the Higgs contribution which was written as $1/6 \times 3/5$ is alternatively equivalent to $1/6 \times 2 \sum(\mathrm{Y})^2$. Taking instead the known value of $\sum(\mathrm{Y})^2 = 2$ for a full matter-partner generation, we apparently get a total $1/6 \times 2 \times N_G \times 2 = \frac{2}{3}N_G$. The net shift is $\delta b_{\mathrm{Y}} = \frac{2}{3}N_G + \frac{3-1}{10}N_H$. All together, the MSSM coefficients, with $(N_G = 3, N_H = 2)$ in the totals, are:

$$\begin{array}{rcccccccc} b_{\mathrm{Y}} & = & 0 & + & 2N_G & + & \frac{3}{10}N_H & \Rightarrow & \frac{33}{5} \\ b_2 & = & -6 & + & 2N_G & + & \frac{1}{2}N_H & \Rightarrow & 1 \\ b_3 & = & -9 & + & 2N_G & + & 0 & \Rightarrow & -3 \end{array} \qquad (17)$$

The need for such measured accounting exists because we must ultimately assign individual shifts to each of the β-coefficients at the threshold mass where each MSSM field enters the renormalization. We provide in Table (2) the resulting detailed breakdown, noting respect for the general principle that the fermionic member of a spin $(0, \frac{1}{2})$ super field doubles the β-contribution of its scalar partner. Multiplicities ($\otimes$) refer to the group theoretic representational degeneracy. The numbers shown are per generation where applicable.

As a practical matter, output of the standard tools such as SSARD, FeynHiggs and ISASUGRA used to determine sparticle masses is not in one-to-one correspondence with the fields listed in Table (2). Table (3) provides β-contributions for all mass content above M_{Z}, tallied for the specific spectral output of these program. Print style symbols and text style field designations are given, as well as the full corresponding MSSM representation. Multiplicities ($\times$) refer to the number of included generations. For values spanning multiple rows, the predicted masses will merged as a geometric mean for purposes of computation. The fields (h^0, H^0, A^0) represent mixtures of the three surviving neutral Higgs elements from the complex doublets $\left(H_u \equiv \begin{pmatrix}0\\ -\end{pmatrix}, H_d \equiv \begin{pmatrix}+\\ 0\end{pmatrix}\right)$ after Z^0 becomes massive in the symmetry breaking. Likewise, $H^{\pm}$ are the two charged Higgs elements surviving after generation of the $W^{\pm}$ masses.

Field	δb_Y	δb_2	δb_3	S-Field	δb_Y	δb_2	δb_3
B^0	0	0	0	$\tilde{B}^0$	0	0	0
$(W^{\pm,0})$	0	$\frac{-22}{3}$	0	$(\tilde{W}^{\pm,0})$	0	$\frac{4}{3}$	0
$g \otimes 8$	0	0	-11	$\tilde{g} \otimes 8$	0	0	2
$u_L \otimes 3$	$\frac{1}{30}$	$\frac{1}{2}$	$\frac{1}{3}$	$\tilde{u}_L \otimes 3$	$\frac{1}{60}$	$\frac{1}{4}$	$\frac{1}{6}$
$d_L \otimes 3$	$\frac{1}{30}$	$\frac{1}{2}$	$\frac{1}{3}$	$\tilde{d}_L \otimes 3$	$\frac{1}{60}$	$\frac{1}{4}$	$\frac{1}{6}$
$u_R^c \otimes 3$	$\frac{8}{15}$	0	$\frac{1}{3}$	$\tilde{u}_R^c \otimes 3$	$\frac{4}{15}$	0	$\frac{1}{6}$
$d_R^c \otimes 3$	$\frac{2}{15}$	0	$\frac{1}{3}$	$\tilde{d}_R^c \otimes 3$	$\frac{1}{15}$	0	$\frac{1}{6}$
ν_L	$\frac{1}{10}$	$\frac{1}{6}$	0	$\tilde{\nu}_L$	$\frac{1}{20}$	$\frac{1}{12}$	0
e_L	$\frac{1}{10}$	$\frac{1}{6}$	0	$\tilde{e}_L$	$\frac{1}{20}$	$\frac{1}{12}$	0
ν_R^c	0	0	0	$\tilde{\nu}_R^c$	0	0	0
e_R^c	$\frac{2}{5}$	0	0	$\tilde{e}_R^c$	$\frac{1}{5}$	0	0
$H_u \otimes 2$	$\frac{1}{10}$	$\frac{1}{6}$	0	$\tilde{H}_u \otimes 2$	$\frac{1}{5}$	$\frac{1}{3}$	0
$H_d \otimes 2$	$\frac{1}{10}$	$\frac{1}{6}$	0	$\tilde{H}_d \otimes 2$	$\frac{1}{5}$	$\frac{1}{3}$	0

Table 2: Detailed breakdown of contributions to the one-loop MSSM β-function coefficients.

Print	Text	MSSM Content	δb_{Y}	δb_2	δb_3
t	t	$(u_L, u_R^c) \times 1$	$\frac{17}{30}$	$\frac{1}{2}$	$\frac{2}{3}$
h^0	HL	(H_u^0, H_d^0)	$\frac{1}{40}$	$\frac{1}{24}$	0
H^0	HH		$\frac{1}{40}$	$\frac{1}{24}$	0
A^0	HA		$\frac{1}{40}$	$\frac{1}{24}$	0
$H^\pm$	H+	(H_u^-, H_d^+)	$\frac{1}{20}$	$\frac{1}{12}$	0
$\tilde{\chi}_1^0$	Z1	$(\tilde{B}^0, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0)$	$\frac{1}{5}$	$\frac{7}{9}$	0
$\tilde{\chi}_2^0$	Z2				
$\tilde{\chi}_3^0$	Z3				
$\tilde{\chi}_4^0$	Z4				
$\tilde{\chi}_1^\pm$	W1	$(\tilde{W}^-, \tilde{W}^+, \tilde{H}_u^-, \tilde{H}_d^+)$	$\frac{1}{5}$	$\frac{11}{9}$	0
$\tilde{\chi}_2^\pm$	W2				
$\tilde{g}$	GL	$\tilde{g}$	0	0	2
$\tilde{e}_L^\pm$	EL	$\tilde{e}_L \times 2$	$\frac{1}{10}$	$\frac{1}{6}$	0
$\tilde{e}_R^\pm$	ER	$\tilde{e}_R^c \times 2$	$\frac{2}{5}$	0	0
$\tilde{\nu}$	SN	$(\tilde{\nu}_L, \tilde{\nu}_R^c) \times 2$	$\frac{1}{10}$	$\frac{1}{6}$	0
$\tilde{\tau}_1^\pm$	TAU1	$(\tilde{e}_L, \tilde{e}_R^c) \times 1$	$\frac{1}{4}$	$\frac{1}{12}$	0
$\tilde{\tau}_2^\pm$	TAU2				
$\tilde{\nu}_\tau$	NTAU	$(\tilde{\nu}_L, \tilde{\nu}_R^c) \times 1$	$\frac{1}{20}$	$\frac{1}{12}$	0
$\tilde{u}_L$	UL	$\tilde{u}_L \times 2$	$\frac{1}{30}$	$\frac{1}{2}$	$\frac{1}{3}$
$\tilde{u}_R$	UR	$\tilde{u}_R^c \times 2$	$\frac{8}{15}$	0	$\frac{1}{3}$
$\tilde{d}_L$	DL	$\tilde{d}_L \times 2$	$\frac{1}{30}$	$\frac{1}{2}$	$\frac{1}{3}$
$\tilde{d}_R$	DR	$\tilde{d}_R^c \times 2$	$\frac{2}{15}$	0	$\frac{1}{3}$
$\tilde{t}_1$	T1	$(\tilde{u}_L, \tilde{u}_R^c) \times 1$	$\frac{17}{60}$	$\frac{1}{4}$	$\frac{1}{3}$
$\tilde{t}_2$	T2				
$\tilde{b}_1$	B1	$(\tilde{d}_L, \tilde{d}_R^c) \times 1$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{3}$
$\tilde{b}_2$	B2				

Table 3: Beta function contributions for the full MSSM field content above M_{Z} are broken down according to the output groupings of standard sparticle program libraries.

We may now proceed with a computation of the sum from Eq. (10) for each of the three gauge couplings as soon as an appropriate sparticle mass spectrum has been specified. We will generally defer here to the suggested post-WMAP CMSSM benchmark scenarios[6] of Ref. [45]. Specifically, we will default to the point B' as characterized by the values following due to its proximity to the region of parameter space favored by a least-squares analysis [46] of predictions for the W boson mass, the effective Weinberg angle, $(g-2)$ of the muon, and the branching ratio for $(b \to s\gamma)$. Recognizing that improvements both in experimental constraint and in the refinement of certain program libraries have transpired since publication of these references, we nevertheless suggest that the present benchmark set sufficiently serves the purpose of collectively establishing an approximate border on variation within plausible parameter space bounds while facilitating ease of comparison with other work.

$$\begin{array}{c} A_0 = 0 \quad ; \quad \mu > 0 \quad ; \quad \tan\beta = 10 \\ m_0 = 60 \quad ; \quad m_{1/2} = 250 \end{array} \tag{18}$$

We will accept at face value the supersymmetric mass textures suggested in [45], which were computed using the SSARD and FeynHiggs library routines[7]. Recognizing also that we will shortly diverge from certain assumptions made there as regards flipped unification and the addition of extra TeV scale multiplets, we again consider the spectrum to be a sufficiently plausible one for our purposes, and judge that any potential discrepancy represents a correction to the correction. For the top quark mass, we adopt the printed 'benchmark' value only for its own threshold factor, rejecting it for all other calculations[8], *e.g.* the Yukawa boundary on the second loop, in favor of the Tevatron Electroweak Working Group world average $m_t = 173.1 \pm 1.3$ [GeV], courtesy of CDF and DØ [47]. We refer back to Eq. (46) for M_Z. Our baseline threshold corrections for the MSSM are then calculated to take the values shown following.

$$\xi_{\rm Y} \approx 3.20 \quad ; \quad \xi_2 \approx 5.48 \quad ; \quad \xi_3 \approx 8.05 \tag{19}$$

We have recently studied [48] the implications of gravity mediated F-theoretic supersymmetry breaking for the electroweak scale gaugino mass relations, presenting two scenarios which are consistent with the latest experimental constraints, including CDMS II. The interplay of these considerations with the topic of proton decay will be taken up in a forthcoming letter.

2. $\mathcal{F}$-lipped $SU(5)$

If the notion of a GUT is to be feasible, then one must necessarily inquire as to candidates for the unified group structure and what representational form the known interactions and fields would take within that group. To contain the SM, $SU(3)_C \times SU(2)_L \times U(1)_Y$, which carry respectively two, one and one diagonal generators, a group of minimal rank four is required. The natural starting point is then $SU(5)$, whose lowest order representations are the singlet **1**, the fundamental **5**, the (anti) symmetric tensors **10** and **15**, the adjoint **24** which transforms as the generators, and their related conjugates.

[6] A partial update (excepting the focus point region and $\mu < 0$) from single to double-primed benchmarks was made in 2005 [44], but the distinction is not significant to our purposes here.

[7] The gluino mass scale appears to be generically larger than that produced by ISASUGRA, but the spectra are otherwise in reasonable agreement.

[8] We likewise prefer the PDG value of the strong coupling α_3 for all calculations.

Within each family, all SM states must be assigned a residence which is compatible with their existing quantum numbers, as shown in Table (4). A charge-parity involution is understood where needed such that all grouped fields will carry a consistent handedness. The six quark states of the left-doublet, color-triplet must then fit in at least an antisymmetric **10**. This leaves four spots open, tidily filled by one right-handed color triplet and a right handed singlet. The remaining color-triplet partners neatly with the electron-neutrino left-doublet as a $\bar{\mathbf{5}}$, canceling the non-Abelian anomaly of the **10**. Not only are the fifteen Standard Model states thus uniquely and compactly represented, but we are gifted additional 'wisdom' in the process. By assignment each of the **10** and $\bar{\mathbf{5}}$ are electrically neutral, thus correlating charge quantization to the number of color degrees of freedom. Furthermore, the apparent masslessness of the neutrinos finds a pleasing justification: there is simply no room remaining in which to house a right-handed component.

$$\begin{matrix} \begin{pmatrix} u \\ d \end{pmatrix}_L \; ; \; u_L^c \; ; \; d_L^c \\ \\ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \; ; \; e_L^c \; ; \; \boxed{\nu_L^c} \end{matrix} \quad \overset{\text{JUST}}{\underset{\text{RIGHT}}{\longrightarrow}} \quad \underset{\bar{\mathbf{5}}}{\begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ \nu_e \end{pmatrix}_L} \; ; \; \underset{\mathbf{10}}{\left(\begin{pmatrix} u \\ d \end{pmatrix}_L \; u_L^c \; e_L^c \right)} \; ; \; \underset{\boxed{\mathbf{1}}}{\boxed{\nu_L^c}}$$

Table 4: The Standard $SU(5)$ charge assignments are 'just right' to compactly house a full generation within a fundamental five-plet and an antisymmetric ten-plet. However, this applies only if if one is willing to either neglect the right-handed neutrino or exile it to a singlet representation.

But experimental evidence cannot any longer allow an agnostic position on neutrino masses. The Super-Kamiokande facility in Japan, which houses $50,000$ metric tonnes of ultra-pure water inside a 40-meter high by 40-meter diameter cylindrical tank faced on all sides by a collection of $13,000$ photomultiplier Čerenkov detectors and shielded beneath $2,700$ meters of earth within the cavity of an old mine, has been diligently studying the problem for many years. By comparing the careful observation of neutrino fluxes with atmospheric origination against the expected detection ratios from known interaction cascades they have borne convincing witness to oscillation between the ν_μ and ν_τ sectors[9]. This phenomenon may only occur when the related states carry non-equivalent masses, of which at least one must then be non-zero. However, chirality can only be an invariant quantum number for massless states, and we are thus compelled to introduce a sixteenth element for accommodation of the right-handed neutrino[10]. It is certainly possible to imagine the new state as a simple singlet outside the main representations already laid down. Surely though it is presumptive to suppose that this right-handed neutrino, while arriving 'last' must also be seated as the 'least'. Every existing position must instead be subject to reassignment. There is indeed then another way, if one is willing to sacrifice charge quantization. We can choose to 'flip' [11] the right-handed quarks placed inside the $\bar{\mathbf{5}}$ and **10** while also swapping e_L^c for ν_L^c.

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[9]Likewise, $\nu_e \leftrightarrow \nu_\mu$ oscillation has been induced from the observation of solar neutrinos by the SNO collaboration.
[10]There is a loophole to this consideration for Standard $SU(5)$ formulations with pure Majorana neutrino masses
[11]For a comprehensive review of Flipped $SU(5)$, please consult [18]

$$f_{\bar{\mathbf{5}}} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ e \\ \nu_e \end{pmatrix}_L \quad ; \quad F_{\mathbf{10}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L \quad d_L^c \quad \nu_L^c \right) \quad ; \quad l_{\mathbf{1}} = e_L^c$$

Table 5: The Flipped $SU(5)$ assignment exchanges the roles of the two right-handed quark triplets and also the two right-handed lepton singlets. The resulting structure is no longer a simple group.

The cost of this flipping is nothing less than the loss of grand unification, as the resulting symmetry group is enlarged to the non-simple variant $SU(5) \times U(1)_X$. As demonstrated in Figure (2), the hypercharge does not descend out of $SU(5)$ together with the nuclear forces, but is instead of an admixture of $U(1)_X$ together with the additional U(1) factor which is emergent from that breaking. If this tune sounds familiar though, it is simply a reprise of the theme played out some fourteen orders of magnitude below in the Glashow-Weinberg-Salam $SU(2)_L \times U(1)_Y$ electroweak 'unification'. Just as it was there no loss to save a true convergence for the future inclusion of color perhaps it is here folly to imagine a full unification which occurs on the border of the Planck mass without waiting on gravity. Just as only experiment could there reject the truly unified but dysfunctional $SU(2)$ model of Georgi and Glashow, between (or against!) these contenders we can again only let phenomenology decide. And here there is no finer judge than the consideration of proton decay.

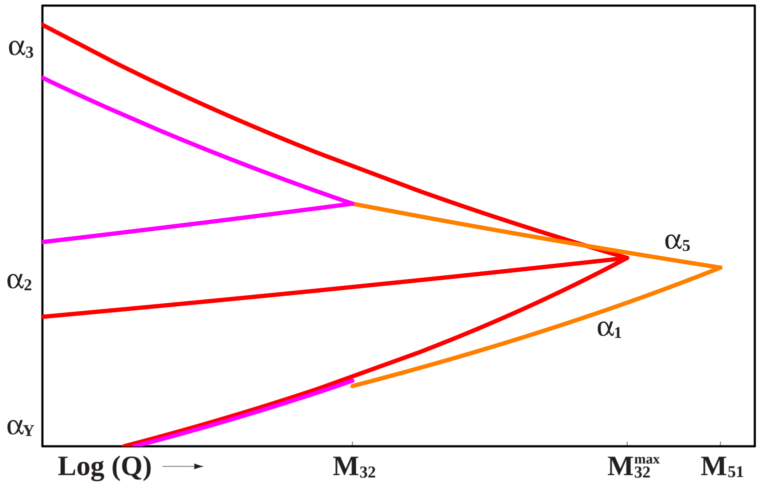


Figure 2: An exaggerated heuristic demonstration of the prominent features of flipped coupling unification. Notice the discontinuity in the (purple) line of $U(1)_Y$, as it remixes between the 'grand unified' $U(1)_{\mathrm{X}}$, and that which emerges out of broken $SU(5)$ at the scale M_{32}. Proceeding upward from this interim stage in orange, $SU(5) \times U(1)_{\mathrm{X}}$ is itself unified at some higher scale M_{51}. For comparison, the Standard $SU(5)$ scenario is shown in red with a single unification at $M_{32}^{\mathrm{max}} \geq M_{32}$, and 'predicting' a larger value for $\alpha_s(M_Z)$.

One of the major predictions of GUTs is that the proton becomes destabilized due to the quark and lepton unification. Pairs of quarks may transform into a lepton and an anti-quark via dimension six operators for the exchange of heavy gauge bosons, and thus the proton may decay into a lepton plus meson final state. Because the masses of heavy gauge bosons are near to the GUT scale, such processes are expected to be very rare. Indeed, proton decay has not yet been seen in the expansive Super-Kamiokande experiment, which currently places a lower bound on the dimension six partial lifetimes around $6 - 8 \times 10^{33}$ years [33].

In Standard $SU(5)$ [13], there exists the Higgs doublet-triplet splitting problem, and the additional threat of proton decay via dimension five operators from the colored Higgsino field (supersymmetric partners of the colored triplet Higgs fields) exchange [32]. These difficulties are solved elegantly in Flipped $SU(5) \times U(1)_X$ [14–16], due to the missing partner mechanism [16], and we thus only need consider dimension six proton decay. This initial salvation from the overly rapid 'ignoble' sparticle channel can however turn subsequently into frustration that allowed portions of the parameter space of minimal Flipped $SU(5)$ models appears to predict lifetimes so long as to be unobservable by even hypothetical proposals for future experiments [29, 30, 32].

In this paper, we consider the testable Flipped $SU(5) \times U(1)_X$ models of Ref. [49] with inclusion of TeV-scale vector-like particles. Such models can be realized within free fermionic string constructions [20] and also F-theory [21–28]. Interestingly, we can solve the little hierarchy problem between the string scale and the GUT scale in the free fermionic constructions [49], and we can explain the decoupling scenario in F-theory model building [27, 28].

The minimal Flipped $SU(5)$ models [14–16]. contain three families of SM fermions as in Table (5), whose quantum numbers under $SU(5) \times U(1)_X$ are as follows, with $i = (1, 2, 3)$.

$$F_i = (\mathbf{10}, \mathbf{1}) \quad ; \quad \overline{f_i} = (\bar{\mathbf{5}}, \mathbf{-3}) \quad ; \quad \overline{l_i} = (\mathbf{1}, \mathbf{5}) \tag{20}$$

To break the GUT and electroweak gauge symmetries, we introduce two pairs of Higgs fields, and also add a SM singlet field Φ.

$$H = (\mathbf{10}, \mathbf{1}) \quad ; \quad \overline{H} = (\overline{\mathbf{10}}, \mathbf{-1}) \quad ; \quad h = (\mathbf{5}, \mathbf{-2}) \quad ; \quad \overline{h} = (\bar{\mathbf{5}}, \mathbf{2}) \tag{21}$$

The particle assignments of Higgs fields are given following, where H_d and H_u are one pair of Higgs doublets in the supersymmetric SM.

$$\begin{aligned} H &= (Q_H, D^c_H, N^c_H) \quad &;& \quad \overline{H} = (\overline{Q}_{\overline{H}}, \overline{D}^c_{\overline{H}}, \overline{N}^c_{\overline{H}}) \\ h &= (D_h, D_h, D_h, H_d) \quad &;& \quad \overline{h} = (\overline{D}_{\overline{h}}, \overline{D}_{\overline{h}}, \overline{D}_{\overline{h}}, H_u) \end{aligned} \tag{22}$$

To break the $SU(5) \times U(1)_X$ gauge symmetry down to the Standard Model, we introduce the following Higgs superpotential at the GUT scale.

$$W_{\mathrm{GUT}} = \lambda_1 HHh + \lambda_2 \overline{HHh} + \Phi(\overline{H}H - M^2_{\mathrm{H}}) \tag{23}$$

There is only one F-flat and D-flat direction, which can always be rotated into orientation with N^c_H and $\overline{N}^c_{\overline{H}}$, yielding $\langle N^c_H \rangle = \langle \overline{N}^c_{\overline{H}} \rangle = M_{\mathrm{H}}$. In addition, the superfields H and $\overline{H}$ are absorbed, acquiring large masses via the supersymmetric Higgs mechanism, except for D^c_H and $\overline{D}^c_{\overline{H}}$. The superpotential terms $\lambda_1 HHh$ and $\lambda_2 \overline{HHh}$ couple the D^c_H and $\overline{D}^c_{\overline{H}}$ with the D_h and $\overline{D}_{\overline{h}}$, respectively, to form the massive eigenstates with masses $2\lambda_1 \langle N^c_H \rangle$ and $2\lambda_2 \langle \overline{N}^c_{\overline{H}} \rangle$. So then, we naturally achieve doublet-triplet splitting due to the missing partner mechanism [16]. Because the triplets in h and $\overline{h}$ only have small mixing through the $\mu h \overline{h}$ term with μ around the TeV scale, we also solve the dimension five proton decay problem from colored Higgsino exchange.

In Flipped $SU(5)\times U(1)_X$, the $SU(3)_C\times SU(2)_L$ gauge couplings are first joined at the scale M_{32}, and the $SU(5)$ and $U(1)_X$ gauge couplings are subsequently unified at the higher scale M_{51}. To separate the M_{32} and M_{51} scales and obtain true string-scale gauge coupling unification in free fermionic models [49] or the decoupling scenario of F-theory [50], we introduce, as elaborated in the following section, vector-like particles which form complete Flipped $SU(5)\times U(1)_X$ multiplets. In order to avoid the Landau pole problem for the strong coupling constant, we can only introduce the following two sets of vector-like particles around the TeV scale [49].

$$Z1 \quad : \quad XF = (\mathbf{10},\mathbf{1})\ ,\ \overline{XF} = (\overline{\mathbf{10}},-\mathbf{1}) \tag{24a}$$

$$Z2 \quad : \quad XF\ ,\ \overline{XF}\ ,\ Xl = (\mathbf{1},-\mathbf{5})\ ,\ \overline{Xl} = (\mathbf{1},\mathbf{5}) \tag{24b}$$

For notational simplicity, we define the Flipped $SU(5)\times U(1)_X$ models with $Z1$ and $Z2$ type sets of vector-like particles as Type I and Type II Flipped $SU(5)\times U(1)_X$ models, respectively. Although we focus in this paper on Type II, results for proton decay are not be found to differ significantly between the Type I and Type II scenarios. The behavior of the two models above M_{32} is however quite distinct, and as shown in Section (6.1), the Type II variation is in this regard preferable.

3. $\mathcal{F}$-theory

3.1. Elements of $\mathcal{F}$-Theory

Recently, semi-realistic Grand Unified Theories (GUTs) have been constructed locally in the F-theory with seven-branes, which can be considered as the strongly coupled formulation of ten-dimensional Type IIB string theory [21–25]. Model building and phenomenological consequences have been studied extensively [27, 28, 31, 51–63]. The known GUTs without additional chiral exotic particles are asymptotically free, and the asymptotic freedom can be translated into the existence of a consistent decompactification limit. Interestingly, the GUT scale $M_{\rm GUT}$ is around 2×10^{16} [GeV] while the reduced Planck scale $M_{\rm Pl}$ is around 2×10^{18} [GeV], so $M_{\rm GUT}/M_{\rm Pl}$ is indeed a small number around 10^{-2}. Thus, it is natural to assume that $M_{\rm GUT}/M_{\rm Pl}$ is small from the effective field theory point of view in the bottom-up approach, and then gravity can be decoupled. In the decoupling limit where $M_{\rm Pl}\to\infty$ while $M_{\rm GUT}$ remains finite, semi-realistic $SU(5)$ models and $SO(10)$ models without chiral exotic particles have been constructed locally.

To decouple gravity and avoid the bulk matter fields on the observable seven-branes, we can show that the observable seven-branes should wrap a del Pezzo n surface dP_n with $n\geq 2$ for the internal space dimensions [23, 24]. A review of del Pezzo n surfaces is provided here as an Appendix. The GUT gauge fields are on the world-volume of the observable seven-branes, and the SM fermions and Higgs fields are localized on the codimension-one curves in dP_n. All the SM fermion Yukawa couplings can be obtained from the triple intersections of the SM fermion and Higgs curves. A brand new feature is that the $SU(5)$ gauge symmetry can be broken down to the SM gauge symmetry by turning on the $U(1)_Y$ flux [23, 24, 58, 61], and the $SO(10)$ gauge symmetry can be broken down to $SU(5)\times U(1)_X$ or $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ by turning on the $U(1)_X$ and $U(1)_{B-L}$ fluxes, respectively [23, 24, 27, 28, 53, 61]. In particular, in the $SO(10)$ models, to eliminate the zero modes of the chiral exotic particles, we must break the $SO(10)$ gauge symmetry down to the Flipped $SU(5)\times U(1)_X$ gauge symmetry [24].

In Flipped $SU(5)\times U(1)_X$ models of $SO(10)$ origin, there are two unification scales: the $SU(2)_L\times SU(3)_C$ unification scale M_{32} and the $SU(5)\times U(1)_X$ unification scale M_{51}, where

M_{32} is in the vicinity the usual GUT scale around 2×10^{16} [GeV]. To address the little hierarchy problem between the GUT scale and string scale, one may introduce extra vector-like particles, achieving models with string-scale $SU(5) \times U(1)_X$ gauge coupling unification [49, 64]. These fields may be naturally associated with a highly restricted class of Flipped $SU(5) \times U(1)_{\mathrm{X}}$ F-theory model building constructions [28], as is the perspective taken in the present work.

In the F-Theory GUTs proposed in Refs. [22, 23], the decoupling scenario under consideration implies that $M_{\mathrm{GUT}}/M_{\mathrm{Pl}}$ is a small number, say $\mathcal{O}$ (0.1) or less. In our Flipped $SU(5)$ models with F-theory derived vectorlike particle content, the analogous statement is rather that the ratio M_{32}/M_{51} is a small number. Satisfaction of this requirement allows for a solution of the monopole problem and a corresponding realization of string scale gauge unification in a free-fermionic model building context. However, because M_{51} is elevated close to the Planck scale, $M_{\mathrm{S}}/M_{\mathrm{Pl}}$ is not a small number in the present constructions, where M_{S} is a scale set by the inverse radius of the Kähler surface wrapped by the GUT seven-branes. Although we therefore cannot decouple gravity, and there may also be corresponding corrections to the gauge kinetic terms, the picture of gauge coupling unification which we present will be undisturbed, since there is an $SO(10)$ gauge symmetry around M_{S} which implies that any corrections will be applied universally. Moreover, our immediately central topic of proton decay is likewise protected, as its dominant physics is unearthed some two orders of magnitude below, at M_{32}. Any shift in the final unified coupling g_{51} will thus be irrelevant to this discussion, and any correction terms applied around the scale M_{51} should be expected to experience a relative suppression of around eight orders of magnitude.

We remark that our construction can indeed be made consistent with both perspectives of decoupling, if one invokes the presence of heavy thresholds to reduce M_{51} to $\mathcal{O}$ (10^{17}) [GeV], keeping $M_{32} \sim \mathcal{O}$ (10^{16}) [GeV], with $M_{\mathrm{Pl}} \sim \mathcal{O}$ (10^{18}) [GeV]. Indeed, this is not merely a hypothetical consideration, as two concrete examples (models Type IA1 and IA2) satisfying this triple hierarchy have been presented in Tables (IX,X) of Ref. [28]. These scenarios thus allow for perturbation analysis in the small expansion parameter M_{51}/M_{Pl}. However, since the dimension six proton decay results are unaffected, we will not consider this possibility any further at the present time.

3.2. $\mathcal{F}$-Theory Model Building

In this section we briefly review F-theory model building [21–25]. The twelve-dimensional F theory is a convenient way to describe Type IIB vacua with varying axion (a)-dilaton (ϕ) field $\tau = a + ie^{-\phi}$. We compactify F-theory on a Calabi-Yau fourfold, which is elliptically fibered $\pi : Y_4 \to B_3$ with a section $\sigma : B_3 \to Y_4$. The base B_3 is the internal spatial dimensions of Type IIB string theory, and the complex structure of the T^2 fibre encodes τ at each point of B_3. The SM or GUT gauge theories are on the world-volume of the observable seven-branes that wrap a complex codimension-one surface in B_3. Denoting the complex coordinate transverse to these seven-branes in B_3 as z, we can write the elliptic fibration in Weierstrass form

$$y^2 = x^3 + f(z)x + g(z)\,, \tag{25}$$

where $f(z)$ and $g(z)$ are sections of $K_{B_3}^{-4}$ and $K_{B_3}^{-6}$, respectively. The complex structure of the fibre is:

$$j(\tau) \;=\; \frac{4(24f)^3}{\Delta} \quad ; \quad \Delta \;=\; 4f^3 + 27g^2 \tag{26}$$

At the discriminant locus $\{\Delta = 0\} \subset B_3$, the torus T^2 degenerates by pinching one of its cycles and becomes singular. For a generic pinching one-cycle $(p, q) = p\alpha + q\beta$ where α and β are one-cycles for the torus T^2, we obtain a (p, q) seven-brane in the locus where the (p, q) string can end. The singularity types of the elliptically fibres fall into the familiar *ADE* classifications, and we identify the corresponding *ADE* gauge groups on the seven-brane world-volume. This is one of the most important advantages for F-theory model building: the exceptional gauge groups appear rather naturally, in contrast perturbative Type II string theory. Subsequently all the SM fermion Yukawa couplings can be generated.

We assume that the observable seven-branes with GUT models on its world-volume wrap a complex codimension-one surface S in B_3, and the observable gauge symmetry is G_S. When $h^{1,0}(S) \neq 0$, the low energy spectrum may contain the extra states obtained by reduction of the bulk supergravity modes of compactification, so we require that $\pi_1(S)$ be a finite group. In order to decouple gravity and construct models locally, the extension of the local metric on S to a local Calabi-Yau fourfold must have a limit where the surface S can be shrunk to zero size. This implies that the anti-canonical bundle on S must be ample. Therefore, S is a del Pezzo n surface dP_n with $n \geq 2$ in which $h^{2,0}(S) = 0$. The Hirzebruch surfaces with degree larger than 2 satisfy $h^{2,0}(S) = 0$ but do not define the fully consistent decoupled models [23, 24].

To describe the spectrum, we have to study the gauge theory of the world-volume on the seven-branes. We start from the maximal supersymmetric gauge theory on $\mathbb{R}^{3,1} \times \mathbb{C}^2$ and then replace $\mathbb{C}^2$ with the Kähler surface S. In order to have four-dimensional $\mathcal{N} = 1$ supersymmetry, the maximal supersymmetric gauge theory on $\mathbb{R}^{3,1} \times \mathbb{C}^2$ should be twisted. It was shown that there exists a unique twist preserving $\mathcal{N} = 1$ supersymmetry in four dimensions, and that chiral matter can arise from the bulk S or the codimension-one curve Σ in S which is the intersection between the observable seven-branes and non-observable seven-brane(s) [23, 24].

In order to have matter fields on S, we consider a non-trivial vector bundle with structure group H_S which is a subgroup of G_S. Then the gauge group G_S is broken down to $\Gamma_S \times H_S$, and the adjoint representation $\mathrm{ad}(G_S)$ of the G_S is decomposed as:

$$\mathrm{ad}(G_S) \to \mathrm{ad}(\Gamma_S) \oplus \mathrm{ad}(H_S) \oplus_j (\tau_j, T_j) \tag{27}$$

Employing the vanishing theorem of the del Pezzo surfaces, we obtain the numbers of the generations and anti-generations by calculating the zero modes of the Dirac operator on S

$$n_{\tau_j} \;=\; -\chi(S, \mathbf{T_j})\,, \quad n_{\tau_j^*} \;=\; -\chi(S, \mathbf{T_j}^*)\,, \tag{28}$$

where $\mathbf{T_j}$ is the vector bundle on S whose sections transform in the representation T_j of H_S, and $\mathbf{T_j}^*$ is the dual bundle of $\mathbf{T_j}$. In particular, when the H_S bundle is a line bundle L, we have:

$$n_{\tau_j} \;=\; -\chi(S, L^j) \;=\; -\Big[1 + \frac{1}{2}\big(\int_S c_1(L^j) c_1(S) + \int_S c_1(L^j)^2\big)\Big] \tag{29}$$

In order to preserve supersymmetry, the line bundle L should satisfy the BPS equation [23]

$$J_S \wedge c_1(L) = 0, \tag{30}$$

where J_S is the Kähler form on S. Moreover, the admissible supersymmetric line bundles on del Pezzo surfaces must satisfy $c_1(L)c_1(S) = 0$, thus, $n_{\tau_j} = n_{\tau_j^*}$ and only the vector-like particles can be obtained. In short, we can not have the chiral matter fields on the world-volume of the observable seven-branes.

Let us consider a stack of seven-branes with gauge group $G_{S'}$ that wrap a codimension-one surface S' in B_3. The intersection of S and S' is a codimension-one curve (Riemann surface) Σ in S and S', and the gauge symmetry on Σ will be enhanced to G_Σ where $G_\Sigma \supset G_S \times G_{S'}$. On this curve, there exists chiral matter from the decomposition of the adjoint representation ad(G_Σ) of G_Σ as follows:

$$\mathrm{ad}(G_\Sigma) = \mathrm{ad}(G_S) \oplus \mathrm{ad}(G_{S'}) \oplus_k (U_k \otimes U'_k) \tag{31}$$

Turning on the non-trivial gauge bundles on S and S', respectively with structure groups H_S and $H_{S'}$, we break the gauge group $G_S \times G_{S'}$ down to the commutant subgroup $\Gamma_S \times \Gamma_{S'}$. Defining $\Gamma \equiv \Gamma_S \times \Gamma_{S'}$ and $H \equiv H_S \times H_{S'}$, we can decompose $U \otimes U'$ into the irreducible representations

$$U \otimes U' = \oplus_k (r_k, V_k), \tag{32}$$

where r_k and V_k are the representations of Γ and H, respectively. The light chiral fermions in the representation r_k are determined by the zero modes of the Dirac operator on Σ. The net number of chiral superfields is given by

$$N_{r_k} - N_{r_k^*} = \chi(\Sigma, K_\Sigma^{1/2} \otimes \mathbf{V}_k), \tag{33}$$

where K_Σ is the restriction of canonical bundle on the curve Σ, and $\mathbf{V}_k$ is the vector bundle whose sections transform in the representation V_k of the structure group H.

In F-theory model building, we are interested in the models where $G_{S'}$ is $U(1)'$, and H_S and $H_{S'}$ are respectively $U(1)$ and $U(1)'$. Then the vector bundles on S and S' are line bundles L and L'. The adjoint representation ad(G_Σ) of G_Σ is decomposed into a direct sum of the irreducible representations under the group $\Gamma_S \times U(1) \times U(1)'$ that can be denoted as $(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})$.

$$\mathrm{ad}(G_\Sigma) = \mathrm{ad}(\Gamma_S) \oplus \mathrm{ad}(G_{S'}) \oplus_j (\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j}) \,. \tag{34}$$

The numbers of chiral superfields in the representation $(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})$ and their Hermitian conjugates on the curve Σ are given by

$$N_{(\mathbf{r_j},\mathbf{q_j},\mathbf{q'_j})} \;=\; h^0(\Sigma, \mathbf{V}_j)\,, \quad N_{(\bar{\mathbf{r}}_\mathbf{j},-\mathbf{q_j},-\mathbf{q'_j})} \;=\; h^1(\Sigma, \mathbf{V}_j)\,, \tag{35}$$

where

$$\mathbf{V}_j \;=\; K_\Sigma^{1/2} \otimes L_\Sigma^{q_j} \otimes L'^{q'_j}_\Sigma \,, \tag{36}$$

where $K_\Sigma^{1/2}$, $L_\Sigma^{r_j}$ and $L'^{q'_j}_\Sigma$ are the restrictions of canonical bundle K_S, line bundles L and L' on the curve Σ, respectively. In particular, if the volume of S' is infinite, $G_{S'} = U(1)'$ is decoupled, and then the index $\mathbf{q'_j}$ can be ignored.

Using the Riemann-Roch theorem, we obtain the net number of chiral superfields in the representation $(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})$ as

$$N_{(\mathbf{r_j},\mathbf{q_j},\mathbf{q'_j})} - N_{(\bar{\mathbf{r}}_\mathbf{j},-\mathbf{q_j},-\mathbf{q'_j})} \;=\; 1 - g + c_1(\mathbf{V}_j)\,, \tag{37}$$

where g is the genus of the curve Σ, and c_1 means the first Chern class.

Moreover, we can obtain the Yukawa couplings at the triple intersection of three curves Σ_i, Σ_j and Σ_k where the gauge group or the singularity type is enhanced further. To have triple intersections, the corresponding homology classes $[\Sigma_i]$, $[\Sigma_j]$ and $[\Sigma_k]$ of the curves Σ_i, Σ_j and Σ_k must satisfy the following conditions:

$$[\Sigma_i] \cdot [\Sigma_j] > 0 \quad ; \quad [\Sigma_i] \cdot [\Sigma_k] > 0 \quad ; \quad [\Sigma_j] \cdot [\Sigma_k] > 0 \tag{38}$$

3.3. F-Theory GUTs

Flipped $SU(5) \times U(1)_X$ models with additional vector-like particles have been studied systematically [23, 24, 27, 28]. In this paper, we supplement those efforts by generic construction of the Georgi-Glashow $SU(5)$ models with additional vector-like particles. In such $SU(5)$ models, we introduce the vector-like particles YF and $\overline{YF}$, and Yf_i and $\overline{Yf}_i$, whose quantum numbers under $SU(5)$ are:

$$YF = \mathbf{10} \quad ; \quad \overline{YF} = \overline{\mathbf{10}} \quad ; \quad Yf_i = \mathbf{5} \quad ; \quad \overline{Yf}_i = \overline{\mathbf{5}} \tag{39}$$

The SM field content from the decomposition of YF, $\overline{YF}$, Yf_i, and $\overline{Yf}_i$ is:

$$\begin{aligned} YF = (XQ, XU^c, XE^c) \quad &; \quad \overline{YF} = (XQ^c, XU, XE) \\ Yf_i = (XD, XL^c) \quad &; \quad \overline{Yf}_i = (XD^c, XL) \end{aligned} \tag{40}$$

Under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, the quantum numbers for the extra vector-like particles are:

$$\begin{aligned} XQ = (\mathbf{3}, \mathbf{2}, \mathbf{\frac{1}{6}}) \quad &; \quad XQ^c = (\overline{\mathbf{3}}, \mathbf{2}, -\mathbf{\frac{1}{6}}) \\ XU = (\mathbf{3}, \mathbf{1}, \mathbf{\frac{2}{3}}) \quad &; \quad XU^c = (\overline{\mathbf{3}}, \mathbf{1}, -\mathbf{\frac{2}{3}}) \\ XD = (\mathbf{3}, \mathbf{1}, -\mathbf{\frac{1}{3}}) \quad &; \quad XD^c = (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{\frac{1}{3}}) \\ XL = (\mathbf{1}, \mathbf{2}, -\mathbf{\frac{1}{2}}) \quad &; \quad XL^c = (\mathbf{1}, \mathbf{2}, \mathbf{\frac{1}{2}}) \\ XE = (\mathbf{1}, \mathbf{1}, -\mathbf{1}) \quad &; \quad XE^c = (\mathbf{1}, \mathbf{1}, \mathbf{1}) \end{aligned} \tag{41}$$

We consider two $SU(5)$ models. The Type I $SU(5)$ model has one pair of vector-like particles YF and $\overline{YF}$, and the Type II $SU(5)$ model has three pairs of vector-like particles Yf_i and $\overline{Yf}_i$. We assume that the observable gauge group on the dP_8 surface S is $SU(5)$. On codimension one curves that are intersections of the observable seven-branes with other non-observable seven-branes, we obtain the SM fermions, Higgs fields, and extra vector-like particles. To break the $SU(5)$ gauge symmetry down to the $SU(3) \times SU(2)_L \times U(1)_Y$ gauge symmetry, we turn on the $U(1)_Y$ flux on S specified by the line bundle L. To obtain the SM fermions, Higgs fields and vector-like particles, we also turn on the $U(1)$ fluxes on the other seven-branes that intersect with the observable seven-branes, and we specify these fluxes by the line bundle L'^n.

We take the line bundle $L = \mathcal{O}_S(E_1 - E_2)^{1/5}$. Note that $\chi(S, L^5) = 0$, *i.e.* we do not have vector-like particles on the bulk S. The curves and homology classes for the matter fields, Higgs fields and vector-like particles, and the gauge bundle assignments for each curve in the Type I and Type II $SU(5)$ models are given in Table (6). From this table, we see that all the SM fermions are localized on the matter curves Σ_F and $\Sigma_{\overline{f}}$, the Higgs fields H_u and H_d are localized on the curves Σ_{Hu} and Σ_{Hd} respectively, and the vector-like particles YF, $\overline{YF}$, Yf_i, and $\overline{Yf}_i$ are localized on the curves Σ_F, $\Sigma_{\overline{F}}$, Σ_f, and $\Sigma_{\overline{f}}$. In the Type I $SU(5)$ model, we choose $n = 1$ and $m = 0$, while in the Type II $SU(5)$ model, we choose $n = 0$ and $m = 3$. In addition, there exist singlets from the intersections of the other seven-branes. It is easy to check that we can realize the SM fermion Yukawa coupling terms in the superpotential. All the vector-like particles can obtain masses by giving vacuum expectation values to the SM singlets at the intersections of the other seven-branes.

Fields	Curves	Class	g_Σ	L_Σ	L'^{n}_Σ
H_u	Σ_{Hu}	$2H - E_1 - E_3$	0	$\mathcal{O}(1)^{1/5}$	$\mathcal{O}(1)^{2/5}$
H_d	Σ_{Hd}	$2H - E_2 - E_3$	0	$\mathcal{O}(-1)^{1/5}$	$\mathcal{O}(-1)^{2/5}$
$10_i + n \times XF$	Σ_F	$2H - E_4 - E_6$	0	$\mathcal{O}(0)$	$\mathcal{O}(3+n)$
$n \times \overline{XF}$	$\Sigma_{\overline{F}}$	$2H - E_5 - E_6$	0	$\mathcal{O}(0)$	$\mathcal{O}(-n)$
$\overline{5}_i + m \times \overline{Xf}_i$	$\Sigma_{\overline{f}}$	$-E_7$	0	$\mathcal{O}(0)$	$\mathcal{O}(-3-m)$
$m \times Xf_i$	Σ_f	$H - E_8$	0	$\mathcal{O}(0)$	$\mathcal{O}(m)$

Table 6: The particle curves and gauge bundle assignments for each curve in the $SU(5)$ models from F-theory. The index $i = 1, 2, 3$. In the Type I $SU(5)$ model, we choose $n = 1$ and $m = 0$. In the Type II $SU(5)$ model, we choose $n = 0$ and $m = 3$.

3.4. Thresholds From Extra Vector-Like Multiplets

Following [49, 65] and references therein, we present the supplementary step-wise contributions to the MSSM one- and two-loop β-function coefficients from the vector-like particles. $\Delta b \equiv (\Delta b_Y, \Delta b_2, \Delta b_3)$, and likewise ΔB are given as complete supermultiplets, including the conjugate representation in each case.

$$\begin{aligned}
\Delta b^{XQ+XQ^c} &= (\frac{1}{5}, 3, 2) \quad ; \quad \Delta b^{XU+XU^c} = (\frac{8}{5}, 0, 1) \\
\Delta b^{XD+XD^c} &= (\frac{2}{5}, 0, 1) \quad ; \quad \Delta b^{XL+XL^c} = (\frac{3}{5}, 1, 0) \\
\Delta b^{XE+XE^c} &= (\frac{6}{5}, 0, 0) \quad ; \quad \Delta b^{XN+XN^c} = (0, 0, 0) \\
\Delta b^{XY+XY^c} &= (5, 3, 2) \quad ; \quad \Delta b^{XT_i+\overline{XT}_i} = (\frac{6}{5}, 0, 0)
\end{aligned} \tag{42}$$

$$\Delta B^{XQ+XQ^c} = \begin{pmatrix} \frac{1}{75} & \frac{3}{5} & \frac{16}{15} \\ \frac{1}{5} & 21 & 16 \\ \frac{2}{15} & 6 & \frac{68}{3} \end{pmatrix} \quad ; \quad \Delta B^{XU+XU^c} = \begin{pmatrix} \frac{128}{75} & 0 & \frac{128}{15} \\ 0 & 0 & 0 \\ \frac{16}{15} & 0 & \frac{34}{3} \end{pmatrix}$$

$$\Delta B^{XD+XD^c} = \begin{pmatrix} \frac{8}{75} & 0 & \frac{32}{15} \\ 0 & 0 & 0 \\ \frac{4}{15} & 0 & \frac{34}{3} \end{pmatrix} \quad ; \quad \Delta B^{XL+XL^c} = \begin{pmatrix} \frac{9}{25} & \frac{9}{5} & 0 \\ \frac{3}{5} & 7 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{43}$$

$$\Delta B^{XE+XE^c} = \begin{pmatrix} \frac{72}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Delta B^{XN+XN^c} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Delta B^{XY+XY^c} = \begin{pmatrix} \frac{25}{3} & 15 & \frac{80}{3} \\ 5 & 21 & 16 \\ \frac{10}{3} & 6 & \frac{68}{3} \end{pmatrix} \quad ; \quad \Delta B^{XT_i+\overline{XT}_i} = \begin{pmatrix} \frac{18}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Scenario	Vector Super-Multiplets	b_Y	b_2	b_3
$SU(5)_0$	None	6.6	1	−3
$SU(5)_I$	$\left(YF \equiv \{XQ, XU^c, XE^c\}_{\mathbf{10}}, \overline{YF}\right)$	9.6	4	0
$SU(5)_{II}$	$3 \otimes \left(Yf \equiv \{XD^c, XL\}_{\mathbf{5}}, \overline{Yf}\right)$	9.6	4	0
$\mathcal{F}$-$SU(5)_0$	None	6.6	1	−3
$\mathcal{F}$-$SU(5)_I$	$\left(XF \equiv \{XQ, XD^c, XN^c\}_{\mathbf{10}}, \overline{XF}\right)$	7.2	4	0
$\mathcal{F}$-$SU(5)_{II}$	$\left(XF \equiv \{XQ, XD^c, XN^c\}_{\mathbf{10}}, \overline{XF}\right) \oplus \left(Xl \equiv \{XE\}_{\mathbf{1}}, \overline{Xl}\right)$	8.4	4	0

Table 7: Light field content and β-function coefficients for the Standard $SU(5)$ and Flipped $SU(5) \times U(1)_X$ models considered in this report, including TeV scale vector-like multiplets from F-theory.

In Table (7), we present the specific field content of the F-theory models which are considered in this report. For both Flipped and Standard $SU(5)$, we begin with a ‘Type 0’ model representing the bare MSSM. As discussed in Section (3.1), avoidance of a Landau pole in the renormalization places very strict limits on the combinations of vector multiplets which may be considered. We study two variations for each GUT category, labeled sequentially as (I,II), adopting the notation of Ref. [31]. There is no immediate connection however between the flipped and standard models which share the designation of Type I or Type II. Moreover, since the possible vector multiplets which we may to choose to populate in Flipped and Standard $SU(5)$ are distinct, as required by the different charge assignments in the two theories, there is no continuous field theoretic transition between the two broad model classes once the heavy fields are implemented. The β-coefficients of the first loop are tallied, starting with the MSSM as presented in Eq. 16, and adding the relevant elements from Eq. (42). The matrices for the second loop are summed in the identical manner using Eq. (43), although we suppress a direct printing of the totals.

4. $\mathcal{F}$-resh Analysis

4.1. Grand Unification With Thresholds

We will now consider the question of how the presence of a sum over internal scales effects the picture of unification, particularly in Flipped $SU(5)$. We start here with the standard set of 3 one-loop equations, including thresholds via the factors[12] ξ_i introduced in Section (1.4).

$$\frac{1}{\alpha_Y} - \frac{1}{\alpha_1} + \frac{\xi_Y}{2\pi} = \frac{b_Y}{2\pi} \ln \frac{M_{32}}{M_Z} \tag{44a}$$

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_5} + \frac{\xi_2}{2\pi} = \frac{b_2}{2\pi} \ln \frac{M_{32}}{M_Z} \tag{44b}$$

$$\frac{1}{\alpha_3} - \frac{1}{\alpha_5} + \frac{\xi_3}{2\pi} = \frac{b_3}{2\pi} \ln \frac{M_{32}}{M_Z} \tag{44c}$$

The mass scale M_{32} of Flipped $SU(5)$ allows that the weak and strong couplings may partially unify to α_5 in isolation, earlier than the full triple unification of Standard $SU(5)$ would occur, and α_1 defines the normalized hypercharge evaluated at that point. As usual:

$$\alpha_Y = \frac{5\,\alpha_{em}(M_Z)}{3\,(1-\sin^2\theta_W)} \tag{45a}$$

$$\alpha_2 = \frac{\alpha_{em}(M_Z)}{\sin^2\theta_W} \tag{45b}$$

$$\alpha_3 \equiv \alpha_s(M_Z) \tag{45c}$$

Whenever numbers are required, we will turn to the precision electroweak measurements compiled by the Particle Data Group [66].

$$\alpha_{em}(M_Z) = \frac{1}{127.925 \pm .016} \quad ; \quad \alpha_s(M_Z) = .1176 \pm .0020 \tag{46}$$
$$\sin^2\theta_W^{\overline{MS}}(M_Z) = .23119 \pm .00014 \quad ; \quad M_Z = 91.1876 \pm .0021 \text{ [GeV]}$$

Interestingly, the strict unification limit $M_{32} \to M_{32}^{max}$ is validated to a surprising accuracy in the first loop using the field content of the MSSM ($b_Y = 33/5$; $b_2 = +1$; $b_3 = -3$) and neglecting thresholds [1].

We may now attempt to absorb the factors of ξ_i in a convenient way, concealing explicit references. It has been a common practice of earlier analysis [2, 29, 30, 42] to account for the thresholds and calculable two-loop corrections simultaneously by shifting $\sin^2\theta_W$ to an effective value.

$$\sin^2\theta_W \Rightarrow \sin^2(\theta_W^{eff}) \equiv \sin^2\theta_W - \delta_{2loop} - \delta_{light} - \delta_{heavy} \tag{47}$$

However, it appears that this approach treats the factors of $\sin^2\theta_W \equiv (1-\cos^2\theta_W)$ from the running of the weak and hypercharge couplings consistently *only if* ($\xi_Y = -3/5\,\xi_2$) and ($\xi_3 = 0$). There is no general protection which applies to these conditions, nor are they found to

[12]The subscripts on b and ξ now revert to traditional usage, labeling the given gauge interaction.

be regularly satisfied in example. Nevertheless, it is possible to let this idea guide a notation[13], which will ease comparison with prior results.

$$\Xi_{\rm Y} \equiv 1 + \frac{\alpha_{\rm em}}{2\pi}\left\{\frac{5}{3}\,\xi_{\rm Y} + \xi_2\right\} \tag{48a}$$

$$\Theta_{\rm W} \equiv \sin^2\theta_{\rm W} + \alpha_{\rm em}\,\xi_2/2\pi \tag{48b}$$

$$\Xi_3 \equiv 1 + \alpha_3\,\xi_3/2\pi \tag{48c}$$

The RGEs with threshold effects accounted now take the form shown following[14]. A reduction fully consistent with prior implicit assumptions is achieved in the limit ($\Xi_{\rm Y} = \Xi_3 = 1$).

$$\frac{3}{5}\,\frac{(\Xi_{\rm Y} - \Theta_{\rm W})}{\alpha_{\rm em}} - \frac{1}{\alpha_1} = \frac{b_{\rm Y}}{2\pi}\ln\frac{M_{32}}{M_{\rm Z}} \tag{49a}$$

$$\frac{\Theta_{\rm W}}{\alpha_{\rm em}} - \frac{1}{\alpha_5} = \frac{b_2}{2\pi}\ln\frac{M_{32}}{M_{\rm Z}} \tag{49b}$$

$$\frac{\Xi_3}{\alpha_3} - \frac{1}{\alpha_5} = \frac{b_3}{2\pi}\ln\frac{M_{32}}{M_{\rm Z}} \tag{49c}$$

We note that Eqs. (49) are intended formally to bridge the $M_{\rm Z}$ and M_{32} scales, and as such the referenced factors of Eqs. (48) include the totality of second order effects over this domain. For example, in Eq.(49c), if α_5 is backed all the way down to $M_{\rm Z}$, we find $\alpha_5(M_{\rm Z}) = \alpha_3(M_{\rm Z}) \div \Xi_3$, which effectively downshifts the origin value of the coupling at $M_{\rm Z}$. We emphasize that this is not an actual departure from the physically measured value of the coupling, but rather a 'visual' artifact of the chosen calculation methodology. Specifically, this methodology is adopted to deliver reliable results at the unknown *upper* boundary of the renormalization range terminated at M_{32}, and having absorbed correction factors at the outset for this complete transit, one should be cautious reinterpreting the results at arbitrarily reduced mass scales.

These three independent algebraic equations allow us to solve for three independent quantities. We will choose α_5, α_1, and depending on application, one of either M_{32} or $\Theta_{\rm W}$. The scale $M_{\rm Z}$ and couplings $\alpha_{(\rm em,3)}(M_{\rm Z})$ are experimentally input, and the β-coefficients $b_{\rm Y,2,3}$ and light thresholds $\delta b_{\rm Y,2,3}$ will be considered as fixed input for the context of a given model. If M_{32} is further input, then the effective shift of (the squared-sine of) the Weinberg angle is determined as output, and any departure from the value stipulated in Eq. (48) will be taken as a sign of ignorance regarding unaccounted heavy thresholds and higher loop corrections.

It is important to recognize that although the Weinberg angle itself is certainly an experimental parameter, $\Theta_{\rm W}$ is undetermined to the extent that the second order factors which it must have absorbed in order to achieve the mandated unification are undetermined. The key point is that every other variable under consideration carries *implicit* dependence on M_{32}, and we must be cautious to avoid substitutions of a given variable between equations evaluated at disparate scales. Indeed, subtle errors of precisely this type have plagued earlier reports with which the present authors have been associated [29, 30]. In particular, Eqs. (56,60b) now supersede their

[13] These definitions will be extended in Section (4.2) to include the correction from the second loop.
[14] It is emphasized that the b_i are to be tallied *inclusive* of all *light* fields with $M_{\rm Z} < M_i \ll M_{32}$.

previously published analogues, as first advertised in [67]. Our current perspective imparts additional caution to the contextual interpretation and use even of expressions, such as the $M_{32}^{\max}\big|_{\Xi_i=1}^{\rm MSSM}$ limit of Eq. (71c), which are formally identical to their predecessors.

If one takes as concrete a specification of the post-$M_{\rm Z}$ spectrum, and diligence is paid to corrections in the second loop, then it is reasonable to extract in converse a prediction for the flipped unification scale M_{32}. In practice, our approach will most often be along this second line, attempting to leverage the enforcement of phenomenological (including cosmological!) restrictions on the parameter space of the constrained MSSM against a prediction for the unification scale M_{32} and related proton lifetime.

To proceed, we will also need to consider the Standard $SU(5)$ scenario of strict unification, *i.e.* $\alpha_1^{\max} = \alpha_5^{\max}$, at an energy scale designated[15] $M_{32}^{\max}$. This specialization yields an *additional* set of three independent equations, which are exactly sufficient to solve for the set of variables remaining: $(M_{32}^{\max}, \alpha_5^{\max}, \Theta_{\rm W}^{\max})$. If the restriction of strict triple unification is relaxed, and M_{32} is allowed to slide downward from its maximal position, the burden which unification imposes on the thresholds within $\Theta_{\rm W}$ can be dispersed.

We begin by taking, in the triple unification limit, the linear combination of Eqs. (49) which eliminates the two traditional variables $\alpha_5^{\max}$ and $\Theta_{\rm W}^{\max}$. The result, inverted to solve for the limiting value of M_{32} is:

$$M_{32}^{\max} = M_{\rm Z} \times \exp\left\{\frac{2\pi}{\alpha_{\rm em}\alpha_3}\left(\frac{3\,\alpha_3\,\Xi_{\rm Y} - 8\,\alpha_{\rm em}\,\Xi_3}{5b_{\rm Y} + 3b_2 - 8b_3}\right)\right\} \tag{50}$$

For the minimal scenario of ($\Xi_{\rm Y} = \Xi_3 = 1$) and MSSM values for the b_i, this reduces:

$$M_{32}^{\max}\Big|_{\Xi_i=1}^{\rm MSSM} = M_{\rm Z} \times \exp\left\{\frac{\pi(3\,\alpha_3 - 8\,\alpha_{\rm em})}{30\,\alpha_{\rm em}\alpha_3}\right\} \simeq 2.089 \times 10^{16}\ [\text{GeV}] \tag{51}$$

Similarly excluding α_5 between just (49b,49c) at *generic* values for M_{32}, and eliminating $M_{\rm Z}$ by use of the expansion

$$\ln\frac{M_{32}}{M_{\rm Z}} = \ln\frac{M_{32}^{\max}}{M_{\rm Z}} - \ln\frac{M_{32}^{\max}}{M_{32}} \quad , \tag{52}$$

we arrive at the following expression, using also Eq. (50) to reach the second form.

$$\begin{aligned} \ln\frac{M_{32}^{\max}}{M_{32}} &= \frac{2\pi}{\alpha_{\rm em}\alpha_3}\left(\frac{\alpha_{\rm em}\,\Xi_3 - \alpha_3\Theta_{\rm W}}{b_2 - b_3}\right) + \ln\frac{M_{32}^{\max}}{M_{\rm Z}} \\ &= \frac{2\pi}{\alpha_{\rm em}\alpha_3}\left(\frac{3\,\alpha_3\Xi_{\rm Y}}{(5b_{\rm Y} + 3b_2 - 8b_3)} - \frac{\alpha_3\Theta_{\rm W}}{(b_2 - b_3)} + \frac{5\,\alpha_{\rm em}\Xi_3(b_{\rm Y} - b_2)}{(b_2 - b_3)(5b_{\rm Y} + 3b_2 - 8b_3)}\right) \end{aligned} \tag{53}$$

This inverts to a 'solution' for the strong coupling at $M_{\rm Z}$.

$$\frac{1}{\alpha_3} = \frac{(5b_{\rm Y} + 3b_2 - 8b_3) \times \left\{\Theta_{\rm W} + \frac{\alpha_{\rm em}(b_2 - b_3)}{2\pi}\,\ln\frac{M_{32}^{\max}}{M_{32}}\right\} - 3\,\Xi_{\rm Y}(b_2 - b_3)}{5\,\alpha_{\rm em}\,\Xi_3\,(b_{\rm Y} - b_2)} \tag{54}$$

[15] The superscript 'max' applied to any parameter indicates evaluation at the extremal scale $M_{32}^{\max}$. It does not necessarily imply that the labeled quantity itself experiences a maximum.

We note that it is not truly consistent to interpret this formula as an independently floating prediction for the strong coupling[16], since other solutions in our current set are dependent upon the experimentally fixed value of α_3. The present is nevertheless useful for visualizing the related slice of parameterizations in M_{32} and the effective Weinberg angle which are compatible with the experimental bounds on α_3. Going again to the minimal limit, Eqs. (53,54) compactify significantly. In particular, Eq. (56) has been applied historically[17] for essentially the purpose just described.

$$\ln \frac{M_{32}^{\max}}{M_{32}} |_{\Xi_i=1}^{\text{MSSM}} = 2\pi \times \left(\frac{1 - 5 \sin^2(\theta_{\text{W}}^{\text{eff}})}{20\, \alpha_{\text{em}}} + \frac{7}{60\, \alpha_3} \right) \tag{55}$$

$$\alpha_3(M_{\text{Z}})|_{\Xi_i=1}^{\text{MSSM}} = \frac{\frac{7}{3}\, \alpha_{\text{em}}}{5 \sin^2(\theta_{\text{W}}^{\text{eff}}) - 1 + \frac{10}{\pi}\, \alpha_{\text{em}} \ln\left(M_{32}^{\max}/M_{32}\right)} \tag{56}$$

Next, we solve Eq. (49c) in the triple unification for $\alpha_5^{\max}$, using Eq. (50) in the logarithm.

$$\alpha_5^{\max} \equiv \alpha_5(M_{32}^{\max}) = \left[\frac{\alpha_{\text{em}} \Xi_3 (5 b_{\text{Y}} + 3 b_2) - 3 \alpha_3 \Xi_{\text{Y}} b_3}{\alpha_{\text{em}} \alpha_3 (5 b_{\text{Y}} + 3 b_2 - 8 b_3)} \right]^{-1} \tag{57}$$

The shift term for generic M_{32} scales can be read off of Eq. (49c) with the aid again of the logarithmic relation from Eq. (52).

$$\frac{1}{\alpha_5} = \frac{1}{\alpha_5^{\max}} + \frac{b_3}{2\pi} \ln \frac{M_{32}^{\max}}{M_{32}} \tag{58}$$

There is a similar expression for the splitting off of α_1 which can be obtained by eliminating Θ_{W} between Eqs. (49a,49b) prior to reading off the 'max' limit and noting that, by definition, $\alpha_1^{\max} \equiv \alpha_5^{\max}$. Reinsertion of Eq. (58) facilitates the choice of reference to one of α_5 or $\alpha_5^{\max}$.

$$\begin{aligned} \frac{1}{\alpha_1} &= \frac{1}{\alpha_5^{\max}} + \frac{b_{\text{Y}} + 3/5(b_2 - b_3)}{2\pi} \ln \frac{M_{32}^{\max}}{M_{32}} \\ &= \frac{1}{\alpha_5} + \frac{5 b_{\text{Y}} + 3 b_2 - 8 b_3}{5 \times 2\pi} \ln \frac{M_{32}^{\max}}{M_{32}} \end{aligned} \tag{59}$$

The minimal scenario limits[18] of Eqs. (57,58,59) are shown below:

$$\alpha_5^{\max}\Big|_{\Xi_i=1}^{\text{MSSM}} = \frac{20\, \alpha_{\text{em}} \alpha_3}{3(\alpha_3 + 4\alpha_{\text{em}})} \simeq 0.0412 \tag{60a}$$

$$\alpha_5|_{\Xi_i=1}^{\text{MSSM}} = \left[\frac{3}{5\alpha_3} + \frac{3}{20\alpha_{\text{em}}} - \frac{3}{2\pi} \ln \frac{M_{32}^{\max}}{M_{32}} \right]^{-1} \tag{60b}$$

$$\alpha_1|_{\Xi_i=1}^{\text{MSSM}} = \left[\frac{3}{5\alpha_3} + \frac{3}{20\alpha_{\text{em}}} + \frac{9}{2\pi} \ln \frac{M_{32}^{\max}}{M_{32}} \right]^{-1} \tag{60c}$$

[16] It is certainly possible to rigorously isolate α_3 as a dependent variable. Although it is not considered profitable here to do so for Flipped $SU(5)$, Eqs. (71) take exactly this view of the triple unification.

[17] The coefficient $\frac{10}{\pi}$ in Eq. (56) corrects the erroneous value $\frac{11}{2\pi}$ which appears in some earlier publications.

[18] The coefficient $-\frac{3}{2\pi}$ in the limit$|_{\Xi_i=1}^{\text{MSSM}}$ given in Eq. (60b) corrects the erroneous published value of $+\frac{33}{56\pi}$.

The triple unification prediction for $\Theta_{\rm W}$ is likewise achieved from Eq. (49b), using Eqs. (50,57) to eliminate reference to $M_{32}^{\rm max}$ and $\alpha_5^{\rm max}$.

$$\Theta_{\rm W}^{\rm max} \equiv \Theta_{\rm W}\left(M_{32}^{\rm max}\right) = \frac{5\alpha_{\rm em}\Xi_3(b_{\rm Y}-b_2)+3\alpha_3\Xi_{\rm Y}(b_2-b_3)}{\alpha_3(5b_{\rm Y}+3b_2-8b_3)} \tag{61a}$$

$$\lim|_{\Xi_i=1}^{\rm MSSM} \Rightarrow \sin^2(\theta_{\rm W}^{\rm eff}) = \frac{1}{5}+\frac{7\,\alpha_{\rm em}}{15\,\alpha_3} \simeq 0.2310 \tag{61b}$$

The surprisingly fine numerical agreement of the minimal scenario limit with the experimental value quoted in Eq. (46) suggests that the light and heavy thresholds, the second loop, and perhaps even a reduction of the unification scale to M_{32}, must effectively conspire to avoid undoing this baseline signal of unification.

Considering M_{32} for the moment to be an independent input, the correction to this expression comes from solving Eq. (53) for $\Theta_{\rm W}^{\rm DEP}$, with the superscript expressed to emphasize that this dependent form of the (sine-squared of the) effective Weinberg angle may take a value either in deficit or surplus of that prescribed by Eq. (48) and the known thresholds. The result reduces correctly in the triple unification limit, and otherwise reveals the shift obtained at generic M_{32}.

$$\Theta_{\rm W}^{\rm DEP} = \Theta_{\rm W}^{\rm max} - \frac{\alpha_{\rm em}(b_2-b_3)}{2\pi}\ln\frac{M_{32}^{\rm max}}{M_{32}} \tag{62a}$$

$$\lim|_{\Xi_i=1}^{\rm MSSM} \Rightarrow \frac{1}{5}+\frac{7\,\alpha_{\rm em}}{15\,\alpha_3}-\frac{2\,\alpha_{\rm em}}{\pi}\ln\frac{M_{32}^{\rm max}}{M_{32}} \tag{62b}$$

In such a case we may choose to attribute any deviation from the independently determined effective angle to the presence of an unknown threshold.

$$\begin{aligned}\Delta\xi_2^{\rm DEP} &\equiv \frac{2\pi}{\alpha_{\rm em}}\left(\Theta_{\rm W}^{\rm DEP}-\Theta_{\rm W}^{\rm IND}\right) \\ &= \frac{2\pi}{\alpha_{\rm em}}\left(\Theta_{\rm W}^{\rm max}-\Theta_{\rm W}^{\rm IND}\right)-(b_2-b_3)\ln\frac{M_{32}^{\rm max}}{M_{32}}\end{aligned} \tag{63}$$

In particular, if one chooses to enforce the strict triple unification of Standard $SU(5)$ via ($\Theta_{\rm W}^{\rm DEP} = \Theta_{\rm W}^{\rm max}$), then Eq. (63) can yield a prediction for the shift $\xi_2^{\rm H}$ which must occur at the heavy scale relative to the thresholds input at the better determined light scale.

$$\xi_2^{\rm H} \equiv \frac{2\pi}{\alpha_{\rm em}}\left(\Theta_{\rm W}^{\rm max}-\Theta_{\rm W}^{\rm light}\right) \tag{64}$$

The natural convention within the established formalism has been to cast the effect of any heavy thresholds onto renormalization of just the second gauge coupling as shown. As hinted previously however, we will also have to make a compensatory shift

$$\xi_{\rm Y}^{\rm H} = -3/5\,\xi_2^{\rm H} \tag{65}$$

to the hypercharge if $\Xi_{\rm Y}$ of Eq. (48) is to be left undisturbed. Whenever plotting, tabulating, or quoting results from the strict triple unification picture, we will always implicitly assume, without immediate concern for their origin, the action of heavy thresholds as specified for internal consistency by Eqs. (64,65).

Of course, enforcing ($\Delta\Xi_i = 0$) can only be an approximation to the reality of simultaneous corrections to the running of all three gauge couplings. Nevertheless, the fixing here of one degree of freedom argues for a parameterization of the cost in terms of a single threshold function. Moreover, a suitably chosen linear combination of the actual ξ_i can constitute under certain conditions a fair rendering of the true dependencies. These issues will be taken up in detail in Section (4.3).

It can also be useful to isolate solutions for $(\alpha_5, \alpha_1, \Theta_W)$ as functions of M_{32} without explicit reference to the 'max' scale parameters from the triple unification. This can be achieved by combination of Eqs. (58,59,62a) and Eqs. (50,57,61a), or more simply, by turning directly to an appropriate remixing of the original Eqs. (49).

$$\frac{1}{\alpha_5} = \frac{\Xi_3}{\alpha_3} - \frac{b_3}{2\pi} \ln \frac{M_{32}}{M_Z} \tag{66a}$$

$$\frac{1}{\alpha_1} = \frac{3\,\Xi_Y}{5\,\alpha_{em}} - \frac{3\,\Xi_3}{5\,\alpha_3} - \frac{b_Y + 3/5\,(b_2 - b_3)}{2\pi} \ln \frac{M_{32}}{M_Z} \tag{66b}$$

$$\Theta_W = \frac{\alpha_{em}\Xi_3}{\alpha_3} + \frac{\alpha_{em}(b_2 - b_3)}{2\pi} \ln \frac{M_{32}}{M_Z} \tag{66c}$$

Or, going to the minimal limit:

$$\frac{1}{\alpha_5}\Big|^{\rm MSSM}_{\Xi_i=1} = \frac{1}{\alpha_3} + \frac{3}{2\pi} \ln \frac{M_{32}}{M_Z} \tag{67a}$$

$$\frac{1}{\alpha_1}\Big|^{\rm MSSM}_{\Xi_i=1} = \frac{3\,(\alpha_3 - \alpha_{em})}{5\,\alpha_{em}\alpha_3} - \frac{9}{2\pi} \ln \frac{M_{32}}{M_Z} \tag{67b}$$

$$\sin^2(\theta_W^{\rm eff})\Big|^{\rm MSSM}_{\Xi_i=1} = \frac{\alpha_{em}}{\alpha_3} + \frac{2\,\alpha_{em}}{\pi} \ln \frac{M_{32}}{M_Z} \tag{67c}$$

We should also in turn consider the inversion of the relationship in Eq. (62a), where all thresholds are considered known, and the discrepancy is attributed instead to the down-shift[19] of M_{32}. Indeed, at the end of the day, the quantity of primary practical interest is often the reduced mass scale M_{32} which plays a decisive role in establishing the rate of proton decay. This value is extremely sensitive to the effects of thresholds, and one must take proper care to employ reasonable estimates of their value. Assuming such, we will shift to the point of view with $(\Theta_W^{\rm DEP} = \Theta_W^{\rm IND})$ as independent input, and M_{32} dependent output.

$$\ln \frac{M_{32}^{\max}}{M_{32}} = \frac{2\pi\left(\Theta_W^{\max} - \Theta_W\right)}{\alpha_{em}(b_2 - b_3)} \tag{68a}$$

$$\lim\Big|^{\rm MSSM}_{\Xi_i=1} \Rightarrow \frac{\pi\left(\sin^2(\theta_W^{\max}) - \sin^2(\theta_W^{\rm eff})\right)}{2\,\alpha_{em}} \tag{68b}$$

We return in the prior and henceforth to suppression of the (DEP/IND) superscripting, emphasizing that the distinction was anyway interpretational rather than material. The defining relation from Eq. (48) applied to both views, admitting that certain of the thresholds there employed might originate, as in Eq. (63), outside the scope of explicitly specified fields.

Eq. (68a) represents a solution for M_{32} in terms of its extremal value $M_{32}^{\max}$ and the corresponding prediction for the effective Weinberg angle at that scale. It may be reinserted directly

[19] It is also possible to share the burden of low energy phenomenological dependencies between the $\xi_i^{\rm H}$ and M_{32}, a freedom which has previously [29, 30] distinguished the flipped unification from its standard counterpart.

in the logarithmic form back into Eqs. (58,59) to realize the analogous expressions for (α_5, α_1). Often however, no direct reference to the 'max' triple unification parameters is of interest. This being the case, the relations may be nicely reduced. We will choose to forgo a direct printing of this equation set in the former style, in favor of the latter.

$$M_{32} = M_Z \times \exp\left\{\frac{2\pi(\alpha_3\Theta_W - \alpha_{em}\Xi_3)}{\alpha_{em}\alpha_3(b_2 - b_3)}\right\} \tag{69a}$$

$$\frac{1}{\alpha_5} = \frac{\alpha_{em}\Xi_3 b_2 - \alpha_3\Theta_W b_3}{\alpha_{em}\alpha_3(b_2 - b_3)} \tag{69b}$$

$$\frac{1}{\alpha_1} = \frac{\frac{3}{5}\alpha_3\Xi_Y(b_2 - b_3) - \alpha_3\Theta_W(b_Y + \frac{3}{5}(b_2 - b_3)) + \alpha_{em}\Xi_3 b_Y}{\alpha_{em}\alpha_3(b_2 - b_3)} \tag{69c}$$

And again for the minimal limit:

$$M_{32}\,|^{\text{MSSM}}_{\Xi_i=1} = M_Z \times \exp\left\{\frac{\pi}{2}\left(\frac{\sin^2(\theta_W^{\text{eff}})}{\alpha_{em}} - \frac{1}{\alpha_3}\right)\right\} \tag{70a}$$

$$\frac{1}{\alpha_5}\,|^{\text{MSSM}}_{\Xi_i=1} = \frac{3\,\sin^2(\theta_W^{\text{eff}})}{4\,\alpha_{em}} + \frac{1}{4\,\alpha_3} \tag{70b}$$

$$\frac{1}{\alpha_1}\,|^{\text{MSSM}}_{\Xi_i=1} = \frac{12 - 45\,\sin^2(\theta_W^{\text{eff}})}{20\,\alpha_{em}} + \frac{33}{20\,\alpha_3} \tag{70c}$$

Returning to the strict triple unification picture, the extra constraint on the merger of α_1 with α_5 has meant that rather than choosing between a solution for either Θ_W or M_{32}, we have determined (the maximal limit of) both. Specifically, we have been compelled to strictly impose some condition akin to Eq. (63) to overcome any shortfall in the effective Weinberg angle. There are however other possibilities if we will instead permit some previously fixed parameter to float. In the spirit of Eq. (56), we may now choose to isolate the strong coupling at the Z-mass as our third dependent function, allowing the shift to an effective value $\alpha_3^{\max} \equiv \alpha_3 + \Delta\alpha_3^{\text{DEP}}$ with $\Delta\xi_2^{\text{DEP}} = 0$.

$$\alpha_3^{\max} = \left[\frac{\Theta_W(5b_Y + 3b_2 - 8b_3) - 3\,\Xi_Y(b_2 - b_3)}{5\,\alpha_{em}(b_Y - b_2)} - \frac{\xi_3}{2\pi}\right]^{-1} \tag{71a}$$

$$\approx \left[\frac{\Theta_W(5b_Y + 3b_2 - 8b_3) - 3\,\Xi_Y(b_2 - b_3)}{5\,\alpha_{em}\,\Xi_3(b_Y - b_2)}\right]^{-1}$$

$$\alpha_5^{\max} = \left[\frac{\Theta_W(5b_Y + 3b_2) - 3\,\Xi_Y b_2}{5\,\alpha_{em}(b_Y - b_2)}\right]^{-1} \tag{71b}$$

$$M_{32}^{\max} = M_Z \times \exp\left\{\frac{2\pi(3\,\Xi_Y - 8\,\Theta_W)}{5\,\alpha_{em}(b_Y - b_2)}\right\} \tag{71c}$$

Although the error bars have been reduced by an order of magnitude since the time when this basic approach was advanced [29] as a way to *predict* α_3, the long 'lever arm' and still significant uncertainty of argue for its selection as dependent parameter over any other outside choice. Nevertheless, our interest in this formulation will primarily be indirect. We will revisit it in Section (4.3) as a guide to our selection of the appropriate linear combination of the $\delta\xi_i$ from which to construct ξ_2^{H} in Standard $SU(5)$.

We emphasize that Eqs. (71b,71c) are in general not numerically equivalent to Eqs. (57,50), being that they place the burden of strict unification onto two different proxies. A correspondence is recovered however, by explicitly forcing evaluation at the 'maximal' (sine-squared) effective Weinberg angle $\Theta_W^{\max}$ from Eq. (61a), whereupon $\alpha_3^{\max}$ likewise reduces to its experimentally assigned value. The minimal limits of the present equation set are:

$$\alpha_3^{\max}\big|_{\Xi_i=1}^{\mathrm{MSSM}} = \frac{7\,\alpha_{\mathrm{em}}}{15\,\sin^2(\theta_W^{\mathrm{eff}}) - 3} \tag{72a}$$

$$\alpha_5^{\max}\big|_{\Xi_i=1}^{\mathrm{MSSM}} = \frac{28\,\alpha_{\mathrm{em}}}{36\,\sin^2(\theta_W^{\mathrm{eff}}) - 3} \tag{72b}$$

$$M_{32}\big|_{\Xi_i=1}^{\mathrm{MSSM}} = M_Z \times \exp\left\{\frac{\pi\left(3 - 8\sin^2(\theta_W^{\mathrm{eff}})\right)}{14\,\alpha_{\mathrm{em}}}\right\} \tag{72c}$$

The underlying message of this section has been simple; we must clearly keep account of the available constraints and matching number of dependent functions, choosing for their solution a properly orthogonalized set of equations. We have presented a compendium of such solution sets, applicable to various situations and needs, for generic β-coefficients and thresholds, and also for the MSSM field limit with all thresholds absorbed into $\sin^2(\theta_W^{\mathrm{eff}})$.

The triple, or Standard $SU(5)$, unification expressions for $(M_{32}^{\max}, \alpha_5^{\max}, \Theta_W^{\max})$ have been given in Eqs. (50,57,61a). The Flipped $SU(5)$ unification is treated either by taking the prior set in conjunction with Eqs. (58,59,62a) to solve for $(\alpha_5, \alpha_1, \Theta_W^{\mathrm{DEP}})$, inserting M_{32} as independent input, or equivalently by using Eqs. (66) which have been solved for the same variables without explicit reference to parameters at the $M_{32}^{\max}$ scale, or finally, as has been our preference in practice, by using Eqs. (69) to solve for $(M_{32}, \alpha_5, \alpha_1)$, taking Θ_W as independent input. Eqs. (71) for $(\alpha_3^{\max}, \alpha_5^{\max}, M_{32}^{\max})$ should not be mixed with the Flipped $SU(5)$ formulae, and indeed even for Standard $SU(5)$ we recommend application of those expressions only in the abstract.

4.2. Grand Unification In The Second Loop

The second order loop corrections are expected to be of comparable significance to the threshold effects. As before, the sharp dependency of τ_p on the unification parameters can magnify even small shifts into the difference between potential detection and stalemate. It is essential to have confidence in the numbers used, and we undertake here again a fresh analysis with full details provided.

We will begin with the renormalization group equations of the MSSM, written to the second loop in the gauge couplings $\alpha_i \equiv g_i^2/4\pi$, and to the first loop for the Yukawa couplings $\lambda_f \equiv y_f^\dagger y_f/4\pi$. The Yukawa index f takes three values (u,d,e) for the up- and down-type quark flavors and for the charged leptons. Each y_f itself is a 3×3 matrix which spans the three generations. Derivatives are taken with respect to $t = \ln(\mu)$, the logarithm of the renormalization scale.

$$\frac{d\alpha_i}{dt} = \frac{b_i\alpha_i^2}{2\pi} + \frac{\alpha_i^2}{8\pi^2}\left[\sum_{j=1}^{3} B_{ij}\alpha_j - \sum_{f=u,d,e} d_i^f\,\mathrm{Tr}\left(\lambda_f\right)\right] \tag{73}$$

$$\frac{d\lambda_u}{dt} = \frac{\lambda_u}{2\pi}\left[3\lambda_u + \lambda_d + 3\mathrm{Tr}(\lambda_u) - \sum_{i=1}^{3} c_i^u \alpha_i\right] \tag{74a}$$

$$\frac{d\lambda_d}{dt} = \frac{\lambda_d}{2\pi}\left[\lambda_u + 3\lambda_d + \mathrm{Tr}(3\lambda_d + \lambda_e) - \sum_{i=1}^{3} c_i^d \alpha_i\right] \tag{74b}$$

$$\frac{d\lambda_e}{dt} = \frac{\lambda_e}{2\pi}\left[3\lambda_e + \mathrm{Tr}(3\lambda_d + \lambda_e) - \sum_{i=1}^{3} c_i^e \alpha_i\right] \tag{74c}$$

The relevant β-coefficients are:

$$b = \left(\frac{33}{5}, 1, -3\right) \quad ; \quad B = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} \tag{75a}$$

$$d^u = \left(\frac{26}{5}, 6, 4\right) \quad ; \quad d^d = \left(\frac{14}{5}, 6, 4\right) \quad ; \quad d^e = \left(\frac{18}{5}, 2, 0\right) \tag{75b}$$

$$c^u = \left(\frac{13}{15}, 3, \frac{16}{3}\right) \quad ; \quad c^d = \left(\frac{7}{15}, 3, \frac{16}{3}\right) \quad ; \quad c^e = \left(\frac{9}{5}, 3, 0\right) \tag{75c}$$

The high entanglement and large dimensionality of these equations prohibits a direct attack. However, a few basic physical approximations will ease the task of numerical computation and even allow for a usable closed form solution. The tremendous falloff of mass between families suggests that the Yukawa couplings for generations one and two may be reasonably ignored here. This reduces each λ_f from a matrix to a simple number, and likewise eliminates the need for tracing over generations. In fact, it will not be so unrealistic to disregard all Yukawa couplings except for the top quark itself, although we will specify explicitly when this second round of cuts is applied.

We can achieve some formal simplification by postulating an ansatz $\alpha_i^{-1} = -(b_i t + \zeta_i)/2\pi$ to extend the one-loop indefinite solution through addition of an undetermined function $\zeta_i(t)$. Inserting this trial form into Eq. (73), cancellations leave a new differential equation for ζ_i which is first order in α_i and λ_f. We emphasize that there is no approximation made in this step itself.

$$\frac{d\zeta_i}{dt} = \frac{1}{4\pi}\left[\sum_{j=1}^{3} B_{ij}\alpha_j - \sum_{f=t,b,\tau} d_i^f \lambda_f\right] \tag{76}$$

If we recursively apply the first order definite solution without thresholds, $\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - b_i/(2\pi) \times \ln(\mu/M_Z)$, the shift in ζ_i from the gauge term of Eq. (76) can be integrated directly without difficulty.

$$\Delta\zeta_i|_{gauge} \approx \sum_{j=1}^{3} \frac{B_{ij}}{4\pi} \times \frac{2\pi}{b_j} \ln\left(1 - \frac{\alpha_j(M_Z)\, b_j}{2\pi} \ln\frac{\mu}{M_Z}\right)^{-1} \simeq \sum_{j=1}^{3} \frac{B_{ij}}{4\pi} \times \frac{2\pi}{b_j} \ln\left(\frac{\alpha_j(\mu)}{\alpha_j(M_Z)}\right) \tag{77}$$

For each $b_j = 0$, the limit $B_{ij}/(4\pi) \times \alpha_j(M_Z) \ln(\mu/M_Z)$ replaces the prior.

To proceed analytically with the Yukawa contribution is somewhat more troublesome. To achieve a suitably compact expression, one must sacrifice all but the top quark contribution as suggested previously, and also apply a simple boundary such as $y_t(M_Z) = 1$. Even less satisfactory however, is the need to further trim back to just a single gauge coupling α and the associated coefficients b and c^t. In these limits, the entire Yukawa sector is reduced to the equation shown following.

$$\frac{d\lambda_t}{dt} = \frac{\lambda_t}{2\pi}\left[6\lambda_t - c^t\alpha\right] \tag{78}$$

This restriction can be made more palatable by the definition of composite constants which represent all three gauge groups simultaneously. A difficulty emerges with this approach, in that the inverse couplings add simply, while the couplings themselves do not. For a majority of the integration domain however, the Taylor expansion applies reasonably well.

$$\frac{\alpha_i(\mu)}{\alpha_i(M_Z)} \approx 1 + \frac{b_i\alpha_i(M_Z)}{2\pi}\ln\frac{\mu}{M_Z} + \ldots \tag{79}$$

In this limit, the sum $\sum_i c_i^t\ \alpha_i(\mu)$ from Eqs. (74) presents no obstacle to the needed reduction. After using Eq. (79) in reverse to restore the original functional form, the 'averaged' constants shown following[20] may be read off.

$$\alpha(M_Z) \equiv \frac{\sum_i c_i^t\ \alpha_i(M_Z)}{\sum_i c_i^t}\ ;\ b \equiv \frac{\sum_i c_i^t\ \sum_i b_i\ c_i^t\ \alpha_i^2(M_Z)}{\left(\sum_i c_i^t\ \alpha_i(M_Z)\right)^2}\ ;\ c^t \equiv \sum_{i=1}^{3} c_i^t \tag{80}$$

Taking again the single loop expression for the recursion of α in Eq. (78), a simple closed form solution for λ_t is possible, which in turn allows for integration of the Yukawa shift from Eq. (76).

$$\Delta\zeta_i|_{Yukawa} \approx -\frac{d_i^t}{4\pi}\times\frac{\pi}{3}\ln\left(1 - \frac{3 - 3\,(\alpha(M_Z)/\alpha(\mu))^{1+c^t/b}}{2\pi(b+c^t)\alpha(M_Z)}\right)^{-1} \tag{81a}$$

$$\lim_{b\to 0} \Rightarrow -\frac{d_i^t}{4\pi}\times\frac{\pi}{3}\ln\left(1 - \frac{3 - 3\,(M_Z/\mu)^{c^t\alpha(M_Z)/2\pi}}{2\pi c^t\alpha(M_Z)}\right)^{-1} \tag{81b}$$

All together, the effect of the second loop is summarized (consult the above for $b = 0$) by the following approximation.

$$\Delta\zeta_i \approx \sum_{j=1}^{3}\frac{B_{ij}}{2b_j}\ln\left(\frac{\alpha_j(\mu)}{\alpha_j(M_Z)}\right) - \frac{d_i^t}{12}\ln\left(1 - \frac{3 - 3\,(\alpha(M_Z)/\alpha(\mu))^{1+c^t/b}}{2\pi(b+c^t)\alpha(M_Z)}\right)^{-1} \tag{82}$$

The form of our trial solution is such that these shifts from the second loop may be applied in a manner directly parallel to the prior handling of the thresholds in Eq. (48). The below will extend and replace that form henceforth. Having made this update, the subsequent expressions of Section (4.1) carry through without modification.

$$\Xi_Y \equiv 1 + \frac{\alpha_{em}}{2\pi}\left\{\frac{5}{3}(\xi_Y - \zeta_Y) + (\xi_2 - \zeta_2)\right\} \tag{83a}$$

$$\Theta_W \equiv \sin^2\theta_W + \alpha_{em}\,(\xi_2 - \zeta_2)/2\pi \tag{83b}$$

$$\Xi_3 \equiv 1 + \alpha_3\,(\xi_3 - \zeta_3)/2\pi \tag{83c}$$

[20] This choice is satisfactory but not unique, due to existence of a symmetry in the solution.

For purposes of computation throughout this report however, we will instead prefer the improved accuracy of a purely numerical solution. Unless otherwise stated, the level of detail used will be as follows. The top *and* bottom quark Yukawa couplings will be considered from the third generation in the first loop. The boundary conditions at M_Z will be given by $y_t = \sqrt{2}m_t/(v_u \equiv v\sin\beta) \approx 0.999$, and $y_b = \sqrt{2}m_b/(v_d \equiv v\cos\beta) \approx 0.242$, with the Higgs vacuum expectation value $v \equiv \sqrt{v_u^2 + v_d^2} = (\sqrt{2}G_F)^{-1/2} \approx 246$ [GeV], using $G_F = 1.16637(1) \times 10^{-5}$[GeV^{-2}] from the PDG [66], $m_t = 173.1(13)$ [GeV] from the TevEWWG [47], $m_b = 4.20(17)$ [GeV] from the PDG, and $\tan\beta \equiv v_u/v_d = 10$ from benchmark scenario [45] B'. All three gauge couplings will be used in the Yukawa renormalization *with* the second loop applied in the recursion. The threshold correction terms, defaulting again to the MSSM spectrum of benchpoint B', will also be applied wherever the gauge couplings α_i are used[21]. Finally, recognizing that the second loop itself influences the upper limit M_{32} of its own integrated contribution, this feedback will be accounted for in the dynamic calculation of the unification scale. The integrity of this construct is verified by comparing the computed value of M_{32} to that produced by Eq. (69a) with the use of (Θ_W, Ξ_3).

Our baseline second loop corrections for the MSSM are then calculated to take the values shown following for the Standard and Flipped $SU(5)$ GUTs. For the triple unification of $SU(5)$, we are bound to also introduce the additional unknown heavy threshold of Eq. (64) to supplement those light threshold factors held over from Eq. (19).

$$\text{Standard} \;:\; \zeta_Y \approx 3.01 \;\;;\;\; \zeta_2 \approx 5.32 \;\;;\;\; \zeta_3 \approx 2.58 \;\;;\;\; \xi_2^H \approx 2.35 \tag{84a}$$

$$\text{Flipped} \;:\; \zeta_Y \approx 2.98 \;\;;\;\; \zeta_2 \approx 5.27 \;\;;\;\; \zeta_3 \approx 2.55 \tag{84b}$$

The standard unification occurs at $M_{32}^{\max} \approx 1.03 \times 10^{16}$ [GeV] with $\alpha_5^{\max} \approx 0.04$, yielding a proton lifetime around 0.95×10^{35} years. The flipped unification by comparison hits at $M_{32} \approx 0.58 \times 10^{16}$ [GeV], for a ratio $M_{32}/M_{32}^{\max} \approx 0.56$, with $\alpha_5 \approx 0.04$, and a proton lifetime near 0.43×10^{35} years. This value is already substantially more rapid than that reported previously [30], with the discrepancy attributable to the additional level of care taken here to calculation of the threshold and second loop effects.

The closed form approximation of Eq. (82) evaluated at identical mass scales compares quite favorably here and also in the extra TeV scale matter scenarios to be discussed shortly, with an apparently systematic tendency to overstate the second loop effects by around ten to fifteen percent. The majority of this difference is traced to numerical recursion of both the thresholds and the second loop, without which the agreement improves to better than a couple of percent. The proton lifetime tends to be underestimated in that approximation with just MSSM content, but overestimated when additional TeV multiplets are included in the running. The amount of disagreement varies, but averages under 20% in magnitude for cases tested.

In all cases, the negative Yukawa sector contributions to the second loop are subordinate to the positive gauge sector term by a substantial multiplicative factor, which ranges between about five and twenty.

4.3. Heavy Threshold Considerations

We turn now to evaluation of the super heavy GUT-scale threshold corrections from the residual Higgs and $SU(5)$ gauge fields. The heavy lifting, so to speak, of this effort has been

[21]There is a minor inconsistency here, as the cumulative effect of the thresholds is applied in full at the outset rather than in steps across the integration.

completed back in Section (1.4), where it was demonstrated, Eq. (13), that such effects can be considered retrospectively of the principal analysis, without upgrading the existing cumulative β-functions, so long as the additional logarithms are compared against the upper rather than the lower mass boundary. Both prescriptions are custom made to order for the present calculation. Indeed, the internal consistency of the view expressed in Eqs. (63,64) which would attribute a shortfall in unification to the action of an as yet undetermined threshold term is implicitly dependent on the β-decoupling. Moreover, the ultra heavy masses under consideration are each naturally tethered by a common vacuum expectation value to the location of the unification point itself, within variations in coupling strength which are typically judged to be of order unity.

From this point of view, we emphasize that the heavy thresholds are of a qualitatively different character than the light thresholds. For the light fields the second order correction was in regards to at which point in the running the given degree of freedom, already included in full to the first order, actually became active. For the heavy fields the second order correction deals with whether the given field exists at all[22]. This distinction between an additive versus a subtractive compensation suggests that GUT scale effects should accordingly be treated somewhat distinctly throughout our process. In particular, we will neglect their second loop, and moreover discard their presence altogether, even in the first loop, during calculation of the second loop contributions of the light fields.

There is a level of mutual consistency to this logic. In calculation of the second loop, Eqs. (49) were treated as continuous functions for all mass scales μ. This is not strictly correct, as the $\delta\xi_i$ are better considered to enter in step-function style after each threshold mass has been crossed. Having passed the point of entry of all relevant thresholds however, the two representations afterward coalesce. Absorption of these discontinuous processes into an adjustment in the 'starting value' of the M_Z gauge couplings precisely characterizes the approach taken with the effective Weinberg angle approach and its related constructs $\Xi_{(Y,3)}$. Since the thresholds considered have entered the running early[23], the region of dispute likewise spans a narrow portion of the integration domain. Allowing this error from the light thresholds to persist in the immediate neighborhood of the lower boundary is considered analogous in spirit to absolute exclusion of the heavy thresholds, whose presence in the continuous function would skew the majority result for the benefit of only that region in closest proximity to the upper boundary. Both surviving errors will be considered to represent comparatively small adjustments to an already small correction term.

The computationally pleasant consequence of all the above is that we may introduce the heavy fields *à la carte*, holding the b_i constant, and calculating only shifts in the existing functions of Eqs. (83), either each in turn or as some appropriately crafted linear combination. We will study both the Standard $SU(5)$ and Flipped $SU(5) \times U(1)_X$ models, establishing the basic technology of analysis and setting up a determination in Section (5.3) of the range of possible consequences which inclusion of the canonical heavy fields may produce.

Let us first consider the Standard $SU(5)$ triple unification. From the perspective of Eqs. (71), we are usually faced, after inclusion of the known light thresholds, with the task of 'lowering α_s' [29] into consistency with its experimental range. We will generally in practice however, instead advocate use of Eqs. (50,57,61a), which treat Θ_W^{max} as a dependent variable, taking $\Delta\alpha_3^{DEP} = 0$. Unification *and* consistency with the central measured value for $\alpha_3(M_Z)$ are secured

[22] This does not necessarily imply that the heavy thresholds are comparatively smaller than the light.

[23] Even logarithmically, even the TeV Vectors are still significantly smaller than M_{32}.

via invocation of unknown heavy thresholds given by Eq. (64). However, as discussed in Section (4.1), the dilemma becomes determination of an effective linear combination $\xi_2^{\rm H}$ from the three ξ_i which any physical threshold term may yield. The merit of the solution for Eqs. (71) now becomes their use as a guide to this selection. It is clear from inspection that no single consolidation of the three gauge thresholds can mimic action of the whole simultaneously for each of $(\alpha_3^{\rm max}, \alpha_5^{\rm max}, M_{32}^{\rm max})$. Specifically, the following applies in the limit of small changes.

$$\frac{\delta\alpha_3^{\rm max}}{\alpha_3^{\rm max}} = \alpha_3^{\rm max} \times \left\{ \frac{(b_2 - b_3)\,\delta\xi_{\rm Y} + (b_3 - b_{\rm Y})\,\delta\xi_2 + (b_{\rm Y} - b_2)\,\delta\xi_3}{2\pi(b_{\rm Y} - b_2)} \right\} \tag{85a}$$

$$\frac{\delta\alpha_5^{\rm max}}{\alpha_5^{\rm max}} = \alpha_5^{\rm max} \times \left\{ \frac{b_2\,\delta\xi_{\rm Y} - b_{\rm Y}\,\delta\xi_2}{2\pi(b_{\rm Y} - b_2)} \right\} \tag{85b}$$

$$\frac{\delta M_{32}^{\rm max}}{M_{32}^{\rm max}} = \frac{\delta\xi_{\rm Y} - \delta\xi_2}{b_{\rm Y} - b_2} \tag{85c}$$

Although the parameters $(\alpha_5^{\rm max},\ M_{32}^{\rm max})$ at the unification scale are more directly relevant to our interest in proton lifetime, they are meaningless if the unification itself is a failure. We will thus direct focus on the variation of $\alpha_3^{\rm max}$ in Eqs. (85), ensuring that it is symmetric under replacement of the actual ξ_i with our choice for $\xi_2^{\rm H}$ and the book-matched hypercharge term $\xi_{\rm Y}^{\rm H}$ of Eq. (65). The point is that we can indeed then consistently lock down the strong coupling as we solve for $\Theta_{\rm W}^{\rm max}$, used next to specify the 'magnitude' of the missing heavy threshold as a single effective parameter. The invariance of $\alpha_3^{\rm max}$ under this simplification ensures that the coupling will not subsequently be displaced from its experimentally established strength. Using the heavy field formulation of the ξ_i from Eq. (13), we will also wish to translate our result into the language of an effective β-coefficient $b_2^{\rm H}$. In keeping with this discussion, the expression for $\xi_2^{\rm H}$, holding the Ξ_i constant while preserving the central value of α_3 as a marker of successful unification, and assuming $5b_{\rm Y} + 3b_2 - 8b_3 \neq 0$, is given following[24]. We emphasize again that this formula is appropriate only[25] for use in Standard $SU(5)$.

$$\xi_2^{\rm H}\Big|^{\rm SU(5)}_{\Delta\Xi_i = 0} \equiv \frac{(b_2 - b_3)\delta\xi_{\rm Y} + (b_3 - b_{\rm Y})\delta\xi_2 + (b_{\rm Y} - b_2)\delta\xi_3}{(b_3 - b_{\rm Y}) - \frac{3}{5}(b_2 - b_3)} \equiv \sum_{\rm heavy} \delta b_2^{\rm H} \times \ln\frac{M_{32}^{\rm max}}{M} \tag{86a}$$

$$= \sum_{\rm heavy} \left\{ \frac{5(b_2 - b_3)\delta b_{\rm Y} + 5(b_3 - b_{\rm Y})\delta b_2 + 5(b_{\rm Y} - b_2)\delta b_3}{5b_{\rm Y} + 3b_2 - 8b_3} \right\} \times \ln\frac{M_{32}^{\rm max}}{M}$$

$$\lim_{\rm MSSM} \Rightarrow \sum_{\rm heavy} \left\{ \frac{1}{3}\,\delta b_{\rm Y} - \frac{4}{5}\,\delta b_2 + \frac{7}{15}\,\delta b_3 \right\} \times \ln\frac{M_{32}^{\rm max}}{M} \tag{86b}$$

The practical effect of activating Standard $SU(5)$ heavy thresholds is, from this perspective, a transverse rescaling of the strong coupling strength at $M_{\rm Z}$. As for the first of Eqs. (85), from

[24] We emphasize again that the bulk b_i reference the light fields *only*, while the δb_i are the incremental additions to the baseline value of each β-coefficient attributable to the heavy fields.

[25] The MSSM limit given in Eq. (86b) is consistent with published [42] results. It has sometimes historically [29, 30] been cross-applied to the flipped unification, a practice which we now discourage.

which the below derives, we again stipulate the limit of small changes.

$$\alpha_3^{\max} \quad \Rightarrow \quad \alpha_3^{\max} \times \left\{ 1 - \xi_2^{\mathrm{H}} \left(\frac{\alpha_3^{\max}}{2\pi} \right) \left(\frac{5b_{\mathrm{Y}} + 3b_2 - 8b_3}{5(b_{\mathrm{Y}} - b_2)} \right) \right\} \tag{87a}$$

$$\lim_{\mathrm{MSSM}} \quad \Rightarrow \quad \alpha_3^{\max} \times \left\{ 1 - \xi_2^{\mathrm{H}} \left(\frac{15\,\alpha_3^{\max}}{14\pi} \right) \right\} \tag{87b}$$

Of course, the cost of favoring the strong coupling here is that predictions for updates to the unified mass and coupling in terms of the effective threshold will be distorted. We strongly advocate deference to the more precise methods presented in Section (1.4) whenever actual β-coefficients and mass scales are known, certainly for the light thresholds, and even for the heavy thresholds. In fact, we most often suggest restricting direct application of even the concept of ξ_2^{H} to situations where it is important only to know that heavy thresholds do exist, the nature of the fields involved being hypothetical, nonspecific or irrelevant.

In this view ξ_2^{H} represents a diagnostic tool for consolidating, abstracting and quantifying the failure of unification rather than a literal implementation of actual thresholds by which a successful unification might be secured. For this purpose and to this end we justify the underlying integrity of the construct, highlighting however the generic limitation that only one of three dependent solutions may be accurately represented. Nevertheless, we shall shortly demonstrate the effectiveness of Eq. (87a) to quickly and intuitively determine what the bulk plausible effects of heavy thresholds could be, and as specific example, whether they might be viable to salvage the triple unification of Standard $SU(5)$.

We turn attention next to the partial unification at M_{32} of Flipped $SU(5)$. Since we are no longer strictly constrained by the triple unification, 'lowering α_s' is no longer a principal concern. To say it another way, there is no obstacle to the meeting of just two non-parallel lines at some distant point, and consequently there is no fundamental reason to require the presence of an unknown heavy threshold ξ^{H}. Nevertheless, we do generally expect that there *will be* some heavy thresholds, and we may now by choice rather than necessity stipulate the presence of such a term. The result will be simply a relocation of the partial unification M_{32} point, and the values of the two couplings at that scale.

As before, we will need to confront the reality that only one of these three parameters may be generically and accurately modeled by a single effective shift term. With the integrity of our low energy phenomenology itself not in jeopardy, we may focus attention instead on fidelity of the GUT scale rendering. With M_{32} being the most sensitive of our three choices to variation in the thresholds, and considering in turn the great sensitivity of the proton lifetime to variation in M_{32}, it will become our key to the definition of an analogue to Eq. (86a) for the Flipped $SU(5)$ context. Taking our cue from the set of Eqs. (69), variations of the dependent parameters are as follows.

$$\frac{\delta M_{32}}{M_{32}} = \frac{\delta\xi_2 - \delta\xi_3}{b_2 - b_3} \tag{88a}$$

$$\frac{\delta\alpha_5}{\alpha_5} = \alpha_5 \times \left\{ \frac{b_3\,\delta\xi_2 - b_2\,\delta\xi_3}{2\pi(b_2 - b_3)} \right\} \tag{88b}$$

$$\frac{\delta\alpha_1}{\alpha_1} = \alpha_1 \times \left\{ \frac{b_{\mathrm{Y}}(\delta\xi_2 - \delta\xi_3)}{2\pi(b_2 - b_3)} - \frac{\delta\xi_{\mathrm{Y}}}{2\pi} \right\} \tag{88c}$$

The effective single parameter shift which correctly mimics the dependency of the flipped unification scale may be simply read off from the first member of the prior. Since the meeting of the second and third couplings occurs independently of the hypercharge, there is no entanglement with $\xi_{\rm Y}^{\rm H}$.

$$\xi_2^{\rm H}\Big|_{\Delta\Xi_i=0}^{\mathcal{F}-{\rm SU}(5)} \quad\equiv\quad (\delta\xi_2-\delta\xi_3) \quad\equiv\quad \sum_{\rm heavy}\delta b_2^{\rm H}\times\ln\frac{M_{32}}{M} \tag{89}$$
$$= \quad \sum_{\rm heavy}\{\delta b_3-\delta b_2\}\times\ln\frac{M_{32}}{M}$$

The practical effect of activating heavy thresholds in Flipped $SU(5)$ is expressed most clearly as a lateral scaling of the unification point.

$$M_{32} \quad\Rightarrow\quad M_{32}\times\exp\left\{\frac{\xi_2^{\rm H}}{(b_2-b_3)}\right\} \quad=\quad M_{32}\times\prod_{\rm heavy}\left(\frac{M}{M_{32}}\right)^{\delta(b_2-b_3)\,/\,(b_2-b_3)} \tag{90a}$$

$$\lim_{\rm MSSM} \quad\Rightarrow\quad M_{32}\times\exp\left\{\frac{\xi_2^{\rm H}}{4}\right\} \tag{90b}$$

The exponential amplification of the unification point's dependency on the heavy thresholds brings a potentially large measure of uncertainty into our calculation. Detailed effects near the GUT scale being largely detached from low energy phenomenological constraint, we have precious little data to guide the selection of a value for this term. Indeed, the plausible range of variation remains sufficient to loom as a lingering threat to swamp out any detailed consideration of better known elements.

The correction presented in Eq. (56) manifests itself also in Eqs. (90), with the coefficient $1/4$ from the MSSM limit given in Eq. (90b) superseding the previously published value of $5/11$. The new value being smaller by nearly a half, the overall uncertainty from the heavy thresholds is trimmed back to roughly its own square-root. Unfortunately, this benefit is immediately countered by substitution of the flipped effective threshold from Eq. (89) for the inapplicable Standard $SU(5)$ form of Eq. (86b). The net result is essentially a wash in comparison to prior calculations.

5. $\mathcal{F}$-ast Proton Decay

5.1. Principal Proton Lifetime Predictions

We present in this section the key tabular and graphical results of our present effort.

Scenario	$\xi_{\rm Y}$	ξ_2	ξ_3	$\zeta_{\rm Y}$	ζ_2	ζ_3	$\Xi_{\rm Y}$	$\Theta_{\rm W}$	Ξ_3	$\Delta\xi_2^{\rm DEP}$	$\Delta\alpha_3^{\rm DEP}$	$\alpha_5^{\rm max}$	$M_{32}^{\rm max}$	$\tau_p^{(e\vert\mu)^+\pi^0}$
$SU(5)_0$	3.20	5.48	8.05	3.01	5.32	2.58	1.001	0.231	1.102	2.35	0.012	0.040	1.03	0.95
$SU(5)_{\rm I}$	10.4	12.7	15.2	8.84	16.9	14.9	0.998	0.226	1.006	3.94	0.022	0.117	1.85	1.15
$SU(5)_{\rm II}$	10.4	12.7	15.2	8.35	12.8	14.0	1.004	0.231	1.023	1.26	0.006	0.115	2.08	1.92

Table 8: Principal results for Standard $SU(5)$, with and without heavy vector multiplets. Mass is in units of [10^{16} GeV]. Lifetimes are given in [10^{35} Y].

The principal numerical results for our study of Standard $SU(5)$, for the MSSM, and also for F-theory models which include extra TeV Scale vector-like multiplets, are accumulated in Table (8). For each scenario under consideration, we present first the ξ_i and ζ_i of Sections (1.4,4.2), which account respectively for the presence of light (including TeV-scale) thresholds and the second loop. These are combined according to Eqs. (83) into an effective sine-squared Weinberg angle and the corresponding factors for the hypercharge and strong coupling. As discussed around Eq. (63), strict triple-unification will not generally be consistent with a given specification of physical thresholds, and one may posit the existence of an unknown (likely heavy) compensating threshold $\Delta\xi_2^{\rm DEP}$ to close the gap. An alternative mechanism for quantifying the failure of unification, as discussed around Eq. (71a), is to lock the thresholds and push any residual discrepancy instead onto a deviation of the strong coupling $\left(\Delta\alpha_3^{\rm DEP} \equiv \alpha_3^{\rm max} - \alpha_{\rm s}(M_{\rm Z})\right)$, now viewed as a dependent parameter. This perspective[26] has an advantage of intuitive immediacy by comparison to experimental uncertainty (0.002) of $\alpha_{\rm s}$. We see in the table that even the best case studied (with Type II vectors) suffers a mismatch of three standard deviations. As discussed in Section (4.3), heavy thresholds are unable to reconcile this disagreement. Finally, we present the unified coupling and mass, and based on these results, the predicted proton lifetime. Second order effects are responsible for reducing the Type 0 unification scale down from its baseline value of $M_{32}^{\rm max} \simeq 2 \times 10^{16}$ [GeV] in Eq. (51). Although the F-theory scenarios are more strongly coupled at unification, this effect is more than offset by something like a relative doubling of their unification mass points, resulting in (slightly) longer overall lifetime predictions. We must emphasize again however, that the central message from the Standard $SU(5)$ scenarios is a failure of the unification process itself, rendering any other conclusions in practice academic.

Scenario	$\xi_{\rm Y}$	ξ_2	ξ_3	$\zeta_{\rm Y}$	ζ_2	ζ_3	$\Xi_{\rm Y}$	$\Theta_{\rm W}$	Ξ_3	α_1	α_5	M_{32}	$\% M_{32}^{\rm max}$	$\tau_p^{(e\vert\mu)^+\pi^0}$
$\mathcal{F}$-SU(5)$_0$	3.20	5.48	8.05	2.98	5.27	2.55	1.001	0.231	1.103	0.039	0.041	0.58	55.8	0.43
$\mathcal{F}$-SU(5)$_{\rm I}$	4.64	12.7	15.2	6.21	16.4	14.3	0.992	0.227	1.018	0.045	0.116	0.68	0.01	0.10
$\mathcal{F}$-SU(5)$_{\rm II}$	7.51	12.7	15.2	6.50	16.4	14.3	0.998	0.227	1.017	0.061	0.116	0.68	1.06	0.10

Table 9: Principal results for Flipped $SU(5)$, with and without heavy vector multiplets. Mass is in units of [10^{16} GeV]. Lifetimes are given in [10^{35} Y].

The collected results for our study of Flipped $SU(5)$ are likewise presented in Table (9). An essential point of distinction from the prior is that this unification of two couplings is intrinsically self consistent[27], and thus has no need for the compensation of unknown thresholds. As before, we see the F-theoretic constructions dramatically lifting the $SU(5)$ coupling at (partial) unification. Since the scale M_{32} now remains relatively stable between the three scenarios, this enhancement of α_5 is able to net something like a four-fold comparative reduction in the overall lifetime. We provide also in this table the relative percentage of the ratio $M_{32}/M_{32}^{\rm max}$. As will be discussed in Section (6.1), the putative point of triple unification $M_{32}^{\rm max}$ for a given Flipped $SU(5) \times U(1)_{\rm X}$ model appears, as a rule of thumb, to target the eventual point of full unification between $\alpha_{\rm X}$ and α_5. As generically expected, the mass scale reductions relative to $M_{32}^{\rm max}$ are more than sufficient to offset the five-fold flipped rate suppression mentioned in Section (1.3), producing a significant downward shift of the proton lifetime for all three flipped models. In the bare MSSM Type 0 scenario, we see a very conventional 'mild' flipping ratio of about 56%. In

[26] We have opted to simply carry over the second loop factor from the original perspective with α_3 independent.

[27] Whether this be virtue (avoidance of fine tuning) or vice (triple unification is more special and thus more suggestive) is open to interpretation.

the models with TeV scale vectors, the flipping becomes extreme[28], dropping to just one percent for Type II, and significantly lower yet for Type I. This corresponds to a significant extension of the (full) unification point, while keeping the initial $SU(5)$ merger, that which is relevant to proton decay, pleasingly low. This constitutes a natural and stable distinction between what might usually be called the GUT scale, and a point of super unification around the reduced Planck mass.

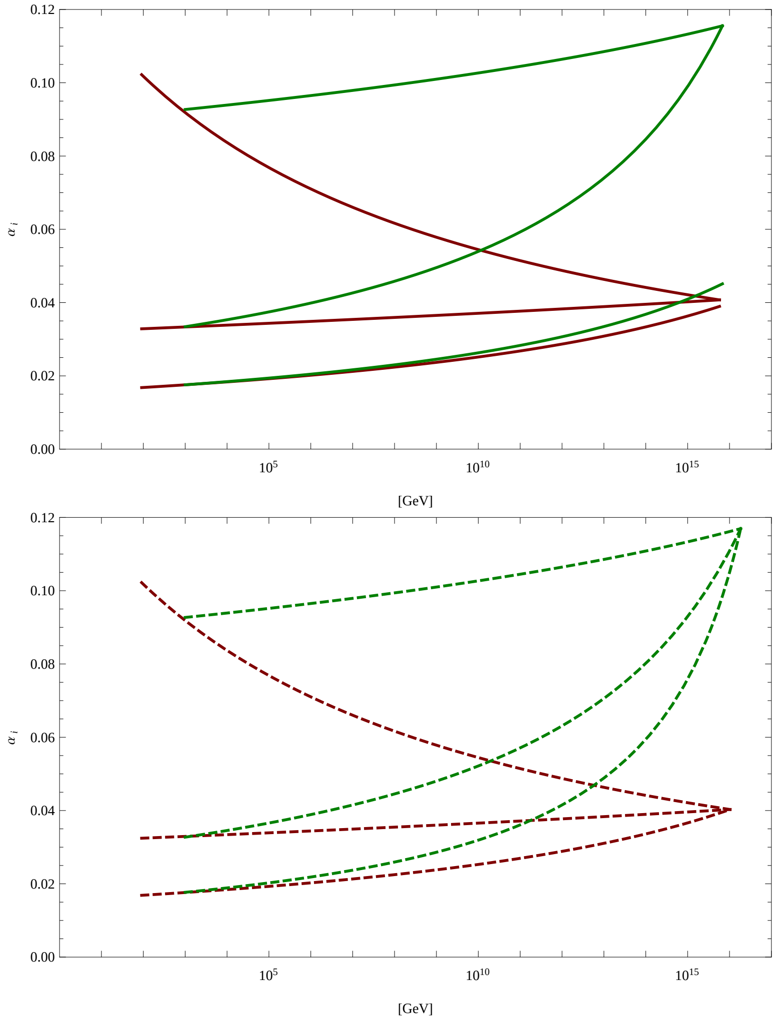


Figure 3: Running of the gauge couplings is compared for $\mathcal{F}$-SU(5)$_0$ (solid red) Vs. $\mathcal{F}$-SU(5)$_{\rm I}$ (solid green) and SU(5)$_0$ (dashed red) Vs. SU(5)$_{\rm I}$ (dashed green). The upper line of each set depicts α_3, with α_2 beneath, and then $\alpha_{\rm Y}$.

[28]It is important to recognize that the limit $M_{32} \rightarrow M_{32}^{\rm max}$ does not in this case actually take you into a continuum with Standard $SU(5)$. This is because the vector multiplets have been assigned specifically for compatibility with flipped charge assignments.

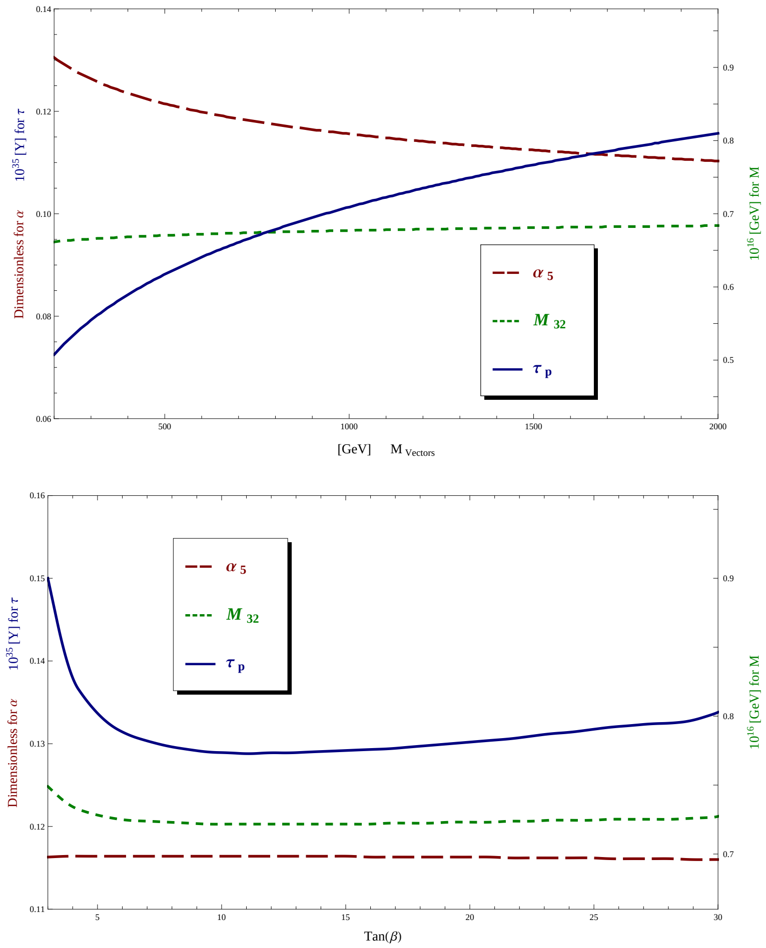


Figure 4: The left-hand numerical scales here perform double duty, read dimensionlessly for α_5, and in units of [10^{35} Y] for the proton partial lifetime $\tau_p^{(e|\mu)^+\pi^0}$. M_{32} is read on the right-hand scale in [10^{16} GeV]. The first example visually explores variation of these parameters with respect to changes in the mass of the F-theory vector-like multiplets of Flipped $SU(5)$ Type II. The vector mass is run from 200 [GeV] up to 2 [TeV], bookending the value of 1 [TeV] to which we otherwise default. The SUSY spectrum is taken at benchmark B' as usual. We see a decline in the coupling strength as the vector mass increases, with the (partial) unification mass holding essentially steady. The result is a modest gradual rise in proton lifetime from left to right, starting some 25% below, and ending some 15% above our baseline value. In the second plot we instead vary the SUSY Higgs parameter $\tan\beta$ from 3 to 30, bounding again our default value of 10 for benchmark B'. We regenerate the SUSY spectrum along this span with ISASUGRA, otherwise leaving the B' input parameters intact. This change of approach with respect to the spectrum is responsible for some overall up-tick in the lifetime. The model again is Flipped $SU(5)$ Type II, with the vector mass locked at 1 [TeV]. In this example we instead see α_5 flatline, while the unification mass declines steeply to a minimum near our default region, thereafter rebounding slowly. The proton lifetime tracks with and amplifies changes in the unification mass, although the overall variation is still mild, the peak within about 15% of the valley for the range considered. The slope approaching the left edge however is steep, and continuing too far outside the graph in either direction we experience pathologies in application of the Yukawa boundary condition for calculation of the second loop.

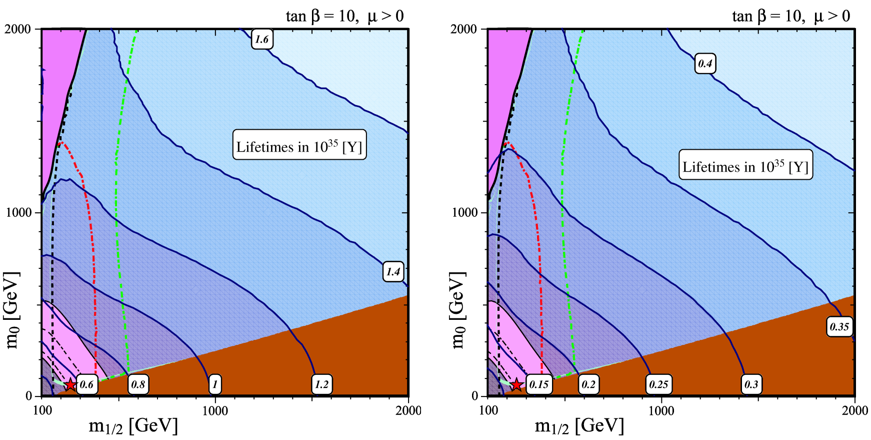


Figure 5: Proton $\tau_p^{(e|\mu)^+\pi^0}$ lifetime contours (dark blue), for Flipped $SU(5)$ first with only MSSM field content, and subsequently including Type II vector-like multiplets, are overlaid onto a background map of phenomenological limits on the CMSSM for $\tan\beta = 10$, and $\mu > 0$, that courtesy of Olive et al. [68]. The orange-brown regions in the lower right of each graphic suffer a charged LSP, while the dark pink at upper left fail electroweak symmetry breaking. The dotted green, red and black lines mark the scalar neutralino-nucleon cross section at 10^{-9} [pB], the Higgs at 114 [GeV], and a chargino mass of 104 [GeV]. The light pink band in the lower left is consistent with measurements of $(g-2)_\mu$, and the narrow turquoise strip marks agreement between the cosmological CDM limits from WMAP and the relic neutralino prediction after coannihilation. Our baseline benchmark B' (red star) sits in the $\mathcal{F}$-ast lower left corner of each diagram.

In Figure Set (3), we comparatively depict the renormalization of gauge couplings for both Standard and Flipped $SU(5)$ models, both with and without the inclusion of F-theory TeV-scale vector multiplets. We expect α_3 to diminish with increasing energy in the MSSM, but the presence of the TeV-scale vectors intervenes to reverse this trend and start the line back upwards in the F-theory related models[29]. The essential flatness of the strong coupling is consistent with our first order expectations for the F-theoretic models with $b_3 = 0$. Prior analysis [29, 30] of thresholds and the 'M_{32}' effect tied variation in proton lifetime strongly to the partial unification scale, with never much change in the coupling between different scenarios. Here however, such a severe discontinuity to all three couplings so early in the running can change things. In fact, it is the leveling out of the strong coupling which forces the flavor coupling to make a greater accommodation, potentially tripling or quadrupling the value of α_{32}, who's effect on the decay is itself to be squared. The consequence is a significant reduction in the $\tau_p^{(e|\mu)^+\pi^0}$ mean lifetime.

Figure Set (4) is a graphical study of variations in the proton lifetime with respect to changes first in the mass of the F-theory vector multiplets, near the TeV order but departing from the value 1, and second in tan(β), the ratio of vacuum expectation values for the up- and down-type Higgs. Figure Set (5) depicts an overlay of proton lifetime contours onto a background of phenomenological constraints on the CMSSM parameter space.

5.2. Deconstruction of Updates

We undertake in this section a comparative forensic deconstruction of prior scenarios and methods of investigation into unification and proton lifetime, isolating the specific updates in data, context, and technique which have culminated in the presently more rapid predictions for decay.

Computational Scenario and Method	$\tau_p^{(e\|\mu)^+\pi^0}$	Δ%
$\mathcal{F}$-SU(5)$_0$ at pt. B with Legacy Threshold and 2nd Loop (2002)	1.44	–
with Corrections to the Isolation of Dependent Parameters	11.1	↑ 670%
with Revisions from the Particle Data Group & pt. B → B'	5.95	↓ 46%
with Light Thresholds Applied to Each Coupling	0.61	↓ 90%
with Detailed Evaluation of the Second Loop	0.43	↓ 29%
with Vector-like Multiplets at 1 [TeV] from $\mathcal{F}$-SU(5)$_{\rm II}$	0.10	↓ 77%
The net change is ↓ 70% for $\mathcal{F}$-SU(5)$_0$ OR ↓ 93% including $\mathcal{F}$-SU(5)$_{\rm II}$ fields		

Table 10: A comparative deconstruction of various model scenarios and computational methods for the determination of proton lifetime. Lifetimes are given in [10^{35} Y].

Table (10) takes Flipped $SU(5)$ with MSSM field content as its baseline, applying the legacy procedures and input data of [30]. Percentage changes[30] are determined sequentially from this

[29] A very mild effective shift in the M_Z scale couplings may be observed graphically, as was discussed in the note following Eqs. (49). With inclusion of the TeV scale vector multiplets however, the effective coupling shift, particularly for α_3, would be quite severe. To alleviate this visual artifact and produce a more realistic plot, we splice the F-theory curves onto the MSSM curves once the 1 TeV boundary has been crossed.

[30] Note for example that: $11.1 \div 1.44 \approx 7.7$; $(7.7 - 1) \times 100\% = +670\%$.

reference, with each methodological adjustment combined with those described preceding. The published benchmark mass spectra from [45, 69], computed by SSARD and FeynHiggs at points (B,B') respectively, are applied at face value.

The corrections described in Section (4.1), specifically as manifest in Eqs. (51,55,60a,60b), taking also the effective shift in α_{Y} of Eq. (45) from Eq. (47), make up the first transition, fully eliminating errors related to the incorrect isolation of dependent parameters. Taken in isolation, this step actually *lengthens* the proton lifetime by a large multiplicative factor of nearly eight times. It will be all downhill from however from this inauspicious start, starting with simple revisions to the central values of M_{Z}-scale parameters reported by the PDG [66], accumulated over the last several years. Together with the transition to the updated benchmark B', this accounts for a not insignificant downward shift by a factor of almost two. The methods, described and applied in Sections (1.4,1.5) respectively, for the correct individual application of light thresholds to each of the three running couplings take the lion's credit for reduction in predicted lifetime, a full order of magnitude. The detailed evaluation of the second loop[31] from Section (4.2), applied again individually to each coupling, yields a modest yet still significant reduction of about a third. The cumulative effect on proton lifetime for Flipped $SU(5)$ with MSSM field content is a cut of seventy percent, due solely to refinements of technique and parameter input.

Including also the F-theory vector-like multiplets of Type II from Section (3.4), proton lifetime drops substantially yet again, now to roughly a quarter of the preceding value. This change is attributed primarily to the strengthening of the α_5 coupling at the partial unification point. At just 1.0×10^{34} Years, this prediction sits coyly on the upper lip of current experimental bounds. The net downward percentage shift from our original baseline number is a rather astonishing 93%. Although experimental uncertainty in parameters at M_{Z} can rescale this result either up or down by a factor of about two, the potentially more significant heavy thresholds appear capable only of elongating the result, upward into the yet unexplored territories. This fact is elaborated upon in the next section, and visually summarized in Figure Set (6), wherein it is furthermore clear that departure from our phenomenologically preferred neighborhood of benchpoint B' should likewise almost certainly slow the decay.

5.3. *Computation of Heavy Thresholds*

We will now make concrete the results and discussion of Section (4.3) by specifically computing the range of possible consequences which may arise from introduction of the canonical GUT scale fields of Standard and Flipped $SU(5)$. Table (11) summarizes the supersymmetric field content for each case [42].

For Standard $SU(5)$, the three relevant mass scales are M_{H_3}, the heavy down-quark type triplet from the $(\mathbf{5}, \overline{\mathbf{5}})$ electroweak-breaking Higgs, M_Σ, representing the octet and triplet from the GUT scale adjoint Higgs **24**, and M_V, representing the (X,Y) gauge fields and the corresponding residual elements of the adjoint Higgs.

For Flipped $SU(5) \times U(1)_{\mathrm{X}}$, $M_{(H_3, \overline{H}_3)}$ are the down-quark type elements of the GUT scale $(\mathbf{10}, \overline{\mathbf{10}})$ Higgs, and M_V again represents the (X,Y) gauge fields, now in combination with the quark-type doublet-triplets remaining from the fundamental Higgs.

Having established the preferred forms of our effective threshold factors in Eqs. (86a,89), the final row of Table (11) presents a tally of the relevant δb_2^{H} for each of the heavy field groupings. It

[31]The relative percentage reductions hold basically steady if the order of application of the modern treatment of the light thresholds and of the second loop is reversed.

	SU(5)			$\mathcal{F}$-SU(5)		
	M_{H_3}	M_Σ	M_V	M_{H_3}	$M_{\overline{H}_3}$	M_V
δb_{Y}	$\frac{2}{5}$	0	−10	$\frac{2}{5}$	$\frac{2}{5}$	−10
δb_2	0	2	−6	0	0	−6
δb_3	1	3	−4	1	1	−4
δb_2^{H}	$\frac{3}{5}$	$-\frac{1}{5}$	$-\frac{2}{5}$	1	1	2

Table 11: β-function coefficients [42] for the GUT-scale fields of Standard and Flipped $SU(5)$. The Effective β-function coefficients b_2^{H} are listed in the final row.

is here the case, both for Standard and Flipped $SU(5)$, that each of the models listed in Table (7) produce numerically identical δb_2^{H}. Although this seems to occur only by simple coincidence, we thus make no effort to distinguish between Types (0,I,II) in the tabulation.

We will adopt the conventional wisdom [29, 42] with regards to mass generation at the heavy scale, *i.e.* that mass M_i is proportionally linked to a common vacuum expectation value $\langle v \rangle_{\mathrm{GUT}}$ by multiplication of the relevant coupling strength λ_i, either gauge or Yukawa. Moreover the relative coupling strengths, combined with any dimensionless numerical factors, should be of a single order in magnitude, usually defined to be within about a factor of three. The largest of the set of heavy masses present should condense at the onset of the breaking of the gauge symmetry, and thus correspond to M_{GUT}, or here M_{32} itself, and specifically M_{32}^{max} for the case of Standard $SU(5)$. All together:

$$M_{32}^{(\mathrm{max})} \div 3 \quad \leq \quad M_i \simeq \lambda_i \langle v \rangle_{\mathrm{GUT}} \quad \leq \quad M_{32}^{(\mathrm{max})} \tag{91}$$

For Standard $SU(5)$, the δb_2^{H} of Table (11) in hand, we may proceed to determine a plausible range for the factor ξ_2^{H} according to the prescription of Eq. (86a). However, there is a most important and well known complication. Namely, Standard $SU(5)$ notoriously suffers from dangerously fast dimension five proton decay mediated by supersymmetric graphs which mix the Higgs $(\mathbf{5}, \overline{\mathbf{5}})$ as required for tuning against the adjoint Higgs **24** to enable doublet-triplet splitting. Suppression of this catastrophic decay places a strong lower bound [32] on the color-triplet Higgs mass M_{H_3}, implying that we must surely take it to be the heaviest member of our set.

We see however, that the triplet Higgs is the only heavy field available to Standard $SU(5)$ with a positive value for the effective β-coefficient b_2^{H}. The negative coefficients for (M_Σ, M_V), along with their positive logarithms, imply that the net contribution to ξ_2^{H} can only itself be negative. This is incompatible with the summary data of Table (8), which specify a positive value between about one and four for the dependent parameter $\Delta\xi_2^{\mathrm{DEP}}$ in order to salvage unification in the various models considered.

As an alternate perspective, taking explicit heavy thresholds contributions from (M_Σ, M_V), each at a third of M_{32}^{max}, with no additional implicit heavy fields, the deviation $\Delta\alpha_3^{\mathrm{max}}$ in the 'predicted' strong coupling increases to $(0.016, 0.027, 0.010)$ for the Standard $SU(5)$ models of

Type (0,I,II) respectively. These results, calculated here by direct reevaluation of Eq. (71a) with all three δb_i of Table (11), agree very nicely with $\delta\alpha_3^{\rm max}$ from Eq. (87a), applying the effective β-coefficient combinations $\delta b_2^{\rm H}$ which are common to all three scenarios studied.

We thus hereby reaffirm the previously advertised [30, 32] conclusion that the triple unification of Standard $SU(5)$ is incapable of simultaneously surviving proton decay limits while maintaining agreement with precision electroweak experiments. This result remains intact with equal force for the Standard $SU(5)$ F-theory variations considered directly herein. We do note carefully however that modern embeddings of supersymmetric $SU(5)$ into M-theory [70, 71] and F-theory [22–25] model building are to be distinguished from the historical Standard $SU(5)$ picture by the presence of alternative scenarios for doublet-triplet splitting, namely the isolation of Higgs doublet/triplet elements on distinct matter curves [24], and suppression of dimension five proton decay.

For Flipped $SU(5) \times U(1)_{\rm X}$, doublet-triplet splitting is achieved naturally via the missing partner mechanism [14–16], so that no threat from dimension five decay exists. Interestingly, the available $\delta b_2^{\rm H}$ from Table (11) are all positive for the flipped models, suggesting that $\xi_2^{\rm H}$ can likewise only be positive, and thus by Eq. (90a) that M_{32} can only be rescaled to a larger value, slowing proton decay. We will quickly check the numbers in order to estimate the size of this effect, taking ($M_{H_3} = M_{\overline{H}_3}$) for simplicity[32].

There are now two options for evaluating the sum from Eq. (89). Either M_V is placed as the heavy member at M_{32}, eliminating its logarithm, or the same happens instead to the pair of triplet Higgs. In the former case, the Higgs each contribute $\delta b_2^{\rm H} = 1$ for a total of 2, and in the latter case, M_V replicates the result identically, yielding a factor of 2 by itself. In either case, to realize the boundary scenario of Eq. (91), we will apply the full factor of three in the mass ratio of the surviving logarithm. The result is then ($\xi_2^{\rm H} \leq 2\ln 3 \approx 2.2$), corresponding to an enlargement[33] of M_{32} by a factor of up to about ($e^{2.2/4} \approx 1.7$). Since the proton lifetime is proportional to the fourth power of this mass, the decay rate can be reduced by a factor as large as nine. This result is fully consistent with direct numerical reevaluation of Eq. (69a), taking the δb_i individually. Its significance is demonstrated graphically in Figure Set (6), first for baseline Flipped $SU(5)$ in the MSSM, and next with TeV-scale vector multiplets of Type II included.

The heavy thresholds represent by far the largest single source of uncertainty in determination of the flipped proton lifetime $\tau_p^{(e|\mu)^+\pi^0}$. In Section (6.1), we will explore the possibility that 'downward pressure' from the true grand unification of $SU(5)$ with $U(1)_{\rm X}$ might suppress excessive upward motion of the scale M_{32}, thereby also constraining runaway uncertainty in the rate of proton decay.

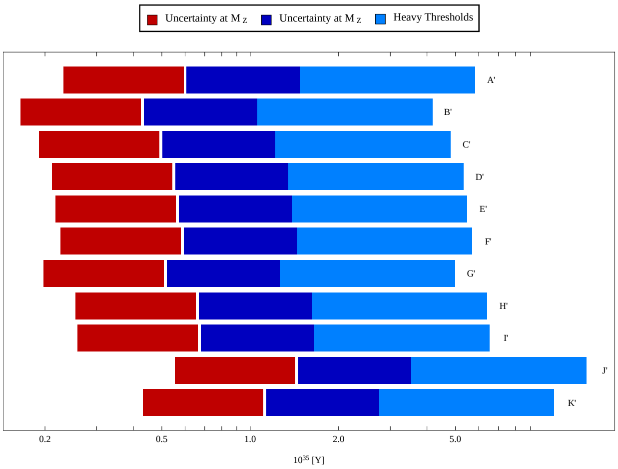
6. $\mathcal{F}$-inale

6.1. The $\mathcal{F}$-inal Unification of $SU(5) \times U(1)_{\rm X}$

The most immediately distinctive characteristics of the Flipped $SU(5)$ 'GUT' are the factorization into the direct product $SU(5) \times U(1)_{\rm X}$, and the existence of a partial unification between (α_3,α_2) at the scale M_{32}, leaving $\alpha_{\rm Y}$ external to the majority field content. M_{32} is shifted

[32]Since $\delta b_i^H = \delta b_i^{\overline{H}}$, there is no loss of generality from the alternate condition sometimes quoted [29, 30] that $M_V \div 3 \leq \sqrt{M_{H_3} \times M_{\overline{H}_3}} \leq 3 \times M_V$

[33]Note that since $(b_2 - b_3) = 4$ applies for each of the flipped scenarios under consideration, the mass rescaling from Eq. (90a) will likewise be universal.



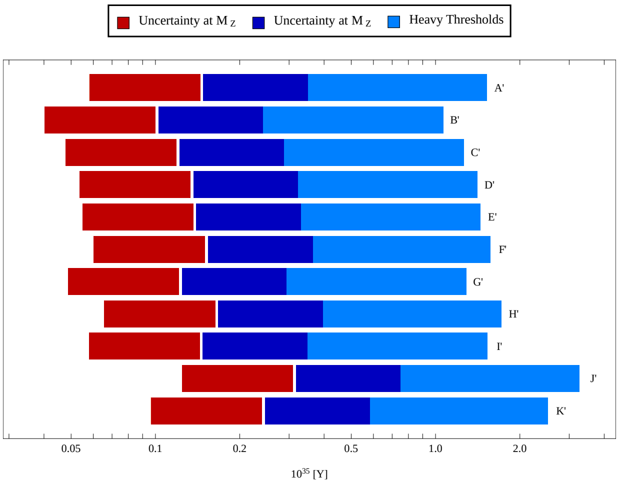


Figure 6: The central value and key uncertainties of the predicted proton lifetime $\tau_p^{(e|\mu)^+\pi^0}$ are depicted for each of the benchmark scenarios (A'-K') for Flipped $SU(5)$ with MSSM field content only (upper) and adding Type II vector-like multiplets (lower), respectively. The central prediction lies at the thin white line dividing the dark red and blue bands which themselves represent the cumulative uncertainty, combined in quadrature, in the electroweak mass scale M_Z and the coupling parameters $(\alpha_{\rm em}, \sin^2\theta_W^{\overline{\rm MS}}, \alpha_s)$ measured at that scale, cf. Eq. (46). The most significant variation is by far attributed to the strong coupling. The light blue bar represents plausible action of the unknown heavy thresholds as described in Sections (4.3,5.3), measured from the central value, and acting only to slow decay in all cases. The second loop is computed once for each benchmark at the central evaluation point. The overall left to right motion of the bar sets gives some idea of the variation attributable to motion within the supersymmetric parameter space. It is a factor somewhat larger than two, comparable to the M_Z-scale uncertainties. Other potential sources of error, such as that from the hadronic matrix elements, are expected to be smaller in magnitude.

downward from its maximal value in conventional $SU(5)$, creating further separation from the fundamental regime of the reduced Planck mass $M_{\rm Pl} \equiv \sqrt{\hbar c/8\pi G_N} \simeq 2.4\times 10^{18}$ [GeV/c^2].

However, this is not the whole story, as we are free to consider that (α_1,α_5) further continue their own running upward toward a meeting at some true grand unified scale M_{51}. In the crossing of a boundary at which the gauge group enlarges (or breaks down depending on direction traversed), there may be residual group elements released which are free to perform a remixing analogous to that of $SU(2)_{\rm L}\times U(1)_{\rm Y'}$ at the electroweak scale, as elaborated in Section (1.5). Here, in crossing the scale M_{32}, the breakdown of $SU(5)$ (rank 4) into $SU(3)\times SU(2)$ ranks (2,1), releases a $U(1)$ (rank 1) symmetry, which remixes with the coupling $(\alpha_1 \equiv \alpha_{\rm Y}(M_{32}))$ of $U(1)_{\rm Y}$.

$$\frac{25}{\alpha_1} = \frac{1}{\alpha_5} + \frac{24}{\alpha_{\rm X}} \tag{92}$$

The $U(1)$ symmetry which emerges from this collision, which we will label $U(1)_{\rm X}$, and the $SU(5)$ coupling are driven upward by renormalization group equations of the familiar form. This process was depicted previously in Figure 3, where the Eq. (92) discontinuity in the $U(1)$ transition across M_{32} is clearly visible. In the spirit of Section (4.2), we will extend renormalization to the the second loop as with Eq. (73), but without the Yukawa sector. Derivatives are taken with respect to $t=\ln(\mu)$, the logarithm of the renormalization scale.

$$\frac{d\alpha_i}{dt} = \frac{b_i\alpha_i^2}{2\pi} + \frac{\alpha_i^2}{8\pi^2}\left[\sum_{j=1}^{3} B_{ij}\alpha_j\right] \tag{93}$$

The one- and two-loop β-coefficients for the the field content of each of the three Flipped $SU(5)$ scenarios under consideration are given following.

$$\mathcal{F}\text{-SU(5)}_0 \quad : \quad b = \left(\tfrac{15}{2}, -5\right)\ ; \quad B = \begin{pmatrix} \frac{33}{4} & 60 \\ \frac{5}{2} & 82 \end{pmatrix} \tag{94a}$$

$$\mathcal{F}\text{-SU(5)}_{\rm I} \quad : \quad b = (8, -2)\ ; \quad B = \begin{pmatrix} \frac{83}{10} & \frac{336}{5} \\ \frac{14}{5} & \frac{776}{5} \end{pmatrix} \tag{94b}$$

$$\mathcal{F}\text{-SU(5)}_{\rm II} \quad : \quad b = \left(\tfrac{37}{4}, -2\right)\ ; \quad B = \begin{pmatrix} \frac{457}{40} & \frac{336}{5} \\ \frac{14}{5} & \frac{776}{5} \end{pmatrix} \tag{94c}$$

Postulating again the ansatz $\alpha_i^{-1} = -(b_i t + \zeta_i)/2\pi$ to extend the one-loop indefinite solution, the undetermined function $\zeta_i(t)$ will represent the totality of the second loop contribution as before, either via the approximate closed form gauge sector solution of Eq. (77), or by direct numerical integration. Although we will not anticipate additional use of thresholds during this final span, we will allow for that generality as well via the natural analogs to Eqs. (83).

$$\Xi_{\rm X} \equiv 1 + \alpha_{\rm X}\,(\xi_{\rm X} - \zeta_{\rm X})/2\pi \tag{95a}$$

$$\Xi_5 \equiv 1 + \alpha_5\,(\xi_5 - \zeta_5)/2\pi \tag{95b}$$

The renormalization group solutions, accurate to the second loop, then mirror Eqs. (49).

$$\frac{\Xi_X}{\alpha_X} - \frac{1}{\alpha_{51}} = \frac{b_X}{2\pi} \ln \frac{M_{51}}{M_{32}} \tag{96a}$$

$$\frac{\Xi_5}{\alpha_5} - \frac{1}{\alpha_{51}} = \frac{b_5}{2\pi} \ln \frac{M_{51}}{M_{32}} \tag{96b}$$

We solve Eqs. (96) for (α_{51},M_{51}), the grand unified coupling and mass scale.

$$\alpha_{51} = \left[\frac{\Xi_5}{(1 - b_5/b_X)\,\alpha_5} + \frac{\Xi_X}{(1 - b_X/b_5)\,\alpha_X} \right]^{-1} \tag{97a}$$

$$M_{51} = M_{32} \times \exp\left\{ \frac{2\pi}{b_X - b_5} \left(\frac{\Xi_X}{\alpha_X} - \frac{\Xi_5}{\alpha_5} \right) \right\} \tag{97b}$$

Scenario	ζ_X	ζ_5	Ξ_X	Ξ_5	α_1	α_5	α_X	M_{32}	α_{51}	M_{51}
$\mathcal{F}$-SU(5)$_0$	0.13	0.16	0.999	0.999	0.039	0.041	0.039	0.58	0.040	1.04
$\mathcal{F}$-SU(5)$_{\text{I}}$	6.01	13.1	0.958	0.758	0.045	0.116	0.044	0.68	0.104	9470
$\mathcal{F}$-SU(5)$_{\text{II}}$	3.29	6.86	0.969	0.873	0.061	0.116	0.060	0.68	0.110	88.1

Table 12: Principal results for the grand unification of Flipped $SU(5) \times U(1)_X$, with and without heavy vector multiplets. Mass is in units of [10^{16} GeV].

The collected results for our study of the grand unification of Flipped $SU(5) \times U(1)_X$ are presented in Table (12). The ζ_i account for the effects of the second loop. Although our calculations are numerical, we again find excellent agreement with the closed form approximation, valid here to about 10%. The second loop is found here to be important, but not decisive. Without the ζ_i, M_{51} would be reduced by about a quarter for model II, and by about a half for model I. These factors are assembled according to Eqs. 95 to form the Ξ_i, in terms of which our solutions to the renormalization group equations are conveniently expressed. We have not considered the possibility of crossing any mass thresholds in the transit from M_{32} to M_{51}, and the ξ_i are thus zero. We provide the prior solutions for (α_1,α_5,M_{32}) for reference, and compute the $U(1)_X$ coupling according to Eq. (92). Finally, we tabulate the grand unified coupling and mass from Eqs. (97).

It is of great interest that string theory generically predicts that any lower energy gauge structures emerge from their 'Super-Unification' at a string scale M_{str} which is significantly depressed from the Planck mass. In weakly coupled heterotic theory, $M_{\text{str}} \simeq 5 \times 10^{17}$ [GeV] is typical. The presence of some (slightly) enlarged extra dimension [72] might indeed be capable of bringing the Super-Unification well down into the $10^{(15-16)}$ [GeV] range, although we favor extreme moderation in the application of this scenario. The broader point is that the assent of the string to make some extension in our direction, encourages our prospects for likewise 'reaching up' to make the identifications[34]:

$$\alpha_{51} \Leftrightarrow \alpha_{\text{str}} \;\; ; \;\; M_{51} \Leftrightarrow M_{\text{str}} \tag{98}$$

[34]Moreover, in the context of heterotic string model building, the string mass and coupling are deeply entangled: $M_{\text{str}} \propto \sqrt{\alpha_{\text{str}}} M_{\text{str}}$.

Examining M_{51} from Table (12), Flipped $SU(5)$ in the bare MSSM shows little promise of realizing this merger. The problem here is the overly close proximity of the couplings (α_1,α_5), so that no 'breathing room' is reserved to allow for a significant second phase of the running. Model Type I with vector multiplets suffers from an opposite problem, catastrophically overshooting the string scale. This problem may be remedied by judicious application of heavy threshold corrections near the M_{32} scale. Model Type II, on the other hand, represents a most appealing result, demonstrated graphically in Figure (7), striking at about 9×10^{17} [GeV], less than a factor of three shy of the reduced Planck mass.

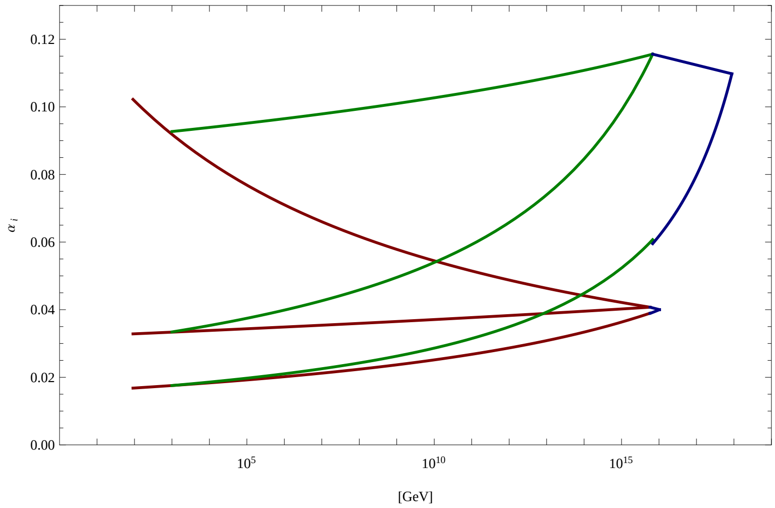


Figure 7: In this plot we follow the Flipped $SU(5) \times U(1)_X$ Type II model (solid green) further upward to a possible super unification (solid blue) just shy of 10^{18} [GeV]. This is nearly two orders of magnitude heavier than the corresponding extension of the Type 0 model (solid red) with MSSM content only. The running is computed to two loops.

Since the grand unification mass M_{51} from Eq. (97b) is directly proportional to the partial unification M_{32}, we expect that the multiplicative transfer described in Eq. (90a) from the presence of heavy thresholds should be carried over directly to the upper mass scale[35], neglecting corrections to the second loop as manifest in the Ξ_i. Ideally, with the baseline prediction already approaching the upper mass boundary, full exhaustion of the heavy thresholds allocated in Section(5.3) would be precluded by 'downward pressure' from the higher scale. However, the allowed rescaling of M_{51} by a factor of around 1.7 does not here present any definite incompatibility with unification around M_{Pl}. Nonetheless, the tendency discussed for M_{str} to itself significantly descend from M_{Pl} suggests that the mechanism is in principle viable, although it is difficult to be more concrete in this case. Reminding ourselves that the heavy thresholds never threatened to lower M_{32} the result could be a significantly tighter constraint from both directions on unification, and in particular the proton lifetime, than we might otherwise have imagined or feared.

It is typical, especially of string derived models, that the constant b_X be conspicuously large, causing α_X to climb much more rapidly than did the hypercharge. This quite general result [73–

[35]Variation in the coupling α_{51} of Eq. (97a) is both relatively more sedate, and of less practical consequence.

75], exists because negative β-function contributions, as would be required to help cancel the large number of positively contributing loops arise, only from non-Abelian self-interactions, of which the group $U(1)$ has, by definition, none. Driven by the large value of b_{X}, the rapid ascent of α_{X} runs the danger of squandering an initial coupling separation, rejoining prematurely with α_5.

It has been natural in the context of Flipped $SU(5)$ to expect that the split couplings continue upward to a true unification. It is in fact only in this way that we can hope to recover the beneficial GUT properties such as a correlated charge quantization and successful prediction[36] of m_b/m_τ. Such memories of a simpler past are naturally inherited if $SU(5) \times U(1)_{\text{X}}$ is descendant[37] from a structure such as $SO(10)$. That the string is willing to provide suitable representations for this purpose in vicinity of the necessarily escalated GUT scale is quite a strong enticement for the union of these pictures.

Being that we are now well approaching energy scales thought to be associated with the onset of large quantum gravitational effects, it is natural to inquire what the superstring may have to say, if anything, on the topic of proton decay. The possibility of modifications to the relevant decay amplitudes by the presence of compact extra dimensions is addressed in Ref. [76]. Interestingly, certain studies [77, 78] suggest overall corrections of order one. In other words, it may likely be correct, as has been our perspective in this report, to view proton decay as a pure particle physics topic. We mention also with interest the suggestion [25, 79] that there may be a logarithmic enhancement to the dimension six amplitude associated with the breaking of GUT groups in F-theoretic constructions.

We see a number of tantalizing conspiracies at work in the models under consideration, particularly model Type II. The dramatic elevation of the GUT coupling, in itself already speeding proton decay, is just right to allow space for a second stage running of some significance. The more traditional mild flipping ratios $M_{32}/M_{32}^{\text{max}}$ in the neighborhood of 50% seem by comparison to be almost 'not worth the effort'. In the TeV-scale vector multiplet models however, we see a dynamically emergent rationale for the closely adjacent yet stably disparate GUT and Planck scales. Any GUT which cannot cast light on the origin and sustenance of this hierarchy seems in comparison phenomenologically pale. In fact, if we stipulate the existence of an upper Super Unification as something akin to 'experimental constraint', the possibility emerges of selecting between string derived models by merit of the supplemental β-functions which they carry, and their effects on the string mass scale and coupling. The loss of grand unification at 10^{16} [GeV] seems a light price for Flipped $SU(5)$ to pay, as we are anyways waiting for the union with gravity itself, already on its doorstep.

6.2. Summary and Conclusion

Proton decay is one of the most unique yet ubiquitous predictions of Grand Unification. Its study draws upon and bears timely relevance to a demonstrably broad palette of topics.

We have considered the proton decay process $p \to (e|\mu)^{+}\pi^{0}$ via dimension six operators for heavy gauge boson exchange. Including uncertainties for light and heavy thresholds, we conclude that a majority of the parameter space for proton decay is indeed within the reach of

[36] Although divergence from this relation for the lighter generations is sometimes mentioned as a failure of Grand Unification, we take the view that competing O(MeV) corrections must in fact be expected to wash out masses in that same order.

[37] It is of great interest that the **16** spinor of $S0(10)$ breaks to the quantum numbers of only the flipped variety $SU(5)$, while the $(\mathbf{5}, \bar{\mathbf{5}})$ of SM Higgs with conventional charges readily emerge from a fundamental **10**.

proposed next-generation experiments such as Hyper-Kamiokande and DUSEL for the Type I and Type II extensions to Flipped $SU(5) \times U(1)_X$. The minimal Flipped $SU(5) \times U(1)_X$ model is also testable if the heavy threshold corrections are small. In particular, detectability of TeV scale vector supermultiplets at the LHC presents an opportunity for cross correlation of results between the most exciting particle physics experiments of the coming decade.

We have significantly upgraded the analysis of gauge coupling unification [29, 30], correcting a subtle inconsistency in usage of the effective Weinberg angle, improving resolution of the light threshold corrections, and undertaking a proprietary determination of the second loop, starting fresh from the standard RGEs, *cf.* [49]. The step-wise entrance of the top quark and supersymmetric particles into the RGE running is now properly accounted to all three gauge couplings individually rather than to a single composite term for the effective shift. In addition to the light M_Z-scale threshold corrections from the superpartner's entry into the RGEs, there may also be shifts occurring near the M_{23} unification point due to heavy Higgs fields and the broken gauge generators of $SU(5)$. The light fields carry strong correlations to cosmology and low energy phenomenology, so that we are guided toward plausible estimates of their mass distribution.

The two-loop contribution has likewise been individually numerically determined for each gauge coupling, including the top and bottom quark Yukawa couplings from the third generation, taken themselves in the first loop. All three gauge couplings are integrated recursively with the second loop into the Yukawa coupling renormalization, with the boundary conditions at M_Z treated correctly for various values of $\tan\beta$. The light threshold correction terms, defaulting to a phenomenologically favored benchmark point B' [44, 45] of the constrained MSSM spectrum space, are included wherever the gauge couplings α_i are used. Recognizing that the second loop itself influences the upper limit M_{32} of its own integrated contribution, this feedback is accounted for in the dynamic calculation of the unification scale. We have also provided a closed form solution which is highly consistent with our numerical results, allowing for transparent 'low tech' precision analysis.

We have provided a comprehensive dictionary of solutions for the relevant unification parameters of both Flipped and Standard $SU(5)$, allowing for generic β-function coefficients, light and heavy threshold factors, and corrections from the second loop. We find that the conjunction of this detailed $\mathcal{F}$-resh analysis with the context of $\mathcal{F}$-theory constructions of $\mathcal{F}$-lipped $SU(5)$ GUTs to result in comparatively $\mathcal{F}$-ast proton decay. Large portions of the lifetime prediction only narrowly evade existing detection limits, and a vast majority of the most plausible range is within the scope of the next generation detectors. Moreover, inclusion of TeV scale vector multiplets in the renormalization significantly magnifies the separation of the flipped scale M_{32}, at which $SU(5)$ breaks and proton decay is established, from M_{51}, the point of $\mathcal{F}$-inal Grand Unification, which can be extended to the order of the reduced Planck mass.

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Appendix A. Brief Review of del Pezzo Surfaces

The del Pezzo surfaces dP_n, where $n = 1, 2, ..., 8$, are defined by blowing up n generic points of $\mathbb{P}^1 \times \mathbb{P}^1$ or $\mathbb{P}^2$. The homological group $H_2(dP_n, Z)$ has the generators

$$H,\ E_1,\ E_2,\ ...,\ E_n\ , \tag{A.1}$$

where H is the hyperplane class for P^2, and E_i, the exceptional divisors at the divergent points, are isomorphic to $\mathbb{P}^1$. The intersection numbers of the generators are:

$$H \cdot H = 1 \quad ; \quad E_i \cdot E_j = -\delta_{ij} \quad ; \quad H \cdot E_i = 0 \tag{A.2}$$

The canonical bundle on dP_n is given by:

$$K_{dP_n} = -c_1(dP_n) = -3H + \sum_{i=1}^{n} E_i \tag{A.3}$$

For $n \geq 3$, we can define the generators as follows, where $i = (1, 2, \ldots, n-1)$.

$$\alpha_i = E_i - E_{i+1} \quad ; \quad \alpha_n = H - E_1 - E_2 - E_3 \tag{A.4}$$

Thus, all the generators α_i are perpendicular to the canonical class K_{dP_n}, and the intersection products are equal to the negative Cartan matrix of the Lie algebra E_n, and can be considered as simple roots. The curves Σ_i in dP_n where the particles are localized must be divisors of S, and the genus for the curve Σ_i is:

$$2g_i - 2 \;=\; [\Sigma_i] \cdot ([\Sigma_i] + K_{dP_k}) \tag{A.5}$$

For a line bundle L on the surface dP_n with

$$c_1(L) = \sum_{i=1}^{n} a_i E_i\ , \tag{A.6}$$

where $a_i a_j < 0$ for some $i \neq j$, the Kähler form J_{dP_n} can be constructed as shown following [23], where $\sum_{i=1}^{k} a_i b_i = 0$ and $b_0 \gg b_i > 0$. By this construction, it is easy to see that the line bundle L solves the BPS equation $J_{dP_k} \wedge c_1(L) = 0$.

$$J_{dP_k} = b_0 H - \sum_{i=1}^{n} b_i E_i, \tag{A.7}$$

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