IC/2004/6
MIFP-03-25
hep-th/0402217

Scalar Potential and Dyonic Strings in 6D Gauged Supergravity

S. Randjbar-Daemi [1] and E. Sezgin [2]

[1] *International Center for Theoretical Physics, 34100 Trieste, Italy*

[2] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics*
Texas A&M University, College Station, TX 77843-4242

Abstract

In this paper we first give a simple parametrization of the scalar coset manifold of the only known anomaly free chiral gauged supergravity in six dimensions in the absence of linear multiplets, namely gauged minimal supergravity coupled to a tensor multiplet, $E_6 \times E_7 \times U(1)_R$ Yang-Mills multiplets and suitable number of hypermultiplets. We then construct the potential for the scalars and show that it has a unique minimum at the origin. We also construct a new BPS dyonic string solution in which $U(1)_R \times U(1)$ gauge fields, in addition to the metric, dilaton and the 2-form potential, assume nontrivial configurations in any $U(1)_R$ gauged $6D$ minimal supergravity coupled to a tensor multiplet with gauge symmetry $G \supseteq U(1)$. The solution preserves 1/4 of the $6D$ supersymmetries and can be trivially embedded in the anomaly free model, in which case the $U(1)$ activated in our solution resides in E_7.

1 Introduction

The most symmetric ground state solutions in all of the higher dimensional supergravity theories, as the low energy limit of the superstring theories or the M theory, are the flat 10 dimensional manifolds ($R^d \times flat \quad torus$) and the pp waves. Such backgrounds have 32 real supersymmetries and are not a suitable starting point for building realistic phenomenology. The Calabi-Yau compactifications have less supersymmetries but also a lesser degree of uniqueness. There exist many of them with a lot of moduli with unknown potentials.

In the gauged minimal supergravity theories in $D = 6$ this is not the case. In fact in these theories the 6-dimensional flat spaces do not solve the supergravity equations. The most symmetric solution is $R^4 \times S^2$ [1] which has been shown recently to be the *unique* maximally symmetric solution of such models [2]. This result has been obtained essentially in the minimal version of such models for which the gauge group is simply $U(1)$, i.e. the $D = 6$ supersymmetric Einstein Maxwell theory, known as the Salam-Sezgin model in the literature [1]. The model by itself is anomalous but it can be embedded into an anomaly-free model [3] with suitable Yang-Mills and hypermatter sector couplings [4, 5].

The uniqueness of the supersymmetric $R^4 \times S^2$ solution should be contrasted with the plethora of solutions of Calabi-Yau type or the ones with exceptional holonomy groups such as G_2 in higher dimensional low energy string theories. For this reason we consider this property as very interesting and believe that such models deserve further study. This is the aim of the present note.

This paper contains two results. First, after giving the multiplet structure of general gauged $N = 1$ supergravity models in $D = 6$, we shall concentrate on the hypermatter sector. This sector contains the scalars and fermions which can be in anomaly free representation of the Yang-Mills gauge groups and therefore are of primary importance in any phenomenological application of such models. So far we know of only one anomaly free gauged minimal supergravity in $D = 6$ [3] in the absence of linear multiplets [1]. For this reason we shall construct in detail the scalar manifold and the potential for the scalars in this particular model. However, our simple parametrization of the quaternionic scalar manifold and the construction of the scalar potential should be applicable in other cases too. The first result of this construction is the observation that the scalar potential admits a unique minimum at the origin. There are no moduli. As we shall see this trivially implies the uniqueness of the $R^4 \times S^2$ solution in the anomaly free full fledged model.

The second main point of the paper is the construction of new dyonic string solutions. These solutions have a rather nontrivial structure and leave 1/4 of the $D = 6$ supersymmetries unbroken. The search for such solutions is motivated by the general philosophy that they can help us to study string and field theories from a non-perturbative, semiclassical point of view.

The choice of the model to be considered here is dictated by anomaly cancellation. It belongs to a general class of (1,0) supergravity models constructed some time ago [4, 5]. It is based on a six-dimensional $(1, 0)$ supergravity theory coupled to a tensor, Yang-Mills and hyper multiplets [3]. At present this is the only known gauged $(1, 0)$ anomaly free supergravity in $D = 6$ in the absence of linear multiplets. Its string or M-theory origin is still not quite well understood,

[1]The linear multiplet is a hypermultiplet in which one of the four scalars is dualized to a 4-form potential. In this case, an additional Green-Schwarz counterterm is possible for anomaly cancellation [6], and this may lead to new anomaly free models.

although some progress has been made in connecting a subset of the model to M-theory in 11-dimensions [7], and Horava-Witten type construction in 7-dimensions [8]. Note, however, that in the latter case the R-symmetry group is not gauged in the resulting $D = 6$ supergravity.

Recently, our model has attracted interest in connection with a possibility of obtaining a small cosmological constant in $D = 4$ [9]. These ideas make use of an extension of the old magnetic monopole solution [10] to a situation in which 3-branes are distributed over a transverse S^2 [11, 12]. In order for the mechanism to work it is required that the supersymmetry breaking on the brane does not propagate to the bulk. This is the weak point of the scheme for a finite volume 2-manifold, as has been argued in a very general context in [13]. We hope that some other variations of the idea will make a dent on this very important unsolved problem.

All the supersymmetric solutions of our model known so far, whether compactifying solutions with a direct product geometry or more involved stringy solutions, use only the $U(1)_R$ gauge fields as part of their ansatz. It seems quite difficult to excite the gauge fields in the non-Abelian $E_6 \times E_7$ component of the gauge group in search of a supersymmetric solution. In this paper we report on one such solution in which the gauge field in a $U(1)$ subgroup of E_7, in addition to that of the $U(1)_R$ factor in the gauge group, is also nonzero. The configuration is quite involved and does not have a 4-dimensional Poincaré invariance. It has a natural dyonic stringy interpretation similar to the one in [14]. In fact it reduces to the solution in [14] upon setting the E_7 gauge field to zero, which in turn generalizes the the $6D$ dyonic strings preserving 1/4 supersymmetry found in [15, 16].

Thus, in our dyonic string solution, in addition to the tensor fields and the gauge field of $U(1)_R$, a $U(1)$ gauge field embedded in E_7 as well as the dilaton field are also active. The only fields which do not participate in the solution are the E_6 gauge fields and the hyperscalars.

The composition of the paper is as follows: In section 2 we give a detailed description of the model with explicit form for the potential in the hyperscalars. In this section we also give the field equations and the supersymmetry transformation rules. In section 3 we show that the absolute minimum of the scalar potential is at $\phi = 0$. This fact excludes a solution of the form $M_4 \times K_2$ with a nonzero vev for any of the gauge fields in $E_6 \times E_7$ and with any unbroken susy. In section 3 we discuss the ansatz for the dyonic string with a nonzero vev for the gauge fields in $U(1)_R \times E_7$ as well as the tensor fields. In section 4 we verify that our ansatz satisfies the field equations and leaves 1/4 of the original supersymmetries, i.e. one complex susy in $1 + 1$ dimensions, unbroken. In the limit of a vanishing E_7 field it reduces to the solution found earlier in [14] where only $U(1)_R$ field was activated. In this section we also show that our solution has a horizon at $r = 0$, while at $r = \infty$ it approaches a cone over a squashed $S^3 \times$ Minkowski$_2$. The dilaton diverges in both limits. Section 5 contains our conclusions. We give the anomaly polynomial in an Appendix correcting the misprints of [3].

2 The Model

2.1 Field Content and the Scalar Manifold

The six-dimensional gauged supergravity model we shall study involves the following $N = (1,0)$ supermultiplets [2]

$$\begin{array}{llll} \text{graviton} & e_M^r & \psi_{M+}^A & B_{MN}^- \\ \text{tensor(dilaton)} & \varphi & \chi_-^A & B_{MN}^+ \\ \text{hypermatter} & \phi^\alpha & \psi_-^a & \\ \text{YangMills} & A_M & \lambda_+^A & \end{array} \tag{2.1}$$

where coordinate basis and tangent space indices are denoted by $M, N, ...$ and $r, s, ...,$ respectively. The antisymmetric tensor potentials, $B_{MN}^{\pm}$, give rise to selfdual and anti-selfdual 3-form field strengths. All the spinors are symplectic Majorana-Weyl, $A = 1,2$ label the doublet of the R symmetry group $Sp(1)_R$ and $a = 1, ..., 912$ labels the 912 dimensional pseudoreal representation of E_7. The chiralities of the fermions are denoted by $\pm$.

The hyperscalars ϕ^α, $\alpha = 1, ..., 912 \times 2$ parameterize the quaternionic Kahler coset

$$\frac{Sp(456,1)}{Sp(456) \times Sp(1)_R} \tag{2.2}$$

The global $Sp(456,1)$ symmetry is broken by the gauging of its compact $E_7 \times U(1)$ subgroup. The composite local $Sp(456) \times Sp(1)_R$ symmetry is left intact by this gauging. Thus, together with the "external" E_6 gauge symmetry, the full symmetry of the model is

$$[E_7 \times E_6 \times U(1)]_{\text{local}} \times [Sp(456) \times Sp(1)]_{\text{composite local}} \tag{2.3}$$

Note that E_7 in $Sp(456)$ is such that the 912 of $Sp(456)$ is irreducible under E_7. This spectrum is anomaly free [3] and so far is the only known anomaly free gauged supergravity model in $D = 6$ in the absence of linear multiplets.

It will prove useful to present the model of [3] in alternative forms. To this end, we need to introduce some notation and outline the building blocks. To begin with, we define the vielbein, $Sp(n)$ and $Sp(1)$ composite connections on the coset via the Maurer-Cartan form as

$$(L^{-1}\partial_\alpha L)^{aA} = V_\alpha^{aA} , \qquad (L^{-1}\partial_\alpha L)^{ab} = A_\alpha^{ab} , \qquad (L^{-1}\partial_\alpha L)^{AB} = A_\alpha^{AB} . \tag{2.4}$$

The vielbeins obey the following relations

$$g_{\alpha\beta} V_{aA}^\alpha V_{bB}^\beta = \Omega_{ab}\epsilon_{AB} , \qquad V_{aA}^\alpha V^{\beta aB} + \alpha \leftrightarrow \beta = g^{\alpha\beta}\delta_A^B , \tag{2.5}$$

[2]There also exists a linear multiplet consisting of a 4-form potential, a symplectic Majorana-Weyl spinor and three real scalars but it is not coupled in the model we study here.

where $g_{\alpha\beta}$ is the metric on the coset and Ω and ϵ are the symplectic invariant antisymmetric matrices. Next, we define the components of the $E_7 \times U(1)_R$ gauged Maurer-Cartan form as

$$(L^{-1}DL)^{aA} = P^{aA} \ , \qquad (L^{-1}DL)^{ab} = Q^{ab} \ , \qquad (L^{-1}DL)^{AB} = Q^{AB} \ , \tag{2.6}$$

where

$$DL = \left(d - g_1 A^3 T^3 - g_7 A^I T^I\right) L \ . \tag{2.7}$$

Here A^3_M and A^I_M $(I = 1, ..., 133)$ are the $U(1)_R \times E_7$ gauge fields and g_1 and g_7 are the corresponding gauge coupling constants. The following relations hold

$$P^{aA} = (\mathcal{D}\phi^\alpha) V^{aA}_\alpha \ , \qquad Q^{ab} = (\mathcal{D}\phi^\alpha) A^{ab}_\alpha \ , \qquad Q^{AB} = (\mathcal{D}\phi^\alpha) A^{AB}_\alpha - g_1 A^3 (T^3)^{AB} \ , \tag{2.8}$$

The covariant derivative can be written as

$$\mathcal{D}_M \phi^\alpha = \partial_M \phi^\alpha - g_1 A^3_M K^{3\alpha} - g_7 A^I_M K^{I\alpha} \ , \tag{2.9}$$

where $K^{3\alpha}$ and $K^{I\alpha}$ are the Killing vectors associated with $E_7 \times U(1)_R \subset Sp(456, 1)$ isometry.

Other building blocks to define the model are certain C-functions on the coset (2.2). These were defined in [5], and studied further in [17] where it was shown that they can be expressed as

$$\begin{aligned} C^3_{AB} &= \left(L^{-1}T^3 L\right)_{AB} \ , \qquad C^I_{AB} \left(L^{-1}T^I L\right)_{AB} \ , \\ C^{aA}_3 &= \left(L^{-1}T^3 L\right)^{aA} \ , \qquad C^{aA}_I = \left(L^{-1}T^I L\right)^{aA} \ , \end{aligned} \tag{2.10}$$

where T^3 and T^I are the anti-hermitian generators of $U(1)_R$ and E_7.

2.2 The Choice of L

From the foregoing description it is clear that the main ingredient in this construction is the section L, which maps the coset G/H to the group manifold G. We begin thus with a brief description of the groups involved here. Firstly, by $Sp(n, 1)$ what is really meant is the group of pseudounitary (2n+2)-dimensional matrices

$$Sp(n+1) \cap SU(2n, 2) \tag{2.11}$$

It is convenient to represent these matrices by $(n+1)$-dimensional arrays whose elements are 2-dimensional matrices,

$$\begin{aligned} g &= (g_\mu{}^\nu) \qquad \mu, \nu = 0, 1, \ldots n \\ &= (g_{\mu A}{}^{\nu B}) \quad A, B = 1, 2 \end{aligned} \tag{2.12}$$

With this notation we have two kinds of metric:

$$\begin{aligned} J &= diag(\sigma_2, \ldots, \sigma_2, -\sigma_2) \\ \eta &= diag(1_2, \ldots, 1_2, -1_2) \end{aligned} \tag{2.13}$$

where σ_2 and 1_2 denote the matrices

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad 1_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The group elements are required to satisfy two conditions

$$\begin{aligned} g^t J g &= J \quad (symplectic) \\ g^\dagger \eta g &= \eta \quad (pseudounitary) \end{aligned} \tag{2.14}$$

where g^t and $g^\dagger$ denote transpose and hermitian conjugate—in the (2n+2)-dimensional sense—respectively. It is straightforward now to show that each 2×2 element in the (n+1)-dimensional array satisfies the reality condition,

$$g_\mu{}^\nu = \sigma_2 g_\mu{}^{\nu\dagger t} \sigma_2 \tag{2.15}$$

It can be interpreted as a real quaternion.

Having defined the group $USp(n,1)$ we may restrict to its maximal compact subgroup, $USp(n)\times USp(1)$ by choosing

$$g_0{}^\mu = g_\mu{}^0 = 0, \ \mu = 1, \ldots, n$$

The matrices $g_0{}^0$ belong to SU(2), i.e.

$$USp(1) = Sp(1) \cap SU(2) = SU(2)$$

To coordinatize the manifold, consider the 'boost',

$$L_\phi = \begin{pmatrix} a + b\phi\phi^\dagger & \phi \\ \phi^\dagger & c \end{pmatrix} \tag{2.16}$$

where ϕ is a $2n \times 2$ matrix, $\phi^\dagger$ is its hermitian conjugate, a, b are real and proportional to the $2n \times 2n$ identity matrix, and c is real and proportional to the 2×2 identity matrix. We can write

$$\phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$$

where the elements of this column are 2×2 matrices satisfying the reality condition given above (and repeated below). It follows that L_ϕ belongs to the group $USp(n,1)$ provided it is also unitary,

$$L_\phi^\dagger \eta L_\phi = \eta$$

This is achieved by choosing

$$
\begin{aligned}
a &= 1 \\
b &= (-1+\sqrt{1+\phi^\dagger\phi})/\phi^\dagger\phi \\
c &= \sqrt{1+\phi^\dagger\phi}
\end{aligned}
\tag{2.17}
$$

In obtaining this result we have used the fact that the 2×2 matrix $\phi^\dagger\phi$ is proportional to the identity 1_2.

In our problem $n = 456$ and thus the scalars have 912×2 complex components. As real quaternions they can be considered as a 456×1 matrix whose elements, ϕ_m, are 2×2 matrices subject to the reality condition

$$
\phi_m^* = \sigma_2\, \phi_m\, \sigma_2, \quad m = 1, \ldots, 456 \tag{2.18}
$$

where σ_2 is the Pauli matrix, then we have a total of $456 \times 4 = 1824$ real components. Thus, the following two notations are equivalent:

$$
\phi^{aA} \quad \leftrightarrow \quad (\phi_m)_{A'}{}^{A} \tag{2.19}
$$

where $a = 1, ..., 912$ and $A, A' = 1, 2$. Using the particular form of the coset representative described above, the following C-functions take a particularly simple form

$$
C^3_{AB} = (1+|\phi|^2)\,(T^3)_{AB}\ , \qquad C^I_{AB} = (\phi^\dagger T^I \phi)_{AB}\ , \tag{2.20}
$$

where $|\phi|^2 \equiv \mathrm{tr}\,\phi^\dagger\phi$. These are the only components we need in order to construct the scalar potential.

2.3 Field Equations and Supersymmetry Transformation Rules

The Lagrangian for the anomaly free model we are studying can be obtained from [4] or [5]. We shall use the latter in the absence of Lorentz Chern-Simons terms and Green-Schwarz anomaly counterterms. Thus, the bosonic sector of the Lagrangian is given by [5]

$$
\begin{aligned}
\mathcal{L} \;=\; & R \ast\mathbb{1} - \tfrac{1}{4}\ast d\varphi\wedge d\varphi - \tfrac{1}{2}e^{\varphi}\ast H\wedge H - \tfrac{1}{2}e^{\frac{1}{2}\varphi}\,\mathrm{tr}\,(\ast F_\wedge F) \\
& - \tfrac{1}{2}\ast\mathcal{D}\phi^\alpha\wedge\mathcal{D}\phi^\beta\, g_{\alpha\beta} - 4\,e^{-\frac{1}{2}\varphi}\,(\mathrm{tr}\, C^2)\ast\mathbb{1}\ ,
\end{aligned}
\tag{2.21}
$$

where

$$
\begin{aligned}
dH &= \tfrac{1}{2}\mathrm{tr}\, F\wedge F \\
\mathrm{tr}\,(\ast F_\wedge F) &\equiv \ast F^3\wedge F^3 + \ast F^I\wedge F^I + \ast F^{I'}\wedge F^{I'}\ , \quad I = 1, ..., 133, \quad I' = 1, 2, ..., 78\ , \\
\mathrm{tr}\, C^2 &\equiv g_1^2 C^3_{AB} C^{3,AB} + g_7^2 C^I_{AB} C^{I,AB}\ .
\end{aligned}
\tag{2.22}
$$

We have let $A_\mu \to A_\mu/\sqrt{v}$ and $g \to g\sqrt{v}$ in the results of [5] to absorb the factors v_1, v_6, v_7 defined in the appendix, so that that the normalizations of the Yang-Mills kinetic terms and the Chern-Simons terms in H are the same as those in [4]. The bosonic field equation following from the above Lagrangian are [4, 5]

$$
\begin{aligned}
R_{MN} &= \tfrac{1}{4}\partial_M\varphi\,\partial_N\varphi + \tfrac{1}{2}e^{\frac{1}{2}\varphi}\,\mathrm{tr}\,(F^2_{MN} - \tfrac{1}{8}F^2\,g_{MN}) + \tfrac{1}{4}e^{\varphi}\,(H^2_{MN} - \tfrac{1}{6}H^2\,g_{MN}) \\
&\quad + \tfrac{1}{2}P^{aA}_M P_{NaA} + e^{-\frac{1}{2}\varphi}(\mathrm{tr}\,C^2)\;g_{MN}\;, \\
\Box\,\varphi &= \tfrac{1}{4}e^{\frac{1}{2}\varphi}\,\mathrm{tr}\,F^2 + \tfrac{1}{6}e^{\varphi}\,H^2 - 4\,e^{-\frac{1}{2}\varphi}\,\mathrm{tr}\,C^2 \\
d(e^{\frac{1}{2}\varphi} * F^3) &= e^{\varphi} *H \wedge F^3 - g_1 * P^{aA}C^3_{aA}\;, \\
d(e^{\frac{1}{2}\varphi} * F^I) &= e^{\varphi} *H \wedge F^I - g_7 * P^{aA}C^I_{aA}\;, \\
d(e^{\frac{1}{2}\varphi} * F^{I'}) &= e^{\varphi} *H \wedge F^{I'}\;, \\
d\,(e^{\varphi} * H) &= 0\;, \\
\mathcal{D}_M P^{MaA} &= 2e^{\varphi}\left(g_1^2 C_3^{AB}(C_3)^a{}_B + g_7^2 C_I^{AB}(C_I)^a{}_B\right)\;.
\end{aligned}
\tag{2.23}
$$

The local supersymmetry transformations of the fermions, up to cubic fermion terms that will not effect our results for the Killing spinors, are given by [5]

$$
\begin{aligned}
\delta\psi_M &= \mathcal{D}_M\varepsilon + \tfrac{1}{48}e^{\frac{1}{2}\varphi}H^+_{NPQ}\Gamma^{NPQ}\Gamma_M\,\varepsilon\;, \\
\delta\chi &= \tfrac{1}{4}\left(\Gamma^M\partial_M\varphi - \tfrac{1}{6}e^{\frac{1}{2}\varphi}H^-_{MNP}\Gamma^{MNP}\right)\varepsilon\;, \\
\delta\lambda^3_A &= -\tfrac{1}{8}F^3_{MN}\Gamma^{MN}\varepsilon_A - g_1 e^{-\frac{1}{2}\varphi}C^3_{AB}\;\varepsilon^B\;, \\
\delta\lambda^I_A &= -\tfrac{1}{8}F^I_{MN}\Gamma^{MN}\varepsilon_A - g_7 e^{-\frac{1}{2}\varphi}C^I_{AB}\;\varepsilon^B\;, \\
\delta\lambda^{I'}_A &= -\tfrac{1}{8}F^{I'}_{MN}\Gamma^{MN}\varepsilon_A\;, \\
\delta\psi^a &= P^{aA}_M\Gamma^M\varepsilon_A\;,
\end{aligned}
\tag{2.24}
$$

where $\mathcal{D}_M\varepsilon_A = \partial_M\varepsilon_A + \frac{1}{4}\omega_{Mrs}\Gamma^{rs}\varepsilon_A + Q_{MA}{}^B\varepsilon_B$. In addition to the constant re-scalings of (A_μ, g) mentioned above, we have also re-scaled $\lambda \to \lambda/\sqrt{v}$ in the results of [5]. Furthermore, the transformation rules for the gauge fermions differ from those in [4], and used in [14], by a field redefinition.

2.4 The Potential and its Minimum

It is convenient to re-write the hyperscalar field equation as

$$g_{\alpha\beta}\mathcal{D}_M\mathcal{D}^M\phi^\beta = \frac{\partial V}{\partial\phi^\alpha} \ . \tag{2.25}$$

Upon the use of (2.20), we obtain the potential

$$\begin{aligned} V(\phi) &= 4\,e^{-\frac{1}{2}\varphi}\,(\mathrm{tr}\,C^2) \\ &= e^{-\frac{1}{2}\varphi}\left[2g_1^2(1+|\phi|^2)^2 - g_7^2\mathrm{tr}\,(\phi^\dagger T^I\phi)^2\right] \ . \end{aligned} \tag{2.26}$$

Observe that since T^I are anti-hermitian, the second term is positive definite by itself, as is the first term. From the above potential, it is obvious that the absolute minimum is at $\phi^\alpha = 0$. Thus the potential has a unique minimum. There are no moduli. Note that if we could set $g_1 = 0$ there could be other nontrivial configurations which could break E_7 spontaneously. However in this particular model g_1 has to be different from zero for the anomaly cancellation. The nonvanishing of g_1 is also the basic reason why the manifold $R^4\times S^2$ is a solution. Essentially, at the minimum of the hyperscalars the exponential potential for the dilaton is given by $2g_1^2e^{-\frac{1}{2}\varphi}$. For a constant dilaton this acts like a 6-dimensional cosmological constant. When a $U(1)$ gauge field assumes a magnetic monopole configuration on S^2 we obtain the solution $R^4\times S^2$.

By examining the susy transformation rules it becomes clear that there can be no product space solution of the form $M_4\times K_2$ with a nonzero vev of any of the non-Abelian gauge fields preserving any amounts of supersymmetries.

The fact that the minimum of the hyperscalar potential is at $\phi = 0$ implies that the E_7 symmetry can not be broken spontaneously by a vev of the hyperscalars at the tree level. The only possibility of a tree level breaking of E_7 (as well as E_6) is to give a vev to the components of vector potential of these groups tangent to the internal manifold S^2. If the monopole sits in $E_6\times E_7$ factor, the configuration is generally unstable, unless the monopole charge is chosen to be the least possible value [18]. Since such configurations also break all the $D=6$ supersymmetries it follows that at the tree level $E_6\times E_7$ and susy break at the same scale.

The mass of the fluctuations of the scalar fields around the minimum at $\phi - 0$ will have two contributions for their masses. The first is the mass term coming from the potential in (2.26) and the second is the KK mass originating from the fact that the scalars are charged with respect to the $U(1)\times E_7$ gauge fields. Therefore a magnetic monopole background sitting in this group will generate a nonzero mass for all the scalars, unless the magnetic charge of the two groups cancel out mutually. If the effective magnetic coupling of a scalar field on S^2 is n, then the mass squared of the lightest Kaluza Klein mode in the expansion of ϕ will be proportional to $|n|/a^2$ where a is the radius of S^2.

On the other hand since the fermions will couple to the magnetic monopole embedded in E_7, there will be many chiral fermions in the low energy spectrum in R^4, exactly in the same manner as in [3] where the monopole in a $U(1)$ subgroup of E_6 gave rise to two families of 16 of $SO(10)$

in R^4. Several models of this type in which the Higgs scalars may originate from the extra components of the gauge field have been studied in detail in [19]

Having established that the minimum of the hyperscalar potential is at $\phi = 0$ the proof of the uniqueness of the $R^4 \times S^2$ solution should follow along the same lines as given in [2] for the Salam-Sezgin model.

3 The Ansatz, the F and H Field Equations and Supersymmetry

In this section we present the dyonic string ansatz and determine the equations that follow from the requirement of F and H field equations, and supersymmetry. Once these equations are satisfied, we show that the Einstein and dilaton field equations are automatically satisfied as well. We then proceed to solve all the required equations in section 4 where we present our dyonic string solution.

The Ansatz

Now we turn to the dyonic string ansatz. But before stating our ansatz we will briefly summarize all the known brane solutions in our model. In [2] it was shown that the most general solution of our model compatible with the Poincaré symmetry in R^4 is a 3-brane with warped metric. The brane is a δ-function singularity which can also be interpreted as a deficit angle in the 2-dimensional transverse space. This solution breaks all the supersymmetries. It reduces to 1/2 supersymmetric solution when the deficit angle vanishes. In [14] solutions of the type $AdS_3 \times S_3$ as well dyonic string solution have been studied. It has also been shown that the $AdS_3 \times S^3$ solution goes over to the maximally symmetric $R^4 \times S^2$ configuration.

Our solution will be a generalization of the dyonic string of [14], in which in addition to the $U(1)_R$ gauge field a $U(1)$ component in E_7 will also be nonzero, in a nontrivial way. We thus start from the following ansatz:

$$
\begin{aligned}
ds_6^2 &= c^2\, dx^\mu\, dx_\mu + a^2\,(\sigma_1^2+\sigma_2^2) + b^2\,\sigma_3^2 + h^2\, dr^2 \ , \\
H &= P\,\sigma_1\wedge\sigma_2\wedge\sigma_3 + u\, d^2x\wedge dr \ , \\
F^3 &= k\,\sigma_1\wedge\sigma_2 , \\
F^7 &= v\,\sigma_1\wedge\sigma_2 + v'\,\sigma_3\wedge dr \ ,
\end{aligned}
\tag{3.1}
$$

where $d^2x = dx^\mu \wedge dx^\nu \epsilon_{\mu\nu}$, k is a constant, a, b, c, h, u, v, P and φ are functions of r, and the σ_i are left-invariant 1-forms on the 3-sphere, satisfying the exterior algebra

$$
d\sigma_i = -\tfrac{1}{2}\epsilon_{ijk}\,\sigma_j\wedge\sigma_k \ . \tag{3.2}
$$

They can be represented, in terms of Euler angles (θ, φ, ψ), by

$$
\sigma_1 + \mathrm{i}\,\sigma_2 = e^{-\mathrm{i}\,\psi}\,(d\theta + \mathrm{i}\,\sin\theta\, d\varphi)\,, \qquad \sigma_3 = d\psi + \cos\theta\, d\varphi \ . \tag{3.3}
$$

The function h which may be removed by a coordinate transformation, $dr' = h(r)\,dr$, will be chosen later to simplify the solution. Locally, we can choose the potential for F^3 to be given by $A^3 = -k\,\sigma_3$, and for F^7 by $A^7 = -v\,\sigma_3$. It is also useful to record

$$\begin{aligned} *F^3 &= \frac{khbc^2}{a^2}\,\sigma_3\wedge dr\wedge d^2x\ , \\ *F^7 &= c^2\left(\frac{v'a^2}{hb}\,\sigma_1\wedge\sigma_2+\frac{vhb}{a^2}\,\sigma_3\wedge dr\right)\wedge d^2x\ , \\ *H &= -\frac{uba^2}{hc^2}\,\sigma_1\wedge\sigma_2\wedge\sigma_3-\frac{Phc^2}{.ba^2}\,d^2x\wedge dr\ . \end{aligned} \tag{3.4}$$

The F and H Field Equations

The H-field equation and the Bianchi identity $dH = \frac{1}{2}\mathrm{tr}\,F^2$ are solved, respectively, by

$$u = -\frac{Q_0hc^2}{ba^2}\,e^{-\varphi}\ , \qquad P = P_0 - \tfrac{1}{2}v^2\ , \tag{3.5}$$

where P_0 and $-Q_0$ are integration constants. The F^3 and F^7 field equations are solved by

$$b^2 = Pe^{\frac{1}{2}\varphi}\ , \qquad v' = \frac{v_0hb}{a^2c^2}e^{-\frac{1}{2}\varphi}\ , \tag{3.6}$$

where v_0 is an integration constant.

Killing Spinor Conditions

Next, we examine the consequences of supersymmetry. Following [14], we impose the following conditions on the which break supersymmetry by a factor of four:

$$\tfrac{1}{2}\Gamma^{12}\varepsilon_A = (T^3)_A{}^B\varepsilon_B\ , \qquad \Gamma_{1234}\,\varepsilon_A = \varepsilon_A\ . \tag{3.7}$$

Thus, the conditions $\delta\lambda^3 = 0$ and $\delta\lambda^7 = 0$ give

$$a^2 = \frac{ke^{\frac{1}{2}\varphi}}{2g_1}\ , \qquad \frac{v'}{v} = \frac{hb}{a^2}\ . \tag{3.8}$$

The condition $\delta\psi^a = 0$ is trivially satisfied, while $\delta\chi = 0$ is solved by

$$\varphi' = -e^{\frac{1}{2}\varphi}\left(\frac{u}{c^2}+\frac{Ph}{ba^2}\right)\ . \tag{3.9}$$

There remains the supersymmetry transformations of the gravitini. To this end, it is useful to note that in the orthonormal frame defined by

$$e^{\tilde{0}} = c\,dt\ ,\quad e^{\tilde{1}} = c\,dx \quad e^1 = a\,\sigma_1 \quad e^2 = a\,\sigma_2 \quad e^3 = b\,\sigma_3\ ,\quad e^4 = h\,dr\ ,$$

the non-vanishing components of the spin connection take the form

$$\begin{aligned}
&\omega_{23} = -\frac{b}{2a^2}\,e^1\ ,\quad \omega_{31} = -\frac{b}{2a^2}\,e^2\ ,\quad \omega_{12} = \left(\frac{b}{2a^2} - \frac{1}{b}\right)\,e^3\ ,\\
&\omega_{14} = \frac{a'}{a\,h}\,e^1\ ,\quad \omega_{24} = \frac{a'}{a\,h}\,e^2\ ,\quad \omega_{34} = \frac{b'}{b\,h}\,e^3\ ,\quad \omega^\mu{}_4 = \frac{c'}{c\,h}\,e^\mu\ .
\end{aligned} \tag{3.10}$$

Using these results, and taking $\varepsilon = \varepsilon(r)$, it follows from $\delta\psi_\mu = 0$, $\delta\psi_i = 0\,(i = 1, 2)$ and $\delta\psi_3 = 0$, respectively, that

$$\frac{c'}{c} = \tfrac{1}{4}e^{\frac{1}{2}\varphi}\left(\frac{u}{c^2} - \frac{Ph}{ba^2}\right)\ , \tag{3.11}$$

$$\frac{a'}{a} = \tfrac{1}{4}e^{\frac{1}{2}\varphi}\left(-\frac{u}{c^2} + \frac{Ph}{ba^2}\right) - \frac{bh}{2a^2}\ , \tag{3.12}$$

$$\frac{b'}{b} = \tfrac{1}{4}e^{\frac{1}{2}\varphi}\left(-\frac{u}{c^2} + \frac{Ph}{ba^2}\right) + \frac{bh}{2a^2} + \frac{(kg_1 - 1)h}{b}\ . \tag{3.13}$$

Finally, $\delta\psi_4 = 0$ gives

$$\varepsilon' = \tfrac{1}{8}e^{\frac{1}{2}\varphi}\left(\frac{u}{c^2} - \frac{Ph}{ba^2}\right)\ . \tag{3.14}$$

Comparing with (3.11), we learn that

$$\varepsilon(r) = c^{1/2}\,\varepsilon_0\ . \tag{3.15}$$

The Einstein and Dilaton Field Equations

Let us begin by writing the Einstein and dilaton equations given in (2.23) as

$$R_{MN} = T_{MN}\ ,\qquad \Box\,\varphi = J\ . \tag{3.16}$$

Substitution of the ansatz into these equations yields rather complicated field equations which have provided in the Appendix. Instead of solving these complicated second order field equations, it is much easier to show that they are automatically satisfied once the Killing spinor conditions, and the F and H-field equations/Bianchi identities are satisfied. To see this, let us first introduce the notation

$$\delta\psi_M = \widetilde{D}_M\varepsilon \ , \qquad \delta\chi = \Delta\varepsilon \ . \tag{3.17}$$

It is then straightforward to show that

$$\begin{aligned}
&\Gamma^N[\widetilde{D}_M, \widetilde{D}_N]\varepsilon \, (R_{MN} - T_{MN}) \; \Gamma^N\varepsilon + X_M \ , \\
&\Gamma^M[\widetilde{D}_M, \Delta]\varepsilon = (\Box\, \varphi - J)\, \varepsilon + Y \ ,
\end{aligned} \tag{3.18}$$

where X_M and Y are expressions which vanish upon the use of the F and H field equations/Bianchi identities. Therefore, the dilaton equation is evidently satisfied, and so is the Einstein equation, once we note that R_{MN} is diagonal for our ansatz, as shown in the Appendix.

4 The Dyonic String Solution

As in [14], we make the gauge choice

$$h = -\frac{2a^2bc^2}{r^3} \ . \tag{4.1}$$

Then, defining the combinations

$$\varphi_{\pm} = \varphi \pm 4\ln c \ , \tag{4.2}$$

we find from (3.9) and (3.11), with the help of (3.5),(3.6) and (3.8) that

$$\begin{aligned}
\varphi'_- &= \frac{4Q_0}{r^3} e^{-\frac{1}{2}\varphi_-} \ , \\
\varphi'_+ &= \frac{4P_0}{r^3}\left(e^{\frac{1}{2}\varphi_+} - \beta^2 e^{-\frac{1}{2}\varphi_+}\right) \ , \\
\beta &= \frac{v_0}{\sqrt{2P_0}} \ ,
\end{aligned} \tag{4.3}$$

These have solutions

$$e^{\varphi} = \beta H_1 \coth(\beta H_2) \ , \qquad c^{-4} = \frac{H_1}{\beta}\tanh(\beta H_2) \ , \tag{4.4}$$

where

$$H_1 = \widetilde{Q}_0 + \frac{Q_0}{r^2} \ , \qquad H_2 = \widetilde{P}_0 + \frac{P_0}{r^2} \ . \tag{4.5}$$

Here, we have introduced the integration constants $\tilde{Q}_0$ and $\tilde{P}_0$. Next, from (3.13), making use of (3.5), (3.6), and (3.8) we find

$$P_0 = \frac{k(1-kg_1)}{2g_1} \ . \tag{4.6}$$

Note that (3.5), (3.6), and (3.8) also yield the results

$$v = \frac{v_0 e^{-\frac{1}{2}\varphi}}{c^2} \ , \qquad P = \frac{P_0}{\cosh^2(\beta H_2)} \ . \tag{4.7}$$

The remaining quantities in the ansatz, namely, the functions (a, b, u) can now be evaluated in terms of (φ, c) via algebraic equations (3.8), (3.6), (3.5) and (4.7). The result for the ansatz can now be summarized as follows:

$$\begin{aligned}
ds_6^2 &= \sqrt{\beta H_1 \coth(\beta H_2)}\left[\frac{dx^\mu dx_\mu}{H_1} + \frac{k}{2g_1}\left(\sigma_1^2+\sigma_2^2\right) + \frac{P_0}{\cosh^2(\beta H_2)}\,\sigma_3^2 + \frac{P_0\beta^2k^2}{g_1^2\sinh^2(\beta H_2)}\frac{dr2}{r^6}\right],\\
e^\varphi &= \beta H_1 \coth(\beta H_2) \ ,\\
H &= \frac{P_0}{\cosh^2(\beta H_2)}\,\sigma_1\wedge\sigma_2\wedge\sigma_3 - d^2x\wedge dH_1^{-1} \ ,\\
F^3 &= k\,\sigma_1\wedge\sigma_2 \ ,\\
F^7 &= \sqrt{2P_0}\left(\tanh(\beta H_2)\,\sigma_1\wedge\sigma_2 + \frac{\beta}{\cosh^2(\beta H_2)}\,\sigma_3\wedge dH_2\right) \ ,
\end{aligned} \tag{4.8}$$

with (4.6) holding, and (H_1, H_2) and β are defined in (4.5) and (4.3), respectively, and (P_0, k, v_0) are constants. Furthermore, from (3.6) (3.8),(4.6) and (4.7), we learn that

$$P_0 \geq 0 \ , \qquad k \leq \frac{1}{g_1} \ . \tag{4.9}$$

In the limit of $\beta \to 0$, the $U(1) \subset E_7$ gauge field vanishes and the non-vanishing fields become

$$\begin{aligned}
ds_6^2 &= \sqrt{\frac{H_1}{H_2}}\left[\frac{1}{H_1}dx^\mu dx_\mu + \frac{k}{2g_1}\left(\sigma_1^2+\sigma_2^2\right) + P_0\,\sigma_3^2 + \frac{P_0^2}{g_1^2(H_2)^2}\frac{dr^2}{r^6}\right],\\
e^\varphi &= \frac{H_1}{H_2} \ ,\\
H &= P_0\,\sigma_1\wedge\sigma_2\wedge\sigma_3 - d^2x\wedge dH_1^{-1} \ ,\\
F3 &= k\,\sigma_1\wedge\sigma_2 \ ,
\end{aligned} \tag{4.10}$$

with (4.6) holding. This is the solution obtained in [14].

Turning to our solution (4.8), in order to study its global structure, following [14] we change to a new radial coordinate ρ related to r as

$$\widetilde{P}_0 + \frac{P_0}{r^2} = \frac{P_0^2}{\rho^4} \,, \tag{4.11}$$

Our solution then is given by (4.8) with

$$H_1 = \left(\widetilde{Q}_0 - \frac{Q_0 \widetilde{P}_0}{P_0}\right) + \frac{Q_0 P_0}{\rho^4} \,, \qquad H_2 = \frac{P_0 2}{\rho^4} \,. \tag{4.12}$$

We now observe that both our metric (4.8) as well as its $\beta \to 0$ limit given in (4.10) take the same form at spatial infinity reached by taking the $\rho \to \infty$ limit, and the resulting metric is

$$ds_6^2 = \frac{4k^2\sqrt{\widehat{Q}_0}}{g_1^2 P_0 2}\left(d\rho^2 + \frac{g_1 P_0}{8k}\,\rho^2\left(\sigma_1^2 + \sigma_2^2 + \frac{2g_1 P_0}{k}\,\sigma_3^2 + \frac{2g_1}{k\,\widehat{Q}_0}\,dx^\mu dx_\mu\right)\right) \,, \tag{4.13}$$

where $\widehat{Q}_0 \equiv \left(\widetilde{Q}_0 P_0 - Q_0 \widetilde{P}_0\right)/P_0$. This metric indeed describes a cone over the product of Minkowski$_2\times$ the squashed 3-sphere [14]. In this limit F_7 vanishes, F_3 and H are finite but the dilaton diverges as $e^\varphi \to \frac{\widehat{Q}_0}{P_0 2}\,\rho^4$.

Our metric (4.8) has a horizon at $\rho = 0$, just as its $\beta \to 0$ limit given in (4.10) does. In this limit we obtain

$$ds_6^2 = \sqrt{\frac{\beta}{Q_0 P_0}}\left[\rho^2 dx^\mu dx_\mu + \frac{kQ_0 P_0}{2g_1\rho^2}\left(\sigma_1^2 + \sigma_2^2 + \frac{2g_1 P_0}{k}\,e^{\frac{-2\beta P_0 2}{\rho^4}}\,\sigma_3^2\right) + \frac{4k^2\beta^2 P_0}{g_1^2}\,e^{\frac{-2\beta P_0^2}{\rho^4}}\,\frac{d\rho^2}{\rho^{12}}\right] \,. \tag{4.14}$$

Furthermore, while H vanishes and F_3, F_7 become constants, the dilaton diverges as $e^\varphi \to \beta Q_0 P_0/\rho^4$ in this limit. Interestingly, taking the $v_0 \to 0$ limit of this near horizon metric (4.14) does not yield the same result as first taking such a limit in the full metric (4.8) and then going to the horizon at $\rho = 0$. In the latter case, as shown in [14], one obtains a direct product of AdS_3 with squashed 3-sphere.

5 Discussion

In this paper we have given the precise form of the potential for the scalars in the hypermatter multiplet of the only known anomaly free gauged $(1,0)$ supergravity in $D = 6$ in the absence of linear multiplets. The model has the gauge group of $E_6 \times E_7 \times U(1)_R$. The hyperscalars are charged with respect to $U(1)_R$ and transform in the 912 dimensional pseudo real representation of E_7. They are singlets of E_6. We showed that the potential has a unique minimum at $\phi = 0$. Despite the fact that there is no obvious mass term in the D=6 action for the scalars their Kaluza Klein tower, on a background of $R^4 \times K_2$ will be all massive due to their $U(1) \times E_7$ charges. The hypermatter fermions on the other hand will give rise to plenty of chiral fermions in $D = 4$. Thus opening the road for a detailed phenomenological study of our model.

One interesting direction in further study of our model would be cosmological investigation along the lines of [20, 21, 22]. The quantum loop of the massive scalars in this model are the natural candidates to stabilize the radius of the compact space in a cosmological context at finite temperature. It has been shown long time ago that the effect of such loops can generate a constant radius for the internal space while the scale factor of our 3-dimensional universe expands according to the standard Friedman Robertson Walker law [23]. It is a very interesting question to seek for an accelerating universe solution. Such solution should exist according to the criteria given in [24].

In this paper we also constructed a dyonic string solution in the same model. Our solution leaves $1/4$ of the original supersymmetries, i.e. one complex supersymmetry in $1+1$ dimensions, unbroken. Furthermore a $U(1)$ component of the E_7 gauge field needs to be nonzero. In fact it assumes a rather complicated form. The solution approaches a cone as $r \to \infty$ over a squashed $S^3\times$ Minkowski$_2$, while at $r=0$ it has a horizon.

Another important question is to complete the search, initiated in [7], for a higher dimensional origin of this gauged supergravity model in $D=6$.

Acknowledgments

We are very grateful to John Strathdee for his collaboration at the early stages of this work. We thank Gary Gibbons, Rahmi Güven, Jim Liu, Hong Lu, H. Nishino, Chris Pope and A. Sagnotti for useful discussions. E.S. would like to thank the Abdus Salam International Center for Theoretical Physics for hospitality. The work of E.S. is supported in part by NSF grant PHY-0314712.

Appendix

The Anomaly Polynomial

There are few misprints in the anomaly polynomial formulae of [3] which we wish to correct here. We begin by listing the individual contributions:

$$
\begin{aligned}
P(\psi_\mu) &= \frac{5}{24}F_1^4 - \frac{19}{96}F_1^2\, trR^2 + \frac{1}{5760}\left[245 \text{ tr } R^4 - \frac{5\times 43}{4}\,(\text{tr } R^2)^2\right] , && (5.1)\\
-2P(\psi_R) &= \frac{1}{24}\ \text{Tr}_{912}\ F^4 + \frac{1}{96}\ \text{Tr}_{912}\ F^2\ \text{tr}R^2 + \frac{912}{5760}\left[\text{tr } R^4 + \frac{5}{4}\,(\text{tr}R^2)^2\right] , && (5.2)\\
-P(\chi_R) &= \frac{1}{24}F_1^4 + \frac{1}{96}F_1^2\, trR^2 + \frac{1}{5760}\left[\text{tr } R^4 + \frac{5}{4}\,(\text{tr } R^2)^2\right] , && (5.3)\\
P(\lambda_L) &= \frac{1}{24}(\ \text{Tr}_{78}\ F^4 + 6\ F_1^2\ \text{Tr}_{78}F^2 + 78F_1^4) \\
&\quad + \frac{1}{24}(\ \text{Tr}_{133}\ F^4 + 6\ F_1^2\ \text{Tr}_{133}\ F^2 + 133F_1^4) + \frac{1}{24}\ F_1^4 \\
&\quad + \frac{1}{96}\ \left[\text{Tr}_{78}\ F^2 + \text{Tr}_{133}\ F^2 + (78+133+1)F_1^2\right]\ \text{tr } R^2 \\
&\quad + \frac{(78+133+1)}{5760}\ \left[\text{tr } R^4 + \frac{5}{4}\,(\text{tr}R^2)^2\right] , && (5.4)
\end{aligned}
$$

where F_1 denotes the $U(1)_R$ field strength. Using the relations between the traces in various representations involved above and the fundamental representations, provided in [3], and adding all the contributions, we find that $P = P(\psi_\mu) + P(\psi_R) + P_(\chi_R) + P(\lambda_R)$ is given by

$$
\begin{aligned}
P \;=\; & -\frac{1}{16}\,(\text{tr}R^2)^2 + \frac{1}{24}\ \text{tr}R^2\ \text{Tr}_{27}F^2 - \frac{1}{8}\ \text{tr}R^2\ \text{Tr}_{56}F^2 + 2\ F_1^2\ trR^2 \\
& + \frac{1}{48}\ \text{Tr}_{27}\ (F^2)^2 + \ F_1^2\ Tr_{27}F^2 - \frac{3}{64}\ (\text{Tr}_{56}\ F^2)^2 \\
& + \frac{3}{4}\ F_1^2\ \text{Tr}_{56}\ F^2 + 9F_1^4\ . && (5.5)
\end{aligned}
$$

The signs of the $(\text{tr}R^2)^2$ terms in (5.2), (5.3), (5.4) and a factor of two in the coefficient of $\text{tr}R^2\ \text{Tr}_{56}F^2$ in (5.5) have been corrected relative to those in [3].

It is possible to factorize this expression and write it as

$$
\begin{aligned}
P &= -\frac{1}{16}\left(\ trR^2 + 4F_1^2 + \frac{1}{3}\ Tr_{27}F^2 + \frac{1}{2}\ Tr_{56}F^2\right)\left(\ trR^2 - 36F_1^2 -\ Tr_{27}F^2 + \frac{3}{2}\ Tr_{56}F^2\right) \\
&\equiv -X_4\tilde{X}_4\ , && (5.6)
\end{aligned}
$$

where, upon writing the traces in the adjoint representations by means of the formula given in [3], we have

$$
\begin{aligned}
X_4 &= \frac{1}{4}\Big(v_L\ trR^2 + v_1\ F_1^2 + \ v_6\ Tr_{78}F^2 + \ v_7\ Tr_{133}F^2\Big) \\
\tilde{X}_4 &= \frac{1}{4}\Big(\tilde{v}_L\ trR^2 + \ \tilde{v}_1\ F_1^2 + \ \tilde{v}_6\ Tr_{78}F^2 + \ \tilde{v}_7\ Tr_{133}F^2\Big)\ ,
\end{aligned}
\tag{5.7}
$$

where $(v_L, v_1, v_6, v_7) = (1, 4, 1/12, 1/6)$ and $(\tilde{v}_L, \tilde{v}_1, \tilde{v}_6, \tilde{v}_7) = (1, -36, -1/4, 1/2)$.

The Einstein and Dilaton Field Equations

Writing the Einstein's equation for the model as $R_{rs} = S_{rs}$, where, we recall that $r, s = \tilde{0}, \tilde{1}, 1, 2, 3, 4$ label the tangent space frame defined in (3), the non-vanishing components of R_{rs} evaluated for the ansatz (3.1) are

$$
\begin{aligned}
R_{\mu\nu} &= -\left[\frac{c'^2}{h^2\,c^2} + \frac{2a'\,c'}{a\,c\,h^2} + \frac{b'\,c'}{b\,c\,h^2} + \frac{1}{c\,h}\Big(\frac{c'}{h}\Big)'\right]\eta_{\mu\nu}\ , \\
R_{11} &= R_{22} = -\frac{2a'\,c'}{a\,c\,h^2} - \frac{a'\,b'}{a\,b\,h^2} - \frac{a'^2}{a^2\,h^2} - \frac{1}{a\,h}\Big(\frac{a'}{h}\Big)' - \frac{b^2}{2a^4} + \frac{1}{a^2}\ , \\
R_{33} &= -\frac{2b'\,c'}{a\,c\,h^2} - \frac{2a'\,b'}{a\,b\,h^2} - \frac{1}{b\,h}\Big(\frac{b'}{h}\Big)' + \frac{b^2}{2a^4}\ , \\
R_{44} &= -\frac{2}{a\,h}\Big(\frac{a'}{h}\Big)' - \frac{1}{b\,h}\Big(\frac{b'}{h}\Big)' - \frac{2}{c\,h}\Big(\frac{c'}{h}\Big)'\ ,
\end{aligned}
\tag{5.8}
$$

while the non-vanishing components of S_{rs} take the form

$$
\begin{aligned}
S_{\mu\nu} &= -\left[\frac{k^2}{8a^4}\,e^{\frac{1}{2}\varphi} + \tfrac{1}{8}\Big(\frac{v^2}{a^4} + \frac{v'^2}{h^2 b^2}\Big)e^{\frac{1}{2}\varphi} + \tfrac{1}{4}\Big(\frac{u^2}{h^2\,c^4} + \frac{P^2}{a^4\,b^2}\Big)\,e^{\varphi} - \tfrac{1}{2}g_1^2\,e^{-\frac{1}{2}\varphi}\right]\eta_{\mu\nu}\ , \\
S_{11} &= S_{22} = \frac{3k^2}{8a^4}\,e^{\frac{1}{2}\varphi} + \Big(\frac{3v^2}{8a^4} - \frac{v'^2}{8h^2 b^2}\Big)e^{\frac{1}{2}\varphi} + \tfrac{1}{4}\Big(\frac{u^2}{h^2\,c^4} + \frac{P^2}{a^4\,b^2}\Big)\,e^{\varphi} + \tfrac{1}{2}g_1^2\,e^{-\frac{1}{2}\varphi}\ , \\
S_{33} &= -\frac{k^2}{8a^4}\,e^{\frac{1}{2}\varphi} - \Big(\frac{v^2}{8a^4} - \frac{3v'^2}{8h^2 b^2}\Big)e^{\frac{1}{2}\varphi} + \tfrac{1}{4}\Big(\frac{u^2}{h^2\,c^4} + \frac{P^2}{a^4\,b^2}\Big)\,e^{\varphi} + \tfrac{1}{2}g_1^2\,e^{-\frac{1}{2}\varphi}\ , \\
S_{44} &= -\frac{k^2}{8a^4}\,e^{\frac{1}{2}\varphi} - \Big(\frac{v^2}{8a^4} - \frac{3v'^2}{8h^2 b^2}\Big)e^{\frac{1}{2}\varphi} - \tfrac{1}{4}\Big(\frac{u^2}{h^2\,c^4} + \frac{P^2}{a^4\,b^2}\Big)\,e^{\varphi} + \tfrac{1}{2}g_1^2\,e^{-\frac{1}{2}\varphi}\ .
\end{aligned}
\tag{5.9}
$$

Finally, writing the dilaton field equation as $\Box\,\varphi = J$, the evaluation of J for our the ansatz (3.1) yields

$$
J = \frac{k^2}{2a^4}\,e^{\frac{1}{2}\varphi} + \tfrac{1}{2}\Big(\frac{v^2}{a^4} + \frac{v'^2}{h^2 b^2}\Big)e^{\frac{1}{2}\varphi} + \Big(\frac{P^2}{a^4\,b^2} - \frac{u^2}{h^2\,c^4}\Big)\,e^{\varphi} - 2g_1^2\,e^{-\frac{1}{2}\varphi}\ . \tag{5.10}
$$

References

[1] A. Salam and E. Sezgin, Phys. Lett. B **147**, 47 (1984).

[2] G. W. Gibbons, R. Güven and C. N. Pope, arXiv:hep-th/0307238.

[3] S. Randjbar-Daemi, A. Salam, E. Sezgin and J. Strathdee, Phys. Lett. B **151**, 351 (1985).

[4] H. Nishino and E. Sezgin, Nucl. Phys. B **278**, 353 (1986).

[5] H. Nishino and E. Sezgin, Nucl. Phys. B **505**, 497 (1997), arXiv:hep-th/9703075.

[6] M. Berkooz, R. G. Leigh, J. Polchinski, J. H. Schwarz, N. Seiberg and E. Witten, Nucl. Phys. B **475**, 115 (1996), arXiv:hep-th/9605184.

[7] M. Cvetic, G. W. Gibbons and C. N. Pope, arXiv:hep-th/0308026.

[8] T. Gherghetta and A. Kehagias, Phys. Rev. D **68**, 065019 (2003), arXiv:hep-th/0212060.

[9] Y. Aghababaie, C. P. Burgess, S. L. Parameswaran and F. Quevedo, arXiv:hep-th/0304256.

[10] S. Randjbar-Daemi, A. Salam and J. Strathdee, Nucl. Phys. B **214**, 491 (1983).

[11] S. M. Carroll and M. M. Guica, arXiv:hep-th/0302067.

[12] I. Navarro, JCAP **0309**, 004 (2003), arXiv:hep-th/0302129. Class. Quant. Grav. **20**, 3603 (2003), arXiv:hep-th/0305014.

[13] G. Dvali, G. Gabadadze and M. Shifman, Phys. Rev. D **67**, 044020 (2003), arXiv:hep-th/0202174.

[14] R. Güven, J. T. Liu, C. N. Pope and E. Sezgin, arXiv:hep-th/0306201.

[15] M. J. Duff, S. Ferrara, R. R. Khuri and J. Rahmfeld, Phys. Lett. B **356**, 479 (1995) [arXiv:hep-th/9506057].

[16] M. J. Duff, H. Lu and C. N. Pope, Phys. Lett. B **378**, 101 (1996) [arXiv:hep-th/9603037].

[17] R. Percacci and E. Sezgin, arXiv:hep-th/9810183.

[18] S. Randjbar-Daemi, A. Salam and J. Strathdee, Phys. Lett. B **124**, 345 (1983) [Erratum-ibid. B **144**, 455 (1984)].

[19] G. R. Dvali, S. Randjbar-Daemi and R. Tabbash, Phys. Rev. D **65** (2002) 064021, arXiv:hep-ph/0102307.

[20] J. M. Cline, J. Descheneau, M. Giovannini and J. Vinet, JHEP **0306**, 048 (2003), arXiv:hep-th/0304147.

[21] T. Bringmann and M. Eriksson, JCAP **0310**, 006 (2003), arXiv:astro-ph/0308498.

[22] T. Bringmann, M. Eriksson and M. Gustafsson, Phys. Rev. D **68**, 063516 (2003), arXiv:astro-ph/0303497.

[23] S. Randjbar-Daemi, A. Salam and J. Strathdee, Phys. Lett. B **135**, 388 (1984).

[24] P. K. Townsend, arXiv:hep-th/0308149.