Massless Higher Spins and Holography

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Abstract

We treat free large N superconformal field theories as holographic duals of higher spin (HS) gauge theories expanded around AdS spacetime with radius R. The HS gauge theories contain massless and light massive AdS fields. The HS current correlators are written in a crossing symmetric form including only exchange of other HS currents. This and other arguments point to the existence of a consistent truncation to massless HS fields. A survey of massless HS theories with 32 supersymmetries in $D = 4, 5, 7$ (where the 7D results are new) is given and the corresponding composite operators are discussed. In the case of AdS_4, the cubic couplings of a minimal bosonic massless HS gauge theory are described. We examine high energy/small tension limits giving rise to massless HS fields in the Type IIB string on $AdS_5 \times S^5$ and M theory on $AdS_{4/7} \times S^{7/4}$. We discuss breaking of HS symmetries to the symmetries of ordinary supergravity, and a particularly natural Higgs mechanism in $AdS_5 \times S^5$ and $AdS_4 \times S^7$ where the HS symmetry is broken by finite $g_{\rm YM}$. In $AdS_5 \times S^5$ it is shown that the supermultiplets of the leading Regge trajectory cross over into the massless HS spectrum. We propose that $g_{\rm YM}^2 = 0$ corresponds to a critical string tension of order $1/R^2$ and a finite string coupling of order $1/N$. In $AdS_7 \times S^4$ we give a rotating membrane solution coupling to the massless HS currents, and describe these as limits of Wilson surfaces in the $A_{N-1}(2,0)$ SCFT, expandable in terms of operators with anomalous dimensions that are asymptotically small for large spin. The minimal energy configurations have semi-classical energy $E = s$ for all s and the geometry of infinitely stretched strings with energy and spin density concentrated at the endpoints.

1 Introduction

The strong form of the Maldacena conjecture states that Type IIB closed string theory on $AdS_5 \times S^5$ with N units of five-form flux and string coupling g_s corresponds to $d = 4$, $\mathcal{N} = 4$ SYM theory with $SU(N)$ gauge group and Yang-Mills coupling $g_{\rm YM}^2 = g_s$ [1, 2, 3]. This conjecture has been primarily tested for $N >> g_{\rm YM}^2 N >> 1$, where supergravity is a valid approximation [4, 5]. It is natural to study the correspondence for $N >> 1 >> g_{\rm YM}^2 N$, and possibly $g_{\rm YM}^2 = 0$, where the SYM theory becomes a theory of a free $SU(N)$ valued $d = 4$, $\mathcal{N} = 4$ vector singletons. At weak 't Hooft coupling $\lambda \equiv g_{\rm YM}^2 N << 1$ the natural gauge invariant operators are composite single-trace operators which can be arranged into 'trajectories' according to the value of the twist $E - s$, where E is the conformal dimension and s is the spin. The twist is the anomalous contribution to E, which becomes small at weak 't Hooft coupling and large N.

A basic observation [6] is the non-intersection principle in a CFT which states that as the coupling varies there cannot be any mixing between operators that are not mixing already at the free level. This applies to both the spectrum of composite operators of 4d SYM in the limit $N >> 1 >> g_{\rm YM}^2 N$ and the spectrum of vertex operators of the sigma model for $N >> g_{\rm YM}^2 N >> 1$. Thus an important test of Maldacena conjecture is to verify that the trajectories of SYM operators with constant twist cross over into the closed string Regge trajectories.

In this paper we shall show that this is indeed the case for the leading trajectories, which consist of the states with minimal E for fixed s. In fact, on the SYM side the leading trajectory, i.e. the operators with minimal twist, consists of bilinear higher spin (HS) tensors. In the free limit, these have twist 2 and the $s \geq 1$ sector coincides with the space of conserved HS currents. General aspects of these currents have been discussed in [7, 8]. The precise spectrum of twist 2 operators and the corresponding HS symmetry algebra extension of the conformal/AdS group was constructed in [9, 10] using group theoretic methods which shows that the twist 2 operators in fact form an irreducible 'gauge' multiplet of the HS algebra.

In [9, 10] it was also shown how to describe the HS gauge multiplet on the bulk side at the level of a linearized AdS field theory containing HS gauge fields as well as other interesting HS fields generalizing the self-dual two-form of the supergravity multiplet contained in the spectrum. This immediately raises the following questions; is it possible to extend this picture to an interacting theory of massless HS fields in AdS_5, and if so, is this theory the result of a consistent truncation of the full closed string theory in the limit $N >> 1 >> g_{\rm YM}^2 N$?

There is increasing evidence for the existence of an interacting 5D massless HS gauge theory [9, 10, 11, 12, 13]. This theory has been constructed at the linearized level [9, 10], and certain cubic interactions of the minimal bosonic theory have already been constructed [12]. The structures involved are natural generalizations of those in $D = 4$, and we expect that a similar development will unfold in $D = 5$.

For those readers not too familiar with massless HS gauge theories, in this paper we review some of their basic properties in dimensions of interest, namely, $D = 4, 5, 7$. The general formulation of interacting massless HS gauge theory has been known in $D = 4$ for quite some time [14] (see, [15] for a review). In testing the free CFT/HS gauge theory correspondence ideas, it is

important to exhibit the couplings of the HS gauge theory. The $D = 4$, $\mathcal{N} = 8$ theory has been examined in great detail in [16, 17]. In $D = 4$ the basic interactions are contained in a minimal bosonic model which can be embedded as a consistent truncation into HS gauge theories with $\mathcal{N} \geq 0$. The explicit couplings of the minimal bosonic model in $D = 4$ are given in a generally covariant curvature expansion scheme in [18, 19]. Here we shall summarize the results of [19] at the level of cubic couplings. The analogous bosonic truncation in $D = 5$ was given in [9] and in $D = 7$ in [20], though the full interactions still remains to be found. In this paper we also give the symmetry algebra and massless spectrum of the $D = 7$, $\mathcal{N} = 2$ HS gauge theory.

The issue of consistent truncation is crucial since the subleading trajectories in the gauge theory correspond to massive AdS fields which are light, meaning that their AdS energies are not separated from the massless ones by a mass-gap. Here it is important to note that regardless of the detailed structure of the bulk interactions, it is still possible [21] to arrange the effective bulk action into a $1/N^2$ expansion such that its extremum reproduces the $1/N^2$ expansion of the correlators of the composite operators of the $SU(N)$ invariant singleton theory. In fact, this expansion remains highly nontrivial even in the limit $g_{\text{YM}}^2 = 0$ [22, 23]. In particular, if one sets to zero all the massive fields on the boundary, then the extremum of the full effective action should reproduce the correlators of the bilinear twist 2 operators. The massive fields may still become excited in the bulk, if massless fields act as sources for massive fields. If this is the case, then the massless HS gauge theory cannot serve as a good approximation for studying these processes, not even as an effective theory since it is not possible to eliminate the light massive fields while preserving locality (the non-localities which one encounters in massless HS theory are not that bad). Thus, for the massless HS gauge theory to be relevant, it must be possible to consistently set the massive fields to zero in the full theory, at least in the leading nontrivial order in the $1/N^2$ expansion.

There are several ways to test this consistent truncation. Firstly, it requires consistent interactions among massless fields, for which there are many indications as already mentioned. Given the consistent equations of motion or action for the massless fields, one must then compute the bulk tree amplitudes, which by definition will only contain massless excitations in the internal lines, and check that they correspond to the correlators of bilinear composite operators computed in the singleton theory [19, 24]. This direct method is technically rather involved, however, and in this paper we instead provide indirect evidence for consistent truncation by examining the nature of the correlators between bilinear operators in singleton theories with large N. We also suggest that the arguments given in [25, 26, 27] for the consistent truncation of Type IIB and eleven-dimensional supergravities on $AdS_{4/7} \times S^{7/4}$ to gauged supergravity carry over to the HS context.

In this paper, we also emphasize the fact that the relations between the closed string parameters g_s and α' in $AdS_5 \times S^5$ and the SYM parameters g_{YM}^2 and N have so far been tested only in the limit $N >> g_s N >> 1$. In this regime the relations can be derived e.g. by first identifying the gauge theory parameters with the closed string parameters in flat 10D spacetime, and then use D3-brane soliton description to interpolate from flat spacetime down to $AdS_5 \times S^5$. Since only 16 supersymmetries are preserved globally by the D3 brane, there may be string corrections to this computation.

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It is important to note that the strong coupling tests of the AdS/CFT duality which are based on exact calculations on the SYM side (see, for example, [28] and references therein) are still limited on the bulk side in that they do not go beyond the leading $\sqrt{\lambda}$ approximation to the closed string theory in AdS background. As $g_{\mathrm{YM}}^2 N$ becomes small (keeping N large), we do not know the precise relations between closed string parameters in AdS and the gauge theory parameters. It is clear that the string coupling g_s decreases and the sigma model coupling α'/R^2 increases as g_{YM}^2 decreases. We shall speculate that the bulk parameters approach critical values as $g_{\mathrm{YM}}^2 = 0$ where the bulk theory is described by closed string theory with coupling $1/N^2$ and a singleton worldsheet CFT based on critical level k affine $PSU(2,2|4)$ algebra, and that the left- and right-moving singleton spin fields can be used in the construction of vertex operators describing massless HS fields in the bulk. The level k is related to the worldsheet sigma model coupling constant, i.e. $\alpha'/R^2 = l_s^2/R^2$. The corrected relations between the closed string parameters in $AdS_5 \times S^5$ and the gauge theory parameters we propose are given by

$$g_{\mathrm{s}} \;=\; f_1(\lambda)g_{\mathrm{YM}}^2 \;, \quad l_{\mathrm{s}} = f_2(\lambda)R \;, \tag{1.1}$$

$$f_1(\lambda) \;\sim\; 1 \;, \quad f_2(\lambda) \sim \lambda^{-1/4} \quad \text{for } \lambda >> 1 \;,$$

$$f_1(\lambda) \;\sim\; \frac{1}{\lambda} \;, \quad f_2(\lambda) \sim 1 \;, \quad \text{for } \lambda << 1 \;. \tag{1.2}$$

Another aspect of massless higher spins and holography which we emphasize in this paper is a Higgs mechanism by which the HS symmetries are spontaneously broken [21] down to the symmetries of ordinary supergravity. This phenomenon is best studied in the case of $AdS_5 \times S^5$, primarily due to the fact that the free boundary SYM theory can be continuously deformed by switching on the coupling constant g_{YM}. As a result, the HS currents with spin $s > 2$ will no longer be conserved. The resulting anomalies in the conservation laws for these currents are encoded in operators which can be coupled to Higgs fields which undergo their landmark shift transformations. Consequently, the Higgs mechanism mentioned above is expected to take place.

Given that the full interacting HS theory theory in 5D is still not known, of course we cannot work out the details of the Higgs mechanism here. However, we do provide kinematic framework for it which suggests that the Higgsing phenomenon takes place in an infinite number of massless $\mathcal{N} = 8$, $D = 5$ multiplets containing HS fields, which together with the supergravity multiplet make up the spectrum of the massless HS gauge theory. In particular, we focus on the Higgsing of the Konishi multiplet, which has $s_{\mathrm{max}} = 4$ and is expected to play an important role in study of the first massive Type IIB closed string level, and outline how the Higgs mechanism can be extended to all the HS multiplets. This phenomenon of anomalies in the HS current conservation laws in the boundary having a holographic dual description in the bulk as spontaneous breaking of the corresponding HS gauge symmetries is similar to a phenomenon of a chiral $U(1)$ anomaly having its gravity dual in a particular AdS_5 supergravity, as has been recently shown in [29].

Switching our discussion to the case of M theory, we first recall that the Maldacena conjecture [1, 4] states the equivalence between M theory on on $AdS_{4/7} \times S^{7/4}$ with N units of 7/4-form

flux on $S^{7/4}$, and superconformal field theories (SCFT) with 16 supercharges describing the low energy dynamics of N parallel coinciding M2/5 branes in flat eleven dimensional spacetime. Apparently, these SCFTs are isolated fixed points of the renormalization groups (RG) that do not admit any marginal deformations, with or without preservation of supersymmetry. Consequently they do not admit any coupling constants and Lagrangian descriptions. The main window for viewing these strongly coupled theories is therefore through the bulk supergravity, which is a valid approximation to M theory at fixed energies provided $N >> 1$. This corresponds to a subset of the SCFT operators with fixed conformal dimensions as $N >> 1$ [1, 4]. Recently other limits of the correspondence based on considering large internal spin have been proposed [30].

In analogy with Type IIB closed string theory on $AdS_5 \times S^5$, it is natural to ask whether M theory on $AdS_{4/7} \times S^{7/4}$ has an unbroken phase in which M theory corrections become relevant at fixed energy and the effective description of the bulk theory becomes a HS gauge theory with holographic dual given by a free SCFT in $d = 3$ or $d = 6$. In other words, we wish to examine whether it is possible to have a 'phase diagram' with two fixed points, one corresponding to the free singleton SCFT describing the unbroken HS phase and another one corresponding to the strongly coupled SCFT describing the broken phase. From the bulk point of view the broken phase is described by membranes interacting in the flat eleven dimensional center of $AdS_{4/7} \times S^{7/4}$, while the unbroken phase, which is specific to AdS, is described by membranes interacting close to the boundary of AdS.

By examining the RG flows on M2/5 branes and D2/4 branes we are led to propose that the relevant free SCFTs in $d = 3, 6$ are described by free $SU(N)$ valued $OSp(8|4)$ singletons and free $SU(N)$ valued $d = 6$, $\mathcal{N} = (2, 0)$ tensor singletons. These theories have of course figured in the literature before (see, for example, [27]), and have been used in many circumstances in order to unravel information about the strongly coupled SCFTs [1]. Our point here is that due to the salient features of the large N limit the free SCFTs make sense on their own as holographic images of the interesting unbroken phases of M theory. Technically speaking, large N implies factorization and $1/N$ expansion of correlators which can be matched with the expansion of the bulk amplitudes in terms of the fundamental Planck scale.

As in the case of the Type IIB theory, an important issue is whether there is a consistent truncation down to a massless sector. The ideas for examining this are similar to those described above for the Type IIB theory. The $D = 4$ case is particularly tractable as in this case we already know the full form of the interactions among massless HS fields, which makes it possible to test directly the consistent truncation without first having to construct the interactions.

An intriguing feature of the proposed unbroken phases of M theory on $AdS_{4/7} \times S^{7/4}$ is that the spectrum is discrete and that there is a finite coupling, $1/N$. Thus the unbroken phases of M theory appears to be on the same footing as the unbroken phase of the Type IIB theory on $AdS_5 \times S^5$. This suggests that the unbroken phases in $AdS_{4/7} \times S^{7/4}$ are theories of M2 branes with fixed tension.

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[1] Free singletons, which form $N - 1$ plets of the Weyl group of $SU(N)$, appear in various 'trivial' IR limits describing stacks of separated branes sitting at certain orbifold singularities [31]. These free singletons should not be confused with the $SU(N)$ valued singletons, though they are curious from the HS perspective and they should presumably be included into the phase diagram as separate HS phases.

To gather further evidence for this, we examine a family of rotating membrane solutions in $AdS_7 \times S^4$ that are curved space generalizations of those given in flat spacetime in [32, 33] and membrane analogs of the string solutions found recently in [34] (which in fact describe the leading Regge trajectory states). The minimal energy configurations have semi-classical energy $E = s$ for all s, and the geometry of infinitely stretched membranes of zero width, whose energy and spin densities are concentrated in the asymptotic region. By examining the supersymmetry enhancement in this region we can further show that the rotating membranes indeed couple to the bilinear HS currents in the SCFT.

There is an important difference between the membrane solitons and the string solitons given in [34]. The string solitons couple to operators whose anomalous dimensions become asymptotically small only for large s, $(E - s)/s \to 0$ as $s\alpha'/R^2 \to \infty$. The membrane solitons, on the other hand, couple to anomaly free operators for any value of s. This is because they arise by taking the limit of zero width which has the dual interpretation of shrinking a Wilson surface which means that the holographic dual flows to the free singleton SCFT in $d = 6$.

We find it rather compelling that relatively simple, free SCFTs contain information about the unbroken, and perhaps more fundamental, phases of Type IIB closed string and M theory. Moreover, this means that the results on free SCFT which are scattered over the literature can now be given a more direct physical interpretation.

In AdS/CFT correspondence, it is important that both the bulk and the boundary theories admit $1/N$ expansions which define the physically relevant, i.e. asymptotically convergent, expansions. In the unbroken HS phase, the bulk side may also admit a strongly coupled closed string/membrane sigma model description, which we propose has large, but fixed, coupling given by a critical tension, as mentioned above. In any event, consistent truncation makes it possible to directly test the AdS/CFT correspondence using only the action for the massless HS fields which does not require strongly coupled sigma model computations.

The breaking of the HS symmetries requires the inclusion of Higgs fields whose interactions require us to go beyond the consistent truncation to massless fields. Whether this can be done at the level of some effective field theoretical construction in the bulk or whether it requires extracting information from the strongly coupled sigma model is not clear at present. Here we can only speculate that the large amount of symmetry present in the unbroken phase should make the critical string and membrane sigma models amenable to exact methods.

This paper is organized as follows. In Section 2, the properties of HS gauge theories in $D = 4, 5, 7$ are reviewed, including their underlying symmetry algebras and field contents. The results for the HS superalgebra and spectrum in $7D$ are new. In Section 3, the composite singleton operators corresponding to the massless states of HS gauge theories, their KK towers and Higgs multiplets are discussed. In Section 4, important aspects of the CFT/HS gauge theory correspondence, and in particular the $1/N$ expansion in the free CFT on the boundary are described. In Section 5, the $5D$ HS gauge theory as the bulk theory arising in the critical limit of Type IIB string theory and a Higgs mechanism breaking the HS gauge symmetries down to those of ordinary supergravity are discussed. In Section 6, first the CFT_3/HS gauge theory correspondence for M theory on $AdS_4 \times S^7$ is described. Then, the minimal bosonic truncation of the theory and its cubic interactions are described. In Section 7, first the CFT_6/HS gauge theory correspondence

for M theory on $AdS_7 \times S^4$ is discussed. Then our rotating membrane solution in $AdS_7 \times S^4$ is given and its properties and relevance to the 7D HS gauge theory are described. Section 8 is devoted to a summary and discussion. In Appendix A, we present several tables which show various sectors of the massless HS gauge theory spectra in $D = 5, 7$. In Appendix B, we summarize the UIRs and BPS states of the maximal AdS superalgebras in $D = 4, 5, 7$. In Appendix C and D, we collect further group theoretical information that is useful for Section 2 and 3.

2   Massless Higher Spin Gauge Theories in $D = 4, 5, 7$

HS gauge theories are generally covariant theories which admit AdS as a vacuum and have an infinite number of local HS supersymmetries based on HS superalgebras which are infinite dimensional extension of the finite dimensional AdS superalgebras [35] . The fundamental UIRs of the HS super algebras in $D = d+1 = 4, 5, 7$ dimensions are ultra-short d-dimensional conformal supermultiplets, which we will refer to as singletons [2]. Gauging of such a HS superalgebra yields a D-dimensional theory based on a massless HS supermultiplet given by the symmetric product of two singletons. In this paper we shall focus our attention on the HS extension of the AdS superalgebras in $D = 4, 5, 7$ with 32 real supersymmetries because these are the most natural ones to explore from the string/M theory point of view. In Section 8, we shall comment on possible extensions to higher D and higher number of supersymmetries.

The massless HS multiplet is an infinite tower of massless AdS supermultiplets with supergravity at the lowest level. One key property is the fact that a HS gauge theory in $D > 3$ cannot be consistently truncated to an AdS supergravity. Basically, this is due to the fact that derivatives of lower spin fields serve as sources for HS fields, and it can also be seen from the structure of the OPE of free field theory stress-energy tensors in $d > 2$ [7]. However, in $D = 4, 5, 7$ there exist minimal bosonic truncations which have remarkably simple physical field content, namely massless fields of spin $s = 0, 2, 4, 6, ...$ described by doubly traceless, symmetric tensors $\phi_{\mu_1...\mu_s}$. The embedding of these theories in their supersymmetric extensions is explained in Tables 1, 2 and 3.

As for the full and covariant (i.e. background independent) interactions among the massless fields, they are known in the 4D theory [14, 18, 19]. A condensed account of how to extract cubic couplings in $D = 4$ will be given in Section 6.2. The fully interacting theories in $D = 5, 7$ have not yet been constructed, though the results obtained so far are promising [12, 9, 10, 20].

We next list the HS superalgebras in $D = 4, 5, 7$, their singleton and massless representations, and how the latter ones are assembled into master 1-form and master 0-form fields. The results in $D = 4, 5$ were obtained in [16, 10]. The minimal bosonic HS algebra in $7D$ was obtained in [20]. The results presented here for its supersymmetric extension are new.

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[2]In $d = 4, 6$, these are usually referred to as doubletons, due to the fact that their oscillator construction is based on two sets of oscillators as opposed to a single set of oscillators used in $d = 3$.

2.1   The $D = 4$, $\mathcal{N} = 8$ Massless HS Gauge Theory

The 4D HS algebra $hs(8|4)$ is realized in terms of oscillators obeying the following algebra [35, 16, 17] [3]

$$y_\alpha \star y_\beta = y_\alpha y_\beta + i\epsilon_{\alpha\beta} , \quad y_\alpha \star \bar{y}_{\dot{\alpha}} = y_\alpha \bar{y}_{\dot{\alpha}} , \quad (y_\alpha)^\dagger = \bar{y}_{\dot{\alpha}} , \tag{2.1}$$

$$\theta^i \star \theta^j = \theta^i \theta^j + \delta^{ij} , \quad (\theta^i)^\dagger = \theta^i , \tag{2.2}$$

where y_α ($\alpha = 1, 2$) is a Weyl spinor which is a Grassmann even generator of a Heisenberg algebra, and θ^i ($i = 1, ..., 8$) is a Grassmann odd generator of an $SO(8)$ Clifford algebra. The $\star$ denotes the associative product between oscillators. The products on the right hand sides are Weyl ordered, so that for example $y_\alpha y_\beta = y_\beta y_\alpha$ and $\theta^i \theta^j = -\theta^j \theta^i$. Using the above contraction rules it is straightforward to compute the $\star$ product between two arbitrary Weyl ordered polynomials of oscillators.

The algebra $hs(8|4)$ consists of arbitrary Grassmann even and anti-hermitian polynomials $P(y, \bar{y}, \theta)$ that are sums of monomials of degree $4\ell + 2$ where $\ell = 0, 1, 2, ...$, which will be referred to as the level index. The Lie bracket between $P, Q \in hs(8|4)$ is given by $[P, Q]_\star$. Thus, denoting by $P^{(\ell)}$ an ℓth level monomial, the commutation relations have the schematic form

$$[P^{(\ell_1)}, P^{(\ell_2)}]_\star = \sum_{|\ell_1 - \ell_2| \leq \ell \leq \ell_1 + \ell_2} P^{(\ell)} . \tag{2.3}$$

In particular, the zeroth level of $hs(8|4)$ is the maximal finite subalgebra $OSp(8|4)$ whose generators schematically take the form

$$\begin{aligned} Q_{\alpha i} &= y_\alpha \theta_i , & \bar{Q}_{\dot{\alpha}i} &= \bar{y}_{\dot{\alpha}} \theta_i , & U_{ij} &= \theta_i \theta_j , \\ M_{\alpha\dot{\beta}} &= y_\alpha \bar{y}_{\dot{\beta}} , & M_{\alpha\beta} &= y_\alpha y_\beta , & M_{\dot{\alpha}\dot{\beta}} &= \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}} . \end{aligned} \tag{2.4}$$

A generator $P^{(\ell)}$ in the ℓth level of $hs(8|4)$ can be expanded as

$$P^{(\ell)}(y, \bar{y}, \theta) = \sum_{\substack{m + n + p \\ = 4\ell + 2}} \frac{1}{m! \, n! \, p!} \, \bar{y}^{\dot{\alpha}_1} \cdots \bar{y}^{\dot{\alpha}_m} y^{\beta_1} \cdots y^{\beta_n} \theta^{i_1} \cdots \theta^{i_p} P_{\dot{\alpha}_1 ... \dot{\alpha}_m \, \beta_1 ... \beta_n \, i_1 ... i_p} . \tag{2.5}$$

The spins of the components are given by $s = \frac{1}{2}(m + n)$. The components with integer spin are Grassmann even and those with half-integer spin are Grassmann odd. Bosons are in the 1, 28

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[3]The algebra $hs(8|4)$ is called $shs^E(8|4)$ in [9] and $shs^E(8, 4|0)$ in [35], where it is also shown to be isomorphic to $ho(8; 8|4)$.

and $35_{\pm}$ irreps of $SO(8)$ and fermions in the 8 and 56 irreps. The reality properties follow from $P^{\dagger} = -P$.

A UIR of $OSp(8|4)$ is denoted by $D(E_0, s; a_1, a_2, a_3, a_4)$, where the notation is explained in Appendix B. The fundamental UIR of $OSp(8|4)$, which is also a UIR of $hs(8|4)$, is the ultra-short singleton [36, 37]

$$D(\tfrac{1}{2}, 0; 0, 0, 0, 1) \oplus D(1, \tfrac{1}{2}; 0, 0, 1, 0) \ . \tag{2.6}$$

By taking products of singletons we obtain further unitary representations of $hs(8|4)$. Two singletons yield $OSp(8|4)$ weight spaces corresponding to massless AdS_4 fields with $E_0 = s + 1$ [38, 39, 40].

The massless sector of the $hs(8|4)$ gauge theory is formulated in terms of an $hs(8|4)$ valued master gauge field $A_\mu(y, \bar{y}, \theta)$ (with expansion given by (2.5)) and a master zero-form $\Phi(y, \bar{y}, \theta)$ in a quasi-adjoint representation of $hs(8|4)$ with expansion

$$\Phi(y, \bar{y}, \theta) = \sum_{\substack{-m+n+p \\ = 0 \bmod 4}} \frac{1}{m!\, n!\, p!}\, \bar{y}^{\dot{\alpha}_1} \cdots \bar{y}^{\dot{\alpha}_m} y^{\beta_1} \cdots y^{\beta_n} \theta^{i_1} \cdots \theta^{i_p} \Phi_{\dot{\alpha}_1 \ldots \dot{\alpha}_m\, \beta_1 \ldots \beta_n\, i_1 \ldots i_p} \ . \tag{2.7}$$

The reality condition on Φ is discussed in detail in [16, 17]. The gauging gives rise to a set of field equations for physical fields (the action still remains to be found) whose spectrum is given by the symmetric product of two singletons which is given in Table 1. The physical spin $s \geq 1$ fields are the gauge fields in $A_\mu(y, \bar{y}, \theta)$ that correspond to $hs(8|4)$ generators in (2.5) satisfying $|m - n| \leq 1$. Those with $m = n$ contain the vierbein and its HS generalizations, while those with $|m - n| = 1$ contain the gravitini and their HS generalizations. The physical fields with $s \leq \tfrac{1}{2}$ arise in $\Phi(y, \bar{y}, \theta)$ as the components in (2.7) with $m + n \leq 1$. The remaining fields in A_μ and Φ are auxiliary and given in terms of derivatives of the independent fields.

So far we have discussed the free massless HS gauge theory. The general formulation of *interacting* massless HS gauge theory has been given in $D = 4$ [14] (see, [15] for a review), and examined in detail for $\mathcal{N} = 8$ [16, 17]. There exists a minimal bosonic truncation of this theory whose spectrum consist the physical states with spin $s = 0, 2, 4, ...$, each occurring once. This theory exhibits the basics of any HS gauge theory rather well and it will be discussed in considerable detail in Section 6.2, which is based on [19].

2.2   The $D = 5$, $\mathcal{N} = 4$ Massless HS Gauge Theory

The 5D HS superalgebra $hs(2, 2|4)$ [10] is realized in terms of the following oscillators [4]

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[4]The algebra $hs(2, 2|4)$ is called $ho_0(1, 0|8)$ in [12].

$\ell \backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	70	56	28	8	1									
1	1+1	8	28	56	70	56	28	8	1					
2					1	8	28	56	70	56	28	8	1	
3									1	8	28	56	70	$\cdots$
4													1	$\cdots$
$\vdots$														

Table 1: The $SO(3,2) \times SO(8)$ content of the symmetric tensor product of two $d = 3$, $\mathcal{N} = 8$ singletons. Each entry refers to the $SO(8)$ content. All $SO(8)$ irreps are irreducible except $70 = 35_+ + 35_-$ and all the states have $E_0 = s+1$ except the scalars in one of the 35-plets at level $\ell = 0$ and one of the scalars at level $\ell = 1$. The representations have been arranged into a tower of $OSp(8|4)$ supermultiplets labeled by a level index ℓ. The zeroth level is the $D = 4$, $\mathcal{N} = 8$ supergravity multiplet with 2^8 degrees of freedom. The level $\ell \geq 1$ multiplets have 2×2^8 degrees of freedom. The spin $s \geq 1$ fields arise in the $hs(8|4)$ valued master gauge field and the spin $s \leq \frac{1}{2}$ arise in the quasi-adjoint master zero-form. The minimal bosonic truncation of the spectrum is obtained by keeping the maximum spin fields at each level and the (non-pseudo) scalar at level $\ell = 1$.

$$y_\alpha \star \bar{y}_\beta = y_\alpha \bar{y}_\beta + C_{\alpha\beta} \ , \quad y_\alpha \star y_\beta = y_\alpha y_\beta \ , \quad (y^\dagger i \Gamma^0 C)_\alpha = \bar{y}_\alpha \ , \tag{2.8}$$

$$\theta^i \star \bar{\theta}_j = \theta^i \bar{\theta}_j + \delta^i_j \ , \quad \theta^i \star \theta^j = \theta^i \theta^j \ , \quad (\theta^i)^\dagger = \bar{\theta}_i \ , \tag{2.9}$$

where y_α $(\alpha = 1,\ldots,4)$ is a Grassmann even Dirac spinor and θ^i $(i = 1,...,4)$ is a Grassmann odd $SO(6) \simeq SU(4)$ spinor. The charge conjugation matrix $C_{\alpha\beta}$ is anti-symmetric. The algebra $hs(2,2|4)$ consists of Grassmann even and anti-hermitian polynomials $P(y,\bar{y},\theta,\bar{\theta})$ that are sums of monomials of degree $4\ell + 2$ $(\ell = 0,1,2,...)$ that are invariant under the $U(1)_Z$ generated by

$$Z = \tfrac{1}{2}(\bar{y}y + \bar{\theta}\theta); \tag{2.10}$$

and traceless in their spinor indices:

$$P^{(\ell)}(y,\bar{y},\theta,\bar{\theta}) = \sum_{\substack{m+n+p+q \\ = 4\ell+2 \\ m+p = n+q}} \frac{1}{m!\,n!\,p!\,q!} \, \bar{y}^{\alpha_1} \cdots \bar{y}^{\alpha_m} y^{\beta_1} \cdots y^{\beta_n} \theta^{i_1} \cdots \theta^{i_p} \bar{\theta}_{j_1} \cdots \bar{\theta}_{j_q} \, P_{\alpha_1 \ldots i_p}{}^{j_1 \ldots j_q} \ , \tag{2.11}$$

where

$$C^{\alpha_1 \beta_1} P_{\alpha_1 \ldots \alpha_m \, \beta_1 \ldots \beta_n \, i_1 \ldots i_p}{}^{j_1 \ldots j_q} = 0 \ , \qquad P_i{}^i = 0 \ . \tag{2.12}$$

The tracelessness of $P_i{}^j$ means the removal of the outer $U(1)_Y$ automorphism generator

$$Y = \bar{\theta}\theta \ . \tag{2.13}$$

The Lie bracket between $P, Q \in hs(2,2|4)$ is given by $[P,Q]_\star/\mathcal{I}$ where $\mathcal{I}$ is the ideal generated by elements of the form

$$\sum_{n=1}^{\infty} P_n(y,\bar{y},\theta,\bar{\theta}) \star \underbrace{Z \star \cdots \star Z}_{n \text{ factors}} \ , \tag{2.14}$$

where P_n are polynomials which are traceless in their spinor indices. The structure of the Lie bracket is similar to (2.3).

The zeroth level of $hs(2,2|4)$ is the maximal finite subalgebra

$$PSU(2,2|4) = PU(2,2|4)/U(1)_Z \ , \tag{2.15}$$

where $PU(2,2|4)$ is the centrally extended superalgebra (with 31 bosonic generators). The $PSU(2,2|4)$ generators are realized schematically as

$$Q_{\alpha i} = y_\alpha \bar{\theta}_i \ , \quad \bar{Q}^i_\alpha = \bar{y}_\alpha \theta^i \ , \quad M_{\alpha\beta} = \bar{y}_\alpha y_\beta - \tfrac{1}{4}C_{\alpha\beta}(\bar{y}y) \ , \quad U^i{}_j = \bar{\theta}^i \theta_j - \tfrac{1}{4}\delta^i_j(\bar{\theta}\theta) \ . \tag{2.16}$$

The Lorentz spin of a generator in (2.11) is given by $(j_L, j_R) = (\tfrac{1}{2}m, \tfrac{1}{2}n)$ and the $U(1)_Y$ charge by $Y = p - q$. The components with integer $j_L + j_R$ are Grassmann even and those with half-integer $j_L + j_R$ are Grassmann odd. Bosons are in the 1_0, 15_0, $20'_0$, 6_2, 10_2 and 1_4 irreps of $SU(4) \times U(1)_Y$ and fermions in the 4_1, 4_3 and 10_3 irreps. The reality properties follow from the condition $P^\dagger = -P$ which, in particular, implies that the irreps with $Y = 0$ are real. The generators of the algebra are summarized in Table 4 and Table 5 in Appendix A.

A UIR of $SU(2,2|4)$ is denoted by $D(E_0, j_L, j_R; a_1, a_2, a_3)_Y$ where the notation is explained in Appendix B. The fundamental UIRs of $SU(2,2|4)$ are the ultra-short singletons given in Table 6 [41] in Appendix A. Due to the modding out of the ideal $\mathcal{I}$ generated by elements of the form (2.14) the fundamental UIR of $hs(2,2|4)$ is the singleton with vanishing Z charge, i.e. the Maxwell supermultiplet [42, 43, 44, 41]

$$D(1,0,0;0,1,0)_0 \oplus D(\tfrac{3}{2},\tfrac{1}{2},0;1,0,0)_{-1} \oplus D(\tfrac{3}{2},0,\tfrac{1}{2};0,0,1)_1$$

$$\oplus D(2,1,0;0,0,0)_{-2} \oplus D(2,0,1;0,0,0)_2 \ . \tag{2.17}$$

By taking products of this multiplet we obtain further unitary representations of $hs(2,2|4)$. In particular, the product of two singletons yields massless AdS_5 fields whose energies, which are given by $E_0 = 2 + j_L + j_R$ saturate the unitarity bound of a continuous series (denoted as series A in Appendix B)[42, 41].

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$\ell \backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	42	48	27	8	1									
1	1	8	28	56	70	56	28	8	1					
2					1	8	28	56	70	56	28	8	1	
3									1	8	28	56	70	$\cdots$
4													1	$\cdots$
$\vdots$														

Table 2: The symmetric tensor product of two $d = 4$, $\mathcal{N} = 4$ SYM singletons arranged into levels $\ell = 0, 1, 2...$ of $PSU(2, 2|4)$ multiplets. The entries denote $USp(8)$ representations: $28 = 27+1$, $56 = 48+8$, $70 = 42+27+1$, which branch under $SU(4) \times U(1)_Y$ as follows: $1 = 1_0$, $8 = 4_1 + \bar{4}_{-1}$, $27 = 15_0 + 6_2 + \bar{6}_{-2}$, $42 = 20'_0 + 10_2 + \bar{10}_{-2} + 1_4 + \bar{1}_{-4}$ and $48 = 20_1 + \bar{20}_{-1} + 4_3 + \bar{4}_{-3}$. Each entry also carries $SO(4) \simeq SU(2)_L \times SU(2)_R \subset SO(4, 2)$ spins (j_L, j_R). The total spin s is defined as $s = j_L + j_R$ and the $U(1)_Y$ charge is given by $Y = 2(j_R - j_L)$. The level $\ell = 0$ multiplet is the $D = 5$, $\mathcal{N} = 8$ supergravity multiplet. The level $\ell = 1$ multiplet is the massless Konishi multiplet. The level $\ell \geq 0$ multiplets have $(4\ell + 1) \times 2^8$ degrees of freedom. The states in the $s \leq \frac{1}{2}$ sector arise as the physical states in the master scalar field Φ, as shown in Table 7. For $s \geq 1$, the states with $Y = 0, \pm 1$ arise in the sector of the master gauge field A_μ corresponding to the generators of $hs(2, 2|4)$ listed in Table 4. Those with $Y = \pm 2, \pm 3, \pm 4$ arise in the master scalar field Φ. With the exception noted in Table 7, these have dual gauge fields corresponding to the generators of $hs(2, 2|4)$ listed in Table 5. The minimal bosonic truncation of the spectrum is obtained by keeping the maximum spin fields at each level and the scalar at level $\ell = 1$.

The massless sector of the $hs(2, 2|4)$ gauge theory is formulated in terms of an $hs(2, 2|4)$ valued master gauge field $A_\mu(y, \bar{y}, \theta, \bar{\theta})$ and a master zero-form $\Phi(y, \bar{y}, \theta, \bar{\theta})$ in a certain quasi-adjoint representation of $hs(8|4)$ [9, 10], which contains the Weyl tensors and the extra 'matter' fields given in Table 7 in Appendix A. The gauging gives rise to physical fields whose spectrum is given by the symmetric product of two singletons given in Table 2. The fields with $|Y| \leq 1$ and $|Y| \geq 2$, $j_L + j_R \geq \frac{1}{2}$ carry $SO(4, 1)$ weights such that the analysis of the curvature constraints in this sector is analogous to that in $D = 4$. In the $|Y| \geq 2$, $j_L + j_R \geq 1$ sector the fields carry $SO(4, 1)$ weights that require a separate analysis. One finds that [10] the physical fields arise as two-form potentials in Φ obeying the odd-dimensional self-duality equation $B_2 = \star dB_2$ or higher-spin analogs of this equation (such equations have been more recently studied in the lightcone gauge in [45]). The gauge fields in A_μ with $|Y| \geq 2$ are auxiliary fields which are related to the independent two-forms in Φ by generalized Hodge dualization rules [10].

So far we have discussed the free massless HS gauge theory. The full interacting theory based on $hs(2, 2|4)$ has not been constructed yet. However, the kinematics established in [10] and summarized above, together with the already established principles [14] that govern the structure of the interacting HS gauge theory in $D = 4$, suggest that the full $5D$ interacting theory is perfectly within reach. Indeed certain cubic interactions of the minimal bosonic HS theory in $5D$ have already been constructed by Vasiliev [12].

2.3   The $D = 7$, $\mathcal{N} = 2$ Massless HS Gauge Theory

The linearized gauge theory of the minimal bosonic HS subalgebra, $hs(8^*)$ in $D = 7$ was introduced in [20]. Here we shall construct its supersymmetric extension $hs(8^*|4)$. This algebra is realized in terms of the following oscillators :

$$y_\alpha \star \bar{y}_\beta = y_\alpha \bar{y}_\beta + C_{\alpha\beta} , \quad y_\alpha \star y_\beta = y_\alpha y_\beta , \quad (y^\dagger i\Gamma^0 C)_\alpha = \bar{y}_\alpha , \tag{2.18}$$

$$\theta^i \star \bar{\theta}^j = \theta^i \bar{\theta}^j + \Omega^{ij} , \quad \theta^i \star \theta^j = \theta^i \theta^j , \quad (\theta^i)^\dagger = \bar{\theta}^j \Omega_{ji} , \tag{2.19}$$

where y_α ($\alpha = 1, \ldots, 8$) is a Grassmann even Dirac spinor and θ^i ($i = 1, \ldots, 4$) is a Grassmann odd Dirac spinor of $SO(5) \simeq USp(4)$. The charge conjugation matrix $C_{\alpha\beta}$ is symmetric and Ω^{ij} is the antisymmetric $USp(4)$ invariant tensor. The algebra $hs(8^*|4)$ consists of Grassmann even and anti-hermitian polynomials $P(y, \bar{y}, \theta, \bar{\theta})$ that are sums of monomials of degree $4\ell + 2$ ($\ell = 0, 1, 2, \ldots$) which are invariant under the $SU(2)_Z$ generated by

$$Z_3 = \tfrac{1}{4}(\bar{y}^\alpha y_\alpha + \bar{\theta}^i \theta_i) , \quad Z_+ = \tfrac{1}{4}(y^\alpha y_\alpha + \theta^i \theta_i) , \quad Z_- = \tfrac{1}{4}(\bar{y}^\alpha \bar{y}_\alpha + \bar{\theta}^i \bar{\theta}_i) . \tag{2.20}$$

and traceless in their spinor indices. The Lie bracket between $P, Q \in hs(8^*|4)$ is given by $[P, Q]_\star / \mathcal{I}$ where $\mathcal{I}$ is the ideal generated by elements of the form

$$\sum_{n=1}^{\infty} P_n^{I_1 \ldots I_n}(y, \bar{y}, \theta, \bar{\theta}) \star Z_{I_1} \star \cdots \star Z_{I_n} , \tag{2.21}$$

where $P_n^{I_1 \ldots I_n}(y, \bar{y}, \theta, \bar{\theta})$ has an expansion in terms of traceless, Weyl ordered multispinors and the $SU(2)_Z$ indices $I_1 \ldots I_n$ are symmetric. The structure of the Lie bracket is again similar to (2.3). The zeroth level of $hs(8^*|4)$ is the maximal finite subalgebra $OSp(8^*|4)$ realized schematically as

$$Q_{\alpha i} = y_\alpha \bar{\theta}_i - \bar{y}_\alpha \theta_i , \quad M_{\alpha\beta} = \bar{y}_{[\alpha} y_{\beta]} , \quad U_{ij} = \theta_{(i} \bar{\theta}_{j)} . \tag{2.22}$$

An ℓth level generator $P^{(\ell)}$ in $hs(8^*|4)$ can be expanded as

$$P^{(\ell)}(y, \bar{y}, \theta, \bar{\theta}) = \sum_{\substack{m+n+p+q \\ = 4\ell+2 \\ m+p=n+q}} \frac{1}{m! \, n! \, p! \, q!} \, \bar{y}^{\alpha_1} \cdots \bar{y}^{\alpha_m} y^{\beta_1} \cdots y^{\beta_n} \theta^{i_1} \cdots \theta^{i_p} \bar{\theta}^{j_1} \cdots \bar{\theta}^{j_q} P_{\alpha_1 \ldots j_q} , \tag{2.23}$$

where the components are traceless in their Lorentz spinor indices and belong to super Young tableaux with two rows of length $2\ell + 1$. A single box in the super Young tableaux represents the superoscillator $\xi^A = (y^\alpha, \theta^i)$ or $\bar{\xi}^A = (\bar{y}^\alpha, \bar{\theta}^i)$. An arbitrary Weyl ordered monomial in these superoscillators corresponds to a super Young tableaux with two rows. The restriction

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$m + n = p + q$ in (2.23) (i.e. equal number of ξ^A and $\bar{\xi}^A$) follows from the condition $[Z_3, P]_\star = 0$, while the condition $[Z_\pm, P]_\star = 0$ rules out super Young tableaux with rows of unequal length. The resulting super Young tableaux of width $2\ell + 1$ splits into a set of Young tableaux of spinors. Each $SO(6, 2)$ Young tableaux branches into a set of Young tableaux of $SO(6, 1)$ spinors. The spinorial $SO(6, 1) \times SO(5)$ Young tableaux can be converted into tensorial ones by multiplying with appropriate Dirac matrices of both groups. The resulting $SO(5)$ irreps are $1_0, 5_0, 10_0, 14_0, 1_2, 5_2, 10_2, 1_4$ in the bosonic sector and $4_1, 16_1, 4_3$ in the fermionic sector, where the subscripts denote the $U(1)_Y$ charge defined as

$$Y = n_{\bar{\theta}} - n_\theta \; , \tag{2.24}$$

with $n_{\bar{\theta}} = q$ and $n_\theta = p$, as specified in the expansion (2.23). The $SO(6, 1)$ highest weights (m_1, m_2, m_3) are given by

$$m_1 = 2\ell + 1 - \tfrac{1}{2}(n_\theta + n_{\bar{\theta}}) \geq m_2 \geq m_3 = \tfrac{1}{2}|Y| \; . \tag{2.25}$$

Note that since P is assumed to be Grassmann even the components in (2.23) with integer weights are Grassmann even and those with half-integer weights are Grassmann odd. The reality properties follow from $P^\dagger = -P$. As a result, all $SO(6, 1) \times SO(5)$ representations obey symplectic reality conditions. For example, the supercharge $Q_{\alpha i}$ obey a symplectic Majorana condition so that it has 32 real components:

$$\bar{Q}_{\alpha i} \equiv (Q_{\beta j})^\dagger (i\Gamma^0 C)_{\beta\alpha} \Omega_{ji} = Q_{\alpha i} \; . \tag{2.26}$$

These results are summarized in Tables 8 and 9 in Appendix A.

A UIR of $OSp(8^*|4)$ is denoted by $D(E_0, J_1, J_2, J_3; a_1, a_2)_Y$ where the notation is explained in Appendix B. The fundamental UIRs of $OSp(8^*|4)$ are the singletons given in Table 10 [46] in Appendix A. The singleton which is singlet of $SU(2)_Z$ also forms an UIR of $hs(8^*|4)$. This singleton is the $(2, 0)$ tensor multiplet [46, 44, 47, 48]

$$D(2, 0, 0, 0; 0, 1)_0 \oplus D(\tfrac{5}{2}, 1, 0, 0; 1, 0)_1 \oplus D(\tfrac{5}{2}, 0, 0, 1; 1, 0)_{-1}$$

$$\oplus D(3, 0, 0, 2; 0, 0)_2 \oplus D(3, 2, 0, 0; 0, 0)_{-2} \; . \tag{2.27}$$

By taking products of this singleton one obtains further unitary representations of $hs(8^*|4)$. In particular, the square yields massless AdS_7 fields with energy $E_0 = 4 + s$ where $s \equiv J_1$. These energies belong to an isolated series (denoted as series B in Appendix B)[48], unlike in $D = 4, 5$ where the massless fields have energies that saturate a continuous series (the continuous series is saturated by lowest weight spaces arising in the product of three singletons).

The superalgebra $hs(8^*|4)$ has a minimal bosonic HS subalgebra $hs(8^*)$ whose representation theory and gauging was described in [20]. We shall assume that the massless sector of the $hs(8^*|4)$ gauge theory is formulated in terms of an $hs(8^*|4)$ valued master gauge field $A_\mu(y, \bar{y}, \theta, \bar{\theta})$ and

$\ell \backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	14	16	15	4	1									
1	1	4	$16'$	24	36	24	$16'$	4	1					
2					1	4	$16'$	24	36	24	$16'$	4	1	
3									1	4	$16'$	24	36	$\cdots$
4													1	$\cdots$
$\vdots$														

Table 3: The symmetric tensor product of two $d = 6$, $\mathcal{N} = (2,0)$ tensor singletons arranged into levels $\ell = 0, 1, 2...$ of $OSp(8^*|4)$ multiplets. The entries denote $SO(5) \times U(1)_Y$ representations as follows: $14 = 14_0$, $16 = 16_1$, $15 = 10_0 + 5_2$, $4 = 4_1$, $16' = 10_0 + 5_2 + 1_2$, $24 = 16_1 + 4_1 + 4_3$, $36 = 14_0 + 5_0 + 1_0 + 10_2 + 5_2 + 1_4$. The $SO(6) \subset SO(6,2)$ highest weights (n_1, n_2, n_3) associated with each entry are given by $n_1 = s$, $n_2 = \frac{1}{2}|Y|$ and $n_3 = \frac{1}{2}Y$. The level $\ell = 0$ multiplet is the $D = 7$, $\mathcal{N} = 2$ supergravity multiplet. The level $\ell \geq 0$ supermultiplets contain $\frac{1}{3}(\ell+1)(2\ell+1)(4\ell+3) \times 2^8$ degrees of freedom. The states with $|Y| \leq 1$, $s \geq 1$ are expected to arise in the sector of the master gauge field A_μ corresponding to the generators given in Table 8. The states with $s \leq \frac{1}{2}$, or $|Y| \geq 2$ and $s \geq 1$, which are listed in Table 11, are expected to arise in a quasi-adjoint master zero-form Φ. With a few low-lying exceptions which are given in Table 11, these are generalized Hodge duals of the $|Y| \geq 2$ sector of the master gauge field A_μ which corresponds to the $hs(8^*|4)$ generators listed in Table 9. The minimal bosonic truncation of the spectrum is obtained by keeping the maximum spin fields at each level and the scalar at level $\ell = 1$.

a master zero-form $\Phi(y, \bar{y}, \theta, \bar{\theta})$ in a quasi-adjoint representation [5] of $hs(8^*|4)$ and that the gauging gives rise to physical fields whose spectrum is given by the symmetric product of two tensor singletons listed in Table 3. The gauge fields with $Y = 0$ carry $SO(6,1)$ weights which are similar to those in the minimal bosonic theory [20]. The gauge fields with $|Y| \geq 1$ carry $SO(6,1)$ weights which are analogous to those carried by the $|Y| \geq 1$ fields in the $hs(2,2|4)$ theory in $D = 5$. Thus we expect that the physical fields with $|Y| \leq 1$, $s \geq 1$ arise in A_μ. The remaining physical fields, which have $s \leq \frac{1}{2}$, or $s \geq 1$ and $|Y| \geq 2$, must arise in Φ and be those given by Table 11 in Appendix A. In particular we expect that the physical fields with $|Y| \geq 2$, $s \geq 1$ arise as three-form potentials in Φ obeying the odd-dimensional self-duality equation $B_3 = \star dB_3$ and their HS analogs, and that the corresponding gauge fields in A_μ are related to these three-forms by generalized Hodge dualization rules analogous to those found in the $hs(2,2|4)$ theory in $D = 5$ [10].

So far we have discussed the free massless HS gauge theory. The interacting theory has not been constructed yet. However, the kinematics of theory established here, together with what we know in lower dimensions about interacting HS gauge theories should help a great deal in such a construction. In particular, there are many parallels with the kinematics of the $5D$ HS gauge theory which is noteworthy.

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[5]This representation was defined for $hs(8^*)$ in [20]. Its generalization to $hs(8^*|4)$ will not be given here.

3 Composite Operators in Singleton Theories

In this section we describe the singleton theories in $d = 3, 4, 6$ with 16 supersymmetries that are of relevance to the HS gauge theories in $D = 4, 5, 7$ described in the previous section. We shall also identify the superfield realization of the HS currents in terms of these singletons, whenever possible.

Explicit expressions for the supersymmetric currents have been constructed so far in $d = 3$, and for a minimal bosonic truncation, in arbitrary dimensions. The bosonic currents are formed out of a set of real scalar singletons and that are primary fields carrying $SO(d, 2)$ lowest weights $(E_0; m_1, \ldots m_{[d/2]}) = (d - 2 + s; s, 0, ..., 0)$, where $s = 0, 2, 4, ...$ These tensors are conserved currents for $s \geq 2$. The minimal bosonic HS theories are still 'maximal' in the sense that the twist $d - 2$ currents with even spin are the only composites which are both conserved and primary. There are conserved currents with $E_0 - s > d - 2$ as well as $E_0 - s = d - 2$ and odd spin, though these can be shown to be descendants of those with twist $d - 2$ and even spin.

3.1 The $d = 3$, $\mathcal{N} = 8$ Singleton and Its Composites

The fundamental UIR of $OSp(8|4)$, which is also a UIR of $hs(8|4)$, is the ultra-short singleton specified in (2.6). This is just the $d = 3, \mathcal{N} = 8$ scalar multiplet, and its superfield realization has been known for sometime. In particular, it has arisen in the superembedding formulation of $M2$-branes [49]. Following [50], let us work with a realization related to the one in [49] by triality. The singleton superfield is then carries a spinor representation of $SO(8)$ and obeys the constraint

$$D_{\alpha i} \Phi_A = (\Gamma_i)_A{}^{\dot{B}} \chi_{\alpha \dot{B}} \ , \tag{3.1}$$

where $\chi_{\alpha \dot{B}}$ is a spinor superfield, $i, A, \dot{B} = 1, ..., 8$ label the $8_v, 8_s, 8_c$ representations of $SO(8)$, respectively, and Γ-matrices are the chirally projected $SO(8)$ Dirac matrices. The singleton superfield Φ_A carries the irrep $D(1/2, 0; 0, 0, 0, 1)$, which belongs to series B and it is BPS 1/2 multiplet. See Appendix B for notation and further details.

Several composite operators built out of two singletons superfields Φ_A and their derivatives are known [50, 51, 52, 53, 54]. Let us identify those which correspond to the spectrum of massless field in the $D = 4$ HS gauge theory based on $hs(8|4)$ as shown in Table 1. The level $\ell = 0$ supercurrent is realized as [50]

$$J_{AB} = \Phi_A \Phi_B - \tfrac{1}{8} \delta_{AB} \Phi^2 \ , \qquad \Phi^2 := \Phi^A \Phi_A \ . \tag{3.2}$$

The superfield J_{AB} carries the irrep $D(1, 0; 0, 0, 2, 0)$ which belongs to series B and it is BPS 1/2 multiplet. Its lowest component carries the 35_s irrep of $SO(8)$. Together with the scalars in 35_c that arise in the θ-expansion, they form the 70-plet corresponding to the 70 scalar fields of level $\ell = 0$ supergravity multiplet in AdS_4 which has 2^8 degrees of freedom.

At level $\ell = 1$, we have the supercurrent [50]

$$J = \Phi^2, \tag{3.3}$$

which, as a consequence of the basic singleton constraint (3.1), obeys [50]

$$D^{ij}J - trace = 0 \ , \qquad D^{ij} := D^{\alpha i}D^{j}_{\alpha} \ . \tag{3.4}$$

The superfield J carries the irrep $D(1, 0; 0, 0, 0, 0)$, which is semi-short IUR that saturates the unitarity bound of series A. Its lowest component is a scalar, and another scalar arises in the θ-expansion. Altogether, 2×2^8 degrees of freedom arise [53] and they correspond to the massless fields of level $\ell = 1$ shown in Table 1.

Finally, the level $\ell \geq 2$ supercurrents can be realized in terms of the singleton superfield as follows [50]

$$J_{\alpha_1 \dots \alpha_{4\ell-4}} = \sum_{k=0}^{2\ell-2} (-1)^k [32i\partial_{(\alpha_1\alpha_2\dots}\partial_{\alpha_{2k-1}\alpha_{2k}}\Phi^A \ \partial_{\alpha_{2k+1}\alpha_{2k+2}}\dots\partial_{\alpha_{4\ell-5}\alpha_{4\ell-4})}\Phi_A \tag{3.5}$$

$$+ (\Gamma^i\Gamma^j)_{AB} \ \partial_{(\alpha_1\alpha_2\dots}\partial_{\alpha_{2k-1}\alpha_{2k}}D^i_{\alpha_{2k+1}}\Phi^A \ D^j_{\alpha_{2k+2}}\partial_{\alpha_{2k+3}\alpha_{2k+4}}\dots\partial_{\alpha_{4\ell-5}\alpha_{4\ell-4})}\Phi^B] \quad .$$

These currents obey the constraint [50]

$$D^{i\alpha}J_{\alpha\alpha_2\dots\alpha_{4\ell-4}} = 0 \ . \qquad \ell \geq 2 \ , \tag{3.6}$$

The superfield $J_{\alpha\alpha_2\dots\alpha_{4\ell-4}}$ carries the irreps $D(2\ell-1, 2\ell-2; 0, 0, 0, 0)$, which is semi-short IUR that saturates the unitarity bound of series A. Its lowest component is the current with spin $s_{min} = 2\ell - 2$ and higher components go up to $s_{\max} = 2\ell + 2$. They correspond to the level $\ell \geq 2$ massless multiplets listed in Table 1.

As discussed in the introduction, and to be elaborated further in Section 5, if interactions can be switched on in the 3d CFT such that the HS gauge symmetry breaks down to $OSp(8|4)$, then the HS currents for level $\ell \geq 1$ will no longer be conserved. Assuming such breaking, we can characterize the anomalies in conservation law for these currents as

$$D^{ij}J - trace = g\Sigma^{ij} - trace \ , \tag{3.7}$$

$$D^{i\alpha}J_{\alpha\alpha_2\dots\alpha_{4\ell-4}} = g\Sigma^i_{\alpha_2\dots\alpha_{4\ell-4}} \ , \tag{3.8}$$

where g is some coupling constant, and the right hand sides denote superfields which are to be determined. These superfields carry the following irreps

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$$\Sigma^{ij} \quad : \qquad\qquad D(2,0;2,0,0,0) \ , \qquad\qquad\qquad (3.9)$$

$$\Sigma^{i}{}_{\alpha_2...\alpha_{4\ell-4}} \quad : \qquad\qquad D(2\ell - \tfrac{1}{2}, 2\ell - \tfrac{5}{2}; 1, 0, 0, 0) \ . \qquad\qquad (3.10)$$

This means that Σ^{ij} describes a BPS 1/8 multiplet and it satisfies the unitarity condition of series B. In [50], a BPS short multiplet of this type is built out of four singletons using harmonic superspace technique. In terms of ordinary superfields we write it as

$$\Sigma^{ij} = (\Gamma_{imnp})_{AB} \ (\Gamma_j{}^{mnp})_{CD} \ \Phi^A \Phi^B \Phi^C \Phi^D - trace \ . \qquad\qquad (3.11)$$

This is just the 35-plet contained in the symmetric product $(8_s \times 8_s \times 8_s \times 8_s)_S$. Since the superfield Σ^{ij} represents a BPS 1/8 multiplet, its components go up to $s_{max} = 7/2$. Therefore, it is natural to consider this superfield as a candidate for coupling to Higgs superfield in the bulk which can be eaten by the massless Konishi multiplet to become massive.

Turning to the candidate anomaly superfield $\Sigma^{i}{}_{\alpha_2...\alpha_{4\ell-4}}$ given in (3.10), we observe that it carries a semi-short IUR that saturates the unitarity bound of series A. In general, such multiplets have been constructed as [50]

$$\mathcal{S}^{[a_i]} \quad = \quad \Phi^2 \, BPS^{[a_i]} \ , \qquad\qquad\qquad (3.12)$$

$$\mathcal{S}^{\{\mu_1...\mu_s\}[a_i]} \quad = \quad J^{\{\mu_1...\mu_s\}} \, BPS^{[a_i]} \ , \qquad\qquad\qquad (3.13)$$

where $BPS^{[a_i]}$ is any one of the BPS short multiplets listed in (2.4)-(2.6), and $J^{\{\mu_1...\mu_s\}}$ is a spin s current. Assuming that the candidate anomaly superfield $\Sigma^{i}{}_{\alpha_2...\alpha_{4\ell-4}}$ belongs to an irreducible representation of $OSp(8|4)$, since it is an 8-plet of $SO(8)$, it requires the BPS 1/8 multiplet $D(1,0;1,0,0,0)$ and a spin $s = 2\ell - \tfrac{1}{2}$ current in (3.13). However, the BPS 1/8 multiplet cannot be built out of one type of singleton field. Thus, the construction of $\Sigma^{i}{}_{\alpha_2...\alpha_{4\ell-4}}$, which is important for a Higgs mechanism that can work at all levels $\ell \geq 1$, remains an open problem.

The BPS multiplets that can be constructed from the product of one type of singletons are all the BPS 1/2 and BPS 1/4 multiplets listed in (2.4) and (2.5), and all those BPS 1/8 multiplets listed in (2.6) with integer s [51] . These multiplets, as well as the semi-short multiplets discussed above which make use of them, are likely to play a significant role in the description of the full HS gauge theory based on $hs(8|4)$. In particular, the KK supermultiplets associated with level ℓ supermultiplets of the massless HS theory are expected to be BPS 1/2 multiplets. For example, the level $\ell = 0$ multiplet and its KK towers are realized as [53]

$$D(k/2, 0; 0, 0, k, 0) \quad : \qquad \Phi_{(A_1} \Phi_{A_2} \cdots \Phi_{A_k)} - \text{traces} \ , \quad k = 2, 3, ... \qquad (3.14)$$

Taking $k = 2$ gives the massless supergravity multiplet and $k = 3, 4, ...$ give their massive KK descendants. Similarly, the semi-short multiplets (3.12) and (3.13) with BPS 1/2 composites carrying the irrep $D(k/2, 0; 0, 0, k, 0)$ are candidates for KK descendants of the level $\ell > 0$ massless multiplets of the HS gauge theory based on $hs(8|4)$.

3.2 The $d = 4$, $\mathcal{N} = 4$ Singleton and Its Composites

The fundamental UIR of $SU(2, 2|4)$, which is also a UIR of $hs(2, 2|4)$, is the ultra-short singleton specified in (2.17). This is the $d = 4, \mathcal{N} = 4$ Maxwell multiplet realized in terms of superfield W_{ij}, where $i = 1, ..., 4$ labels the 4-plet of $SU(4)$ and $W^{ij} = -Wji$. [6] It satisfies the following constraints and reality condition

$$D_\alpha^{(i} W^{j)k} = 0 , \quad \bar{D}_{\alpha i} W^{jk} - \text{trace} = 0 ,$$

$$W_{ij} \equiv (W^{ij})^\dagger = \tfrac{1}{2} \epsilon_{ijkl} W^{kl} . \tag{3.15}$$

The singleton superfield W_{ij} carries the irrep $D(1, 0, 0; 0, 1, 0)$. It belongs to series C and it describes a BPS 1/2 multiplet. There are several papers which deal with the construction of the composite operators built out of the Maxwell (or SYM) singleton. See, for example, [55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65]. Here we shall follow closely the treatment of [50].

For each state in the spectrum of the HS gauge theory listed in Table 2, one can construct the corresponding conserved current out of two Maxwell singletons and their derivatives. To begin with, the level $\ell = 0$ supercurrent is contained in the superfield $J_{ij,kl}$, which is in $20'$ of $SU(4)$ and is given by [55, 56]

$$J_{ij,kl} = W_{ij} W_{kl} - \tfrac{1}{12} \epsilon_{ijkl} W^{mn} W_{mn} . \tag{3.16}$$

Defining $W^a \equiv (\Gamma^a)_{ij} W^{ij}$ ($a = 1, 2, ..., 6$), where Γ^a are the chirally projected $SO(6)$ Dirac matrices, the current superfield (3.16) can equivalently be written as $J_{ab} = W_a W_b - \tfrac{1}{6} \delta_{ab} W_c W_c$.

Defining $J_{ij}^{kl} = \epsilon^{klmn} J_{ij,mn}$, on the other hand, it obeys the constraint [56]

$$D_\alpha^m J_{ij}^{kl} = \chi_{\alpha ij}^{mkl} + \delta_{[i}^m \lambda_{\alpha j]}^{kl} + \delta_{[i}^{[k} \lambda_{\alpha j]}^{l]m} , \tag{3.17}$$

where λ and χ are both totally anti-symmetric in lower and upper indices and totally traceless. The superfield $J_{ij,kl}$ carries the irrep $D(2, 0, 0; 0, 2, 0)$. It belongs to belongs to series C and it describes a BPS 1/2 multiplet. Its components can be shown to contain the composite operators that correspond to the level $\ell = 0$ supergravity multiplet shown in Table 2, and that the components with spin $s \geq 1$ are conserved currents.

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[6]This is the unique singleton multiplet of $PSU(2, 2|4)$ and it has vanishing $U(1)_Z$ central charge. The centrally extended $PU(2, 2|4)$ superalgebra admits an infinite number of singleton multiplets. These have $j_R = 0$ (their complex conjugates have $j_L = 0$), and $E_0 = j_L + 1$. Each singleton multiplet forms a massless UIR of the $d = 4, \mathcal{N} = 4$ Poincaré superalgebra and is characterized by central charge $\ell = 2|Z| = 0, 1, 2, ...$ Viewed as massless states of $d = 4$ Poincaré group, they carry Lorentz spin (i.e. maximum $SO(2)$ helicity) $s = j_L$. The first three levels of the singleton spectrum shown in Table 6 are special because they are the only singleton multiplets which contain scalar fields. They are $D(1, 0, 0; 0, 1, 0)$, $D(1, 0, 0; 1, 0, 0)$ and $D(1, 0, 0; 0, 0, 0)$ with Z charges $(0, 1/2, 1)$ and they can be described by superfields (W_{ij}, W^i, W), respectively. The level $\ell \geq$ singletons $D(\tfrac{\ell}{2}, \tfrac{\ell}{2} - 1, 0; 0, 0, 0)$ have central charge $Z = \tfrac{\ell}{2}$ and can be described by superfield $\omega_{\alpha_1 ... \alpha_{\ell-2}}$. The constraints satisfied by all singleton superfields can be found in [50].

The level $\ell = 1$ supercurrent is also a special one and is known as the massless Konishi multiplet. It has the simple form [56]

$$J = W_{ij}W^{ij} \ . \tag{3.18}$$

As a result of the basic singleton constraint (3.15), this current obeys the constraint

$$D^{ij}J = 0 \ , \qquad D^{ij} := D^{\alpha(i}D^{j)}_{\alpha} \ . \tag{3.19}$$

This multiplet has 5×2^8 and they precisely correspond to the level $\ell = 1$ massless states shown in Table 2. It is characterized by the irrep $D(2,0,0;0,0,0)$ carried by its lowest component. It is a semi-short multiplet which saturates the unitarity bound of series A.

In the Poincaré limit, the states are labelled by the little group $SO(3) \times SU(4)$. Denoting the irreps by R_s, where R is denotes an $USp(8)$ irrep (which should be decomposed into $SU(4)$ irreps)and s is the $SO(3)$ spin, the level $\ell = 1$ massless Konishi multiplet can be obtained by tensoring the level $\ell = 0$ supergravity multiplet with an $SU(4)$ singlet spin $s = 2$ state as follows:

$$\text{Massless Konishi}: \qquad (42_0 + 48_{1/2} + 27_1 + 8_{3/2} + 1_2) \times 1_2 =$$

$$1_0 + 8_{1/2} + (27+1)_1 + (48+8)_{3/2} + (42+27+1)_2$$

$$+(48+8)_{5/2} + (27+1)_3 + 8_{7/2} + 1_4 \tag{3.20}$$

The massless multiplets arising at level $\ell \geq 2$ in the spectrum shown in Table 2 are generic in their structure. The corresponding conserved currents are contained in a superfield

$$J_{\mu_1\mu_2...\mu_{2\ell-2}} \ , \qquad \ell \geq 2 \ , \tag{3.21}$$

which obey the constraints [50]

$$(\bar{\sigma}^{\mu_1})_{\dot{\alpha}\beta} D^{i\beta} J_{\mu_1\mu_2...\mu_{2\ell-2}} = 0 \ , \qquad (\sigma^{\mu_1})_\alpha{}^{\dot\beta} \bar{D}_{i\dot\beta} J_{\mu_1\mu_2...\mu_{2\ell-2}} = 0 \ . \tag{3.22}$$

The superfield $J_{\mu_1\mu_2...\mu_{2\ell-2}}$ carries the irrep $D(4\ell-2, 2\ell-2, 2\ell-2; 0,0,0)$ and its components have spins that range from $(2\ell-2)$ to $(2\ell+2)$. This superfield saturates the unitarity bound of series A and it describes a semi-short multiplet.

The explicit construction of all the supercurrents in terms of Maxwell singleton is straightforward but tedious exercise which apparently has not been carried so far. They are known, however, for the minimal bosonic truncation of the massless HS gauge theory in $D = 5$ discussed above. They take the form [8, 13]

$$j_{\mu_1\cdots\mu_{2\ell-2}} = \sum_{k=0}^{2\ell-2} \frac{(-1)^k}{(k!)^2!((s-k)!)^2} \partial_{\mu_1}\cdots\partial_{\mu_k}\phi^* \partial_{\mu_{k+1}}\cdots\partial_{\mu_{2\ell-2}}\phi - \text{traces} \ . \tag{3.23}$$

So far we have considered free SYM singletons. Switching on the SYM interactions, the currents listed above for $\ell \geq 1$ will no longer be conserved. The resulting anomalies can be characterized as follows

$$D^{ij} J = \sqrt{\lambda} \Sigma^{ij} \ ,$$

$$(\bar{\sigma}^{\mu_1})_{\dot{\alpha}\beta} D^{i\beta} J_{\mu_1 \mu_2 ... \mu_{\ell-2}} = \sqrt{\lambda} \Sigma^i_{\mu_2 ... \mu_{2\ell-2}, \dot{\alpha}} \ ,$$

$$(\sigma^{\mu_1})_{\alpha}{}^{\dot{\beta}} \bar{D}_{i\dot{\beta}} J_{\mu_1 \mu_2 ... \mu_{2\ell-2}} = \sqrt{\lambda} \Sigma_{i\mu_2 ... \mu_{\ell-2}, \alpha} \ , \tag{3.24}$$

where the constant normalization factor is introduced for later convenience (see Section 5).

The superfields on the right hand side carry the following UIRs of $SU(2,2|4)$

$$\Sigma^{ij} \quad : \quad D(3,0,0;2,0,0) \ , \tag{3.25}$$

$$\Sigma_{\mu_2 ... \mu_{2\ell-2}, \dot{\alpha}} \quad : \quad D(2\ell - \tfrac{3}{2}, \ell - \tfrac{3}{2}, \ell - 1; 0, 0, 1) \ , \tag{3.26}$$

$$\Sigma_{\mu_2 ... \mu_{2\ell-2}, \alpha} \quad : \quad D(2\ell - \tfrac{3}{2}, \ell - 1, \ell - \tfrac{3}{2}; 1, 0, 0) \ . \tag{3.27}$$

In the interacting SYM singleton theory the anomaly superfield Σ^{ij} takes the well known form (see, for example, [61, 65]):

$$\Sigma^{ij} = \frac{4}{N^{3/2}} \operatorname{Tr} W^{k(i} W^{j)\ell} W_{k\ell} \ , \tag{3.28}$$

where the constant normalization factor is introduced for later convenience (see Section 5). This superfield belongs to series B and it describes a BPS 1/8 multiplet. Consequently its components go up to $s_{max} = 7/2$ and therefore it is a candidate for coupling to Higgs superfield in the bulk which can be eaten by the massless Konishi multiplet to become massive. All the components of the massive Konishi multiplet of $PSU(2,2|4)$ have been tabulated in [57].

The candidate anomaly superfields $\Sigma^i_{\alpha_2 ... \alpha_{2\ell-2}, \alpha}$, on the other hand carries a semi-short IUR that satisfy the unitarity bounds of series A or B. In general, such multiplets have been constructed as [50]

$$\mathcal{S}^{[a_i]} = \Phi^2 \, BPS^{[a_i]} \ ,$$

$$\mathcal{S}^{\{\mu_1 ... \mu_s\}[a_i]} = J^{\{\mu_1 ... \mu_s\}} \, BPS^{[a_i]} \ , \tag{3.29}$$

where $BPS^{[a_i]}$ is any one of the BPS operators listed in (2.11)-(2.13), and $J^{\{\mu_1 ... \mu_s\}}$ is a spin s current, to be constructed out of the free SYM singleton in our case. For the BPS 1/2 and BPS 1/4 cases, both of the above operators saturate the series A unitarity bound (2.8),

while in the case of BPS 1/8, they belong to series B. Assuming that the candidate anomaly superfield $\Sigma^i_{\alpha_2\dots\alpha_{2\ell-2},\alpha}$ carries an irreducible representation, and given that it is in (100) of $SU(4)$, attempting to construct it as in (3.29) requires the use of BPS 1/8 multiplet $D(3/2,0,0;1,0,0)$, as follows from (2.13). However, these BPS multiples cannot be built out of SYM singletons alone [50].

The BPS multiplets that can be constructed out of products of SYM singleton alone are all the BPS 1/2 and BPS 1/4 multiplets listed in (2.11) and (2.12), and all those BPS 1/8 multiplets listed in (2.13) with integer r [51]. These multiplets, and the semi-short multiplets discussed above which make use of them, are likely to play a role in finding the massive states of the full HS gauge theory based on $hs(2,2|4)$. In particular, the KK supermultiplets associated with level ℓ supermultiplets of the massless HS theory are expected to make use of the BPS 1/2 states. For example, the level $\ell = 0$ multiplet and its KK towers are realized as [3, 58, 59]

$$D(k,0;0,k,0) \ : \qquad W_{(a_1}W_{a_2}\cdots W_{a_k)} - \text{traces} \ , \quad k = 2,3,... \tag{3.30}$$

Setting $k = 2$ gives the massless supergravity multiplet and $k = 3, 4, ...$ their massive KK descendants. Similarly, the semi-short multiplets (3.12) and (3.13) involving the BPS 1/2 composites carrying the irrep $D(k,0;0,k,0)$ are candidates for KK descendants of the level $\ell > 0$ massless multiplets of the HS gauge theory based on $hs(2,2|4)$.

3.3   The $d = 6$, $\mathcal{N} = (2,0)$ Tensor Singleton and Its Composites

The fundamental UIRs of $OSp(8^*|4)$ are the singletons given in Table 10 in Appendix A [48, 46]. Each row in the Table denotes an irreducible singleton multiplet. The superfield realization of the $6d$ singletons have been studied by several authors. Here we shall follow [50, 51] where several references to earlier literature can also be found. There exist several papers on the construction of the composite operators out of the 6d singletons as well; see [66, 50, 51, 67], for example.

There exist an infinite set of singletons of $OSp(8^*|4)$. They are shown in Table 10 and listed in Appendix B. The $(2,0)$ tensor singleton is the only one which is singlet under an $SU(2)_Z$ defined in Section 2.3. Here we shall focus our attention to the level $\ell = 0$ singleton described by the superfield W^{ij} which forms the tensor multiplet of $d = 6, \mathcal{N} = (2,0)$ Poincaré supersymmetry, since all the HS gauge theory states will be formed out of them. To begin with, we shall take a single copy of the tensor multiplet. Abelian nature of the singletons is essential for the construction of conserved currents. The superfield W_{ij} satisfies the following constraints and reality condition [66]

$$D^{(i}_\alpha W^{j)k} = 0 \ , \qquad \bar{W}_{ij} = \Omega_{ik}\Omega_{jl}W^{kl} \ . \tag{3.31}$$

The singleton superfield W_{ij} carries the irrep $D(2;0,0,0;0,1)$ which belongs to series D and it is BPS 1/2 supermultiplet. For each state in the spectrum of the HS gauge theory listed in Table 3, one can construct the corresponding conserved current out of two tensor singletons and their

derivatives. To begin with, the level $\ell = 0$ supercurrent is contained in the superfield $J_{ij,kl}$, which is in 14-plet of $USp(4)$ and is given by

$$J_{ij,kl} = W_{ij}W_{kl} - \tfrac{1}{6}\Omega_{k[i}\Omega_{j]\ell}W^{mn}W_{mn} . \tag{3.32}$$

Defining $W^a \equiv (\Gamma^a)_{ij} W^{ij}$ ($a = 1, 2, ..., 5$), where Γ^a are the $SO(5)$ Dirac matrices, the current superfield (3.32) can equivalently be written as $J_{ab} = W_a W_b - \tfrac{1}{6}\delta_{ab} W_c W_c$.

The superfield $J_{ij,kl}$ carries the irrep $D(4; 0, 0, 0; 0, 2)$, which belongs to series D and it describes a BPS 1/2 multiplet. This is the level $\ell = 0$ supergravity multiplet shown in Table 3.

The level $\ell = 1$ supercurrent is similar to the ones in $d = 3, 4$ and it takes the form

$$J = W_{ij}W^{ij} . \tag{3.33}$$

This current obeys the constraint [50]

$$\epsilon^{\alpha\beta\gamma\delta} D_\alpha^{(i} D_\beta^j D_\gamma^{k)} J = 0 . \tag{3.34}$$

The superfield J carries the irrep $D(4; 0, 0, 0; 0, 0)$. It has 14×2^8 components and it can be obtained group theoretically by tensoring the level $\ell = 0$ supergravity multiplet with the graviton state which has 14 degrees of freedom. It is a semi-short multiplet which belongs to series B.

The massless multiplets arising at level $\ell \geq 2$ in the spectrum shown in Table 2 are generic and the corresponding conserved currents are contained in the superfield

$$J_{\alpha_1...\alpha_{2\ell-2},\beta_1...\beta_{2\ell-2}} , \qquad \ell \geq 2 , \tag{3.35}$$

where the α and β indices are symmetrized separately. These current superfields obey the constraint [50]

$$\epsilon^{\delta\gamma\alpha_1\beta_1} D_\gamma^i J_{\alpha_1...\alpha_{2\ell-2},\beta_1...\beta_{2\ell-2}} = 0 . \tag{3.36}$$

The superfield $J_{\alpha_1...\alpha_{2\ell-2},\beta_1...\beta_{2\ell-2}}$ carries the irrep $D(2\ell + 2; 0, 2\ell - 2, 0; 0, 0)$. It is a semi-short multiplet which belongs to series B.

An explicit construction of these supercurrents in terms of the $(2, 0)$ tensor singleton apparently has not been carried out so far. They are known, however, for the minimal bosonic truncation of the massless HS gauge theory in $D = 7$ discussed earlier. They take the form [8, 13]

$$j_{\mu_1\cdots\mu_{2\ell-2}} = \sum_{k=0}^{2\ell-2} \frac{(-1)^k}{k!(k+1)!(s-k)!(s-k+1)!} \partial_{\mu_1}\cdots\partial_{\mu_k}\phi^* \partial_{\mu_{k+1}}\cdots\partial_{\mu_{2\ell-2}}\phi - \text{traces} . \tag{3.37}$$

So far we have considered free $(2,0)$ tensor singletons. Interactions for multi-copies of these singletons are not known and they are expected to be radically different than those familiar from ordinary field theory. These interactions are also expected to break the HS gauge symmetries down to those of level $\ell = 0$ supergravity. Let us characterize the break-down in the conservation laws of the supercurrents of level $\ell \geq 1$ as follows

$$\epsilon^{\alpha\beta\gamma\delta} \, D_\alpha^{(i} D_\beta^j D_\gamma^{k)} \, J = g \Sigma^{\delta ijk} \ , \tag{3.38}$$

$$\epsilon^{\delta\gamma\alpha_1\beta_1} \, D_\gamma^i J_{\alpha_1...\alpha_{2\ell-2},\beta_1...\beta_{2\ell-2}} = g \Sigma^{\delta i}_{\alpha_2...\alpha_{2\ell-2},\beta_1...\beta_{2\ell-2}} \ , \tag{3.39}$$

where g is some coupling constant. Unlike in the cases of $d = 3, 4$, here we see that the representation content of the candidate anomaly superfields do not correspond to any BPS short or semi-short multiplets listed in Appendix B. Of course, here we are assuming that these anomaly superfields are irreducible. Their computation from first principles may in principle reveal that they are reducible, and possibly derivatives of some irreducible superfields. The nature of the anomaly superfields should also reflect the fact that there there are no local non-abelian interactions for tensor fields that can be described by continuous deformations of the free theory [90]. This is a qualitative difference between $d = 6$ and $d = 3, 4$, where the free fields admit SYM deformations (after dualization of a scalar in $d = 3$).

The semi-short multiplets, as in 3d and 4d cases, have also been constructed in terms of building blocks discussed above, and they take the form [50]

$$\mathcal{S}^{[a_i]} \ = \ \Phi^2 \, BPS^{[a_i]} \ , \tag{3.40}$$

$$\mathcal{S}^{\{\mu_1...\mu_s\}[a_i]} \ = \ J^{\{\mu_1...\mu_s\}} \, BPS^{[a_i]} \ , \tag{3.41}$$

where $BPS^{[a_i]}$ is any one of the BPS operators listed in (2.20) and (2.21), and $J^{\{\mu_1...\mu_s\}}$ is a spin s current, which is to be constructed out of the free $(2,0)$ tensor singleton in our case. Both of these saturate the unitary bound of series B.

The BPS multiplets that can be constructed out of products of the tensor singleton alone are all the BPS 1/2 multiplets listed in (2.20) and all those BPS 1/4 multiplets listed in (2.21) with integer q[51]. In particular, the level $\ell = 0$ multiplet and its KK towers are realized as [50]

$$D(2k,0,0,0;0,k) \ : \qquad W_{(a_1} W_{a_2} \cdots W_{a_k)} - \text{traces} \ , \quad k = 2, 3, ... \tag{3.42}$$

As in the cases of 3d and 4d, here too, setting $k = 2$ gives the massless supergravity multiplet and $k = 3, 4, ...$ give their massive KK descendants. Similarly, the semi-short multiplets (3.40) and (3.41) with BPS 1/2 composites carrying the irrep $D(2k,0,0,0;0,k)$ are candidates for KK descendants of the level $\ell > 0$ massless multiplets of the HS gauge theory based on $hs(8^*|4)$.

4 Higher Spin Gauge Theory and Holography

We shall first discuss some general features of HS gauge theory/singleton correspondence before we turn to the cases of interest in Type IIB string theory and M theory. In particular, the properties of the free boundary CFT's which indicate that the massless HS gauge theories in the bulk provide effective descriptions of the full HS gauge theories truncated to their massless sector will be emphasized.

Consider a CFT_d consisting of N' supersingletons W^i, where $i = 1, ..., N'$ is an internal index and each W^i belongs to some singleton representation of the superconformal group. Let each singleton belongs to an irreducible representation of some internal symmetry group G and consider G invariant composite operators $\mathcal{O}$. Our first basic assumption is that the correlation functions of invariant composite operators factorize as $N' \to \infty$. For example, by using the operator product expansion, a four-point function $< \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 >$ can be decomposed as

$$< \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 > = < \mathcal{O}_1 \mathcal{O}_2 > < \mathcal{O}_3 \mathcal{O}_4 > + < \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 >_{\text{conn}} , \tag{4.1}$$

$$< \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 >_{\text{conn}} = \sum_r \frac{< \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_r > < \mathcal{O}_r \mathcal{O}_3 \mathcal{O}_4 >}{< \mathcal{O}_r \mathcal{O}_r >} , \tag{4.2}$$

where the disconnected terms are the contributions from the unit operator and the connected terms are the contributions from the remaining operators. The factorization means that the the connected terms are suppressed by powers of $1/N'$:

$$\frac{< \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 >_{\text{conn}}}{< \mathcal{O}_1 \mathcal{O}_2 > < \mathcal{O}_3 \mathcal{O}_4 >} \to 0 \quad \text{as } N' \to \infty . \tag{4.3}$$

In general, there can be several parameters in addition to N' in CFT_d. Fortunately, supersymmetry puts considerable amount of constraint on these possibilities. With application to Type IIB string and M theory in mind, we shall assume that $G = SU(N)$ and consider $SU(N)$ valued singleton scalar superfields denoted by W^I, $I = 1, ..., n$. In this case we have $N' = N^2 - 1$ and the singletons transform in the fundamental representation of the R-symmetry group $SO(n)$. For the cases of interest, namely in $d = 3, 4, 6$, we have in mind the R-symmetry groups $SO(8), SO(6)$ and $SO(5)$, respectively, which correspond to 16 ordinary plus 16 special supersymmetries in the CFT_d. The $SU(N)$ valued singletons in $d = 4$ are adequate for discussing the tensionless limit of the Type IIB theory on $AdS_5 \times S^5$. The extent to which $SU(N)$ valued singletons in $d = 3, 6$ may encode the properties of (an unbroken phase of) M theory on $AdS_{4/7} \times S^{7/4}$ is discussed in Sections 6 and 7.

The basic composite operators in CFT_d are primary *bilinear* single-trace operators $\mathcal{O}_{(2)r}$, where the index r labels collectively the set of $SO(d, 2) \times R$-representations involved [7, 8]. These operators do not mix with any other operators and provide conserved HS currents with spin $s \geq 1$, and certain composite operators of lower spin $s < 1$. Together they form an HS multiplet

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that corresponds in a one-to-one fashion to an HS multiplet of physical massless bulk fields [7], $\phi_{(2)r}$. In the supersymmetric singleton models of special interest to Type IIB/M theory the bilinear primaries are discussed in Section 3 and the corresponding massless spectra are listed in Tables 1, 2 and 3.

The free CFT_d also contains composite operators which are pth order monomials in the basic singleton and its derivatives. Those composites which are not normal ordered products of other composites as $N' \to \infty$ are interpreted as massive single-particle states in AdS. We shall denote these operators and the corresponding massive bulk fields by $\mathcal{O}_{(p)r}$ and $\phi_{(p)r}$, respectively, where $p \geq 3$ and r is an additional set of indices labeling the $SO(d, 2) \times R$ weights. The massiveness means that there is no shortening of the associated $SO(d, 2)$ weight spaces. This implies that the massive operators are not conserved and hence there are no gauge symmetries associated with the corresponding massive AdS fields. However, as discussed in the previous section, some of the massive operators belong to shortened supermultiplets, provided that the superconformal weights saturate certain unitarity bounds or belong to discrete series. This is the case, for example, for 1/2 BPS KK modes and the Higgs multiplets listed in the previous section.

For fixed p the space of massive operators $\mathcal{O}_{(p)r}$ clearly decomposes into irreducible HS multiplets, though the representation theory of HS algebras, such as their root structure, has not yet been developed far enough to characterize the precise 'lowest' weights carried by these multiplets (see [20] for a discussion of this point).

Composite operators which are normal ordered products of other composite operators as $N' \to \infty$ are interpreted as many-particle states. In the case of $SU(N)$ valued singletons, the single-particle states, $\mathcal{O}_{(p)r}$ ($p = 2, 3, ...$) are given in the large N limit by single-trace operators. The n-particle states, which we shall denote by $\mathcal{O}_{(p_1,...,p_n)r}$ are given in this limit by multi-trace operators in the form of normal ordered products of single trace operators $\mathcal{O}_{(p_i)r_i}$ and their derivatives, $p_i = 2, 3, ..., i = 1, ..., n$.

For finite N there is mixing between the single-trace and multi-trace operators [7, 23]. This is because n-particle states in the bulk couple to operators that diagonalize the two-point function:

$$< \mathcal{O}_R \mathcal{O}_S > = \eta_{RS} , \tag{4.4}$$

where $R = (p_1, \ldots, p_n)r$ and η_{RS} is an N-independent diagonal matrix. For example, consider the minimal bosonic truncation based on a single $SU(N)$ valued singleton field W. The bilinear and tri-linear composites, which have to be single-traces, do not mix. However, the quartic composites do mix, and they do so as follows. The diagonal scalar states of energy $\Delta = 2d - 4$ are given schematically by

$$\mathcal{O}_{(4)} = J_{(4)} + f J_{(2,2)} , \quad \mathcal{O}_{(2,2)} = J_{(2,2)} - \frac{2f}{1 + f^2} J_{(4)} , \tag{4.5}$$

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[7]In the minimal bosonic truncation this dictionary has been extended to also include local currents corresponding to the auxiliary HS gauge fields of the bulk theory [8]. This offers an opportunity to compute bulk amplitudes in a first order formalism.

where $f(N) = \frac{a}{N} + \frac{b}{N^3}$, with a and b being some constants, and

$$J_{(4)} \sim \frac{1}{N^2} : \text{tr}(W^4) : , \qquad J_{(2,2)} \sim : \text{tr}(W^2)\text{tr}(W^2) : , \qquad (4.6)$$

are assumed to be normalized such that

$$< J_{(4)}J_{(4)} >= \Delta^4 , \quad < J_{(4)}J_{(2,2)} >= f(N)\Delta^4 , \quad < J_{(2,2)}J_{(2,2)} >= \Delta^4 , \qquad (4.7)$$

where $\Delta = |x|^{-d+2}$ is the singleton propagator.

Having introduced the main notation and kinematics, we now continue with the discussion of the factorization of correlators as $N \to \infty$. From (4.4) it follows that as $N \to \infty$ a general n-point correlator either vanishes if n is odd or can be written as the sum of products of $n/2$ two-point functions. Thus, in the limit $N \to \infty$ the singleton CFT_d describes an 'anti-holographic' bulk theory of free n-particle states corresponding to $\mathcal{O}_{(p_1,...,p_n)r}$. For finite N, the $1/N$ corrections to the singleton CFT give rise to nontrivial connected parts of the correlation functions which we wish to represent as anti-holographic interactions. To be more precise we wish to examine whether the $SU(N)$ valued singleton field theory is the holographic dual of an interacting $(d+1)$-dimensional theory based on an effective action, consisting of a bulk term plus a boundary term

$$\Gamma_{\text{eff}}[\phi_{(p)r}] = \Gamma_{\text{eff,bulk}}[\phi_{(p)r}] + \Gamma_{\text{eff,boundary}}[\phi_{(p)r}] , \qquad (4.8)$$

which admit perturbative expansions in powers of $1/N$ around an AdS_{d+1} vacuum[21]. The boundary term plays a role in representing certain correlators, such as the extremal correlators discussed below, that cannot be reproduced from a bulk action. This boundary term is needed because the variational principle requires $\Gamma_{\text{eff}}[\phi_{(p)r}]$ to be stationary when the fields are varied subject to Dirichlet conditions. The variation of $\Gamma_{\text{eff,boundary}}[\phi_{(p)r}]$ should therefore cancel the total derivatives from the variation of $\Gamma_{\text{eff,bulk}}[\phi_{(p)r}]$ that give rise to boundary terms that involve normal derivatives of the variations. For example, $\Gamma_{\text{eff,bulk}}[\phi_{(p)r}]$ is expected to contain an ordinary R-term for the spin 2 fields and consequently that $\Gamma_{\text{eff,boundary}}[\phi_{(p)r}]$ contains the corresponding Brown-York term.

In the case of 16 supersymmetries (4.8) can be expressed formally as

$$e^{i\Gamma_{\text{eff}}[\phi_{(p)r}(V)]} =< e^{i\sum_{p,r} \int d^d x d^{16}\theta \mathcal{O}_{(p)r}V_{(p)r}} > , \qquad (4.9)$$

where the effective action on the left hand side is evaluated subject to boundary conditions dictated by superconformal tensors $V_{(p)r}$. These superfields are prepotentials for super Weyl multiplets containing the boundary conditions on the AdS curvatures.

The correlators of composite operators on the right hand side of (4.9) are well-behaved functions of the insertion points as long as they are separated. However, as these points coincide, the correlators are in general rather badly behaved distributions. Thus a more careful definition of the generating functional of correlators requires the choice of a regularization scheme. This

leaves room for anomalous effects, even though the singleton theory is free, which may serve the purpose of selecting critical field content and number of supersymmetries. In other words, consistency of the right hand side of (4.9) in the case of a free SCFT in d dimensions with finite sources for composite operators should be about as restrictive as consistency of an interacting SCFT in d dimensions. Moreover, the fact that a successful definition of (4.9) in principle would give rise to a consistent bulk theory including quantum gravity [8] suggests that only the special supersymmetric singletons corresponding to limits of string/M theory will be viable in the above sense. Thus we shall assume that ultimately (4.9) makes sense only for free SCFTs in $d \leq 6$ with less than or equal to 16 supersymmetries [9]. We address these issues further below when we discuss the subleading $1/N$ corrections to the definition of the vacuum used in the correlator on the right hand side of (4.9).

The generating functional makes sense only as an asymptotic expansion in $1/N$ in which a given order is a formal power series expansion in $\phi_{(p)r}$, which has a finite radius of convergence by the combinatorial counting rules for double line diagrams of fixed topology. From the normalization (4.4) and assuming that $< \mathcal{O} >= 0$ it follows that as far as the $1/N$ counting goes the effective action has the form

$$\Gamma_{\text{eff}}[\phi] = \phi^2 + \frac{1}{N} f_3\left(\frac{1}{N^2}\right) \phi^3 + \frac{1}{N^2} f_4\left(\frac{1}{N^2}\right) \phi^4 + \cdots ,$$

$$f_n\left(\frac{1}{N^2}\right) \sim 1 + \mathcal{O}\left(\frac{1}{N^2}\right) . \tag{4.10}$$

The singleton field theory determines $\Gamma_{\text{eff}}[\Phi_{(p)r}]$ up to non-linear field redefinitions of the type $\phi \rightarrow \phi + \frac{1}{N}\phi^2 + \cdots$. After rescaling the fields as

$$\phi = N\Phi , \tag{4.11}$$

we define the classical action as follows

$$\Gamma_{\text{eff}}[\Phi] = \Gamma_{\text{cl}}[\Phi] + \mathcal{O}(1/N^2) , \tag{4.12}$$

$$\Gamma_{\text{cl}}[\Phi] = \frac{N^2}{R^{d-1}} \int d^{d+1}x \mathcal{L}(\Phi, R\partial\Phi, (R\partial)^2\Phi, \ldots) + \text{boundary term} , \tag{4.13}$$

[8]As the basic mechanism behind holography is general covariance, this raises the question whether holography exhibits any new features as general covariance is extended by HS symmetries. To analyze this, we presumably need to refine our present, mainly algebraic, understanding of HS symmetries by formulating these in a more geometric language, perhaps by extending the set of spacetime coordinates as to realize HS gauge transformations as extended reparametrizations [11].

[9]Massless HS fields admit background independent self-interactions in $D = 4$, and it is most likely that this is the case for all D (though interactions in $D > 7$ bring in symplectic spacetime symmetries). However, the theories of massless HS fields in higher dimensions are presumably not consistent truncations of quantum consistent theories.

where R is the AdS radius. We can now state the properties of the HS gauge theories as follows. They possess:

a) a set of one-particle states forming HS multiplets

b) a corresponding set of 'vertex operators' of a free CFT_d;

c) a fundamental mass scale, $1/R$ where R is the AdS radius, and a fundamental expansion parameter, l_{Pl}/R where the Planck length l_{Pl} determines the normalization of the effective AdS action to be [10]

$$\frac{1}{l_{\text{Pl}}^{d-1}} = \frac{N'}{R^{d-1}} \ . \tag{4.14}$$

Given these facts we would like to determine the effective action $\Gamma_{\text{eff}}[\phi_{(p)r}]$ from a set of bulk interactions, without any direct reference to the boundary singleton. The basic issue is whether the interactions can be derived from a string or membrane sigma model, that can be coupled to the HS background fields. The mass-scale of the HS spectrum is set by the AdS radius R, which is suggestive of a sigma-model with a fixed critical tension of order 1 in units where the AdS radius $R = 1$, as we shall discuss further in Section 5,6 and 7.

Due to the absence of mass-gap it is not possible to separate the massless fields, $\phi_{(2)r}$, from the massive AdS fields, $\phi_{(p)r}$, $p > 2$, by taking a low energy limit. In a local process in AdS with energies of the order $E \sim n/R$, $n >> 1$, the massive modes with $E_0 < n/R$ behave essentially as the KK modes which arise in an AdS compactification of string/M theory. Thus the only reasonable possibility in which the massless modes can be separated from the massive modes in a HS theory is by consistent truncation to the massless sector [11], which is similar to what happens in the (maximally supersymmetric) sphere compactifications of Type IIB and eleven-dimensional supergravities. There are examples, however, of compact manifolds, such as $T^{1,1}$, where the higher dimensional supergravity theory does not admit a consistent truncation despite the fact that there does exist a lower-dimensional gauged supergravity. [12]

Thus we propose that the HS gauge theories in $D = 4, 5, 7$ with gauge groups $hs(8|4)$, $hs(2, 2|4)$ and $hs(8^*|4)$ admit consistent truncation down to the corresponding massless theories, which we described in Section 2. This consistent truncation can be directly tested by verifying that the massless bulk theory reproduces exactly the correlators of the corresponding bilinear operators in the singleton theory. This is a nontrivial test since nothing is known about higher-dimensional covariant description of the HS theory so far.

Consistent truncation of the full HS gauge theory to its massless sector requires that there are no terms in the effective bulk action of the form $\int \phi_{(p)} \phi_{(2)} \cdots \phi_{(2)}$ for $p \geq 3$. Let us show this in the

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[10]In the case of $SU(N)$ valued singletons $N' = N^2 - 1$, which means that the Planck constant in the bulk is given by $\hbar = 1/N^2$. The $1/N$ corrections to the bulk theory are therefore weighted by positive integer powers of the Planck's constant.

[11]We thank L. Rastelli for helpful discussions on this point.

[12]We thank C. Pope for pointing this to us.

case of scalar bulk fields. Then the corresponding singleton correlators are non-zero provided that $\Delta_{(p)} \leq n\Delta_{(2)}$ where $n \geq 2$ is the number of massless fields. The case $\Delta_{(p)} = n\Delta_{(2)}$ is called an extremal correlator. The extremality condition implies $p = 2n$ and in that case it is straightforward to use free field contraction rules to show that

$$< \mathcal{O}_{(p)}(x)\mathcal{O}_{(2)}(x_1)\cdots\mathcal{O}_{(2)}(x_n) >= \prod_{i=1}^{n}(\Delta(x - x_i))^2 \ , \tag{4.15}$$

where $\Delta(x) = |x|^{-d+2}$ is the singleton propagator. Consider, on the other hand, the bulk integral

$$I = \int \frac{d^{d+1}z}{z_0^{d+1}} K_{\Delta_{(p)}}(z,x) K_{\Delta_{(2)}}(z,x_1)\cdots K_{\Delta_{(2)}}(z,x_n) \ , \tag{4.16}$$

where $K_{\Delta}(z,x_i)$ is the standard bulk-to-boundary propagator. $K_{\Delta}(z,x_i) \sim z_0^{d-\Delta}\delta^d(z - x_i)$ for small z_0, and as $z \to x_i$,

$$I \sim \int \frac{dz_0}{z_0} z_0^{-\Delta+n\Delta_{(2)}} \prod_{i=1}^{n}(\Delta(x - x_i))^2 \ . \tag{4.17}$$

Thus, in the extremal case this integral diverges logarithmically, and the residue of the pole, treating Δ as a variable, has the same structure as the extremal correlation function. By assumption, the anti-holographic dual should, however, give rise to finite amplitudes. The resolution is that a term which diverges logarithmically is scale-invariant, which means that it can be represented equivalently by a boundary term which is finite. Thus extremal correlators give rise to couplings that are boundary terms and therefore they do not upset consistent truncation.

A similar argument applies to the near-extremal case, when $d - 2 < \Delta < n\Delta_{(2)}$. Here the integral I is finite, but the dependence on the x's is not of the same form as the singleton CFT correlator. There are exchange diagrams, though, with the correct structure of the x-dependence [5]. Thus the near-extremal correlators must be represented anti-holographically in terms of exchange diagrams, and there cannot be any contact term in the bulk action that can upset consistent truncation we are examining.

The above evidence for consistent truncation is similar to the one given for ordinary Type IIB supergravity $AdS \times S^5$ [25, 26] and eleven dimensional supergravity on $AdS_{4/7} \times S^{7/4}$ [27]. The main difference is that whereas the arguments in SUGRA only holds for 1/2 BPS states, the arguments given here for HS theory hold for more general operators since the holographic dual is by assumption a singleton.

To provide further evidence for consistent truncation, we examine the correlator of four massless scalar operators $\mathcal{O}_i = \mathcal{O}_{(2)}(x_i)$, $i = 1,...,4$. Using free field theory contraction rules it can be written on manifestly crossing symmetric form as

$$< \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4 > \quad = \quad \eta_{12}\eta_{34} + \eta_{14}\eta_{23} + \eta_{13}\eta_{24} + < \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4 >_{\text{conn}} \ , \tag{4.18}$$

$$< \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4 >_{\text{conn}} = A^{(s,t)}_{1234} + A^{(t,u)}_{1324} + A^{(u,s)}_{1243} , \tag{4.19}$$

where

$$A^{(x,y)}_{ijkl} =<: \mathcal{O}_i\mathcal{O}_k : \; : \mathcal{O}_j\mathcal{O}_l :>_{\text{conn}} \tag{4.20}$$

and x and y denote in which of the s-, t- and u-channels the quantity $A^{(x,y)}_{ijkl}$ has singularities. In the limit x_{12}, $x_{34} \to 0$, the correlator can be expanded in the s-channel by using the OPE

$$\mathcal{O}_1\mathcal{O}_2 = \eta_{12} + C_{12}{}^{(2)r}\mathcal{O}_{(2)r}(x_2) + C_{12}{}^{(4)r}\mathcal{O}_{(4)r}(x_2) + C_{12}{}^{(2,2)r}\mathcal{O}_{(2,2)r}(x_2) , \tag{4.21}$$

where we recall that $\mathcal{O}_{(2)r}$ denotes the set of all primary bilinear single-trace operators labeled by an index r, and $\mathcal{O}_{(4)r}$ and $\mathcal{O}_{(2,2)r}$ are as given in (4.5). The resulting s-channel expansion is given by

$$< \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4 >_{s-\text{ch}} = \eta_{12}\eta_{34} + C_{12}{}^{(2)r}C_{34,(2)r} + < \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4 >_{s-\text{ch,finite}} , \tag{4.22}$$

where $C_{RST} = C_{RS}{}^U\eta_{UT} =< \mathcal{O}_R\mathcal{O}_S\mathcal{O}_T >$, and $< \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4 >_{s-\text{ch,finite}}$, which is finite in the s-channel, is given by

$$< \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4 >_{s-\text{ch,finite}} = <: \mathcal{O}_1\mathcal{O}_2 : \; : \mathcal{O}_3\mathcal{O}_4 :>= \eta_{13}\eta_{24} + \eta_{14}\eta_{23} + A^{(t,u)}_{1324} , \tag{4.23}$$

$$A^{(t,u)}_{1324} = C_{12}{}^{(4)r}C_{34,(4)r} + C_{12}{}^{(2,2)r}C_{34,(2,2)r} . \tag{4.24}$$

The structure of (4.24) appears to be problematic for consistent truncation, and cannot be ignored in the large N limit as follows from

$$C_{(2)(2)}{}^{(2)r} \sim \frac{1}{N} , \quad C_{(2)(2)}{}^{(4)r} \sim \frac{1}{N} , \quad C_{(2)(2)}{}^{(2,2)r} \sim 1 . \tag{4.25}$$

It is possible, however, to write (4.24) in a more tractable form as a manifestly crossing symmetric sum of terms involving only exchange of bilinear operators. To this end we first note that the crossing symmetry of the singleton theory implies that the complete s-channel expansion (4.22) is equal in the sense of analytical continuation to the complete t- and u-channel expansions in the limits x_{14}, $x_{23} \to 0$ or x_{13}, $x_{24} \to 0$, respectively. Thus (4.22) must contain contributions that are singular in the t- and u-channels. From the form of (4.19) we therefore deduce that the singular part of (4.22) actually must consist of two separate contributions, one which becomes singular in the t-channel and another one which becomes singular in the u-channel. We also see that the problematic term in (4.22) must have singularities in both the t- and u-channels, which by crossing symmetry should describe massless exchanges. In fact, from (4.21) and free field theory contraction rules it follows that

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$$A_{1234}^{(s,t)} \equiv \; <: \mathcal{O}_1 \mathcal{O}_3 : \; : \mathcal{O}_2 \mathcal{O}_4 :>_{\text{conn}} = \tfrac{1}{2} C_{12}^{(2)r} C_{34,(2)r} + \tfrac{1}{2} C_{32}^{(2)r} C_{14,(2)r} \;, \qquad (4.26)$$

$$A_{1324}^{(t,u)} \equiv \; <: \mathcal{O}_1 \mathcal{O}_2 : \; : \mathcal{O}_3 \mathcal{O}_4 :>_{\text{conn}} = \tfrac{1}{2} C_{13}^{(2)r} C_{24,(2)r} + \tfrac{1}{2} C_{14}^{(2)r} C_{23,(2)r} \;, \qquad (4.27)$$

$$A_{1243}^{(u,s)} \equiv \; <: \mathcal{O}_1 \mathcal{O}_4 : \; : \mathcal{O}_2 \mathcal{O}_3 :>_{\text{conn}} = \tfrac{1}{2} C_{12}^{(2)r} C_{43,(2)r} + \tfrac{1}{2} C_{13}^{(2)r} C_{42,(2)r} \;. \qquad (4.28)$$

To compute the first term in (4.26) we use (4.21) to expand the single contraction connecting $\mathcal{O}_1$ to $\mathcal{O}_2$ in terms of $C_{12}^{(2)r} \mathcal{O}_{(2)r}$ and similarly for 3 and 4. The remaining two contractions that contribute to the connected part give rise to $\tfrac{1}{2} \eta_{rs}$, where the factor of $\tfrac{1}{2}$ arises due to the normal ordering prescription which forbids contractions connecting 1 with 2 and 3 with 4, respectively. The second term in (4.26) contains the single contractions connecting 1 to 4 and 3 to 2. The relations (4.27)and (4.28) are obtained analogously. Eqs. (4.26)and (4.28) imply that the finite contribution (4.24) can be rewritten in terms of partial wave expansions involving only exchange of bilinear operators in the crossed channels. Thus the complete four-point correlator can be written in a manifestly crossing symmetric form involving only massless partial waves:

$$< \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 > \; = \; \eta_{12} \eta_{34} + \eta_{23} \eta_{14} + \eta_{24} \eta_{13} + < \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 >_{\text{conn}} \;, \qquad (4.29)$$

$$< \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 >_{\text{conn}} \; = \; C_{12}^{(2)s} C_{34,(2)s} + C_{23}^{(2)t} C_{14,(2)t} + C_{13}^{(2)u} C_{42,(2)u} \;. \qquad (4.30)$$

This generalizes so that any correlator of bilinear operators can be written as a manifestly channel duality invariant sum of conformal blocks involving only exchange of bilinear operators.

The test of holography requires that the result (4.30) is consistent with that obtained from the corresponding Witten diagram that uses the classical action of the HS gauge theory truncated to its massless sector. Now it has been shown in [68] that a Witten diagram with four external scalars and exchange of an internal scalar ϕ equals the sum of the conformal block with exchange of the scalar operator $\mathcal{O}$ coupling to ϕ *plus* terms which have the same structure as, but do not exactly agree with, the conformal blocks with exchange of operators corresponding to the two-particle states formed out of the external scalar states. In [68] it has also been shown that a Witten contact diagram with four external scalars has the form of two-particle exchange.

Thus (4.30) has the form required by consistent truncation, provided that the quartic bulk interactions in the cases of interest lead to cancellation of the parts in bulk four-point amplitudes that have the structure of conformal blocks with massless two-particle state exchange. The remaining terms, which come from the Witten diagrams with exchange of massless bulk fields, can then be written in manifestly *s-t-u* channel duality invariant form as conformal blocks with exchange of the corresponding bilinear operators, as in (4.30). Thus, the bulk side of the story remains to be established. It would be interesting to examine to what extent the requirement that two-particle partial waves must cancel determines the structure of higher order interactions in the action for massless fields. We shall return to this point below in discussing the interaction ambiguity in the massless sector.

Having gathered evidence for the consistent truncation, let us now proceed to explore some of its consequences. In the above discussion, we have implicitly made the assumption that the correlators in the free singleton theory are given by ordinary vacuum expectation values on a conformal plane. Let us assume that this is indeed correct in the large N limit. The generating functional for correlators of bilinear operators is then given for large N by a one-loop functional determinant, i.e. the connected n-point correlators are planar diagrams that scale like $N^{-(n-2)}$. Thus, the effective bulk action for the massless fields is 'classical' and takes the form

$$\Gamma_{\rm cl}[\Phi_{(2)r}] = \frac{N^2}{R^{d-1}} \int d^{d+1}x \mathcal{L}(\Phi_{(2)r}, R\partial\Phi_{(2)r}, (R\partial)^2\Phi_{(2)r}, \ldots) + \text{boundary terms} , \qquad (4.31)$$

where R is the AdS radius. The Lagrangian contains higher derivative interactions and the quadratic part is ghost and tachyon free. It is important to note that the quantity $R\nabla$ is not small in an expansion around AdS.

By construction, both $\Gamma_{\rm cl}[\Phi_{(2)r}]$, and the full classical action $\Gamma_{\rm cl}[\Phi_{(p)r}]$ defined in (4.12), reproduce the correlators of bilinear operators holographically to the leading orders in the $1/N$ expansion, i.e. the extrema of the two actions are equal provided the massive modes $\Phi_{(p)r}$, $p \geq 3$ are set to zero at the boundary of AdS. The consistent truncation can now be phrased as the stronger condition

$$\Gamma_{\rm cl}[\Phi_{(2)r}] = \Gamma_{\rm eff}[\Phi_{(p)r}, \Phi_{(3)} = 0, \ldots] . \qquad (4.32)$$

This offers the following possibility to test consistent truncation directly. Based on the results in $D = 4$, we expect that HS gauge symmetry together with the requirement of manifest local Lorentz symmetry determines a family of actions

$$S[\phi_{(2)r}; \mathcal{V}] \qquad (4.33)$$

for massless fields where $\mathcal{V}$ represents a set of arbitrary parameters. As explained in Section 6.2, and in more detail in [19], there exist an interaction ambiguity in the $4D$ HS gauge theory which involves the introduction of an odd function $\mathcal{V}(x) = \sum_{n=1}^{\infty} b_{2n+1} x^{2n+1}$. Already the simplest choice $\mathcal{V}(x) = b_1 x$ gives rise to a highly nontrivial model with a structure of the type indicated in (4.31). The n'th order term in $\mathcal{V}(x)$ results in higher order derivative corrections starting at order $2n + 2$ in the Lagrangian. Thus, in $D = 4$ the consistent truncation (4.32) implies a specific choice $\mathcal{V}(x) = \mathcal{V}_\Gamma(x)$ such that

$$\Gamma_{\rm cl}[\phi_{(2)r}] = S[\phi_{(2)r}; \mathcal{V}_\Gamma] . \qquad (4.34)$$

Thus, consistent truncation means that there exists a set of parameters $\mathcal{V}_\Gamma$ for which the extremum of $S[\phi_{(2)r}; \mathcal{V}_\Gamma]$ corresponds to the generating functional of correlators of bilinear operators in the singleton theory. A perturbative scheme for obtaining the interactions in $D = 4$ to any desired order is given in [18, 19], and described in Section 6 for the case of quadratic terms in the field equations. We are still lacking the description of the full interactions for massless

fields in $D > 4$, though we expect that the basic building blocks are of the kind described in [9, 12, 10, 20].

The supersymmetric HS gauge theories in $D = 4, 5, 7$ can be truncated consistently to a minimal bosonic HS theory with massless and massive fields. Moreover the massless minimal bosonic theory is a consistent truncation of the massless supersymmetric HS theory. Hence, if the truncation of the massive modes is consistent in the supersymmetric theory then this must also be the case in the minimal bosonic theory. In particular, in $D = 4$ the interaction ambiguities in the supersymmetric theory and the minimal bosonic theory are parametrized by the same function $\mathcal{V}$.

Let us now examine more closely the qualitative behavior of the $1/N$ dependence of some singleton correlators involving higher than second order traces of singletons. For example, the connected part of the correlator of four cubic scalar single-trace operators, $\mathcal{O}_{(3)} \sim \frac{1}{N^{3/2}} : \mathrm{tr}(W^3) :$ (these operators do not mix with any other operators) contains both planar and non-planar double-line graphs which scale like $1/N^2$ and $1/N^4$, respectively, in the large N limit. Another interesting example is the correlator of three $\mathcal{O}_{(4)}$ operators, which contain $1/N$ and $1/N^3$ contributions. In the supersymmetric case, one can arrange the cyclic orders of R-symmetry indices carried by the singletons to cancel the leading $1/N$ contribution, and thus the corresponding cubic coupling in the effective action [22]. Hence, the full singleton theory encodes information about a nontrivial $1/N$ expansion of the anti-holographic dual.

As we have already mentioned, we think of these corrections as being generated by a quantum theory in the bulk which is generated by a string theory or some other sigma model which can be coupled to the massless HS fields. From this point of view, it would be natural to have subleading $1/N$ corrections also to the interactions in the massless sector, so that (4.31) would only be valid for large N. We would also expect corrections to $\Gamma_{\mathrm{eff}}[\Phi_{(p)r}]$ which violate the consistent truncation (4.32). These effects do not arise, however, if we treat the correlators in the singleton theory as ordinary vacuum expectation values of operators inserted on the conformal plane.

We conclude this section by speculating on possible subleading in $1/N$ corrections to the free singleton correlators on the right hand side of (4.9). To this end, let us assume that the free singleton theory in question is an actual limit of a CFT describing the low energy dynamics of open string modes in string theory or 'open membrane' modes in M theory. For concreteness, let us consider the case of the $SU(N)$ invariant singleton theory that arises as a limit of the $d = 4$, $\mathcal{N} = 4$ SYM theory. For finite open string length the prescription for computing open string theory amplitudes is to attach open string vertex operators to open string boundaries and sum over all open string fluctuations. This includes virtual processes including formation of closed string loops. A closed string loop can be created by inserting a 'sewing operator'

$$R_s = \sum_s V_s(z) \bar{V}^s(0) \tag{4.35}$$

on the string worldsheet where the sum runs over a complete set of physical closed string states. In taking the low energy limit leading to the conformal SYM theory, the physical effect of the

sewing operation is included into the $1/N$ expansion of the SYM theory with finite g_{YM}^2. Thus the limit $g_{YM}^2 \to 0$ is not smooth in the sense that the closed string sewing operations, which are present for any finite g_{YM}^2 are absent for $g_{YM}^2 = 0$, simply because there are no virtual processes in the singleton theory that leads to the addition of internal 'handles' in the $1/N$ expansion. This is reminiscent of the fact that the deformation of the free singleton theory corresponding to switching on finite g_{YM}^2 cannot be described directly at the level of the composite operators built from the singleton superfield, which contains the abelian field strength but not any explicit gauge potential. In fact, this requires that we introduce gauge couplings by hand, after which g_{YM}^2 can be shifted to any finite value by marginal deformations.

The above arguments suggest that we modify the definition of the generating functional in the singleton theory by working with full singleton correlators given schematically by

$$< \mathcal{O}_1 \cdots \mathcal{O}_n >_{\text{full}} \quad = \quad \sum_k \frac{1}{k!} < R^k \mathcal{O}_1 \cdots \mathcal{O}_n > , \tag{4.36}$$

where the (super)conformally invariant singleton sewing operator R is defined as the sum over a complete set single trace operators describing a virtual closed string process:

$$R \quad = \quad \sum_p \int \frac{d^d x \, d^d y}{|x - y|^{2d}} \eta^{rs}(x - y) \mathcal{O}_{(p)r}(x) \mathcal{O}_{(p)s}(y) . \tag{4.37}$$

Since each power of R adds an extra power of $1/N^2$, the above definition does not affect the classical limit though it yields the desired nontrivial subleading $1/N$ corrections to the correlators. The insertion of R formally corresponds to taking a trace, which in turn implies that the correlation function becomes periodic along a cycle on the conformal plane. In string theory, R_s acts similarly, and has the geometric effect of adding a handle to the two-dimensional worldsheet. This suggests that R insertions describe large fluctuations of the D3 brane worldvolume in the singleton limit. As in the closed string theory, the consistency of the sewing operation in the free singleton theory may lead to restrictions on the spacetime superdimension.

In summary, we propose to use HS symmetries in diverse dimensions to determine actions (or field equations) for massless HS multiplets up to certain well-defined interaction ambiguities and then to compare the resulting Witten amplitudes with correlators of bilinear operators in corresponding large N singleton theories. The next step in this program is to explain the consistent singleton/HS correspondences as limits of string and M theories, which in particular require the identifications of possible schemes for breaking HS symmetries.

We emphasize that the tests of CFT/AdS in the HS regime involve a free CFT on the boundary, unlike the tests in the supergravity regime where the boundary CFT is strongly coupled. This is possible due to the proposed consistent truncation and the fact that there still remains the expansion parameter $1/N$.

It is not clear exactly how the state of affairs will change once the HS symmetries are broken. In Section 3 we have identified candidate Higgs multiplets in $d = 3, 4$. Presumably this can be done also in $d = 6$ provided that we develop the proper mathematical language for describing

the interactions on the M5 brane. In general, we expect that the Higgsing upsets the consistent truncation to the massless sector alone. Moreover, it is not obvious if there exists a generalized consistent truncation scheme that retains the massless, Higgs and other relevant massive fields. In any event, it will be interesting to see whether HS field theoretic methods can be used to describe the Higgsing or one has to resort to some more basic definition of the bulk interactions, based on some sigma model. We believe it is too early to make any conclusive remarks on this, though it seems possible to describe couplings between massless HS fields and Higgs fields, which should form HS multiplets fitting into master fields of the type discussed in Section 2.

In Sections 5-7 we shall discuss these issues in more detail, and case by case for the theories described in Section 2.

5 Type IIB on $AdS_5 \times S^5$ and 5D Higher Spin Gauge Theory

According to the strong version of the Maldacena conjecture [1, 4, 5] $d = 4$, $\mathcal{N} = 4$ SYM theory with $SU(N)$ gauge group, gauge coupling g_{YM}^2 and 't Hooft coupling $\lambda = Ng_{\mathrm{YM}}^2$ is equivalent to Type IIB string theory on $AdS_5 \times S^5$ of radius R with string coupling g_s and string length l_s given by

$$
\begin{aligned}
g_s &= f_1(\lambda)g_{\mathrm{YM}}^2 , & f_1(\lambda) &\sim 1 & &\text{for } \lambda >> 1 , \\
l_s &= f_2(\lambda)R , & f_2(\lambda) &\sim \lambda^{-1/4} & &\text{for } \lambda >> 1 .
\end{aligned}
\tag{5.1}
$$

For large λ, these relations are deduced by interpolating between the $AdS_5 \times S^5$ vacuum with radius R and dilaton $e^\phi = g_s$, and the ten-dimensional Minkowski vacuum, using the classical D3-brane solution with harmonic function $H(r) = 1 + 4\pi Ng_s l_s^4 r^{-4}$. The functions $f_{1,2}(\lambda)$ account for possible string corrections to the interpolating region, where only 16 supersymmetries are preserved. The Type IIB string/4d SYM correspondence is an AdS/CFT correspondence whereby the 4d SYM theory is identified as the holographic dual of the Type IIB closed string theory. The closed string theory is based on a non-linear sigma-model with coupling constant l_s/R. A (dimensionless) closed string amplitude $A^{(\mathrm{str})}$ has the doubly asymptotic expansion

$$
A^{(\mathrm{str})} = \sum_{g=0}^\infty g_s^{2g-2} A_g^{(\mathrm{str})}(l_s/R) ,
\tag{5.2}
$$

where the amplitude $A_g^{(\mathrm{str})}(l_s/R)$, which is obtained from worldsheet perturbation theory on a Riemann surfaces of fixed genus g, is given by an asymptotic expansion in l_s/R. The 5D Planck length is given by

$$
\frac{1}{l_{\mathrm{Pl}}^3} = \frac{N^2}{R^3} .
\tag{5.3}
$$

Thus the perturbative string expansion in $AdS_5 \times S^5$ makes sense provided that

$$N >> 1 \ , \quad g_s << 1 \ , \quad l_s << R \ . \tag{5.4}$$

The 't Hooft expansion of the corresponding correlation function $A^{(\text{SYM})}$ in the SYM theory reads

$$A^{(\text{SYM})} = \sum_{g=0}^{\infty} N^{2-2g} A_g^{(\text{SYM})}(\lambda) \ , \tag{5.5}$$

where the amplitude $A_g^{(\text{SYM})}(\lambda)$ is obtained from double-line Feynman graphs with fixed topology and is given by an analytical expansion in λ. Hence the conjectured correspondence $A^{(\text{str})} = A^{(\text{SYM})}$ can be examined order-by-order in string loop expansion and SYM $1/N$ expansion, leading to a set of strong/weak coupling dualities between $A_g^{(\text{str})}(l_s/R)$ and $A_g^{(\text{SYM})}(\lambda)$.

As discussed earlier, it has been proposed that the HS gauge theory emerges in the description of the Type IIB string theory on $AdS_5 \times S^5$ in the limit [22, 23, 9, 21]

$$g_s \to 0 \ , \quad l_s \to \infty \ ; \quad N >> 1, \quad R \text{ fixed} \ . \tag{5.6}$$

In this limit the dual free SYM theory is described by an $SU(N)$ valued $d = 4$, $\mathcal{N} = 4$ SYM singleton. As discussed in the previous section, the bulk physics is conjectured to be an HS gauge theory in $5D$ which admits a consistent truncation to an effective action $\Gamma_{\text{cl}}[\Phi_{(2)r}]$ for massless fields. The HS gauge group $hs(2,2|4)$ and its massless gauge theory has been described in [10]. We emphasize that there should be direct agreement between the individual terms in the $1/N$ expansions of massless gauge theory amplitudes and the correlators of bilinear currents in the free CFT as described in (4.34) (without having to first obtain strong coupling results).

There still remains the task of constructing the full interacting HS gauge theory in $5D$, though cubic interactions for massless spin $s = 2, 4, 6, ...$ fields have already been constructed in [12]. These form a subset of the cubic interactions of the minimal bosonic truncation S_{bos} of $S[\phi_{(2)}; \mathcal{V}]$ provided that it is consistent to set the scalar field ϕ in S_{bos} equal to zero at the cubic level. This requirement means that S_{bos} must not have any cubic interactions that are linear in ϕ and quadratic in spin $s \geq 2$ fields. On the other hand, from the known stress-energy tensor OPEs (see, for example, eq. (4.58) in [7]), it follows that the effective action $\Gamma_{\text{eff}}[\phi_{(2)}]$ should give rise to a non-zero cubic graviton-graviton-scalar amplitude. Thus the scalar can only be consistently truncated at the cubic level if this amplitude is represented by a boundary term in $\Gamma_{\text{eff}}[\phi_{(2)}]$, i.e. if the correlator in question is extremal or near-extremal. Whether or not this is the case remains to be seen.

We next discuss breaking of the HS symmetry. The level $\ell = 0$ supergravity multiplet of the massless spectrum of the $hs(2,2|4)$ theory contains a dilaton, φ which is an $SU(4)$ singlet with energy $\Delta = 4$ and AdS mass $m^2 = 0$. Since $m^2 = 0$ it is consistent to give φ a VEV in the linearized theory, and we shall assume that this is possible also in the full HS gauge theory. This

corresponds to switching on a finite g^2_{YM} in the 4d SYM theory. As result the 4d supercovariant derivative D^i_α becomes also gauge covariant. This does not upset the stress-energy conservation law (3.17), as it is first order in the superderivative, while it breaks the Konishi multiplet conservation law (3.19), which is second order in derivatives. Using the relation

$$D^{ij}W^{kl} = -2g_{\mathrm{YM}}[W^{k(i}, W^{j)l}] \ , \tag{5.7}$$

which follows from the superspace formulation of the $\mathcal{N} = 4$ SYM system in 4d, one finds that the anomalous conservation law for the Konishi current is given by (see, for example, [61, 65]):

$$D^{ij}J = \frac{4g_{\mathrm{YM}}}{N}\mathrm{tr}\, W^{k(i}W^{j)l}W_{kl} \equiv \sqrt{\lambda}\Sigma^{ij} \ . \tag{5.8}$$

The operator Σ^{ij} belongs to the massive Higgs multiplet with $s_{\mathrm{max}} = \frac{7}{2}$ discussed in Section 3. Thus, for finite g^2_{YM} the anomalous conservation law (5.8) describes how Σ^{ij} is 'eaten' by the massless Konishi operator J to form a massive operator which belongs to the long massive Konishi multiplet with $s_{\mathrm{max}} = 4$ containing 2^{16} states. The coupling between the corresponding bulk fields, which are described on the boundary by prepotentials V and V_{ij}, and the massless Konishi operator J and its Higgs descendant Σ^{ij} is described by

$$S_{\mathrm{boundary}} = \int d^4x d^{16}\theta \left(JV + \Sigma^{ij}V_{ij}\right) \ . \tag{5.9}$$

For finite g^2_{YM}, the action S_{boundary} is invariant under modified gauge transformations involving a Stückelberg shift transformation of the massive Higgs field,

$$\delta V = D^{ij}\Lambda_{ij} \ , \quad \delta V_{ij} = -\sqrt{\lambda}\Lambda_{ij} \ . \tag{5.10}$$

We thus expect that for finite $< \varphi >= g_s$ the effective action $\Gamma_{\mathrm{eff}}[\phi_{(p)r}]$ contains kinetic terms of the schematic form $|d\phi_{(2)}|^2 + |d\phi_{(3)} + \sqrt{\lambda}\phi_{(2)}|^2$, describing a single massive gauge field with non-critical mass [21]

$$m^2 - m^2_{\mathrm{crit}} \sim \frac{\lambda}{R^2} \ , \tag{5.11}$$

where $(D^2 - m^2_{\mathrm{crit}})\phi = 0$ for an AdS massless field ϕ.

As discussed in Section 3, the massive spectrum also contains 1/2 BPS massive states that have the interpretation of KK modes built on the massless HS multiplets. We shall assume that the Higgs mechanism can be described at the level of KK towers as well, and that the remaining massive HS multiplets can be organized into massive HS multiplets and their KK towers. This picture is suggestive of a covariant theory in $D = 10$ with 'critical' length scale l_{10} and coupling constant $g = 1/N$ which admits $AdS_5 \times S^5$ with radius $R = l_{10}$ as a vacuum. Since HS interactions in AdS spaces blow up in the flat limit for finite g, we do not expect the

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10D HS theory to admit 10D Minkowski space as a vacuum for finite $g > 0$. For $g = 0$ we get a quadratic Lagrangian, however, which is second order in derivatives, and as it contains no positive powers of R, it does admit a flat space limit. Thus, the tensionless limit of the Type IIB string theory in 10D flat spacetime is trivial.

Higgsing of the critical theory leads to a non-critical theory with $l_{10} < R$ which for $l_{10} << R$ should be identified with Type IIB string theory in $AdS_5 \times S^5$ with $l_s \sim l_{10}$. For small l_s/R the spectrum of string states with AdS energy (measured in units of $1/R$) satisfying the condition $E << R^2/l_s^2$, and spin $s << R^2/l_s^2$, can be obtained by KK reducing the 10D Minkowski space spectrum on S^5 by means of group theoretical methods (at the classical level these states are described by 'short' strings with energy $E = Rl/l_s^2$ and length $l << R$). In particular, for fixed $SO(4) \times SO(6)$ highest weight, the worldsheet Hamiltonian has a ground state which is the 'lightest' state carrying that highest weight. The lightest states states correspond to the leading Regge trajectory in 10D Minkowski space and form supermultiplets in both 10D Minkowski space and in $AdS_5 \times S^5$ with $s_{\max} = 2, 4, 6, ...$ In 10D Minkowski space these arise at closed string level $\ell = \frac{1}{2} s_{\max} - 1$ (see, for example, [69]), where all multiplets are massive except for level $\ell = 0$ where the supergravity multiplet resides. For example, the lightest $s_{\max} = 4$ multiplet is the massive Konishi multiplet which resides at level $\ell = 1$.

As l_s/R varies from $l_s/R << 1$ to $l_s/R >> 1$ the different Regge trajectories do not mix [6] even though the five-form flux and other terms of order 1 in units of R will become comparable to the mass-term. This follows from the fact that in an exact CFT that admits a perturbative formulation, such as the worldsheet theory and the boundary SYM theory, there cannot be mixing between two operators that do not mix in the free theory. Note that such an admixture would require the introduction of a mass-parameter in the perturbative formulation, which is not compatible with conformal invariance.

Indeed, there is an exact agreement between the supermultiplet structures of the leading Regge trajectory for large string tension and the set of massless states of the critical $hs(2, 2|4)$ theory, such that the level ℓ multiplet on the leading Regge trajectory flows, after reversed Higgsing, to the level ℓ multiplet of the massless spectrum given in Table 2.

We have already argued in Section 4 and 5 that there should exist a consistent truncation of the full $hs(2, 2|4)$ theory down to its massless sector. There is no analogous truncation of the non-critical string theory down to the leading Regge trajectory because the lightest states of level $\ell \geq 1$ consist of massless states plus Higgs states. The Higgs states belong to the massive sector of the $hs(2, 2|4)$ theory and therefore break the consistent truncation.

Since the HS symmetries are broken spontaneously it would be interesting to construct a HS field theoretic description in AdS of the couplings between the massless fields and the Higgs fields. Clearly the master field formalism described in Section 2 should be useful in doing this, though one presumably needs to invoke some additional information, perhaps from the structure of the factorization of the SYM correlation functions for $\lambda << 1$. Thus we should try to find a HS action $S(\Phi_{(2)r}, H_r; \mathcal{V}, \mathcal{M})$ for massless fields $\Phi_{(2)r}$ and Higgs fields H_r, where $\mathcal{V}$ are the parameters describing the gauge interactions, as will be discussed in Section 6.2, and $\mathcal{M}$ are the parameters describing the coupling of the gauge multiplet to the massive Higgs fields. We can then study the issue of whether the 'weak/weak' version of the AdS/CFT correspondence,

which is valid for the massless sector at $\lambda = 0$, can be generalized to include the leading Regge trajectory for $\lambda > 0$.

The connection between the leading Regge trajectory at small l_s and the bilinear HS currents in the SYM theory at small λ has also been made by the authors of [34] who give 'long string' solutions to the worldsheet sigma model in the limit $l_s/R << 1$. These solutions describe states on the the leading Regge trajectory with spins $s >> R^2/l_s^2 >> 1$ and AdS energies $E \sim s + R^2/l_s^2 \log(sl_s^2/R^2) \sim s$, which couple to bilinear HS currents in the SYM theory. These operators arise in the OPE of Wilson lines making up the boundaries of worldsheets of infinitely long strings on the leading Regge trajectory. The leading Regge trajectory also contains 'short strings' which have spins $s << R^2/l_s^2$ and energies $E \sim \sqrt{s}R/l_s$. Since $E/s \sim 1$ for long strings, and $E/s >> 1$ for short strings, the long ones cannot decay into large number of short ones. Furthermore, in any Regge trajectory there should be long string with large spins and asymptotically small anomalous dimensions, suggesting that string interactions in the limit $s >> R^2/l_s^2 >> 1$ have a consistent description in terms of long strings with $E \sim s$. Upon increasing l_s/R, we expect a short string state with fixed s to become long for large enough l_s/R. In the limit in which $\lambda \to 0$ on the SYM side, there should exist a $hs(2, 2|4)$ invariant worldsheet sigma model describing closed string interactions in the bulk corresponding to the free SYM theory.

From the above discussions we are led to propose that there is a cross-over from large to small λ in the expressions for the AdS string length and string coupling in terms of the gauge theory quantities given in (5.1), such that

$$f_1(\lambda) \sim 1/\lambda , \quad f_2(\lambda) \sim 1 + \mathcal{O}(\lambda) \quad \text{for } \lambda << 1 . \tag{5.12}$$

Then $l_s/R \sim 1$ and $g_s \sim 1/N$ as $\lambda \to 0$. This suggests that the $hs(2, 2|4)$ higher spin gauge theory is described by a string theory which has a left-moving and right-moving $PSU(2, 2|4)$ KM algebra with critical level $k = k_{\text{crit}} \sim 1$ which admits a singleton representation and an affine $hs(2, 2|4)$ extension. To be more precise, the critical value for the level should be such that there exists a maximally reducible Verma module based on the singleton which contains a maximal number of null-states. In fact, it has been shown [70] that the affine $SO(3, 2) \simeq Sp(4)$ algebra admits singleton-like representations for $k = 5/2$. It would be interesting to generalize this result to $SO(D - 1, 2)$ and supersymmetric cases. For critical level the closed string spectrum then contains physical massless HS states states formed by multiplying a left-moving and a right-moving singleton. The algebra $hs(2, 2|4)$ can be identified with the following coset

$$hs(2, 2|4) = Env(PSU(2, 2|4))/\mathcal{R} , \tag{5.13}$$

where $\mathcal{R}$ is a certain ideal generated by elements in $Env(PSU(2, 2|4))$ which vanish identically when the $PSU(2, 2|4)$ generators are realized in terms of a single super-oscillator as described in Section 2.1. For $k = k_{\text{crit}}$ this construction should lift to the affine case. The symmetry enhancement from AdS group to HS algebra for critical level, i.e. critical radius in units of fixed string length, would be similar in spirit to the $SU(2)$ enhancement occurring at the self-dual radius for string theory on a circle.

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The possibility to realize massless higher spins directly in the bulk as products of left-moving and right-moving singleton representations at critical KM level is rather appealing. Perhaps the close resemblance between the HS gauge theories in $D = 4, 5, 7$ is an indication of that singletons play a similar role on critical membranes in $D = 4, 7$.

6 M Theory on $AdS_4 \times S^7$ and 4D Higher Spin Gauge Theory

6.1 Holography

Already in [71] it was observed that the $OSp(8|4)$ singleton may play a role in the description of the supermembrane on $AdS_4 \times S^7$. In [40] the quantization of the $d = 3$, $\mathcal{N} = 8$ singleton theory corresponding to a single membrane was shown to yield the infinite set of massless HS fields contained in the symmetric tensor product of two singleton weight spaces [38]. Moreover, it was conjectured in [40] that these massless states, as well as the massive states contained in the higher order tensor products, arise in the supermembrane theory [13]. Subsequently, the group theoretical HS/singleton connection was utilized in [35] and the fully interacting massless HS field equations in $D = 4$ were constructed in [14]. In the light of [1], the 4D HS/singleton connection found in [40] was revived as an actual AdS/CFT correspondence in [16, 17].

Importantly, the role of large N discussed in Section 4 was not emphasized in these early formulations of the correspondence. Thus we need to refine the formulation of the correspondence by identifying the appropriate dependence on N of the free $OSp(8|4)$ singleton.

Let us first recall the Maldacena conjecture [1] on the correspondence between M theory on $AdS_4 \times S^7$ with N units of 7-form flux on S^7 and the low energy dynamics of N parallel $M2$ branes in flat eleven-dimensional spacetime, which is described by a strongly coupled $d = 3$, $\mathcal{N} = 8$ CFT with $SO(8)_R$ symmetry [1, 4]. This theory defines a nontrivial IR fixed point of $d = 3$, $\mathcal{N} = 8$ SYM theory with $SU(N)$ gauge group. The resulting $SO(7)_R$-invariant flow has an anti-holographic description as a D2 brane near-horizon geometry, which is reliable in the UV where the dilaton is small. In the IR the dilaton blows up and the IIA solution lifts to the $SO(8)_R$ invariant $AdS_4 \times S^7$ near horizon region of a stack of N coinciding M2 branes. The resulting anti-holographic description of the strongly coupled SCFT is conjectured to be M theory on $AdS_4 \times S^7$. For large N the membrane tension scales like

$$T_{M2} = \frac{1}{l_{M2}^3} \sim \frac{\sqrt{N}}{R^3} \ , \tag{6.1}$$

where R is the AdS radius, and the 4D Planck length is given by

$$\frac{1}{l_{\text{Pl}}^2} = \frac{N^{3/2}}{R^2} \ . \tag{6.2}$$

---

[13]To describe the S^7 compactified M theory all higher tensor products are needed. The resulting theory lives on the double cover of AdS_4 times S^7. It is consistent to truncate the theory to only even powers of the singleton. This corresponds to M theory on the single cover of AdS_4 times $S^7/\mathsf{Z}_2 \simeq \mathsf{RP}^7$.

Hence, for large N,

$$R >> l_{M2} >> l_{\mathrm{Pl}} . \qquad (6.3)$$

For AdS energies E obeying $1 << E << R/l_{\mathrm{M2}}$ the low-energy dynamics of the anti-holographic dual is conjectured [1] to be described by $D = 4$, $\mathcal{N} = 8$ gauged supergravity. In particular, it follows from the normalization (6.2) that the strongly coupled SCFT has $\sim N^{\frac{3}{2}}$ massless degrees of freedom for large N [72, 73].

In the UV limit of the D2 brane geometry the dilaton e^ϕ vanishes and the 10D gravitational curvature diverges (which one might interpret as the appearance of the new massless HS states that we shall define below). The D2 brane field theory becomes a $SU(N)$ invariant theory of free 3d super Maxwell multiplets. Here we note that the Yang-Mills coupling in the dual SYM theory on the stack of N coinciding D2-branes, $g_{\mathrm{YM}}^2 = g_s/l_s$, is held fixed in taking the near-horizon limit. This coupling also coincides with the 'local' Yang-Mills coupling on a stack of probe D2-branes placed at energy scale u in the near-horizon region, $g_{\mathrm{YM}}^2(u) \equiv e^{\phi(u)}\sqrt{-g_{00}(u)/l_s^2} = g_{\mathrm{YM}}^2$, as required for interpreting the stack of probe branes as describing a Higgs branch of the dual SYM theory. Thus both the dilaton and running string length vanishes in the UV limit, which is why we can trust the free $SU(N)$ field theory even though the gravitational curvature diverges.

Dualizing the vector fields and using g_{YM}^2 to rescale the fields, we obtain a free $SU(N)$ valued $OSp(8|4)$ singleton $\Phi^i \in \mathbf{8}_v$ described by the $SO(8)_R$ invariant Lagrangian [14]

$$\int d^3x \, \mathrm{tr}\left((\partial\Phi^i)^2 + \mathrm{fermions} \right) . \qquad (6.4)$$

Conversely, assuming that this Lagrangian describes a fixed point on the membrane we can break $SO(8)_R \to SO(7)_R$ by taking the M theory to have a finite radius R_{11} and take Φ^8 to be periodic:

$$\Phi^8 \sim \Phi^8 + g , \qquad (6.5)$$

where the radius g is a constant with dimension $1/2$ which we identify as $g = R_{11}/(l_{11})^{3/2}$ and l_{11} is the eleven-dimensional Planck length. We recover the free $OSp(8|4)$ invariant singleton in the decompactification limit $R_{11} \to \infty$. We may instead use g to dualize Φ^8 and introduce Yang-Mills interactions with $g_{\mathrm{YM}} = g$. The effective coupling is $g_{\mathrm{eff}}^2 = g^2/u$, where u is the 3d energy scale, and as a result the theory now decompactifies in the IR [1, 4, 31, 74, 25]. Thus we have two decompactification limits, the free $SU(N)$ valued $OSp(8|4)$ singleton field theory which resides in the UV and the strongly coupled $SO(8)_R$ invariant $d = 3$, $\mathcal{N} = 8$ SCFT in the IR.

Thus it is natural to describe the low energy dynamics of M2 branes in terms of an UV fixed point of free $SU(N)$ valued $OSp(8|4)$ singletons and an IR fixed point of strongly coupled $OSp(8|4)$

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[14]The singleton consists of 8 scalars in $\mathbf{8}_v$ and 8 spinors in $\mathbf{8}_s$ of $SO(8)_R$. By triality one can also obtain a singleton multiplet in which the scalars are in $\mathbf{8}_s$ and the spinors in $\mathbf{8}_c$.

singletons. We note that the number of massless degrees of freedom indeed decreases along the RG flow, from N^2 to $N^{3/2}$.

We conjecture that the free singleton theory at the UV fixed point mentioned above is the holographic dual of the $hs(4|8)$ gauge theory which admits the massless $hs(4|8)$ gauge theory described in Section 2.2 as a consistent truncation. This theory describes an unbroken phase of M theory with N units of M2 brane charge. The strongly coupled fixed point is the holographic image of a broken phase, which admits an effective supergravity description at low energies.

There are also IR fixed points containing free $OSp(8|4)$ singletons forming $N-1$ dimensional representation of the Weyl group of $SU(N)$ [31]. These are curious points from the point of view of HS dynamics, and it may be that one should also include them as nontrivial points in the phase diagram.

As discussed in the previous section, the unbroken phase of the Type IIB theory on $AdS_5 \times S^5$ arises either as the critical limit $\lambda \to 0$ at fixed E and s, or as the high energy limit $s >> \sqrt{\lambda}$ at fixed λ and $N >> 1$. Moreover, as we shall see in the next section, the unbroken phase of M theory on $AdS_7 \times S^4$ arises at high energies whereby certain membrane solitons propagate close to the boundary of AdS_7. This suggests that also the unbroken phase of M theory on $AdS_4 \times S^7$ arises in a high energy limit in which bulk membranes couple to HS operators in the strongly coupled $SCFT_3$ with asymptotically small anomalous dimensions, $(E-s)/s \to 0$, as $s \to \infty$. The four-form flux in the AdS_4 directions results ensures the M2-brane equations admit spherical membrane solutions in $AdS_4 \times S^7$ [75, 76, 77]. These solutions carry internal $SO(8)$ spin, and are hence closely related to the matrix-model found in the pp-wave limit [30]. It is natural to expect that these solutions can be deformed into time-dependent membrane solutions carrying also AdS spin, in analogy with the string solutions in AdS_3 with NS-fluxes [78]. We also expect the anomalous part of the energy to be minimized and certain fractional supersymmetry to be restored by taking large AdS radius, i.e. large bulk energies, such that the solution couples to the conserved HS currents of the $hs(8|4)$ theory. The fact that the holographic dual resides at a UV fixed point should be encoded into the local geometry of the solution and to how it minimizes the AdS energy, as in the case of the rotating membrane in $AdS_7 \times S^4$.

It will be interesting to examine the above picture in more detail and in particular to examine the fluctuation spectrum about this solution, where we expect to find some critical membrane theory with fixed tension, and perhaps singletons in the worldvolume, giving rise to the massless HS states.

We expect that the Higgsing of the massless HS fields and the resulting spontaneous breaking of the $hs(4|8)$ is described by a radially dependent solution to the HS theory which is the anti-holographic dual of the 3d SYM flow obtained by switching on a finite g_{YM}^2 as discussed above. It will be interesting to see whether HS field theoretic methods are still relevant for describing this solution, which would then yield 'weak/weak' correspondence between the HS theory coupled to Higgs sector and the SYM theory with expansions in both $1/N$ and g_{YM}^2. It may also be necessary to exhibit in more detail the nature of the above-mentioned critical membrane.

6.2 Cubic Couplings in the 4D Higher Spin Gauge Theory

In this section we shall outline the structure of the minimal bosonic HS gauge theory in $D = 4$ which is a consistent truncation of the supersymmetric HS theory discussed in Section 2.1. The spectrum consists of massless fields with spin $s = 0, 2, 4, ...$, each occurring once. The underlying algebra, called $hs(4)$, is an infinite dimensional extension of the bosonic AdS_4 group. Similar truncation exists also in $D = 5, 7$ at the spectrum level but only in $D = 4$ a full interacting theory is known, both supersymmetric and minimal bosonic.

The $4D$ minimal bosonic model is of great interest because it is the simplest interacting HS gauge theory (with propagating HS degrees of freedom), and yet it exhibits all the essential principles that underlie such theories. It is a very good starting point for finding ways to construct the $D = 5, 7$ HS gauge theories as well. Moreover, it is amenable to calculations and it is possible to test directly in this model the consistent truncation of the kind discussed in Section 4 which is required for the holography picture to make sense. Here, we will not go as far as carrying out these tests [24] but we will nonetheless exhibit the structure the couplings to give the reader an idea about how they actually look like, as well as providing enough ingredients to facilitate the required holography computations.

Here we shall focus our attention on the quadratic terms in all the field equations, which, of course, mean all the cubic couplings at the action level. In an accompanying paper [19], we shall give a more detailed treatment involving an expansion scheme where the gravitational gauge fields are treated exactly and the gravitational curvatures and the HS gauge fields as weak perturbations to all orders. The 4D HS/3d singleton correspondence in the $hs(4)$ theory at the level of quadratic field equation/cubic action will be provided elsewhere [24].

The massless field equations (including general interaction ambiguities) have been given in [14] and studied in more detail in [16, 17, 18] and more recently in [19]. These studies are based on a curvature expansion scheme. The most important step in the expansion scheme is the linearized analysis which shows that all auxiliary fields are non-propagating. As a result it is possible to solve iteratively for the auxiliary fields and obtain the physical field equations to any order. In fact, this scheme yields field equations in terms of only the physical fields.

The HS spin algebra $hs(4)$ is obtained from $hs(4|8)$ defined in Section 2.1 by setting the fermionic generators θ^i equal to zero. To describe the field equations in 4D spacetime, which has coordinates x^μ, one introduces an auxiliary set of coordinates $(z^\alpha, \bar{z}^{\dot\alpha})$ which are Grassmann even spinors that are non-commutative in nature, and consider extensions $\varphi(x; z, \bar{z})$ of the basic spacetime fields $\varphi(x)$. One then imposes an integrable curvature constraint in the extended space, whose $(x; z, \bar{z})$-components determine the $(z, \bar{z})$ dependence of the extended fields $\varphi(x; z, \bar{z})$ in terms of "initial" conditions $\phi(x)$. Setting $z = \bar{z} = 0$ in the remaining x-components of the curvature constraint leads to reduced curvature constraints in spacetime, which are integrable by construction and one can show that they contain the physical field equations of the HS gauge theory. Since $(z, \bar{z})$ are non-commutative, the reduced constraints contain interactions even though the original constraint in $(x; z, \bar{z})$ space has a simple form.

The basic building blocks of the theory are a master 0-form Φ and a master 1-form

$$\widehat{A} = dx^\mu \widehat{A}_\mu + dz^\alpha \widehat{A}_\alpha + d\bar{z}^{\dot\alpha} \widehat{A}_{\dot\alpha} \ , \tag{6.6}$$

where the hats are used to indicate quantities that depend on $(z, \bar{z})$. The hatted fields are given as expansions order by order in z and $\bar{z}$, with expansion coefficients which are functions of $(x, y, \bar{y})$ with the $(y, \bar{y})$ expansions determined by suitable group theoretical conditions. These conditions are engineered such that at $z = \bar{z} = 0$, the pulled-back components

$$A_\mu = \widehat{A}_\mu|_{Z=0} \ , \quad \Phi = \widehat{\Phi}|_{Z=0} \tag{6.7}$$

define an $hs(4)$ valued spacetime one-form and a spacetime zero-form in a certain quasi-adjoint representation of $hs(4)$ [18]:

$$A_\mu(x; y, \bar{y}) \;=\; \frac{1}{2i} \sum_{m+n=2 \bmod 4} \frac{1}{m!n!} \bar{y}^{\dot\alpha_1} \cdots \bar{y}^{\dot\alpha_m} y^{\alpha_1} \cdots y^{\alpha_n} A_{\mu\alpha_1\ldots\alpha_n\dot\alpha_1\ldots\dot\alpha_m}(x) \ , \tag{6.8}$$

$$\Phi(x; y, \bar{y}) \;=\; \sum_{|m-n|=0 \bmod 4} \frac{1}{m!n!} \bar{y}^{\dot\alpha_1} \cdots \bar{y}^{\dot\alpha_m} y^{\alpha_1} \cdots y^{\alpha_n} \Phi_{\alpha_1\ldots\alpha_n\dot\alpha_1\ldots\dot\alpha_m}(x) \ . \tag{6.9}$$

The curvature constraints giving rise to the spacetime field equations read

$$\widehat{F} \;=\; d\widehat{A} + \widehat{A} \star \widehat{A} = \tfrac{i}{4} dz^\alpha \wedge dz_\alpha \widehat{\Phi} \star \kappa + \tfrac{i}{4} d\bar{z}^{\dot\alpha} \wedge d\bar{z}_{\dot\alpha} \widehat{\Phi} \star \bar{\kappa} \ , \tag{6.10}$$

$$\widehat{D}\widehat{\Phi} \;=\; d\widehat{\Phi} + \widehat{A} \star \widehat{\Phi} - \widehat{\Phi} \star \bar{\pi}(\widehat{A}) = 0 \ , \tag{6.11}$$

where the operators $\kappa, \bar{\kappa}$ are defined as

$$\kappa = \exp(iy^\alpha z_\alpha) \ , \qquad \bar{\kappa} = \kappa^\dagger = \exp(-i\bar{y}^{\dot\alpha} \bar{z}_{\dot\alpha}) \ , \tag{6.12}$$

the π-map, and its complex conjugate $\bar{\pi}$, acting on an arbitrary polynomial $f(y, \bar{y}; z, \bar{z})$ are defined as

$$\pi(f(y, \bar{y}; z, \bar{z})) = f(-y, \bar{y}; -z, \bar{z}) \ , \qquad \bar{\pi}(f(y, \bar{y}; z, \bar{z})) = f(y, -\bar{y}; z, -\bar{z}) \ , \tag{6.13}$$

and the $\star$-product between two arbitrary polynomials $f(y, \bar{y}, x; \bar{z})$ and $g(y, \bar{y}; z, \bar{z})$ is defined as

$$f * g = f \exp\left[i \left(\frac{\overleftarrow{\partial}}{\partial z_\alpha} + \frac{\overleftarrow{\partial}}{\partial y_\alpha} \right) \left(\frac{\overrightarrow{\partial}}{\partial z^\alpha} - \frac{\overrightarrow{\partial}}{\partial y^\alpha} \right) + i \left(\frac{\overleftarrow{\partial}}{\partial \bar{z}_{\dot\alpha}} - \frac{\overleftarrow{\partial}}{\partial \bar{y}_{\dot\alpha}} \right) \left(\frac{\overrightarrow{\partial}}{\partial \bar{z}^{\dot\alpha}} + \frac{\overrightarrow{\partial}}{\partial \bar{y}^{\dot\alpha}} \right) \right] g \ . \tag{6.14}$$

The constraints (6.10) and (6.11) have the gauge symmetry

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$$\delta\widehat{A} = d\widehat{\epsilon} + [\widehat{A}, \widehat{\epsilon}]_\star \,, \qquad \delta\widehat{\Phi} = \widehat{\epsilon} \star \widehat{\phi} - \widehat{\Phi} \star \widehat{\epsilon} \,. \tag{6.15}$$

Given the initial conditions (6.7), the components of the constraints (6.10-6.11) which have at least one α or $\dot{\alpha}$ index can be solved by expanding $\widehat{A}$ and $\widehat{\Phi}$ in powers of Φ, which contains curvatures and the scalar field, as follows:

$$\widehat{\Phi} = \sum_{n=1}^{\infty} \widehat{\Phi}^{(n)} \,, \quad \widehat{A}_\alpha = \sum_{n=1}^{\infty} \widehat{A}_\alpha^{(n)} \,, \quad \widehat{A}_\mu = \sum_{n=0}^{\infty} \widehat{A}_\mu^{(n)} \,, \tag{6.16}$$

where

$$\widehat{\Phi}^{(n)}|_{Z=0} = \begin{cases} \Phi \,, & n = 1 \\ 0 \,, & n = 2, 3, ... \end{cases} \tag{6.17}$$

$$\widehat{A}_\mu^{(n)}|_{Z=0} = \begin{cases} A_\mu \,, & n = 0 \\ 0 \,, & n = 1, 2, 3, ... \end{cases} \tag{6.18}$$

$$\widehat{A}_\alpha|_{Z=0} = 0 \,, \tag{6.19}$$

and $\widehat{\Phi}^{(n)}$ $(n = 2, 3, ...)$, $\widehat{A}_\alpha^{(n)}$ $(n = 1, 2, 3, ...)$ and $\widehat{A}_\mu^{(n)}$ $(n = 2, 3, ...)$ are nth order in Φ. Note that $\widehat{A}_\mu^{(n)}$ are linear in A_μ. The condition (6.19) is a physical gauge condition, which can be imposed by using the gauge symmetry (6.15).

As shown in detail in [19], one first solves iteratively the constraints $\widehat{F}_{\mu\alpha} = \widehat{F}_{\alpha\beta} = \widehat{F}_{\dot{\alpha}\dot{\beta}} = 0$ and $D_\alpha\widehat{\Phi} = 0$ to determine the Φ expansions of $\widehat{A}_\mu, \widehat{A}_\alpha$ and $\widehat{\Phi}$, which schematically take the form

$$\widehat{A}_\mu = \widehat{A}_\mu[A_\mu, \Phi] \,, \qquad \widehat{A}_\alpha = \widehat{A}_\alpha[\Phi] \,, \qquad \widehat{\Phi} = \widehat{\Phi}[\Phi] \,. \tag{6.20}$$

Having solved the Z-space part of (6.10) and (6.11), the remaining constraints $\widehat{F}_{\mu\nu} = 0$ and $\widehat{D}_\mu\widehat{\Phi} = 0$ yield spacetime field equations of the form

$$F_{\mu\nu} = -\sum_{n=1}^{\infty} \sum_{j=0}^{n} \left(\widehat{A}_{[\mu}^{(j)} \star \widehat{A}_{\nu]}^{(n-j)} \right)|_{Z=0} \,, \tag{6.21}$$

$$D_\mu\Phi = \sum_{n=2}^{\infty} \sum_{j=1}^{n} \left(\widehat{\Phi}^{(j)} \star \pi(\widehat{A}_\mu^{(n-j)}) - \widehat{A}_\mu^{(n-j)} \star \widehat{\Phi}^{(j)} \right)|_{Z=0} \,, \tag{6.22}$$

where $F = dA + A \star A$ and $D\Phi = d\Phi + A \star \Phi - \Phi \star \bar{\pi}(A)$.

Next, we define the physical scalar ϕ and expand the master gauge field $A_\mu(x, y, \bar{y})$ as

$$\Phi|_{Y=0} = \phi. \tag{6.23}$$

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As for the vierbein and HS gauge fields, requiring that they transform homogeneously under Lorentz transformation, one is led to the following expansion scheme for the master gauge field

$$A_\mu = e_\mu + \omega_\mu + W_\mu + \left(i\omega_\mu^{\alpha\beta}\, \widehat{A}_\alpha \star \widehat{A}_\beta - \text{h.c} \right)_{Z=0} , \tag{6.24}$$

where the vielbein and the Lorentz connections are defined as

$$e_\mu = \frac{1}{2i} e_\mu^{\alpha\dot\alpha} y_\alpha \bar{y}_{\dot\alpha} , \qquad \omega_\mu = \frac{1}{4i}\left(\omega_\mu^{\alpha\beta} y_\alpha y_\beta + \text{h.c} \right) , \tag{6.25}$$

and W_μ contains the fields with spin $s = 4, 6, 8, \dots$ and their corresponding auxiliary fields.

We are now ready to state the result for the cubic couplings [19]. They give rise to quadratic terms in the field equations given by [19]

$$(\nabla^2 + 2)\,\phi = \left(\nabla^\mu P_\mu^{(2)} - \tfrac{i}{2}(\sigma^\mu)^{\alpha\dot\alpha} \frac{\partial}{\partial y^\alpha} \frac{\partial}{\partial \bar{y}^{\dot\alpha}} P_\mu^{(2)} \right)_{Y=0} , \tag{6.26}$$

$$(\sigma^{\mu\nu\rho})^{\dot\beta}_\alpha \, \mathcal{R}_{\nu\rho\,\dot\beta\dot\gamma} = (\sigma^{\mu\nu\rho})^{\dot\beta}_\alpha \left(\frac{\partial}{\partial \bar{y}^{\dot\beta}} \frac{\partial}{\partial \bar{y}^{\dot\gamma}} J_{\nu\rho}^{(2)} \right)_{Y=0} , \tag{6.27}$$

$$(\sigma^{\mu\nu\rho})^{\dot\beta}_{\alpha_1} \, F^{(1)}_{\nu\rho\,\alpha_2\dots\alpha_{s-1}\dot\beta\dot\gamma\dot\alpha_2\dots\dot\alpha_{s-1}} = (\sigma^{\mu\nu\rho})^{\dot\beta}_{\alpha_1} \left(\frac{\partial}{\partial y^{\alpha_2}} \cdots \frac{\partial}{\partial y^{\alpha_{s-1}}} \frac{\partial}{\partial \bar{y}^{\dot\beta}} \cdots \frac{\partial}{\bar{y}^{\dot\alpha_{s-1}}} J_{\nu\rho}^{(2)} \right)_{Y=0} \tag{6.28}$$

where $\mathcal{R}_{\mu\nu\dot\alpha\dot\beta} \equiv F_{\mu\nu\dot\alpha\dot\beta}$ is the (self-dual part of) the AdS_4 valued Riemann curvature, while the curvature associated with spin $s = 4, 6, 8, \dots$ fields is defined as

$$
\begin{aligned}
F^{(1)}_{\nu\rho\,\alpha_2\dots\alpha_{s-1}\dot\beta\dot\gamma\dot\alpha_2\dots\dot\alpha_{s-1}} = {}& 2\nabla_{[\nu} W_{\rho]\alpha_2\dots\alpha_{s-1}\dot\beta\dot\gamma\dot\alpha_2\dots\dot\alpha_{s-1}} \\
& - (s-2)(\sigma_{\nu\rho}\sigma_\mu)_{\alpha_2}{}^\delta\, W_{\mu\,\alpha_3\dots\alpha_{s-1}\dot\beta\dot\gamma\dot\delta\dot\alpha_2\dots\dot\alpha_{s-1}} \\
& - s(\sigma_\mu\sigma_{\nu\rho})_{\dot\beta}{}^{\dot\gamma}\, W_{\mu\,\gamma\alpha_2\alpha_3\dots\alpha_{s-1}\dot\gamma\dot\alpha_2\dots\dot\alpha_{s-1}} .
\end{aligned}
\tag{6.29}
$$

The covariant derivatives in (6.26) and (6.29) are with respect to lo the Lorentz connection ω. Furthermore, in (6.28) and (6.29), separate symmetrization in the dotted and undotted indices is understood. Further definitions are

$$
\begin{aligned}
P_\mu^{(2)} = {}& \Phi \star \bar{\pi}(W_\mu) - W_\mu \star \Phi \\
& + \left[\Phi \star \bar{\pi}(\widehat{e}_\mu{}^{(1)}) - \widehat{e}_\mu^{(1)} \star \Phi + \widehat{\Phi}^{(2)} \star \bar{\pi}(e_\mu) - e_\mu \star \widehat{\Phi}^{(2)} \right]_{Z=0}
\end{aligned}
\tag{6.30}
$$

$$J^{(2)}_{\mu\nu} = -\left(\left[\widehat{e}^{(1)}_\mu \star \widehat{e}^{(1)}_\nu + \{e_\mu, \widehat{e}^{(2)}_\nu\}_\star + \{e_\mu, \widehat{W_\nu}^{(1)}\}_\star + \{W_\mu, \widehat{e}^{(1)}_\nu\}_\star\right]_{Z=0}\right.$$

$$\left. + \left[iR_{\mu\nu}{}^{\alpha\beta}\widehat{A}^{(1)}_\alpha \star \widehat{A}^{(1)}_\beta + \text{h.c.}\right]_{Z=0} + W_\mu \star W_\nu\right) + \left(\mu \leftrightarrow \nu\right), \tag{6.31}$$

and the hatted quantities occurring in the above equations are given by [19]

$$\widehat{A}^{(1)}_\alpha = -\frac{i}{2}z_\alpha \int_0^1 tdt\; \Phi(-tz,\bar{y})\kappa(tz,y) , \tag{6.32}$$

$$\widehat{A}^{(2)}_\alpha = z_\alpha \int_0^1 tdt\left(\widehat{A}^{(1)\beta} \star \widehat{A}^{(1)}_\beta\right)_{z\to tz,\bar{z}\to t\bar{z}} \tag{6.33}$$

$$+ \bar{z}^{\dot\beta}\int_0^1 tdt\left[\widehat{A}^{(1)}_\alpha, \widehat{A}^{(1)}_{\dot\beta}\right]_{z\to tz,\bar{z}\to t\bar{z}}$$

$$\widehat{W}^{(1)}_\mu = -i\int_0^1 \frac{dt}{t}\left(\left[\frac{\partial W_\mu}{\partial y^\alpha}, \widehat{A}^{\alpha(1)}\right]_\star + \left[\widehat{A}^{\dot\alpha(1)}, \frac{\partial W_\mu}{\partial \bar{y}^{\dot\alpha}}\right]_\star\right)_{z\to tz,\bar{z}\to t\bar{z}}$$

$$\widehat{\Phi}^{(2)} = z^\alpha \int_0^1 dt\left[\Phi \star \bar{\pi}(\widehat{A}^{(1)}_\alpha) - \widehat{A}^{(1)}_\alpha \star \Phi\right]_{t\to tz,\bar{z}\to t\bar{z}} \tag{6.34}$$

$$+ \bar{z}^{\dot\alpha}\int_0^1 dt\left[\Phi \star \pi(\widehat{A}^{(1)}_{\dot\alpha}) - \widehat{A}^{(1)}_{\dot\alpha} \star \Phi\right]_{t\to tz,\bar{z}\to t\bar{z}}$$

$$\widehat{e}^{(1)}_\mu = -ie^{\alpha\dot\alpha}_\mu \int_0^1 \frac{dt}{t}\left(\left[\bar{y}_{\dot\alpha}, \widehat{A}^{(1)}_\alpha\right]_\star + \left[\widehat{A}^{(1)}_{\dot\alpha}, y_\alpha\right]_\star\right)_{z\to tz,\bar{z}\to t\bar{z}} ,$$

$$\widehat{e}^{(2)}_\mu = -ie^{\alpha\dot\alpha}_\mu \int_0^1 \frac{dt}{t}\left(\left[\bar{y}_{\dot\alpha}, \widehat{A}^{(2)}_\alpha\right]_\star + \left[\widehat{A}^{(2)}_{\dot\alpha}, y_\alpha\right]_\star\right)_{z\to tz,\bar{z}\to t\bar{z}} ,$$

$$-e^{\alpha\dot\alpha}_\mu \int_0^1 \frac{dt}{t}\int_0^1 \frac{dt'}{t'}\left[\widehat{A}^{\beta(1)} \star \left(\frac{\partial}{\partial z^\beta} - \frac{\partial}{\partial y^\beta}\right)\left(\left[\bar{y}_{\dot\alpha}, \widehat{A}^{(1)}_\alpha\right]_\star + \left[\widehat{A}^{(1)}_{\dot\alpha}, y_\alpha\right]_\star\right)_{z\to t'z,\bar{z}\to t'\bar{z}}\right.$$

$$+\widehat{A}^{\dot\beta(1)} \star \left(\frac{\partial}{\partial \bar{z}^{\dot\beta}} + \frac{\partial}{\partial \bar{y}^{\dot\beta}}\right)\left(\left[\bar{y}_{\dot\alpha}, \widehat{A}^{(1)}_\alpha\right]_\star + \left[\widehat{A}^{(1)}_{\dot\alpha}, y_\alpha\right]_\star\right)_{z\to t'z,\bar{z}\to t'\bar{z}}$$

$$+\left(\frac{\partial}{\partial z^\beta} + \frac{\partial}{\partial y^\beta}\right)\left(\left[\bar{y}_{\dot\alpha}, \widehat{A}^{(1)}_\alpha\right]_\star + \left[\widehat{A}^{(1)}_{\dot\alpha}, y_\alpha\right]_\star\right)_{z\to t'z,\bar{z}\to t'\bar{z}} \star \widehat{A}^{\beta(1)}$$

$$\left. +\left(\frac{\partial}{\partial \bar{z}^{\dot\beta}} - \frac{\partial}{\partial \bar{y}^{\dot\beta}}\right)\left(\left[\bar{y}_{\dot\alpha}, \widehat{A}^{(1)}_\alpha\right]_\star + \left[\widehat{A}^{(1)}_{\dot\alpha}, y_\alpha\right]_\star\right)_{z\to t'z,\bar{z}\to t'\bar{z}} \star \widehat{A}^{\dot\beta(1)}\right]_{z\to tz,\bar{z}\to t\bar{z}} .$$

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In the above formulae, the replacement of $(z, \bar{z})$ by $(tz, t\bar{z})$ is to be made inside the integrals and *after* performing the $\star$ products. Note also the quantity $\hat{\mathcal{A}}_\alpha^{(1)}$ is a basic building block which occurs in many of the formulae above and that it is first order in Φ.

It is important to note that not all the fields occurring in (6.8) and (6.9) are independent. An analysis of the constraints (6.10) and (6.11) shows a) $\Phi_{\alpha_1...\alpha_{2s}}(s = 2, 4, ...)$ are the Weyl tensors which can be in terms of the curvatures, b) $\Phi_{\alpha(m)\dot\alpha(n)}$ for $m + n > 2$ can be solved in terms of ϕ, the Weyl tensors and their derivatives, c) $\omega_\mu^{\alpha\beta}$ is, of course, the Lorentz spin connection which can be solved in terms of the vierbein $e_\mu^{\alpha\dot\alpha}$, and d) $W_{\mu\alpha(m)\dot\alpha(n)}$ for $|m - n| \geq 2$ are auxiliary gauge fields which can be solved in terms of the physical fields $W_{\alpha(s-1)\dot\alpha(s-1)}$ [19].

The general solution for the auxiliary fields is given by [19]

$$
W_{\alpha\dot\alpha,\beta_1...\beta_m\dot\beta_1...\dot\beta_n} = \frac{2}{m+1}\widehat{\nabla}W_{\dot\beta_1\dot\beta_2,\alpha\beta_1...\beta_m\dot\alpha\dot\beta_3...\dot\beta_n} + \epsilon_{\dot\alpha\dot\beta_1}\frac{2n}{n+1}\left[\frac{n-1}{m+n+2}\widehat{\nabla}W_{\dot\beta_2}{}^{\dot\gamma}{}_{,\alpha\beta_1...\beta_m\dot\gamma\dot\beta_3...\dot\beta_n}\right.
$$

$$
\left. +\frac{n+1}{m+n+2}\widehat{\nabla}W_{(\alpha}{}^{\gamma}{}_{,\beta_1...\beta_m)\gamma\dot\beta_2...\dot\beta_n} - \frac{m}{(m+1)(m+2)}\epsilon_{\alpha\beta_1}\widehat{\nabla}W^{\gamma\delta}{}_{,\gamma\delta\beta_2...\beta_m\dot\beta_2...\dot\beta_n}\right]
$$

$$
+m\epsilon_{\alpha\beta_1}\xi_{\beta2...\beta_m\dot\alpha\dot\beta_1...\dot\beta_n} , \qquad n > m \geq 0 , \tag{6.35}
$$

$$
\Phi_{\alpha_1...\alpha_m\dot\alpha_1...\dot\alpha_n} = -i\widehat{\nabla}_{\alpha_1\dot\alpha_1}\Phi_{\alpha_2...\alpha_m\dot\alpha_2...\dot\alpha_n} , \tag{6.36}
$$

where the the modified covariant derivatives are defined by

$$
\widehat{\nabla}W_{\alpha\beta,\gamma_1...\gamma_m\dot\gamma_1...\dot\gamma_n} = \tfrac{1}{2}(\sigma^{\mu\nu})_{\alpha\beta}\left(\nabla_\mu W_{\nu,\gamma_1...\gamma_m\dot\gamma_1...\dot\gamma_n} - \tfrac{1}{2}J^{(2)}_{\mu\nu,\gamma_1...\gamma_m\dot\gamma_1...\dot\gamma_n}\right) , \tag{6.37}
$$

$$
\widehat{\nabla}_{\alpha_1\dot\alpha_1}\Phi_{\alpha_2...\alpha_m\dot\alpha_2...\dot\alpha_n} = \left(\nabla_{\alpha_1\dot\alpha_1}\Phi_{\alpha_2...\alpha_m\dot\alpha_2...\dot\alpha_n} - P^{(2)}_{\alpha_1\dot\alpha_1,\alpha_2...\alpha_m\dot\alpha_2...\dot\alpha_n}\right) , \tag{6.38}
$$

and separate total symmetrization of dotted and undotted indices is understood. Since J and P depend on the auxiliary fields, eqs. (6.35) and (6.36) must be iterated within the curvature expansion scheme. This leads to explicit expressions of all auxiliary components of W_μ and Φ in terms of the remaining physical fields.

Further comments about the above results are in order:

1) The z-dependence of all the fields involved are exhibited. The above results are explicit and the remaining task is reduced to performing certain star products and doing some elementary parameter integrals. These steps, as well as the derivation of the above results and their generalization to all orders will be provided elsewhere [19].

2) It is easy to rewrite the field equation (6.27) for the graviton as [15]

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[15]We have set the AdS radius $R = 1$ but it is straightforward to re-introduce R by dimensional analysis in which the master 0-form and the master 1-form fields are dimensionless.

$$R_{\mu\nu}(\omega) - g_{\mu\nu} = \left[(\sigma_\mu{}^\lambda)^{\alpha\beta} \left(\frac{\partial}{\partial y^\alpha} \frac{\partial}{\partial y^\beta} J^{(2)}_{\lambda\nu} \right)_{Y=0} + (\mu \leftrightarrow \nu) + \text{h.c.} \right] , \tag{6.39}$$

where $R_{\mu\nu}(\omega)$ is the Ricci tensor obtained from the Riemann tensor associated with the Lorentz connection ω_μ. It is important to note that this connection contains torsion as can be seen from (6.35) and (6.37) which for $m = 0, n = 2$ give

$$\omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e) + \kappa_\mu{}^{ab} , \tag{6.40}$$

where $\kappa_\mu{}^{ab}$ is the con-torsion tensor related to the torsion tensor $T_{\mu\nu}{}^a$ as

$$\kappa_\mu{}^{ab} = T_\mu{}^{ab} - T_\mu^{ba} + T^{ab}{}_\mu , \tag{6.41}$$

where

$$T_{\mu\nu}{}^a = (\sigma^a)_{\alpha\dot\beta} \left(\frac{\partial}{\partial y_\alpha} \frac{\partial}{\partial \bar y_{\dot\beta}} J^{(2)}_{\mu\nu} \right)_{Y=0} . \tag{6.42}$$

3) The elimination of the auxiliary fields by means of the equations (6.35) and (6.36) gives rise to higher derivative interactions. In particular, in a given spin sector, the auxiliary fields are $W_{\mu\alpha_1...\alpha_k\dot\alpha_{k+1}...\dot\alpha 2s-2}$ with $k = 0, 1, ..., s/2$ and they are related to the physical fields $W_{\mu\alpha(s-1)\dot\alpha(s-1)}$ schematically as

$$W_{\mu,\alpha(m)\dot\alpha(n)} \sim \partial^{|m-n|/2} W_{\mu,\alpha(s-1)\dot\alpha(s-1)} , \qquad m + n = 2s - 2 . \tag{6.43}$$

Similarly, the components $\Phi_{\alpha(m)\dot\alpha(n)}$ of the master scalar field are related to the Weyl tensors which are purely chiral, their derivatives as well as the derivatives of the scalar as (taking $m > n$ without loss of generality)

$$\Phi_{\alpha(m)\dot\alpha(m)} \sim \partial^m \phi ,$$

$$\Phi_{\alpha(m)\dot\alpha(n)} \sim \partial^{(m-n)/2} \Phi_{\alpha(m-n)} , \qquad m - n = 0 \bmod 4 . \tag{6.44}$$

4) Whether the master constraints (6.10) and (6.11) are unique is an important question. In fact, there exist a generalization of (6.10) in which [19] we let

$$\widehat\Phi \star \kappa \;\rightarrow\; \mathcal{V}(\widehat\Phi \star \kappa) , \qquad \widehat\Phi \star \bar\kappa \;\rightarrow\; \bar{\mathcal{V}}(\widehat\Phi \star \bar\kappa) , \tag{6.45}$$

where $\mathcal{V}(X)$ is a $\star$-function, with its complex conjugate $\bar{\mathcal{V}}(X^\dagger) = (\mathcal{V}(X))^\dagger$. In [19] we argue that this function must be of the form

$$\mathcal{V}(X) = \sum_{n=0}^{\infty} b_{2n+1} \, X^{2n+1} \ , \qquad |b_1| = 1 \ . \tag{6.46}$$

A similar interaction ambiguity is expected to arise in HS theories yet to be constructed in $5D$ and $7D$ as well, and implications of this are discussed in Section 4, in the context of 5D HS gauge theory and holography. In particular, we argued that the freedom in choosing b_{2n+1} is important in order to find the precise agreement between the bulk amplitudes and the boundary correlators required for the massless theory to be a consistent truncation.

7 M Theory on $AdS_7 \times S^4$ and 7D Higher Spin Gauge Theory

The low-energy dynamics of a stack of N parallel coinciding M5 branes in flat eleven dimensional spacetime is described by a strongly coupled $d = 6$, $\mathcal{N} = (2,0)$ SCFT with $SO(5)_R$ symmetry group [1, 4], known as the the $A_{N-1}(2,0)$ theory [79, 80, 81, 31]. The theory is conjectured to have no marginal operators, which means that it describes an isolated UV fixed point of the renormalization group. It is not known whether the theory has any relevant operators which preserve the R-symmetry (the supergravity dual description provides relevant operators which break the R-symmetry). Conversely, starting in the IR with a number, N' say, of free $d = 6$, $\mathcal{N} = (2,0)$ tensor singletons, it is not known how to describe non-abelian interactions among tensor fields; in fact, there are no local perturbations with this effect [82] [16]. This is believed to reflect the fact that open membranes ending on coinciding M5 branes give rise to tensionless closed strings and that the proper language for formulating the dynamics on the fivebrane is therefore not ordinary field theory but rather some nonlocal extension of it.

However, if we are willing to give up 6d covariance, then we can use lower-dimensional RG flows based on ordinary interacting field theories to define the $A_{N-1}(2,0)$ theory [1, 4, 31, 25, 74]. In particular, circle reductions of the 6d theory describes RG flows of 4d and 5d SYM theories with $SU(N)$ gauge group. The $SO(4)_R$ invariant RG_5 flow has a Type IIA supergravity dual description in terms of the near horizon region of a D4 brane solution. In the UV limit the dilaton diverges and the solution uplifts to the $AdS_7 \times S^4$ near horizon region of the stack of M5 branes. The resulting anti-holographic description of the $A_{N-1}(2,0)$ theory is conjectured to be M theory on $AdS_7 \times S^4$ [1]. For large N the membrane tension scales like

$$T_{M2} \sim \frac{N}{R^3} \ , \tag{7.1}$$

where R is the AdS radius, and the 7D Planck length is given by

$$\frac{1}{l_{\text{Pl}}^5} = \frac{N^3}{R^5} \ . \tag{7.2}$$

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[16] A single tensor multiplet admits self-interactions, such as for example those describing the motion of a single M5 brane [83, 84, 85].

For large N the Planck length is much smaller than the M2 length scale, l_{M2} which in in turn is much smaller than the radius. Thus, for energies E obeying $1 << E << R/l_{M2}$ the low-energy dynamics of the anti-holographic dual is described by $D = 7$, $\mathcal{N} = 2$ gauged supergravity. The $A_{N-1}(2,0)$ theory has been conjectured to admit an expansion in terms of integer powers of $1/N$ which factorize for large N [4] [17] From (7.2) it follows that the $A_{N-1}(2,0)$ theory has $\sim N^3$ massless degrees of freedom for large N which contain the $N-1$ massless $(2,0)$ tensor multiplets of the 'Higgs branch' of the theory.

In the IR limit of the D4 brane geometry the dilaton e^ϕ vanishes and the gravitational curvature diverges. As for the D2-brane discussed in Section 6.1, the dual SYM coupling $g_{YM}^2 = g_s l_s$ is held fixed in taking the near horizon limit and equals the local Yang-Mills coupling $g_{YM}^2(u) \equiv e^{\phi(u)}/\sqrt{-g_{00}(u)/l_s^2} = g_{YM}^2$. Hence the local string length diverges in the IR (unlike the case of the D2 brane where the local string length disappears together with the dilaton in the UV). Hence, naively the D4 brane field theory becomes a free $SU(N)$ valued $d = 5$, $\mathcal{N} = 2$ Maxwell theory with $SO(5)_R$ symmetry and finite Yang-Mills coupling g_{YM}^2. This theory can be made scale invariant by absorbing g_{YM}^2 into the fields, but this symmetry is superficial since it cannot be lifted to superconformal invariance.

Instead a more natural interpretation is that superconformal invariance is restored by uplifting to a free $SU(N)$ valued $d = 6$, $\mathcal{N} = (2,0)$ tensor singleton described by the superconformal action[18]

$$S_6 = \int d^6 x \, \mathrm{tr} \left(|d\Phi^a|^2 + |dB|^2 + \text{fermions} \right) , \tag{7.3}$$

where Φ^a ($a = 1, ..., 5$) and $B_{\mu\nu}$ have dimension 2. The superspace formulation of this theory is described in more detail in Section 3.3. The 6d superconformal invariance can be spontaneously broken by compactifying (7.3) on a circle of radius R_{11} (after which R_{11} can of course be used to rescale the fields):

$$S_{5+1} = \frac{1}{R_{11}} \int d^5 x \, \mathrm{tr} \left(|d\phi^a| + |dA|^2 + \text{KK modes and fermions} \right) , \tag{7.4}$$

where all bosonic fields have dimension 1. Taking S_{5+1} as the generic starting point for describing the fivebrane dynamics there are thus two ways in which the theory can decompactify and become superconformally invariant: either by directly taking $R_{11} \to \infty$ which yields back S_6; or by throwing away the KK modes and switching on the Yang-Mills interaction with $g_{YM}^2 = R_{11}$, which then reaches the $A_{N-1}(2,0)$ limit in the UV limit $R_{11} \to \infty$. Note that the Yang-Mills deformation does not lead to loss of degrees of freedom since the KK modes are exchanged with monopoles with mass proportional to $g_{YM}^2 = R_{11}^{-1}$.

We conclude that it is natural to describe the low energy dynamics of N coinciding M5 branes in terms of an IR fixed point of free $SU(N)$ valued $d = 6$, $\mathcal{N} = (2,0)$ tensor singletons and a

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[17]From (7.1) it follows that M theory on $AdS_7 \times S^4$ has an expansion in terms of integer powers of $1/T_{M2}$ rather than integer powers of the 7D Plank's constant. The same remark applies to M theory on $AdS_4 \times S^7$, which has M2 tension given by (6.1) and has been conjectured to have an expansion in terms of integer powers of $1/\sqrt{N}$[4].

[18]Tensor self-duality and supersymmetry can be restored at the level of the field equations [85].

UV fixed point given by the $A_{N-1}(2,0)$ theory in the unbroken HS phase. We note that the number of massless degrees of freedom indeed decreases along the RG flow, from N^3 to N^2.

We conjecture that the free singleton theory at the IR fixed point mentioned above is the holographic dual of an $hs(8^*|4)$ gauge theory which admits a consistent truncation to the massless $hs(8^*|4)$ gauge theory in $D = 7$ described in Section 2.2. This theory describes an unbroken phase of M theory with N units of M5 brane charge. The strongly coupled fixed point is the holographic image of a broken phase which admits an effective supergravity description at low energies.

As in the case of M theory on $AdS_4 \times S^7$, there are also curious IR fixed points consisting of $N-1$ free tensor multiplets acted upon by the Weyl group of $SU(N)$ [31] which should also be included as nontrivial points in the phase diagram of M theory on $AdS_7 \times S^4$.

We can motivate further our proposal by constructing and examining the properties of 'long membrane' solutions to the M2-brane action [86]

$$S_{M2} = N \int d^3\sigma \sqrt{-\det\gamma} + N \int C_3 , \qquad (7.5)$$

where we have set the fermions equal to zero, $\gamma_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X^N g_{MN}$ and C_3 is the pull-back of the M-theory three-form potential which has non-zero components only in S^4. The worldvolume field equations are

$$\partial_\alpha(\sqrt{-\gamma}\gamma^{\alpha\beta}\partial_\beta X^M g_{MQ}) - \frac{1}{2}\sqrt{-\gamma}\gamma^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N \partial_Q g_{MN} + \epsilon^{\alpha\beta\gamma}\partial_\alpha X^M \partial_\beta X^N \partial_\beta X^P H_{QMNP} = 0 , \qquad (7.6)$$

where $H_4 = dC_3$. In order to describe the solution, which is similar to the string solution of [34], we use the global coordinates in AdS_7:

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho(d\theta^2 + \sin^2\theta(d\phi^2 + \sin^2\phi d\Omega_3^2)) . \qquad (7.7)$$

The $M2$ brane worldvolume coordinates are (τ, σ, φ) and our rotating membrane solution is given by

$$t = \tau , \quad \rho = \rho(\sigma) , \quad \theta = \theta(\varphi) , \quad \phi = \omega\tau , \quad \text{fixed point in } S^3 , \qquad (7.8)$$

where the membrane has the topology of a cylinder $-1 < \sigma < 1$, $0 \le \varphi < 2\pi$, which has been flattened such that the portion with $0 < \varphi < \pi$ is folded on top of the portion with $\pi < \varphi < 2\pi$. The induced metric becomes

$$ds^2 = -(\cosh^2\rho - \omega^2\sinh^2\rho\sin^2\theta)d\tau^2 + (\rho')^2 d\sigma^2 + \sinh^2\rho(\theta')^2 d\varphi^2 , \qquad (7.9)$$

where $\rho' \equiv d\rho/d\sigma$ and $\theta' \equiv d\theta/d\varphi$. It is straightforward to verify the nontrivial components of the field equations (7.6) which are the t, ρ, θ, ϕ component. The energy and spin of the configuration (7.8) are given by

$$E = 4N \int_0^{\rho_0} d\rho \int_{\theta_1}^{\theta_2} d\theta \frac{\cosh^2 \rho \sinh \rho}{\sqrt{\cosh^2 \rho - \omega^2 \sinh^2 \rho \sin^2 \theta}} , \tag{7.10}$$

$$s = 4N\omega \int_0^{\rho_0} d\rho \int_{\theta_1}^{\theta_2} d\theta \frac{\sinh^3 \rho \sin^2 \theta}{\sqrt{\cosh^2 \rho - \omega^2 \sinh^2 \rho \sin^2 \theta}} , \tag{7.11}$$

where

$$\theta_1 = \theta(0) = \theta(2\pi) , \quad \theta_2 = \theta(\pi) , \quad \rho_0 = \rho(\pm 1) . \tag{7.12}$$

The solution which minimizes the energy for a fixed spin and fixed width $\ell = \theta_2 - \theta_1$ (we shall minimize the energy with respect to the width below) is obtained by centering $\theta(\varphi)$ around $\theta = 0$ and maximizing the extension in the ρ-direction by taking

$$\coth \rho_0 = \omega . \tag{7.13}$$

If we assume that ℓ is small then

$$E = \frac{\ell N}{\omega^2} {}_2F_1[2, 1; 3/2; 1/\omega^2] , \tag{7.14}$$

$$s = \frac{2\ell N}{3\omega^3} {}_2F_1[2, 2; 5/2; 1/\omega^2] . \tag{7.15}$$

For $\omega \gg 1$ this describes short membranes with length $\rho_0 \sim 1/\omega$ and energy and spins given by

$$E^3 = 8\ell N \ s^2 , \quad E, s \ll \ell N . \tag{7.16}$$

In flat eleven-dimensional spacetime an analogous relation holds between mass and spin for all values of the spin (in flat space this relation follows from dimensional analysis). Thus, in flat spacetime the mass is minimized for given spin by sending $\ell \to 0$ and $\omega \to 0$ (keeping s fixed). The flat space spectrum therefore contains massless states arbitrary spin, which can be thought of as infinitely long, thin string-like membranes which are virtually at rest.

In fact, long ago bosonic open membrane (a disk) rotating simultaneously about two axis was considered in [32] where the relation a relation like (7.16) was derived. Such solutions are possible for $D \geq 5$. Later, this solution was generalized in [33] for the $D = 11$ supermembrane [86], by gluing two copies of the open membrane of [32] along their edges to obtain a 'pancake'

membrane. The zero-point energy of this membrane was studied by these authors and later in [87]. It was conjectured in [33] that the (semi-classical) energy-angular momentum relation of the kind (7.16) would be modified by an integral or half integral number due to the fact that the fermionic coordinates of the supermembrane also carry intrinsic angular momentum. See [88] for a review of this fascinating subject.

Going back to $AdS_7 \times S^4$, for slow rotation, $\omega \sim 1$, $\omega > 1$ and finite width ℓ, the solution (7.8) describes long membranes whose energy and spin now obeys

$$E - s = \frac{3\pi^{2/3}}{2^{1/3}}(\ell N)^{2/3} s^{1/3} \ , \quad E, s >> \ell N \ . \tag{7.17}$$

For $\omega \to 1$ the energy and spin diverges and the rotating membrane develops a boundary given by a folded closed string of length ℓ which trace out a Wilson surface in the stack of five-branes. Thus, the long membranes of width ℓ with finite energy describe operators in the the $A_{N-1}(2,0)$ theory which arise in the operator product expansion of the Wilson surface. The shape of the Wilson surface together with (2.21) suggest that its expansion contains bilinear higher spin operators which have asymptotically small anomalous dimensions, $(E - s)/s << 1$ for high spin, $s >> \ell N >> 1$. In the limit $s \to \infty$ their interactions should be equivalent to those described by the singletons.

Suppose there is no boundary condition which fixes ℓ to a finite value. The prescription is then to vary ℓ keeping s fixed as to minimize E. The minimal energy configuration for given spin s is obtained by taking $\ell \to 0$, $\omega \to 1$ which results in an infinitely long string-like membrane with energy $E = s$ (the ratio E/s is larger for short wide membranes than for long thin ones). Note that this geometry is assumed for any value of s, unlike in the case of the Type IIB closed string which became infinitely long only as $s/\sqrt{\lambda} \to \infty$. As $\ell \to 0$ the dual Wilson surface collapses and the higher derivative corrections to the $A_{N-1}(2,0)$ theory becomes suppressed, resulting in a flow down to the free tensor theory describing the unbroken phase with $hs(8^*|4)$ gauge symmetry.

Let us examine the supersymmetry of this solution. The condition for worldvolume supersymmetry is [75]

$$\Gamma\epsilon = \epsilon \ , \quad \Gamma = \frac{1}{\sqrt{-\det\gamma}}\frac{1}{3!}\epsilon^{\alpha\beta\gamma}\partial_\alpha X^M \partial_\beta X^N \partial_\gamma X^N \ , \tag{7.18}$$

and that ϵ is the Killing spinor of the $AdS_7 \times S^4$ background. An important property of these Killing spinors is that as we approach the boundary of AdS_7, i.e. as $\rho \to \infty$, they become an eigenstate of a constant Γ-matrix as follows [75]

$$\tilde{\Gamma}\epsilon = \epsilon \ , \quad \tilde{\Gamma} = \Gamma_{012345} \ , \tag{7.19}$$

where Γ_a are flat Dirac matrices and $a = 0, ..., 5$ are the indices tangent to the boundary of AdS_7. We have relabeled the coordinates of AdS_7 as

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$$(t, \phi, \theta, \psi', \theta', \phi', \rho) \rightarrow (x_0, x_1, ..., x_5, \rho) \ , \tag{7.20}$$

where (ψ', θ', ϕ') are the S^3 angles. Now, inserting the solution (7.8) into the definition of Γ in (7.18) gives

$$\Gamma = \frac{\left(c\Gamma_0 + (\omega s)\sin\theta\Gamma_1\right)\Gamma_{62}}{\sqrt{c^2 - \omega^2 s^2 \sin^2\theta}} \ . \tag{7.21}$$

where $c = \cosh\rho$ and $s = \sinh\rho$. Next, we find that $[\Gamma, \tilde{\Gamma}] = 0$. Therefore the worldvolume supersymmetries can be written as

$$\epsilon = (1 + \Gamma)(1 + \tilde{\Gamma})\eta \ , \tag{7.22}$$

for arbitrary η. We conclude that in the limit $\ell \rightarrow 0, \omega \rightarrow 1, \rho \rightarrow \rho_0 \rightarrow \infty$ keeping s fixed, the solution (7.8) preserves the 8 supersymmetries described by (7.22). The remaining 8 supersymmetries are broken, which means that the limiting solutions belong to semi-short multiplets which we identify as the massless HS multiplets arising in the tensor product of two tensor singletons listed in Table 3. Hence the corresponding anomaly free operators in the dual SCFT must fall into the same semi-short multiplet. This suggests that the dual operators are the Konishi-like superfields (3.33) and (3.35) containing the conserved HS currents described Section 3.

In the above limit the energy and the spin of the membrane accumulate at its ends which in turn move along light cones at the the boundary of AdS_7. The condition (7.22) has a natural interpretation as the supersymmetry condition for an intersection between the five-brane and the boundary of an open membrane. This suggests that the relevant part of the membrane dynamics are the fluctuations in this asymptotic region. It will be interesting to study more carefully the fluctuation spectrum about these worldvolume singletons, and in particular to examine whether they exhibit features such as fixed critical tension and discrete spectrum.

From the fact that the membrane interactions are concentrated at the boundary of the AdS spacetime we conclude that the $hs(8^*|4)$ gauge theory is a high energy limit of M theory on $AdS_7 \times S^4$.

We remark that the rotating membrane limit is a Lorentzian analog of the pp-wave limit on $AdS_7 \times S^4$ [30] which can be thought of as a collapsed membrane rotating around the equator of S^4. A difference that might be important is that the collapsed membrane has spherical topology while the rotating membrane has cylindrical topology.

One important test of the free $CFT_6/7D$ HS gauge theory correspondence is the matching of the holographic Weyl anomaly [73]. It was shown in [89] that the 6d trace-anomaly of (free) tensor singletons does not match the holographic anomaly computed in gauged supergravity in $D = 7$ [73]. To be more precise, the relative strength of the Euler invariant and the remaining invariant differs in the two cases by a factor of 4/7. It was argued in [89], however, that the trace anomaly of any CFT_6 picks up contributions from four-point stress-energy correlators.

This makes the 6d trace anomaly sensitive to the actual interactions of the bulk theory, unlike in e.g. $d = 4$, where the trace anomaly can be computed at weak coupling. Thus the 6d free field trace anomaly should be compared with the holographic anomaly of the corresponding 7D massless $hs(8^*|4)$ gauge theory (which does not admit any consistent truncation to gravity). Note that the corrections from massless HS exchange in the bulk are higher order in derivatives but of the same order in $1/N$. The matching of the overall strength is a consequence of the normalization (4.14), though the interactions should correct the factor of 4/7 [89].

Finally we comment on the breaking of HS gauge symmetries in AdS_7. The situation is much less clear here than in $D = 4, 5$, basically due to the fact that we do not know how to deform the boundary theory. As was discussed in Section 3.3, we do not have any candidates for the Higgs fields, which may have to do with the fact that we are writing local expressions whereas a more drastic, perhaps nonlocal construction, is what is actually required to break the HS symmetries in $D = 7$. Another important difference between the massless spectra in $D = 7$ and in $D = 4, 5$ is the fact that the latter saturate the unitarity bound for UIRs belonging to certain continuous series of the corresponding AdS supergroups, while the former belongs to an isolated series (see Section 3). In fact, this can be used to show that there can be no continuous (marginal) deformations taking the free SCFT to the strongly coupled fixed point [90]. Another curious fact is that the massless HS theory described in Section 2.3 does not make use of the 'massless' states which saturate the unitarity bound for UIRs belonging to the continuous series A. These operators are described by cubic tensor singletons, and it will be interesting to attempt to incorporate these into the master field formulation for the 7D HS theory described in Section 2.3.

8   Summary and Discussion

We have proposed that Type IIB string theory with N units of D3-brane charge and M theory with N units of M2-brane or M5-brane charge have unbroken phases described by HS gauge theories which admit consistent truncations to massless HS gauge theories in $D = 4, 5, 7$ with holographic duals given by $SU(N)$ valued scalar singleton theories in $d = 3, 4, 6$ with 16 super-symmetries. The corresponding HS algebras are

$$hs(8|4) \supset OSp(8|4) , \tag{8.1}$$

$$hs(2, 2|4) \supset PSU(2, 2|4) , \tag{8.2}$$

$$hs(8^*|4) \supset OSp(8^*|4) , \tag{8.3}$$

which are described in Section 2 together with the corresponding massless HS gauge theories. These theories also contain massive fields, some of which are Higgs fields that can be eaten by the massless fields. Both massless and massive fields also have KK towers which can be used to re-construct the spectrum of the Type IIB string and M theory in appropriate limits as discussed in more detail in Section 5.

In the case of Type IIB string on $AdS_5 \times S^5$, we have conjectured that the $hs(2,2|4)$ gauge theory arises in a critical limit of the Type IIB theory in which

$$g_s \sim 1/N , \qquad l_s \sim R \qquad \text{fixed } R , \quad N >> 1 , \qquad (8.4)$$

and that this limit corresponds to the free $4d, \mathcal{N} = 4, SU(N)$ SYM in which $g_{YM} = 0$. This means that the relations between the closed string parameters in $AdS_5 \times S^5$ and the gauge theory parameters for $\lambda >> 1$, which are read off from the D3-brane solution obtained in the supergravity approximation, are renormalized, as discussed in Section 5, and summarized in (1.1) and (1.2).

In the case of M theory on $AdS_{4/7} \times S^{7/4}$, we have conjectured the holographic boundary theories to be a $SU(N)$ valued $OSp(8|4)$ singleton field theory which resides at a UV fixed point in $3d$, and a free $SU(N)$ valued $(2,0)$ tensor singleton field theory residing at a IR fixed point in $6d$.

The spectrum of massless states in all the HS gauge theories discussed here have the universal property that they all arise in the symmetric product of two singletons. This motivates a world-sheet sigma model description of these theories based on an affine extension of AdS superalgebras in $D = 4, 5, 7$ with critical KM levels leading to left-moving and right-moving singleton Verma modules with a maximal number of null-states. In this respect, the existence of a singleton-like representations of affine $SO(3,2)$ with level $k_{\text{crit}} = 5/2$ found in [70] is encouraging.

The idea of obtaining the massless states of a $D = 4, \mathcal{N} = 8$ HS theory starting from the free $OSp(8|4)$ singleton theory, which in turn was obtained from the eleven dimensional supermembrane on $AdS_4 \times S^7$, already appeared long ago [40]. We recall that all the massless fields in this theory, with the exception of a pseudoscalar, satisfy the energy-spin relation $E_0 = s + 1$. More recently, long rotating strings that extend to the boundary of AdS_5 and couple to operators which are asymptotically anomaly free, i.e. $(E - s)/s \to 0$ as $E, s \to \infty$, have been studied [34].

Motivated by above the considerations, we have found rotating long membrane solutions (7.8) to the equations which describe the M2-brane in $AdS_7 \times S^4$ background. These membranes have width ℓ and the geometry of infinitely stretched strings with energy and spin density concentrated at the end points. They satisfy the semi-classical energy-spin relation $E = s$. A feature not present in the string case is that the energy is minimized for fixed spin by sending the angular velocity $\omega \to 1$ and the width $\ell \to 0$ keeping s fixed, resulting in infinitely long membranes with string-like geometry and semi-classical energy $E = s$. In Section 7, we have interpreted these as the lowest weight states of the massless supermultiplets of the 7D HS gauge theory discussed in Section 2 (see Table 3). Further aspects of this picture, especially the quantization issue, remain to be studied.

It would also be interesting to study the spherical membrane in AdS_4 and examine whether it admits 'breathing' and 'rotation' modes similar to those of strings in AdS_3 with NS-fluxes [78].

As there is effectively no separation in AdS energy between the massless HS fields and the massive HS fields, we have proposed that the massless HS theories (based on HS extension of the 32 supercharge AdS_{d+2} superalgebras in $d = 3, 4, 6$) arise as a result of consistent truncation of the full HS theories. This proposal can be tested explicitly since for large N, the singleton theory

and the HS gauge theory can be compared order by order in the $1/N$ expansion: consistent truncation implies that the massless HS theory action is the generating functional of correlators of bilinear operators. Indeed, a correlation function of four bilinear operators in a singleton theory can be written in a manifestly s-t-u symmetric form in terms of two- and three-point functions involving only bilinear operators, as discussed in Section 4.

We have also examined mechanisms for spontaneous breaking of HS gauge symmetry down to the symmetries underlying ordinary supergravity. In $D = 4, 5$ the 'order parameter' for breaking of HS gauge symmetry is the holographic Yang-Mills coupling. In $4d$ this is a marginal deformation which corresponds to a finite dilaton VEV in the bulk. The broken theory has an AdS vacuum in which the broken gauge fields have non-critical masses $m^2 - m^2_{\text{crit}} \sim N g^2_{\text{YM}}/R^2$. Using the non-intersection principle we argue these cross over into the leading Regge trajectory as $N g^2_{\text{YM}}$. We have also identified the Higgs multiplets at arbitrary level in the HS spectrum, and the realization of the level-one Higgs multiplet in terms of composite operators (i.e. anomaly multiplets) in the free singleton SCFT.

Also in $3d$, where the Yang-Mills coupling is a relevant perturbation, we have identified the Higgs multiplets at arbitrary level in the HS spectrum, and the realization of the level-one Higgs multiplet in terms of composite operators (i.e. anomaly multiplets) in the free $OSp(8|4)$ singleton field theory.

In $D = 7$ we do not know what is the order parameter for breaking HS gauge symmetry, nor have we identified the Higgs multiplets. This is presumably related to the fact that the massless gauge fields in $D = 7$ belong to the discrete B series (see (2.16) in Appendix B). We believe this issue should have a simple resolution in a framework where the nature of the mysterious interactions on the fivebrane is well understood. We stress that the Higgsing of the 7D HS gauge theory is dual to weak irrelevant perturbations of the tensor theory in the IR, which should be describable using a field theoretic, perhaps non-local, construction in $6d$. One may also speculate that the continuous A series (see (2.15)) could play a role in this, since the corresponding fields can be Higgsed, which signals the existence of the corresponding anomaly multiplets. This, in turn, would provide valuable data on the details of the interactions in $6d$.

An interesting open problem is to use the HS gauging techniques described in Section 2 and 6 to construct interactions between massless HS fields and Higgs fields. Clearly, the issue of consistent truncation becomes moot once we include (massive) Higgs fields. It is therefore a challenge to examine whether some generalized truncation scheme, perhaps of the type described in [27], may temper the fluctuations in the massive sector.

In testing various aspects of the AdS/HS gauge theory correspondences discussed in this paper, it will be very useful to develop a deeper understanding of the geometrical nature of HS interactions, possibly formulating them in a generalized superembedding approach. This would provide a universal tool for studying the HS dynamics [91] which would not only simplify the task of coupling Higgs master fields to HS gauge theories but also yield a superfield formulation [91] that would simplify the treatment of the bulk interaction and the computations of the attendant Witten diagrams. On the boundary side, the existing literature on the OPE computations involving free fields should be extended to cases where subleading in $1/N$ contributions will arise [22]. We have described few examples of such correlators in Section 4.

In this paper we have focused our attention on HS gauge theories in $D = 4, 5, 7$. No doubt these results can be extended to AdS_6 as well. In $D = 3$ the HS gauge fields do not propagate physical degrees of freedom. Nonetheless, physical matter fields of spin $s = 0, 1/2$ can be coupled to massless HS gauge theory [92, 93]. The advantage here is that an action principle is known and the mathematics is much simpler than in higher dimensions. It would be interesting to study this model in the context of massless higher spins and holography.

At the algebraic level there is in principle no bound on the number of supersymmetries in HS gauge theories and we expect consistent massless interactions for any $\mathcal{N}$ in $D \leq 7$, though certain restrictions follow from the requirement of an R symmetry neutral vierbein [91]. As discussed in Section 4, the restrictions on the spacetime superdimension are instead expected to be related to the consistency of the full HS quantum theory, including both massless and massive states, which requires the full generating functional (4.9) of the free singleton SCFT with finite sources for composite operators. Effectively, the condition that this quantity exists is expected to be as restrictive in the free singleton SCFT as in the (strongly) interacting singleton SCFT. This may lead to the restriction that the holographic dual cannot have more than 16 supersymmetries in $d \leq 6$. Similar restrictions should follow from the quantum consistency of the yet to be constructed dual bulk sigma models. In Section 4, similar effects were argued to arise in the holographic theory due to insertions of sewing operators in the free singleton field theory required for unitarity.

Another particularly interesting class of singleton CFTs, which we have not considered here, are the free $4d$ conformal HS theories constructed in [11]. Here the singleton field is a master field comprising an infinite set of ordinary singletons which together form an irreducible representation of a HS extension of the d-dimensional conformal group. In that case the relevant HS symmetry algebra is an infinite dimensional extension of $Sp(8, R)$ which contains the AdS group in $5D$.

To conclude, we believe that the remarkable algebraic and geometric structures underlying HS gauge symmetry are natural extensions of supergravity and will be important guides towards the true foundations of string and M theory. In particular, the simplicity of their holographic duals together with the fact that the bulk physics can still be phrased in a relatively simple language is both gratifying and compelling. Clearly much remains to be done in this subject which may be viewed as being still in its infancy.

Acknowledgements

ideas on critical singleton closed string theory. This research project has been supported in part by NSF Grant PHY-0070964.

A Spectra of Massless Higher Spin Gauge Theories in $D = 5, 7$

In this Appendix, we tabulate the spectra of singletons and the generators of the super HS groups and the field content of the master scalar fields in AdS_5 and AdS_7. The case of $D = 4$ is relatively simpler and has been presented in Section 2.1. The spectrum of physical states described by the master gauge fields in $D = 4, 5, 7$ are also given in Section 2.

$\ell \backslash s$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	15	4	1									
1	$16'$	24	36	24	$16'$	4	1					
2			1	4	$16'$	24	36	24	$16'$	4	1	
3							1	4	$16'$	24	36	$\cdots$
4											1	$\cdots$
$\vdots$												

Table 4: The $hs(2, 2|4)$ generators with $Y = 0, \pm 1$ arranged into levels labeled by $\ell = \frac{1}{4}(n_y + n_{\bar{y}} + n_\theta + n_{\bar{\theta}} - 2)$. The entries are $SU(4) \times U(1)_Y$ representations as follows: $15 = 15_0$, $4 = 4_1$, $1 = 1_0$, $16' = 15_0 + 1_0$, $24 = 20_1 + 4_1$ and $36 = 20'_0 + 15_0 + 1_0$, where the $U(1)_Y$ charge is defined by $Y = n_y - n_{\bar{y}}$. The $SO(4, 1)$ content is given by the highest weights $m_1 \geq m_2 \geq \frac{1}{2}|Y|$ where $m_1 = \frac{1}{2}(n_y + n_{\bar{y}})$. Upon gauging, these generators give rise to spin $s = m_1 + 1$ gauge fields which can be used to write a canonical set of covariant curvature constraints. As a result the gauge fields for $m_2 \geq \frac{1}{2}|Y| + 1$, $s \geq 2$ are auxiliary while those for $m_2 = \frac{1}{2}|Y|$ contain physical degrees of freedom.

$\ell \backslash s$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
1	16	4	6							
2			6	4	$16 + 1$	4	6			
3						6	4	$16 + 1$		$\cdots$
$\vdots$										

Table 5: The $hs(2, 2|4)$ generators with $Y = \pm 2, \pm 3, \pm 4$. The entries are $SU(4) \times U(1)_Y$ representations as follows: $16 = 10_2 + 6_2$, $4 = 4_3$, $6 = 6_2$ and $1 = 1_4$. Further notation is defined in Table 4. These generators are associated with gauge fields dual to generalized anti-symmetric tensor fields contained in the scalar master field Φ; see Table 7 for $s \geq 1$.

$\lvert Z\rvert \backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\cdots$
0	6	4	1					
$\frac{1}{2}$	4	$6+1$	4	1				
1	1	4	6	4	1			
$\frac{3}{2}$		1	4	6	4	1		
2			1	4	6	4	1	
$\vdots$					$\vdots$			

Table 6: The $d=4$, $\mathcal{N}=4$ singletons. The quantity Z is the $SU(2,2\vert 4)$ central charge carried by the supermultiplet. The entries in the Table denote $SU(4)$ representations. Each entry carries an $SO(4) \subset SO(6) \subset SO(6,2)$ representation $(j_L,0)$, and their complex conjugates $(0,j_R)$. The states for each value of $\lvert Z\rvert$ form a single massless irrep of $d=4, \mathcal{N}=4$ Poincaré superalgebra, and the states carry spin $s=j_L$. For all the states $E_0 = s+1$, where E_0 is the lowest AdS_5 energy. There exists an outer automorphism group $U(1)_Y$ of $SU(2,2\vert 4)$, and the $U(1)_Y$ charges of 6, 4 and 1 are 0, ± 1 and ± 2, respectively. The $Z=0$ multiplet is the $d=4$, $\mathcal{N}=4$ SYM singleton multiplet which has $8+8$ degrees of freedom. All the other singleton multiplets have $16+16$ degrees of freedom. For superfield realization of all the singletons listed in this Table, see Section 3.2.

$\ell \backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	42	48	$\underline{6}$											
1	1	8	$\underline{6}$	$\underline{4}$	$16+\underline{1}$	4	6							
2					6	4	$16+1$	4	6					
3									6	4	$16+1$			$\cdots$
$\vdots$														

Table 7: The physical fields contained in the master scalar field Φ arising in the $hs(2,2\vert 4)$ gauge theory in $D=5$. The entries are the following $SU(4) \times U(1)_Y$ representations for $s<1$: $42 = 20'_0 + 10_2 + \bar{10}_{-2} + 1_4 + \bar{1}_{-4}$, $48 = 20_1 + \bar{20}_{-1} + 4_3 + \bar{4}_{-3}$, $8 = 4_1 + \bar{4}_{-1}$ and 1_0; for $s \geq 1$: 6_2, 4_3, $16 = 10_2 + 6_2$ and 1_4. The spin $s \geq 1$ sector is realized in the field theory in terms of two-form potentials and their higher spin generalizations. These fields obey self-duality in $D=5$ and have dual one-form gauge fields corresponding to the generators given in Table 5, with the exception of the underlined representations, which have no one-form duals. Here the form degree refers to the number of curved indices as opposed to the tangential multi-spinor indices arising from the $(y,\bar{y})$-expansion.

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$\ell\backslash s$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	10	4	1									
1	10	20	20′	20	10	4	1					
2			1	4	10	20	20′	20	10	4	1	
3							1	4	10	20	20′	$\cdots$
4											1	$\cdots$
$\vdots$												

Table 8: The $hs(8^*|4)$ generators with $Y = 0, \pm 1$ arranged into levels labelled by $\ell = \frac{1}{4}(n_y + n_{\bar{y}} + n_\theta + n_{\bar{\theta}} - 2)$. The entries are $SO(5) \times U(1)_Y$ representations as follows: $10 = 10_0$, $4 = 4_1$, $1 = 1_0$, $20 = 16_1 + 4_1$ and $20′ = 14_0 + 5_0 + 1_0$, where the $U(1)_Y$ charge is defined by $Y = n_y - n_{\bar{y}}$. The $SO(6,1)$ content is labelled by highest weights $m_1 \geq m_2 \geq m_3 = \frac{1}{2}|Y|$ where $m_1 = \frac{1}{2}(n_y + n_{\bar{y}})$. Upon gauging, these generators give rise to spin $s = m_1 + 1$ gauge fields which can be used to write a canonical set of covariant curvature constraints. As a result the gauge fields for $m_2 \geq \frac{1}{2}|Y| + 1$, $s \geq 2$ are auxiliary while those for $m_2 = \frac{1}{2}|Y|$ contain physical degrees of freedom.

$\ell\backslash s$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
1	15	4	6							
2				6	4	16	4	6		
3								6	4	16 $\cdots$
$\vdots$										

Table 9: The $hs(8^*|4)$ generators with $Y = \pm 2, \pm 3, \pm 4$. The entries are $SO(6) \times U(1)_Y$ representations as follows: $15 = 5_2 + 10_2$, $4 = 4_3$, $6 = 5_2 + 1_2$ and $16 = 10_2 + 5_2 + 1_4$. These generators are associated with gauge fields dual to generalized anti-symmetric three-form tensor fields contained in the scalar master field Φ; see Table 11 for $s \geq 1$. Further notation is defined in Table 8.

$|Z|\backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\cdots$
0	5	4	1					
$\frac{1}{2}$	4	6	4	1				
1	1	4	6	4	1			
$\frac{3}{2}$		1	4	6	4	1		
2			1	4	6	4	1	
$\vdots$					$\vdots$			

Table 10: The $d = 6$, $\mathcal{N} = (2,0)$ singletons. The quantity Z denotes the $SU(2)_Z$ spin defined in Section 2.3. The entries denote $USp(4)_Y \simeq SO(5) \times U(1)_Y$ representations, which are irreducible except $6 = 5+1$. The $U(1)_Y$ charges of 1, 4, 5 are 0, ± 1 and ± 2, respectively. The $SO(6)$ highest weights (n_1, n_2, n_3) associated with each entry are given by $n_1 = n = 2 = n_3 = s$, and the AdS_7 energy by $E_0 = s + 2$. The level $\ell = 0(Z = 0)$ multiplet is the $d = 6$, $\mathcal{N} = (2,0)$ tensor singleton; see Section (3.3) for superfield realization of all the singletons shown in the Table, and composites formed out of the tensor singleton.

$\ell\backslash s$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\cdots$
0	14_0	16_1	$\underline{5}_2$											
1	1_0	4_1	$\underline{6}_2$	$\underline{4}_3$	$15_2 + \underline{1}_4$	4_3	6_2							
2						6_2	4_3	$15_2 + 1_4$	4_3	6_2				
3										6_2	4_3	$15_2 + 1_4$		$\cdots$
$\vdots$														

Table 11: The physical fields expected to arise in the master scalar field Φ in the $hs(8^*|4)$ gauge theory in $D = 7$. The entries are $SO(6) \times U(1)_Y$ representations, where $6 = 5+1$ and $15 = 10+5$. The spin $s \geq 1$ sector is expected to be realized in Φ in terms of three-form potentials and their higher spin generalizations. These fields obey self-duality in $D = 7$ and have dual one-form gauge fields corresponding to the generators given in Table 9, with the exception of the underlined representations, which have no one-form duals. Here the form degree refers to the number of curved indices as opposed to the tangential multi-spinor indices arising from the $(y, \bar{y})$-expansion.

2   UIRs of AdS Superalgebras in $D = 4, 5, 7$

In this Appendix, we define notation for the irreps of AdS superalgebras in $D = 4, 5, 7$, and list these irreps as well as the BPS short supermultiplets. These results are especially useful for the discussions of Section 2 and 3.

2.1   The UIRs of $OSp(8|4)$ and BPS Multiplets

Recall that a UIR of $OSp(8|4)$ is a multiplet of $SO(3,2) \times SO(8)$ UIR's denoted by

$$D(E_0, s; a_1, a_2, a_3, a_4) , \tag{2.1}$$

where E_0 is the minimum eigenvalue of the AdS energy generator M_{05}, s denotes $SO(3) \subset SO(3,2)$ spin and (a_1, a_2, a_3, a_4) are the Dynkin labels of the $SO(8)$ irrep carried by the lowest energy state. There exist two series of supermultiplets [48]:

$$\text{A)} \qquad E_0 \geq 1 + s + a_1 + a_2 + \tfrac{1}{2}(a_3 + a_4) , \tag{2.2}$$

$$\text{B)} \qquad E_0 = a_1 + a_2 + \tfrac{1}{2}(a_3 + a_4) , \qquad s = 0 . \tag{2.3}$$

These are the irreps carried by the lowest components of the supermultiplets, and the entire $OSp(8|4)$ supermultiplets are obtained by acting with supercharges.

The lowest components of the massless supermultiplets shown in Table 1 saturate the unitarity bound of series A as $E_0 = s + 1$, except in level $\ell = 0$ supergravity multiplet, in which case $D(1, 0; 0, 0, 2, 0)$ belongs to series B. The discrete series B contains the BPS multiplets. In particular, the singleton multiplet is characterized by the irrep $D(1/2, 0; 0, 0, 0, 1)$ carried by its lowest component and it belongs to series B. It can be described by a suitably constrained superfield. Taking a suitably symmetrized and constrained product of $2E_0$ singleton superfields one can construct BPS superfields whose lowest components carry the following irreps [50]

$$\text{BPS } 1/2 \quad : \quad D\left(p/2, 0; 0, 0, p, 0\right) , \tag{2.4}$$

$$\text{BPS } 1/4 \quad : \quad D\left((p + 2q)/2, 0; 0, q, p, 0\right) , \tag{2.5}$$

$$\text{BPS } 1/8 \quad : \quad D\left((p + 2q + 3r + 4s)/2, 0; r + 2\ell, q, p, r\right) . \tag{2.6}$$

All of these belong to series B. In particular, the lowest components of the KK towers of the level $\ell = 0$ supergravity multiplet carry the irrep $D(k/2, 0; 0, 0, k, 0)$ for $k = 3, 4, ...$ [53].

2.2   The UIRs of $PSU(2, 2|4)$ and BPS Multiplets

A UIR of $SU(2, 2|4)$ consists of UIRs of $SO(4, 2) \times SO(6)$ denoted by

$$D(E_0, j_L, j_R; a_1, a_2, a_3)_Y \ , \tag{2.7}$$

where E_0 is the eigenvalue of the AdS energy generator M_{06} and (j_L, j_R) label the $SO(4) \subset SO(4,2)$ irrep, (a_1, a_2, a_3) denote the Dynkin labels of the $SO(6) \simeq SU(4)$ R-symmetry irrep carried by the minimum energy states and Y denotes the outer $U(1)_Y$ automorphism charge, which will often be suppressed when it is vanishing. There exist three series of supermultiplets [42]:

A) $E_0 \geq 2 + J_L + J_R + a_1 + a_2 + a_3 \ , \quad J_L - J_R \geq \frac{1}{2}(a_3 - a_1)$ \hfill (2.8)

B) $E_0 = \frac{1}{2}(a_1 + 2a_2 + 3a_3) \geq 2 + 2J_L + \frac{1}{2}(3a_1 + 2a_2 + a_3) \ , \quad J_R = 0$ \hfill (2.9)

which implies

$E_0 \geq 1 + J_L + a_1 + a_2 + a_3 \ , \quad 1 + J_L \leq \frac{1}{2}(a_3 - a_1)$

C) $E_0 = 2a_1 + a_2 \ , \quad a_3 = a_1 \ , \quad J_L = J_R = 0$ \hfill (2.10)

In the case of series B, irreps with $(J_L \leftrightarrow J_R, a_1 \leftrightarrow a_3)$ must also be included. The irreps listed above are carried by the lowest components of the supermultiplets, and the entire $PSU(2,2|4)$ supermultiplets are obtained by acting with supercharges.

The lowest components of the massless supermultiplets shown in Table 2 saturate the unitarity bound of series A as $E_0 = s + 2$, with $J_L = J_R = s/2$, except in level $\ell = 0$ supergravity multiplet in which case $D(2,0,0;0,2,0)$ belongs to series C. The discrete series C contains the BPS multiplets. In particular the Maxwell singleton multiplet is characterized by $D(1,0,0;0,1,0)$ carried by its lowest component and it belongs to series C. It can be described by a suitably constrained superfield. Taking a properly symmetrized and constrained product of E_0 singletons superfields one can construct BPS superfields whose lowest components carry the following irreps [50]

BPS 1/2   : $D(p,0,0;0,p,0) \ ,$ \hfill (2.11)

BPS 1/4   : $D(p + 2q, 0, 0; q, p, q) \ ,$ \hfill (2.12)

BPS 1/8   : $D(p + 2q + 3r, 0, 0; q, p, q + 2r) \ .$ \hfill (2.13)

The BPS 1/2 and BPS 1/4 multiplets belong to series C, and the BPS 1/8 multiplets belong to series B. The KK towers of the level $\ell = 0$ supergravity are the BPS 1/2 multiplets given by $D(k, 0, 0; 0, k, 0)$ with $k = 3, 4, \dots$ [3, 58, 59].

There exists an extensive literature on the OPEs of various BPS 1/2 operators. The UIRs which can appear in these OPEs belong to series A with $J_L = J_R = s/2$, and series C [51].

2.3 The UIRs of $OSp(8^*|4)$ and BPS Multiplets

A UIR of $OSp(8^*|4)$ consists of UIRs of $SO(6,2) \times USp(4)$ denoted by

$$D(E_0, J_1, J_2, J_3; a_1, a_2)_Y , \tag{2.14}$$

where E_0 is the eigenvalue of the AdS energy generator M_{08}, (J_1, J_2, J_3) denote the Dynkin labels of the $SU(4) \simeq SO(6) \subset SO(6,2)$ irrep, (a_1, a_2) denote the Dynkin labels of the $USp(4)$ irrep carried by the minimum energy states and Y denotes the outer $U(1)_Y$ automorphism charge, which will often be suppressed when it is vanishing. There exist four series of supermultiplets [48]:

$$\text{A)} \quad E_0 \geq 6 + \tfrac{1}{2}(J_1 + 2J_2 + 3J_3) + 2(a_1 + a_2) , \tag{2.15}$$

$$\text{B)} \quad E_0 = 4 + \tfrac{1}{2}(J_1 + 2J_2) + 2(a_1 + a_2) , \quad J_3 = 0 , \tag{2.16}$$

$$\text{C)} \quad E = 2 + \tfrac{1}{2}J_1 + 2(a_1 + a_2) , \quad J_3 = J_2 = 0 , \tag{2.17}$$

$$\text{D)} \quad E_0 = 2(a_1 + a_2) , \quad J_3 = J_2 = J_1 = 0 . \tag{2.18}$$

These are the irreps carried by the lowest components of the supermultiplets, and the entire $OSp(8^*|4)$ supermultiplets are obtained by acting with supercharges.

The lowest components of the massless supermultiplets shown in Table 3 have $E_0 = s + 4$ and belong to series B, while the level $\ell = 0$ supergravity multiplet carries the irrep. $D(4, 0, 0, 0; 0, 2)$ which belongs to series D. The discrete series D contains the BPS multiplets. The singletons are contained in series C and D. The superfields in terms of which they are realized, and the UIRs carried by their lowest components are as follows [48, 46, 50]:

$$
\begin{array}{lll}
\text{D)} & W^{ij} & D(2, 0, 0, 0; 0, 1) \\
\text{D)} & W^i & D(2, 0, 0, 0; 1, 0) \\
\text{C)} & W & D(2, 0, 0, 0; 0, 0) \\
\text{C)} & \omega_{\alpha_1 \dots \alpha_{\ell-2}} & D(\tfrac{\ell}{2} + 1, \ell - 2, 0, 0; 0, 0)
\end{array}
\tag{2.19}
$$

The index $i = 1, ..., 4$ labels the 4-plet of $USp(4)$, the index $\alpha = 1, ..., 4$ labels the chiral spinor of $SO(6)$, $W^{ij} = -W^{ji}$ and symplectic traceless, $\Omega^{ij}W_{ij} = 0$, and $\omega_{\alpha_1 \dots \alpha_{\ell-2}}$ is totally symmetric in its indices (see Table 10 for further details). The superspace constraints imposed on these superfields can be found in [50]. The superfield W_{ij} represents the well known $(2, 0)$ tensor singleton and it is singlet under an $SU(2)_Z$ group defined in Section 2.3. The singleton superfields $(W^i, W, \omega_{\alpha_1 \dots \alpha_{\ell-2}})$, on the other hand, carry $SU(2)_Z$ spins $(1/2, 1, \ell/2)$, respectively. These are the level $\ell = 1, 2$ and $\ell \geq 3$ singletons shown in Table 10.

Taking suitably symmetrized and constrained products of singleton superfields, one can construct BPS superfields whose lowest components carry the following UIRs [50]:

$$\text{BPS } 1/2: \qquad D(2p, 0, 0, 0; 0, p) \ , \tag{2.20}$$

$$\text{BPS } 1/4: \qquad D(2p + 4q, 0, 0, 0; 2q, p) \ . \tag{2.21}$$

Both of these belong to series D. The KK towers of the level $\ell = 0$ supergravity are the BPS 1/2 multiplets given by $D(2k, 0, 0, 0; 0, k)$ with $k = 3, 4, \dots$ [50].

The OPEs of BPS 1/2 operators have been studied [51, 90]. The supermultiplets that can appear in the OPE of two BPS 1/2 operators belong to series A with $(J_1, J_2, J_3) = (0, s, 0)$; series B with $(J_1, J_2) = (0, s)$ and $E_0 = 4 + s + 2(a_1 + a_2)$; series C with $J_1 = 0$ and $E_0 = 2 + 2(a_1 + a_2)$, and series D [51].

3 Labeling of $USp(8)$, $SU(4)$, $USp(4)$ and $SO(8)$ Irreps

3.1 $USp(8)$

The highest weight state (HWS) labels (n_1, n_2, n_3, n_4) of $USp(8)$ satisfy $n_1 \geq n_2 \geq n_3 \geq n_4$ and are related to the Dynkin labels $[a_1, a_2, a_3, a_4]$ as follows:

$$n_1 = a_1 + a_2 + a_3 + a_4 \ , \qquad n_2 = a_2 + a_3 + a_4 \ , \qquad n_3 = a_3 + a_4 \ , \qquad n_4 = a_4 \ .$$

3.2 $SU(4) \sim SO(6)$

The HWS labels of $SU(4)$ irreps are (n_1, n_2, n_3). They satisfy $n_1 \geq n_2 \geq n_3$ and they are related to the Dynkin labels $[a_1, a_2, a_3]$ as follows:

$$n_1 = a_1 + a_2 + a_3 \ , \qquad n_2 = a_2 + a_3 \ , \qquad n_3 = a_3 \ .$$

The $SO(6)$ HW labels by (m_1, m_2, m_3) obey $m_1 \geq m_2 \geq |m_3|$ and they are related to the $SO(6)$ Dynkin labels $[b_1, b_2, b_3]$ as

$$m_1 = b_1 + \tfrac{1}{2}(b_2 + b_3) \ , \qquad m_2 = \tfrac{1}{2}(b_2 + b_3) \ , \qquad m_3 = \tfrac{1}{2}(-b_2 + b_3) \ .$$

These are related to the $SU(4)$ HW labels (n_1, n_2, n_3) and $SU(4)$ Dynkin labels $[a_1, a_2, a_3]$ as

$$
\begin{array}{lll}
m_1 = \tfrac{1}{2}(n_1 + n_2 - n_3) & m_2 = \tfrac{1}{2}(n_1 - n_2 + n_3) & m_3 = \tfrac{1}{2}(-n_1 + n_2 + n_3) \\
b_1 = a_2 & b_2 = a_1 & b_3 = a_3
\end{array}
$$

3.3 $USp(4) \sim SO(5)$

The $USp(4)$ irreps have the HWS labels (n_1, n_2) which satisfy $n_1 \geq n_2 \geq$ and they are related to the Dynkin labels $[a_1, a_2]$ as

$$n_1 = a_1 + a_2 , \qquad n_2 = a_2$$

The irreps of $SO(5)$ have HW labels (m_1, m_2) which satisfy $m_1 \geq m_2 \geq 0$ and they are related to the $SO(5)$ Dynkin labels $[b_1, b_2]$ as

$$m_1 = b_1 + \tfrac{1}{2} b_2 , \qquad m_2 = \tfrac{1}{2} b_2$$

These are related to the $USp(4)$ HW labels (n_1, n_2) and $USp(4)$ Dynkin labels $[a_1, a_2]$ as

$$m_1 = \tfrac{1}{2}(n_1 + n_2) \qquad m_2 = \tfrac{1}{2}(n_1 - n_2)$$
$$b_1 = a_2 \qquad b_2 = a_1$$

3.4 $SO(8)$

The irreps of $SO(8)$ have HWS labels (n_1, n_2, n_3, n_4) which satisfy $n_1 \geq n_2 \geq 0 \geq n_3 \geq |n_4|$ and they are related to the $SO(8)$ Dynkin labels $[a_1, a_2, a_3, a_4]$ as

$$n_1 = a_1 + a_2 + \tfrac{1}{2}(a_3 + a_4) , \qquad n_2 = a_2 + \tfrac{1}{2}(a_3 + a_4) , \qquad n_3 = \tfrac{1}{2}(a_3 + a_4) , \qquad n_4 = \tfrac{1}{2}(-a_3 + a_4) .$$

4   Compact and Non-compact Bases for $SO(d, 2)$

We write the $SO(d, 2)$ algebra in canonical form as ($A = 0, \ldots, d, d + 2$):

$$[M_{AB}, M_{CD}] = i\eta_{BC} M_{AD} + 3 \text{ more} , \tag{4.1}$$

where $\eta = \text{diag}(-, +, ..., +, -)$. The compact basis, which is suitable for describing physical AdS fields, consists of the AdS energy $E = -M_{0,d+2}$, the $SO(d)$ generators M_{ij} ($i = 1, ..., d$) and the spin-boosts $L_i^{\pm} = M_{i,d+2} \mp i M_{0i}$, which shift the AdS energy by ± 1. In compact basis the $SO(d, 2)$ weight spaces $D(E_0; m_1, \ldots m_{[d/2]})$ are obtained by acting with L_i^+ on lowest weight states, which have minimal energy $E = E_0$ and carry $SO(d)$ highest weights $(m_1, \ldots m_{[d/2]})$. Note that the label m_1 is the $SO(3) \subset SO(3, 2)$ spin in the case of AdS_4 and the sum $j_L + j_R$ of $SU(2)_L \times SU(2)_R \simeq SO(4) \subset SO(4, 2)$ spins in the case of AdS_5. The non-compact

basis, which is suitable for describing conformal fields, consists of the dilatation generator $D = M_{d,d+2}$, the $SO(d-1,1)$ generators $M_{\mu\nu}$ ($\mu = 0, 1, \ldots, d-1$), and the d-dimensional momentum $P_\mu = M_{\mu d} + M_{\mu,d+2}$ and generator of special conformal transformations $K_\mu = M_{\mu d} - M_{\mu,d+2}$. The compact basis $(E, M_{ij}, L_i^\pm)$ and non-compact basis $(D, M_{\mu\nu}, K_\mu, P_\mu)$ are related [94] by a similarity transformation executed by the (non-unitary) operator

$$S = \exp i L_d^+ \ , \tag{4.2}$$

with the following properties

$$SDS^{-1} = -iE + \frac{1}{2}L_d^- \ , \tag{4.3}$$

$$SM_{0a}S^{-1} = -iM_{a,d} - \frac{i}{2}L_a^- \ , \qquad SM_{ab}S^{-1} = M_{ab} \ , \tag{4.4}$$

$$SK_0S^{-1} = -\frac{i}{2}L_d^- \ , \qquad SK_aS^{-1} = -\frac{1}{2}L_a^- \ , \tag{4.5}$$

where we have split the indices as follows

$$i = \overbrace{1, 2, ..., d-1}^{a}, d \ , \qquad \mu = 0, \overbrace{1, 2, ..., d-1}^{a} \ . \tag{4.6}$$

Hence $(d+1)$-dimensional time-evolution and spatial rotation are equivalent to d-dimensional dilatation and Lorentz rotation. Thus

$$S^{-1}D(E_0; m_1, \ldots, m_{[d/2]}) = \mathcal{O}_\Delta(0)|0\rangle$$

where $|0\rangle$ is the vacuum of the CFT$_d$ and $\mathcal{O}_\Delta(x) = e^{ix^\mu P_\mu}\mathcal{O}_\Delta(0)e^{-ix^\mu P_\mu}$ is a conformal tensor with scaling dimension

$$\Delta = E_0$$

and Lorentz spin given by $(m_1, \ldots, m_{[d/2]})$.

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