Published for SISSA by Springer

Received: *January 4, 2015*
Revised: *April 15, 2015*
Accepted: *May 22, 2015*
Published: *June 17, 2015*

Singly-spinning black rings in $D = 5$ U$(1)^3$ supergravity

H. Lü,[a] **C.N. Pope,**[b,c] **Justin F. Vázquez-Poritz**[d,e] **and Zhibai Zhang**[d,e]

[a] *Department of Physics, Beijing Normal University, Beijing 100875, China*

[b] *George P. & Cynthia Woods Mitchell Institute for Fundamental Physics and Astronomy, Texas A&M University, College Station, TX 77843, U.S.A.*

[c] *DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 OWA, U.K.*

[d] *New York City College of Technology, The City University of New York, Brooklyn NY 11201, U.S.A.*

[e] *The Graduate School and University Center, The City University of New York, New York NY 10016, U.S.A.*

E-mail: mrhonglu@gmail.com, pope@physics.tamu.edu, jvazquez-poritz@citytech.cuny.edu, zzhang@citytech.cuny.edu

Abstract: We construct black ring solutions in five-dimensional U$(1)^3$ supergravity which carry three dipole charges, three electric charges and one angular momentum parameter. These solutions are written in a form that is sufficiently compact that their global and thermodynamic properties can be studied explicitly. In particular, we find that the Smarr formula is obeyed regardless of whether or not conical singularities are present, whereas the first law of thermodynamics holds only in the absence of conical singularities. We also present black ring solutions with three background magnetic fields.

Keywords: Black Holes, Supergravity Models, Spacetime Singularities

ArXiv ePrint: 1411.1413

doi:10.1007/JHEP06(2015)112

Contents

1 Introduction

Black holes in five dimensions can have event horizons with $S^1 \times S^2$ topology, and are known as black rings in order to distinguish them from topologically spherical black holes (see [1] for a review). The existence of black ring solutions demonstrated that black hole uniqueness is violated in five dimensions. The neutral black ring in pure gravity was presented in [2], and charged black rings in supergravity were found in [3–11].

Various generalizations have been constructed, a spherical black hole surrounded by a black ring or multiple concentric rings [7, 12], a doubly-spinning black ring [13], and two concentric orthogonal rotating black rings [14], as well as approximate solutions describing thin, neutral black rings in any dimension $D \geq 5$ in asymptotically flat spacetime [15] and asymptotically (anti)-de Sitter spacetime [16]. However, while the most general supersymmetric solution has been known for quite some time [6, 8–10], the general nonextremal solution in five-dimensional U(1)^3 supergravity has remained elusive until recently [17]. This nonextremal black ring has three independent dipole and electric charges, along with five additional parameters. If one imposes the condition that there are no conical singularities present, then the remaining parameters correspond to three electric charges, three dipole charges, two angular momenta and the mass.

One solution-generating technique that has been used to construct a number of the above black ring solutions is the inverse scattering method, which was first adapted to Einstein's equations in [18, 19]. In this paper, we shall make use instead of a solution-generating technique that involves performing a dimensional reduction, and which hinges on the relation between black rings and the C-metric solution. In particular, the neutral black ring can be obtained by lifting the Euclideanized Kaluza-Klein C-metric of [20] to five dimensions on a timelike direction [2, 21].

A three-charge generalization of the C-metric has recently been constructed in STU gauged supergravity [22]. In the ungauged limit, the Euclidean-signature version of this solution can be lifted on a timelike direction to give a black ring solution with three independent dipole charges and one non-vanishing rotation parameter. Dipole black rings were first found in [5]. Furthermore, three independent electric charges can be generated by repeatedly lifting this solution to six dimensions and applying boosts. The resulting black ring solutions generalize those found in [11], for which only two out of the three dipole charges were independent parameters. In a similar manner, one can repeatedly lift the C-metric solution to five dimensions and apply $\mathrm{SL}(2,R)$ transformations to get a black ring solution with background magnetic fields, at the expense of altering the asymptotic geometry.

In order to avoid closed timelike curves for these solutions with a single angular momentum parameter, it turns out that one must turn off the electric charges. Moreover, one must impose a constraint in order to avoid conical singularities. Then one is left with solutions which have five nontrivial parameters corresponding to the mass, one nonvanishing angular momentum and three dipole charges. In addition to this, one can turn on three independent background magnetic fields, in which case the solution is no longer asymptotically flat but rather asymptotically approaches a five-dimensional Melvin fluxbrane [23].

The solutions discussed in this paper all have a single non-vanishing rotation parameter, as opposed to the black ring presented in [17] which has two independent angular momenta. In particular, we expect that our black ring with three dipole charges and three electric charges arises as a specialization of the one in [17]. The relatively compact form of our solution enables one to verify its validity analytically, rather than numerically as done in [17]. Moreover, the various properties of the solution, its physical and thermodynamic quantities and the relations between them can be studied explicitly.

This paper is organized as follows. In the next section, we demonstrate how black rings with three dipole charges can be obtained from the C-metric solutions. We then use solution-generating techniques involving dimensional reductions to add three electric charges, as well as three background magnetic fields. In section 3, we study the global properties of the five and six-dimensional solutions in the Ricci-flat limit. In section 4, we perform the global analysis and study the thermodynamics of the general black ring solutions. Conclusions are presented in section 5. Lastly, in an appendix, we compile the dimensional reductions that relate the six, five and four-dimensional supergravity theories used for the purposes of solution generating.

JHEP06(2015)112

2 Black rings in five-dimensional $U(1)^3$ supergravity

2.1 From C-metrics to dipole black rings

Our starting point is the four-dimensional ungauged supergravity theory that is known as the STU model, which can be obtained via a dimensional reduction from six-dimensional string theory. It has an $SL(2,R)\times SL(2,R)\times SL(2,R)$ global symmetry, corresponding to– when discretized– the (S,T,U) duality symmetries of the string theory. The theory contains four U(1) vector fields $F_I = dA_I$ and three complex scalars $\tau_i = \chi_i + \mathrm{i}e^{\phi_i}$. The axions χ_i can be consistently truncated out, provided that one imposes the conditions

$$F_I \wedge F_J = 0\,, \qquad I \neq J\,. \tag{2.1}$$

The truncated bosonic Lagrangian is given by

$$e_4^{-1}\mathcal{L}_4 = R_4 - \tfrac{1}{2}\sum_{i=1}^{3}(\partial\phi_i)^2 - \tfrac{1}{4}\sum_{I=1}^{4} e^{\vec{a}_I\cdot\vec{\phi}}F_I^2\,, \tag{2.2}$$

where the dilaton vectors $\vec{a}_I$ satisfy

$$\vec{a}_I\cdot\vec{a}_J = 4\delta_{IJ} - 1\,, \qquad \sum_{I=1}^{4}\vec{a}_I = 0\,. \tag{2.3}$$

Our starting point will be the charged C-metric solutions obtained in [22], in the ungauged limit $g = 0$. In this case the solutions can be written as

$$\begin{aligned} ds_4^2 &= \frac{1}{\alpha^2(y-x)^2}\left[\frac{1}{\sqrt{\tilde{U}}}\left(G(y)d\tau^2 - \frac{\mathcal{H}(y)}{G(y)}dy^2\right) + \sqrt{\tilde{U}}\left(\frac{\mathcal{H}(x)}{G(x)}dx^2 + G(x)d\varphi^2\right)\right], \\ e^{\vec{a}_I\cdot\vec{\phi}} &= \frac{U_I^2}{\sqrt{\tilde{U}}}\,, \qquad F_I = \frac{4Q_I}{h_I(y)^2}\,dy\wedge d\tau\,, \qquad U_I = \frac{h_I(y)}{h_I(x)}\,, \qquad \tilde{U} = U_1U_2U_3U_4\,, \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} h_I(\xi) &= 1 + \alpha q_I\xi\,, \qquad \mathcal{H}(\xi) = \prod_{I=1}^{4} h_I(\xi)\,, \\ G(\xi) &= \mathcal{H}(\xi)\left(b_0 + \sum_{i=1}^{4}\frac{16Q_i^2}{\alpha^2 q_i\prod_{j\neq i}(q_j - q_i)}\frac{1}{h_i(\xi)}\right). \end{aligned} \tag{2.5}$$

The general solution contains 10 parameters: (α, b_0, q_I, Q_I) where $I = 1,\ldots,4$. The parameter b_0 can always be scaled to have one or another of the discrete values $(-1,0,1)$. The form of the solution is invariant under

$$x = \frac{\tilde{x}}{1+b\tilde{x}}\,, \qquad y = \frac{\tilde{y}}{1+b\tilde{y}}\,, \tag{2.6}$$

which enables one to set one of the four scalar charges q_I to any value, including zero (the STU model has three independent scalars after all). In our construction, the parameters

JHEP06(2015)112

Q_I are independent of the scalar charges q_I. Therefore, we could set $Q_I = 0$ and obtain C-metric solutions supported solely by the scalar charges. On the other hand, for generic Q_I we cannot set the scalar charges to zero, as one would have expected from the theory. When all of the electric charge parameters are equal, namely $Q_I \equiv Q$, we can set all $q_I \equiv q$, the three scalar fields all vanish and we recover the charged C-metric solution of the Einstein-Maxwell theory. While conical singularities can be avoided only for a single point in the parameter space of the dilaton C-metric in [20], conical singularities are absent for a range of parameters for the present C-metric solutions. We will discuss global properties after we lift the solutions to five dimensions.

We shall be lifting the C-metric solution to a dipole black ring solution in five dimensions, but before that a number of preliminary steps are needed. First, we use electromagnetic duality to map the C-metric solution to one that has magnetic charges, for which

$$e^{\vec{a}_I\cdot\vec{\phi}} = \frac{\sqrt{\tilde{U}}}{U_I^2}\,, \qquad F_I = \frac{4Q_I}{h_I(x)^2}\, dx\wedge d\varphi\,. \tag{2.7}$$

Next, we Wick rotate $\tau \to \mathrm{i}\psi$ so that the C-metric has Euclidean signature. Then we dualize one of the gauge fields so that it can play the role of a Kaluza-Klein vector along a timelike direction:

$$\mathcal{F} = e^{\vec{a}_4\cdot\vec{\phi}} {*F_4} = \frac{4Q_4}{h_4^2}\, dy\wedge d\psi \qquad \Longrightarrow \qquad \mathcal{A} = \frac{4Q_4 y}{h_4(y)} d\psi\,. \tag{2.8}$$

Note that the kinetic term for the corresponding field strength now has the "wrong" sign in the Lagrangian, which is given by (A.11). However, this is not an issue once we lift to five dimensions. Using the reduction ansatz (A.10), we lift the solution along a timelike direction. The resulting five-dimensional solution is given by

$$\begin{aligned}
ds_5^2 &= -\frac{U_4}{(U_1U_2U_3)^{\frac{1}{3}}}\left(dt + \frac{4Q_4 y}{h_4(y)}d\psi\right)^2 \\
&\quad + \frac{1}{\alpha^2(x-y)^2}\left[\frac{1}{U_4(U_1U_2U_3)^{\frac{1}{3}}}\left(-G(y)d\psi^2 - \frac{\mathcal{H}(y)}{G(y)}dy^2\right)\right. \\
&\quad \left. + (U_1U_2U_3)^{\frac{2}{3}}\left(\frac{\mathcal{H}(x)}{G(x)}dx^2 + G(x)d\varphi^2\right)\right], \\
F_i &= \frac{4Q_i}{h_i^2(x)}dx\wedge d\varphi\,, \qquad e^{\vec{b}_i\cdot\vec{\phi}} = \frac{(U_1U_2U_3)^{\frac{2}{3}}}{U_i^2}\,, \qquad U_i = \frac{h_i(y)}{h_i(x)}\,.
\end{aligned} \tag{2.9}$$

This is a black ring solution, and it generalizes the dipole black ring found in [5] such that all three dipole charges are independent parameters. The (x,φ) subspace corresponds to the S^2 part of its $S^2\times S^1$ topology, and the S^1 is associated with ψ.

In the special case that Q_i and q_i are related as follows:

$$q_i = -\mu s_i^2\,, \qquad Q_I = \tfrac{1}{4}\mu s_i c_i\,, \qquad s_i = \sinh\delta_i\,, \qquad c_i = \cosh\delta_i\,, \tag{2.10}$$

we find that

$$G(\xi) = b_0\mathcal{H}(\xi) - \xi^2(1+\alpha\mu\xi)\,. \tag{2.11}$$

This specialisation enables us to take a Ricci-flat limit of the black ring solution, by setting $\delta_1 = \delta_2 = \delta_3 = 0$, and we shall return to it in section 3.

2.2 Adding electric charges

Using the reduction ansatz (A.2), we lift the black ring solution (2.9) to six dimensions on the z direction. Then we perform a Lorentz boost $t \to c_1 t + s_1 z$, $z \to c_1 z + s_1 t$, where $c_i = \cosh\delta_i$, $s_i = \sinh\delta_i$. The reduction back to five dimensions along the boosted z direction generates the first electric charge, which is associated with the KK vector A_3. Once we apply the discrete symmetry to interchange the gauge fields A_1 and A_3 (and rotate the dilatons accordingly), the electric charge is now associated with A_1. Next, we repeat the process of lifting to six dimensions, performing a boost with parameter δ_2 and reducing back to five dimensions in order to generate a second electric charge. Then we dualise the 2-form potential B in (A.2) to a 1-form potential A_2 and use the discrete symmetry again to interchange A_3 and A_2, so that the second electric charge is now associated with A_2. Dualising the resulting A_2 to a 2-form potential B again, lifting to six dimensions and boosting with parameter δ_3, we again reduce back to five dimensions. Finally, we dualise the 2-form B to a new 1-form A_2, thereby arriving at the black ring solution with three independent electric charges. After making a convenient transformation of the time coordinate, and a relabelling of the various functions in the solution, the 3-charge black ring takes the following form:

$$ds_5^2 = -\frac{U_4}{U^{1/3}(H_1H_2H_3)^{2/3}}\,(dt+\omega)^2 + (H_1H_2H_3)^{1/3}\,ds_4^2\,, \tag{2.12}$$

where

$$\begin{aligned}
\omega &= -4x\Big(\frac{s_1c_2c_3Q_1}{h_1(x)} + \frac{c_1s_2c_3Q_2}{h_2(x)} + \frac{c_1c_2s_3Q_3}{h_3(x)} + \frac{s_1s_2s_3Q_4}{h_4(x)}\Big)d\varphi \\
&\quad +4y\Big(\frac{c_1s_2s_3Q_1}{h_1(y)} + \frac{s_1c_2s_3Q_2}{h_2(y)} + \frac{s_1s_2c_3Q_3}{h_3(y)} + \frac{c_1c_2c_3Q_4}{h_4(y)}\Big)d\psi\,, \\
ds_4^2 &= \frac{1}{\alpha^2(x-y)^2}\left[\frac{1}{U_4U^{\frac13}}\left(-G(y)d\psi^2 - \frac{\mathcal{H}(y)}{G(y)}dy^2\right) + U^{\frac23}\left(\frac{\mathcal{H}(x)}{G(x)}dx^2 + G(x)d\varphi^2\right)\right]
\end{aligned} \tag{2.13}$$

and

$$H_i = 1 + \left(1 - \frac{U_i^2\,U_4}{U}\right)s_i^2\,, \qquad U = U_1U_2U_3\,. \tag{2.14}$$

The scalar fields are given by

$$e^{\vec{b}_i\cdot\vec{\phi}} = \frac{H_i^2}{U_i^2}\left(\frac{U}{H_1H_2H_3}\right)^{2/3}\,, \tag{2.15}$$

and the gauge potentials are

$$\begin{aligned}
A_1 \;&=\; \frac{s_1c_1}{H_1}\left(1 - \frac{U_1U_4}{U_2U_3}\right)dt\, + \\
&\quad + \frac{4x}{H_1}\left(\frac{c_1c_2c_3Q_1}{h_1(x)} + \frac{s_1s_2c_3Q_2U_1U_4}{U_3h_2(y)} + \frac{s_1c_2s_3Q_3U_1U_4}{U_2h_3(y)} + \frac{c_1s_2s_3Q_4}{h_4(x)}\right)d\varphi \\
&\quad - \frac{4y}{H_1}\left(\frac{s_1s_2s_3Q_1U_4}{U_2U_3h_1(x)} + \frac{c_1c_2s_3Q_2}{h_2(y)} + \frac{c_1s_2c_3Q_3}{h_3(y)} + \frac{s_1c_2c_3Q_4U_1}{U_2U_3h_4(x)}\right)d\psi\,,
\end{aligned} \tag{2.16}$$

with A_2 and A_3 being obtained from A_1 by cycling the indices 1, 2 and 3 on all quantities appearing in (2.16).

2.3 Adding background magnetic fields

We can use a similar solution-generating procedure as in the previous section to obtain an Ernst-like generalization of the C-metric solution. Using the reduction ansatz (A.7), we lift the C-metric solution (2.4) to five dimensions on the spacelike direction z. Next, we perform the coordinate transformation $\varphi \to \varphi + Bz$ and reduce back to four dimensions, where we find that B parameterizes the strength of a background magnetic field. Applying the discrete symmetry to interchange gauge fields, we keep repeating this procedure until we have generated a solution with four background magnetic fields, given by

$$
\begin{aligned}
ds_4^2 &= \frac{1}{\alpha^2(y-x)^2}\prod_{I=1}^4\sqrt{\frac{\Lambda_I}{U_I}}\left[\left(G(y)d\tau^2-\frac{\mathcal{H}(y)}{G(y)}dy^2\right)+\prod_{I=1}^4 U_I\left(\frac{\mathcal{H}(x)}{G(x)}dx^2+\frac{G(x)}{\prod_{I=1}^4\Lambda_I}d\varphi^2\right)\right],\\
A_I &= -\frac{1}{B_I\Lambda_I}\left(1+\frac{4B_IQ_Ix}{h_I(x)}\right)d\varphi\,, \qquad e^{\vec{a}_I\cdot\vec{\phi}} = \frac{\prod_{J=1}^4\sqrt{\Lambda_J U_J}}{\Lambda_I^2 U_I^2}\,,\\
\Lambda_I &= \left(1+\frac{4B_IQ_Ix}{h_I(x)}\right)^2+\frac{B_I^2G(x)}{\alpha^2(x-y)^2U_I^2}\,.
\end{aligned}
\tag{2.17}
$$

The various functions appearing here are defined in (2.5). The C-metric solution (2.4) is recovered for vanishing magnetic field parameters B_I. Note that one can tune the values of the B_I so as to avoid a conical singularity, even if one is present in the corresponding C-metric solution (meaning that all B_I are taken to zero with all other parameters held fixed). Thus, the background magnetic fields associated with B_I play a role analogous to the cosmic string in the C-metric itself, by providing the force necessary to accelerate the black hole. (See [24] for a discussion of externally magnetised charged black hole solutions in STU supergravity.)

We can Wick rotate $\tau \to i\psi$ in the solution (2.17) and then lift it to five dimensions on a timelike direction using the metric ansatz (A.10). This yields a dipole black ring with background magnetic fields, given by

$$
\begin{aligned}
ds_5^2 &= \frac{\Lambda^{\frac{2}{3}}}{\alpha^2(x-y)^2}\left[\frac{1}{U_4U^{\frac{1}{3}}}\left(-G(y)d\psi^2-\frac{\mathcal{H}(y)}{G(y)}dy^2\right)+U^{\frac{2}{3}}\left(\frac{\mathcal{H}(x)}{G(x)}dx^2+\frac{G(x)}{\Lambda}d\varphi^2\right)\right]\\
&\quad -\frac{U_4}{(\Lambda U)^{\frac{1}{3}}}\left(dt+\frac{4Q_4y}{h_4(y)}d\psi\right)^2,\\
A_I &= -\frac{1}{B_I\Lambda_I}\left(1+\frac{4B_IQ_Ix}{h_I(x)}\right)d\varphi\,, \qquad e^{\vec{b}_i\cdot\vec{\phi}} = \frac{(\Lambda U)^{\frac{2}{3}}}{(\Lambda_iU_i)^2}\,,\\
U &\equiv U_1U_2U_3\,, \qquad \Lambda \equiv \Lambda_1\Lambda_2\Lambda_3\,,
\end{aligned}
\tag{2.18}
$$

and $i = 1, 2, 3$. Note that this solution has three independent magnetic field parameters, since in five dimensions one can get rid of B_4 by performing the reverse of the coordinate transformation that was used to generate it in the first place.

JHEP06(2015)112

3 Global analysis of Ricci-flat solutions

3.1 $D = 5$ Ricci-flat metric

Before turning to the general black ring solution, we shall first study the global structure of its Ricci-flat limit, as a warm-up exercise. If we set $Q_1 = Q_2 = Q_3 = 0 = q_1 = q_2 = q_3$, then the solution (2.9) becomes Ricci-flat and takes the form

$$\begin{aligned} ds_5^2 = & -\frac{h_4(y)}{h_4(x)}\left(dt+\frac{4Q_4 y}{h_4(y)}d\psi\right)^2+\frac{1}{\alpha^2(x-y)^2}\left[\frac{h_4(x)}{h_4(y)}\left(-G(y)d\psi^2-\frac{h_4(y)}{G(y)}dy^2\right)\right. \\ & \left.+\left(\frac{h_4(x)}{G(x)}dx^2+G(x)d\varphi^2\right)\right], \end{aligned} \tag{3.1}$$

where $h_4(\xi) = 1+\alpha q_4\xi$. Setting $q_i = 0$ can be subtle, due to diverging terms in the expression for $G(\xi)$. Instead, we substitute the above form of the solution directly into the equations of motion in order to obtain the most general solution for $G(\xi)$. This is given by

$$G(\xi) = (c_0+c_1\xi+c_2\xi^2)h_4(\xi)-\frac{16Q_4^2\xi^2}{q_4^2}. \tag{3.2}$$

We can then shift c_0, c_1, c_2 to get rid of the ξ^2 factor in the Q_4^2 term, and shift ξ such that $h_4(\xi) \sim \xi$. This results in the expression

$$G(\xi) = (c_0+c_1\xi+c_2\xi^2)\xi-\frac{16Q_4^2}{\alpha^2 q_4^4}, \tag{3.3}$$

in which all the parameters of the solution are shown explicitly. Note that $G(\xi)$ cannot be written in this form for the general black ring solution. The curvature singularities are located at $x = \infty$, $y = \infty$ and $h_4(x) = 0$, and the asymptotic region is at $x = y$.

Consider the coordinate transformations

$$x \to \frac{\tilde{x}-1}{2q_4^2}, \qquad y \to \frac{\tilde{y}-1}{2q_4^2}, \qquad t \to -\sqrt{a_0}(q_4^2\tilde{t}+\psi), \tag{3.4}$$

where a_0 is related to Q_4 by $Q_4 = \frac{1}{2}\sqrt{a_0}q_4^2$. Upon setting $\alpha = 2q_4$, we find that the metric can be written as

$$\begin{aligned} ds_5^2 &= \frac{1}{(x-y)^2}\left[\frac{x dx^2}{4G(x)}+G(x)d\varphi^2-\frac{x dy^2}{4G(y)}-\frac{xG(y)d\psi^2}{y}\right]-\frac{a_0 y}{x}(dt+y^{-1}d\psi)^2, \\ G(\xi) &= -a_0+a_1\xi+a_2\xi^2+a_3\xi^3, \end{aligned} \tag{3.5}$$

where the parameters (a_1, a_2, a_3) are related to (c_0, c_1, c_2) in (3.3) by

$$c_0 = \frac{a_1+a_2+a_3}{q_4^2}, \qquad c_1 = 2(a_2+2a_3), \qquad c_2 = 4a_3q_4^2. \tag{3.6}$$

Then performing the triple Wick rotation

$$\psi \to \mathrm{i}\psi, \qquad \varphi \to \mathrm{i}\varphi, \qquad t \to \mathrm{i}t, \tag{3.7}$$

JHEP06(2015)112

together with $a_0 \to -a_0$, we find that the metric (3.5) has the form given by (2.13)-(2.14) in [29]. Note that the above triple Wick rotation is equivalent to a double Wick rotation on (x, y), which corresponds to taking $a_3 \to -a_3$. Since $a_3 = -\mu^2 < 0$ in the black ring analysis of [29], we can let $a_3 = \mu^2 > 0$ and hence

$$G(\xi) = \mu^2(\xi - \xi_1)(\xi - \xi_2)(\xi - \xi_3)\,, \qquad a_0 = \mu^2 \xi_1 \xi_2 \xi_3 > 0\,. \tag{3.8}$$

We shall consider two choices for coordinate ranges.

3.1.1 Case 1

We can consider $x \in [\xi_1, \xi_2]$ and $y \in [\xi_2, \xi_3]$ with $\xi_3 > \xi_2 > \xi_1 > 0$, so that $G(x) \geq 0$ and $G(y) \leq 0$. In contrast to the point $y = 0$ for the solution discussed in [29], in the present case there is no ergo-region. In fact, it turns out that this case describes a smooth soliton which has a region with closed timelike curves (CTC's).

3.1.2 Case 2

Alternatively, we can consider

$$G(\xi) = -\mu^2(\xi - \xi_0)(\xi - \xi_1)(\xi - \xi_2)\,, \qquad a_0 = -\mu^2 \xi_0 \xi_1 \xi_2 > 0\,, \tag{3.9}$$

with $x \in [\xi_1, \xi_2]$, $y \in [\xi_0, \xi_1]$ and $\xi_0 < 0 < \xi_1 < \xi_2$, so that $G(x) \geq 0$ and there is a horizon at $y = \xi_0 < 0$. Then $y = 0$ constitutes an ergo-region. This is rather similar to the case considered in [29], even though one needs to perform a triple Wick rotation in order to relate them.

Following the analysis in [29], we define

$$\xi_0 = -\eta_0^2\,, \qquad \xi_1 \equiv \eta_1^2 < \xi_2 \equiv \eta_2^2\,, \tag{3.10}$$

such that $a_0 = \mu^2 \eta_0^2 \eta_2^2 \eta_3^2$ and all η_i are positive with $\eta_1 < \eta_2$. The ϕ direction collapses at $x = \eta_1^2$ and $x = \eta_2^2$. In order to avoid a conical singularity, we must have

$$\eta_0 = \sqrt{\eta_1 \eta_2}\,, \tag{3.11}$$

as well as the periodicity condition $\Delta\phi_2 = 2\pi$, where

$$\varphi_2 = \mu^2(\eta_2 - \eta_1)(\eta_1 + \eta_2)^2 \varphi\,. \tag{3.12}$$

In order to avoid CTC's at $y = \xi_1$, we take

$$t \to t - \frac{\psi}{\eta_1^2}\,. \tag{3.13}$$

Then in order to avoid a conical singularity at $y = \xi_1$ we must have $\Delta\phi_1 = 2\pi$, where

$$\phi_1 = \mu^2(\eta_2 - \eta_1)(\eta_1 + \eta_2)^2 \psi\,. \tag{3.14}$$

In order to determine the horizon, we first need to find the asymptotic region where t is appropriately defined. This requires that

$$t \to \frac{t}{\mu(\eta_1\eta_2)^{\frac{3}{2}}} \,. \tag{3.15}$$

Making a coordinate transformation

$$\frac{\sqrt{x-\xi_1}}{x-y} = \frac{\mu(\eta_2+\eta_1)\sqrt{\eta_2-\eta_1}}{\sqrt{\eta_1}}\, r\cos\theta \,, \qquad \frac{\sqrt{\xi_1-y}}{x-y} = \frac{\mu(\eta_2+\eta_1)\sqrt{\eta_2-\eta_1}}{\sqrt{\eta_1}}\, r\sin\theta \,, \tag{3.16}$$

and then letting $r \to \infty$ yields

$$ds^2 = -dt^2+dr^2+r^2(d\theta^2+\cos^2\theta\, d\phi_2^2+\sin^2\theta\, d\phi_1^2) \,. \tag{3.17}$$

On the horizon $y = \xi_0 = -\eta_0^2$, the null Killing vector is given by

$$\ell = \frac{\partial}{\partial_t}+\Omega_\psi \frac{\partial}{\partial_\psi} \,, \qquad \Omega_\psi = \frac{\mu(\eta_2^2-\eta_1^2)\sqrt{\eta_1}}{\eta_2} \,, \tag{3.18}$$

and the surface gravity is

$$\kappa = \mu\eta_1(\eta_1+\eta_2) \,. \tag{3.19}$$

Therefore, this solution indeed describes a black ring. This solution, in slightly different coordinates, was first shown to describe a black ring in [2].

3.2 $D = 6$ Ricci-flat metric

Using the reduction ansatz (A.2) to lift the solution (2.9) to six dimensions yields

$$\begin{aligned} ds_6^2 &= -\frac{U_4}{\sqrt{U_1U_2}}\left(dt+\frac{4Q_4 y}{h_4(y)}d\psi\right)^2+\frac{\sqrt{U_1U_2}}{U_3}\left(dz+\frac{4Q_3 x}{h_3(x)}d\varphi\right)^2 \\ &\quad +\frac{1}{\alpha^2(x-y)^2}\left[\frac{1}{U_4\sqrt{U_1U_2}}\left(-G(y)d\psi^2-\frac{\mathcal{H}(y)}{G(y)}dy^2\right)\right. \\ &\quad \left. +U_3\sqrt{U_1U_2}\left(\frac{\mathcal{H}(x)}{G(x)}dx^2+G(x)d\varphi^2\right)\right] \,, \\ \hat{B} &= \frac{4Q_1 x}{h_1(x)}d\varphi\wedge dz+\frac{4Q_2 y}{h_2(y)}dt\wedge d\psi \,, \qquad e^{\sqrt{2}\,\phi_1} = \frac{U_2}{U_1} \,. \end{aligned} \tag{3.20}$$

As in the five-dimensional case, it is subtle to take the Ricci-flat limit of this solution by setting $Q_1 = Q_2 = 0 = q_1 = q_2$, since information is lost upon setting $Q_1 = Q_2$ and then $q_1 = q_2$. However, this can be remedied by first taking the specialization

$$q_i = -\mu s_i^2 \,, \qquad Q_i = \mu s_i c_i \,, \qquad s_i = \sinh\delta_i \,, \qquad c_i = \cosh\delta_i \,, \qquad i = 1, 2. \tag{3.21}$$

Then setting $s_i = 0$ and renaming the integration constants yields

$$G(\xi) = (b_0+b_1 x)h_3(\xi)h_4(\xi)+\frac{16Q_4^2\, h_3(\xi)}{q_4^3(q_3-q_4)}+\frac{16Q_3^2\, h_4(\xi)}{q_3^3(q_4-q_3)} \,. \tag{3.22}$$

This expression can also be obtained by inserting the form of the solution (3.20) directly into the equations of motion. Redefining (b_0, b_1) appropriately results in

$$\begin{aligned}
ds_6^2 &= -U_4\left(dt+\frac{4Q_4 y}{h_4(y)}d\psi\right)^2+\frac{1}{U_3}\left(dz+\frac{4Q_3 x}{h_3(x)}d\varphi\right)^2 \\
&\quad+\frac{1}{\alpha^2(x-y)^2}\left[\frac{1}{U_4}\left(-G(y)d\psi^2-\frac{\mathcal{H}(y)}{G(y)}dy^2\right)+U_3\left(\frac{\mathcal{H}(x)}{G(x)}dx^2+G(x)d\varphi^2\right)\right], \\
G(\xi) &= (b_0+b_1 x)h_3(\xi)h_4(\xi)+\frac{16Q_4^2\, h_3(\xi)\xi^2}{q_4(q_3-q_4)}+\frac{16Q_3^2\, h_4(\xi)\xi^2}{q_3(q_4-q_3)}\,. \qquad (3.23)
\end{aligned}$$

Taking $q_i \to 1/q_i$ and then shifting (x, y) so that $U_4(x, y) = y/x$ enables one to redefine parameters such that the metric can be written as

$$\begin{aligned}
ds^2 &= -\frac{y}{x}\left(dt+Q_4 y^{-1}d\psi\right)^2+\frac{x+q_3}{y+q_3}\left(dz+\frac{Q_3}{x+q_3}d\varphi\right) \\
&\quad+\frac{1}{(x-y)^2}\left[-\frac{x}{y}\left(G(y)d\psi^2+\frac{\mathcal{H}(y)}{G(y)}dy^2\right)\right. \\
&\quad\left.+\frac{y+q_3}{x+q_3}\left(\frac{\mathcal{H}(x)}{G(x)}dx^2+G(x)d\varphi^2\right)\right], \qquad (3.24)
\end{aligned}$$

where

$$G(\xi) = (c_0+c_1\xi)\xi(\xi+q_3)+q_3^{-1}Q_3^2\xi-q_3^{-1}Q_4^2(\xi+q_3)\,, \qquad \mathcal{H}(\xi) = \xi(\xi+q_3)\,. \qquad (3.25)$$

Note also that the q_4 parameter is also trivial and can be absorbed. Curvature singularities occur at $x = \pm\infty$, $y = \pm\infty$, $x = 0$ and $y = -q_3$. We can express

$$G(\xi_i) = -\mu^2(\xi-\xi_0)(\xi-\xi_1)(\xi-\xi_2)\,, \qquad (3.26)$$

where $\xi_0 = -\eta_0^2 < 0$ and $\xi_i = \eta_i^2$ $(i = 1, 2)$ with $\eta_2 > \eta_1$. In this parametrization, we have

$$Q_4 = \mu\eta_0\eta_1\eta_2\,, \qquad Q_3 = \mu\sqrt{(\eta_0^2-q_3)(\eta_1^2+q_3)(\eta_2^2+q_3)}\,. \qquad (3.27)$$

We can consider the coordinate ranges $x \in [\xi_1, \xi_2]$ and $y \in [\xi_0, \xi_1]$ so that $G(x) \geq 0$ and $G(y) \leq 0$. Next, we make coordinate transformations

$$\begin{aligned}
t &\to t+\frac{2\eta_0\eta_2\sqrt{\eta_1^2+q_3}}{(\eta_0^2+\eta_1^2)(\eta_2^2-\eta_1^2)\mu}\varphi_1\,, & \psi &= \frac{2\eta_1\sqrt{\eta_1^2+q_3}}{(\eta_0^2+\eta_1^2)(\eta_2^2-\eta_1^2)\mu}\varphi_1\,, \\
z &\to z-\frac{2\eta_1\sqrt{(\eta_0^2-q_3)(\eta_2^2+q_3)}}{(\eta_0^2+\eta_1^2)(\eta_2^2-\eta_1^2)\mu}\varphi_2\,, & \varphi &= \frac{2\eta_1\sqrt{\eta_0^2-q_3}}{(\eta_0^2+\eta_1^2)(\eta_2^2-\eta_1^2)\mu}\varphi_2\,. \qquad (3.28)
\end{aligned}$$

In the new coordinates, the absence of conical singularities at $x = \xi_1$ and $x = \xi_2$ requires that

$$\Delta\phi_2 = 2\pi = \Delta\phi_1\,. \qquad (3.29)$$

The absence of a conical singularity at $x = \xi_2$ implies that $\Delta z = 2\pi$. The horizon is located at $y = \xi_0$. The asymptotic region $x = \xi_1 = y$ has the geometry $(\mathrm{Mink})_5 \times S^1$, where S^1

corresponds to the z direction. The horizon topology is $S^3\times S^1$, where the S^1 lies along the ϕ_1 direction and the S^3 corresponds to the (x,ϕ_2,z) directions.

In order for the asymptotic region to have the geometry $(\text{Mink})_6$ instead of $(\text{Mink})_5\times S^1$, we need to decompactify the z direction, by taking $Q_3=0$, which corresponds to setting $q_3=\eta_0^2$. One can then attempt to avoid a conical singularity at $x=\xi_{1,2}$ by taking the periodicity condition $\Delta\phi_2=2\pi$. However, it turns that this requires that $q_3=0$, and so a conical singularity cannot be avoided.

4 Global properties and thermodynamics of black rings

4.1 Black rings with triple dipole charges

We shall now study the global structure of black rings that carry three dipole charges and obtain the first law of thermodynamics. As it is written in (2.9), the local solution is over-parameterized. It is advantageous to make the reparametrization

$$q_i=\frac{1}{\tilde q_4+\tilde q_i}\,,\qquad Q_i=-\frac{\widetilde Q_i}{4(\tilde q_4+\tilde q_i)^2\zeta}\,,\qquad Q_4=-\frac{\widetilde Q_4}{4\tilde q_4^2\zeta}\,,\qquad i=1,2,3, \tag{4.1}$$

where $\zeta=\sqrt{\tilde q_4(\tilde q_4+\tilde q_1)(\tilde q_4+\tilde q_2)(\tilde q_4+\tilde q_3)}$, followed by the coordinate transformation

$$x=\tilde x-\tilde q_4\,,\qquad y=\tilde y-\tilde q_4\,,\qquad \varphi=\zeta\tilde\varphi\,,\qquad \psi=\zeta\tilde\psi\,. \tag{4.2}$$

We then drop the tilde and redefine $h_i(\xi)$ as

$$h_i(\xi)\equiv\xi+q_i\,,\qquad i=1,2,3\,, \tag{4.3}$$

and $h_4(\xi)=\xi$. It turns out that the U_i are given by the same expressions as before, namely

$$U_i=\frac{h_i(y)}{h_i(x)}\,,\qquad U=U_1U_2U_3\,. \tag{4.4}$$

The scalar fields and gauge potentials are now given by

$$e^{\vec b_i\cdot\vec\phi}=\frac{U^{\frac23}}{U_i^2}\,,\qquad A_i=\frac{Q_i}{h_i(x)}d\varphi\,,\qquad i=1,2,3. \tag{4.5}$$

The metric can now be written as

$$ds_5^2=U^{-\frac13}\left(-\frac{y}{x}(dt+Q_4y^{-1}d\psi)^2+ds_4^2\right), \tag{4.6}$$

$$ds_4^2=\frac{1}{(x-y)^2}\left[\frac{x}{y}\left(-G(y)d\psi^2-\frac{\mathcal H(y)}{G(y)}dy^2\right)+U\left(\frac{\mathcal H(x)}{G(x)}dx^2+G(x)d\varphi^2\right)\right], \tag{4.7}$$

where

$$\begin{aligned}\mathcal H(\xi)&=\xi\,h_1(\xi)h_2(\xi)h_3(\xi)\,,\\ G(\xi)&=\mathcal H(\xi)\bigg(b_0+\frac{Q_1^2}{q_1(q_1-q_2)(q_1-q_3)h_1(\xi)}+\frac{Q_2^2}{q_2(q_2-q_1)(q_2-q_3)h_2(\xi)}\\ &\qquad+\frac{Q_3^2}{q_3(q_3-q_1)(q_3-q_2)h_3(\xi)}-\frac{Q_4^2}{q_1q_2q_3\xi}\bigg)\,.\end{aligned} \tag{4.8}$$

An advantage of this parameterization is that q_4 drops out from the solution completely. We have also set $\alpha = 1$ without loss of generality.

We are now in the position to study the global structure of the solution. The asymptotic region is located at $x = y$. Curvature singularities arise when either $h_i(x)$ or $h_i(y)$ vanishes or when either x or y diverges. Thus, we should arrange that the ranges of the coordinate x and y are confined to intervals that are specified by adjacent roots of $G(\xi)$, within which the $h_i(\xi)$ are non-vanishing. It is therefore more convenient to express $G(\xi)$ in terms of four roots, namely

$$G(\xi) = -\mu^2(\xi-\xi_1)(\xi-\xi_2)(\xi-\xi_3)(\xi-\xi_4)\,. \tag{4.9}$$

Here we let $b_0 = -\mu^2 < 0$. The charge parameters Q_i and Q_4 can be expressed in terms of μ and the four roots as

$$\begin{aligned} Q_i^2 &= \mu^2(q_i+\xi_1)(q_i+\xi_2)(q_i+\xi_3)(q_i+\xi_4) \geq 0\,, \\ Q_4^2 &= \mu^2\xi_1\xi_2\xi_3\xi_4 \geq 0\,. \end{aligned} \tag{4.10}$$

As with the Ricci-flat metrics discussed in the previous section, the signs of $G(x) \geq 0$ and $G(y) \leq 0$ should be such that the metric has the proper signature. Without loss of generality, we consider $x \in [\xi_3, \xi_4]$ and $y \in [\xi_2, \xi_3]$.

The reality of Q_4 leads to two possibilities. The first one is where $0 < \xi_1 < \xi_2 < \xi_3 < \xi_4$. After a careful analysis, we find that the absence of naked curvature power-law singularities, together with the reality condition on Q_i, implies that the solution has an unavoidable naked conical singularity. This conclusion may not be too surprising, given that we would otherwise have a rotating black hole without an ergo-region. The natural location of the ergo-region is $y = 0$, which leads to the second case, for which

$$\xi_1 < \xi_2 < 0 < \xi_3 < \xi_4\,. \tag{4.11}$$

This ensures that the ergo-region at $y = 0$ lies in the range $y \in [\xi_2, \xi_3]$. The metric has no naked power-law curvature singularities provided that

$$\begin{aligned} h_i(x) > 0 \qquad &\text{for} \qquad x \in [\xi_3, \xi_4]\,, \\ h_i(y) > 0 \qquad &\text{for} \qquad y \in [\xi_2, \xi_3]\,. \end{aligned} \tag{4.12}$$

Now we ensure that the solution does not have conical singularities. The null Killing vectors with unit Euclidean surface gravity at $x = \xi_3$ and $x = \xi_4$ are given by

$$\ell_{x=\xi_3} = \alpha_3\partial_\varphi\,, \qquad \ell_{x=\xi_4} = \alpha_4\partial_\varphi\,, \tag{4.13}$$

where

$$\alpha_3 = \frac{2\sqrt{\xi_3(\xi_3+q_1)(\xi_3+q_2)(\xi_3+q_3)}}{\mu^2(\xi_3-\xi_1)(\xi_3-\xi_2)(\xi_4-\xi_3)}\,, \quad \alpha_4 = \frac{2\sqrt{\xi_4(\xi_4+q_1)(\xi_4+q_2)(\xi_4+q_3)}}{\mu^2(\xi_4-\xi_1)(\xi_4-\xi_2)(\xi_4-\xi_3)}\,. \tag{4.14}$$

The absence of conical singularities then requires that $\alpha_3 = \alpha_4$. This condition and (4.12) can be shown to be simultaneously satisfied for the appropriate choice of parameters. As an example, consider $\xi_1 = -2$, $\xi_2 = -1$, $\xi_3 = 1$ and $\xi_4 = 2$, for which the absence of conical singularities requires

$$\frac{(q_1+2)(q_2+2)(q_3+2)}{2(q_1+1)(q_2+1)(q_3+1)} = 1\,. \tag{4.15}$$

This can be solved, for example, with $q_1 = 2$, $q_2 = 3$ and $q_3 = 4$. All the $q_i \geq -2$, so that $Q_i \geq 0$, $h_i(x) > 0$ and $h_i(y) > 0$. Thus, there are no singularities in the region of interest.

Once we have established that $\alpha_3 = \alpha_4$ and that the charge parameters Q_i are real, there are no further conditions on the parameters. The absence of a conical singularity at $y = \xi_3$ tells us the appropriate period for the coordinate ψ, and the horizon condition gives the temperature and entropy. However, the algebraic expression for $\alpha_3 = \alpha_4$ is rather complicated to solve, which means that quantities such as mass, charges and temperature can be quite complicated.

To proceed, it is convenient to rewrite the roots as

$$\xi_1 = -\eta_1^2\,, \qquad \xi_2 = -\eta_2^2\,, \qquad \xi_3 = \eta_3^2\,, \qquad \xi_4 = \eta_4^2\,, \tag{4.16}$$

with $\eta_1 > \eta_2 > 0$ and $\eta_4 > \eta_3 > 0$. The avoidance of naked conical singularities requires that

$$\frac{\eta_4(\eta_1^2+\eta_3^2)(\eta_2^2+\eta_3^2)\sqrt{(q_1+\eta_4^2)(q_2+\eta_4^2)(q_3+\eta_4^2)}}{\eta_3(\eta_1^2+\eta_4^2)(\eta_2^2+\eta_4^2)\sqrt{(q_1+\eta_3^2)(q_2+\eta_3^2)(q_3+\eta_3^2)}} = 1\,. \tag{4.17}$$

We define a new set of coordinates, given by

$$\begin{aligned} &\varphi = a\,\varphi_2\,, \qquad \psi = a\,\varphi_1\,, \qquad t \to t - \frac{Q_4 a}{\eta_3^2}\varphi_1\,, \\ &a = \frac{2\eta_3\sqrt{(q_1+\eta_3^2)(q_2+\eta_3^2)(q_3+\eta_3^2)}}{\mu^2(\eta_1^2+\eta_3^2)(\eta_2^2+\eta_3^2)(\eta_4^2-\eta_3^2)}\,. \end{aligned} \tag{4.18}$$

Then the periods of ϕ_1 and ϕ_2 are both 2π. Note that the shift in t ensures that only the spatial coordinate ϕ_1 collapses to zero size at $y = \xi_3$, and hence we avoid naked closed timelike curves (CTC's).

On the horizon at $y = \xi_2$, the null Killing vector is

$$\ell = \partial_t + \Omega_+ \partial_{\phi_1}\,, \qquad \Omega_+ = \frac{\eta_2^2\eta_3^2}{a(\eta_2^2+\eta_3^2)Q_4}\,. \tag{4.19}$$

The (Euclidean) surface gravity is given by

$$\kappa^2 = -\frac{\mu^4\eta_2^2\eta_3^2(\eta_1^2-\eta_2^2)^2(\eta_2^2+\eta_4^2)^2}{4(q_1-\eta_2^2)(q_2-\eta_2^2)(q_3-\eta_2^2)Q_4^2}\,. \tag{4.20}$$

For the solution to describe a black object with a horizon, we must have $\kappa^2 < 0$. It is worth checking that this condition can indeed be satisfied. As an example, we take

$$\xi_1 = -2\,, \qquad \xi_2 = -1\,, \qquad \xi_3 = 1\,, \qquad \xi_4 = 4\,. \tag{4.21}$$

An acceptable set of q_i with $i = 1, 2, 3$ must satisfy (4.17) and all have $q_i > -2$. Such solutions do in fact exist; for instance,

$$q_1 = \tfrac{7}{3}\,, \qquad q_2 = \tfrac{9}{4}\,, \qquad q_3 = \tfrac{68971}{27389}\,. \tag{4.22}$$

For this choice of parameters the Q_i are real, $h_i(x)$ and $h_i(y)$ are positive definite in the regions of concern, $\kappa^2 < 0$ and χ is real. Thus, a well-behaved black ring exists. The temperature and entropy are given by

$$T = \frac{\kappa}{2\pi}\,, \qquad S = \frac{\pi^2 a^2 Q_4(\eta_4^2-\eta_3^2)\sqrt{(q_1-\eta_2^2)(q_2-\eta_2^2)(q_3-\eta_2^2)}}{\eta_2\eta_3^2(\eta_2^2+\eta_4^2)}\,. \tag{4.23}$$

When $\eta_1 = \eta_2$, the temperature vanishes, corresponding to the extremal limit.

The asymptotic region is located at $x = \xi_3 = y$. To see this, we make the coordinate transformation

$$\frac{\sqrt{x-\xi_3}}{x-y} = b\,r\cos\theta\,, \qquad \frac{\sqrt{\xi_3-y}}{x-y} = b\,r\sin\theta\,,$$
$$b^2 = \frac{\mu^2(\eta_1^2+\eta_3^2)(\eta_2^2+\eta_3^2)(\eta_4^2-\eta_3^2)}{4\eta_3^2(\eta_3^2+q_1)(\eta_3^2+q_2)(\eta_3^2+q_3)}\,, \tag{4.24}$$

and then we take $r \to \infty$. The asymptotic geometry is five-dimensional Minkowski spacetime, with the metric written as

$$ds_5^2 = -dt^2+dr^2+r^2(d\theta^2+\sin^2\theta d\phi_1^2+\cos^2\theta d\phi_2^2)\,. \tag{4.25}$$

From the asymptotic falloffs of the metric, we can read off the ADM mass and the angular momenta as

$$M = \frac{\pi}{8b^2}\left(\frac{3}{\eta_3^2} - \frac{1}{\eta_3^2+q_1} - \frac{1}{\eta_3^2+q_2} - \frac{1}{\eta_3^2+q_3}\right), \tag{4.26}$$

and

$$J_{\phi_1} = \frac{\pi a Q_4}{4b^2\eta_3^4}\,, \qquad J_{\phi_2} = 0\,, \tag{4.27}$$

respectively. The dipole charges are given by

$$D_i = \tfrac{1}{8}\int F_i = \tfrac{1}{4}\pi a Q_i\left(\frac{1}{q_i+\eta_3^2} - \frac{1}{q_i+\eta_4^2}\right). \tag{4.28}$$

The only quantities left to determine are the dipole potentials Φ_{D_i}, which requires the dualization of A^i_μ to $B^i_{\mu\nu}$. Since $F_i\wedge F_j = 0$, we do not need to worry about the FFA term in the Lagrangian when performing the dualization. We find

$$e^{\vec{b}_i\cdot\vec{\phi}} {*F_i} = \frac{aQ_i}{(y+q_i)^2} dt\wedge d\phi_1\wedge dy\,, \qquad B_i = -\frac{aQ_i}{y+q_i} dt\wedge d\phi_1\,. \tag{4.29}$$

The 2-form potential difference between the horizon and the asymptotic region is then given by

$$\Phi_{D_i} = aQ_i\left(\frac{1}{q_i-\eta_2^2} - \frac{1}{q_i+\eta_3^2}\right). \tag{4.30}$$

Having obtained all of the thermodynamic quantities, it is straightforward to verify that the first law of the thermodynamics,

$$dM = TdS+\Omega_+ dJ_{\phi_1} + \sum_{i=1}^{3} \Phi_{D_i} dD_i\,, \tag{4.31}$$

is obeyed. The Smarr formula is given by

$$M = \tfrac{3}{2}TS + \tfrac{3}{2}\Omega_{\phi_1}J_{\phi_1} + \sum_{i=1}^{3}\tfrac{1}{2}\Phi_{D_i}D_i\,. \tag{4.32}$$

Interestingly enough, the Smarr formula is actually valid even without imposing the condition (4.17) that ensures the absence of naked conical singularities.

After performing the various changes of variables discussed in this section, the solution can be written in terms of the eight parameters: μ, q_i and η_I, where $i = 1,2,3$ and $I = 1,\ldots,4$. However, the solution is over-parameterized by two trivial parameters. While the μ parameter can be absorbed by a "trombone" scaling of the metric and other fields, the second extra parameter is more subtle. Although the five-dimensional theory has only two scalars, the solution has three scalar charges q_i, one of which is therefore trivial. Thus, removing μ and q_3, we are left with six nontrivial parameters: q_1, q_2 and η_I. After imposing the condition (4.17) to avoid naked conical singularities, we are left with five parameters associated with the mass, one non-vanishing angular momentum, and three dipole charges. Dipole black rings were first found in [5].

The horizon geometry has the metric

$$\begin{aligned} ds_3^2 = \left(\frac{(x+q_1)(x+q_2)(x+q_3)}{(q_1-\eta_2^2)(q_2-\eta_2^2)(q_3-\eta_2^2)^2}\right)^{\frac{1}{3}} &\Bigg[\frac{(q_1-\eta_2^2)(q_2-\eta_2^2)(q_3-\eta_3^2)\,x}{\mu^2(x+\eta_1^2)(x+\eta_2^2)^3(\eta_4^2-x)(x-\eta_3^2)}dx^2 \\ &+\frac{a^2\mu^2(q_1-\eta_2^2)(q_2-\eta_2^2)(q_3-\eta_3^2)(x+\eta_1^2)(\eta_4^2-x)(x-\eta_3^2)}{(x+q_1)(x+q_2)(x+q_3)(x+\eta_2^2)}d\phi_2^2 \\ &+\frac{a^2Q_4^2(\eta_2^2+\eta_3^2)^2}{\eta_2^2\eta_3^4\,x}d\phi_1^2\Bigg]\,. \end{aligned} \tag{4.33}$$

Since $x \in [\eta_3^2, \eta_4^2]$, it is clear that the horizon topology is $S^2\times S^1$. With non-vanishing dipole charges, the black ring solution does not have a limit with a spherical horizon.

The black ring solution has an Extremal Vanishing Horizon (EVH) case for which the near-horizon geometry has an AdS_3 factor and is contained within the large class of near-horizon geometries that have been studied and classified in [25–27]. According to the proposed EVH/CFT correspondence, there is a two-dimensional CFT description of the low-energy excitations of the black ring in this case [28].

4.2 Electrically-charged black rings and naked CTC's

We shall now study the global structure of the black ring solutions with three dipole charges and three electric charges, which was obtained in section 2.2. Note that these solutions generalize those found in [11], for which only two out of the three dipole charges were independent parameters.

As before, it is convenient to make the reparametrizations (4.1) and the coordinate transformations (4.2). Furthermore, we make a redefinition of the time coordinate,

$$\begin{aligned} t = \tilde{t} &- \left(\frac{s_1c_2c_3\widetilde{Q}_1}{\tilde{q}_4+\tilde{q}_1}+\frac{c_1s_2c_3\widetilde{Q}_2}{\tilde{q}_4+\tilde{q}_2}+\frac{c_1c_2s_3\widetilde{Q}_3}{\tilde{q}_4+\tilde{q}_3}+\frac{s_1s_2s_3\widetilde{Q}_4}{\tilde{q}_4}\right)\tilde{\varphi} \\ &+\left(\frac{c_1s_2s_3\widetilde{Q}_1}{\tilde{q}_4+\tilde{q}_1}+\frac{s_1c_2s_3\widetilde{Q}_2}{\tilde{q}_4+\tilde{q}_2}+\frac{s_1s_2c_3\widetilde{Q}_3}{\tilde{q}_4+\tilde{q}_3}+\frac{c_1c_2c_3\widetilde{Q}_4}{\tilde{q}_4}\right)\tilde{\psi}\,. \end{aligned} \tag{4.34}$$

JHEP06(2015)112

We may now drop the tilde, and redefine $h_i(\xi)$ as in (4.3). The functions H_i take the same form, namely

$$H_i = 1+\left(1-\frac{yU_i^2}{xU}\right)s_i^2\,, \tag{4.35}$$

where U_i and U are given by (4.4). The scalar fields retain the same form given by (2.15). The metric is now given by

$$ds_5^2 = \frac{1}{U^{1/3}(H_1H_2H_3)^{2/3}}\left(-\frac{y}{x}\,(dt+\omega)^2+H_1H_2H_3\,ds_4^2\right), \tag{4.36}$$

where ds_4^2 is precisely the same as (4.7) and ω is

$$\begin{aligned}\omega = &-\left(\frac{s_1c_2c_3Q_1}{h_1(x)}+\frac{c_1s_2c_3Q_2}{h_2(x)}+\frac{c_1c_2s_3Q_3}{h_3(x)}+\frac{s_1s_2s_3Q_4}{x}\right)d\varphi\\ &+\left(\frac{c_1s_2s_3Q_1}{h_1(y)}+\frac{s_1c_2s_3Q_2}{h_2(y)}+\frac{s_1s_2c_3Q_3}{h_3(y)}+\frac{c_1c_2c_3Q_4}{y}\right)d\psi\,.\end{aligned} \tag{4.37}$$

The gauge potentials are

$$\begin{aligned}A_1 = &\frac{s_1c_1}{H_1}\left(1-\frac{U_1U_4}{U_2U_3}\right)dt+\\ &+\frac{1}{H_1}\left(\frac{c_1c_2c_3Q_1}{h_1(x)}+\frac{s_1s_2c_3Q_2U_1U_4}{U_3h_2(y)}+\frac{s_1c_2s_3Q_3U_1U_4}{U_2h_3(y)}+\frac{c_1s_2s_3Q_4}{x}\right)d\varphi\\ &-\frac{1}{H_1}\left(\frac{s_1s_2s_3Q_1U_4}{U_2U_3h_1(x)}+\frac{c_1c_2s_3Q_2}{h_2(y)}+\frac{c_1s_2c_3Q_3}{h_3(y)}+\frac{s_1c_2c_3Q_4U_1}{U_2U_3\,x}\right)d\psi\,,\end{aligned} \tag{4.38}$$

with A_2 and A_3 being obtained from A_1 by cycling the indices 1, 2 and 3 on all quantities appearing in (4.38). Note that, aside from the coordinate transformations and reparametrizations, the A_i obtained above are related to those in (2.16) by gauge transformations.

It is of interest to note that the dipole charges are magnetic and are associated with the (x,φ) directions. Adding the electric charges has the effect of producing angular momentum in the φ direction as well. However, this also has the undesirable side effect of producing naked CTC's. To see this explicitly, it is useful to note that ds_4^2 is identical to that in the previous subsection and $G(\xi)$ can be expressed by (4.9) with (4.11). The null Killing vector associated with the collapsing circles at $x=\xi_3$ and $x=\xi_4$ are given by

$$\ell_{x=\xi_3} = \alpha_3\partial_\varphi+\beta_3\partial_t\,,\qquad \ell_{x=\xi_4} = \alpha_4\partial_\varphi+\beta_4\partial_t\,. \tag{4.39}$$

The absence of CTC's requires that

$$\frac{\beta_3}{\alpha_3} = \frac{\beta_4}{\alpha_4}\,, \tag{4.40}$$

in which case we can shift $t\to t+\gamma\varphi$, for an appropriate constant γ, such that the null Killing vectors do not involve the newly-defined time. For black rings with dipole charges but no electric charges, $\beta_3=0=\beta_4$ and hence there are no naked CTC's. By contrast, we find that for black rings with both dipole and electric charges, the condition (4.40) cannot be satisfied and the solutions have naked CTC's. This parallels the situation for the five-dimensional black hole solutions studied in [29], for which naked CTC's appeared when the electric charges were nonvanishing. Turning on two independent angular momenta might result in black ring solutions with dipole and electric charges but without naked CTC's.

4.3 In background magnetic fields

We shall now consider some of the global properties of the solution with external magnetic fields, with the metric (2.18). This solution asymptotes to a five-dimensional Melvin fluxbrane, which is a higher-dimensional analog of Melvin's four-dimensional flux tube solution and describes the self-gravity of the background magnetic fields [23].

We use the same parameterizations as in section 4.1, with $G(\xi)$ given by (4.9) and with the coordinate ranges $x \in [\xi_3, \xi_4]$ and $y \in [\xi_2, \xi_3]$. In order to avoid a conical singularity in the (x, ϕ) subspace, we must have

$$\prod_{i=1}^{3} \left(\frac{\xi_4 + q_i + B_i Q_i (\xi_4 - q_4)}{\xi_3 + q_i + B_i Q_i (\xi_3 - q_4)} \sqrt{\frac{\xi_3 + q_i}{\xi_4 + q_i}} \right) = \frac{(\xi_4 - \xi_1)(\xi_4 - \xi_2)}{(\xi_3 - \xi_1)(\xi_3 - \xi_2)} \sqrt{\frac{\xi_3}{\xi_4}} \,, \tag{4.41}$$

along with the appropriate periodicity for ψ.

First consider the case where $0 < \xi_1 < \xi_2 < \xi_3 < \xi_4$. While it is not possible to satisfy the condition (4.41) and the reality condition on the charge parameters Q_i at the same time for vanishing B_i, both conditions can be satisfied simultaneously when the B_i are turned on. A sample solution is given by

$$\begin{aligned} &B_1 = -1\,, \quad B_2 = -2\,, \quad B_3 = -3\,, \quad \xi_1 = 1\,, \quad \xi_2 = 2\,, \quad \xi_3 = 3\,, \quad \xi_4 = 4\,, \\ &\mu = 0.2\,, \quad q_2 = 1\,, \quad q_3 = 1.5\,, \quad q_4 = 2.5\,, \quad q_1 = 2.09789\,. \end{aligned} \tag{4.42}$$

Thus, at the expense of altering the asymptotic geometry, turning on background magnetic fields can have the effect of removing conical singularities for black rings, in much the same way as it does for Ernst solutions [20, 30].

Background magnetic fields also enable us to have multiple branches of solutions. For instance, in the case where $\xi_1 < \xi_2 < 0 < \xi_3 < \xi_4$ with vanishing B_i, a sample solution is given by

$$\xi_1 = -2\,, \quad \xi_2 = -1\,, \quad \xi_3 = 1\,, \quad \xi_4 = 2\,, \quad q_1 = 2\,, \quad q_2 = 3\,, \quad q_3 = 4\,. \tag{4.43}$$

Note that if only q_1 were to be left unspecified then the condition (4.41) would uniquely determine its value in terms of the other parameters. On the other hand, for nonvanishing B_i, sample solutions are given by

$$\begin{aligned} &B_1 = 1\,, \quad B_2 = 2\,, \quad B_3 = 3\,, \quad \xi_1 = -2\,, \quad \xi_2 = -1\,, \quad \xi_3 = 1\,, \quad \xi_4 = 2\,, \\ &\mu = 1\,, \quad q_2 = 2\,, \quad q_3 = 3\,, \quad q_4 = 4\,, \quad q_1 = 2.03367 \text{ or } 0.975202\,. \end{aligned} \tag{4.44}$$

Due to the presence of the B_i, using the condition (4.41) to solve for q_1 in terms of the other specified parameters yields two different solutions, both of which satisfy the reality condition on the charge parameters Q_i.

5 Conclusions

We have constructed black ring solutions in five-dimensional $\mathrm{U}(1)^3$ supergravity, carrying three independent dipole charges, three electric charges and one non-vanishing angular

JHEP06(2015)112

momentum. We have also presented black ring solutions with three background magnetic fields. These various solutions have been obtained by lifting the Euclidean C-metric solution of four-dimensional ungauged STU supergravity [22] to five dimensions on a timelike direction, and then using solution-generating techniques involving dimensional reductions to add electric charges or background magnetic fields. We find that adding the electric charges gives rise to black rings with naked CTC's.

We expect that the solutions without the background magnetic fields should arise as special cases of the black ring solutions obtained in [17] if one of the angular momenta is set to zero. We have expressed this specialization in a form that is sufficiently compact that its various physical properties can be investigated explicitly. In particular, its global structure has been analyzed and the conditions determined in order for conical singularities and Dirac string singularities to be absent. Expressions for its mass, dipole charges, electric charges and angular momentum have been obtained, as well as the temperature and entropy. Moreover, we have analyzed the thermodynamics, finding that the Smarr formula is obeyed regardless of whether or not conical singularities are present. By contrast, the first law of thermodynamics is obeyed only in those cases where conical singularities are absent.

The four-dimensional Ernst-like generalization of the C-metric solution obtained in this paper can be Wick rotated to a Euclidean instanton that describes the pair creation of black holes in magnetic fields. This generalizes the one-parameter family of instantons in [20, 31, 32] to multiple parameters. This substantially enhances the families of explicit examples for the creation of maximally entangled black holes, which have recently been proposed to be connected by some kind of Einstein-Rosen bridge [33].

The exact time-dependent C-metric solution constructed in [34] can be embedded in STU supergravity which, in the ungauged limit, can be lifted to five dimensions. It would be interesting to analyze the global structures of these five-dimensional solutions, especially with the prospect of finding time-dependent black rings.

Acknowledgments

We are grateful to Mboyo Esole for helpful conversations. The work of C.N.P. was supported in part by DOE grant DE-FG02-13ER42020; the work of H.L. was supported in part by NSFC grants 11175269, 11475024 and 11235003; the work of J.F.V.P. was supported in part by a PSC-CUNY Award.

A Dimensional reductions

We present the Kaluza-Klein dimensional reductions that have been used to relate the four-dimensional C-metrics with the five-dimensional black ring solutions, as well as for the purposes of generating electric charges and background magnetic fields. We start with the $D = 6$ theory whose Lagrangian is given by

$$e_6^{-1}\mathcal{L}_6 = R_6 - \tfrac{1}{2}(\partial\phi_1)^2 - \tfrac{1}{12}e^{\sqrt{2}\phi_1}\hat{H}^2\,, \tag{A.1}$$

where $\hat{H} = d\hat{B}$. Consider the reduction ansatz

$$\begin{aligned} ds_6^2 &= e^{-\frac{1}{\sqrt{6}}\phi_2} ds_5^2 + e^{\frac{3}{\sqrt{6}}\phi_2}(dz + A_3)^2 \,, \\ \hat{B} &= B + A_1 \wedge dz \,, \end{aligned} \tag{A.2}$$

where

$$F_2 = e^{-b_2\cdot\vec{\phi}} {*H} \,, \qquad H = dB - A_1 \wedge A_3 \,. \tag{A.3}$$

This yields the Lagrangian for the bosonic sector of five-dimensional U(1)^3 supergravity, given by

$$e_5^{-1}\mathcal{L}_5 = R_5 - \tfrac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{4}\sum_{i=1}^{3} e^{\vec{b}_i\cdot\vec{\phi}} \hat{F}_i^2 + \mathcal{L}_{FFA} \,, \tag{A.4}$$

where $\vec{\phi} = (\phi_1, \phi_2)$ and the dilaton vectors $\vec{b}_i$ are given by

$$\vec{b}_1 = \left(\sqrt{2}, -\tfrac{2}{\sqrt{6}}\right) \,, \qquad \vec{b}_2 = \left(-\sqrt{2}, -\tfrac{2}{\sqrt{6}}\right) \,, \qquad \vec{b}_3 = \left(0, \tfrac{4}{\sqrt{6}}\right) \,, \tag{A.5}$$

which obey

$$\vec{b}_i\cdot\vec{b}_j = 4\delta_{ij} - \tfrac{4}{3} \,, \qquad \sum_{i=1}^{3} \vec{b}_i = 0 \,. \tag{A.6}$$

Next, we perform a dimensional reduction to the four-dimensional U(1)^4 theory.[1] Reducing on a spacelike direction with the metric ansatz

$$ds_5^2 = e^{-\frac{1}{\sqrt{3}}\phi_3} ds_4^2 + e^{\frac{2}{\sqrt{3}}\phi_3}(dz + A_4)^2 \,, \tag{A.7}$$

yields

$$e_4^{-1}\mathcal{L}_4 = R_4 - \tfrac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{4}\sum_{i=1}^{4} e^{\vec{a}_i\cdot\vec{\phi}} \hat{F}_i^2 \,, \tag{A.8}$$

where we are now taking $\vec{\phi} = (\phi_1, \phi_2, \phi_3)$. The dilaton vectors $\vec{a}_i$ are given by

$$\vec{a}_i = (\vec{b}_i, \tfrac{1}{\sqrt{3}}) \,, \qquad \vec{a}_4 = (0, 0, -\sqrt{3}) \,, \tag{A.9}$$

and they satisfy the conditions in (2.3).

Alternatively, we can reduce on the timelike direction with the metric ansatz

$$ds_5^2 = e^{-\frac{1}{\sqrt{3}}\phi_3} ds_4^2 - e^{\frac{2}{\sqrt{3}}\phi_3}(dt - \mathcal{A})^2 \,, \tag{A.10}$$

which gives

$$e_4^{-1}\mathcal{L}_4 = R_4 - \tfrac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{4}\sum_{i=1}^{3} e^{\vec{a}_i\cdot\vec{\phi}} \hat{F}_i^2 + \frac{1}{4} e^{-\vec{a}_4\cdot\vec{\phi}} \mathcal{F}^2 \,. \tag{A.11}$$

If we perform a Hodge dualization on the Kaluza-Klein vector, namely

$$e^{-\vec{a}_4\cdot\vec{\phi}} {*\mathcal{F}} = F_4 \,, \tag{A.12}$$

then the kinetic term changes sign and the four-dimensional Lagrangian can be expressed in the more symmetric form given by (A.8).

[1] Note that we are truncating out the three axions that would arise in the reductions of the three five-dimensional gauge fields. This truncation is consistent with the equations of motion provided that we restrict our attention to solutions for which $F^i \wedge F^j = 0$.

JHEP06(2015)112

References

[1] R. Emparan and H.S. Reall, *Black rings*, *Class. Quant. Grav.* **23** (2006) R169 [hep-th/0608012] [INSPIRE].

[2] R. Emparan and H.S. Reall, *A rotating black ring solution in five-dimensions*, *Phys. Rev. Lett.* **88** (2002) 101101 [hep-th/0110260] [INSPIRE].

[3] H. Elvang, *A charged rotating black ring*, *Phys. Rev.* **D 68** (2003) 124016 [hep-th/0305247] [INSPIRE].

[4] H. Elvang and R. Emparan, *Black rings, supertubes and a stringy resolution of black hole nonuniqueness*, *JHEP* **11** (2003) 035 [hep-th/0310008] [INSPIRE].

[5] R. Emparan, *Rotating circular strings and infinite nonuniqueness of black rings*, *JHEP* **03** (2004) 064 [hep-th/0402149] [INSPIRE].

[6] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, *A supersymmetric black ring*, *Phys. Rev. Lett.* **93** (2004) 211302 [hep-th/0407065] [INSPIRE].

[7] J.P. Gauntlett and J.B. Gutowski, *Concentric black rings*, *Phys. Rev.* **D 71** (2005) 025013 [hep-th/0408010] [INSPIRE].

[8] I. Bena and N.P. Warner, *One ring to rule them all . . . and in the darkness bind them?*, *Adv. Theor. Math. Phys.* **9** (2005) 667 [hep-th/0408106] [INSPIRE].

[9] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, *Supersymmetric black rings and three-charge supertubes*, *Phys. Rev.* **D 71** (2005) 024033 [hep-th/0408120] [INSPIRE].

[10] J.P. Gauntlett and J.B. Gutowski, *General concentric black rings*, *Phys. Rev.* **D 71** (2005) 045002 [hep-th/0408122] [INSPIRE].

[11] H. Elvang, R. Emparan and P. Figueras, *Non-supersymmetric black rings as thermally excited supertubes*, *JHEP* **02** (2005) 031 [hep-th/0412130] [INSPIRE].

[12] H. Elvang and P. Figueras, *Black Saturn*, *JHEP* **05** (2007) 050 [hep-th/0701035] [INSPIRE].

[13] A.A. Pomeransky and R.A. Sen'kov, *Black ring with two angular momenta*, hep-th/0612005 [INSPIRE].

[14] H. Elvang and M.J. Rodriguez, *Bicycling black rings*, *JHEP* **04** (2008) 045 [arXiv:0712.2425] [INSPIRE].

[15] R. Emparan, T. Harmark, V. Niarchos, N.A. Obers and M.J. Rodriguez, *The phase structure of higher-dimensional black rings and black holes*, *JHEP* **10** (2007) 110 [arXiv:0708.2181] [INSPIRE].

[16] M.M. Caldarelli, R. Emparan and M.J. Rodriguez, *Black rings in (Anti)-deSitter space*, *JHEP* **11** (2008) 011 [arXiv:0806.1954] [INSPIRE].

[17] A. Feldman and A.A. Pomeransky, *General triple charged black ring solution in supergravity*, *JHEP* **08** (2014) 059 [arXiv:1406.4617] [INSPIRE].

JHEP06(2015)112

[18] V.A. Belinsky and V.E. Zakharov, *Integration of the Einstein equations by the inverse scattering problem technique and the calculation of the exact soliton solutions*, *Sov. Phys. JETP* **48** (1978) 985 [*Zh. Eksp. Teor. Fiz.* **75** (1978) 1953] [INSPIRE].

[19] V.A. Belinsky and V.E. Sakharov, *Stationary gravitational solitons with axial symmetry*, *Sov. Phys. JETP* **50** (1979) 1 [INSPIRE].

[20] F. Dowker, J.P. Gauntlett, D.A. Kastor and J.H. Traschen, *Pair creation of dilaton black holes*, *Phys. Rev.* **D 49** (1994) 2909 [hep-th/9309075] [INSPIRE].

[21] A. Chamblin and R. Emparan, *Bubbles in Kaluza-Klein theories with space-like or time-like internal dimensions*, *Phys. Rev.* **D 55** (1997) 754 [hep-th/9607236] [INSPIRE].

[22] H. Lü and J.F. Vázquez-Poritz, *C-metrics in gauged STU supergravity and beyond*, *JHEP* **12** (2014) 057 [arXiv:1408.6531] [INSPIRE].

[23] G.W. Gibbons and D.L. Wiltshire, *Space-Time as a Membrane in Higher Dimensions*, *Nucl. Phys.* **B 287** (1987) 717 [hep-th/0109093] [INSPIRE].

[24] M. Cvetič, G.W. Gibbons, C.N. Pope and Z.H. Saleem, *Electrodynamics of black holes in STU supergravity*, *JHEP* **09** (2014) 001 [arXiv:1310.5717] [INSPIRE].

[25] H. Golchin, M.M. Sheikh-Jabbari and A. Ghodsi, *Dual 2d CFT identification of extremal black rings from holes*, *JHEP* **10** (2013) 194 [arXiv:1308.1478] [INSPIRE].

[26] A. Ghodsi, H. Golchin and M.M. Sheikh-Jabbari, *More on five dimensional EVH black rings*, *JHEP* **09** (2014) 036 [arXiv:1407.7484] [INSPIRE].

[27] S. Sadeghian, M.M. Sheikh-Jabbari and H. Yavartanoo, *On classification of geometries with* $SO(2,2)$ *symmetry*, *JHEP* **1410** (2014) 81 [arXiv:1409.1635] [INSPIRE].

[28] M.M. Sheikh-Jabbari and H. Yavartanoo, *EVH black holes, AdS3 throats and EVH/CFT proposal*, *JHEP* **10** (2011) 013 [arXiv:1107.5705] [INSPIRE].

[29] H. Lü, J. Mei and C.N. Pope, *New black holes in five dimensions*, *Nucl. Phys.* **B 806** (2009) 436 [arXiv:0804.1152] [INSPIRE].

[30] F.J. Ernst, *Removal of the nodal singularity of the C-metric*, *J. Math. Phys.* **17** (1976) 515.

[31] D. Garfinkle and A. Strominger, *Semiclassical Wheeler wormhole production*, *Phys. Lett.* **B 256** (1991) 146 [INSPIRE].

[32] D. Garfinkle, S.B. Giddings and A. Strominger, *Entropy in black hole pair production*, *Phys. Rev.* **D 49** (1994) 958 [gr-qc/9306023] [INSPIRE].

[33] J. Maldacena and L. Susskind, *Cool horizons for entangled black holes*, *Fortsch. Phys.* **61** (2013) 781 [arXiv:1306.0533] [INSPIRE].

[34] H. Lü and J.F. Vázquez-Poritz, *Dynamic C metrics in gauged supergravities*, *Phys. Rev.* **D 91** (2015) 064004 [arXiv:1408.3124] [INSPIRE].

JHEP06(2015)112