# $B_{s}-\bar{B}_{s}$ Mixing and Its Implication for $b \rightarrow s$ Transitions in Supersymmetry 

Richard Arnowitt ${ }^{1 *}$, Bhaskar Dutta ${ }^{1 \dagger}$, Bo $\mathrm{Hu}^{2 \ddagger}$ and Sechul $\mathrm{Oh}^{3 \S}$<br>${ }^{1}$ Department of Physics, Texas AछM University, College Station, TX, 77845, USA<br>${ }^{2}$ Department of Physics, Nanchang University, Jiangxi 330047, China<br>${ }^{3}$ Natural Science Research Institute, Yonsei University, Seoul 120-479, Korea


#### Abstract

We investigate the effect of the current measurement of the neutral $B_{s}$ meson mass difference, $\Delta M_{B_{s}}$, on SUGRA models which have non-zero values of the soft breaking terms $\left(m_{L L, R R}^{2}\right)_{23}$ and $A_{23}^{u, d}$ at the GUT scale. We use non-zero values of these parameter to explain the $B \rightarrow K \pi$ puzzle and find that even after satisfying the experimental result on $\Delta M_{B_{s}}$ and the branching ratio (BR) of $b \rightarrow s \gamma$ we still can explain the puzzle. Further we show that in this parameter space it is possible to accommodate the large BR of $B \rightarrow \eta^{\prime} K$ and the current experimental data for CP asymmetries of $B \rightarrow \eta^{\prime} K^{0}$ and $B \rightarrow \phi K^{0}$. The predicted value of $\sin \left(2 \beta^{\text {eff }}\right)_{\eta^{\prime} K^{0}}$ is about $0.52-0.67$.


[^0]Flavor changing $b \rightarrow s$ transitions are particularly interesting for new physics (NP) searches using $B$ meson decays. In the standard model (SM) these transitions can occur only at the loop-level so that they are particulary sensitive to NP effects. So far, a few possible indications to NP effects through $b \rightarrow s$ transitions have been reported by experimental collaborations such as BaBar and Belle. Among them is the recent $B \rightarrow K \pi$ puzzle: i.e., discrepancies between the SM predictions and the experimental results for the direct and mixing-induced CP asymmetries and the branching ratios (BRs) in $B \rightarrow K \pi$ modes whose dominant quark level processes are $b \rightarrow s q \bar{q}(q=u, d)[1,2,3]$. The measurements of the CP asymmetries in $B_{d} \rightarrow \eta^{\prime} K$ and $B_{d} \rightarrow \phi K$ modes as well as the rather large BR for $B \rightarrow \eta^{\prime} K$ and $B \rightarrow \eta K$ also have drawn a lot of attention, due to their possible deviation from the SM predictions [1, 4]. The (dominant) subprocess of these modes is the $b \rightarrow s s \bar{s}$ transition.

Recently, the CDF collaboration has reported a new result for another interesting observable relevant to the $b \rightarrow s$ transition: the mass difference between the neutral $B_{s}$ states that characterizes the $B_{s}-\bar{B}_{s}$ mixing phenomenon. The CDF result is [5]

$$
\begin{equation*}
\Delta M_{B_{s}}=17.33_{-0.21}^{+0.42} \text { (stat.) } \pm 0.07 \text { (syst.) } \mathrm{ps}^{-1} \tag{1}
\end{equation*}
$$

The $\mathrm{D} \emptyset$ collaboration has also recently provided a new result [6]:

$$
\begin{equation*}
17 \mathrm{ps}^{-1}<\Delta M_{B_{s}}<21 \mathrm{ps}^{-1} \quad(90 \% \text { C.L. }) \tag{2}
\end{equation*}
$$

These experimental results are consistent with the SM estimation. Therefore, these new experimental results are expected to provide important constraints on NP beyond the SM [7]. Motivated by these new results, some theoretical studies have been done to search for NP effects [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

In the SM , the mass difference in the $B_{s}$ system is given by

$$
\begin{equation*}
\Delta M_{B_{s}}^{\mathrm{SM}}=\frac{G_{F}^{2} M_{W}^{2}}{6 \pi^{2}} M_{B_{s}} \hat{\eta}_{B} \hat{B}_{B_{s}} f_{B_{s}}^{2}\left|V_{t b} V_{t s}^{*}\right|^{2} S_{0}\left(x_{t}\right) \tag{3}
\end{equation*}
$$

where the NLO short-distance QCD correction gives $\hat{\eta}_{B}=0.552$ and $S_{0}\left(x_{t}\right)=2.463$ [19]. The non-perturbative quantities $\hat{B}_{B_{s}}$ and $f_{B_{s}}$ are the bag parameter and the decay constant, respectively. The best fit for $\Delta M_{B_{s}}^{\mathrm{SM}}$ is given by [20, 21]

$$
\begin{equation*}
\Delta M_{B_{s}}^{\mathrm{SM}}=21.5 \pm 2.6 \mathrm{ps}^{-1} \quad[\mathrm{UTfit}], \quad \Delta M_{B_{s}}^{\mathrm{SM}}=21.7_{-4.2}^{+5.9} \mathrm{ps}^{-1} \quad[\text { CKMfitter }] \tag{4}
\end{equation*}
$$

In a recent paper [14], this mass difference is found to be $23.4 \pm 3.8 \mathrm{ps}^{-1}$ using HPQCD and JLQCD data for $f_{B_{s}} \sqrt{\hat{B}_{B_{s}}}$.

In this letter, we study the neutral $B_{s}$ meson mixing effect in supersymmetry (SUSY): specifically in the supergravity (SUGRA) model. Then, using the constraints obtained from $\Delta M_{B_{s}}$, we focus on how to resolve all the possible current anomalies observed in hadronic $B \rightarrow P P(P$ denotes a pseudoscalar meson) decays through the $b \rightarrow s$ transitions, such as $B \rightarrow K \pi, B \rightarrow \eta^{\prime} K$. The current experimental data is listed in Table 1.

We consider the SUGRA model with the simplest possible non-universal soft terms which is the simplest extension of the minimal SUGRA (mSUGRA) model. In the SUGRA model, the superpotential and soft SUSY breaking terms at the grand unified theory (GUT) scale are given by

$$
\begin{align*}
\mathcal{W}= & Y^{U} Q H_{2} U+Y^{D} Q H_{1} D+Y^{L} L H_{1} E+\mu H_{1} H_{2} \\
\mathcal{L}_{\text {soft }}= & -\sum_{i} m_{i}^{2}\left|\phi_{i}\right|^{2}-\left[\frac{1}{2} \sum_{\alpha} M_{\alpha} \bar{\lambda}_{\alpha} \lambda_{\alpha}+B \mu H_{1} H_{2}\right. \\
& \left.+\left(A^{U} Q H_{2} U+A^{D} Q H_{1} D+A^{L} L H_{1} E\right)+\text { H.c. }\right] \tag{5}
\end{align*}
$$

where $E, U$ and $D$ are respectively the lepton, up-quark and down-quark singlet superfields, $L$ and $Q$ are the $\mathrm{SU}(2)_{L}$ doublet lepton and quark superfields, and $H_{1,2}$ are the Higgs doublets. $\phi_{i}$ and $\lambda_{\alpha}$ denote all the scalar fields and gaugino fields, respectively.

The SUSY contributions appear at loop order. In our calculation, we do not use the mass insertion approximation, but rather do a complete calculation [22, 23]. We assume the breakdown of the universality to accommodate the $b \rightarrow s$ transition data. While we satisfy this data, we also have to be careful to satisfy other data, e.g., $b \rightarrow s \gamma$.

We use the following boundary conditions at the GUT scale:

$$
\begin{equation*}
\left(m_{\left(Q_{L L}, U_{R R}, D_{R R}\right)}^{2}\right)_{i j}=m_{0}^{2}\left[\delta_{i j}+\left(\Delta_{\left(Q_{L L}, U_{R R}, D_{R R}\right)}\right)_{i j}\right], \quad A_{i j}^{(u, d)}=A_{0}\left(Y_{i j}^{(u, d)}+\Delta A_{i j}^{(u, d)}\right) \tag{6}
\end{equation*}
$$

where $i, j=1,2,3$ are the generation indices. The SUSY parameters can have phases at the GUT scale: $M_{i}=\left|M_{1 / 2}\right| e^{i \theta_{i}}$ (the gaugino masses for the $U(1), S U(2)$ and $S U(3)$ groups, $i=1,2,3), A_{0}=\left|A_{0}\right| e^{i \alpha_{A}}$ and $\mu=|\mu| e^{i \theta_{\mu}}$. However, we can set one of the gaugino phases to zero and we choose $\theta_{2}=0$. The electric dipole moments (EDMs) of the electron and neutron can now allow the existence of large phases in the theory [24]. In our calculation, we use $O(1)$ phases but calculate the EDMs to make sure that current bounds $\left(\left|d_{e}\right|<1.2 \times 10^{-27} \mathrm{ecm}\right.$ [25] and $\left|d_{n}\right|<6.3 \times 10^{-26} \mathrm{ecm}$ [26]) are satisfied.

We evaluate the squark masses and mixings at the weak scale by using the above boundary conditions at the GUT scale. The RGE evolution mixes the non-universality of type LR (A

TABLE I: Experimental data on the CP-averaged branching ratios ( $\overline{\mathcal{B}}$ in units of $10^{-6}$ ), the direct CP asymmetries $\left(\mathcal{A}_{C P}\right)$, and the effective $\sin (2 \beta)$ ( $\beta$ is the angle of the unitarity triangle) for $B \rightarrow P P$ decays 1].

| BR | Average | CP asymmetry | Average |
| :---: | :---: | :---: | :---: |
| $\overline{\mathcal{B}}\left(B^{ \pm} \rightarrow K^{0} \pi^{ \pm}\right)$ | $24.1 \pm 1.3$ | $\mathcal{A}_{C P}\left(K^{0} \pi^{ \pm}\right)$ | $-0.02 \pm 0.04$ |
| $\overline{\mathcal{B}}\left(B^{ \pm} \rightarrow K^{ \pm} \pi^{0}\right)$ | $12.1 \pm 0.8$ | $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{0}\right)$ | $+0.04 \pm 0.04$ |
| $\overline{\mathcal{B}}\left(B^{0} \rightarrow K^{ \pm} \pi^{\mp}\right)$ | $18.9 \pm 0.7$ | $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{\mp}\right)$ | $-0.115 \pm 0.018$ |
| $\overline{\mathcal{B}}\left(B^{0} \rightarrow K^{0} \pi^{0}\right)$ | $11.5 \pm 1.0$ | $\mathcal{A}_{C P}\left(K^{0} \pi^{0}\right)$ | $+0.001 \pm 0.155$ |
|  |  | $\sin \left(2 \beta^{\mathrm{eff}}\right)_{K_{s} \pi^{0}}$ | $+0.34 \pm 0.29$ |
| $\overline{\mathcal{B}}\left(B^{ \pm} \rightarrow \eta^{\prime} K^{ \pm}\right)$ | $69.7_{-2.7}^{+2.8}$ | $\sin \left(2 \beta^{\mathrm{eff}}\right)_{\eta^{\prime} K^{0}}$ | $+0.50 \pm 0.09$ |
|  |  | $\sin \left(2 \beta^{\mathrm{eff}}\right)_{\phi K^{0}}$ | $+0.47 \pm 0.19$ |

terms) via $d m_{Q}^{Q_{L L, R R}} 2 / d t \propto A_{u(d)}^{\dagger} A_{u(d)}$ terms and creates new LL and RR contributions at the weak scale. We then evaluate the Wilson coefficients from all these new contributions. We have both chargino and gluino contributions arising due to the LL, LR, RR up type and down type squark mixing. These contributions affect the following Wilson coefficients $C 3-C 9, C_{7 \gamma}$ and $C_{8 g}$. The chargino contributions affect mostly the electroweak penguins (C7 and C9) and the dipole penguins, where as the gluino penguin has the largest contribution to the dipole penguins due to the presence of an enhancement factor $m_{\tilde{g}} / m_{b}$ (The gluino contribution also affects the QCD penguins). We include all contributions in our calculation.

For calculation of the relevant hadronic matrix elements, we adopt the QCD improved factorization. This approach allows us to include the possible non-factorizable contributions, such as vertex corrections, penguin corrections, hard spectator scattering contributions, and weak annihilation contributions [27].

The neutral $B$ meson mass difference involves gluino and chargino diagrams in SUSY [28]. In mSUGRA, with universal boundary condition, the chargino diagram has the dominant contribution. Once we introduce mixing in the (2,3)-sector of the $m_{L L, R R}^{2}$ or $A_{L R}$ soft breaking terms, the mass difference gets enhanced and we get large contributions from the gluino diagrams. The $B \rightarrow \pi K$ puzzle can not be solved using just the mSUGRA boundary condition. In order to explain the $B \rightarrow K \pi$ puzzle, we have noticed that the flavor violating terms in the $(2,3)$-sector of the soft breaking masses are needed [3].


FIG. 1: $A_{C P}\left(K^{ \pm} \pi^{\mp}\right)$ versus $\Delta M_{B_{s}} / \Delta M_{B_{d}}$ in the SUGRA model. The parameters are described in the text.

In order to investigate the effect of the neutral $B_{s}$ mixing on the $b \rightarrow s$ transitions, we first try to fit the $B \rightarrow K \pi$ data using $A_{23}^{u, d},\left(m_{L L, R R}^{2}\right)_{23}$ at the GUT scale. The constraint from the BR of $b \rightarrow s \gamma$ is also included. We vary $m_{1 / 2}$ in the range $(350-500) \mathrm{GeV}$ [corresponding to gluino mass of $(1-1.5) \mathrm{TeV}], A_{0}=-800 \mathrm{GeV}, \Delta_{\left(Q_{L L}, U_{R R}, D_{R R}\right)} \sim 0-0.3$, $\Delta A_{23}^{(u, d)}=0-0.3, m_{0}=300 \mathrm{GeV}$ and $\tan \beta=40$. The $\Delta$ 's also have $\mathrm{O}(1)$ phases. The magnitudes of $\Delta$ 's get reduced at the weak scale compared to the GUT scale since the squark masses get a contribution from $m_{1 / 2}$ in the RGEs.

In Fig 1, we plot $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{\mp}\right)$ versus $\Delta M_{B_{s}} / \Delta M_{B_{d}}$, where $\Delta M_{B_{d}}$ is the mass difference between the neutral $B_{d}$ states. The experimental value for the $\Delta M_{B_{d}}$ is $0.507 \pm 0.005 \mathrm{ps}^{-1}$. In the SM,

$$
\begin{equation*}
\frac{\Delta M_{s}^{\mathrm{SM}}}{\Delta M_{d}^{\mathrm{SM}}}=\frac{M_{B_{s}}}{M_{B_{d}}} \xi^{2}\left|\frac{V_{t s}}{V_{t d}}\right|^{2} \tag{7}
\end{equation*}
$$

where $\xi \equiv \frac{f_{B_{s}} \sqrt{\hat{B}_{s}}}{f_{B_{d}} \sqrt{\hat{B}_{d}}}$. In the plot, we used $\xi=1.18$ and the CKM phase $\gamma=61.1^{\circ} \pm 4.5^{\circ}$ [20]. We find that the $2 \sigma$ experimental range about the central value of the ratio $\Delta M_{B_{s}} / \Delta M_{B_{d}}=$ 34.66 rules out a lot of model points. In order to extract the valid points, we include the error of $\xi=1.23 \pm 0.06[10]$ (consistent with the value of $\xi=1.21_{-0.035}^{+0.047}$ using the JLQCD and the HPQCD calculations in Ref. [14]), and calculate the BRs and the CP asymmetries of different $B \rightarrow K \pi$ modes. We also calculate the BR of $B \rightarrow \eta^{\prime} K$ and $\sin \left(2 \beta^{\mathrm{eff}}\right)_{\eta^{\prime} K^{0}}$ as well as $\sin \left(2 \beta^{\text {eff }}\right)_{\phi K^{0}}$ for the allowed model points.

The recent experimental data for the CP-averaged BRs of $B \rightarrow K \pi$ may indicate a


FIG. 2: $R_{c}-R_{n}$ versus $R_{c}$ in the SUGRA model.


FIG. 3: $A_{C P}\left(K^{ \pm} \pi^{0}\right)$ versus $A_{C P}\left(K^{ \pm} \pi^{\mp}\right)$ in the SUGRA model.
possible deviation from the prediction of the SM:

$$
\begin{equation*}
R_{c} \equiv \frac{2 \overline{\mathcal{B}}\left(B^{ \pm} \rightarrow K^{ \pm} \pi^{0}\right)}{\overline{\mathcal{B}}\left(B^{ \pm} \rightarrow K^{0} \pi^{ \pm}\right)}=1.00 \pm 0.09, \quad R_{n} \equiv \frac{\overline{\mathcal{B}}\left(B^{0} \rightarrow K^{ \pm} \pi^{\mp}\right)}{2 \overline{\mathcal{B}}\left(B^{0} \rightarrow K^{0} \pi^{0}\right)}=0.79 \pm 0.08 . \tag{8}
\end{equation*}
$$

It has been claimed that within the $\mathrm{SM}, R_{c} \approx R_{n}$ [29, 30]. But, the data show the pattern $R_{c}>R_{n}$, which would indicate the enhancement of the electroweak (EW) penguin and/or the color-suppressed tree contributions [2]. In Fig. 2, we plot $R_{c}-R_{n}$ versus $R_{c}$ and find that $R_{c}>R_{n}$ can be satisfied.

Also, in the conventional prediction of the $\mathrm{SM}, \mathcal{A}_{C P}\left(K^{ \pm} \pi^{0}\right)$ is expected to be almost the same as $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{\mp}\right)$. In particular, they would have the same sign. However, the current data show that $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{0}\right)$ differs by $3.5 \sigma$ from $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{\mp}\right)$. In Fig. 3, we plot $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{0}\right)$


FIG. 4: $\sin \left(2 \beta^{\mathrm{eff}}\right)_{K_{s} \pi^{0}}$ versus $A_{C P}\left(K^{ \pm} \pi^{\mp}\right)$ in the SUGRA model.


FIG. 5: $\sin \left(2 \beta^{\text {eff }}\right)_{\eta^{\prime} K^{0}}$ versus the BR of $B^{ \pm} \rightarrow \eta^{\prime} K^{ \pm}$in the SUGRA model.
versus $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{\mp}\right)$ and find that the signs can be different for the points allowed by the neutral $B$ mixing data.

The predicted $\sin \left(2 \beta^{\mathrm{eff}}\right)_{K_{S} \pi^{0}}$ is shown in Fig. 4. We find that the minimum value is 0.7 . The present experimental data still have large errors so that future results will confirm/rule out our model.

The experimental BRs of $\mathcal{B}\left(B \rightarrow \eta^{\prime} K\right)$ are large compared to the conventional SM predictions. In Fig. 5, we plot $\sin \left(2 \beta^{\text {eff }}\right)_{\eta^{\prime} K^{0}}$ versus the BR of $B^{ \pm} \rightarrow \eta^{\prime} K^{ \pm}$. These decay modes get SUSY contributions since we are using non-zero values of $\left(m_{L L, R R}^{2}\right)_{23}$ and $A_{23}$ and the BR gets enhanced. The values of $\sin \left(2 \beta^{\mathrm{eff}}\right)_{\eta^{\prime} K^{0}}$ is allowed by the experimental value which

TABLE II: The CP-averaged branching ratios ( $\overline{\mathcal{B}}$ in units of $10^{-6}$ ), the direct CP asymmetries $\left(\mathcal{A}_{C P}\right)$, and the effective $\sin (2 \beta)$ for $m_{1 / 2}=450 \mathrm{GeV}, m_{0}=300 \mathrm{GeV}$ and $\tan \beta=40,\left|\Delta_{23 L L}\right|=$ $0.48,\left|\Delta A_{23}^{d}\right|=0.1,\left|\Delta A_{23}^{u}\right|=0.3$.

| BR | Average | CP asymmetry | Average |
| :---: | :---: | :---: | :---: |
| $\overline{\mathcal{B}}\left(B^{ \pm} \rightarrow K^{0} \pi^{ \pm}\right)$ | 23.8 | $\mathcal{A}_{C P}\left(K^{0} \pi^{ \pm}\right)$ | -0.03 |
| $\overline{\mathcal{B}}\left(B^{ \pm} \rightarrow K^{ \pm} \pi^{0}\right)$ | 11.1 | $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{0}\right)$ | 0.013 |
| $\overline{\mathcal{B}}\left(B^{0} \rightarrow K^{ \pm} \pi^{\mp}\right)$ | 19.6 | $\mathcal{A}_{C P}\left(K^{ \pm} \pi^{\mp}\right)$ | -0.10 |
| $\overline{\mathcal{B}}\left(B^{0} \rightarrow K^{0} \pi^{0}\right)$ | 11.4 | $\mathcal{A}_{C P}\left(K^{0} \pi^{0}\right)$ | -0.11 |
|  |  | $\sin \left(2 \beta^{\mathrm{eff}}\right)_{K_{s} \pi^{0}}$ | +0.8 |
| $\overline{\mathcal{B}}\left(B^{ \pm} \rightarrow \eta^{\prime} K^{ \pm}\right)$ | 72 | $\sin \left(2 \beta^{\mathrm{eff}}\right)_{\eta^{\prime} K^{0}}$ | +0.6 |

has a smaller error than that of $\sin \left(2 \beta^{\mathrm{eff}}\right)_{\phi K^{0}}$. The value of $\sin \left(2 \beta^{\mathrm{eff}}\right)_{\phi K^{0}}$ is predicted to be around $(0.55-0.70)$ and the BRs of $B^{ \pm} \rightarrow \phi K^{ \pm}$and $B^{0} \rightarrow \phi K^{0}$ are around $(7-9) \times 10^{-6}$ and $(6.5-8.5) \times 10^{-6}$ respectively in our calculation and $\mathcal{B}(b \rightarrow s \gamma)$ is $(2-4.5) \times 10^{-4}$. We also find that $\mathcal{B}(B \rightarrow \eta K)$ is around $3 \times 10^{-6}$. The CP asymmetries for $B^{ \pm} \rightarrow \phi K^{ \pm}$and $B^{ \pm} \rightarrow \eta^{\left({ }^{\prime}\right)} K^{ \pm}$are -0.1 to 0.1 and close to 0 , respectively. The $\operatorname{Arg}\left[M(12)_{B_{s}}\right]$ is less than $5^{\circ}$ for our model points.

It is possible to obtain a fit for the experimental results even without using $m_{L L}^{2}$ contribution at al. The nonzero values of $A_{23}^{u, d}$ parameters generate the dipole penguin and the ( $Z$-mediated) electroweak penguin diagrams. As a representative example, we present the BRs and CP asymmetries for a specific model point in Table II to show that all these different experimental results can be explained by one model point using just $A_{23}^{u, d}$. The parameters of the model point are given at the GUT scale by $m_{1 / 2}=450 \mathrm{GeV}, m_{0}=350 \mathrm{GeV}$, $A_{0}=-800 \mathrm{GeV}, \Delta A_{23}^{d}=0.1 e^{-2.0 i}, \Delta A_{23}^{u}=0.48 e^{1.1 i}$, and we choose $\tan \beta=40$. We find that the BRs and CP asymmetries are all within one sigma of the experimental results except for $\sin \left(2 \beta^{\mathrm{eff}}\right)_{K_{s} \pi^{0}}$ which is about $1.6 \sigma$ away (this deviation is lowered when we include $m_{L L}^{2}$ contribution). The QCD parameters for this fit are $\rho_{A}=2$ and $\phi_{A}=2.75$, where $\rho_{A}$ and $\phi_{A}$ are defined by $X_{A} \equiv \int_{0}^{1} \frac{d x}{x} \equiv\left(1+\rho_{A} e^{i \phi_{A}}\right) \ln \frac{m_{B}}{\Lambda_{h}}$ [27]. The ratio $\Delta M_{B_{s}} / \Delta M_{B_{d}}$ is 34.3 for this model point. The EDMs are following: $\left|d_{e}\right|=2.48 \times 10^{-29} \mathrm{e} \mathrm{cm}$ and $\left|d_{n}\right|=8.6 \times 10^{-28} \mathrm{e} \mathrm{cm}$. The BR of $b \rightarrow s \gamma$ is $4.2 \times 10^{-4}$.

The origin of the $\left(m_{L L, R R}^{2}\right)_{23}$ terms are natural in the grand unifying models which explain
neutrino masses. For example, if right handed neutrinos exist, $\operatorname{SU}(5)$ might generate the term $Y_{\nu} \overline{5} \bar{N} 5_{H}$, where $\overline{5}$ has $d_{i}^{c}$ and the lepton $L$ doublet, $\bar{N}$ is the singlet right handed neutrinos and $5_{H}$ contains the SM Higgs doublet (along with the colored Higgs fields. $Y_{\nu}$ has a flavor structure in order to explain the neutrino masses and bilarge-mixing angles. Now these couplings introduce flavor violation to the soft masses ( $\tilde{d}^{c}$ and $\tilde{l}$ ) via the RGEs, $\frac{d m^{2}}{d t} \propto m^{2} Y_{\nu} Y_{\nu}^{\dagger}$. In this model the $m_{i j, 5}^{2}$ terms for $i \neq j$ can be generated 31]. These terms get introduced between the GUT scale and the string scale due to the RGEs. One expects these flavor violating terms also in the $\mathrm{SO}(10)$ type models 32]. The right handed neutrinos there belong to the fundamental 16 representation of $\mathrm{SO}(10)$ and produce these flavor violating terms in the soft masses. The flavor structures of the Dirac and Majorana coupling arise from the neutrino mixing matrix. The $A_{i j}$ terms (for $i \neq j$ ) also get contributions from the flavor structure of $Y_{\nu}$ due to the quark-lepton unification. Similar flavor violating effects in the soft terms are also present in the Pati-Salam type models [33]. In this case, the quark-lepton unification can happen at the intermediate scale and the flavor violating Majorana coupling $f \psi_{\overline{4}, 1,2} \psi_{\overline{4}, 1,2} \Delta_{10,1,3}\left(\psi_{\overline{4}, 1,2}\right.$ contains right handed neutrinos along with the right handed quarks and leptons, $\Delta_{10,1,3}$ is the new Higgs field ) is responsible for right handed neutrino Majorana masses. Now the RGEs involving these couplings between the intermediate scale and the grand unifying scale can easily introduce flavor violating terms in the squark and the slepton masses.

In conclusion, we find that the current experimental results on the neutral $B_{s}$ meson mass difference have introduced strict constraint on the the SUGRA parameter space for flavor mixing terms $A_{23}$ and $\left(m_{L L, R R}^{2}\right)_{23}$ in the soft SUSY breaking terms. These flavor violating soft breaking terms are natural in the grand unifying models. In order to explain the $B \rightarrow K \pi$ puzzle, $A_{23}$ and $\left(m_{L L, R R}^{2}\right)_{23}$ are needed. We show that it is still possible to explain the $B \rightarrow K \pi$ puzzle even after satisfying the new Tevatron result on $\Delta M_{B_{s}}$. The model used here contains three complex nonuniversal soft breaking terms ( $\Delta_{23, L L}, \Delta A_{23}^{u, d}$ ), though an acceptable fit can be obtained using just $\Delta A_{23}^{u, d}$. This allows us to calculate 19 observables of the $B$ system ( 9 observables in the $B \rightarrow \pi K$ modes, 4 observables in $B \rightarrow \phi K$ modes, 5 observables in the $B \rightarrow \eta^{(\prime)} K$ modes and $\left.\mathcal{B}(b \rightarrow s \gamma)\right)$. The future results on $\sin \left(2 \beta^{\mathrm{eff}}\right)_{K_{s} \pi^{0}}$ and $A_{C P}\left(K^{ \pm} \pi^{\mp}\right)$ are crucial to probe this model. Finally, we find that the large $\mathcal{B}\left(B \rightarrow \eta^{\prime} K\right)$ can be explained in this parameter space with $\sin \left(2 \beta^{\text {eff }}\right)_{\eta^{\prime} K^{0}}$ near the current experimental result which is $2 \sigma$ away from $\sin \left(2 \beta^{\mathrm{eff}}\right)_{J / \psi K}$.

## ACKNOWLEDGEMENTS

The work of B.H. was supported in part by the National Nature Science Foundation of China (No. 10505011). The work of S.O. was supported by the Korea Research Foundation Grant (KRF-2004-050-C00005).

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[^0]:    * arnowitt@physics.tamu.edu
    $\dagger$ dutta@physics.tamu.edu
    $\ddagger$ bohu@ncu.edu.cn
    § scoh@phya.yonsei.ac.kr

