

A K-Theory Anomaly Free Supersymmetric Flipped $SU(5)$ Model from Intersecting Branes.

C.-M. Chen^{♠ 1}, G. V. Kraniotis^{♠ 2}, V. E. Mayes^{♠ 3},
D. V. Nanopoulos^{♠,♣,◇ 4}, and J. W. Walker^{♠ 5}

♠ *George P. and Cynthia W. Mitchell Institute for Fundamental Physics,
Texas A&M University, College Station TX, 77843, USA*

♣ *Astroparticle Physics Group, Houston Advanced Research Center (HARC)
Mitchell Campus, Woodlands, TX 77381, USA*

◇ *Academy of Athens, Division of Natural Sciences,
28 Panepistimiou Avenue, Athens 10679, Greece*

ABSTRACT

We construct an $N = 1$ supersymmetric three-family flipped $SU(5)$ model from type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at general angles. The model is constrained by the requirement that Ramond-Ramond tadpoles cancel, the supersymmetry conditions, and that the gauge boson coupled to the $U(1)_X$ factor does not get a string-scale mass via a generalised Green-Schwarz mechanism. The model is further constrained by requiring cancellation of K-theory charges. The spectrum contains a complete grand unified and electroweak Higgs sector, however the latter in a non-minimal number of copies. In addition, it contains extra matter both in bi-fundamental and vector-like representations as well as two copies of matter in the symmetric representation of $SU(5)$.

¹cchen@physics.tamu.edu

²kraniotis@physics.tamu.edu

³eric@physics.tamu.edu

⁴dimitri@physics.tamu.edu

⁵jwalker@physics.tamu.edu

1 Introduction

One of the main goals of string phenomenology is to derive standard model physics in a convincing way and to embed the latter into a unified description of gravitational and gauge forces.

In the intersecting D-branes approach in type II string theory [1, 2, 3, 4], gravity is mediated in the entire 10-dimensional bulk by the exchange of closed strings. On the other hand, the gauge and matter fields are localized respectively on the D-brane, and at pairwise D-brane intersections, and correspond to open string excitations. One of the outstanding issues is the breaking of supersymmetry. There are at least two scenarios of how supersymmetry breaking can be realized: In the first case, the various open string sectors break supersymmetry while the closed string sector may preserve supersymmetry. In order to solve the hierarchy problem this scenario normally requires the existence of large extra dimensions, transverse to the D-branes of the standard model [5, 6, 7]. In the second scenario, all open string sectors preserve $N = 1$ supersymmetry, and all D-branes together with the orientifold planes, are mutually supersymmetric. Then the closed string sector breaks supersymmetry, which manifests itself as soft supersymmetry-breaking terms in the effective action of the open string matter fields. The spontaneous supersymmetry breaking can be achieved by internal background fluxes of closed string field strengths $\langle G_{ijk} \rangle \neq 0$ [8, 9, 10].

In this latter case, one in general has to introduce in addition to D6-branes orientifold O6-planes, which can be regarded as branes of negative RR-charge and tension. For a general Calabi-Yau compact space these orientifold planes wrap special Lagrangian 3-cycles calibrated with respect to the real part of the holomorphic 3-form Ω_3 of the Calabi-Yau compact space. In order to preserve $N = 1$ supersymmetry all the D6-branes must be calibrated by the 3-form Ω_3 . Calibrated geometries lead to volume minimization in homology. Adopting this philosophy for solving the gauge hierarchy problem and other instabilities of non-supersymmetric models from intersecting branes, we recently constructed a three-generation $N = 1$ supersymmetric flipped $SU(5)$ model [11], whose gauge symmetry included $SU(5) \times U(1)_X$ symmetry, from type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes. For other examples of Grand Unified Models as well as for a complete set of references in the field, the reader should consult [12, 13].

The flipped $SU(5) \times U(1)_X$ model [14, 15] is a well motivated example of a Grand Unified Theory (GUT), and had been extensively studied in the closed string era of the heterotic compactifications [16, 17]. From the theoretical point of view this motivation was coming from the fact that its symmetry breaking requires only **10** and $\overline{\mathbf{10}}$ Higgs representations at the grand unification scale, as well as **5** and $\overline{\mathbf{5}}$ Higgs representations at the electroweak scale, which were consistent with the representations of $SU(5)$ allowed by the unitarity condition with gauge group at level 1 [18, 19]¹. From the phenomenological point of view flipped $SU(5) \times U(1)_X$ [14, 15]

¹Thus attempts to embed conventional grand unified theory (GUT) groups such as $SU(5)$ or

has a number of attractive features in its own right [20]. For example, it has a very elegant missing-partner mechanism for suppressing proton decay via dimension-5 operators [15], and is probably the simplest GUT to survive experimental limits on proton decay [21]. These considerations motivated the derivation of a number of flipped $SU(5)$ models from constructions using fermions on the world sheet [16, 17]. Also, non-supersymmetric flipped $SU(5)$ models have been produced in [22] using D6-branes wrapping toroidal 3-cycles and when the wrapping space is the T^6/\mathbb{Z}_3 orbifold.

The wrapping numbers of the various stacks in [11] were constrained by the requirement of RR-tadpole cancellation as well as the supersymmetry conditions. Tadpole cancellation ensures the absence of non-abelian anomalies in the emergent low-energy quantum field theory. A generalised Green-Schwarz mechanism ensures that the gauge bosons associated with all anomalous $U(1)$ s acquire string-scale masses [23], but the gauge bosons of some non-anomalous $U(1)$ s can also acquire string-scale masses [24]; in all such cases the $U(1)$ group survives as a global symmetry. Thus we had also to ensure the flipped $U(1)_X$ group remained a gauge symmetry by requiring that its gauge boson did *not* get such a mass. The gauge symmetry of the model included a $USp(2) \cong SU(2)$ factor associated with the presence of filler branes (D-branes that wrap 3-cycles which are invariant under the orientifold action). The low energy spectrum of the model we constructed was free from any $SU(2)$ global gauge anomalies (associated with the fourth non-trivial homotopy group of $SU(2)$: $\pi^4(SU(2)) = \mathbb{Z}_2$) since the number of the corresponding fermion doublets was even [25]². The model however, suffered from a number of serious phenomenological drawbacks. Among them, the global $U(1)$ symmetries of the model, that arise after the G-S anomaly cancellation mechanism, did forbid some of the Yukawa couplings required for mass generation, as well as the couplings responsible for the elegant solution of the doublet-triplet splitting problem in flipped $SU(5)$ [11]. The model also included a lot of exotic matter both in bi-fundamental and vector-like representations, as well as two copies of matter in the symmetric representation of $SU(5)$. Furthermore, three adjoint (24-plets) ($N = 1$) chiral multiplets were also present.

It has been argued by a number of authors [26, 27, 28, 29], that for the D-branes to consistently wrap the 3-cycles of the compact space, additional conditions on their wrapping numbers have to be satisfied beyond those described above, which stem from the K-theory interpretation of D-branes. In particular, it has been argued that often it is K-theory which fully classifies the RR-charges of D-branes and not the ordinary homology theory³. This approach was also motivated by the work in [32] in which the non-BPS D-branes were constructed as bound states of brane-anti-brane

$SO(10)$ in heterotic string required more complicated compactifications, but none of these has been completely satisfactory. Constructions with the minimal option to embed just the standard model gauge group, were plagued with at least extra unwanted $U(1)$ factors.

²This was the case since the number of intersections of the filler branes to the other stacks of D6-branes was even.

³For some computations of the K-theory groups, that make use of Bott's periodicity theorem and Atiyah's real K-theory [30], for D-branes on top of orientifold planes see [31].

pairs. These constructions were interpreted in terms of K-theory in [27]. The model constructed in [11] did not satisfy all the additional constraints from K-theory derived in [29] for the particular orientifold background.

It is the purpose of this Letter, to incorporate these additional constraints, which ensure that discrete K-theory charge cancels, into the search for building a viable flipped $SU(5)$ model. Interestingly, as we shall see in the main body of the paper, although the new model constructed, which is compatible with the above consistency requirements, does contain a lot of extra matter, it does not contain exotic matter other than two copies in the symmetric ($\mathbf{15}$) representation of $SU(5)$ in the spectrum on the sector of D6-branes at generic angles (i.e. those that wrap 3-cycles not invariant under the anti-holomorphic involution). However, the model still possesses exotic matter from the intersections of filler branes with the other stacks of D6-branes, which is charged under both the flipped $SU(5)$ and filler branes gauge symmetries.

The material of this Letter is organized as follows. In section 2, we provide the consistency as well as the other conditions described above for the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold which are used for the construction of our model. In section 3, after a short introduction to basic flipped $SU(5)$ phenomenology we present the model, which is a consistent solution of all the constraints we described above including those from K-theory. Finally, section 4 is used for our conclusions.

2 Definitions and Conditions for Intersecting Brane Models on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold

2.1 Basic Configuration

Consider type IIA theory on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, where the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ generators θ, ω act on the complex coordinates (z_1, z_2, z_3) of T^6 as

$$\begin{aligned}\theta &: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3) \\ \omega &: (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3)\end{aligned}\tag{1}$$

This $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ structure was first introduced in [9] and further studied in [12]⁴, and we will use the same notations here. We implement an orientifold projection ΩR , where Ω is the world-sheet parity, and R acts as

$$R : (z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3)\tag{2}$$

With the wrapping numbers (n^i, m^i) along the canonical basis of homology one-cycles $[a_i]$ and $[b_i]$ the complete cycle on a T^2 is given by $n^i[a_i] + m^i[b_i]$. Note that a tilted complex structure is allowed by setting $[a'_i] \equiv [a_i] + \frac{1}{2}[b_i]$ so we rewrite

⁴See also [33].

the expression of the one-cycle by $n^i[a'_i] + 2^{-\beta_i}l^i[b_i]$ where $l^i = m^i$, $\beta_i = 0$ if the i th torus is not tilted and $l^i = 2m^i + n^i$, $\beta_i = 1$ if it is tilted.

Therefore the homology three cycle $[\Pi_a]$ for a stack a of D6-brane and its orientifold image $[\Pi'_a]$ can be written as

$$[\Pi_a] = \prod_{i=1}^3 (n_a^i[a_i] + 2^{-\beta_i}l_a^i[b_i]), \quad [\Pi_{a'}] = \prod_{i=1}^3 (n_a^i[a_i] - 2^{-\beta_i}l_a^i[b_i]) \quad (3)$$

and the O6-plane associated with the four orientifold projections ΩR , $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$ is

$$\begin{aligned} [\Pi_{O6}] &= [\Pi_{\Omega R}] + [\Pi_{\Omega R\omega}] + [\Pi_{\Omega R\theta\omega}] + [\Pi_{\Omega R\theta}] \\ &= 2^3[a_1][a_2][a_3] - 2^{3-\beta_2-\beta_3}[a_1][b_2][b_3] \\ &\quad - 2^{3-\beta_1-\beta_3}[b_1][a_2][b_3] - 2^{3-\beta_1-\beta_2}[b_1][b_2][a_3] \end{aligned} \quad (4)$$

It is convenient for model building purposes to use a set of parameters introduced in [12]

$$\begin{aligned} A_a &= -n_a^1 n_a^2 n_a^3, \quad B_a = n_a^1 l_a^2 l_a^3, \quad C_a = l_a^1 n_a^2 l_a^3, \quad D_a = l_a^1 l_a^2 n_a^3 \\ \tilde{A}_a &= -l_a^1 l_a^2 l_a^3, \quad \tilde{B}_a = l_a^1 n_a^2 n_a^3, \quad \tilde{C}_a = n_a^1 l_a^2 n_a^3, \quad \tilde{D}_a = n_a^1 n_a^2 l_a^3 \end{aligned} \quad (5)$$

2.2 The Spectrum

Chiral matter particles are formed from open strings with two ends attaching on different stacks. The multiplicity (\mathcal{M}) of the corresponding bi-fundamental representation is given by the intesection numbers (I_{ab}) between different stacks of branes by using Graßmann algebra $[a_i][b_j] = -[b_j][a_i] = \delta_{ij}$ and $[a_i][a_j] = -[b_j][b_i] = 0$. It should be noted that the initial $U(N_a)$ gauge group supported by a stack of N_a identical D6-branes is broken down by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry to a subgroup $U(N_a/2)$ [9]. In Table 1 we exhibit the generic chiral spectrum for the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold for D6-branes at generic angles [12]. A zero intersection number between two branes implies that the branes are parallel on at least one torus. At such kind of intersection additional non-chiral (vector-like) multiplet pairs from $ab + ba$, $ab' + b'a$, and $aa' + a'a$ sectors can arise [34]. The multiplicity of these non-chiral multiplet pairs is given by the remainder of the intersection product, neglecting the null sector. For example, if $(n_a^1 l_b^1 - n_b^1 l_a^1) = 0$ in $I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^3 (n_a^i l_b^i - n_b^i l_a^i)$,

$$\mathcal{M} \left[\left(\frac{N_a}{2}, \frac{\overline{N_b}}{2} \right) + \left(\frac{\overline{N_a}}{2}, \frac{N_b}{2} \right) \right] = \prod_{i=2}^3 (n_a^i l_b^i - n_b^i l_a^i) \quad (6)$$

where we have assumed that, $k \equiv \beta_1 + \beta_2 + \beta_3 = 0$, i.e. all tori are untilted.

Strings stretching between a brane in stack a and its mirror image a' yield chiral matter in the antisymmetric and symmetric representations of the group $U(N_a/2)$ with multiplicities

$$\mathcal{M}((A_a)_L) = \frac{1}{2}I_{aO6}, \quad \mathcal{M}((A_a + S_a)_L) = \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6}) \quad (7)$$

so that the net total of antisymmetric and symmetric representations are those in Table 1. Also

$$I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k} \prod_{i=1}^3 n_a^i l_a^i \quad (8)$$

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k} (\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a) \quad (9)$$

This distinction is critical, as we require independent use of the paired multiplets such as $(\mathbf{10}, \overline{\mathbf{10}})$ which are masked in the corresponding expressions given in Table 1.

Sector	Representation
aa	$U(N_a/2)$ vector multiplet and 3 adjoint chiral multiplets
$ab + ba$	$\mathcal{M}(\frac{N_a}{2}, \frac{\overline{N_b}}{2}) = I_{ab} = 2^{-k} \prod_{i=1}^3 (n_a^i l_b^i - n_b^i l_a^i)$
$ab' + b'a$	$\mathcal{M}(\frac{N_a}{2}, \frac{N_b}{2}) = I_{ab'} = -2^{-k} \prod_{i=1}^3 (n_a^i l_b^i + n_b^i l_a^i)$
$aa' + a'a$	$\mathcal{M}(\text{Anti}_a) = \frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO6}) = -2^{1-k} [(2A_a - 1)\tilde{A}_a - \tilde{B}_a - \tilde{C}_a - \tilde{D}_a]$ $\mathcal{M}(\text{Sym}_a) = \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6}) = -2^{1-k} [(2A_a + 1)\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a]$

Table 1: General chiral spectrum on D6-branes at generic angles, for $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold.

2.3 Consistency and Supersymmetry Conditions

The Ramond-Ramond tadpole cancellation needs to be satisfied by requiring the total homology cycle charge of D6-branes and O6-planes to vanish, namely we require

$$\sum_a N_a [\Pi_a] + \sum_a N_a [\Pi_{a'}] - 4[\Pi_{O6}] = 0 \quad (10)$$

By introducing the filler branes wrapping cycles along the four O6-planes, we can rewrite the above equation in terms of the parameters defined in (5) as

$$\begin{aligned} -2^k N^{(\Omega R)} + \sum_a N_a A_a &= -2^k N^{(\Omega R \omega)} + \sum_a N_a B_a = \\ -2^k N^{(\Omega R \theta \omega)} + \sum_a N_a C_a &= -2^k N^{(\Omega R \theta)} + \sum_a N_a D_a = -16 \end{aligned} \quad (11)$$

Although the total non-Abelian anomaly cancels automatically when the RR-tadpole conditions are satisfied, additional mixed anomalies like the mixed gravitational anomalies which generate massive fields are not trivially zero [9]. These anomalies are cancelled by a generalized Green-Schwarz (G-S) mechanism which involves

untwisted Ramond-Ramond forms. The couplings of the four untwisted Ramond-Ramond forms B_2^i to the $U(1)$ field strength F_a of each stack a are

$$\begin{aligned} N_a \tilde{B}_a \int_{M4} B_2^1 \wedge \text{tr} F_a, & \quad N_a \tilde{C}_a \int_{M4} B_2^2 \wedge \text{tr} F_a \\ N_a \tilde{D}_a \int_{M4} B_2^3 \wedge \text{tr} F_a, & \quad N_a \tilde{A}_a \int_{M4} B_2^4 \wedge \text{tr} F_a \end{aligned} \quad (12)$$

These couplings determine the linear combinations of $U(1)$ gauge bosons that acquire string scale masses via the G-S mechanism. In flipped $SU(5) \times U(1)_X$, the symmetry $U(1)_X$ must remain an unbroken gauge symmetry so that it may mix to help generate the standard model hypercharge after the breaking of $SU(5)$. Therefore, we must ensure that the gauge boson of the flipped $U(1)_X$ group does not receive such a mass. The $U(1)_X$ is a linear combination (to be identified in section 3.2) of the $U(1)$ s from each stack :

$$U(1)_X = \sum_a c_a U(1)_a \quad (13)$$

The corresponding field strength must be orthogonal to those that acquire G-S mass. Thus we demand :

$$\begin{aligned} \sum_a c_a N_a \tilde{B}_a = 0, & \quad \sum_a c_a N_a \tilde{C}_a = 0 \\ \sum_a c_a N_a \tilde{D}_a = 0, & \quad \sum_a c_a N_a \tilde{A}_a = 0 \end{aligned} \quad (14)$$

The condition to preserve $N = 1$ supersymmetry in four dimensions is that the rotation angle of any D-brane with respect to the orientifold plane is an element of $SU(3)$ [1, 9]. Considering the angles between each brane and the R-invariant axis of i^{th} torus θ_a^i , we require $\theta_a^1 + \theta_a^2 + \theta_a^3 = 0 \pmod{2\pi}$. This means $\sin(\theta_a^1 + \theta_a^2 + \theta_a^3) = 0$ and $\cos(\theta_a^1 + \theta_a^2 + \theta_a^3) = 1 > 0$. We define

$$\tan \theta_a^i = \frac{2^{-\beta_i} l_a^i R_2^i}{n_a^i R_1^i} \quad (15)$$

where R_2^i and R_1^i are the radii of the i^{th} torus. The above supersymmetry conditions can be recast in terms of the parameters defined in (5) as follows [12]:

$$\begin{aligned} x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0 \\ A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0 \end{aligned} \quad (16)$$

where x_A, x_B, x_C, x_D are complex structure parameters composed of R_2^i and R_1^i , all of which share the same sign [12]. In what follows we consider the case $k = 0$.

2.4 The K-Theory Constraints

In the previous section, the consistency conditions for having a model free of RR tadpoles were stated. These conditions essentially translate into constraints on the allowed homology cycles. However, as we discussed in the introduction it has been argued that it is K-theory which fully classifies the RR-charges of D-branes and not the ordinary homology theory [26, 27, 28, 29, 35]. Because of this, there are additional consistency constraints related to cancellation of the K-theory charges [28, 29]. These additional constraints are not visible through homology. In this Letter, we improve upon our earlier work [11] with a model that satisfies these K-theory constraints as well as all the other conditions for the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold described in section 2.3.

In type I superstring theory there exist non-BPS D-branes carrying non-trivial K-theory \mathbb{Z}_2 charges. To avoid this anomaly it is required that in compact spaces these non-BPS branes must exist in an even number [28]. If we consider a type I non-BPS D7-brane ($\widehat{D7}$ -brane), we may regard it as a pair of D7-brane and its world-sheet parity image $\overline{D7}$ -brane in type IIB theory, i.e. $\widehat{D7} = D7 + \overline{D7}/\Omega$. There are three different kinds of non-BPS ($\widehat{D7}$)-branes, denoted as $\widehat{D7}_i$, where $i = 1, 2, 3$ labels the two-torus where the $\widehat{D7}$ does not wrap. By construction there are three pairs $D7_i, \overline{D7}_i$ in type IIB theory [29]. These D7-brane pairs, as well as other D-brane pairs in type IIB theory, can be explicitly expressed by the homology 3-cycles in type IIA theory as listed in Table 2 ⁵.

It is reasonable to take the branes in Table 2 as a basis of a magnetized model (obviously they are in terms of the homology one-cycles). We can see that a general D6-brane three-cycle in type IIA theory is composed of these brane pairs, i.e., a general D6-brane is a linear combination of these brane pairs, which is why we should take the K-theory constraints into account since the numbers of the pairs given by wrapping numbers are not trivially even.

We do not have to worry about the K-theory charge contributed by D5 and D9-branes since the RR-tadpole conditions in (11) guarantee the even numbers if we choose the number of the filler branes to be even, which is not difficult to achieve. The real problem comes from D3 and D7-branes, though they do not contribute to the standard RR charges. The K-theory conditions for a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold were

⁵In type IIB picture $D5_i$ stands for a D5-brane wrapping the i^{th} two torus.

derived in [29] and are given by

$$\begin{aligned}
\sum_a N_a l_a^1 l_a^2 l_a^3 &= \sum_a N_a \tilde{A}_a = 0 \pmod{4} \\
\sum_a N_a l_a^1 n_a^2 n_a^3 &= \sum_a N_a \tilde{B}_a = 0 \pmod{4} \\
\sum_a N_a n_a^1 l_a^2 n_a^3 &= \sum_a N_a \tilde{C}_a = 0 \pmod{4} \\
\sum_a N_a n_a^1 n_a^2 l_a^3 &= \sum_a N_a \tilde{D}_a = 0 \pmod{4}
\end{aligned} \tag{17}$$

These constraints turn out to be more clear if the additional three D5-branes or D9-brane are introduced as ‘‘probes’’ [28]. These branes wrap cycles along the O6-planes so they satisfy supersymmetry automatically and form USp groups. The sum of intersection numbers between these probe branes and the general D6-branes should be even (mod 4 in our case) in order to cancel the global gauge anomaly [25]. For example,

$$\sum_a N_a [\Pi_{D5_1}][\Pi_a] = \sum_a N_a l_a^1 n_a^2 n_a^3 = 0 \pmod{4} \tag{18}$$

which is exactly the same as the second equation in (17). Though we add these extra branes to detect the K-theory charges, they are still exterior to our original model and do not contribute to the determined RR-tadpole cancellation configuration.

D3-brane	$\Pi_{D3} = ([b_1])([b_2])([b_3])$	$\Pi_{\overline{D3}} = (-[b_1])(-[b_2])(-[b_3])$
D5-brane	$\Pi_{D5_1} = ([a_1])([b_2])([b_3])$	$\Pi_{\overline{D5_1}} = ([a_1])(-[b_2])(-[b_3])$
	$\Pi_{D5_2} = ([b_1])([a_2])([b_3])$	$\Pi_{\overline{D5_2}} = (-[b_1])([a_2])(-[b_3])$
	$\Pi_{D5_3} = ([b_1])([b_2])([a_3])$	$\Pi_{\overline{D5_3}} = (-[b_1])(-[b_2])([a_3])$
D7-brane	$\Pi_{D7_1} = ([b_1])([a_2])([a_3])$	$\Pi_{\overline{D7_1}} = (-[b_1])([a_2])([a_3])$
	$\Pi_{D7_2} = ([a_1])([b_2])([a_3])$	$\Pi_{\overline{D7_2}} = ([a_1])(-[b_2])([a_3])$
	$\Pi_{D7_3} = ([a_1])([a_2])([b_3])$	$\Pi_{\overline{D7_3}} = ([a_1])([a_2])(-[b_3])$
D9-brane	$\Pi_{D9} = ([a_1])([a_2])([a_3])$	$\Pi_{\overline{D9}} = ([a_1])([a_2])([a_3])$

Table 2: Brane pairs of Type IIB theory without B-field and their corresponding homology classes of 3-cycles in type IIA picture.

3 Model Building for a Flipped $SU(5)$ GUT

3.1 Basic Flipped $SU(5)$ Phenomenology

In a flipped $SU(5) \times U(1)_X$ [14, 15] unified model, the electric charge generator Q is only partially embedded in $SU(5)$, *i.e.*, $Q = T_3 - \frac{1}{5}Y' + \frac{2}{5}\tilde{Y}$, where Y' is the $U(1)$

internal $SU(5)$ and \tilde{Y} is the external $U(1)_X$ factor. Essentially, this means that the photon is ‘shared’ between $SU(5)$ and $U(1)_X$. The Standard Model (SM) plus right handed neutrino states reside within the representations $\mathbf{5}$, $\mathbf{10}$, and $\mathbf{1}$ of $SU(5)$, which are collectively equivalent to a spinor $\mathbf{16}$ of $SO(10)$. The quark and lepton assignments are flipped by $u_L^c \leftrightarrow d_L^c$ and $\nu_L^c \leftrightarrow e_L^c$ relative to a conventional $SU(5)$ GUT embedding:

$$\bar{f}_{\mathbf{5},-\frac{3}{2}} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ e \\ \nu_e \end{pmatrix}_L ; \quad F_{\mathbf{10},\frac{1}{2}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L d_L^c \nu_L^c \right) ; \quad l_{\mathbf{1},\frac{5}{2}} = e_L^c \quad (19)$$

In particular this results in the $\mathbf{10}$ containing a neutral component with the quantum numbers of ν_L^c . We can break spontaneously the GUT symmetry by using a $\mathbf{10}$ and $\overline{\mathbf{10}}$ of superheavy Higgs where the neutral components provide a large vacuum expectation value, $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle$,

$$H_{\mathbf{10},\frac{1}{2}} = \{Q_H, d_H^c, \nu_H^c\} ; \quad \bar{H}_{\overline{\mathbf{10}},-\frac{1}{2}} = \{Q_{\bar{H}}, d_{\bar{H}}^c, \nu_{\bar{H}}^c\}. \quad (20)$$

The electroweak spontaneous breaking is generated by the Higgs doublets H_2 and \bar{H}_2

$$h_{\mathbf{5},-1} = \{H_2, H_3\} ; \quad \bar{h}_{\mathbf{5},1} = \{\bar{H}_2, \bar{H}_3\} \quad (21)$$

Flipped $SU(5)$ model building has two very nice features which are generally not found in typical unified models: (i) a natural solution to the doublet (H_2)-triplet(H_3) splitting problem of the electroweak Higgs pentaplets h, \bar{h} through the trilinear coupling of the Higgs fields: $H_{\mathbf{10}} \cdot H_{\mathbf{10}} \cdot h_{\mathbf{5}} \rightarrow \langle \nu_H^c \rangle d_H^c H_3$, and (ii) an automatic see-saw mechanism that provide heavy right-handed neutrino mass through the coupling to singlet fields ϕ , $F_{\mathbf{10}} \cdot \bar{H}_{\overline{\mathbf{10}}} \cdot \phi \rightarrow \langle \nu_{\bar{H}}^c \rangle \nu^c \phi$.

The generic superpotential W for a flipped $SU(5)$ model will be of the form :

$$\lambda_1 F F h + \lambda_2 F \bar{f} \bar{h} + \lambda_3 \bar{f} l^c h + \lambda_4 F \bar{H} \phi + \lambda_5 H H h + \lambda_6 \bar{H} \bar{H} \bar{h} + \dots \in W \quad (22)$$

the first three terms provide masses for the quarks and leptons, the fourth is responsible for the heavy right-handed neutrino mass and the last two terms are responsible for the doublet-triplet splitting mechanism [15].

3.2 Model Building

This model is similar to the one given in [11], however the K-theory constraints are satisfied. In this Letter, we present an example with 6+1 stacks of branes. The first stack has the same set of wrapping numbers as in our previous model [11]. We also have a stack with $N^{(\Omega R)} = 8$ filler branes which give rise to a $USp(8)$ gauge group. The gauge symmetry of the (6+1)-stack model, whose wrapping numbers are

presented in Table 3, is $U(5) \times U(1)^5 \times USp(8)$, and the structure parameters of the wrapping space are

$$x_A = 1, \quad x_B = 2, \quad x_C = 10, \quad x_D = 1 \quad (23)$$

The intersection numbers are listed in Table 4, and the resulting spectrum in Table 5.

The singlet (under the $SU(5)$ symmetry) representation e_L^c , now comes from the bi-fundamentals, namely from the intersection (cf) and we choose the $\mathbf{5}$ and $\bar{\mathbf{5}}$ Higgs pentaplets from a non-chiral intersection (ab') . There is less exotic matter in this model, though we still have two copies of $\bar{\mathbf{15}}$ which is unavoidable since we need $\bar{\mathbf{10}}$ in the Higgs sector. Matter charged under both the $SU(5) \times U(1)_X$ and $USp(8)$ gauge symmetries is also present, as is evident from Table 4.

The $U(1)_X$ is

$$U(1)_X = \frac{1}{2} (U(1)_a - 5U(1)_b + 5U(1)_c + 5U(1)_d - 5U(1)_e - 5U(1)_f) \quad (24)$$

while the other anomaly-free and massless combinations $U(1)_Y$ is

$$U(1)_Y = U(1)_b - U(1)_c + U(1)_d - U(1)_e \quad (25)$$

The remaining four global $U(1)$ s from the Green-Schwarz mechanism are given respectively by

$$\begin{aligned} U(1)_1 &= -10U(1)_a + 2U(1)_b + 2U(1)_c - 2U(1)_d - 2U(1)_e - 2U(1)_f \\ U(1)_2 &= -2U(1)_b - 2U(1)_c \\ U(1)_3 &= 8U(1)_b + 8U(1)_c + 4U(1)_d + 4U(1)_e \\ U(1)_4 &= 20U(1)_a + 8U(1)_b + 8U(1)_c + 4U(1)_f \end{aligned} \quad (26)$$

stack	N_a	(n_1, l_1)	(n_2, l_2)	(n_3, l_3)	A	B	C	D	\tilde{A}	\tilde{B}	\tilde{C}	\tilde{D}
a	$N = 10$	$(0, -1)$	$(-1, -1)$	$(-1, -2)$	0	0	-2	-1	2	-1	0	0
b	$N = 2$	$(-1, -1)$	$(-1, 1)$	$(1, 4)$	-1	-4	4	-1	4	1	-1	4
c	$N = 2$	$(-1, -1)$	$(-1, 1)$	$(1, 4)$	-1	-4	4	-1	4	1	-1	4
d	$N = 2$	$(-1, 1)$	$(1, 0)$	$(-1, -2)$	-1	0	-2	0	0	-1	0	2
e	$N = 2$	$(-1, 1)$	$(1, 0)$	$(-1, -2)$	-1	0	-2	0	0	-1	0	2
f	$N = 2$	$(0, 1)$	$(1, 1)$	$(-1, -2)$	0	0	-2	-1	2	-1	0	0
filler	$N^{(\Omega R)} = 8$	$(1, 0)$	$(1, 0)$	$(1, 0)$	-1	0	0	0	0	0	0	0

Table 3: Wrapping numbers and their consistent parameters.

stk	N	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	f1
a	10	2	-2	-4	0(6)	-4	0(6)	0(1)	4	0(1)	4	0(0)	0(8)	2
b	2	32	0	-	-	0(0)	32	4	0(6)	4	0(6)	4	0(6)	4
c	2	32	0	-	-	-	-	4	0(6)	4	0(6)	4	0(6)	4
d	2	2	-2	-	-	-	-	-	-	0(0)	0(8)	0(1)	4	0
e	2	2	-2	-	-	-	-	-	-	-	-	0(1)	4	0
f	2	2	-2	-	-	-	-	-	-	-	-	-	-	2

Table 4: List of intersection numbers. The number in parenthesis indicates the multiplicity of non-chiral pairs.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$2U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_Y$
(10, 1)	3	2	0	0	0	0	0	2	-20	0	0	40	0
$(\bar{5}_a, 1_b)$	3	-1	1	0	0	0	0	-6	12	-2	8	-12	1
$(1_c, \bar{1}_f)$	3	0	0	1	0	0	-1	10	4	-2	8	4	-1
(10, 1)	1	2	0	0	0	0	0	2	-20	0	0	40	0
$(\bar{10}, 1)$	1	-2	0	0	0	0	0	-2	20	0	0	-40	0
$(5_a, 1_b)^*$	1	1	1	0	0	0	0	-4	-8	-2	8	28	1
$(\bar{5}_a, \bar{1}_b)^*$	1	-1	-1	0	0	0	0	4	8	2	-8	-28	-1
$(1_b, 1_c)$	4	0	1	1	0	0	0	0	4	-4	16	16	0
$(\bar{15}, 1)$	2	-2	0	0	0	0	0	-2	20	0	0	-40	0
$(\bar{10}, 1)$	1	-2	0	0	0	0	0	-2	20	0	0	-40	0
$(5_a, 1_b)$	1	-1	1	0	0	0	0	-6	12	-2	8	-12	1
$(\bar{5}_a, 1_c)$	4	-1	0	1	0	0	0	4	12	-2	8	-12	-1
$(5_a, 1_d)$	4	1	0	0	1	0	0	6	-12	0	4	20	1
$(5_a, 1_e)$	4	1	0	0	0	1	0	-4	-12	0	4	20	-1
$(1_b, 1_c)$	28	0	1	1	0	0	0	0	4	-4	16	16	0
$(1_b, \bar{1}_d)$	4	0	1	0	-1	0	0	-10	4	-2	4	8	0
$(1_b, \bar{1}_e)$	4	0	1	0	0	-1	0	0	4	-2	4	8	2
$(1_b, \bar{1}_f)$	4	0	1	0	0	0	-1	0	4	-2	8	4	1
$(1_c, \bar{1}_d)$	4	0	0	1	-1	0	0	0	4	-2	4	8	-2
$(1_c, \bar{1}_e)$	4	0	0	1	0	-1	0	10	4	-2	4	8	0
$(1_c, \bar{1}_f)$	1	0	0	1	0	0	-1	10	4	-2	8	4	-1
$(1_d, 1_f)$	4	0	0	0	1	0	1	0	-4	0	4	4	1
$(1_e, 1_f)$	4	0	0	0	0	1	1	-10	-4	0	4	4	-1
$(\bar{1}, \bar{1})$	2	0	0	0	-2	0	0	-10	4	0	-8	0	-2
$(\bar{1}, \bar{1})$	2	0	0	0	0	-2	0	10	4	0	-8	0	2
$(\bar{1}, \bar{1})$	2	0	0	0	0	0	-2	10	4	0	0	-8	0
$(5_a, 1_b)^*$	5	1	1	0	0	0	0	-4	-8	-2	8	28	1
$(\bar{5}_a, \bar{1}_b)^*$	5	-1	-1	0	0	0	0	4	8	2	-8	-28	-1
Additional non-chiral Matter													
USp(8) Matter													

Table 5: The spectrum of $U(5) \times U(1)^5 \times USp(8)$, or $SU(5) \times U(1)_X \times U(1)_Y \times USp(8)$, with the four global $U(1)$ s from the Green-Schwarz mechanism. The $\star d$ representations stem from vector-like non-chiral pairs.

4 Conclusions

In this Letter we have constructed a particular $N = 1$ supersymmetric three-family model whose gauge symmetry includes $SU(5) \times U(1)_X$, from type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at general angles. The model satisfies all the consistency requirements of string theory and in addition the constraints arising from the K-theory interpretation of D-branes.

The spectrum contains a complete grand unified theory and electroweak Higgs sector, the latter however, in a non-minimal number of copies. In addition, it contains extra matter both in bi-fundamental and vector-like representations as well as two copies of matter in the symmetric representation of $SU(5)$. Chiral matter charged under both the $SU(5) \times U(1)_X$ and $USp(8)$ gauge symmetries is also present, as is evident from Table 4. Furthermore, three adjoint ($N = 1$) chiral multiplets are provided from the aa sector [9].

The global symmetries, that arise after the G-S anomaly cancellation mechanism, forbid some of the Yukawa couplings required for mass generation, for instance terms like FFh . However, by the same token the term HHh is also forbidden. We note that such a term is essential for the doublet-triplet splitting solution mechanism in flipped $SU(5)$. On the other hand, the global $U(1)$ symmetries do not forbid the coupling $Ff\bar{h}$, responsible for the up-type (t, c, u) quark mass terms. Neutrino Yukawas ($F\bar{H}\phi$), and the coupling $\bar{f}l^c h$ are also absent at the trilinear level, due to the global $U(1)$ symmetries. Nevertheless, it should not escape our notice that while these global $U(1)$ symmetries are exact to all orders in perturbation theory, they can be broken explicitly by non-perturbative instanton effects [36], thus providing us with the possibility of recovering the appropriate superpotential couplings. Another interesting approach toward generating these absent Yukawa couplings may entail the introduction of type IIB flux compactifications [10]. This exceeds the scope of our current Letter, but shall be further investigated in an upcoming publication.

An additional important avenue for future research is avoidance of the chiral supermultiplets in the adjoint representation from the aa sector. These multiplets are associated with the moduli space of deformations of special Lagrangian submanifolds and their number is equal to the first Betti number of the 3-cycle that the stack a of D6-branes wrap [37]. Mechanisms for blocking these problematic particles for phenomenology include constructing 3-cycles of the wrapping space with zero first Betti-number (such as S^3 or $\mathbb{R}P^3$ ⁶) or other rigid 3-cycles, as has been recently discussed in [38]. Alternatively, it may be possible to simply give masses to these particles at very high energies. These issues however, are also beyond the scope of this Letter and shall be investigated in a future publication.

⁶Such Lagrangian manifolds are additionally interesting because they are stable and volume minimizing under Hamiltonian deformations [39].

5 Acknowledgments

The work of G. V. K. and D. V. N. is supported by DOE grant DE-FG03-95-Er-40917.

References

- [1] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B480 (1996) 265, hep-th/9606139
- [2] C. Bachas, hep-th/9503030
- [3] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, JHEP 0010 (2000) 006, hep-th/0007024; R. Blumenhagen, V. Braun, B. Körs and D. Lüst, JHEP 0207 (2002) 026, hep-th/0206038; hep-th/0210083; R. Blumenhagen, B. Körs, D. Lüst and T. Ott, Nucl. Phys. B616 (2001) 3, hep-th/0107138
- [4] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, J. Math. Phys. 42 (2001) 3103, hep-th/0011073; G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, JHEP 0102 (2001) 047, hep-ph/0011132
- [5] D. Cremades, L. E. Ibáñez and F. Marchesano, Nucl.Phys. B643 (2002) 93, hep-th/0205074
- [6] D. Bailin, G. V. Kraniotis and A. Love, Phys. Lett. B530 (2002) 202, hep-th/0108131; D. Bailin, G. V. Kraniotis and A. Love, Phys. Lett. B547 (2002) 43, hep-th/0208103; D. Bailin, G. V. Kraniotis and A. Love, Phys. Lett. B553 (2003) 79, hep-th/0210219; D. Bailin, G. V. Kraniotis and A. Love, JHEP 0302:052 (2003), hep-th/0212112
- [7] G. Honecker, JHEP 0201 (2002) 025, hep-th/0201037
- [8] M. Graña, Phys. Rev. D67 (2003) 066006, hep-th/0209200; D. Lüst, S. Reffert and S. Stieberger, Nucl. Phys. B 706 (2005) 3, hep-th/0406092; P.G. Camara, L. E. Ibáñez, A.M. Uranga, Nucl.Phys. B 689 (2004) 195, hep-th/0311241
- [9] M. Cvetič, G. Shiu and A. M. Uranga, Nucl. Phys. B615 (2001) 3, hep-th/0107166; M. Cvetič, P. Langacker and G. Shiu, hep-ph/0205252, hep-th/0206115
- [10] M. Cvetič, T. Li and T. Liu, hep-th/0501041
- [11] C.-M Chen, G. V. Kraniotis, V. E. Mayes, D. V. Nanopoulos and J. W. Walker, Phys. Lett. B611 (2005) 156, hep-th/0501182.
- [12] M. Cvetič, I. Papadimitriou, G. Shiu, Nucl. Phys. B659, 193 (2003), hep-th/0212177
- [13] D. Lüst, Class. Quantum Grav. 21 (2004) S1399, hep-th/0401156 ; R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, hep-th/0502005
- [14] S. M. Barr Phys. Lett. B112 (1982) 219; J. P. Derendinger, J. E. Kim and D. V. Nanopoulos Phys. Lett. B139 (1984), 170

- [15] I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B194 (1987) 231
- [16] I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B 205 (1988) 459; Phys. Lett. B 208 (1988) 209; Phys. Lett. B213 (1988) 562, Addendum; Phys. Lett. B231 (1989) 65.
- [17] J. L. Lopez, D. V. Nanopoulos and K. Yuan, Nucl. Phys. B399 (1993) 654
- [18] P. Goddard and D. Olive, Intern. J. Mod. Phys. A1 (1986) 303
- [19] H. Dreiner, J. L. Lopez, D. V. Nanopoulos and D. R. Reiss, Phys. Lett. B216 (1989) 283; J. R. Ellis, J. L. Lopez and D. V. Nanopoulos, Phys. Lett. B245 (1990) 375
- [20] For a review see: D. V. Nanopoulos, hep-ph/0211128
- [21] J. R. Ellis, D. V. Nanopoulos and J. W. Walker, Phys. Lett. B550 (2002) 99, hep-ph/0205336
- [22] J. R. Ellis, P. Kanti, D. V. Nanopoulos, Nucl. Phys. B647 (2002) 235, hep-th/0206087
- [23] L. E. Ibáñez, R. Rabadán and A. M. Uranga, Nucl. Phys. B542 (1999) 112, hep-th/9808139
- [24] L. E. Ibáñez, F. Marchesano and R. Rabadan, JHEP 0111 (2001) 002, hep-th/0105155
- [25] E. Witten, Phys. Lett. B117 (1982) 324
- [26] R. Minasian and G. Moore, JHEP 9711 (1997) 002
- [27] E. Witten, JHEP 9812 (1998) 019
- [28] A. M. Uranga, Nucl. Phys. B 598 (2001) 225, hep-th/0011048
- [29] F. Marchesano and G. Shiu, JHEP 0411 (2004) 041, hep-th/0409132
- [30] M. F. Atiyah, Quart. J. Math. Oxford (2) 17 (1966), 367
- [31] S. Gukov, Commun.Math.Phys. 210 (2000) 621
- [32] A. Sen, JHEP 06 (1998) 007; JHEP 08 (1998) 010
- [33] M. Berkooz and R.G. Leigh, Nucl. Phys. B483 (1997) 187; S. Forste, G. Honecker and R. Schreyer, Nucl. Phys. B593 (2001) 127
- [34] M. Cvetič, P. Langacker and G. Shiu, Phys. Rev. D66, 066004 (2002)

- [35] M. Cvetič, P. Langacker, T.-J. Li and T. Liu, Nucl. Phys. B709 (2005) 241, hep-th/0407178
- [36] S. Kachru, S. Katz, A. Lawrence and J. McGreevy, Phys. Rev. D 62, (2000) 026001; A. M. Uranga, Class. Quant. Grav. 20 (2003) S373, hep-th/0301032
- [37] R. C. McLean, Comm. Anal. Geom. 6 (1998), 705
- [38] R. Blumenhagen, M. Cvetič, F. Marchesano and G. Shiu, JHEP 0503 (2005) 050, hep-th/0502095; E. Dudas and C. Timirgaziu, Nucl. Phys. B716 (2005) 65, hep-th/0502085.
- [39] Y-G Oh, Math.Z. 212 (1993) 175