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FAST TRACK PAPER

A construct of internal multiples from surface data only: the concept of virtual seismic events

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SUMMARY

We here describe one way of constructing internal multiples from surface seismic data only. The key feature of our construct of internal multiples is the introduction of the concept of virtual seismic events. Virtual events here are events, which are not directly recorded in standard seismic data acquisition, but their existence allows us to construct internal multiples with scattering points at the sea surface; the standard construct of internal multiples does not include any scattering points at the sea surface.

The mathematical and computational operations invoked in our construction of virtual events and internal multiples are similar to those encountered in the construction of free-surface multiples based on the Kirchhoff or Born scattering theory. For instance, our construct operates on one temporal frequency at a time, just like free-surface demultiple algorithms; other internal multiple constructs tend to require all frequencies for the computation of an internal multiple at a given frequency. It does not require any knowledge of the subsurface nor an explicit knowledge of specific interfaces that are responsible for the generation of internal multiples in seismic data. However, our construct requires that the data be divided into two, three or four windows to avoid generating primaries. This segmentation of the data also allows us to select a range of periods of internal multiples that one wishes to construct because, in the context of the attenuation of internal multiples, it is important to avoid generating short-period internal multiples that may constructively average to form primaries at the seismic scale.

Key words: Born scattering, internal multiples, Kirchhoff scattering, scattering diagrams, virtual seismic events.

1 INTRODUCTION

The present seismic imaging techniques require, for their applications, that seismic data contain only primaries. To fulfil this requirement, we need to attenuate free surface and internal multiples from our seismic data prior to imaging them. So far, most efforts by the petroleum seismology community have been limited to the attenuation of free-surface multiples. The few examples of techniques for attenuating internal multiples will be discussed later in this section. Our objective here is to present a construct of internal multiples, which can be computed in the same mode as free-surface multiples at a similar cost in CPU time and computational storage.

Fig. 1 shows an example of primaries, free-surface multiples and internal multiples. We can see that internal multiples are seismic events with no bounce at the free surface but with a bounce between two interfaces other than the free surface. So the fundamental difference between internal multiples and free-surface multiples, from the scattering theory point of view, is that the path of a free-surface multiple contains at least one scattering point at the sea surface, whereas the path of an internal multiple does not contain any scattering point at the sea surface. This fundamental difference between internal multiples and free-surface multiples can be cast into the following four points:

(i) The interface generating free-surface multiples, which is the air–water interface (a free surface), is well defined and unique in its kind in our geological models. On the contrary, the internal multiples can be generated at any interface in the subsurface. Therefore, possible interfaces generating internal multiples are in theory almost endless. Moreover, the notion of interface in the context of internal multiples is totally different from that of the sea-surface interface in the context free-surface multiples. Rock formations in the subsurface result in the breakdown of pre-existing rocks, leading to fractures and bedding in rock formations, and thus making it quite difficult to clearly define internal-multiple-generating interfaces. Even potential internal multiple generators like the top and bottom of basalt might sometimes be difficult to define for this reason. Obviously, the seafloor (i.e. the water–solid interface in the case

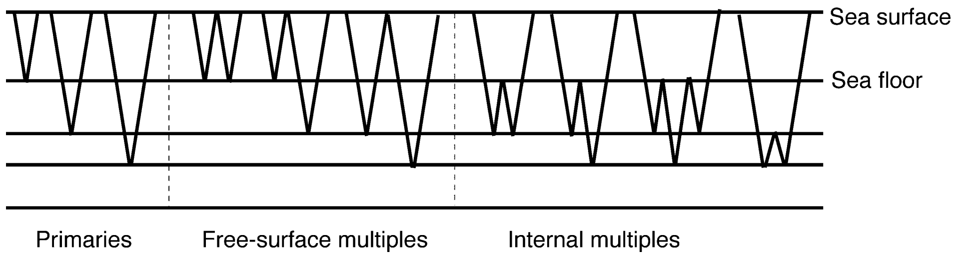

Figure 1. Examples of primaries, free-surface multiples and internal multiples.

of the hard seafloor) is an exception; it is a well-defined internal-multiple-generating interface.

(ii) We know that the smallest period of free-surface multiples is the two-way traveltime in the water column. This period is a fundamental feature, which allows us to distinguish between primaries and free-surface multiples. For practical purposes, we require this period to be longer than the duration of the source signature. In the case of internal multiples, the period of multiples can be very short (less than the typical seismic temporal sampling interval, which is 4 ms) or very long, because we have heterogeneities at almost all scales in the subsurface, as well logs and core samples have shown. Actually, primaries themselves are generally an average of short-period internal multiples. In other words, the shortest period of internal multiples is not as clearly defined as that of free-surface multiples, and it is a parameter that we need to control to avoid modifying our primary signals.

(iii) In towed-streamer experiments, for instance, sensors are located near the sea surface and, therefore, can be extrapolated to the sea surface to coincide with the sea-surface scattering points of free-surface multiples. In essence, that is what we do in the construction of free-surface multiples based on the Kirchhoff theory (e.g. Ikelle *et al.* 2003). Unfortunately, all the scattering points in the context of internal multiples, as depicted in the classical scattering diagrams in Fig. 1, are located in the subsurface. The extrapolations of these scattering points to the surface already required the knowledge of the model of the subsurface. So one of the fundamental challenges in the attenuation of internal multiples that we will be addressing in this paper is how to attenuate them without any knowledge of the subsurface. We will introduce the concept of virtual events, which will allow us to redraw the scattering diagrams of internal multiples with scattering points at the free surface.

Our goal in this paper is to formulate a construct of internal multiples, which does not require any knowledge of the subsurface. We will also seek to include in this formulation ways of selecting the range of periods of internal multiples that one might wish to construct. Before we start addressing this goal, let us put our work in this paper in some context with respect to previous contributions. Efforts to attenuate internal multiples (sometimes called interbed multiples) can be traced to the 1950s (e.g. Hansen 1948; Robinson 1957; Schneider *et al.* 1965). The predictive deconvolution was then the method of choice for attenuating internal multiples. This method is essentially based on the assumption that multiple events are periodic and primary events are not. This assumption holds quite well in zero-offset data, stack data and moveout-corrected CMP data for relatively flat models of the subsurface. That is why the predictive deconvolution was very successful in the 1960s and 1970s when seismic processing was generally limited to stacked data from relatively flat models of the subsurface. Unfortunately, this assumption breaks down when the medium is multidimensional.

The more recent efforts are those of Berkhout & Verschuur (1997) and Weglein *et al.* (1997). Both of these works are multidimensional and do not make any assumption about the periodicity of multiples. Berkhout and Verschuur's approach, which is generally known as the feedback method, attenuates internal multiples related to a specific interface that is responsible for the generation of internal multiples. This method requires the selection of the multiple generating reflector or a velocity model of the subsurface. When the internal-multiple-generating reflector is specified, the feedback method becomes computationally equivalent to free-surface multiple attenuation. In the Weglein *et al.* approach,

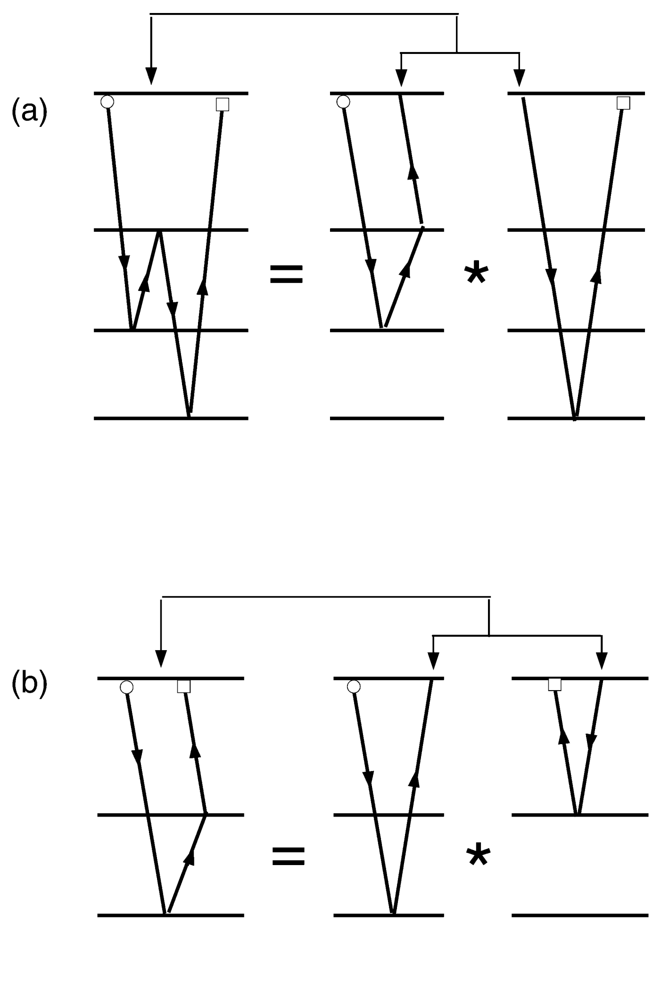

Figure 2. (a) A combination of virtual seismic data with standard towed-streamer data produces internal multiples. (b) An illustration of the construction of virtual seismic data.

internal-multiple-generating interfaces are not explicitly specified and can attenuate all internal multiples of a given order at once through an inverse scattering series. However, their solution is quite expensive compared to the feedback method or free-surface multiple attenuation. In fact, unlike the free-surface multiple attenuation method, the Weglein *et al.* method requires all frequencies for the computation of an internal multiple at a given frequency. Hence it does not contain the key computational feature of the free-surface multiple methods based on the Kirchhoff or Born scattering theory.

Our approach here differs from those of Berkhout & Verschuur (1997) and Weglein *et al.* (1997) because it leads to a construct of internal multiples, which is both computationally similar to the construct of free-surface multiples and does not require any specific interpretative interface or velocity model. However, our construct requires that we divide data into segments to control the range of periods of internal multiples that one might wish to generate and to avoid generating primaries, as we will see later.

2 CONSTRUCTION OF VIRTUAL EVENTS AND INTERNAL MULTIPLES

Fig. 2 illustrates one possible construct of internal multiples. First of all, let us remark that scattering diagrams are drawn with arrows, which is unusual. The scattering diagrams are generally drawn without arrows because it is generally assumed in the petroleum seismology community that the ray paths in these diagrams track the direction of wave propagation. We have explicitly included the arrows in the scattering diagrams in our construct of internal multiples because our diagrams sometimes do not follow the path of the wave propagation.

The construct of internal multiples has two steps. The first consists of constructing a virtual event (Fig. 2b). We call this event virtual because it comes into existence to facilitate merely our physical construct of internal multiples. More specifically, it allows us to redraw the scattering diagram of internal multiples with a scattering point at the free-surface (Fig. 2a). Let me emphasize that virtual events are not directly recorded in seismic data. Moreover, virtual events can be temporarily allowed to violate Snell's laws and the other laws governing the energy partition at an interface. Notice that the construction of this virtual event requires a time inversion of one of the two fields that we are combining for this construct. In other words, the scattering operation, which is generally carried out as a convolution in most free-surface multiples, must be replaced by a correlation type in order to take into account this time inversion. However, the combination of the virtual event with normal seismic events is carried out with standard convolution operations.

3 A NUMERICAL ILLUSTRATION OF THE CONSTRUCTION OF VIRTUAL EVENTS AND OF INTERNAL MULTIPLES FOR 1-D MEDIA

For a 1-D medium and when working in the f–k domain, the mathematical expressions of our construct of virtual events and internal multiples can be described as follows. Let $D_A(k, \omega)$ be the f–k version of a CMP gather $d_a(x, t)$. We partition our data into $d_0(x, t)$ and $d_0'(x, t)$. We will later discuss the reason for this partition and how it can be performed. The field of virtual events, which we will denote $D_V(k, \omega)$ in the f–k domain, can be obtained as follows:

$$D_V(k, \omega) = D_0^*(k, \omega) D_A(k, \omega). \quad (1)$$

The asterisk denotes the complex conjugate.

Fig. 3 shows the data used to illustrate our construct of internal multiples. We have divided these data into two parts: $d_0(x, t)$ and $d_0'(x, t)$. The data $d_0(x, t)$ consist of three primaries. The corresponding field of virtual events is shown in Fig. 4, that is, the inverse Fourier transform of $D_V(k, \omega)$ with respect to t and x. We will denote this field by $d_V(x, t)$. The field $d_V(x, t)$ contains two

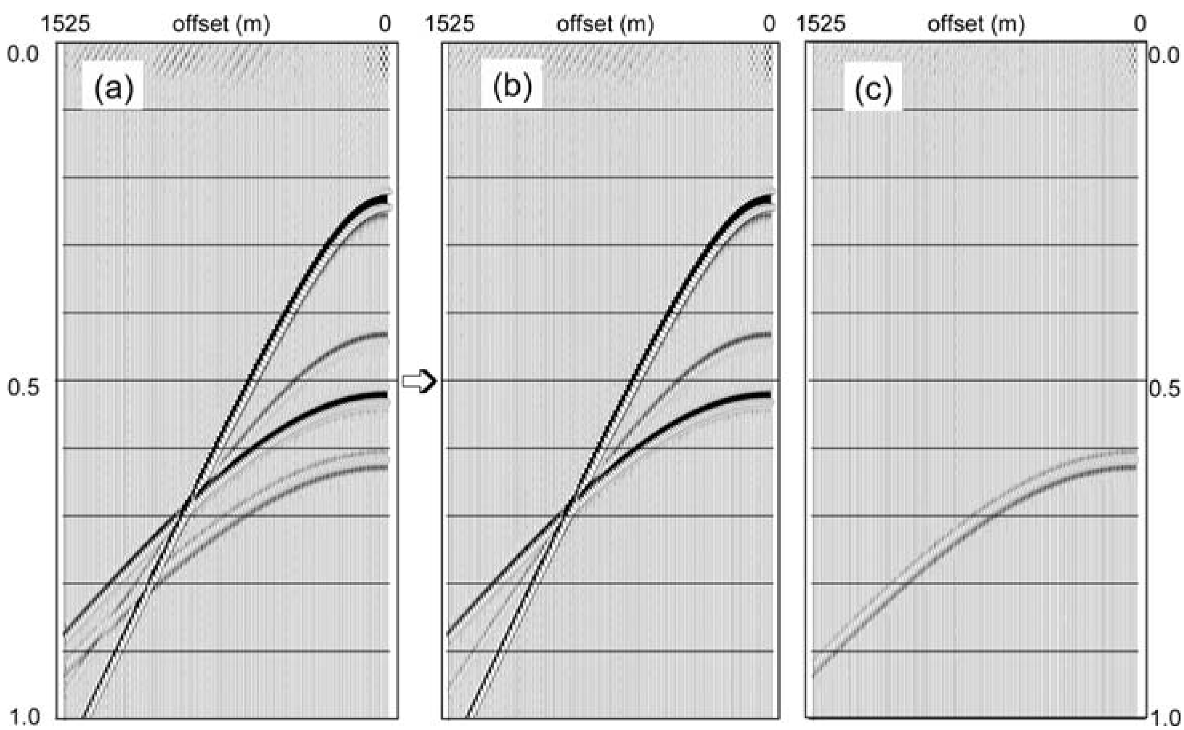

Figure 3. (a) 1-D synthetic data consisting of four primaries. We have divided these data into two parts: $d_0(x, t)$ and $d_0'(x, t)$. (b) is $d_0(x, t)$ and (c) $d_0'(x, t)$.

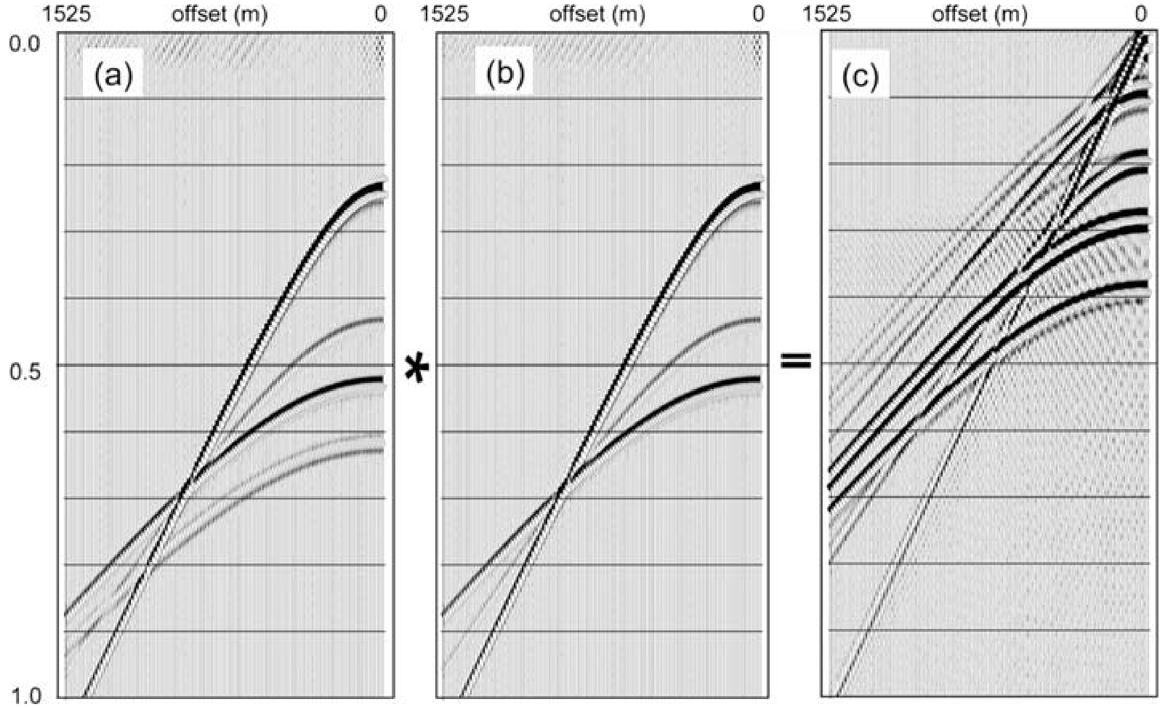

Figure 4. An illustration of the construction of virtual seismic data as a multidimensional correlation of the actual data with $d_0(x, t)$. (a) is the actual data in Fig. 3(a), (b) is $d_0(x, t)$ and (c) is the field of virtual seismic events.

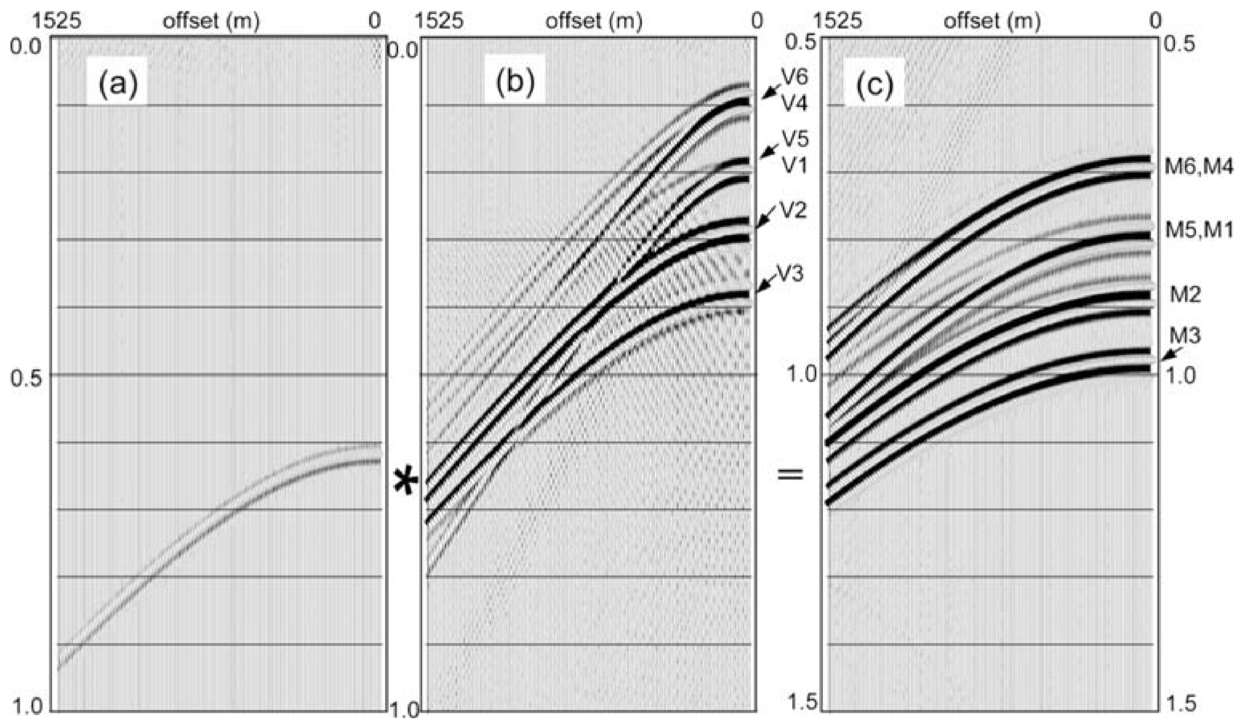

Figure 5. An illustration of the construction of internal multiples as a multidimensional convolution of the field of virtual events (without the apparent direct-wave arrivals) with the data $d_0'(x, t)$. (a) is $d_0'(x, t)$, (b) is $d_v'(x, t)$ and (c) is the field of predicted internal multiples $d_I(x, t)$. The nomenclature of the events in (b) and (c) is given in Fig. 6. Notice also that the field of predicted internal multiples is displayed for a time window between 0.5 s and 1.5 s, whereas the data $d_0'(x, t)$ and the field of virtual events $d_v'(x, t)$ are displayed for the time window between 0.0 s and 1.0 s.

types of events: the autocorrelation of each of the three primaries and the crosscorrelations between the primaries. In the x–t domain, the events corresponding to autocorrelation are all grouped into the apparent direct wave in Fig. 4(c), which can easily be eliminated. The events corresponding to the crosscorrelation between the primaries constitute the virtual events that we are interested in generating. So Fig. 5(b) represents the desired field of virtual events, which we will denote as $d_V'(x, t)$. Notice that we have retained only the positive time in our computation of $d'_V(x, t)$ to avoid creating non-virtual events.

Let us now consider another part of the data located below $d_0(x, t)$, which we have denoted $d'_0(x, t)$. If $D'_0(k, \omega)$ and $D'_V(k, \omega)$ are the f–k versions of $d'_0(x, t)$ and $d'_V(x, t)$, respectively, we can obtain the field of internal multiples as follows:

$$D_I(k, \omega) = D'_0(k, \omega) D'_V(k, \omega). \tag{2}$$

Fig. 5(c) shows that we have effectively predicted all internal multiples whose first or last bounces are below the second primaries. Note that in eq. (1) as well as in eq. (2), the computation of virtual events and internal multiples can be carried on one temporal frequency, say ω_1, at a time. The computations of these events for ω_1 are totally independent of the other frequencies. Thus they can be carried out in parallel over various frequencies. It is nice to see that our computation of virtual events and internal multiples preserves this key computational features of free-surface demultiple algorithms.

The results in Figs 3–5 are also captured in Fig. 6 by using the scattering diagrams. So the reason why we decided to divide the data in $d_0(x, t)$ and $d'_0(x, t)$ is now easy to understand. We need to partition our data to avoid generating primaries; for instance, if we use $d_0(x, t)$ instead of $d'_0(x, t)$ in the computation of internal multiples in eq. (2), by drawing a diagram similar to the one in Fig. 6(b), we can see that we will end up reconstructing primaries in addition to constructing internal multiples. This division need not be a very careful operation; we simply have to avoid generating primaries. Moreover, there are very few internal-multiple-generating reflectors (e.g. seafloor, top of salt and top of basalt). Therefore, this partition may be required only once or twice per data set in order to remove all significant internal multiples. We will further discuss

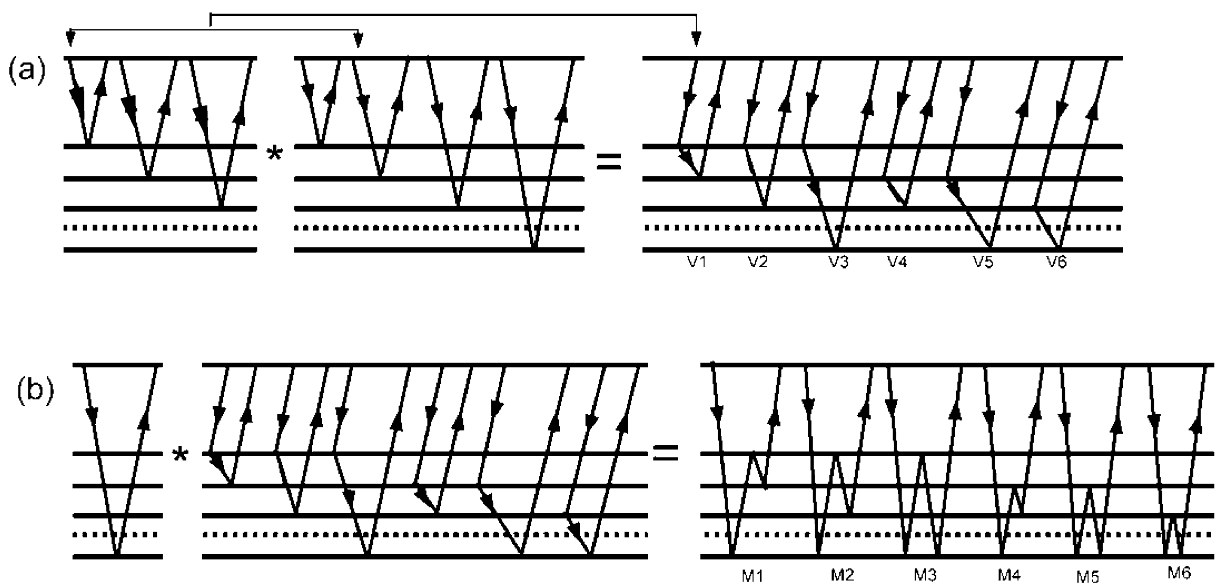

Figure 6. (a) Scattering diagrams of virtual events in Fig. 5(b). (b) Scattering diagrams of internal multiples in Fig. 5(c).

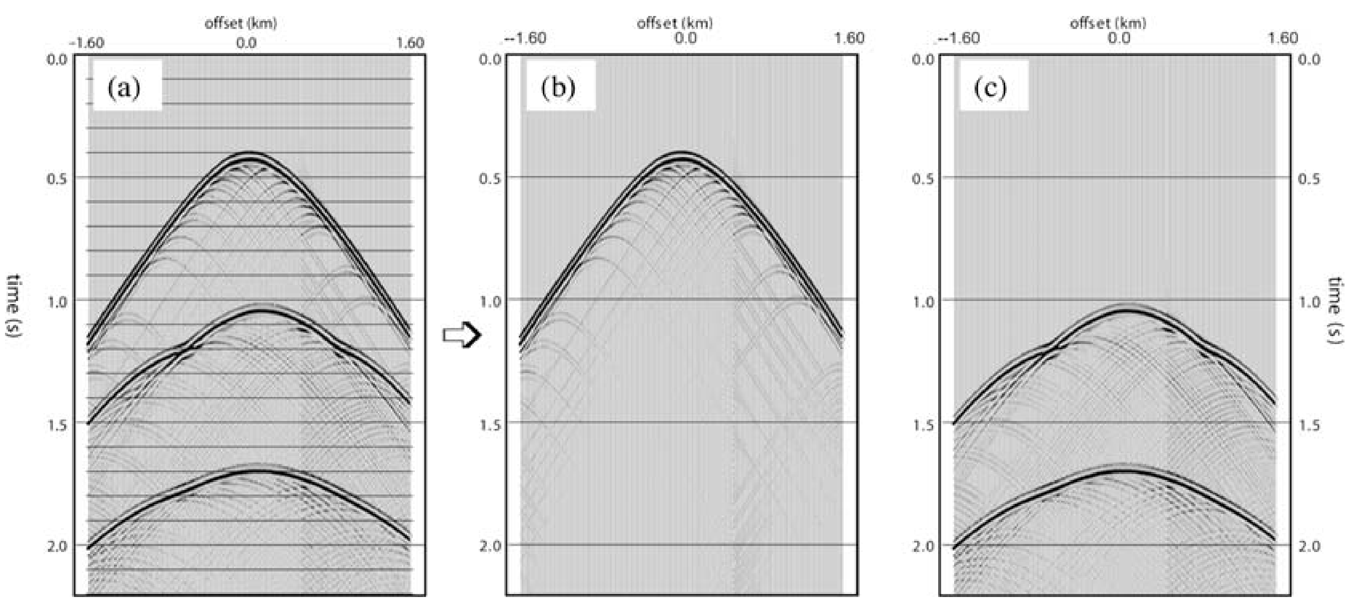

Figure 7. (a) One shot gather of 2-D synthetic data consisting of three primaries. We have divided these data into two parts: d'_0 and d_0. (b) is d_0 and (c) d'_0.

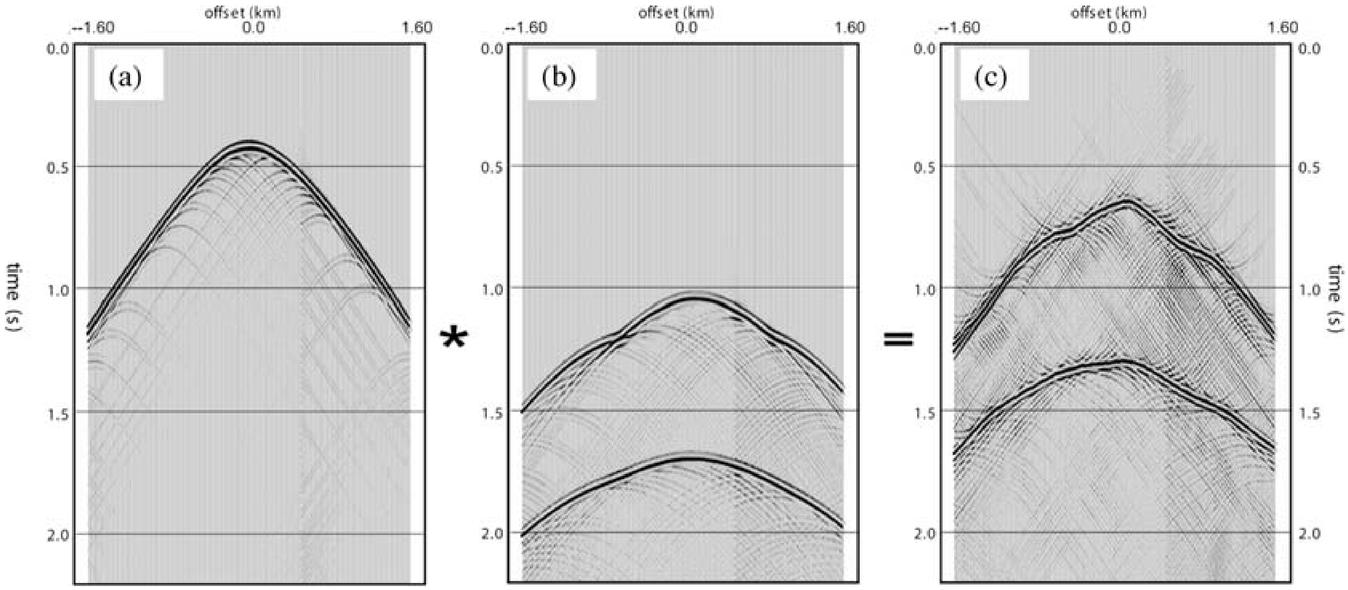

Figure 8. An illustration of the construction of virtual seismic data as a multidimensional correlation of the d_0 with d'_0. (a) is the data in Fig. 7(b), (b) is the data in Fig. 7(c) and (c) is the field of virtual seismic events.

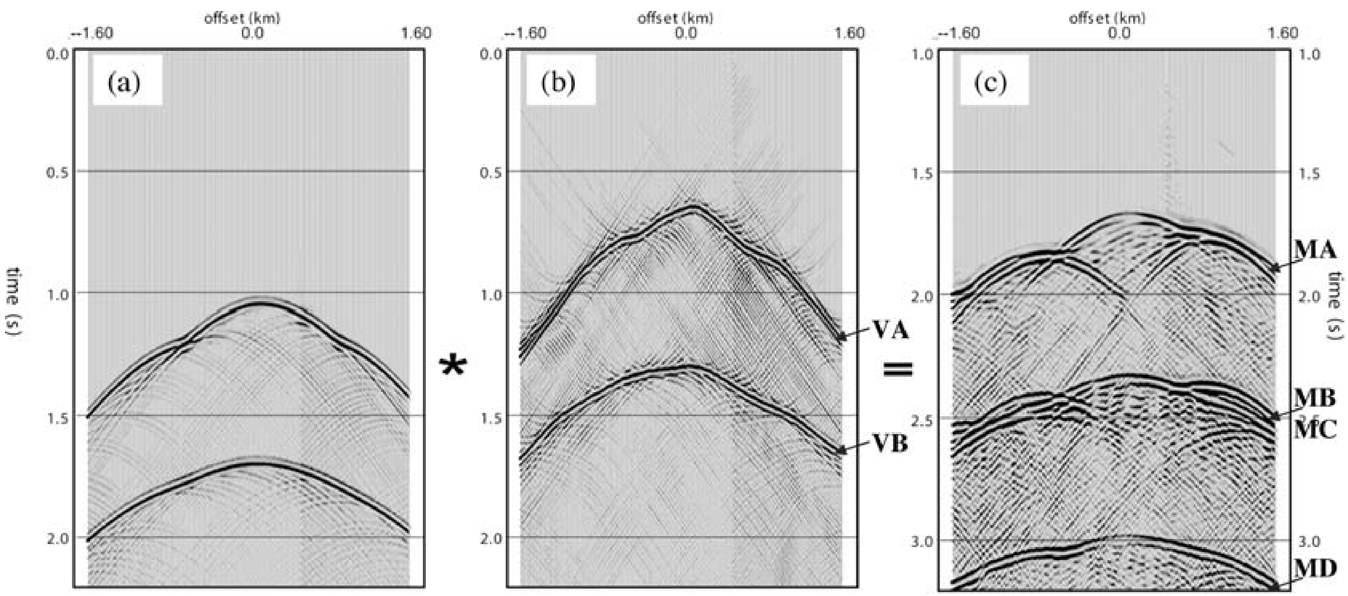

Figure 9. An illustration of the construction of internal multiples as a multidimensional convolution of the field of virtual events with the data d'_0 in Fig. 7(c). (a) is the data in Fig. 7(c), (b) is the field of virtual events, and (c) is the field of predicted internal multiples. The nomenclature of the events in (b) and (c) is given in Fig. 10. Notice also that the field of predicted internal multiples is displayed for a time window between 1.0 s and 3.2 s, whereas the data d_0 and the field of virtual events d_V are displayed for the time window between 0.0 s and 2.2 s.

the issues related to the division of the data in the next two sections of this paper.

The particular strategy in how the data must be partitioned for the construction of virtual events and internal multiples will depend on processing objectives like imaging of multiples, imaging of virtual events or attenuation of internal multiples. We cannot describe a specific strategy for each of these potential processing objectives in a single paper. We will limit ourselves in the remainder of this paper to answering the following four questions that may help in developing strategies for specific processing objectives:

(i) Is the construct of virtual events and internal multiples that we have just described valid for multidimensional media?

(ii) Can we avoid generating the apparent direct wave in Fig. 4?

(iii) How can we generate all internal multiples associated with a particular data set?

(iv) How can we avoid generating very short-period internal multiples, which may sometimes be part of our primaries at the seismic scale?

In the next section we will focus on the first two questions. The following section will address questions (iii) and (iv). It will also

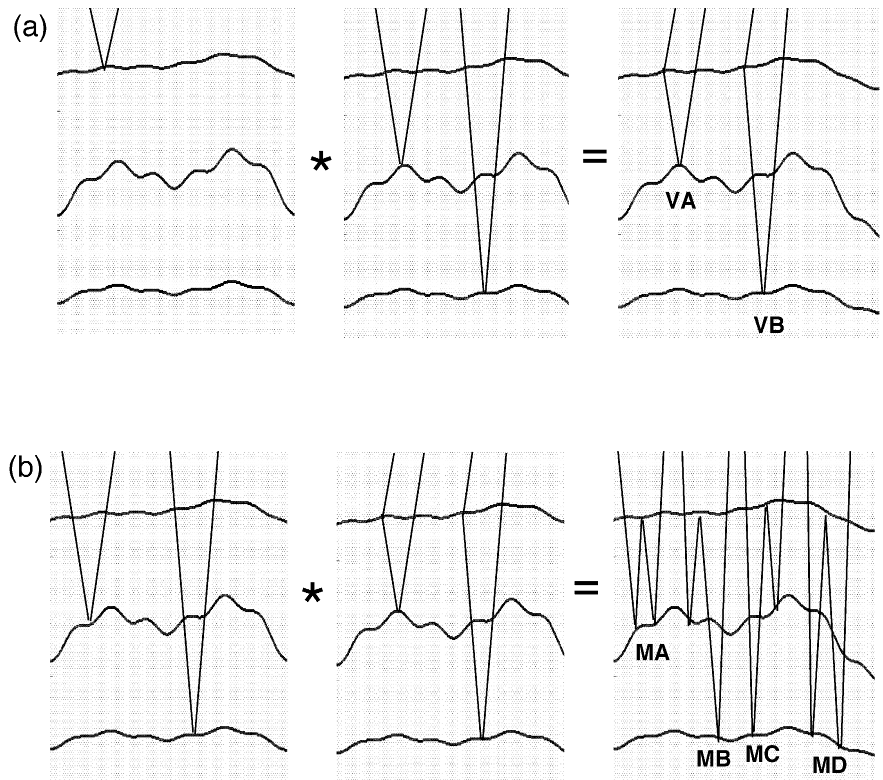

Figure 10. (a) Scattering diagrams of virtual events in Fig. 9(b). (b) Scattering diagrams of internal multiples in Fig. 9(c).

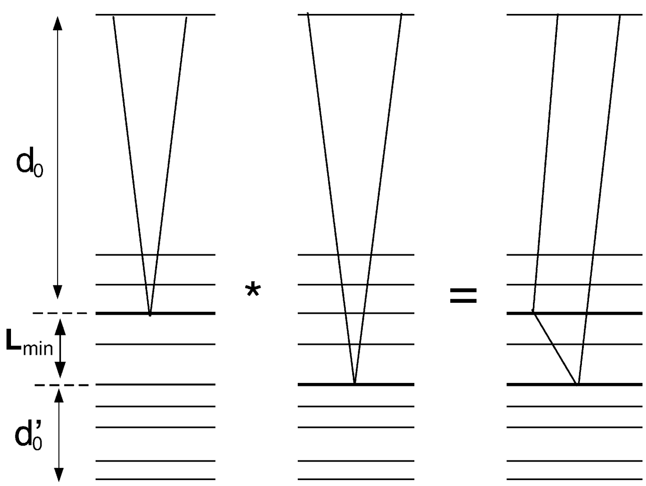

Figure 11. An illustration of the gap between d_0 and d_0', which allows us to control the smallest period of internal multiples that we can generate.

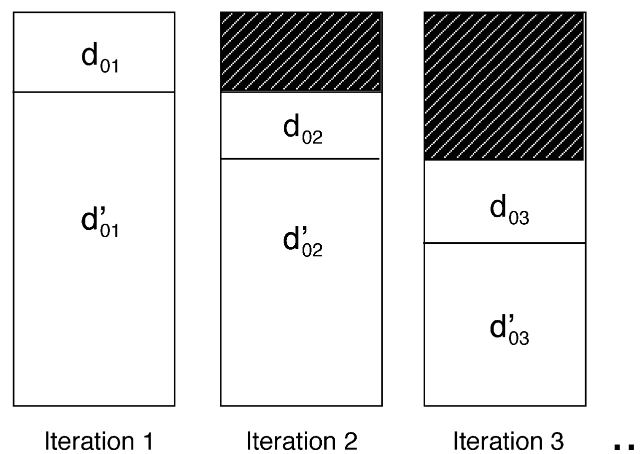

Figure 12. An illustration of how we can progressively move the bottom internal-multiple generator (BIMG) to generate and attenuate several classes of internal multiples. This process is carried out iteratively. In the first iteration, we predict and attenuate all the internal multiples, which have at least one bounce above the BIMG1 and at least one below the BIMG1. Using the output of this iteration as our new data, we then move the BIMG deeper to a new position: the BIMG2. We partition our new data in d_{02} and d_{02}', as depicted in Fig. 12. Notice that d_{02} does not include data above the BIMG1. Then we predict and attenuate all the internal multiples, which have at least one bounce located between the BIMG1 and the BIMG2, one bounce below the BIMG2 and so on.

discuss some minor computational issues related to the segmentation of the data.

4 A NUMERICAL ILLUSTRATION OF THE CONSTRUCTION OF VIRTUAL EVENTS AND OF INTERNAL MULTIPLES FOR 2-D MEDIA

For a 2-D medium and when working in the $\omega - x$ domain, the mathematical expressions of our construct of virtual events and internal multiples can be described as follows. Let $d_A(x_s, \omega, x_r)$ be the seismic data in which x_s and x_r represent the shot points and receiver points, respectively. We partition our data into $d_0(x_s, \omega, x_r)$ and $d_0'(x_s, \omega, x_r)$. The field of virtual events, which we will denote $d_V(x_s, \omega, x_r)$ in the $\omega - x$ domain, can be obtained as follows:

$$d_V(x_s, \omega, x_r) = \int d_0^*(x_s, \omega, x)\, d_A(x, \omega, x_r)\, dx. \quad (3)$$

Again, the asterisk denotes the complex conjugate. If we want to avoid generating the apparent direct wave depicted in Fig. 4(c), we can alternatively compute the field of virtual events as follows:

$$d_V(x_s, \omega, x_r) = \int d_0^*(x_s, \omega, x)\, d_0'(x, \omega, x_r)\, dx. \qquad (4)$$

In other words, we have replaced $d_A(x_s, \omega, x_r)$ in eq. (3) with $d_0'(x_s, \omega, x_r)$. By doing so, we eliminate all the autocorrelations of primaries, and we are left only with the crosscorrelations between the primaries that constitute the virtual events that we are interested in generating.

The field of virtual events produced by eq. (3) is obviously different from the one produced by eq. (4). However, as we will discuss in the next section, either formula can be used recursively to produce all desired virtual events and subsequently all desired internal multiples. The attractiveness of the formula in eq. (4) over the one in eq. (3) is that it does not generate an apparent direct wave.

Because we illustrated the formula in eq. (3) in its 1-D form in the previous section, let us use our 2-D example to illustrate the formula in eq. (4). Fig. 7 shows one of the shot gathers of the data used to illustrate our construct of internal multiples. We have divided these data into two parts: $d_0(x_s, t, x_r)$ and $d_0'(x_s, t, x_r)$. The corresponding field of virtual events is shown in Fig. 8. Notice that, contrary to Fig. 4, the field $d_V(x_s, t, x_r)$ in Fig. 8 does not contain an apparent direct wave. Notice also the strange look of some virtual events in Fig. 8, especially the diffraction events, which seem to have been totally time inverted. One way of explaining this strange look is to approach the operation in eq. (4) as a kind of a migration operation without stack in which the migration operator is computed with an incorrect velocity model. Let $d_0(x_s, \omega, x)$ be the migration operator and $d_0'(x, \omega, x_r)$ be the data in this hypothetical migration. The velocity model associated with the data $d_0(x_s, \omega, x)$ is obviously different from the one associated with the data $d_0'(x, \omega, x_r)$. Therefore, the crosscorrelation operation in eq. (4), which performs the migration operation in this case, has overcorrected the data, hence producing the strange-looking image in Fig. 8.

Let us now turn to the construction of internal multiples. We can obtain the field of internal multiples as follows:

$$d_I(x_s, \omega, x_r) = \int d_0'(x_s, \omega, x)\, d_V(x, \omega, x_r)\, dx. \qquad (5)$$

Fig. 9 shows that after convolution (5), we recover the normal shape of data. As expected, we have effectively predicted all four internal multiples in this case. The scattering diagrams in Fig. 10 describe these four multiples.

5 DISCUSSION

5.1 Controlling the shortest period of internal multiples to be predicted

Well logs and core samples have shown that there are heterogeneities in the subsurface at almost all scales. Therefore, the period of internal multiples can be quite small. Actually a number of primaries in the seismic data are generally just averages of short-period internal multiples. So it is important to develop ways of selecting the range of periods of internal multiples that one may wish to generate. In particular, we would like to make sure that we are generating internal multiples that are part of primary events at the seismic scale. One way of selecting the shortest periods of internal multiples that we can generate using the algorithm in eqs (4) and (5) is to introduce a gap between $d_0(x_s, \omega, x)$ and $d_0'(x, \omega, x_r)$, as described in Fig. 11. As illustrated in this figure, the length of such a gap defines the shortest period of internal multiples that we can generate.

5.2 Attenuation of internal multiples

As we have seen in Figs 6 and 10, by segmenting the data we cannot predict all the possible internal multiples in a given data set by using eqs (4) and (5). However, we sometimes need to construct more than the multiples in Figs 6 and 10, especially when the processing objective is to attenuate internal multiples. An iterative approach along the lines of the one described in Fig. 12 can be used to generate and attenuate all significant internal multiples. The basic idea is to continuously move the boundary between $d_0(x_s, \omega, x)$ and $d_0'(x, \omega, x_r)$ at each iteration. We will call this boundary the bottom internal-multiple generator (BIMG).

Let us expand further on the particular scheme described in Figs 12 and 13. At the first iteration, we predict and attenuate all the internal multiples which have at least one bounce above the first BIMG (which we denote BIMG1) and at least one below the BIMG1; in Fig. 12 the data above the BIMG1 are denoted $d_{01}(x_s, \omega, x)$, and the data below the BIMG1 are denoted $d_{01}'(x, \omega, x_r)$. The output of this iteration is used as the data for the next iteration. In the second iteration, we move the BIMG deeper, to a new position, say, the BIMG2, and define new fields $d_{02}(x_s, \omega, x)$ and $d_{02}'(x, \omega, x_r)$, as depicted in Fig. 12. Notice that $d_{02}(x_s, \omega, x)$ does not include data above the BIMG1. Then we predict and attenuate all the internal multiples, which have at least one bounce between the BIMG1 and the BMG2, one bounce below the BIMG2 and so on. The scattering diagrams in Fig. 13 illustrate the first two iterations of this process. This figure is based on eqs (4) and (5). Similar diagrams can be produced by using eqs (3) and (5). In practice, there are very few internal-multiple-generating reflectors; the classic ones are the seafloor, the top and bottom of salt and the top and bottom of basalt. Therefore, only two or three iterations may be required per data set in order to attenuate all significant internal multiples (see also Ikelle 2003, 2004).

As illustrated in various examples in this paper, the kinematic of predicted multiples is an obvious correction. To ensure an effective removal of predicted internal multiples from the data, it is important to make the amplitude of virtual events consistent with those of the data by replacing, for example, in eq. (3), d_0^* with d_0^{-1}. The field d_0^{-1} is defined as follows:

$$\int_{-\infty}^{\infty} dx'\, d_0^{-1}(x_s, \omega, x')\, d_0(x', \omega, x_r) = \delta(x_s - x_r). \qquad (6)$$

The numerical aspects of the computations of d_0^{-1} are discussed in Ikelle (2005). For example, we described in that paper a way of computing d_0^{-1} through a process of migration and demigration without the need to inverse any matrix.

5.3 Separation of seismic data at the BIMG location

Notice that the separation of seismic data at the BIMG location does not require any special smoothing technique, as we are going to end up convolving the truncated data with the field of virtual events. This convolution allows us to smooth any rough edges that the separation of data at the BIMG location might have created.

For long-offset data, some events may have their trajectories crossing the BIMG. In other words, one portion of an event may be located above the BIMG, and the other portion of the same event may be located below the BIMG. This separation is not a problem;

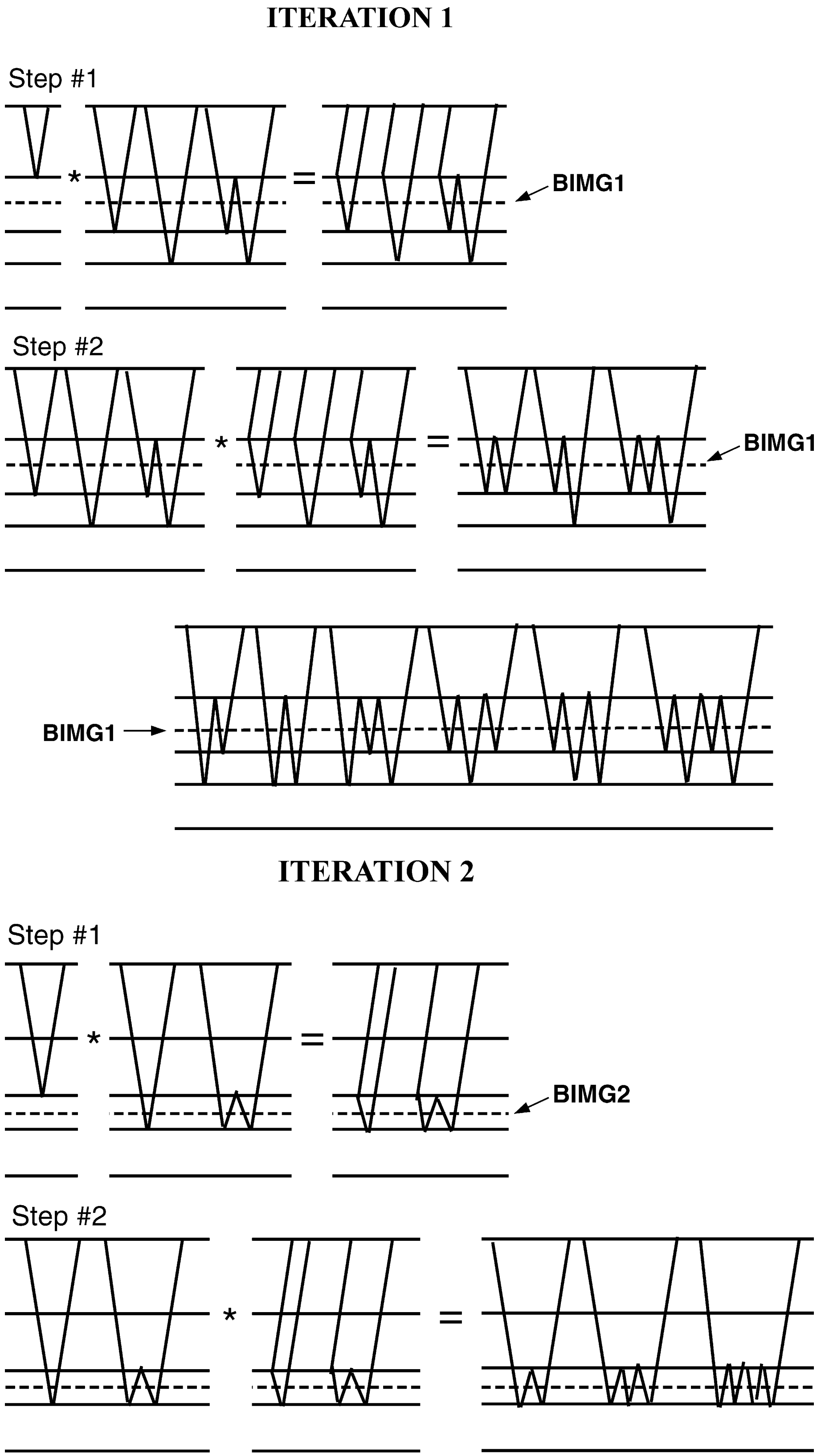

Figure 13. An illustration with scattering diagrams of the first two iterations of the iterative process described in Fig. 12. The first iteration predicts and attenuates all the internal multiples, which have at least one bounce above the BIMG1 and at least one below the BIMG1. The output of this iteration is used as the data for the second iteration. So in the second iteration, we predict and attenuate all the internal multiples, which have at least one bounce located between the BIMG1 and the BIMG2, one bounce below the BIMG2 and so on.

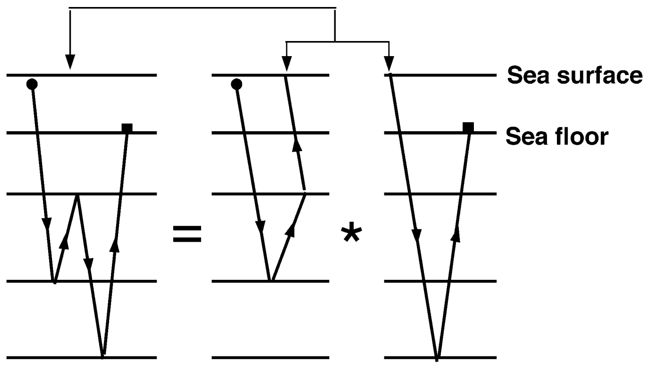

Figure 14. An example of construction of OBS internal multiples as a combination of virtual events with OBS data. Notice that this combination is independent of OBS receivers, therefore, the OBS internal multiple of each component of OBS data is constructed separately. Moreover, this construct is not affected by any potential poor coupling of geophones that may occur during OBS acquisition.

the portion of the event located above the BIMG will be used to predict internal multiples in one iteration, and the second portion of the event located below the BIMG will be used in the next iteration to predict the second set of internal multiples associated with the event located below the BIMG. In other words, the fact that some complex events may not fall completely above the BIMG or completely below the BIMG is another reason why the iterative process described in Figs 12 and 13 is necessary. Watts and Ikelle (2005) show examples of this point for complex models containing salt bodies.

In this paper, we have made our separation directly using the modelling to focus the discussion entirely on the construction of virtual events and of internal multiples.

5.4 Internal multiples in OBS

The concept of virtual events introduced here can also be used to construct OBS internal multiples. As illustrated in Fig. 14, OBS internal multiples can be constructed as a multidimensional convolution of the virtual events with OBS data. Note that just as in the case of the construction of the OBS free-surface multiples (see Ikelle 1999a,b), in which we combine towed-streamer data and OBS data to construct multiples. We also need to combine virtual events constructed from towed-streamer data with OBS data in order to construct internal multiples. Note also that just as in the case of OBS free-surface multiples, the OBS internal multiple of each component of OBS data is constructed separately because the scattering integral is carried out OBS shot points instead of receiver points. Moreover, this construct is not affected by any potential poor coupling of geophones that may occur during OBS acquisition.

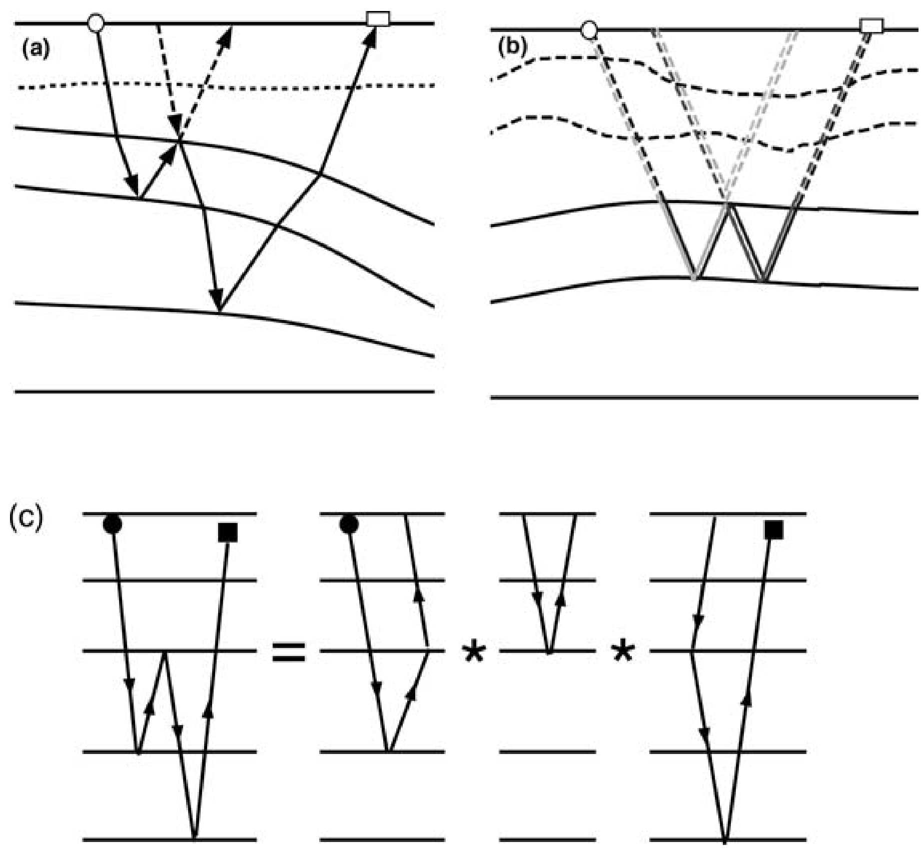

Figure 15. (a) A construction of internal multiples based on the downgoing continuation operation (adapted from Jakubowicz 1998). (b) A construction of internal multiples as a combination of two primaries minus a third primary (adapted from Landa *et al.* 1999). (c) One way of constructing internal multiples with virtual events that can be related to the construct in (a) and (b). Note that the downgoing continuation operations in (c) are performed by the primaries sandwiched between the two virtual events.

5.5 Linking the concept of virtual events to other constructs of internal multiples

During the review process for this paper, an internal-multiple constructs presented at a conference (Jakubowicz 1998) and another published by Landa *et al.* (1999) were drawn to our attention. Figs 15(a) and (b) show the diagram of Jakubowicz and Landa *et al.*, respectively. Fig. 15(a) is based on the downgoing continuation proposed by Berkhout & Verschuur (1997), with the difference that Jakubowicz's diagram does not require the velocity model. Fig. 15(b) can be described as a combination of two primaries minus a third primary. Fig. 15(c) shows how these diagrams can be constructed from virtual events.

6 CONCLUSIONS

We have described a construct of virtual events and internal multiples that is based on the classical computational operations encountered in the construction of free-surface multiples. Our construct uses only surface data and does not require any knowledge of the subsurface.

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