

Superaligned Beta Decay of Nuclei with $A \geq 62$: The Limiting Effect of Weak Gamow-Teller Branches

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The most precise value of V_{ud} , which is obtained from superallowed nuclear β decay, leads to a violation of CKM unitarity by 2.2σ . Experiments are underway on two continents to test and improve this result through decay studies of odd-odd $N = Z$ nuclei with $A \geq 62$. We show, in a series of illustrative shell-model calculations, that numerous weak Gamow-Teller branches are expected to compete with the superallowed branch in each of these nuclei. Though the total Gamow-Teller strength is significant, many of the individual branches will be unobservably weak. Thus, new techniques must be developed if reliable ft -values are to be obtained with 0.1% precision for the superallowed branches.

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One of the most exacting tests of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is provided by nuclear β -decay. Precise measurements of the ft -values for superallowed β -transitions between $T = 1$ analog 0^+ states are used to determine G_V , the vector coupling constant; this, in turn, yields V_{ud} , the up-down element of the CKM matrix. In contradiction to the Standard Model, the result from current world data violates CKM unitarity by more than two standard deviations [1]: *viz.* $V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9968 \pm 0.0014$. The potential significance of this outcome has drawn attention to the reliability of small theoretical corrections that must be applied to each experimental ft -value in order to extract G_V . In particular, there are numerous experimental programs now under way [2, 3, 4, 5, 6, 7, 8, 9, 10] to study the superallowed decays of odd-odd $N = Z$ nuclei with $A \geq 62$, where the charge-dependent correction terms are expected to be larger than among the lower- Z nuclei where all previous measurements have been made [1, 11].

Here, we report illustrative calculations demonstrating that these heavier nuclei will also exhibit a serious complication that is not present among the lower- Z nuclei. This complication – the presence of numerous weak Gamow-Teller β -decay branches that, in total, will compete with the superallowed branch – must ultimately limit the precision achievable on any superallowed ft -value in this mass region. It is particularly important to recognize that many of these branches can be below the threshold for conventional γ -ray detection and could easily be ignored. If they are ignored, any ft -value quoted for the superallowed branch could certainly not be relied upon to the 0.1% precision required for a demanding test of the charge-dependent corrections. Our results have a similar impact on measurements of non-analog $0^+ \rightarrow 0^+$ β -transitions in nuclei with $A \geq 62$. However, they do *not* indicate any problems with the superallowed ft -values previously measured for lighter ($A \leq 54$) nuclei. Thus, the non-unitarity result stands unaltered.

In general, G_V can be obtained from the measured ft -

value of a $0^+ \rightarrow 0^+$ β -transition between $T = 1$ analog states via the relationship [1]

$$\mathcal{F}t \equiv ft(1 + \delta_R)(1 - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^V)}, \quad (1)$$

with

$$\begin{aligned} K/(\hbar c)^6 &= 2\pi^3 \hbar \ln 2 / (m_e c^2)^5 \\ &= (8120.271 \pm 0.012) \times 10^{-10} \text{GeV}^{-4} \text{s}, \quad (2) \end{aligned}$$

where f is the statistical rate function, t is the partial half-life for the transition, δ_C is the isospin-symmetry-breaking correction, δ_R is the transition-dependent part of the radiative correction and Δ_R^V is the transition-independent part. Here we have also defined $\mathcal{F}t$ as the “corrected” ft -value. Even though the three calculated correction terms, Δ_R^V , δ_R and δ_C , are all of order 1%, their estimated uncertainties ($\sim 0.1\%$) actually dominate the uncertainty in the derived value of G_V . Thus, improvements in the unitarity test must be sought through improvements in the precision of these calculations rather than improvements in the experimental input.

Because the leading terms in the radiative corrections are well founded in QED [1], attention has focused more on the isospin symmetry-breaking correction, δ_C . It is somewhat smaller than the radiative corrections but depends strongly on the structure of the nuclear states involved. (Actually, a small component of the δ_R term also depends on nuclear structure but, for our present purposes, that component need not be explicitly identified.) The term δ_C comes about because the Coulomb and charge-dependent nuclear forces break isospin symmetry between the analog initial and final states in superallowed β decay. Although small, this term is clearly very important: for the nine precisely-measured superallowed transitions in nuclei between ^{10}C and ^{54}Co , the transition-to-transition variations that appear in the uncorrected experimental ft -values would not themselves pass the CVC test. It is only after the δ_C (and δ_R) corrections have been applied, yielding the corrected $\mathcal{F}t$ -values,

that the results are constant to three parts in 10^4 . This in itself can be considered some validation of the δ_C calculations, providing we accept CVC in the first place, but a more precise test of their validity would be welcome.

The important role of the δ_C correction in the unitarity test has focused recent attention on odd-odd $T_z = 0$ nuclei with $A \geq 62$ since the calculated δ_C corrections for their superallowed β -decay transitions are computed to be relatively large [12, 13]. The interest in these nuclei has also coincided with the emergence of new radioactive-beam facilities, which for the first time have made it possible, at least in principle, to produce them in statistically significant quantities. The expected values of δ_C , as calculated by Ormand and Brown [12], are in the 1.0 to 1.8% range, depending on the nucleus in question and on the calculational method adopted; that is, three to five times larger than the δ_C values for the nine precisely measured cases with $A \leq 54$. The main reason for this dramatic increase is the predominance of the $2p$ shell-model orbital in nuclei with $A \geq 62$. The radial wave function for this orbital has a node, while the important orbitals in the lighter nuclei are all nodeless, and this has a strong impact on the matrix elements of the Coulomb interaction. Because the range of calculated δ_C values would be considerably increased by the inclusion of nuclei with $A \geq 62$, it is argued that the calculations could be tested more stringently against CVC by precise ft -value measurements in that mass region.

To measure an ft -value, three quantities are required from experiment: the transition energy, Q_{EC} , which is used in calculating f ; the half-life, $t_{1/2}$, of the parent nuclide and the branching ratio for the superallowed transition, which together yield the partial half-life, t . Recent experimental activity on the $A \geq 62$ superallowed emitters includes the Q -value [5], half-life [2], and branching ratios [3, 4] for ^{74}Rb decay; the branching ratios for ^{62}Ga decay [6, 7, 8]; and the half-lives of heavier nuclei up to ^{98}In [9, 10]. Most of the results from these measurements are not yet at the required 0.1% level of precision, but the quality is improving steadily.

The purpose of this letter is to point out an important – and complicating – property of the decays of these $A \geq 62$ nuclei. As with the lighter odd-odd $T_z = 0$ nuclei, we expect their ground-state-to-ground-state superallowed branch to be predominant; however, unlike the lighter cases, that branch will not constitute $\geq 99.94\%$ of the total decay rate, but instead will amount to around 99.0% for $62 \leq A \leq 74$, and somewhat less for the heavier nuclei. What makes this difference critical is that the remaining $\sim 1\%$ β -decay strength is expected to be spread over numerous Gamow-Teller transitions, of which all those stronger than, say, 0.01% will have to be identified and measured in order to determine the superallowed branching ratio to the required 0.1% precision. The existence of these Gamow-Teller branches simply follows from the fact that, as one moves to heavier and heav-

ier $T_z = 0$ nuclei in the same $A = 4n + 2$ sequence, the β -decay Q -value increases, thus opening up a larger and larger energy window for β decay. At the same time, the density of 1^+ states in the daughter also increases, as does their structural complexity, with the result that weak Gamow-Teller branches become abundant. The deleterious effects of numerous weak Gamow-Teller transitions have been remarked in the study of much heavier exotic nuclei [14] but their potential impact on precise superallowed ft -values has not been noted before.

To quantify these ideas, and to illustrate the nature of the problem, we have mounted a series of shell-model calculations for decays of the four $4n + 2$ nuclei with $62 \leq A \leq 74$ and, for comparison purposes, the three cases with $46 \leq A \leq 54$, where precise data already exist. Since these are just illustrative calculations, the model spaces were kept fairly modest. For $46 \leq A \leq 54$, we took a ^{40}Ca core with a $(f_{7/2})^{n-r}(p_{3/2}, f_{5/2}, p_{1/2})^r$ model space truncated to $r \leq 3$. We used standard effective interactions, KB3 [15, 16] and FPM13 [17], but, because of the truncations we readjusted their centroids to reproduce the experimental splitting between the ground-state 0^+ and the first-excited 0^+ state, a key datum for superallowed beta decay.

For nuclei with $62 \leq A \leq 74$, we use the model space $(p_{3/2}, f_{5/2}, p_{1/2})^n$, which is built on a ^{56}Ni core with an effective interaction from Koops and Glaudemans [18] based on the modified surface-delta interaction (MSDI). For $A = 62, 66$ and 70 , this interaction puts the excited 0^+ close to its observed location, but in ^{74}Kr it fails badly, placing the state at 2.5 MeV, compared to the experimentally known [19] 0.5 MeV excitation. Thus, for $A = 74$, it is essential to include configurations involving the $g_{9/2}$, $d_{5/2}$ and possibly the $g_{7/2}$ orbitals. Such calculations quickly become unmanageable, so we limited the d, g -shell occupation to two nucleons and tuned the effective interaction (MSDI') to reproduce the energy of the first-excited 0^+ state.

Clearly, such simple calculations cannot possibly be expected to reproduce the properties of the $62 \leq A \leq 74$ nuclei in detail. It is well known that ^{74}Kr is difficult to describe in the shell model since it is in a region where deformation effects are growing. Indeed projected Hartree-Fock-Bogoliubov calculations [20] point to oblate-prolate shape coexistence in the low-energy spectrum. Furthermore, even in the lighter $A = 62, 66$ and 70 nuclei, the $g_{9/2}$ orbit is becoming important, particularly for high-spin states as demonstrated by Vincent *et al.*[21] for ^{62}Ga . However, since our simple calculations already demonstrate considerable complexity in the beta decay of all these nuclei – the principal thrust of this communication – our conclusions will not be altered by larger calculations with additional orbitals, which can only serve to increase this complexity.

In Table I we present the results of these shell-model calculations. In the sixth column, we identify how many

1^+ states in the daughter $T_z = 1$ nuclei are calculated to have an excitation energy, E_x , less than the electron-capture Q -value, Q_{EC} . For each of these 1^+ states we computed the Gamow-Teller transition probability, $B(GT)$, and the partial width, Γ , with

$$\Gamma \propto B(GT) f, \quad (3)$$

where $f = f_+ + f_{ec}$ is the sum of the statistical rate functions for positron decay and electron capture. The proportionality constant is fixed from the ground-state-to-ground-state superallowed branch, with the result that the Gamow-Teller branching ratios, BR , are given by

$$BR = \frac{\Gamma}{\Gamma_0} = \frac{B(GT) f}{2f_0}, \quad (4)$$

where Γ_0 and f_0 are the width and statistical rate function for decay to the ground state. We sum these branching ratios over all the 1^+ states in the Q -value window and present the results in column seven.

TABLE I: Summed Gamow-Teller branching fractions in the superallowed decay of selected $A = 4n + 2$ nuclei.

Parent Nucleus	Q_{EC} (MeV)	Shell model	First 1^+ state		# of 1^+ states ^c	Total GT branching ^c (%)
			Expt. (MeV)	Theo. (MeV)		
^{46}V	7.051	FPMI3	3.73	4.18	7	0.027
		KB3		2.34	10	0.020
^{50}Mn	7.632	FPMI3	3.63	3.91	16	0.013
		KB3		3.54	35	0.019
^{54}Co	8.243	FPMI3	(3.84) ^a	4.20	23	0.006
		KB3		4.17	75	0.024
^{62}Ga	9.171	MSDI	(3.16) ^a	2.48	110	0.28
^{66}As	9.57 ^b	MSDI	(3.24) ^a	2.27	255	0.67
^{70}Br	9.97 ^b	MSDI	(3.14) ^a	2.71	325	1.59
^{74}Rb	10.418	MSDI	(3.2) ^b	2.69	180	0.72
		MSDI'		2.76	> 400	0.92

^aLowest daughter state listed in ENSDF data file [23] whose spin-parity is unassigned.

^bAssumed similar to the other $A \geq 62$ nuclei.

^cThese results have been derived by our shifting the theoretical 1^+ spectrum so that the energy of the lowest 1^+ state agrees with the experimental value or estimate.

For the three well known cases with $46 \leq A \leq 54$, there are relatively few Gamow-Teller transitions predicted and their total strength is a barely significant 0.025% (or less) of the total β -decay. This is in excellent qualitative agreement with experiment. All three nuclei can be produced prolifically and their simple decays have been studied experimentally [22] with a sensitivity to branches as small as 0.001%. The total observed Gamow-Teller branching was 0.011% (in one branch), 0.058% (in two) and $\leq 0.001\%$ for the decays of ^{46}V , ^{50}Mn and ^{54}Co respectively. These branches are already incorporated in the

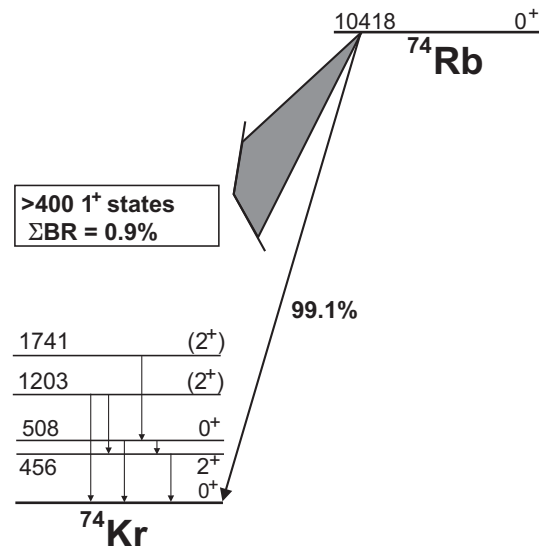


FIG. 1: Decay scheme of ^{74}Rb . All known low-spin states [3, 23, 24] are shown for ^{74}Kr . The 1^+ states and β -branching ratios are those calculated with MSDI'.

current analysis [1] of superallowed β -decay data. Thus, our calculations offer no correction whatsoever to those data.

However, for the nuclei with $62 \leq A \leq 74$, where new measurements are underway, our calculations indicate that the Gamow-Teller branching fraction is substantially larger, ranging from 0.3% in ^{62}Ga to 1.6% in ^{70}Br , and certainly cannot be ignored. Furthermore, it is also important to recognize that these are accumulated branching fractions, the sum of many individual branches. The largest single (non-superallowed) branch calculated in each case is about one-third of the total: 0.1% to the sixth 1^+ state in ^{62}Zn ; 0.2% to the third 1^+ state in ^{66}Ge ; 1% to the third 1^+ state in ^{70}Se ; and 0.3% to the fifth 1^+ state in ^{74}Kr . The remaining two-thirds of the Gamow-Teller strength in each decay is spread over a large number of states: for example, in the case of ^{74}Rb decay, there are 20 transitions with individual branching ratios above 0.005%. To date, none of the 1^+ daughter states has even been located and, in most cases, the β -decay branches feeding them will be below normal detection sensitivity for such exotic nuclei. See Fig. 1.

There is clear experimental support for these predictions of complexity. First, the observed decays of ^{62}Ga [6] and ^{74}Rb [3] show evidence for the population of states in their daughters that could not be fed directly by allowed β -decay but must have been populated by unobserved γ transitions from weakly fed states at higher excitation. Second, multiple Gamow-Teller transitions of the type we describe have been observed [23] in the decays of odd-odd 0^+ nuclei with $N \neq Z$ in this mass region: ^{64}Ga , ^{66}Ga and ^{78}Rb . All three exhibit complex decays with a minimum of 9, 12 and 23 significant Gamow-Teller transitions respectively, which populate 1^+ states in their daughters.

All three have Q_{EC} values that are *lower* than the $N = Z$ nuclei listed in Table I and the measured $\log ft$ -values are between 5 and 8, quite comparable to those calculated for the latter.

For the superallowed emitters with $46 \leq A \leq 54$, the 1^+ states populated in their daughters de-excite by γ -ray (and conversion-electron) emission, so the Gamow-Teller branching ratios are normally obtained experimentally from the intensities of observed β -delayed γ -rays. If most of the β -fed 1^+ states de-excite through the first excited 2^+ state in the daughter, then the $E2$ γ -ray from that 2^+ state to the ground state might serve as a “collector”, whose intensity approximates the total Gamow-Teller β intensity. To examine this possibility, we have again used the shell model to calculate all the de-excitation gamma-ray transition probabilities. The results appear in Table II where, for each decay, we list the fraction of β -fed 1^+ states that de-excite through the first 2^+ state. Evidently, the excited 2^+ state does act to some extent as a collector but it misses enough of the Gamow-Teller strength that a measurement of its intensity alone would not be sufficient for a precise quantitative determination of the ground-state superallowed branching ratio.

TABLE II: Fraction of summed Gamow-Teller strength decaying through the lowest 2^+ state.

Parent Nucleus	Shell model	Summed GT $BR(\%)$	Fraction decaying via 2^+ state
^{62}Ga	MSDI	0.28	80%
^{66}As	MSDI	0.67	70%
^{70}Br	MSDI	1.59	63%
^{74}Rb	MSDI	0.72	47%
	MSDI'	0.92	56% ^a

^aWe have taken the excited 0_2^+ state to de-excite 44% by electron-conversion to the ground state and 56% to the first 2_1^+ state as determined in ref. [3].

These results clearly indicate that if superallowed β -decay ft -values are to be determined for $A \geq 62$ nuclei with a precision better than, say, 0.5%, then new techniques will have to be developed to incorporate the effects of many weak Gamow-Teller transitions. Total absorption spectrometry has the potential to accomplish this goal, but whether it can do so with sufficient precision is an unanswered question. For these heavy nuclei to become useful in testing δ_C calculations, the development of such new techniques will have to become a priority.

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