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We study the parameter space of D -dimensional cosmological Einstein gravity together with quadratic curvature terms. In $D > 4$ there are in general two distinct (anti)-de Sitter vacua. We show that, for an appropriate choice of the parameters, there exists a critical point for one of the vacua, with only massless tensor, but neither massive tensor nor scalar, gravitons. At criticality, the linearized excitations have formally vanishing energy (as do black hole solutions). A further restriction of the parameters gives a one-parameter cosmological Einstein plus Weyl² model with a unique vacuum, whose Λ is determined.

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I. INTRODUCTION

Adding new types of terms—and their associated parameters—to an action can lead, in the extended parameter space, to solutions with properties qualitatively different from those of their components. In the gravitational models we consider here, such phenomena have already been found. Examples include two, maximally symmetric, vacua in Einstein-Hilbert gravity with quadratic curvature terms in dimensions $D > 4$ [1]; “partial masslessness” [2], in which degrees of freedom are lost and gauge invariance “emerges” for massive higher spins $s > 1$ in AdS backgrounds with suitable mass^2/Λ ratios, and the tuning of masses in the “new massive gravity” (NMG) in $D = 3$ [3,4]. Another example where the tuning of parameters leads to new effects is the so-called “chiral gravity” [5], which is a special case of topologically massive gravity in AdS [6]. Most recently [7], still wider critical effects were found in $D = 4$ systems combining the Einstein-Hilbert action, a cosmological term, and terms quadratic in the curvature. The latter work motivates our extension to

generic dimensions, where the interplay between the multiple parameters will provide novel effects, some relying on the explicit cosmological term. For example, as we shall see, multiple vacua and “tuned” masslessness will now interact. We will find these critical models by studying their conserved charges, especially the energy, of black holes, as well as their excitation spectrum.

II. FIELD EQUATIONS

The most general quadratic gravity model is

$$I = \int d^D x \sqrt{-g} \left[\frac{1}{\kappa} (R - 2\Lambda_0) + \alpha R^2 + \beta R^{\mu\nu} R_{\mu\nu} + \gamma (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2) \right]. \quad (1)$$

We follow the notation and conventions of [8,9], where relevant details of field equations, curvature linearizations, and conserved charges may also be found. For generic values of the parameters, there are two distinct (A)dS vacua,¹ with metrics satisfying $R_{\mu\nu} = \frac{2\Lambda}{D-2} g_{\mu\nu}$, where Λ is determined by the quadratic equation

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¹Depending upon the choice of parameters, and assuming $\Lambda_0 < 0$, there can be either two AdS vacua or else one AdS and one dS. If Λ_0 is positive, there will be either two dS vacua or one dS and one AdS. If $\Lambda_0 = 0$ there will be one Minkowski vacuum and either an AdS or a dS vacuum.

$$\frac{\Lambda - \Lambda_0}{2\kappa} + f\Lambda^2 = 0, \quad (2)$$

$$f \equiv (D\alpha + \beta) \frac{(D-4)}{(D-2)^2} + \gamma \frac{(D-3)(D-4)}{(D-1)(D-2)}.$$

This phenomenon was first observed [1] in Einstein-Lovelock gravity ($\alpha = \beta = 0$, $\Lambda_0 = 0$), where Minkowski and (A)dS vacua coexist. The source-free linearized equation of motion for the metric fluctuations around an (A)dS vacuum, $h_{\mu\nu} \equiv g_{\mu\nu} - \bar{g}_{\mu\nu}$, becomes

$$c\mathcal{G}_{\mu\nu}^L + (2\alpha + \beta) \left(\bar{g}_{\mu\nu} \bar{\square} - \bar{\nabla}_\mu \bar{\nabla}_\nu + \frac{2\Lambda}{D-2} \bar{g}_{\mu\nu} \right) R^L + \beta \left(\bar{\square} \mathcal{G}_{\mu\nu}^L - \frac{2\Lambda}{D-1} \bar{g}_{\mu\nu} R^L \right) = 0, \quad (3)$$

where c is given by

$$c \equiv \frac{1}{\kappa} + \frac{4\Lambda D}{D-2} \alpha + \frac{4\Lambda}{D-1} \beta + \frac{4\Lambda(D-3)(D-4)}{(D-1)(D-2)} \gamma,$$

and the linearization of the Einstein tensor $\mathcal{G}_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu}$ [defined using the effective Λ of the system's vacuum state(s), not the bare parameter Λ_0 in the action] is given by

$$\mathcal{G}_{\mu\nu}^L = R_{\mu\nu}^L - \frac{1}{2} \bar{g}_{\mu\nu} R^L - \frac{2\Lambda}{D-2} h_{\mu\nu}.$$

Here, the linearized Ricci tensor $R_{\mu\nu}^L$ and scalar curvature $R^L = (g^{\mu\nu} R_{\mu\nu})^L$ are given by

$$R_{\mu\nu}^L = \frac{1}{2} (\bar{\nabla}^\sigma \bar{\nabla}_\mu h_{\nu\sigma} + \bar{\nabla}^\sigma \bar{\nabla}_\nu h_{\mu\sigma} - \bar{\square} h_{\mu\nu} - \bar{\nabla}_\mu \bar{\nabla}_\nu h),$$

$$R^L = -\bar{\square} h + \bar{\nabla}^\sigma \bar{\nabla}^\mu h_{\sigma\mu} - \frac{2\Lambda}{D-2} h.$$

Taking the trace of (3) gives

$$\left[(4\alpha(D-1) + D\beta) \bar{\square} - (D-2) \left(\frac{1}{\kappa} + 4f\Lambda \right) \right] R^L = 0. \quad (4)$$

We see that the D'Alembertian operator is removed if α and β are chosen to obey

$$4\alpha(D-1) + D\beta = 0; \quad (5)$$

then R^L is constrained to vanish, provided also that

$$\frac{1}{\kappa} + 4f\Lambda \neq 0. \quad (6)$$

(See [9], and the discussion below.) As in [7], we shall choose the (compatible) gauge $\bar{\nabla}^\mu h_{\mu\nu} = \bar{\nabla}_\nu h$, which then leads to $R^L = -\frac{2\Lambda}{D-2} h$. Imposing the condition (5), which eliminates the scalar mode [see Eq. (30) of [9]], one has $h = 0$; hence $h_{\mu\nu}$ further satisfies the conditions

$$\bar{\nabla}^\mu h_{\mu\nu} = 0, \quad h = 0. \quad (7)$$

The linearized Ricci, $R_{\mu\nu}^L$, and Einstein, $\mathcal{G}_{\mu\nu}^L$, tensors reduce to

$$R_{\mu\nu}^L = \frac{2D\Lambda}{(D-1)(D-2)} h_{\mu\nu} - \frac{1}{2} \bar{\square} h_{\mu\nu},$$

$$\mathcal{G}_{\mu\nu}^L = \frac{2\Lambda}{(D-1)(D-2)} h_{\mu\nu} - \frac{1}{2} \bar{\square} h_{\mu\nu}.$$

The equations of motion simplify to the product of second order operators annihilating $h_{\mu\nu}$:

$$-\frac{\beta}{2} \left(\bar{\square} - \frac{4\Lambda}{(D-1)(D-2)} - M^2 \right) \times \left(\bar{\square} - \frac{4\Lambda}{(D-1)(D-2)} \right) h_{\mu\nu} = 0, \quad (8)$$

where

$$M^2 \equiv -\frac{1}{\beta} \left(c + \frac{4\Lambda\beta}{(D-1)(D-2)} \right). \quad (9)$$

The massless and massive modes satisfy

$$\left(\bar{\square} - \frac{4\Lambda}{(D-1)(D-2)} \right) h_{\mu\nu}^{(m)} = 0, \quad (10)$$

$$\left(\bar{\square} - \frac{4\Lambda}{(D-1)(D-2)} - M^2 \right) h_{\mu\nu}^{(M)} = 0,$$

respectively. Stability requires $M^2 \geq 0$, and the case where $M^2 = 0$, i.e.,

$$c + \frac{4\Lambda\beta}{(D-1)(D-2)} = 0, \quad (11)$$

defines the critical point.

III. ENERGY

We now show that at the critical point, the mass and angular momenta of all asymptotically Kerr-AdS and Schwarzschild-AdS black holes vanish.² We can calculate the conserved charges of any such asymptotically Kerr-AdS space, as discussed in [11]. Letting ξ_μ be a Killing vector of AdS, the conserved charges associated with this Killing vector will be given by [8]

$$Q^\mu(\xi) = \left(c + \frac{4\Lambda\beta}{(D-1)(D-2)} \right) \int_{\mathcal{M}} d^{D-1} x \sqrt{-\bar{g}} \bar{\xi}_\nu \mathcal{G}_L^{\mu\nu} + (2\alpha + \beta) \int_{\partial\mathcal{M}} dS_i \sqrt{-g} \{ \bar{\xi}^\mu \bar{\nabla}^i R_L + R_L \bar{\nabla}^\mu \bar{\xi}^i - \bar{\xi}^i \bar{\nabla}^\mu R_L \} + \beta \int_{\partial\mathcal{M}} dS_i \sqrt{-g} \{ \bar{\xi}_\nu \bar{\nabla}^i \mathcal{G}_L^{\mu\nu} - \bar{\xi}_\nu \bar{\nabla}^\mu \mathcal{G}_L^{i\nu} - \mathcal{G}_L^{\mu\nu} \bar{\nabla}^i \bar{\xi}_\nu + \mathcal{G}_L^{i\nu} \bar{\nabla}^\mu \bar{\xi}_\nu \}. \quad (12)$$

Here, \mathcal{M} is a spacelike hypersurface in the asymptotically AdS space, and $\partial\mathcal{M}$ denotes its boundary. The first integral

²Actually, unlike for cosmological Einstein theory [10], explicit Kerr-AdS type solutions for these models are as yet unknown. Such solutions presumably exist, and they will approach the standard Kerr-AdS metrics at large distance where the effects of the higher-order curvature terms become negligible.

can also be written as a boundary term, see [8,12]. In asymptotically AdS spaces, only the first term survives. [We recall that the charges in (12) are measured at infinity; their detailed form as volume integrals is given by the spatial integral of the field equations' nonlinear terms.] At the critical point then, all charges, including the energy (Q^0) formally vanish. For example, the energy of the Schwarzschild–AdS solution, with asymptotics $h_{00} = h^{rr} = (\frac{r_0}{r})^{D-3}$, is [8]

$$E = \left(c + \frac{4\Lambda\beta}{(D-1)(D-2)} \right) \frac{(D-2)}{2} \Omega_{D-2} r_0^{D-3}, \quad (13)$$

where Ω_{D-2} is the solid angle on the $D-2$ sphere. Specifically, in four dimensions, $\kappa = 8\pi G_N$ and $r_0 = 2G_N m$, and then $E = [1 + 16\pi G_N(4\alpha + \beta)]m$. At the critical point (11), we see that in all dimensions, $E = 0$.

Note that in $D = 3$, the critical theory reduces to the NMG at the ‘‘Proca’’ point. Namely, $8\alpha + 3\beta = 0$ gives the NMG theory, and the criticality condition $M^2 = 0$ gives $\Lambda_0 = \frac{3}{\kappa\beta}$ which is exactly the point where the linearized NMG can be written explicitly as a unitary massive spin-1 theory with mass squared $-\frac{8}{\kappa\beta}$ (see [4], and Sec. V.A in [3]). (For AdS and with $\beta > 0$, one should set $\kappa < 0$ since $\Lambda = \frac{2}{\kappa\beta}$.)

IV. LINEAR EXCITATIONS

Let us now consider the energy of the excitations, expressed as explicit volume integrals, by constructing the Hamiltonian for the theory, as in [5,7]. The quadratic Lagrangian for the metric perturbations of (1) around an AdS vacuum is given by simply multiplying the left-hand side of (3) by $-\frac{1}{2}h^{\mu\nu}$; hence it vanishes on-shell. With the parameter choice (5), and in the transverse-traceless gauge (7), the quadratic action takes the form

$$\begin{aligned} I_2 = & -\frac{1}{2} \int d^D x \sqrt{-\bar{g}} \left[-\frac{\beta}{2} \bar{\square} h^{\mu\nu} \bar{\square} h_{\mu\nu} \right. \\ & - \frac{1}{2} \left(\frac{4\Lambda\beta}{(D-1)(D-2)} - c \right) \bar{\nabla}^\rho h^{\mu\nu} \bar{\nabla}_\rho h_{\mu\nu} \\ & \left. + \frac{2\Lambda c}{(D-1)(D-2)} h^{\mu\nu} h_{\mu\nu} \right]. \end{aligned} \quad (14)$$

The canonical momenta, using the Ostrogradsky formalism for higher-order Lagrangians, are defined and computed as follows:

$$\begin{aligned} \Pi_{(1)}^{\mu\nu} & \equiv \frac{\delta \mathcal{L}_2}{\delta \dot{h}_{\mu\nu}} - \bar{\nabla}_0 \left(\frac{\delta \mathcal{L}_2}{\delta [\frac{\partial}{\partial t}(\bar{\nabla}_0 h_{\mu\nu})]} \right) \\ & = -\frac{\sqrt{-\bar{g}}}{2} \bar{\nabla}^0 \left[-\left(\frac{4\Lambda\beta}{(D-1)(D-2)} - c \right) h^{\mu\nu} + \beta \bar{\square} h^{\mu\nu} \right], \\ \Pi_{(2)}^{\mu\nu} & \equiv \frac{\delta \mathcal{L}_2}{\delta [\frac{\partial}{\partial t}(\bar{\nabla}_0 h_{\mu\nu})]} = \frac{\sqrt{-\bar{g}}}{2} \beta \bar{g}^{00} \bar{\square} h^{\mu\nu}. \end{aligned}$$

The Hamiltonian can be written as

$$\begin{aligned} H & \equiv \int d^{D-1} x \left[\Pi_{(1)}^{\mu\nu} \dot{h}_{\mu\nu} + \Pi_{(2)}^{\mu\nu} \frac{\partial}{\partial t} (\bar{\nabla}_0 h_{\mu\nu}) \right] \\ & - \int \sqrt{-g} \mathcal{L}_2 d^{D-1} x, \end{aligned} \quad (15)$$

where we use for the implicit AdS background metric the time-independent form

$$\begin{aligned} d\bar{s}^2 & = \frac{(D-1)(D-2)}{2(-\Lambda)} [-\cosh^2 \rho dt^2 + d\rho^2 \\ & + \sinh^2 \rho d\Omega_{D-2}^2]. \end{aligned}$$

Substituting the canonical momenta into (15) gives the Hamiltonian for the quadratic fluctuations. The energies of the various (linearized) excitations could be obtained directly from (15), but a more convenient starting point is to note that the Hamiltonian H is time independent, since the Lagrangian has no explicit time dependence. We are therefore free to write H as its time average, $H = \langle H \rangle = T^{-1} \int_0^T dt H$. The advantage of doing this is that we can then perform integrations by parts for time derivatives. This allows us to manipulate (15) into a more convenient form. The Lagrangian term $\int \sqrt{-g} \mathcal{L}_2 d^{D-1} x$, being proportional to the equation of motion (8), does not contribute to the on-shell energies. Substituting the expressions (10) for the massless and the massive modes into the time average of the Hamiltonian (15), we find, after an integration by parts, that their energies are given by

$$\begin{aligned} E_m & = -\frac{1}{2T} \left(c + \frac{4\Lambda\beta}{(D-1)(D-2)} \right) \\ & \times \int d^D x \sqrt{-\bar{g}} (\dot{h}_{\mu\nu}^{(m)} \bar{\nabla}^0 h_{(m)}^{\mu\nu}), \end{aligned} \quad (16)$$

$$\begin{aligned} E_M & = \frac{1}{2T} \left(c + \frac{4\Lambda\beta}{(D-1)(D-2)} \right) \\ & \times \int d^D x \sqrt{-\bar{g}} (h_{\mu\nu}^{(M)} \bar{\nabla}^0 h_{(M)}^{\mu\nu}), \end{aligned} \quad (17)$$

where the time integrations are understood to be over the interval T . The excitation energies E_m and E_M have opposite signs, as expected from their fourth-order origin [13], but at the critical point (11), they vanish. The integral itself for the massless modes is known to be negative, since in pure Einstein gravity for which $\alpha = \beta = \gamma = 0$, the energies E_m are known to be positive. The integral for the massive modes is also expected to be negative, away from $M^2 = 0$, at least for small mass.

We started with a theory with four parameters, namely $(\Lambda_0, \alpha, \beta, \gamma)$, not counting κ , since it can be scaled out. The critical point is defined by the two conditions (5) and (11), implying that any two of the four parameters can be eliminated. Note, however, that we must also require that (6) hold at the critical point, since otherwise, R_L , a gauge invariant object, would be left undetermined in the theory.

As we already remarked, there are in general two distinct AdS vacua in the theory with given values for the four parameters $(\Lambda_0, \alpha, \beta, \gamma)$, corresponding to the two roots of the quadratic equation (2). Our specialization to the critical case turns the massive spin-2 modes massless for just one of these vacua. In the other vacuum, the spectrum still contains massive as well as massless spin-2 modes, with excitation energies of opposite signs. Whether it is the massive spin-2 modes or the massless spin-2 modes that have the negative energy depends upon the detailed choice of parameters. Note that if the parameters are chosen such that the massless spin-2 modes have positive energy, then it will also be the case that black holes in the noncritical vacuum will have positive mass.

For some purposes, in order to avoid solving the quadratic equation (2) for Λ , it is convenient instead to view it as a linear equation for the parameters expressed in terms of the critical value of Λ . We take α and γ as the free parameters for $D > 4$. From (5), one determines β in terms of α . Then, from (11), one obtains the unique critical vacuum with a cosmological constant

$$\Lambda_{\text{crit}} = -\frac{D(D-1)(D-2)}{4\kappa[(D-1)(D-2)^2\alpha + D(D-3)(D-4)\gamma]}. \quad (18)$$

To determine the corresponding critical Λ_0 , we can use (2) to get

$$\Lambda_0 = -\frac{D^2(D-1)(D-2)[(D-1)(D-2)\alpha + (D-3)(D-4)\gamma]}{8\kappa[(D-1)(D-2)^2\alpha + D(D-3)(D-4)\gamma]^2}. \quad (19)$$

The other (noncritical) vacuum has the cosmological constant

$$\Lambda_{\text{noncrit}} = \frac{D[(D-1)(D-2)\alpha + (D-3)(D-4)\gamma]\Lambda_{\text{crit}}}{(D-4)[(D-1)(D-2)\alpha + D(D-3)\gamma]}. \quad (20)$$

As a result, the two-parameter critical theory is given by the action

$$I = \int d^D x \sqrt{-g} \left[\frac{1}{\kappa} (R - 2\Lambda_0) + \alpha R^2 - \frac{4(D-1)}{D} \alpha R^{\mu\nu} R_{\mu\nu} + \gamma (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2) \right], \quad (21)$$

where the bare cosmological constant is given by (19). The theory has a critical vacuum (18), while there is a noncritical vacuum (20).

Finally, one can overcome the two-vacuum problem, that one of them necessarily allows negative-energy excitations, using one last allowed (consistent as we shall see) reduction, to a single parameter. We require $f = 0$ in (2), whose only solution is thereby $\Lambda = \Lambda_0$. Our four parameters $(\alpha, \beta, \gamma, \Lambda_0)$, are now constrained by the three conditions (2) and (11), plus scalar mode suppression (5), to obey

$$\begin{aligned} \beta &= -\frac{4\alpha(D-1)}{D}, \\ \Lambda &= \Lambda_0 = -\frac{D}{8\kappa\alpha}, \\ \alpha &= -\frac{\gamma D(D-3)}{(D-1)(D-2)}. \end{aligned} \quad (22)$$

Note that Λ_0 is also fixed in terms of the remaining parameter. The general action (1) now reduces to Einstein-Weyl form with a single arbitrary constant γ , since Λ is also fixed:

$$I = \int d^D x \sqrt{-g} \left[\frac{1}{\kappa} (R - 2\Lambda_0) + \gamma C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} \right], \quad (23)$$

$$\Lambda = \Lambda_0 = \frac{(D-1)(D-2)}{8\kappa\gamma(D-3)}.$$

Here, $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, for which

$$C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - \frac{4}{D-2} R^{\mu\nu} R_{\mu\nu} + \frac{2}{(D-1)(D-2)} R^2.$$

Physically, it is clear *a priori* that only the combination (23) has a single vacuum: the Weyl tensor, being insensitive to conformally related metrics, such as different (A)dS or flat ones, does not contribute directly to (2). We also emphasize that while condition (6) is automatically satisfied by taking $f = 0$, attempting instead to force the two roots of (2) to be equal would violate it.

It is interesting to compare our maximally restricted Einstein-Weyl model (23) with the similarly constrained Einstein-Lovelock model, namely, the action (1) without Ricci² and R^2 discussed in [1]. There too one must specialize to a single vacuum, and fix the cosmological constant to leave one free parameter. (Otherwise, one of the two vacua is always unstable.) The excitation here is just the single massless tensor of (cosmological) Einstein gravity, as the Lovelock terms quadratic in h vanish for all D and don't contribute.

V. CONCLUSIONS

We have constructed a two-parameter theory of gravity in D dimensions, which admits a critical AdS vacuum with only a pair of massless spin-2 modes. For $D = 3$, the critical theory reduces to the NMG at the Proca point, while for $D = 4$, it reduces to the critical theory constructed recently in [7]. In four dimensions, once the condition (5) for the elimination of the scalar mode is imposed, the theory has a unique AdS vacuum solution. For dimensions $D > 4$, by contrast, the theory in general has two distinct AdS vacua, which cannot simultaneously be rendered critical for any choice of parameters. A further specialization of parameters was then made, leading to a one-parameter critical theory with a unique, fixed Λ , AdS vacuum. The surprising confluence of so many unexpected properties at our models' critical points may repay further investigation.

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