

## New $D = 6$ , $\mathcal{N} = (1, 1)$ Gauged Supergravity with Supersymmetric $(\text{Minkowski})_4 \times S^2$ Vacuum

J. Kerimo and H. Lü<sup>1</sup>

*George P. and Cynthia W. Mitchell Institute for Fundamental Physics,  
Texas A&M University, College Station, TX 77843-4242, USA*

### ABSTRACT

We obtain a new gauged  $D = 6$ ,  $\mathcal{N} = (1, 1)$  pure supergravity by a generalised consistent Kaluza-Klein reduction of M-theory on  $K3 \times R$ . The reduction requires a conspiratory gauging of both the Cremmer-Julia type global (rigid) symmetry and the homogeneous rescaling symmetry of the supergravity equations of motion. The gauged supergravity is different from the Romans  $D = 6$  gauged supergravity in that the four vector fields in our new theory are all abelian. We show that it admits a supersymmetric  $(\text{Minkowski})_4 \times S^2$  vacuum, which can be lifted to  $D = 11$  where it becomes the near-horizon geometry of two intersecting M5-branes wrapping on a supersymmetric two-cycle of K3.

---

<sup>1</sup> Research supported in part by DOE grant DE-FG03-95ER40917.

# 1 Introduction

It has long been known that the Salam-Sezgin model of  $D = 6$ ,  $\mathcal{N} = (1, 0)$  Einstein-Maxwell gauged supergravity admits a supersymmetric  $(\text{Minkowski})_4 \times S^2$  vacuum with the chiral  $\mathcal{N} = 1$  four-dimensional supersymmetry [1]. It was recently shown that the six-dimensional theory admits a fully consistent dimensional reduction on the 2-sphere [2], giving rise to a massless sector that comprises a supergravity multiplet, an  $SU(2)$  Yang-Mills multiplet and a scalar multiplet, with a vanishing cosmological constant. The supersymmetry and the absence of the cosmological constant make it an interesting model for phenomenology [3, 4, 2].

However, the higher-dimensional origin of the six-dimensional theory remains elusive. It is a particular case of a more general class of supergravities constructed in [5], but it cannot be obtained from the truncation of the Romans  $D = 6$   $\mathcal{N} = (1, 1)$  gauged supergravity [6]. A potential difficulty is associated with the fact that the scalar potential is positive definite, which rules out the possibility of a sphere reduction, which is a typical origin of gauged supergravities. Recently, it was proposed that the six-dimensional theory should be viewed as an effective theory on the boundary of  $\text{AdS}_7$ ; however, the reduction is not consistent in the bulk [7, 8].

In this paper we obtain a new  $D = 6$ ,  $\mathcal{N} = (1, 1)$  gauged supergravity, which has also a  $(\text{Minkowski})_4 \times S^2$  vacuum, from a consistent *generalised* Kaluza-Klein reduction of  $D = 11$  supergravity on  $\text{K3} \times R$ . The dimensional reduction comprises two steps. The first step is to compactify M-theory on a K3 manifold, which gives rise to, as the low-energy effective theory, the  $D = 7$ ,  $\mathcal{N} = 1$  supergravity coupled to matter.<sup>1</sup> We shall concentrate on the pure supergravity multiplet, which consists of the metric, the dilaton, one rank-2 antisymmetric tensor, three vectors, and their fermionic counterparts. This theory has a Cremmer-Julia type global (rigid) symmetry which shifts the dilaton by a constant. The Lagrangian is invariant provided that appropriate constant scalings of the tensor and vector fields are performed. The  $D = 7$  theory has also a homogeneous scaling symmetry, with the following transformation

---

<sup>1</sup>The consistency of the reduction of M-theory on K3 is questionable. It is consistent however if one restricts to the pure supergravity multiplet, which does not turn on the scalars parameterising the K3.

rule

$$ds^2 \rightarrow \Omega^2 ds^2, \quad A_{(n)} \rightarrow \Omega^n A_{(n)}, \quad \Psi_M \rightarrow \Omega^{\frac{1}{2}} \Psi_M, \quad (1)$$

where  $A_{(n)}$  is an  $n$ -form gauge potential and  $\Psi_M$  is the gravitino, with  $M$  being the curved index. This transformation has the effect of scaling the whole Lagrangian, and therefore it is not a symmetry of the action, but only of the equations of motion. It was shown in [9] that both symmetries can be gauged in a Kaluza-Klein circle reduction, where the constant parameters of the global symmetry transformation rules are taken to be linearly dependent on the reduction circle coordinate. It was furthermore shown that this procedure of reduction is consistent; it is called “generalised Kaluza-Klein reduction” in [9].

The motivation for us to consider such a generalised reduction ansatz is that the gauging of the homogeneous rescaling symmetry has the effect of generating a positive cosmological constant [9]. Thus, in the second step, we apply such a generalised reduction on the minimal  $D = 7$  supergravity and obtain a new  $D = 6$   $\mathcal{N} = (1, 1)$  gauged supergravity coupled to a vector multiplet. We find that the matter vector multiplet can remarkably be truncated out by a certain choice of the two cosmological parameters associated with the gauging of the two symmetries, giving rise to a pure  $D = 6$ ,  $\mathcal{N} = (1, 1)$  gauged supergravity.

The new theory is different from the Romans  $D = 6$ ,  $\mathcal{N} = (1, 1)$  gauged supergravity in that the four vectors in our new theory are all abelian instead of being  $SU(2) \times U(1)$  Yang-Mills fields. An interesting feature of the  $D = 6$ ,  $\mathcal{N} = (1, 1)$  theory that we are going to present is that it also admits a supersymmetric  $(\text{Minkowski})_4 \times S^2$  vacuum, with the  $\mathcal{N} = 2$  four-dimensional supersymmetry. Owing to the consistency of our reductions, we can lift the solution back to  $D = 11$ , where it becomes the near-horizon geometry of two intersecting M5-branes wrapping on a supersymmetric two-cycle of K3. The solution of the two intersecting M5-branes preserves  $\frac{1}{4}$  of the maximal supersymmetry.

Since the compactification of M-theory on K3 is well understood, we consider in section 2 the second step of the reduction, namely the generalised Kaluza-Klein reduction of the minimal supergravity in  $D = 7$  on  $S^1$ . We obtain the new  $\mathcal{N} = (1, 1)$  gauged supergravity in six dimensions coupled to a vector multiplet. We show that by setting equal the two arbitrary mass parameters, the vector multiplet can consistently

be truncated out, giving rise to the pure  $D = 6$ ,  $\mathcal{N} = (1, 1)$  gauged supergravity. In addition, we show that a further truncation in the bosonic equations of motion to a subsector is possible, whose Lagrangian turns out to be identical to the bosonic sector of the Salam-Sezgin model. However, this truncation is not supersymmetric. In section 3, we obtain the supersymmetric  $(\text{Minkowski})_4 \times S^2$  vacuum solution. We first lift the solution back to  $D = 7$ , where it becomes the near-horizon geometry of the 3-brane in  $D = 7$ . It can then further be lifted back to  $D = 11$ , and the solution becomes the near horizon geometry of two intersecting M5-branes wrapping a supersymmetric two-cycle of the K3. We provide further comments and conclusions in section 4.

## 2 New gauged $\mathcal{N} = (1, 1)$ supergravity from $D = 7$

Minimal supergravity in seven dimensions consists of the metric, a dilaton, a 2-form antisymmetric tensor and three vectors, together with their fermionic counterparts. The Lagrangian for the bosonic sector is given by

$$\hat{e}^{-1}\mathcal{L} = \hat{R} - \frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{12}e^{\frac{4}{\sqrt{10}}\hat{\phi}}\hat{G}_{(3)}^2 - \frac{1}{4}e^{\frac{2}{\sqrt{10}}\hat{\phi}}(\hat{F}_{(2)}^i)^2, \quad (2)$$

where  $\hat{G}_{(3)} = d\hat{B}_{(2)} + \frac{1}{2}\hat{F}_{(2)}^i \wedge \hat{A}_{(1)}^i$ ,  $\hat{F}_{(2)}^i = d\hat{A}_{(1)}^i$  and  $i = 1, 2, 3$ . The theory possesses the following global (rigid) symmetry

$$\begin{aligned} \hat{\phi} &\rightarrow \hat{\phi} + \sqrt{10}\lambda_1, & d\hat{s}^2 &\rightarrow e^{2\lambda_2} d\hat{s}^2, \\ \hat{B}_{(2)} &\rightarrow e^{-2\lambda_1+2\lambda_2} \hat{B}_{(2)}, & \hat{A}_{(1)}^i &\rightarrow e^{-\lambda_1+\lambda_2} \hat{A}_{(1)}^i. \end{aligned} \quad (3)$$

The transformation associated with  $\lambda_1$  leaves the Lagrangian invariant, and therefore describes a symmetry of the Lagrangian. On the other hand, the transformation associated with  $\lambda_2$  scales the whole Lagrangian, and so it is not a symmetry of the Lagrangian, or even the action, but a symmetry of the equations of motion.

It was shown in [9] that these symmetries can be gauged in the Kaluza-Klein circle reduction, since we can let  $\lambda_i$  depend linearly on the  $S^1$  coordinate  $z$ . Following [9], we consider the following generalised  $S^1$  reduction ansatz

$$d\hat{s}_7^2 = e^{2m_2 z} \left( e^{2\alpha\varphi} ds_6^2 + e^{2\beta\varphi} (dz + \mathcal{A}_{(1)})^2 \right),$$

$$\begin{aligned}
\hat{B}_{(2)} &= e^{2(m_2-m_1)z} (B_{(2)} + B_{(1)} \wedge dz), \\
\hat{A}_{(1)}^i &= e^{(m_2-m_1)z} (A_{(1)}^i + \chi^i dz), \\
\hat{\phi} &= \phi + \sqrt{10} m_1 z,
\end{aligned} \tag{4}$$

where  $\alpha^2 = \frac{1}{40}$  and  $\beta = -4\alpha$ . When  $m_1 = 0 = m_2$ , the ansatz becomes the one for the standard  $S^1$  Kaluza-Klein reduction. We shall next present the detailed reduction and show that the generalised reduction ansatz (4) is consistent with the equations of motion of the  $D = 7$  minimal supergravity. The resulting  $D = 6$  theory is somewhat complicated, involving mass parameters  $m_1$  and  $m_2$ . One reason for this complication is the occurrence of the prefactors for  $\hat{B}_{(2)}$  and  $\hat{A}_{(1)}^i$  in the reduction ansatz (4). However, the prefactors vanish if we set  $m_1 = m_2 \equiv m$ . In this case, the system becomes much simpler. In particular, it is possible to consistently truncate out the axionic fields  $\chi^i$  and set equal the Kaluza-Klein and winding vectors  $B_{(1)}$  and  $\mathcal{A}_{(1)}$ . Furthermore, one combination of the dilaton and  $\varphi$  can be set to zero. The resulting theory is a new  $D = 6$ ,  $\mathcal{N} = (1, 1)$  pure gauged supergravity.

First, we find that the field strengths  $\hat{G}_{(3)}$  and  $\hat{F}_{(2)}^i$  can be expressed as

$$\begin{aligned}
\hat{G}_{(3)} &= e^{2(m_2-m_1)z} \left( G_{(3)} + G_{(2)} \wedge (dz + \mathcal{A}_{(1)}) \right), \\
\hat{F}_{(2)}^i &= e^{(m_2-m_1)z} \left( F_{(2)}^i + L_{(1)}^i \wedge (dz + \mathcal{A}_{(1)}) \right),
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
G_{(3)} &\equiv dB_{(2)} + \frac{1}{2} F_{(2)}^i \wedge A_{(1)}^i - dB_{(1)} \wedge \mathcal{A}_{(1)} - 2(m_2 - m_1) B_{(2)} \wedge \mathcal{A}_{(1)} - \frac{1}{2} \chi^i F_{(2)}^i \wedge \mathcal{A}_{(1)}, \\
G_{(2)} &\equiv dB_{(1)} + \frac{1}{2} \chi^i F_{(2)}^i - \frac{1}{2} L_{(1)}^i \wedge A_{(1)}^i + \frac{1}{2} \chi^i L_{(1)}^i \wedge \mathcal{A}_{(1)} + 2(m_2 - m_1) B_{(2)}, \\
F_{(2)}^i &\equiv dA_{(1)}^i - d\chi^i \wedge \mathcal{A}_{(1)} + (m_2 - m_1) A_{(1)}^i \wedge \mathcal{A}_{(1)}, \\
L_{(1)}^i &\equiv d\chi^i - (m_2 - m_1) A_{(1)}^i.
\end{aligned} \tag{6}$$

Thus, we see that for general values of  $m_1$  and  $m_2$ , the vector fields  $A_{(1)}^i$  and the tensor field  $B_{(2)}$  become massive, eating the axions  $\chi^i$  and the winding vector  $B_{(1)}$  respectively. Their masses are proportional to  $|m_2 - m_1|$ , and therefore vanish when  $m_1 = m_2$ .

Substituting the reduction ansatz into the equations of motion in  $D = 7$ , we find that they can all be satisfied provided that the lower dimensional fields satisfy the

following equations of motion

$$\begin{aligned}
\nabla^\theta (e^{a\phi-4\alpha\varphi} G_{\mu\nu\theta}) &= (3m_2 + 2m_1) (e^{a\phi-4\alpha\varphi} G_{\mu\nu\theta} \mathcal{A}^\theta - e^{a\phi+6\alpha\varphi} G_{\mu\nu}), \\
\nabla^\nu (e^{a\phi+6\alpha\varphi} G_{\mu\nu}) &= \frac{1}{2} e^{a\phi-4\alpha\varphi} G_{\mu\nu\theta} F^{\nu\theta} + (3m_2 + 2m_1) e^{a\phi+6\alpha\varphi} G_{\mu\nu} \mathcal{A}^\nu, \\
\nabla^\nu (e^{\frac{1}{2}a\phi-2\alpha\varphi} F_{\mu\nu}^i) &= -\frac{1}{2} e^{a\phi-4\alpha\varphi} G_{\mu\nu\theta} F^{i\nu\theta} - e^{a\phi+6\alpha\varphi} G_{\mu\nu} L^{i\nu} \\
&\quad + (4m_2 + m_1) e^{\frac{1}{2}a\phi-2\alpha\varphi} F_{\mu\nu}^i \mathcal{A}^\nu - (4m_2 + m_1) e^{\frac{1}{2}a\phi+8\alpha\varphi} L_\mu^i, \\
\nabla^\mu (e^{\frac{1}{2}a\phi+8\alpha\varphi} L_\mu^i) &= \frac{1}{2} e^{\frac{1}{2}a\phi-2\alpha\varphi} F_{\mu\nu}^i \mathcal{F}^{\mu\nu} + \frac{1}{2} e^{a\phi+6\alpha\varphi} G_{\mu\nu} F^{i\mu\nu} \\
&\quad + (4m_2 + m_1) e^{\frac{1}{2}a\phi+8\alpha\varphi} L_\mu^i \mathcal{A}^\mu, \\
\nabla^\nu (e^{-10\alpha\varphi} \mathcal{F}_{\mu\nu}) &= \frac{1}{2} e^{a\phi-4\alpha\varphi} G_{\mu\nu\theta} G^{\nu\theta} - e^{\frac{1}{2}a\phi-2\alpha\varphi} F_{\mu\nu}^i L^{i\nu} + 5m_2 e^{-10\alpha\varphi} \mathcal{A}^\nu \mathcal{F}_{\mu\nu} \\
&\quad + \sqrt{10} m_1 (\partial_\mu \phi - \sqrt{10} m_1 \mathcal{A}_\mu) - 10m_2 (\beta \partial_\mu \varphi - m_2 \mathcal{A}_\mu), \\
\Box \phi &= \frac{1}{3\sqrt{10}} e^{a\phi-4\alpha\varphi} G_{(3)}^2 + \frac{1}{2\sqrt{10}} e^{\frac{1}{2}a\phi-2\alpha\varphi} (F_{(2)}^i)^2 + \frac{1}{\sqrt{10}} e^{a\phi+6\alpha\varphi} G_{(2)}^2 \\
&\quad + \frac{1}{\sqrt{10}} e^{\frac{1}{2}a\phi+8\alpha\varphi} (L_{(1)}^i)^2 + 5m_2 \mathcal{A}^\mu \partial_\mu \phi - 5\sqrt{10} m_1 m_2 (\mathcal{A}^2 + e^{10\alpha\varphi}) \\
&\quad + \sqrt{10} m_1 \nabla_\mu \mathcal{A}^\mu, \\
-\beta \Box \varphi &= -\frac{1}{30} e^{a\phi-4\alpha\varphi} G_{(3)}^2 - \frac{1}{20} e^{\frac{1}{2}a\phi-2\alpha\varphi} (F_{(2)}^i)^2 + \frac{3}{20} e^{a\phi+6\alpha\varphi} G_{(2)}^2 \\
&\quad - \frac{1}{4} e^{-10\alpha\varphi} \mathcal{F}_{(2)}^2 + \frac{2}{5} e^{\frac{1}{2}a\phi+8\alpha\varphi} (L_{(1)}^i)^2 - 5\beta m_2 \mathcal{A}^\mu \partial_\mu \varphi \\
&\quad + 5m_2^2 \mathcal{A}^2 - m_2 \nabla_\mu \mathcal{A}^\mu + 5m_1^2 e^{10\alpha\varphi}, \\
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R &= \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{4} (\partial\varphi)^2 g_{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi - \frac{1}{4} (\partial\phi)^2 g_{\mu\nu} \\
&\quad + \frac{1}{2} e^{-10\alpha\varphi} (\mathcal{F}_{\mu\theta} \mathcal{F}_\nu^\theta - \frac{1}{4} g_{\mu\nu} \mathcal{F}_{(2)}^2) + \frac{1}{4} e^{a\phi-4\alpha\varphi} (G_{\mu\theta\lambda} G_\nu^{\theta\lambda} - \frac{1}{6} g_{\mu\nu} G_{(3)}^2) \\
&\quad + \frac{1}{2} e^{\frac{1}{2}a\phi-2\alpha\varphi} (F_{\mu\theta}^i F_\nu^{i\theta} - \frac{1}{4} g_{\mu\nu} (F_{(2)}^i)^2) + \frac{1}{2} e^{a\phi+6\alpha\varphi} (G_{\mu\theta} G_\nu^\theta - \frac{1}{4} g_{\mu\nu} G_{(2)}^2) \\
&\quad + \frac{1}{2} e^{\frac{1}{2}a\phi+8\alpha\varphi} (L_\mu^i L_\nu^i - \frac{1}{2} g_{\mu\nu} (L_{(1)}^i)^2) - 5\alpha m_2 (\mathcal{A}^\theta \partial_\theta \varphi g_{\mu\nu} - \mathcal{A}_\mu \partial_\nu \varphi - \mathcal{A}_\nu \partial_\mu \varphi) \\
&\quad + \frac{1}{2} \sqrt{10} m_1 (\mathcal{A}^\theta \partial_\theta \phi g_{\mu\nu} - \mathcal{A}_\mu \partial_\nu \phi - \mathcal{A}_\nu \partial_\mu \phi) - 5(m_2^2 - m_1^2) \mathcal{A}_\mu \mathcal{A}_\nu \\
&\quad - \frac{5}{2} m_2 (\nabla_\mu \mathcal{A}_\nu + \nabla_\nu \mathcal{A}_\mu - 2\nabla_\theta \mathcal{A}^\theta g_{\mu\nu}) - 5(2m_2^2 + \frac{1}{2} m_1^2) (\mathcal{A}^2 + e^{10\alpha\varphi}) g_{\mu\nu}, \quad (7)
\end{aligned}$$

where  $a = 4/\sqrt{10}$  and  $\mathcal{F}_{(2)} = d\mathcal{A}_{(1)}$ . Clearly, these are the bosonic equations of motion for the new  $D = 6$ ,  $\mathcal{N} = (1, 1)$  gauged supergravity coupled to a vector multiplet.

It would be interesting to obtain the pure  $D = 6$ ,  $\mathcal{N} = (1, 1)$  gauged supergravity by truncating out the matter vector multiplet. To do this, we make an orthonormal rotation for the scalar  $\phi$  and  $\varphi$ , namely

$$a\phi - 4\alpha\varphi = \sqrt{2}\phi_1, \quad 4\alpha\phi + a\varphi = \sqrt{2}\phi_2. \quad (8)$$

It now becomes clear that if we set  $m_1 = m_2 \equiv m$ , we can perform the following consistent truncation

$$\phi_2 = 0, \quad \chi^i = 0, \quad \mathcal{A}_{(1)} = -B_{(1)} \equiv \frac{1}{\sqrt{2}} A_{(1)}. \quad (9)$$

The equations of motion for the remaining fields are then given by

$$\begin{aligned} \nabla^\theta \left( e^{\sqrt{2}\phi_1} G_{\mu\nu\theta} \right) &= \frac{5m}{\sqrt{2}} \left( e^{\sqrt{2}\phi_1} G_{\mu\nu\theta} A^\theta + e^{\frac{1}{\sqrt{2}}\phi_1} F_{\mu\nu} \right) \\ \nabla^\nu \left( e^{\frac{1}{\sqrt{2}}\phi_1} F_{\mu\nu} \right) &= -\frac{1}{2} e^{\sqrt{2}\phi_1} G_{\mu\nu\theta} F^{\nu\theta} + \frac{5}{\sqrt{2}} m e^{\frac{1}{\sqrt{2}}\phi_1} F_{\mu\nu} A^\nu \\ \nabla^\nu \left( e^{\frac{1}{\sqrt{2}}\phi_1} F_{\mu\nu}^i \right) &= -\frac{1}{2} e^{\sqrt{2}\phi_1} G_{\mu\nu\theta} F^{i\nu\theta} + \frac{5}{\sqrt{2}} m e^{\frac{1}{\sqrt{2}}\phi_1} F_{\mu\nu}^i A^\nu \\ \square\phi_1 &= \frac{1}{6\sqrt{2}} e^{\sqrt{2}\phi_1} G_{(3)}^2 + \frac{1}{4\sqrt{2}} e^{\frac{1}{\sqrt{2}}\phi_1} \left( F_{(2)}^2 + (F_{(2)}^i)^2 \right) + \frac{5}{\sqrt{2}} m A^\mu \partial_\mu \phi_1 \\ &\quad + \frac{5}{2} m \nabla_\mu A^\mu - \frac{25}{\sqrt{2}} m^2 \left( \frac{1}{2} A^2 + e^{-\frac{1}{\sqrt{2}}\phi_1} \right), \\ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R &= \frac{1}{2} (\partial_\mu \phi_1 \partial_\nu \phi_1 - \frac{1}{2} (\partial\phi_1)^2 g_{\mu\nu}) + \frac{1}{4} e^{\sqrt{2}\phi_1} (G_{\mu\theta\lambda} G_\nu^{\theta\lambda} - \frac{1}{6} g_{\mu\nu} G_{(3)}^2) \\ &\quad + \frac{1}{2} e^{\frac{1}{\sqrt{2}}\phi_1} (F_{\mu\theta} F_\nu^\theta - \frac{1}{4} g_{\mu\nu} F_{(2)}^2) + \frac{1}{2} e^{\frac{1}{\sqrt{2}}\phi_1} (F_{\mu\theta}^i F_\nu^{i\theta} - \frac{1}{4} g_{\mu\nu} (F_{(2)}^i)^2) \\ &\quad + \frac{5m}{4} (A^\theta \partial_\theta \phi_1 g_{\mu\nu} - A_\mu \partial_\nu \phi_1 - A_\nu \partial_\mu \phi_1) \\ &\quad - \frac{5m}{2\sqrt{2}} (\nabla_\mu A_\nu + \nabla_\nu A_\mu - 2\nabla_\theta A^\theta g_{\mu\nu}) - \frac{25}{2} m^2 \left( \frac{1}{2} A^2 + e^{-\frac{1}{\sqrt{2}}\phi_1} \right) g_{\mu\nu}, \end{aligned} \quad (10)$$

where  $F_{(2)} = dA_{(1)}$ ,  $F_{(2)}^i = dA_{(1)}^i$  and  $G_{(3)} = dB_{(2)} + \frac{1}{2} F_{(2)} \wedge A_{(1)} + \frac{1}{2} F_{(2)}^i \wedge A_{(1)}^i$ .

Thus we have obtained the full bosonic equations of motion for the pure supergravity multiplet of the new  $D = 6$ ,  $\mathcal{N} = (1, 1)$  gauged supergravity. Note that all the four vectors are abelian, but with the gauge symmetry of  $A_{(1)}$  broken owing to the higher-order interactions. This is different from the Romans  $D = 6$  gauged supergravity where the four vectors are the  $SU(2) \times U(1)$  Yang-Mills fields. Furthermore, our new theory has a positive-definite scalar potential  $V = 25m^2 e^{-\frac{1}{\sqrt{2}}\varphi}$ .

The supersymmetry of the theory is straightforward. It was shown in [9, 10] that the generalised dimensional reductions are consistent with supersymmetry. This can be seen from the fact that the full  $D = 7$  equations of motion, including the fermions, are invariant under the dilaton shifting symmetry and the homogeneous scaling symmetry.

As in the case discussed in [9], the theory (10) obtained from the generalised reduction does not have a Lagrangian formalism. This can be seen from the fact that if there were a Lagrangian, it would from the Einstein equation of motion in (10) have

the term  $m^2 A^2$ . On the other hand, the equation of motion for  $A_{(1)}$  indicates that such a term should not exist. At first sight, one might conclude that  $A_{(1)}$  is massive, but the equation of motion for  $A_{(1)}$  clearly shows that it is a massless field.

A few remarks are needed at this stage. Owing to the overall  $z$ -dependent scaling factor in the ansätze (4) and (11), the coordinate  $z$  cannot be viewed as a circle coordinate. Thus the theory is not compactified. To resolve such a problem, it was proposed in [11, 12] that one can modify the original supergravity by introducing an auxiliary field associated with the gauging of the scaling symmetry, which can be identified with the reduction coordinate in the dimensional reduction. The auxiliary field always appear in the equations through a derivative in the modified theory, and can therefore be defined as a circle coordinate in the reduction. Locally, this approach is the same as our generalised circular reduction, but globally, the internal direction is a circle instead of a real line. In fact, if we consider in our example the string frame, then there is no  $z$ -dependence in the metric when  $m_1 = m_2$ , and so  $z$  can be viewed as a circular coordinate at least from the metric point of view. An alternative approach is to introduce a delta function singularity à la Randall-Sundrum. We can then replace the prefactor in the metric  $e^{2mz}$  by  $e^{-2m|z|}$ . By doing this, the volume of the internal direction will be finite even though  $z$  is a non-compact coordinate. Consequently, the gravity will be localised on the brane located at  $z = 0$ . The exponential nature of the warp factor in the conformal-frame metric implies that the effect of localisation is strong with a mass gap. It would be interesting to study further if the delta function singularity in this procedure can be smoothed out.

It is interesting to note that we can consistently truncate out the  $A_{(1)}$  field in the pure bosonic equations of motion. Then, the reduction ansatz becomes

$$\begin{aligned} d\hat{s}_7^2 &= e^{2mz} \left( e^{-\frac{1}{5\sqrt{2}}\phi} ds_6^2 + e^{\frac{2\sqrt{2}}{5}\phi} dz^2 \right), \\ \hat{B}_{(2)} &= B_{(2)}, \quad \hat{A}_{(1)}^i = A_{(1)}^i, \quad \hat{\phi} = \frac{2}{\sqrt{5}}\phi + \sqrt{10}mz. \end{aligned} \quad (11)$$

Note that, if we consider the string frame, there is no  $z$ -dependence in the metric. The  $z$ -dependence in the reduction appears then only in the dilaton. The equations of motion for the remaining fields can arise from the Lagrangian

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12}e^{\sqrt{2}\phi}G_{(3)}^2 - \frac{1}{4}e^{\frac{1}{\sqrt{2}}\phi}(F_{(2)}^i)^2 - 25m^2 e^{-\frac{1}{\sqrt{2}}\phi}, \quad (12)$$



where  $G_{(3)} = dB_{(2)} + \frac{1}{2}F_{(2)}^i \wedge A_{(1)}^i$  and  $F_{(2)}^i = dA_{(1)}^i$ . Intriguingly, this is precisely the Lagrangian for the bosonic sector of the  $D = 6$ ,  $\mathcal{N} = (1, 0)$  gauged supergravity constructed in [1], coupled to two vector multiplets.<sup>2</sup> However, unfortunately the truncation is not supersymmetric. This can be seen by setting the parameter  $m = 0$ , in which case the truncation from  $N = (1, 1)$  pure supergravity to  $N = (1, 0)$  requires setting to zero all the four vector fields.

### 3 M-theory interpretation

One intriguing feature of the  $D = 6$ ,  $\mathcal{N} = (1, 0)$  gauged supergravity is that it admits a supersymmetric  $(\text{Minkowski})_4 \times S^2$  vacuum. In the new  $\mathcal{N} = (1, 1)$  theory, we also have such a vacuum solution, given by

$$\begin{aligned} ds^2 &= dx^\mu dx^\nu \eta_{\mu\nu} + \frac{1}{25m^2} d\Omega_2^2, \\ F_{(2)} &= \frac{\sqrt{2}}{5m} \Omega_{(2)}, \quad \phi = 0, \end{aligned} \tag{13}$$

where we have turned on one of the three vector field strengths  $F_{(2)}^i$ . Lifting this solution back to  $D = 7$ , it becomes the near-horizon limit of a 3-brane supported by one of the vector field strengths  $\hat{F}_{(2)}^i$ . To see this, let us start with the 3-brane, given by

$$\begin{aligned} d\hat{s}_7^2 &= H^{-\frac{2}{5}} dx^\mu dx^\nu \eta_{\mu\nu} + H^{\frac{8}{5}} (dr^2 + r^2 d\Omega_2^2), \\ \hat{F}_2 &= \sqrt{2} Q \Omega_{(2)}, \quad e^\phi = H^{-\frac{2}{\sqrt{10}}}, \end{aligned} \tag{14}$$

where  $H = 1 + Q/r$ . In the decoupling (or near-horizon) limit, we have  $H = Q/r$ . Taking the charge parameter  $Q$  to be  $Q = 5m$ , and making a coordinate transformation  $Q/r = e^{-5mz}$ , the solution (14) becomes

$$\begin{aligned} d\hat{s}_7^2 &= e^{2mz} (dx^\mu dx^\nu \eta_{\mu\nu} + \frac{1}{25m^2} d\Omega_2^2 + dz^2), \\ \hat{F}_{(2)} &= \frac{\sqrt{2}}{5m} \Omega_{(2)}, \quad \hat{\phi} = \sqrt{10} m z. \end{aligned} \tag{15}$$

---

<sup>2</sup>Note that the bosonic sector of the minimal  $D = 6$ ,  $\mathcal{N} = (1, 0)$  gauged supergravity consists of the metric, a dilaton, a tensor and a vector, which, when ungauged, can be viewed as pure  $\mathcal{N} = (1, 0)$  supergravity coupled to a tensor and a vector multiplet

This fits exactly the reduction ansatz (11), giving rise to precisely the lower dimensional solution (13). It is worth mentioning that the solution (15) can also be viewed as a domain wall with a  $(\text{Minkowski})_4 \times S^2$  world-volume.

The Killing spinor for the 3-brane is given by

$$\hat{\epsilon} = e^{\frac{1}{2}mz} \epsilon_0, \quad (16)$$

where  $\epsilon_0$  is a constant spinor satisfying  $\hat{\Gamma}_{z12} \epsilon_0 = \epsilon_0$ , where 1 and 2 are the labels for the vielbein of  $S^2$ . Since the supersymmetry transformation rule of the gravitino in  $D = 7$  has the form  $\delta\hat{\psi}_\mu \rightarrow \hat{\mathcal{D}}_\mu \hat{\epsilon}$ , it follows that the generalised reduction for the supersymmetry transformation parameter  $\hat{\epsilon}$  is given by  $\hat{\epsilon} = e^{\frac{1}{2}mz} \epsilon_0$ . Thus the Killing spinor of the 3-brane (16) fits precisely the reduction ansatz, giving rise to a constant Killing spinor  $\epsilon_0$  for the  $(\text{Minkowski})_4 \times S^2$  vacuum in  $D = 6$ .

We can further lift the solution back to  $D = 11$ , where it becomes the near-horizon structure of two intersecting M5-branes. As in the above, we start with the two intersecting M5-branes in  $D = 11$ :

$$\begin{aligned} ds_{11}^2 &= (H_1 H_2)^{-1/3} \left( dx^\mu dx^\nu \eta_{\mu\nu} + H_2 (dz_1^2 + dz_2^2) + H_1 (dz_3^2 + dz_4^2) \right. \\ &\quad \left. + H_1 H_2 (dr^2 + r^2 d\Omega_2^2) \right), \\ F_{(4)} &= (Q_1 dz_3 \wedge dz_4 + Q_2 dz_1 \wedge dz_2) \wedge \Omega_{(2)}, \end{aligned} \quad (17)$$

with  $H_i = 1 + Q_i/r$ . Setting  $Q_1 = Q_2 = Q$ , the solution in the near-horizon limit becomes

$$\begin{aligned} ds_{11}^2 &= \rho^{2/3} \left( dx^\mu dx^\nu \eta_{\mu\nu} + Q^2 \frac{d\rho^2}{\rho^2} + Q^2 d\Omega^2 \right) + \rho^{-1/3} ds_4^2, \\ F_{(4)} &= Q J_{(2)} \wedge \Omega_{(2)}. \end{aligned} \quad (18)$$

Here we can replace the 4-torus  $ds_4^2$  by a Ricci-flat K3 manifold, and  $J_{(2)}$  is a self-dual harmonic 2-form in the K3. It is straightforward to see that the  $D = 11$  solution (18) becomes (13) in  $D = 6$  by first reducing on the K3 manifold followed by the generalised Kaluza-Klein reduction.

It is interesting to note that only by taking the decoupling or near-horizon limit does the brane solution fit the reduction ansatz. This is different from the usual Kaluza-Klein circle reduction where the whole solution can be reduced instead of just the near-horizon limit. Thus the standard  $S^1$  reduction can be viewed as a special

case of a DeWitt group-manifold reduction, whose consistency is guaranteed, whilst the generalised Kaluza-Klein reduction can be viewed as a special case of a Pauli sphere reduction, where the consistency requires conspiracies. (A discussion of the terminology is contained in [13].)

Of course, the  $\mathcal{N} = 1$  supergravity in  $D = 7$  can also be obtained from a  $T^3$  reduction of the heterotic string theory, which is S-dual to M-theory on K3. The vector field strengths  $F_{(2)}^i$  in the minimal  $D = 7$  supergravity come from setting equal the three Kaluza-Klein and the three winding vectors. It follows that the 3-brane in  $D = 7$  can be lifted to the  $D = 10$  heterotic theory as an intersection of the heterotic 5-brane and Taub-NUT.

In [14], a more general class of dyonic strings were obtained in the  $D = 6$ ,  $\mathcal{N} = (1, 0)$  gauged supergravity. Identical solutions exist in our new gauged supergravity. It is also straightforward to lift these solutions to  $D = 7$  and hence further back to  $D = 11$ . The  $D = 11$  structure is somewhat complicated, supported by the 4-form field strength that carries three magnetic charges and one electric charge; the solution is thus interpreted as an intersection of three M5-branes and one M2-brane.

## 4 Conclusions and discussions

In this paper we have performed generalised Kaluza-Klein reduction of M-theory on  $K3 \times R$  and obtained a new  $D = 6$ ,  $\mathcal{N} = (1, 1)$  gauged supergravity. The theory differs from the Romans gauged supergravity in that in the new theory all the four vector fields are abelian, and it has in addition a positive definite scalar potential. We find that the theory admits a  $(\text{Minkowski})_4 \times S^2$  vacuum solution, which can be embedded in the 3-brane (15) in seven dimensions, which itself can be viewed as intersecting M5-branes wrapping on a supersymmetric two-cycle of K3 in  $D = 11$ . Clearly, the orders of the reductions of the 3-brane can be reversed, by performing the  $S^2$  reduction first, which gives rise to a  $D = 5$  domain wall, with a  $(\text{Minkowski})_4$  world-volume. We finally arrive at the four-dimensional Minkowski spacetime by performing a brane-world Kaluza-Klein reduction introduced in [15]. (See also, [16, 17, 18, 19, 20].)

In fact, if we truncate out the Kaluza-Klein and winding vectors, we expect that the above reduction can be performed on the theory instead of just on a specific

solution. First, we expect that there should be a consistent reduction of the minimal  $D = 7$  supergravity on  $S^2$ . To see this, we can study the global symmetry of the theory reduced on  $T^2$ . If we for simplicity set two of the three vector fields in  $D = 7$  to zero, then the resulting  $D = 5$  theory has a global  $O(2, 3)$  T-duality symmetry, with the scalars parameterising the coset  $O(2, 3)/(SO(2) \times SO(3))$ . Clearly, we can gauge the  $SO(3)$  maximal compact subgroup, which is exactly the isometry group of  $S^2$ . This is indicative of a consistent  $S^2$  reduction of the seven-dimensional theory [21]. The resulting gauged  $D = 5$  supergravity will have a negative exponential-scalar potential, which can support a domain wall solution. We can then perform the brane-world Kaluza-Klein reduction to obtain a four-dimensional  $\mathcal{N} = 1$  supergravity coupled to an  $SU(2)$  vector multiplet and a scalar multiplet, exactly like the one obtained from an  $S^2$  reduction of the  $D = 6$  chiral gauged supergravity [2]. The fact that the brane-world Kaluza-Klein reduction can give rise to Yang-Mills fields while keeping the cosmological constant vanishing has been demonstrated in [20].

It is also interesting to note that in our earlier approach, the four dimensional theory arises first from the generalised Kaluza-Klein reduction on  $R$ , and then the standard  $S^2$  reduction, in which case, the reduction makes use of a gauging of the homogeneous scaling symmetry. If we instead perform the  $S^2$  reduction first, and then the brane-world reduction, it would appear that we do not need to appeal to the homogeneous scaling symmetry. Clearly, the two approaches are equivalent. One feature in common is that in both approaches, the reduction involves warp factors. Thus our first approach is nothing more than giving a symmetry interpretation of the warp factor in the reduction ansatz. In fact, the near-horizon structure of the 3-brane given by (15) in  $D = 7$  can be viewed as a domain wall written in the conformal frame, with the world-volume being  $(\text{Minkowski})_4 \times S^2$ . Thus the generalised dimensional reduction can be viewed as a special case of the brane-world reduction.

## Acknowledgment

We are grateful to James Liu and Chris Pope for useful discussions.

## References

- [1] A. Salam and E. Sezgin, *Chiral Compactification on Minkowski $\times S^2$  of  $N = 2$  Einstein-Maxwell Supergravity In Six-Dimensions*, Phys. Lett. **B147**, 47 (1984).
- [2] G.W. Gibbons and C.N. Pope, *Consistent  $S^2$  Pauli reduction of six-dimensional chiral gauged Einstein-Maxwell supergravity*, hep-th/0307052.
- [3] Y. Aghababaie, C.P. Burgess, S.L. Parameswaran and F. Quevedo, *SUSY breaking and moduli stabilization from fluxes in gauged 6D supergravity*, JHEP **0303**, 032 (2003), hep-th/0212091.
- [4] Y. Aghababaie, C.P. Burgess, S.L. Parameswaran and F. Quevedo, *Towards a naturally small cosmological constant from branes in 6D supergravity*, hep-th/0304256.
- [5] H. Nishino and E. Sezgin, *Matter And Gauge Couplings Of  $N = 2$  Supergravity In Six-Dimensions*, Phys. Lett. **B144**, 187 (1984).
- [6] L.J. Romans, *The  $F(4)$  Gauged Supergravity In Six-Dimensions*, Nucl. Phys. **B269**, 691 (1986).
- [7] S. Cucu, H. Lü and J.F. Vazquez-Poritz, *A supersymmetric and smooth compactification of M-theory to  $AdS_5$* , hep-th/0303211.
- [8] S. Cucu, H. Lü and J.F. Vazquez-Poritz, *Interpolating from  $AdS_{D-2} \times S^2$  to  $AdS_D$* , hep-th/0304022.
- [9] I.V. Lavrinenko, H. Lü and C.N. Pope, *Fibre bundles and generalised dimensional reductions*, Class. Quant. Grav. **15**, 2239 (1998), hep-th/9710243.
- [10] E. Bergshoeff, T. de Wit, U. Gran, R. Linares and D. Roest, *(Non-)Abelian gauged supergravities in nine dimensions*, JHEP **0210**, 061 (2002), hep-th/0209205.
- [11] P.S. Howe, N.D. Lambert and P.C. West, *A new massive type IIA supergravity from compactification*, Phys. Lett. **B416**, 303 (1998), hep-th/9707139.

- [12] A. Chamblin and N.D. Lambert, *De Sitter space from M-theory*, Phys. Lett. **B508**, 369 (2001), hep-th/0102159.
- [13] M. Cvetič, G.W. Gibbons, H. Lü and C.N. Pope, *Consistent group and coset reductions of the bosonic string*, hep-th/0306043.
- [14] R. Guven, J.T. Liu, C.N. Pope and E. Sezgin, *Fine tuning and six-dimensional gauged  $N = (1, 0)$  supergravity vacua*, hep-th/0306201.
- [15] H. Lü and C.N. Pope, *Branes on the brane*, Nucl. Phys. **B598**, 492 (2001), hep-th/0008050.
- [16] M. Cvetič, H. Lü and C.N. Pope, *Brane-world Kaluza-Klein reductions and branes on the brane*, J. Math. Phys. **42**, 3048 (2001), hep-th/0009183.
- [17] M.J. Duff, J.T. Liu and W.A. Sabra, *Localization of supergravity on the brane*, Nucl. Phys. **B605**, 234 (2001), hep-th/0009212.
- [18] I.Y. Park, C.N. Pope and A. Sadrzadeh, *AdS braneworld Kaluza-Klein reduction*, Class. Quant. Grav. **19**, 6237 (2002), hep-th/0110238.
- [19] J.L. Lehnert and K.S. Stelle,  *$D = 5$  M-theory radion supermultiplet dynamics*, hep-th/0210228.
- [20] J.T. Liu, H. Lü and C.N. Pope, *The radion mode in consistent brane-world reductions*, hep-th/0212037.
- [21] M. Cvetič, H. Lü and C.N. Pope, *Consistent Kaluza-Klein sphere reductions*, Phys. Rev. **D62**, 064028 (2000), hep-th/0003286.
- [22] P.M. Cowdall, H. Lü, C.N. Pope, K.S. Stelle and P.K. Townsend, *Domain walls in massive supergravities*, Nucl. Phys. **B486**, 49 (1997), hep-th/9608173.